Analysis of Cardiovascular Hemodynamics Using a 3-Windkessel (3-WK) Model: Insights from Pediatric and Adult Data

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Abstract

This study investigates the cardiovascular hemodynamics using a 3-Windkessel (3-WK) model with three pairs of (Q, P) data. One pair consists of patient-specific measurements, while the other two pairs are synthetic, generated from a computer model. The analysis includes calculating key cardiovascular parameters such as cardiac output (CO, in L/min), stroke volume (SV, in mL/beat), and vascular resistance (R, in mmHg·min/L) for each case.

Additionally, the 3-WK model will be utilized to evaluate pressure (P) from the prescribed blood flow rate (Q). A simulated pressure value (P3-WK) will be generated and compared to the measured or synthetic pressure (Pmeas). The study will compute four types of errors(average point-to-point relative error, mean pressure error, systolic pressure error, and diastolic pressure error)based on the definitions established by Boileau et al. (2015). Furthermore, the influence of varying distal resistance (R2) in the 3-WK model will be analyzed by adjusting R2 by ±25%. This research aims to enhance the understanding of cardiovascular dynamics across different patient profiles.

Keywords:

3-Windkessel model, Cardiovascular dynamics, Stroke volume, Vascular resistance, Pediatric vs. Adult cardiovascular models, Blood flow modeling.

1 INTRODUCTION

Cardiovascular modeling plays a critical role in understanding the dynamics of blood flow and pressure regulation. In this study, we use the 3-Windkessel (3-WK) model to analyze pressure (P) and flow (Q) data from pediatric and adult cardiovascular systems. The model incorporates arterial compliance and vascular resistance to simulate pressure waveforms, which are compared to measured data from patients and synthetic cases. Our aim is to quantify the cardiovascular parameters such as cardiac output (CO), stroke volume (SV), and vascular resistance (R), and evaluate the impact of changes in distal resistance (R2) on pressure profiles.

2 MATERIALS AND METHODS

The project consists of three different analyses. Firstly, we have selected the provided (P, Q) data from three patients to calculate several cardiovascular parameters such as cardiac output (CO), stroke volume (SV), and vascular resistance (R). For this, we imported our data into MATLAB and plotted the stroke volume graphs to visualize and better understand the data. Then, we applied the equations indicated on the following page.

In order to shorten the code included in the report, we will only present the data for case 16, but for cases 17 and 18, we would follow the exact same process.

In the second section, we need to analyze the 3-WK models for each (P, Q) pair. By prescribing the blood flow rate (Q), the pressure (P) will be evaluated using the 3-WK model. Therefore, a simulated pressure will be generated, which we will then compare with the measured pressure. To do this, we used the equation of Pin(t) provided in the project statement.

It computes the pressure (Pin_16) at different time steps based on given system parameters and inputs such as flow rate (Q_16), resistance (R1_16, R2_16), capacitance (C_16), and output pressure (Pout_16). It simulates the dynamic response of the system where pressure is influenced by both flow and resistance-capacitance behavior. Over time, the system accumulates the effects of past flow, and the pressure decays according to an exponential model. The result is a pressure profile (Pin_16) that reflects these dynamics, with the last portion of the profile extracted for further analysis (Pin_16ult).

Later, to evaluate the extent to which the model represents reality, the following four errors will be calculated: the average point-to-point relative error, the error in mean pressure, the error in systolic pressure, and the error in diastolic pressure. The errors are defined according to the definitions given in the work by Boileau et al. (2015).

In the third section, we need to analyze the influence of the distal resistance of the 3-WK model, R2, on the cardiovascular system. To do this, the blood flow rate Q will be prescribed, and a new pressure P will be evaluated. For each of the pairs, R2 will be increased by 25% and decreased by 25%. The main purpose is to analyze how modifying the resistance (R2_16) affects the pressure response over time.

The code simulates and visualizes the behavior of the system's input pressure (Pin_16) over time when the system's resistance (R2_16) is increased and decreased. It plots two different scenarios:

1. The pressure response when the resistance R2 is increased by 25%.
2. The pressure response when the resistance R2 is decreased by 25%.

The comparison with a reference pressure (PP_16) helps in analyzing how the system behaves under different resistance conditions.

Then, the code extracts the final segment of two pressure arrays, Pin_16dec (for decreased resistance) and Pin_16inc (for increased resistance), corresponding to the last length(t_16) time steps. It stores these final values in two new arrays, Pin_16dec_ult and Pin_16inc_ult. This allows the final portion of pressure data for both increased and decreased resistance cases to be analyzed or compared over a shorter time range.

