IN480 Larstruct Module

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Contents

Introduction

This module of LAR-CC library is about a hierarchical structures with LAR. Hierarchical models of complex assemblies are generated by an aggregation of subassemblies, each one defined in a local coordinate system, and relocated by affine transformations of coordinates.

In this module there are:

- The **Affine transformations** are based on elementary matrices for affine transformation of vectors in any dimensional vector space, including translation, scaling and rotation;
- The **Struct iterable class**, starting from an array representable geometric object, generates a new object representing the initial one in an alternative way, by means of specific fields attribution, e.g. body, box, etc..
- The **structure to LAR conversion** is based on functions for embedding two-dimensional LAR model in 3D space and remove duplicate faces, vertices and cells of geometric object.

In the next sections the main functions of the module will be shown through the API. Furthermore, each function will be proposed to convert code from the Python language to Julia, parallelization, some unit tests and the study of execution times.

1 API

- **larApply**(Matrix)(Tuple)→ Tuple,

  through the affine matrix given in input, it performs an affine transformation and returns in output a tuple containing two arrays: the list of the transformed coordinates of the vertices and a cell vector.

- **evalStruct**(Struct)→ Array,

  analyzes the elements contained in the Struct object and returns an array containing the main data structures. Each structure is described by an array containing two arrays: the list of the transformed coordinates of the vertices and a cell vector.

- **vcode**(Int)(Array)→String,

  generates the function that approximates each element of the data structure to which it is applied. the approximation occurs with precision given by the entire input data. It transforms the data structure obtained in string to be returned to output.

- **struct2lar**(Struct)→ Array,

  it extracts the elements composing the input and combines them into a single element represented by the output array, where: the first element contains the array of the coordinates of the vertices of all the objects present, the second contains vectors of larger cells without duplicates of vertices. If we are in 3D: the third element contains the 3D cells, this array can add elements depending on how many were added to the first one.

- **larRemoveVertices**(Array, Array)→ (Array,Array)

  takes in input arrays of vertices and cells and returns them private of their duplicates

- **larEmbed**(Int)(Tuple)→ Tuple, if the input n is such that n¿0 the function generated increases n the size of the object to which it is applied, otherwise it reduces the size of n.

- **larBoundary**(Struct)→ (Array,Array,Array)

  apply struct2lar to the input, if the output has dimension 3, the function returns a tuple containing the 3 outputs, otherwise it returns a string containing an error message.

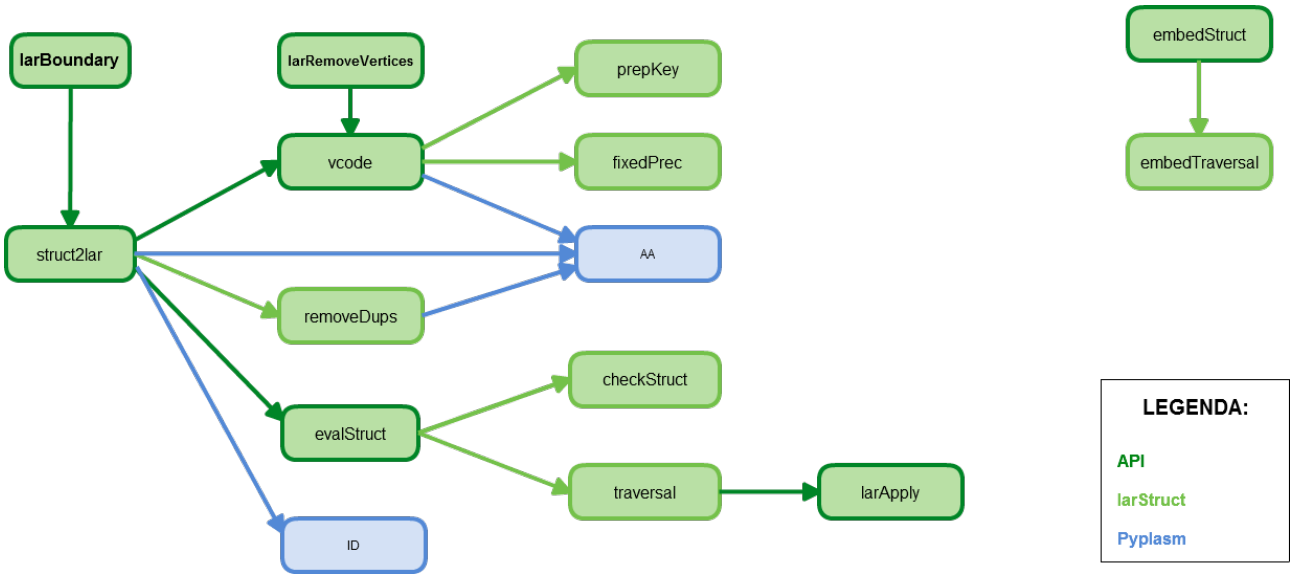
- **embedStruct**(Int)(Struct)→ Struct,

  creates a copy of the object to which it is applied, increases its size, as much as the value entered in input and returns the new object created.

Internal Functions

- **checkStruct**(Array)→ Int

- **traversal**(Matrix,Array,Struct,Array)→ Array
- **prepKey**(String)→String
- **fixedPrec**(Int)→ String
- **removeDups**(Array)→ Array



2 Implementation

This section shows the translation from the Python language to Julia, of some of the most important module functions. The Results section is divided into two parts: the first shows the performance graphs of the functions, run on the 'Tesla' computer with 30 processors, where the input is increased by increasing the body of the objects of Struct type, through the function:

```
function addn2D(n,model)
    body=[ ]
    for i in range(1,n)
        el=[ ]
        matrix=rand(1:3)
        if matrix==1
            x=rand(1:10)/10
            y=rand(1:10)/10
            append!(el,larApply(t(x,y))(model))
            append!(body,[el])
        elseif matrix ==2
            x=rand(1:10)/10
            y=rand(1:10)/10
            append!(el,larApply(s(x,y))(model))
            append!(body,[el])
        elseif matrix ==3
            x=rand(1:10)
            append!(el,larApply(r(pi/x))(model))
            append!(body,[el])
        end
    end
    a=Struct(body)
    return a
end
```

In the second part the graphs refer to functions run on a personal computer with 4 processors, where the increase in input was made starting from a function external to the " larCuboids " module by:

```
using PyCall
@pyimport larlib as lar
l=[ ]
for i in range(0,3)
    push!(l,PyObject(lar.larCuboids([10^i,10^i])))
    push!(l,PyObject(lar.larCuboids([2*10^i,2*10^i])))
    push!(l,PyObject(lar.larCuboids([5*10^i,5*10^i])))
end
```

The function to calculate the execution time is:

```
function Time(f::Function,args)
    @elapsed f(args...)
    t=[ ]
    for i in range(1,10)
        push!(t,@elapsed f(args...))
    end
    m=mean(t)
    return m
end
```

The requirements for running scripts are:

- using Base.Test
- using Plots
- using Distributions

from the Julia terminal on Tesla:

```
addprocs(29)
```

or from the shell run Julia with the command:

```
julia -p 30
```

from the Julia terminal on personal computer:

```
addprocs(3)
```

or from the shell run Julia with the command:

```
julia -p 4
```

2.1 checkStruct

2.1.1 Conversion

Python

```
def checkStruct(lst):
    obj = lst[0]
    if(isinstance(obj,tuple) or isinstance(obj,list)):
        dim = len(obj[0][0])
    elif isinstance(obj,Model): 30
        dim = obj.n
    elif isinstance(obj,Mat):
        dim = obj.shape[0]-1
    elif isinstance(obj,Struct):
        dim = len(obj.box[0])
    return dim
```

Julia

```
function checkStruct(lst)
    obj = lst[1]
    if isa(obj,Matrix)
        dim=size(obj)[1]-1
    elseif(isa(obj,Tuple) || isa(obj,Array))
        dim=length(obj[1][1])
    elseif isa(obj,Struct)
        dim=length(obj.box[1])
    end
    return dim
end
```

2.1.2 Parallelization

```
function pcheckStruct(lst)
    obj = lst[1]
    if isa(obj,Matrix)
        dim=size(obj)[1]-1
    elseif(isa(obj,Tuple) || isa(obj,Array))
        dim=length(obj[1][1])
    elseif isa(obj,pStruct)
        dim=length(obj.box[1])
    end
    return dim
end
```

2.1.3 Unit-Test

```
@testset "checkStruct" begin
    list=([[0.575,-0.175],[0.575,0.175],[0.925,-0.175],[0.925,0.175]],
```

```
    [[0,1,2,3]])
@test checkStruct(list)==length(list[1][1][1])
@test typeof(checkStruct(list))==Int
end
```

