Further Pure 3 Notes

Jacob Santry

June 28, 2017

Contents

Group Theory

1.1 Background Work

Set Notation

$$A = \{\text{Colours of the rainbow}\}$$

$$\mathit{Red} \underset{\substack{\uparrow \\ \text{Is an element of.}}}{\in} A$$

$$\text{or } B\{2, 5, 11, 17\}$$

$$\text{or } C = \{x \underset{\substack{\uparrow \\ \text{Such that.}}}{:} x > 2\}$$

Special sets

$\mathbb{R}$=$\{$Real numbers$\}$
$\mathbb{C}$=$\{$Complex numbers$\}$
$\mathbb{Z}$=$\{$Integers$\}$
$\mathbb{Z}^+$=$\{$Real numbers$\}$

$$\left[\text{also } \mathbb{N} = \{\text{Natural numbers}\}\right]$$

$\mathbb{Q}$=$\{$Rational numbers$\}$

Binary Operations

The operation, * , acts on the two elements of a set to produce a unique "product" (i.e. outcome).

Properties of Binary Operations

1. Commutivity: an operation, *, is communative iff

$$x * y = y * x \quad \forall x, y.$$

2. Associativity: operation, *, is associative iff

$$x * (y * z) = (x * y) * z \quad \forall x, y, z$$

3. Distributive property: operation, *, is distributive over operation $\circ, iff$

$$x * (y \circ z) = (x * y) \circ (x * z) \quad \forall x, y, z$$

1.2 Modulo Arithmetic

Aritmetic modulo n returns the remainder after division by n.

e.g.

$$3 +_6 5 = 2$$

$$3 \times_6 5 = 3$$

both of these calculations are restricted to the set $\mathbb{Z} = \{0,1,2,3,4,5\}$
so $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$

Definitions of a group