2.1 Equations

Cardiovascular Parameter Calculations:

Cardiac Output (CO): $CO = HR \times SV$

Stroke Volume (SV): $SV = \frac{\int_0^t Q(t)dt}{t}$

Vascular Resistance (R): $R = \frac{Pa}{CO}$

The 3-Element Windkessel model:

For an input flow waveform Qin(t), the pressure developed at the inlet of the 3-WK model is given by:

$$P_{\text{in}}(t) = [P_{\text{in}}(t_0) - R_1 Q_{\text{in}}(t_o) - P_{\text{out}}]e^{-\frac{(t-t_0)}{R_2 C}} + R_1 Q_{\text{in}}(t) + P_{\text{out}} + \frac{e^{-\frac{t}{R_2 C}}}{C} \int_{t_0}^{t} e^{\frac{t'}{R_2 C}} Q_{\text{in}}(t')\text{d}t'$$

Error calculation:

Point-to-point average error:

$$\varepsilon_{\text{PP}}(\%) = \frac{1}{N}\sqrt{\sum_{n=1}^{N}\left(\frac{P_n^{\text{sim}} - P_n^{\text{meas}}}{P_n^{\text{meas}}}\right)^2} \cdot 100$$

Error in the mean value:

$$\varepsilon_{\text{avg}}(\%) = \frac{\text{mean}(P_n^{\text{sim}}) - \text{mean}(P_n^{\text{meas}})}{\text{mean}(P_n^{\text{meas}})} \cdot 100$$

Error in systolic value:

$$\varepsilon_{\text{sys}}(\%) = \frac{\max(P_n^{\text{sim}}) - \max(P_n^{\text{meas}})}{\max(P_n^{\text{meas}})} \cdot 100$$

Error in diastolic value:

$$\varepsilon_{\text{dia}}(\%) = \frac{\min(P_n^{\text{sim}}) - \min(P_n^{\text{meas}})}{\min(P_n^{\text{meas}})} \cdot 100$$

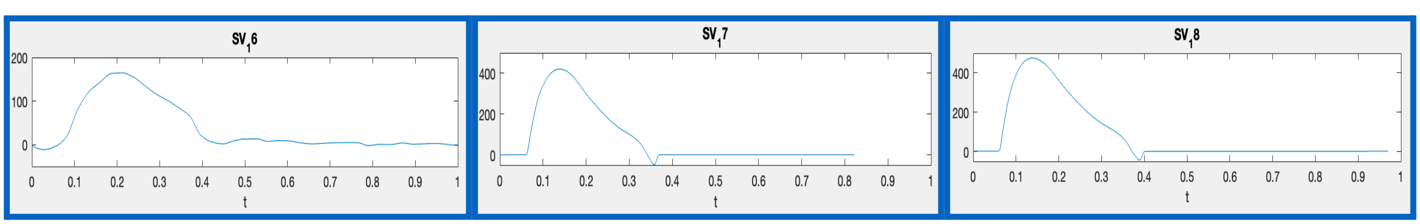
3 RESULTS

1. Results of the first analysis:

Table 1: Results of Cardiac Output, Stroke Volume and Vascular Resistance

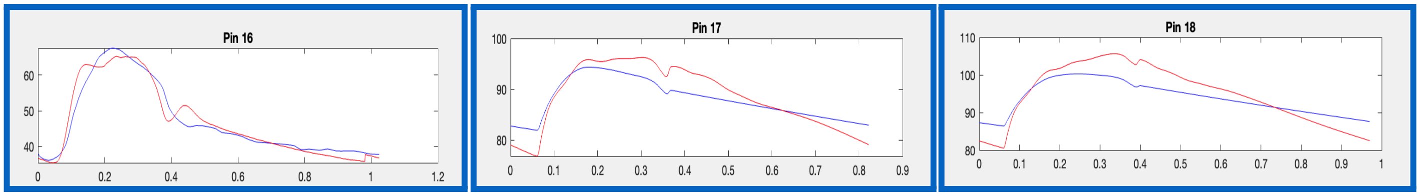
	Cardiac Output	Stroke Volume	Vascular Resistance
16	2.2403	0.0382	20.9462
17	4.8407	0.0663	18.3067
18	5.1494	0.0832	18.4240