2.2 larApply

2.2.1 Conversion

Python

```
def t(*args):
    d = len(args)
    mat = scipy.identity(d+1)
    for k in range(d):
        mat[k,d] = args[k]
    return mat.view(Mat)

def s(*args):
    d = len(args)
    mat = scipy.identity(d+1)
    for k in range(d):
        mat[k,k] = args[k]
    return mat.view(Mat)

def r(*args):
    args = list(args)
    n = len(args)
    if n== 1: # rotation in 2D
        angle = args[0]; cos = COS(angle); sin = SIN(angle)
        mat = scipy.identity(3)
        mat[0,0] = cos;      mat[0,1] = -sin;
        mat[1,0] = sin;      mat[1,1] = cos;
    if n== 3: # rotation in 3D
        mat = scipy.identity(4)
        angle = VECTNORM(args); axis = UNITVECT(args)
        cos = COS(angle); sin = SIN(angle)
        if axis[1]==axis[2]==0.0:     # rotation about x
            mat[1,1] = cos;      mat[1,2] = -sin;
            mat[2,1] = sin;      mat[2,2] = cos;
        elif axis[0]==axis[2]==0.0:     # rotation about y
            mat[0,0] = cos;      mat[0,2] = sin;
            mat[2,0] = -sin;      mat[2,2] = cos;
        elif axis[0]==axis[1]==0.0:     # rotation about z
            mat[0,0] = cos;      mat[0,1] = -sin;
            mat[1,0] = sin;      mat[1,1] = cos;
        else:          # general 3D rotation (Rodrigues' rotation formula)
            I = scipy.identity(3) ; u = axis
            Ux = scipy.array([
            [0,          -u[2],       u[1]],
```

```
            [u[2],          0,      -u[0]],
            [-u[1],      u[0],          0]])
            UU = scipy.array([
            [u[0]*u[0],     u[0]*u[1],     u[0]*u[2]],
            [u[1]*u[0],     u[1]*u[1],     u[1]*u[2]],
            [u[2]*u[0],     u[2]*u[1],     u[2]*u[2]]])
            mat[:3,:3] = cos*I + sin*Ux + (1.0-cos)*UU
        return mat.view(Mat)

    def larApply(affineMatrix):
        def larApply0(model):
            if isinstance(model,Model):
                V = scipy.dot(array([v+[1.0] for v in model.verts]),
                affineMatrix.T).tolist()
                V = [v[:-1] for v in V]
                CV = copy.copy(model.cells)
                return Model((V,CV))
            elif isinstance(model,tuple) or isinstance(model,list):
                if len(model)==2: V,CV = model
                elif len(model)==3: V,CV,FV = model
                V=scipy.dot([list(v)+[1.0]for v in V],affineMatrix.T).tolist()
                if len(model)==2: return [v[:-1] for v in V],CV
                elif len(model)==3: return [v[:-1] for v in V],CV,FV
            return larApply0
```

Julia

```
function t(args...)
    d=length(args)
    mat=eye(d+1)
    for k in range(1,d)
        mat[k,d+1]=args[k]
    end
    return mat
end

function s(args...)
    d=length(args)
    mat=eye(d+1)
    for k in range(1,d)
        mat[k,k]=args[k]
    end
    return mat
end
```

```
function r(args...)
    args = collect(args)
    n = length(args)
    if n== 1 # rotation in 2D
        angle = args[1]; COS = cos(angle); SIN = sin(angle)
        mat = eye(3)
        mat[1,1] = COS;      mat[1,2] = -SIN;
        mat[2,1] = SIN;      mat[2,2] = COS;
    end
    if n== 3 # rotation in 3D
        mat = eye(4)
        angle = norm(args); axis = normalize(args)
        COS = cos(angle); SIN= sin(angle)
        if axis[2]==axis[3]==0.0     # rotation about x
            mat[2,2] = COS;      mat[2,3] = -SIN;
            mat[3,2] = SIN;      mat[3,3] = COS;
        elseif axis[1]==axis[3]==0.0   # rotation about y
            mat[1,1] = COS;      mat[1,3] = SIN;
            mat[3,1] = -SIN;      mat[3,3] = COS;
        elseif axis[1]==axis[2]==0.0     # rotation about z
            mat[1,1] = SIN;      mat[1,2] = -SIN;
            mat[2,1] = COS;      mat[2,2] = COS;
        else
            I=eye(3); u=axis
            Ux=[0 -u[3] u[2] ; u[3] 0 -u[1] ; -u[2] u[1] 1]
            UU =[u[1]*u[1]     u[1]*u[2]    u[1]*u[3];
            u[2]*u[1]     u[2]*u[2]    u[2]*u[3];
            u[3]*u[1]     u[3]*u[2]    u[3]*u[3]]
            mat[1:3,1:3]=COS*I+SIN*Ux+(1.0-COS)*UU
        end
    end
    return mat
end

function larApply(affineMatrix)
    function larApply0(model)
        if length(model)==2
            V,CV=model
        elseif length(model)==3
            V,CV,FV = model
        end
        V1=Array{Float64}[]
        for (k,v) in enumerate(V)
```

```
            append!(v,[1.0])
            push!(V1,vec((v')*transpose(affineMatrix)))
            pop!(V[k])
            pop!(V1[k])
              end
        if length(model)==2
            return V1,CV
        elseif length(model)==3
            return V1,CV,FV
        end
    end
    return larApply0
end
```

2.2.2 Parallelization

The functions for the affine transformation matrices are the same as in the sequential case.

```
@everywhere function plarApply(affineMatrix)
          function plarApply0(model)
              if length(model)==2
                 V,CV=deepcopy(model)
              elseif length(model)==3
                      V,CV,FV = deepcopy(model)
              end
             V1=Array{Float64}[ ]
             V1=@sync @parallel (append!) for v in V
                          append!(v,[1.0])
                         [collect(vec((v')*transpose(affineMatrix)))]
                  end
                  for v in V1
                            pop!(v)
                  end
               if length(model)==2
                  return fetch(V1),CV
               elseif length(model)==3
                  return V1,CV,FV
                end
            end
            return plarApply0
end
```

2.2.3 Unit-Test

```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
cubes=([[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],[1,1,0],[1,1,1]],
       [[0,1,2,3,4,5,6,7]])

@testset "larApply Tests" begin
    @testset "2D" begin
        @testset "larApply Translation 2D" begin
        @test typeof(larApply(t(-0.5,-0.5))(square))==Tuple{Array{Array{
                    Float64,N} where N,1},Array{Array{Int64,1},1}}
        @test larApply(t(-0.5,-0.5))(square)==([[-0.5,-0.5],[-0.5,0.5],
             [0.5,-0.5],[0.5,0.5]],[[0,1,2,3]])
    end
        @testset "larApply Scaling 2D" begin
            @test typeof(larApply(s(-0.5,-0.5))(square))==Tuple{Array{Array{
                      Float64,N} where N,1},Array{Array{Int64,1},1}}
            @test larApply(s(-0.5,-0.5))(square)==([[0.0,0.0],[0.0,-0.5],
                    [-0.5,0.0],[-0.5,-0.5]],[[0,1,2,3]])
        end
        @testset "larApply Rotation 2D" begin
            @test typeof(larApply(r(0))(square))==Tuple{Array{Array{
                      Float64,N}  where N,1},Array{Array{Int64,1},1}}
            @test larApply(r(0))(square)==square
        end
    end

    @testset "3D" begin
        @testset "larApply Translation 3D" begin
            @test typeof(larApply(t(-0.5,-0.5,-0.5))(cubes))==Tuple{Array{
                  Array{Float64,N} where N,1},Array{Array{Int64,1},1}}
            @test larApply(t(-0.5,-0.5,-0.5))(cubes)==([[-0.5,-0.5,-0.5],
                  [-0.5,-0.5,0.5],[-0.5,0.5,-0.5],[-0.5,0.5,0.5],
                  [0.5,-0.5,-0.5],[0.5,-0.5,0.5],[0.5,0.5,-0.5],
                  [0.5,0.5,0.5]],[[0,1,2,3,4,5,6,7]])
        end

        @testset "larApply Scaling 3D" begin
            @test typeof(larApply(s(-0.5,-0.5,-0.5))(cubes))==Tuple{Array{
                  Array{Float64,N} where N,1},Array{Array{Int64,1},1}}
            @test larApply(s(-0.5,-0.5,-0.5))(cubes)==([[0.0,0.0,0.0],
                    [0.0,0.0,-0.5],[0.0,-0.5,0.0],[0.0,-0.5,-0.5],
                    [-0.5,0.0,0.0],[-0.5,0.0,-0.5],[-0.5,-0.5,0.0],
                    [-0.5,-0.5,-0.5]],[[0,1,2,3,4,5,6,7]])
        end
```

