Core 3 Notes

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June 30, 2017

Contents

Differentiation

1.1 C1 Differentiation reminder

$$y = ax^n$$

$$\Rightarrow \frac{dy}{dx} = anx^{n-1}$$

Remember
$\frac{dy}{dx}$ is a measure of the rate of change of y compared to x. e.g. $\frac{dy}{dx}$=3 then y increases by 3 for every increase of 1 in x.
If two lines are perpendicular the the product of their gradients is -1.

1.2 C2 Differentiation

Chain rule (or "function of function rule")

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Ex1

$$y = (3x+2)^5$$

$$\text{let } u = 3x+2$$

$$\Rightarrow y = u^5$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = 5u^4 \times 3$$

$$= 5u^4 \times 3$$

$$= 15u^4$$

$$= 15(3x+2)^4$$

Ex2

$$y = \sqrt{4x^2 - 1}$$

$$= (4x^2-1)^{\frac{1}{2}}$$

$$\text{let } u = 4x^2 - 1$$

$$\Rightarrow y = u^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{1}{2}u^{-\frac{1}{2}} \times 8x$$

$$\frac{4x}{(4x^2-1)^{\frac{1}{2}}}$$

Ex3

$$y = \frac{1}{\sqrt[3]{5-4x}}$$

$$= (5-4x)^{-\frac{1}{3}}$$

$$\text{let } u = 5-4x$$

$$y = u^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\frac{1}{3}U^{\frac{-4}{3}} \times -4$$

$$= \frac{4}{3}u^{-\frac{4}{3}}$$

$$= \frac{4}{3}(5-4x)^{-\frac{4}{3}}$$

General Rule
If

$$y = \left[f(x)\right]^n$$

then

$$\frac{dy}{dx} = n\left[f(x)\right]^{n-1} \times f'(x)$$

$$y = (4x^2+9)^5$$

$$\frac{dy}{dx} = 5(4x^2+9)^4 \times 8x$$

$$= 40x(4x^2+9)^4$$

One particular example of our general rule is

$$\text{if } y = (ax+b)^n$$

$$\text{then } \frac{dy}{dx} = an(ax+b)^{n-1}$$

and by considering intergration as the reverse of our differentiation we cna deduce that

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \quad \text{for } n \neq -1$$

e.g.

1.
$$\int (2x+3)^5 dx = \frac{(2x+2)^6}{2x+6} + c$$
$$= \frac{(2x+3)^4}{12} + c$$

2.
$$\int_{-\frac{1}{3}}^{2} \frac{1}{\sqrt[3]{3x+2}} dx = \int_{-\frac{1}{3}}^{2} (3x+2)^{-\frac{1}{3}} dx$$
$$= \left[\frac{(3x+2)^{\frac{2}{3}}}{2}\right]_{-\frac{1}{3}}^{2}$$
$$= 2 - \frac{1}{2} = \frac{3}{2}$$

The link between $\frac{dy}{dx}$ and $\frac{dx}{dy}$

If $\frac{dy}{dx} = 4$ then y is increasing 4 times as fast as x. This would mean that x is increasing $\frac{1}{4}$ times as fast as y.

$$\text{i.e.} \frac{dy}{dx} = \frac{1}{4}$$

Hence

$$\boxed{\frac{dy}{dx} = \frac{1}{\frac{dy}{dx}}}$$

1. (Volume of a sphere $= \frac{4}{3}\pi r^3$)
   The radius of a sphere is increasing at 2cms^{-1}.
   Find the rate of increase of volume at the instant that the radius is 5cm.

$$\rightarrow \frac{dv}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2$$

$$= 4\pi r^2 \times 2$$

when $r = 5$
$\frac{dV}{dt} = 200\pi cm^3 s^{-1}$

2. (Surface area of a sphere $= 4\pi r^2$) The surface area of a sphere is decreasing at $40\text{cm}^2\text{s}^{-1}$. Find the rate of change of the radius at the instant the radius is 4cm.

$$\frac{dr}{dt} = \frac{dr}{dS} \times \frac{dS}{dt} \quad \text{Where S=surface area}$$

$$S = 4\pi r^2$$

$$\frac{1}{\frac{dS}{dr}} \times \frac{dS}{dt}$$

$$\frac{dS}{dt} = 8\pi r$$

$$\frac{1}{8\pi r} \times -40$$

When r=4, $\frac{dr}{dt} = \frac{-40}{32\pi} = \frac{-5}{4\pi}$
rate of decrease of radius $= \frac{5}{4\pi} cms^{-1}$

Exponential functions
Exponential functions have a variable as a power.

Ex1

$$Q = 2 \times 3^t$$

t	0	1	2	3	4
Q	2	6	18	54	162

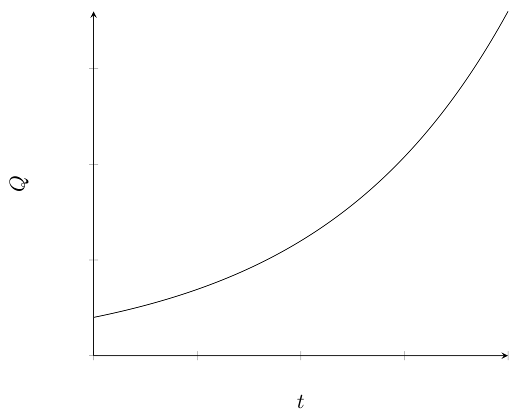
Ex2

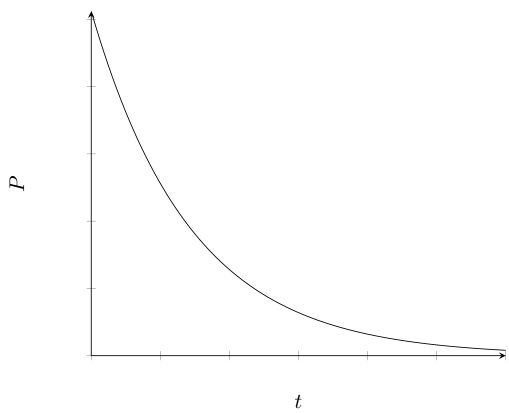
$$P = 256 \times \left(\frac{1}{2}\right)^t$$

t	0	1	2	3	4
Q	256	128	64	32	16

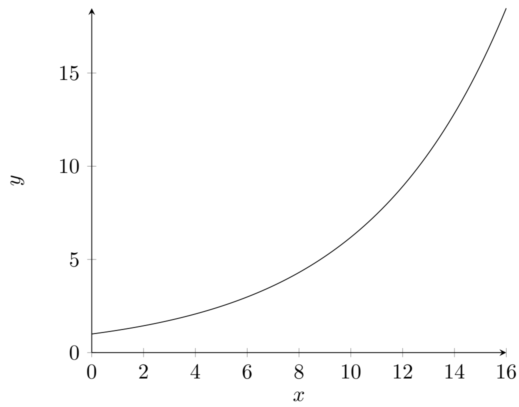
Note- If $y = a^2$ then

1. $y = a$ When $t = 0$
2. For each equal interval of t the value of y is multiplied by the same amount.





Exponential Growth/Decay - the rate of increase/decrease of the quantity is proportional to the amount present at that time.



This is quite hard...