TRANSPORT PHENOMENA

27/09/2024

PRESSURE-DRIVEN FLOW THROUGH A CYLINDRICAL TUBE

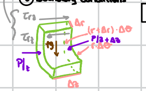
① Coordinate system (r, θ, z)

★ LA TENSIÓN CORTANTE → SENTIDO OPUESTO AL FLUJO ($\vec{z}$)

τ_{r} → ⊥ r
τ_{rz} → DIRECCIÓN 'z'

| como $\Delta r + r > r$ ~ τ_{rz} ⇄ τ_{rz}.

② Boundary conditions



$v(R)=0$

$\vec{v} = v_r \cdot \vec{e_r} + v_\theta \cdot \vec{e_\theta} + v_z \cdot \vec{e_z}$ (≠0)

$v_z(r)$
$P(z)$ ⟶ la presión ↓↓ a medida que ( ) ?

$F_g = \rho g$ ('volumen de control')

$F_g = \rho g (r \cdot \Delta\theta \cdot \Delta r \cdot \Delta\theta)$

EJEMPLO ANTERIOR

$F_g = \rho \cdot g (\Delta x \Delta y \Delta z)$

③ Momentum balance z

{Sum of external forces} = {momentum flow out} − {momentum flow IN} = 0 ⤳

$\left(\rho \cdot v_z|_z \cdot v_z|_z \cdot \Delta r \cdot r\Delta\theta \;\middle|\; v_z|_z = v_z|_{z+\Delta z} \right)$

$-(P|_{z+\Delta z} - P|_z) \cdot \Delta r \cdot r \cdot \Delta\theta + \tau_{rz}|_{r+\Delta r} (r+\Delta r)\Delta\theta \cdot \Delta z - \tau_{rz}|_r \cdot r\,\Delta\theta \cdot \Delta z = 0$

$-(P|_{z+\Delta z} - P|_z) \cdot \Delta r \cdot r \cdot \Delta\theta + \left[(r+\Delta r)\tau_{rz}|_{r+\Delta r} - r \cdot \tau_{rz}|_r\right] \Delta\theta \cdot \Delta r = 0$

÷ Vol.

$Vol = r \cdot \Delta\theta \cdot \Delta x \cdot \Delta z$

$-\frac{P|_{z+\Delta z} - P|_z}{\Delta z} + \frac{1}{r} \cdot \frac{(r+\Delta r)\tau_{rz}|_{r+\Delta r} - r \cdot \tau_{rz}|_r}{\Delta r} = 0$

$-\frac{\Delta p}{\Delta z} + \frac{1}{r} \frac{\Delta(r \cdot \tau_{rz})}{\Delta r} = 0$

$$\boxed{-\frac{dp}{dz} + \frac{1}{r} \cdot \frac{d(r \cdot \tau_{rz})}{dr} = 0}$$

⤳ $\underbrace{\frac{dp}{dz}}_{f(z)} = \underbrace{\frac{1}{r} \cdot \frac{d(r \cdot \tau_{rz})}{dr}}_{f(r)} = C_1 = -\frac{\Delta p}{L}$

$$\boxed{\frac{1}{r} \cdot \frac{d(r \cdot \tau_{rz})}{dr} = -\frac{\Delta p}{L}}$$

$\frac{dp}{dz}$ = cte

$d(r \cdot \tau_{rz}) = \frac{-\Delta p}{L} \cdot r \cdot dr$

$r \cdot \tau_{rz} = -\frac{\Delta p}{2L} \cdot r^2 + C_2$

$\tau_{rz} = -\frac{\Delta p}{2L} r + \frac{C_2}{r}$ (=0) ⤳ $C_2 = 0$, τ_{rz} would tend to ∞. ⟶ $\tau_{rz} = -\frac{\Delta p}{2L} r$ FOR ANY FLUID.

- For a Newtonian fluid:

$\tau_{rz} = \mu \cdot \frac{dv_z}{dr}$

$\mu \cdot \frac{dv_z}{dr} = -\frac{\Delta p}{2L} r$

$dv_z = -\frac{\Delta p}{2\mu L} r \cdot dr$

$v_z(r) = -\frac{\Delta p}{4\mu L} r^2 + C_3$

(Boundary condition) BC ⟶ $0 = -\frac{\Delta p}{4\mu L} \cdot R^2 + C_3$

$$\boxed{C_3 = \frac{\Delta p}{4\mu L} R^2}$$

$$\boxed{v_z(r) = \frac{\Delta p}{4\mu L} R^2 \left(1 - \left(\frac{r}{R}\right)^2\right) \qquad \tau_{rz} = -\frac{\Delta p}{2L} r}$$

↑tensión → ↓ flujo (vel) ⓘ?

(↓ viscoso → ↑ vel)

veloc. flujo ↓↓.