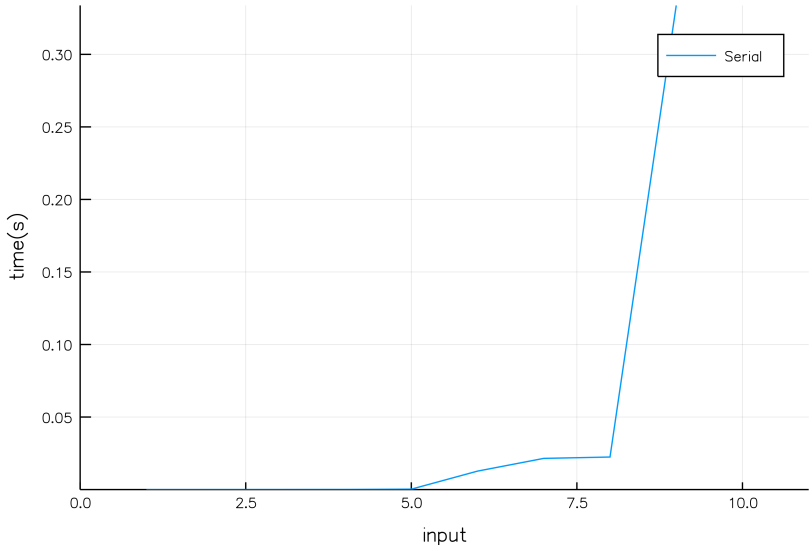
```
        @testset "larApply Rotation 3D" begin
            @test typeof(larApply(r(pi,0,0))(cubes))==Tuple{Array{Array{
                    Float64,N} where N,1},Array{Array{Int64,1},1}}
            @test larApply(r(pi,0,0))(cubes)[1] ≈ [[0.0,0.0,0.0],[0.0,-1.22465e-16,-1.0],
                    [0.0,-1.0,1.22465e-16],[0.0,-1.0,-1.0],[1.0,0.0,0.0],
                    [1.0,-1.22465e-16,-1.0],[1.0,-1.0,1.22465e-16],
                    [1.0,-1.0,-1.0]]
        end
    end

square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
cubes=([[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],[1,1,0],
        [1,1,1]],[[0,1,2,3,4,5,6,7]])
@testset "plarApply Tests" begin
    @testset "2D" begin
        @testset "plarApply Translation 2D" begin
            @test typeof(plarApply(t(-0.5,-0.5))(square))==Tuple{Array{
                    Array{Float64,1},1},Array{Array{Int64,1},1}}
            @test plarApply(t(-0.5,-0.5))(square)==([[-0.5,-0.5],[-0.5,0.5],
                    [0.5,-0.5],[0.5,0.5]],[[0,1,2,3]])
        end

        @testset "plarApply Scaling 2D" begin
            @test typeof(plarApply(s(-0.5,-0.5))(square))==Tuple{Array{
                    Array{Float64,1},1},Array{Array{Int64,1},1}}
            @test plarApply(s(-0.5,-0.5))(square)==([[0.0,0.0],[0.0,-0.5],
                    [-0.5,0.0],[-0.5,0.5]],[[0,1,2,3]])
        end

        @testset "plarApply Rotation 2D" begin
            @test typeof(plarApply(r(0))(square))==Tuple{Array{Array{
                    Float64,1},1},Array{Array{Int64,1},1}}
            @test plarApply(r(0))(square)==square
        end
    end
    @testset "3D" begin
        @testset "plarApply Translation 3D" begin
            @test typeof(plarApply(t(-0.5,-0.5,-0.5))(cubes))==Tuple{Array{
                    Array{Float64,1},1},Array{Array{Int64,1},1}}
            @test plarApply(t(-0.5,-0.5,-0.5))(cubes)==([[-0.5,-0.5,-0.5],
                    [-0.5,-0.5,0.5],[-0.5,0.5,-0.5],[-0.5,0.5,0.5],
```

```
                [0.5,-0.5,-0.5],[0.5,-0.5,0.5],[0.5,0.5,-0.5],
                [0.5,0.5,0.5]],[[0,1,2,3,4,5,6,7]])
    end
    @testset "plarApply Scaling 3D" begin
      @test typeof(plarApply(s(-0.5,-0.5,-0.5))(cubes))==Tuple{Array{
                Array{Float64,1},1},Array{Array{Int64,1},1}}
      @test plarApply(s(-0.5,-0.5,-0.5))(cubes)==([[0.0,0.0,0.0],
                [0.0,0.0,-0.5],[0.0,-0.5,0.0],[0.0,-0.5,-0.5],
                [-0.5,0.0,0.0],[-0.5,0.0,-0.5],[-0.5,-0.5,0.0],
                [-0.5,-0.5,-0.5]],[[0,1,2,3,4,5,6,7]])
    end
    @testset "plarApply Rotation 3D" begin
      @test typeof(plarApply(r(pi,0.0,0.0))(cubes))==Tuple{Array{Array{
                    Float64,1},1},Array{Array{Int64,1},1}}

      @test plarApply(r(pi,0,0))(cubes)[1]≈[[0.0,0.0,0.0],[0.0,-1.22465e-16,1.0],

            [0.0,-1.0,1.22465e-16],[0.0,-1.0,-1.0],[1.0,0.0,0.0],
            [1.0,-1.22465e-16,-1.0],[1.0,-1.0,1.22465e-16],
            [1.0,-1.0,-1.0]]
     end
   end
end
```

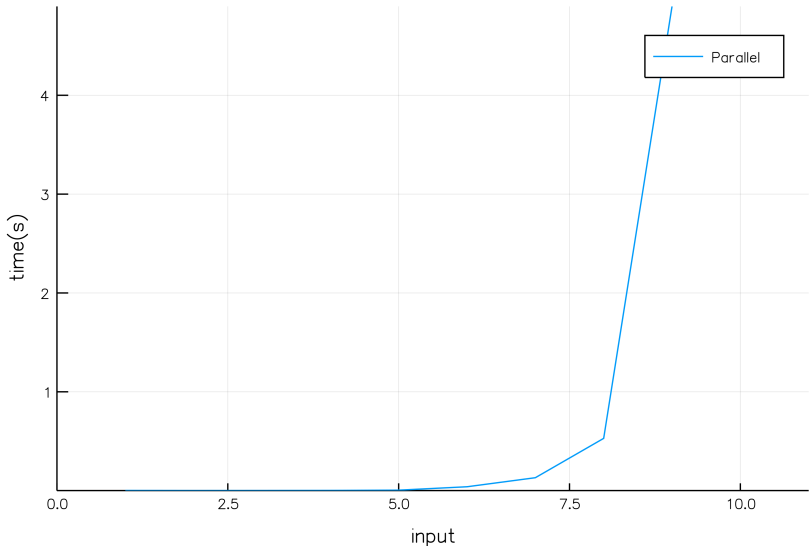
2.2.4 Results

PC

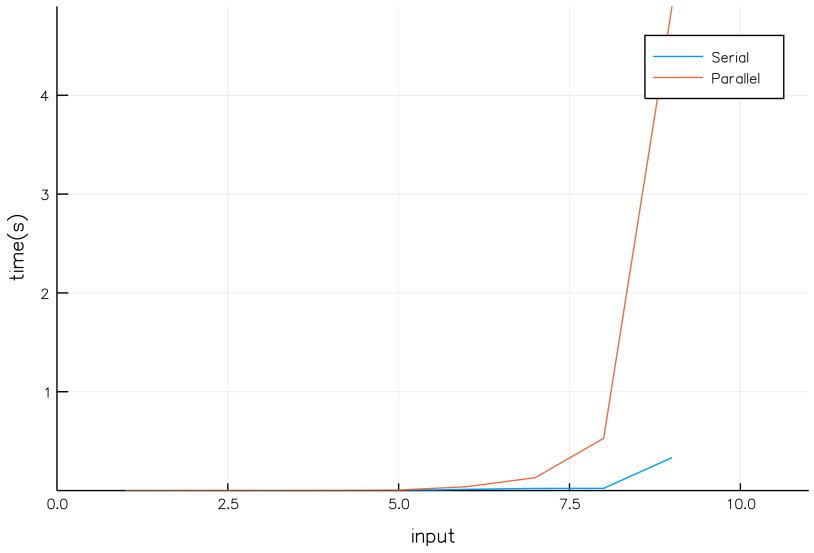
```
times=[ ]
ptimes=[ ]
for i in range(1,length(l))
    append!(times,Time(larApply(t(-0.5,-0.5)),l[i]))
end
for i in range(1,length(l))
    append!(ptimes,Time(plarApply(t(-0.5,-0.5)),l[i]))
end

plot(times,xlabel="input",xlims=(0,length(times)+2),
       ylabel="time(s)",label=["Serial"])
```




```
plot(ptimes,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Parallel"])
```



```
plot([times,ptimes],xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Serial","Parallel"])
```



2.3 box

2.3.1 Conversion

Python

```
def box(model):
    if isinstance(model,Mat): return []
    elif isinstance(model,Struct):
        dummyModel = copy.deepcopy(model)
        dummyModel.body = [term if (not isinstance(term,Struct))
                else [term.box,[[0,1]]]  for term in model.body]
            listOfModels = evalStruct( dummyModel )
            theMin,theMax = box(listOfModels[0])
            for theModel in listOfModels[1:]:
                modelMin, modelMax = box(theModel)
                theMin = [val if val<theMin[k] else theMin[k]
                for k,val in enumerate(modelMin)]
                theMax = [val if val>theMax[k] else theMax[k]
                for k,val in enumerate(modelMax)]
            return [theMin,theMax]
        elif isinstance(model,Model):
            V = model.verts
        elif (isinstance(model,tuple) or isinstance(model,list)) and
            (len(model)==2 or len(model)==3):
            V = model[0]
        coords = TRANS(V)
        theMin = [min(coord) for coord in coords]
        theMax = [max(coord) for coord in coords]
        return [theMin,theMax]
```

julia

```
function box(model)
    if isa(model,Matrix)
        return [ ]
    elseif isa(model,Struct)
        dummyModel=deepcopy(model)
        dummyModel.body=Any[ ]
        for term in model.body
            if isa(term,Struct)
                push!(dummyModel.body,[term.box,[0,1]])
            else
                push!(dummyModel.body,term)
            end
        end
```

```
        listOfModels=evalStruct(dummyModel)
        theMin,theMax=box(listOfModels[1])
        for theModel in listOfModels[2:end]
            modelMin,modelMax= box(theModel)
            for (k,val) in enumerate(modelMin)
                if val < theMin[k]
                    theMin[k]=val
                end
            end
            for (k,val) in enumerate(modelMax)
                if val > theMax[k]
                    theMax[k]=val
                end
            end
        end
        return Array[theMin,theMax]
    elseif (isa(model,Tuple)||isa(model,Array)) &&
            (length(model)==2||length(model)==3)
        V=model[1]
        theMin=[ ]
        theMax=[ ]
        for j in range(1,length(V[1]))
            Min=V[1][j]
            Max=V[1][j]
            for i in range(1,length(V))
                Min=min(Min,V[i][j])
                Max=max(Max,V[i][j])
            end
            push!(theMin,Min)
            push!(theMax,Max)
        end
        return Array[theMin,theMax]
    end
end
```