Figure 1: Plot of the Stroke Volume



2. Results of the second analysis:

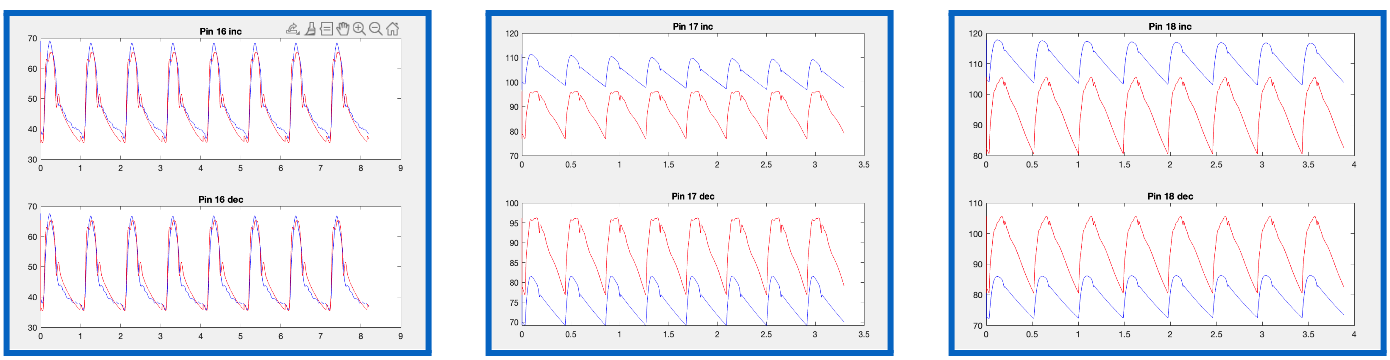
Figure 2: Pressure Pin over time



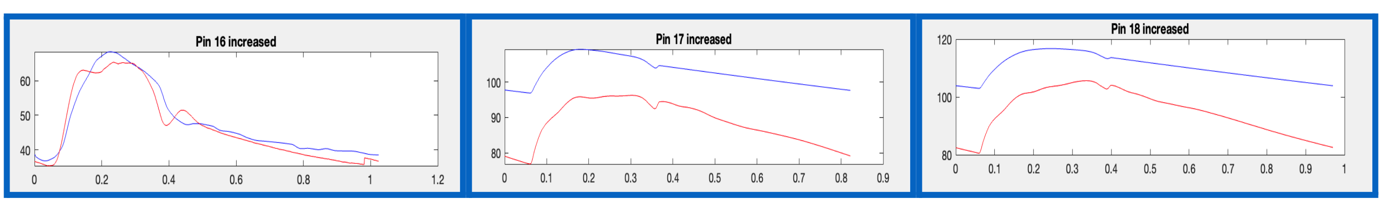
2.1 Error calculation:

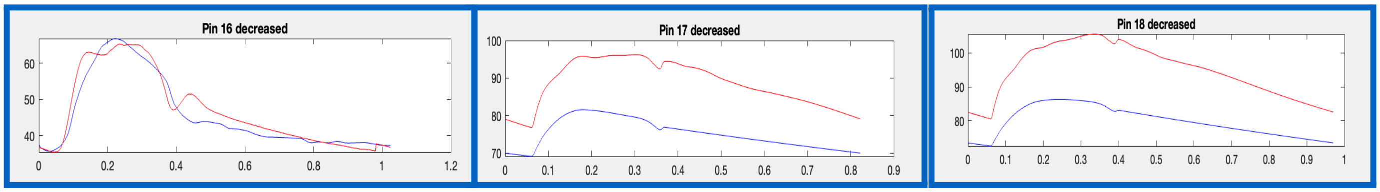
Error definition	Point-to-point average	Mean value	Systolic value	Diastolic value
16	0.0029	3.2026e-04	3.4486	2.0100
17	0.0117	1.7466	1.9559	6.6341
18	0.0127	1.1086	5.0639	7.3183

3. Results of the third analysis:



3.2:





4 DISCUSSION

According to the results we obtained, we realized cardiac output and stroke volume increase together, reflecting a more efficient heart pumping more blood with each beat. Moreover, as vascular resistance decreases, the cardiovascular system adjusts to accommodate the increased cardiac output, ensuring that blood pressure doesn't rise excessively. The balance between cardiac output and vascular resistance reflects the body's ability to maintain stable circulation, likely through mechanisms like vasodilation or changes in heart function.

In addition, we also realized the impact of resistance on pressure dynamics. The two plots illustrate the response of the system when the resistance R2 is increased (to 1.25 * R2) and decreased (to 0.75 * R2). In the case of patient 16, the blue curve for increased resistance (Pin_16inc) demonstrates a notable initial rise in pressure, followed by a gradual stabilization. This behavior aligns with expectations; as resistance increases, the flow rate diminishes, leading to a higher pressure build-up initially before the system equilibrates. In contrast, the decreased resistance scenario (Pin_16dec) showcases a more rapid increase in input pressure, likely due to the reduced opposition to flow, resulting in greater pressure accumulation within the system.

As it is explained in the conclusions, the data of patients 17 and 18 were generated by a computer, so the resulting plots exhibit distinct characteristics compared to those derived from real patient data.

By focusing on the last recorded values in the vectors Pin_16dec_ult and Pin_16inc_ult, we can clearly assess the system's performance, eliminating the influence of initial values. These final pressure readings facilitate better visualization and analysis of the pressure dynamics.

5 CONCLUSIONS

Stroke volume, the amount of blood ejected by the heart with each contraction, varies between children and adults due to differences in body size, heart structure, and metabolic needs. In infants, the average stroke volume is lower, typically around 3 to 5 milliliters per beat, reflecting their smaller heart size and lower blood volumes children grow, their stroke volume increases, reaching approximately 50 to 70 milliliters per beat in older children and adolescents. In adults, the normal stroke volume generally ranges from 60 to 100 milliliters per beat, depending on factors such as body composition, fitness level, and overall health. Understanding these variations helps in assessing cardiovascular function and developing appropriate medical care across different age groups. With this information in mind, we can conclude that the patient 16 is a child, so it´s data is real-measured, while the patients 17 and 18 are adults with computer-generated data.

6 REFERENCES

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