Core 3 Notes

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Contents

Differentiation

1.1 C1 Differentiation reminder

$$y = ax^n$$
$$\Rightarrow \frac{dy}{dx} = anx^{n-1}$$

Remember
$\frac{dy}{dx}$ is a measure of the rate of change of y compared to x. e.g. $\frac{dy}{dx}$=3 then y increases by 3 for every increase of 1 in x.
If two lines are perpendicular the the product of their gradients is -1.

1.2 C2 Differentiation

Chain rule (or "function of function rule")

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Ex1

$$y = (3x+2)^5$$
$$\text{let } u = 3x+2$$
$$\Rightarrow y = u^5$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$\frac{dy}{dx} = 5u^4 \times 3$$
$$= 5u^4 \times 3$$
$$= 15u^4$$
$$= 15(3x+2)^4$$

Ex2

$$y = \sqrt{4x^2-1}$$
$$= (4x^2-1)^{\frac{1}{2}}$$
$$\text{let } u = 4x^2-1$$
$$\Rightarrow y = u^{\frac{1}{2}}$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$\frac{1}{2}u^{-\frac{1}{2}} \times 8x$$
$$\frac{4x}{(4x^2-1)^{\frac{1}{2}}}$$

Ex3

$$y = \frac{1}{\sqrt[3]{5-4x}}$$
$$= (5-4x)^{-\frac{1}{3}}$$
$$\text{let } u = 5-4x$$
$$y = u^{-\frac{1}{3}}$$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$= -\frac{1}{3}U^{\frac{-4}{3}} \times -4$$
$$= \frac{4}{3}u^{-\frac{4}{3}}$$
$$= \frac{4}{3}(5-4x)^{-\frac{4}{3}}$$

General Rule
If

$$y = \big[f(x)\big]^n$$

then

$$\frac{dy}{dx} = n\big[f(x)\big]^{n-1} \times f'(x)$$

$$y = (4x^2 + 9)^5$$

$$\frac{dy}{dx} = 5(4x^2 + 9)^4 \times 8x$$

$$= 40x(4x^2 + 9)^4$$

One particular example of our general rule is

$$\text{if } y = (ax + b)^n$$

$$\text{then } \frac{dy}{dx} = an(ax + b)^{n-1}$$

and by considering intergration as the reverse of our differentiation we cna deduce that

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + c \quad \text{for } n \neq -1$$

e.g.

1.
$$\int (2x + 3)^5 dx = \frac{(2x + 2)^6}{2x + 6} + c$$
$$= \frac{(2x + 3)^4}{12} + c$$

2.
$$\int_{-\frac{1}{3}}^{2} \frac{1}{\sqrt[3]{3x + 2}} dx = \int_{-\frac{1}{3}}^{2} (3x + 2)^{-\frac{1}{3}} dx$$
$$= \left[\frac{(3x + 2)^{\frac{2}{3}}}{2}\right]_{-\frac{1}{3}}^{2}$$
$$= 2 - \frac{1}{2} = \frac{3}{2}$$

The link between $\frac{dy}{dx}$ and $\frac{dx}{dy}$

If $\frac{dy}{dx} = 4$ then y is increasing 4 times as fast as x. This would mean that x is increasing $\frac{1}{4}$ times as fast as y.

$$\text{i.e.} \frac{dy}{dx} = \frac{1}{4}$$

Hence

$$\boxed{\frac{dy}{dx} = \frac{1}{\frac{dy}{dx}}}$$

1. (Volume of a sphere $= \frac{4}{3}\pi r^3$)
   The radius of a sphere is increasing at 2cms^{-1}.
   Find the rate of increase of volume at the instant that the radius is 5cm.
2. (Surface area of a sphere $= 4\pi r^2$)