2.3.2 Parallelization

```
function pbox(model)
    if isa(model,Matrix)
        return [ ]
    elseif isa(model,pStruct)
        dummyModel=deepcopy(model)
        dummyModel.body=Any[ ]
```

```
    @sync for term in model.body
        if isa(term,pStruct)
            push!(dummyModel.body,[term.box,[0,1]])
        else
            push!(dummyModel.body,term)
        end
    end
    listOfModels=pevalStruct(dummyModel)
    theMin,theMax=pbox(listOfModels[1])
    @sync for theModel in listOfModels[2:end]
        modelMin,modelMax= pbox(theModel)
        @async begin
            for (k,val) in enumerate(modelMin)
                if (val < theMin[k])
                    theMin[k]=val
                end
            end
            for (k,val) in enumerate(modelMax)
                if (val > theMax[k])
                    theMax[k]=val
                end
            end
        end
    end
    return Array[theMin,theMax]
elseif (isa(model,Tuple) ||isa(model,Array))&&
        (length(model)==2 || length(model)==3)
    V=model[1]
    theMin=[ ]
    theMax=[ ]
    @sync for j in range(1,length(V[1]))
        Min=V[1][j]
        Max=V[1][j]
        for i in range(1,length(V))
            Min=min(Min,V[i][j])
            Max=max(Max,V[i][j])
        end
        @async begin
            push!(theMin,Min)
            push!(theMax,Max)
        end
    end
    return Array[theMin,theMax]
```

```
    end
end
```

2.3.3 Unit-Test

These Unit-Test can be runned after the definition of the Struct type on the section 2.5

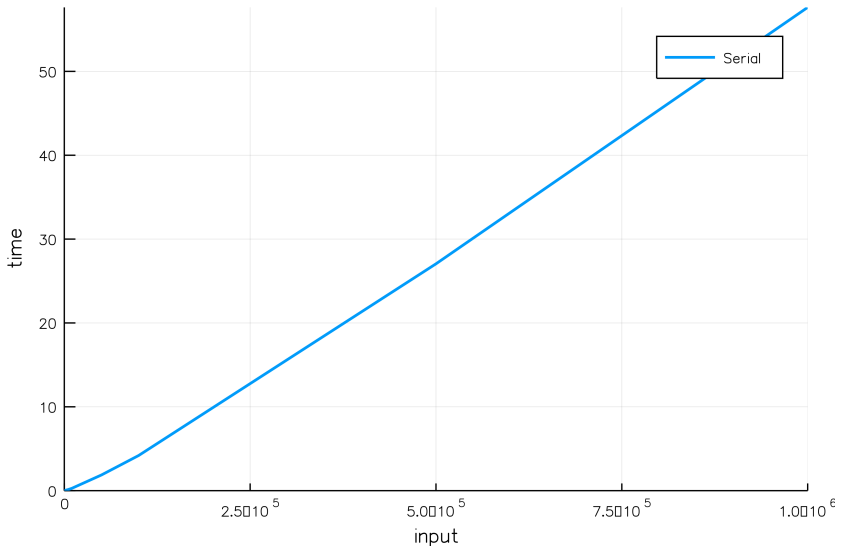
```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
cubes=([[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],[1,1,0],
        [1,1,1]],[[[0],[1],[2],[3],[4],[5],[6],[7]],[[0,1],[2,3],[4,5],
        [6,7],[0,2],[1,3],[4,6],[5,7],[0,4],[1,5],[2,6],[3,7]],
        [[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],
        [1,3,5,7]],[[0,1,2,3,4,5,6,7]]])

@testset "box Tests" begin
  @testset "box Tests 2D" begin
    @test typeof(box(square))==Array{Array,1}
    @test length(box(square))==2
    @test length(box(square)[1])==2
  end
  @testset "box Tests 3D" begin
    @test typeof(box(cubes))==Array{Array,1}
    @test length(box(cubes))==2
    @test length(box(cubes)[1])==3
  end
end

square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
cubes=([[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],[1,1,0],
        [1,1,1]],[[[0],[1],[2],[3],[4],[5],[6],[7]],[[0,1],[2,3],[4,5],
        [6,7],[0,2],[1,3],[4,6],[5,7],[0,4],[1,5],[2,6],[3,7]],
        [[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],
        [1,3,5,7]],[[0,1,2,3,4,5,6,7]]])

@testset "pbox Tests" begin
        @testset "pbox Tests 2D" begin
                @test typeof(pbox(square))==Array{Array,1}
                @test length(pbox(square))==2
                @test length(pbox(square)[1])==2
        end
        @testset "pbox Tests 3D" begin
                @test typeof(pbox(cubes))==Array{Array,1}
                @test length(pbox(cubes))==2
                @test length(pbox(cubes)[1])==3
        end
```

end

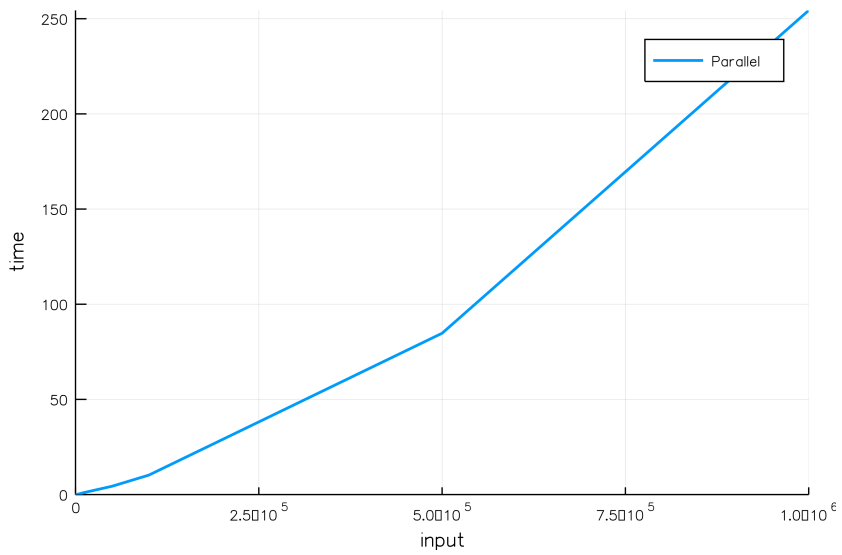
2.3.4 Results

Tesla

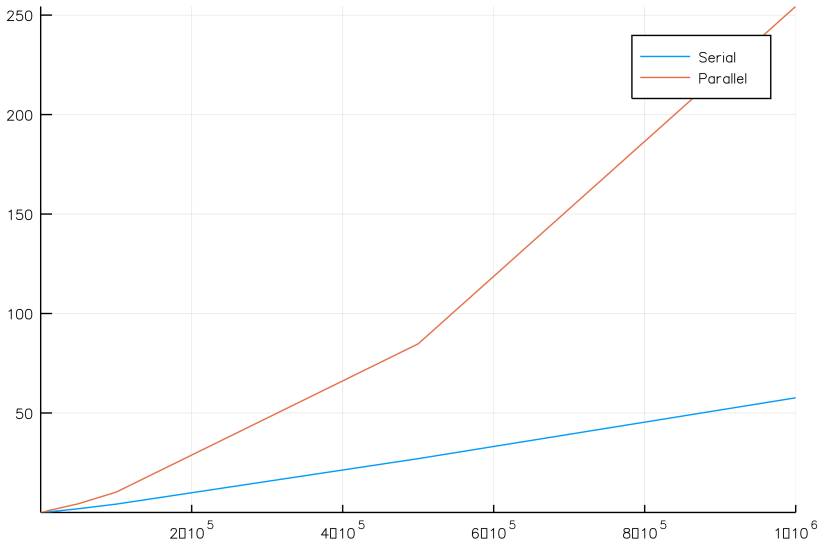
```
input=[1,10,50,10^2,5*10^2,10^3,5*10^3,10^4,5*10^4,10^5,5*10^5,10^6]
function timeFstruct(f::Function,pf::Function,model,input)
        t=Array{Float64}(length(input))
        pt=Array{Float64}(length(input))
        for i in range(1,length(input))
            structo=addn2D(input[i],model)
            pstructo=pStruct(structo.body)
            f(structo)
            pf(pstructo)
            t[i]=@elapsed f(structo)
            pt[i]=@elapsed pf(pstructo)
        end
        return t,pt
end

y,yp=timeFstruct(box,pbox,square,input)
p=plot(y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```



