Quantum Computation on the Toric Code

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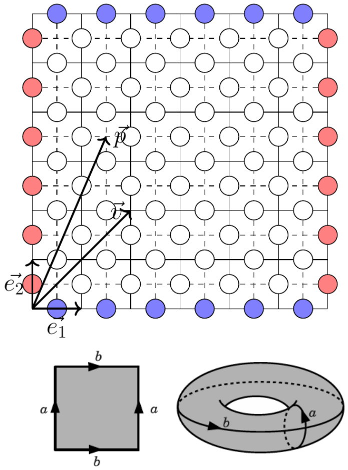
Outline:

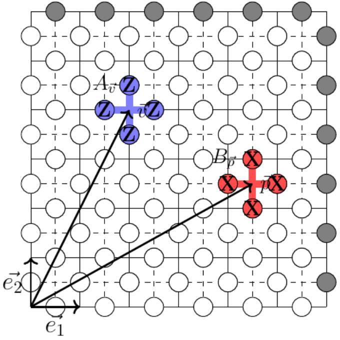
1. Square Lattice
2. Hamiltonian of the System
3. Ground States
4. Encoding of the Qubits
5. QEC on the Toric Code

Square Lattice



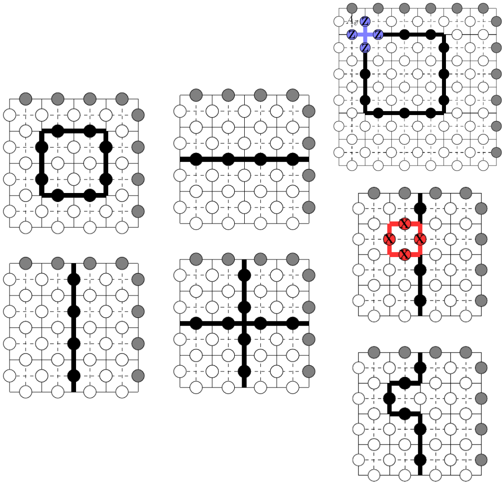
- Square and dual lattice with periodic boundary conditions;
- spin-$\frac{1}{2}$ particles, each belonging to $\mathscr{H} = \mathbb{C}^2$;
- edges;
- vertices and plaquettes.

Hamiltonian of the Toric Code



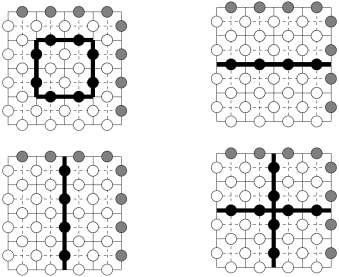
- vertex and plaquette operators:
  $A_{\vec{v}} = \prod_{e \in star(\vec{v})} Z_e$,
  $B_{\vec{p}} = \prod_{e \in bdy(\vec{p})} X_e$
  where :
  $Z_e = \mathbb{I}_{E \setminus e} \otimes \sigma_e^z$,
  $X_e = \mathbb{I}_{E \setminus e} \otimes \sigma_e^x$;
- commutation properties;
- $H = -\sum_{\vec{v} \in V} A_{\vec{v}} - \sum_{\vec{p} \in P} B_{\vec{p}}$ acting on the Hilbert space of the system $\mathscr{H} = \bigotimes_{e \in E} \mathbb{C}^2$;
- compute the ground state(s): find all the eigenstates of $A_{\vec{v}}$ and $B_{\vec{p}}$ with $+1$ eigenvalue.

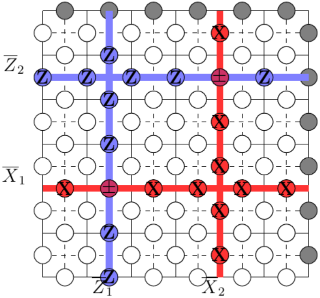
Ground States



- Eigenstates of $A_{\vec{v}}$;
- eigenstates of $B_{\vec{p}}$ are not eigenstates of $A_{\vec{v}}$ by themselves. Instead a completely symmetric superposition of all the eigenstates of $A_{\vec{v}}$, belonging to the same class, is an eigenstate of $B_{\vec{p}}$;
- degenerate ground state: 4 classes of topologically protected eigenstates.

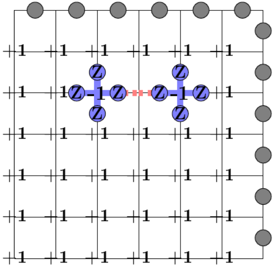
How do we Encode Qubits?

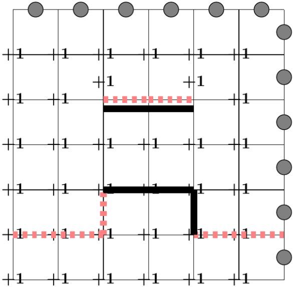




- Relabel the 4 ground states: $|00\rangle, |10\rangle, |01\rangle$ and $|11\rangle$;
- logical qubits are encoded as non-local excitations in the lattice (gates), which are protected against local errors by the vertex and plaquette operators;
- in order to have 2 logical qubits we need $2N^2$ physical qubits (spin).

QEC on the Toric Code





- Error Syndrome: properties of operators allow simultaneous measurements that help us to detect endpoints of error strings;
- there exists a natural error correction algorithm using minimum distance between quantum states: apply error correcting strings;
- protection against local errors of the order of $n < \frac{N}{2}$;
- strategies to reduce probability of obtaining irreversible errors.