```
pp=plot(yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```

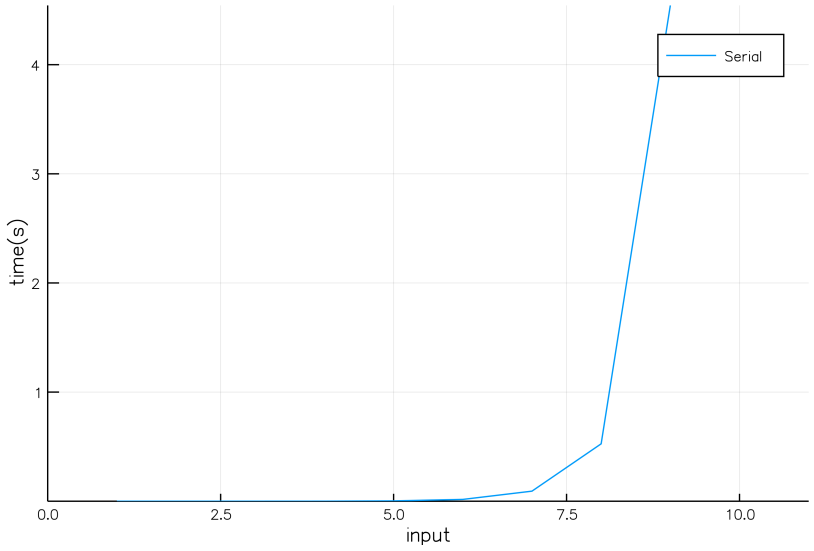



```
yc=[y,yp]
pc=plot(yc,label=["Serial" "Parallel"])
```

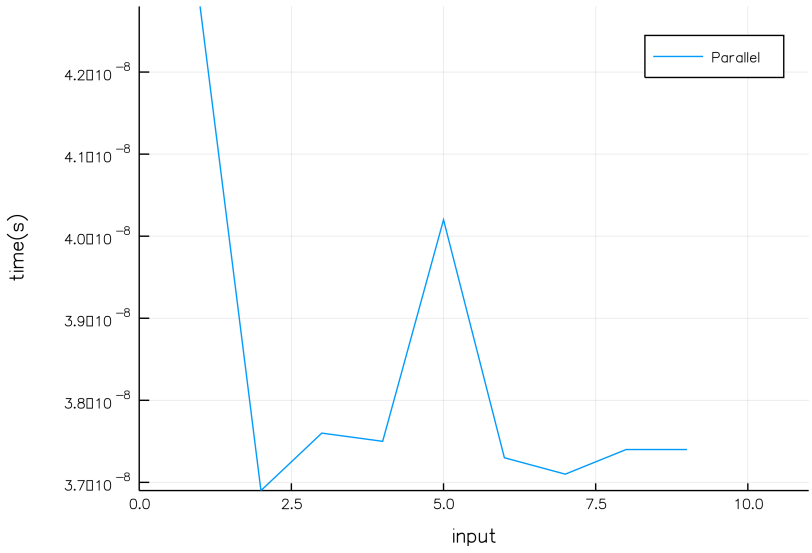


PC

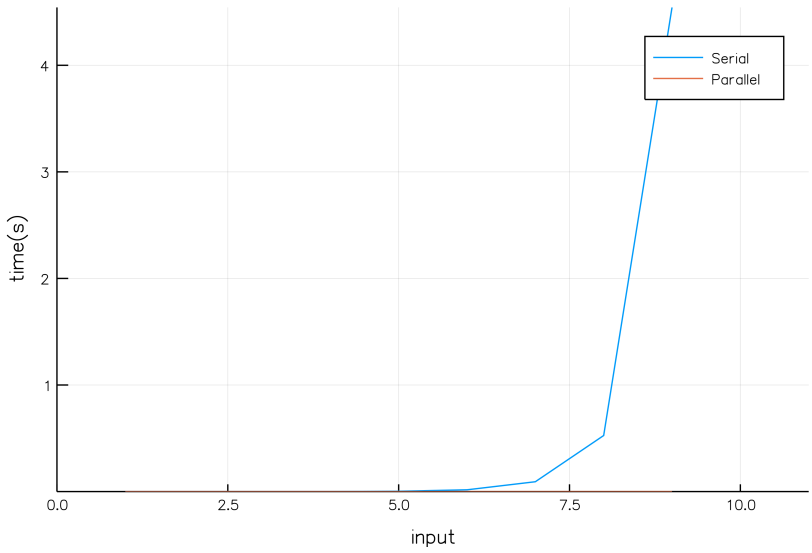
```
times=[ ]
ptimes=[ ]
input=[ ]
for i in range(1,length(l))
    push!(input,Struct([repeat([l[i]],outer=i)...]))
end
for i in range(1,length(input))
    append!(times,Time(box,input[i]))
end
for i in range(1,length(input))
    append!(ptimes,Time(pbox,input[i]))
end

plot(times,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Serial"])
```



```
plot(ptimes,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Parallel"])
```




```
plot([times,ptimes],xlabel="input",xlims=(0,length(ptimes)+2),
        ylabel="time(s)",label=["Serial","Parallel"])
```



2.4 traversal

2.4.1 Conversion

Python

```
def traversal(CTM, stack, obj, scene=[]):
    for i in range(len(obj)):
        if isinstance(obj[i],Model):
scene += [larApply(CTM)(obj[i])]
elif(isinstance(obj[i],tuple) or isinstance(obj[i],list)) and
(len(obj[i])==2 or len(obj[i])==3):
            scene += [larApply(CTM)(obj[i])]
        elif isinstance(obj[i],Mat):
            CTM = scipy.dot(CTM, obj[i])
        elif isinstance(obj[i],Struct):
            stack.append(CTM)
            traversal(CTM, stack, obj[i], scene)
            CTM = stack.pop()
    return scene
```

Julia

```
function traversal(CTM,stack,obj,scene=[])
    for i in range(1,len(obj))
        if isa(obj.body[i],Matrix)
            CTM = CTM * obj.body[i]
        elseif (isa(obj.body[i],Tuple) || isa(obj.body[i],Array)) &&
                (length(obj.body[i])==2 || length(obj.body[i])==3)
                l=larApply(CTM)(obj.body[i])
                push!(scene,l)
            elseif isa(obj.body[i],Struct)
                push!(stack,CTM)
                traversal(CTM,stack,obj.body[i],scene)
                CTM = pop!(stack)
            end
        end
        return scene
    end
```

2.4.2 Parallelization

```
@everywhere function ptraversal(CTM,stack,obj,scene=[])
    @sync for i in range(1,len(obj))
        if isa(obj.body[i],Matrix)
```

```
            CTM = CTM*obj.body[i]
        elseif (isa(obj.body[i],Tuple) || isa(obj.body[i],Array)) &&
                (length(obj.body[i])==2 || length(obj.body[i])==3)
                l=plarApply(CTM)(obj.body[i])
                push!(scene,l)
        elseif isa(obj.body[i],pStruct)
            push!(stack,CTM)
            ptraversal(CTM,stack,obj.body[i],scene)
            CTM = pop!(stack)
        end
    end
    return scene
end
```

2.4.3 Unit-Test

These tests can be runned after the definition of the Struct type on the section 2.5

```
@testset "traversal" begin
    square=([[0, 0], [0, 1], [1, 0], [1, 1]], [[0, 1, 2, 3]])
    @everywhere structure=Struct([square])
    @everywhere dim=checkStruct(structure.body)
    @test length(traversal(eye(dim+1),[],structure,[]))==length(structure.body)
    @test typeof(traversal(eye(dim+1),[],structure,[]))==Array{Any,1}
end
```

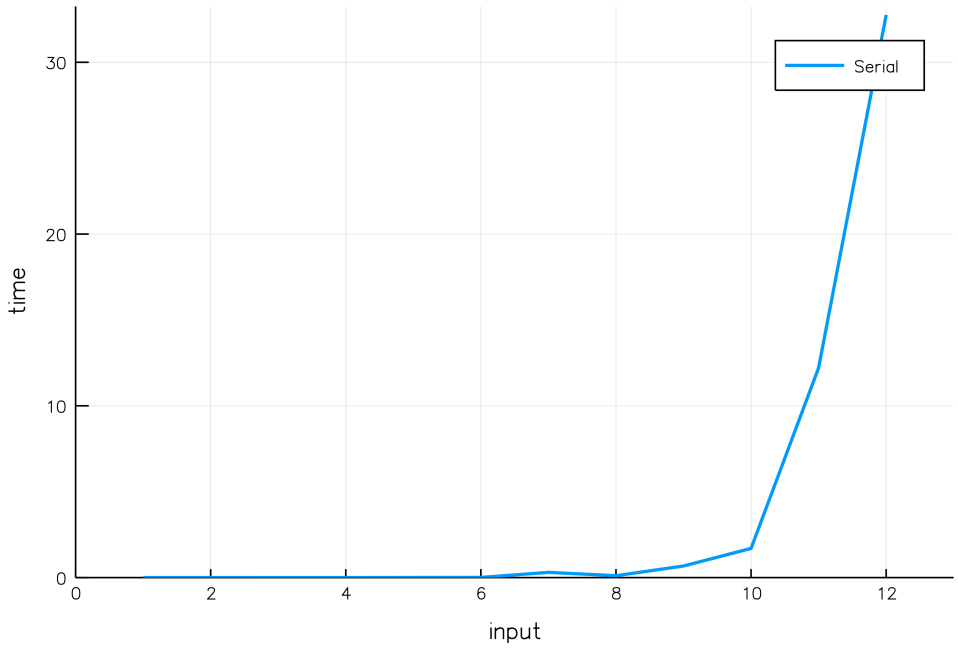
```
 square=([[0, 0], [0, 1], [1, 0], [1, 1]], [[0, 1, 2, 3]])
 structure=pStruct([square])
 dim=pcheckStruct(structure.body)
@testset "ptraversal" begin
  @test length(ptraversal(eye(dim+1),[],structure,[]))==length(structure.body)
  @test typeof(ptraversal(eye(dim+1),[],structure,[]))==Array{Any,1}
end
```

2.4.4 Results

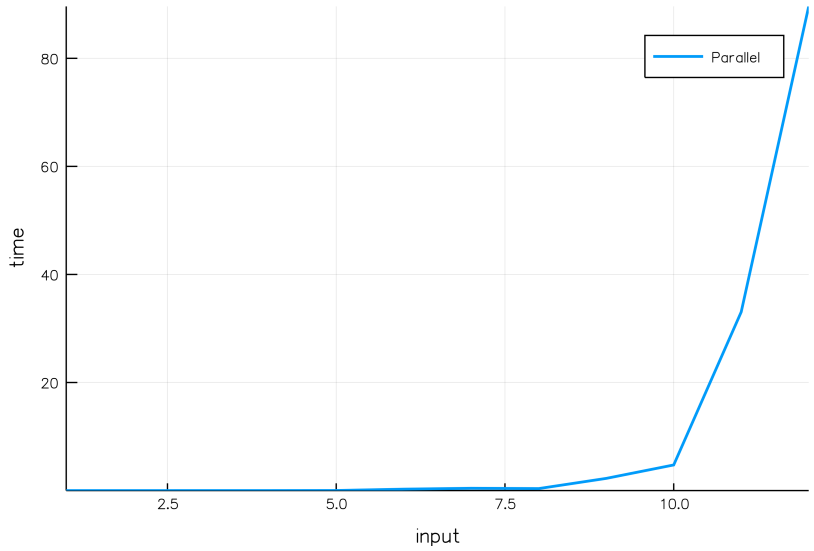
Tesla

```
input=[1,10,50,10^2,5*10^2,10^3,5*10^3,10^4,5*10^4,10^5,5*10^5,10^6]

function timeTraversal(model,input)
    t=Array{Float64}(length(input))
    pt=Array{Float64}(length(input))
```

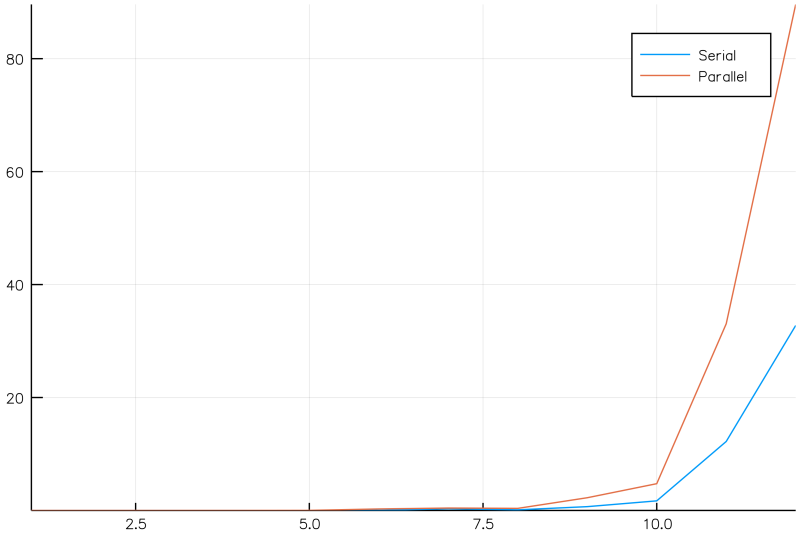
```
    for i in range(1,length(input))
        structo=addn2D(input[i],model)
        pstructo=pStruct(structo.body)
        dim=checkStruct(structo.body)
        traversal(eye(dim+1),[],structo,[])
        ptraversal(eye(dim+1),[],pstructo,[])
        pt[i]=@elapsed ptraversal(eye(dim+1),[],pstructo,[])
        t[i]=@elapsed traversal(eye(dim+1),[],structo,[])
    end
    return t,pt
end

y,yp=timeTraversal(square,input)
p=plot(y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```



```
pp=plot(yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```

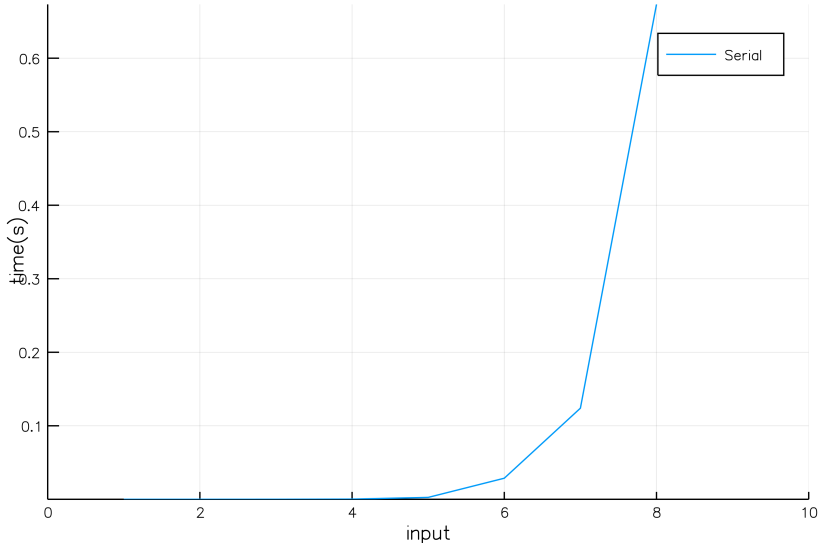



```
yc=[y,yp]
pc=plot(yc,label=["Serial" "Parallel"])
```

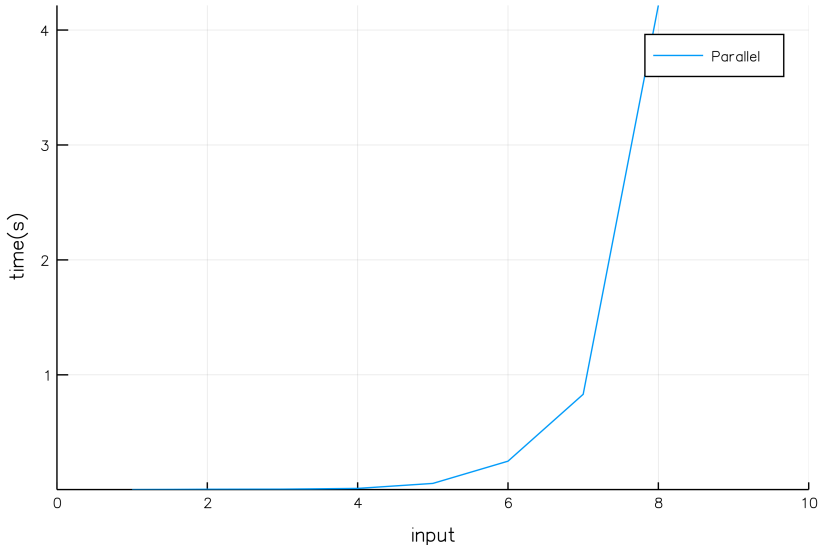


PC

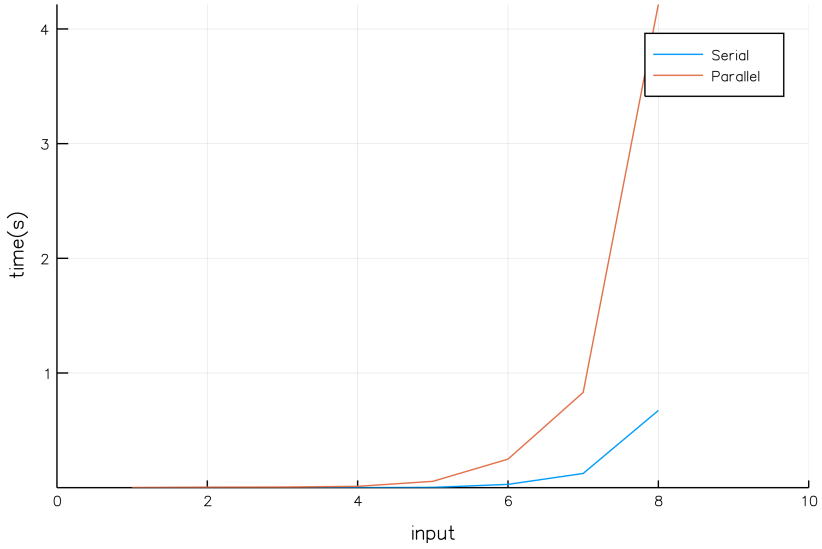
```
times=[ ]
ptimes=[ ]
input=[ ]
dim=[ ]
for i in range(1,length(l)-1)
    push!(input,Struct([repeat([l[i]],outer=i)...]))
end
for i in range(1,length(l)-1)
    append!(dim,checkStruct(input[i].body))
end
for i in range(1,length(input))
    args=(eye(dim[i]+1),[],input[i],[])
    append!(times,Time(traversal,(args...)))
end
for i in range(1,length(input))
    args=(eye(dim[i]+1),[],input[i],[])
    append!(ptimes,Time(ptraversal,(args...)))
end

plot(times,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Serial"])
```




```
plot(ptimes,xlabel="input",xlims=(0,length(times)+2),
    ylabel="time(s)",label=["Parallel"]
```




```
plot([times,ptimes],xlabel="input",xlims=(0,length(ptimes)+2),
    ylabel="time(s)",label=["Serial","Parallel"])
```



2.5 Struct

2.5.1 Conversion

Python

```
def evalStruct(struct):
    dim = checkStruct(struct.body)
    CTM, stack = scipy.identity(dim+1), [ ]
    scene = traversal(CTM, stack, struct, [ ])
    return scene

class Struct:
    def __init__(self,data=None,name=None,category=None):
        if data==None or data==[ ]:
            self.body = [ ]
        else:
            #self.body = [item for item in data if item != None]
            self.body = [item for item in data]
            self.box = box(self)
            self.dim = len(self.box[0])
        if name != None:
            self.name = str(name)
        else:
            self.name = str(id(self))
        if category != None:
            self.category = str(category)
        else:
            self.category = "feature"
        def __name__(self):
            return self.name
        def __category__(self):
            return self.category
        def __iter__(self):
            return iter(self.body)
        def __len__(self):
            return len(list(self.body))
        def __getitem__(self,i):
            return list(self.body)[i]
        def __setitem__(self,i,value):
            self.body[i] = value
        def __print__(self):
            return "<Struct name: %s>" % self.__name__()
        def __repr__(self):
            return "<Struct name: %s>" % self.__name__()
```

```
    def set_name(self,name):
        self.name = str(name)
    def clone(self,i=0):
        from copy import deepcopy
        newObj = deepcopy(self)
        if i != 0: newObj.name = self.name + "_" + str(i)
        return newObj
    def set_category(self,category):
        self.category = str(category)
```

Julia

```
function evalStruct(self)
    dim = checkStruct(self.body)
    CTM, stack = eye(dim+1), []
    scene = traversal(CTM, stack, self, [])
    return scene
end

type Struct
        body::Array
        box
        name::AbstractString
        dim
        category::AbstractString

        function Struct()
                self=new([],Nullable{Any},"new",Nullable{Any},"feature")
                self.name=string(object_id(self))
                return self
        end
        function Struct(data::Array)
                self=Struct()
                self.body=data
                self.box=box(self)
                self.dim=length(self.box[1])
                return self
        end
        function Struct(data::Array,name)
                self=Struct()
                self.body=[item for item in data]
                self.box=box(self)
                self.dim=length(self.box[1])
                self.name=string(name)
                return self
```

```
        end
        function Struct(data::Array,name,category)
                self=Struct()
                self.body=[item for item in data]
                self.box=box(self)
                self.dim=length(self.box[1])
                self.name=string(name)
                self.category=string(category)
                return self
        end
end
        function name(self::Struct)
                return self.name
        end
        function category(self::Struct)
                return self.category
        end
        function len(self::Struct)
                return length(self.body)
        end
        function getitem(self::Struct,i::Int)
                return self.body[i]
        end
        function setitem(self::Struct,i,value)
                self.body[i]=value
        end
        function pprint(self::Struct)
                return "<Struct name: $(self.__name__())"
        end
        function set_name(self::Struct,name)
                self.name=string(name)
        end
        function clone(self::Struct,i=0)
                newObj=deepcopy(self)
                if i!=0
                        newObj.name="$(self.__name__())_$(string(i))"
                end
                return newObj
        end
        function set_category(self::Struct,category)
                self.category=string(category)
        end
```

2.5.2 Parallelization

```
function pevalStruct(self)
    dim = pcheckStruct(self.body)
    CTM, stack = eye(dim+1), []
    scene = ptraversal(CTM, stack, self, [])
    return scene
end

@everywhere type pStruct
        body::Array
        box
        name::AbstractString
        dim
        category::AbstractString

        function pStruct()
                self=new([],Nullable{Any},"new",Nullable{Any},"feature")
                self.name=string(object_id(self))
                return self

        end
        function pStruct(data::Array)
                self=pStruct()
                self.body=data
                self.box=pbox(self)
                self.dim=length(self.box[1])
                return self
        end
        function pStruct(data::Array,name)
                self=pStruct()
                self.body=[item for item in data]
                self.box=pbox(self)
                self.dim=length(self.box[1])
                self.name=string(name)
                return self
        end
        function pStruct(data::Array,name,category)
                self=pStruct()
                self.body=[item for item in data]
                self.box=pbox(self)
                self.dim=length(self.box[1])
                self.name=string(name)
                self.category=string(category)
```

```
            return self
        end
end
        function name(self::pStruct)
            return self.name
        end
        function category(self::pStruct)
            return self.category
        end
        function len(self::pStruct)
            return length(self.body)
        end
        function getitem(self::pStruct,i::Int)
            return self.body[i]
        end
        function setitem(self::pStruct,i,value)
            self.body[i]=value
        end
        function pprint(self::pStruct)
            return "<Struct name: $(self.__name__())"
        end
        function set_name(self::pStruct,name)
            self.name=string(name)
        end
        function clone(self::pStruct,i=0)
            newObj=deepcopy(self)
            if i!=0
                newObj.name="$(self.__name__())_$(string(i))"
            end
            return newObj
        end
        function set_category(self::pStruct,category)
            self.category=string(category)
        end
```

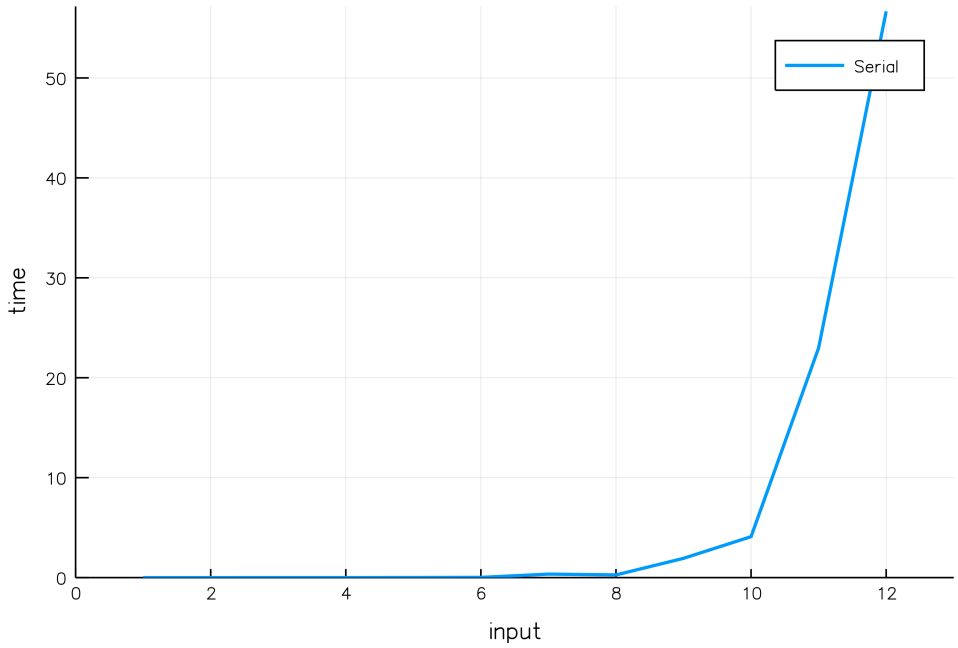
2.5.3 Unit-Test

```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])

@testset "Struct Tests" begin
    @test Struct([square]).body==[square]
    @test Struct([square]).dim==length(square[1][1])
    @test Struct([square]).box==[[0,0],[1,1]]
end
```

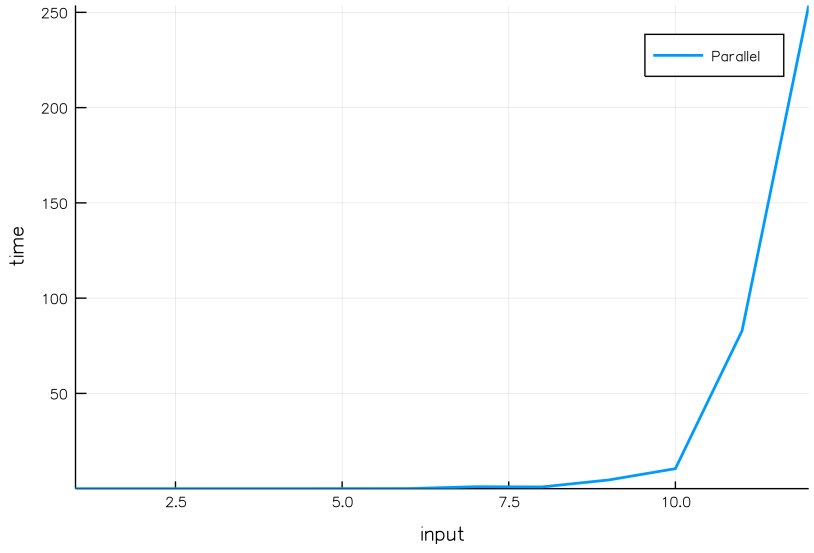
```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
@testset "pStruct Tests" begin
    @test pStruct([square]).body==[square]
        @test pStruct([square]).dim==length(square[1][1])
        @test pStruct([square]).box==[[0,0],[1,1]]
        @test pStruct([square]).category=="feature"
        @test pStruct([square],"quadrato").name=="quadrato"
    end
```

2.5.4 Results

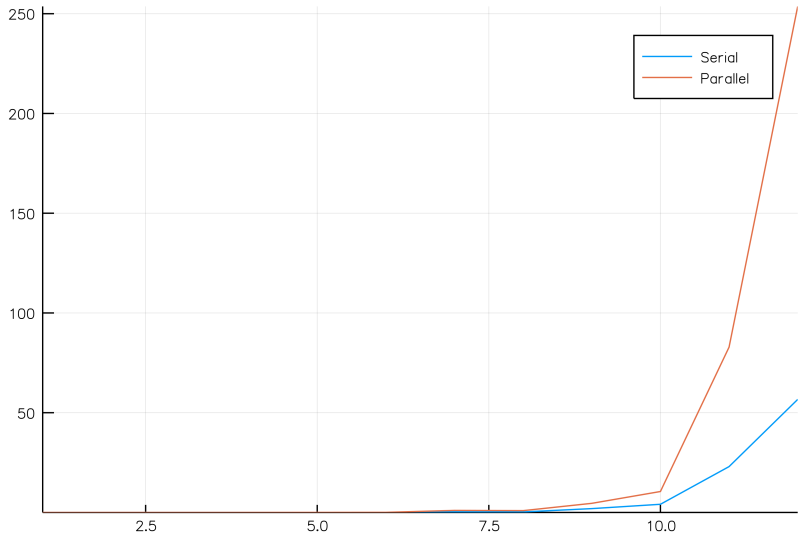
```
input=[1,10,50,10^2,5*10^2,10^3,5*10^3,10^4,5*10^4,10^5,5*10^5,10^6]
function timeStruct(model,input)
        t=Array{Float64}(length(input))
        pt=Array{Float64}(length(input))
        for i in range(1,length(input))
            structo=addn2D(input[i],model)
            Struct(structo.body)
            pStruct(structo.body)
            t[i]=@elapsed  Struct(structo.body)
            pt[i]=@elapsed pStruct(structo.body)
        end
        return t,pt
end

y,yp=timeStruct(square,input)
p=plot(y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
       ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```




```
pp=plot(yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```



```
yc=[y,yp]
pc=plot(yc,label=["Serial" "Parallel"])
```



2.6 embedTraversal

2.6.1 Conversion

Python

```
def embedTraversal(cloned, obj,n,suffix):
    for i in range(len(obj)):
        if isinstance(obj[i],Model):
            cloned.body += [obj[i]]
        elif (isinstance(obj[i],tuple) or isinstance(obj[i],list)) and
              (len(obj[i])==2):
              V,EV = obj[i]
              V = [v+n*[0.0] for v in V]
              cloned.body  += [(V,EV)]
        elif (isinstance(obj[i],tuple) or isinstance(obj[i],list)) and
                (len(obj[i])==3):
                V,FV,EV = obj[i]
                V = [v+n*[0.0] for v in V]
                cloned.body  += [(V,FV,EV)]
            elif isinstance(obj[i],Mat):
                mat = obj[i]
                d,d = mat.shape
                newMat = scipy.identity(d+n*1)
                for h in range(d-1):
                    for k in range(d-1):
                        newMat[h,k] = mat[h,k]
                    newMat[h,d-1+n*1] = mat[h,d-1]
                cloned.body  +=  [newMat.view(Mat)]
            elif isinstance(obj[i],Struct):
                newObj = Struct()
                newObj.box = hstack((obj[i].box, [n*[0],n*[0]]))
                newObj.name = obj[i].name+suffix
                newObj.category = obj[i].category
                cloned.body +=[embedTraversal(newObj, obj[i], n, suffix)]
            return cloned
```

Julia

```
function embedTraversal(cloned,obj,n,suffix)
    for i in range(1,len(obj))
        if isa(obj.body[i],Matrix)
            mat=obj.body[i]
            d,d=size(mat)
            newMat=eye(d+n*1)
            for h in range(1,d-1)
```

```
            for k in range(1,d-1)
                newMat[h,k]=mat[h,k]
            end
            newMat[h,d-1+n*1]=mat[h,d-1]
        end
        append!(cloned.body,newMat)
    elseif (isa(obj.body[i],Tuple) ||isa(obj.body[i],Array))&&
            length(obj.body[i])==3
            V,FV,EV=obj.body[i]
            dimadd=fill([0.0],n)
            for k in dimadd
                for v in V
                    append!(v,k)
                end
            end
            append!(cloned.body,[(V,FV,EV)])
        elseif (isa(obj.body[i],Tuple) ||isa(obj.body[i],Array))&&
                length(obj.body[i])==2
                V,EV=deepcopy(obj.body[i])
                dimadd=fill([0.0],n)
                for k in dimadd
                    for v in V
                        append!(v,k)
                    end
                end
                append!(cloned.body,[(V,EV)])
            elseif isa(obj.body[i],Struct)
                newObj=Struct()
                newObj.box=hcat((obj.body[i].box,[fill([0],n),fill([0],n)]))
                newObj.category=obj.body[i].category
                append!(cloned.body,embedTraversal(newObj,obj.body[i],n,suffix))
            end
        end
        return cloned
    end
```

2.6.2 Parallelization

```
@everywhere function pembedTraversal(cloned,obj,n,suffix)
    for i in range(1,len(obj))
        if (isa(obj.body[i],Matrix) || isa(obj.body[i],SharedArray))
            mat=obj.body[i]
            d,d=size(mat)
```

```
    newMat=eye(d+n*1)
    @sync begin
        for h in range(1,d-1)
            @async begin
                for k in range(1,d-1)
                    newMat[h,k]=mat[h,k]
                end
            end
            newMat[h,d-1+n*1]=mat[h,d-1]
        end
    end
    append!(cloned.body,newMat)
elseif (isa(obj.body[i],Tuple) ||isa(obj.body[i],Array))&&
        length(obj.body[i])==2
    V,EV=deepcopy(obj.body[i])
    dimadd=fill([0.0],n)
    @sync begin
        for k in dimadd
            @async begin
                for v in V
                    append!(v,k)
                end
            end
        end
    end
    append!(cloned.body,[(V,EV)])
elseif (isa(obj.body[i],Tuple) ||isa(obj.body[i],Array))&&
        length(obj.body[i])==3
    V,FV,EV=obj.body[i]
    dimadd=fill([0.0],n)
    @sync begin
        for k in dimadd
            @async begin
                for v in V
                    append!(v,k)
                end
            end
        end
    end
    append!(cloned.body,[(V,FV,EV)])
elseif isa(obj.body[i],pStruct)
    newObj=pStruct()
    @async begin
```

```
                newObj.box=hcat((obj.body[i].box,[fill([0],n),fill([0],n)]))
                newObj.category=obj.body[i].category
                append!(cloned.body,embedTraversal(newObj,obj.body[i],n,suffix))
            end
        end
    end
    return cloned
end
```

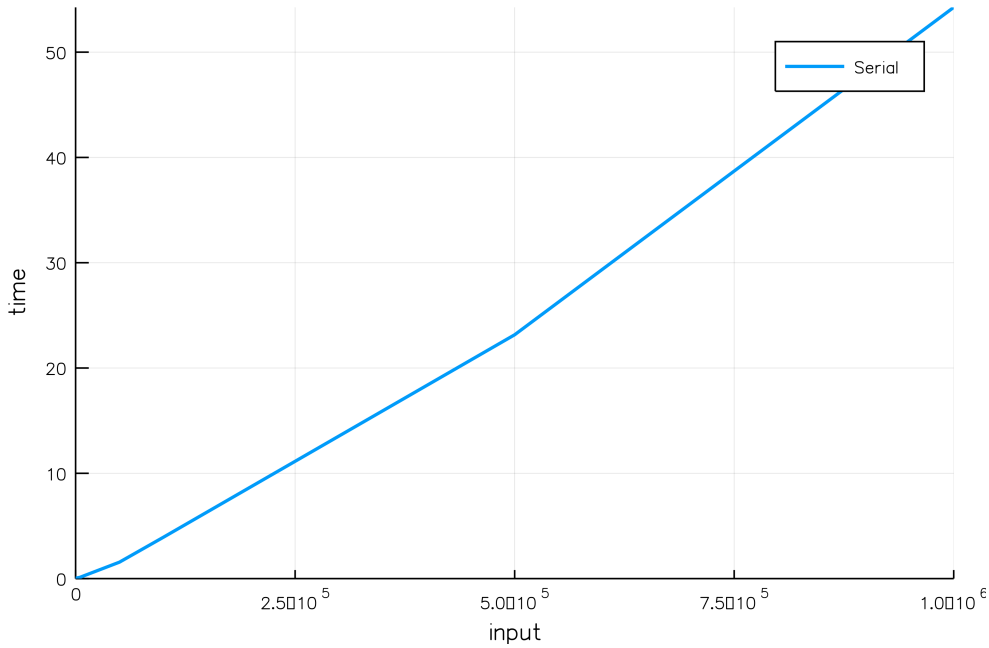
2.6.3 Unit-Test

```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
x=Struct([square])

@testset "embedTraversal Tests" begin
    @test length(embedTraversal(deepcopy(x),deepcopy(x),1,"New").body[2][1][1])
          ==length(x.body[1][1][1])+1
    #in this case n=1, but generally:
    # length(length(embedTraversal(x,x,1,"New")=length(x.body[1][1][1])+n
    @test length(embedTraversal(deepcopy(x),deepcopy(x),3,"New").body[2][1][1])
          ==length(x.body[1][1][1])+3
    @test typeof(embedTraversal(deepcopy(x),deepcopy(x),1,"New"))==Struct
end

square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
x=pStruct([square])

@testset "pembedTraversal Tests" begin
    @test length(pembedTraversal(deepcopy(x),deepcopy(x),1,"New").body[2][1][1])
          ==length(x.body[1][1][1])+1
    #in this case n=1, but generally:
    # length(length(embedTraversal(x,x,1,"New")=length(x.body[1][1][1])+n
    @test length(pembedTraversal(deepcopy(x),deepcopy(x),3,"New").body[2][1][1])==
          length(x.body[1][1][1])+3
    @test typeof(pembedTraversal(deepcopy(x),deepcopy(x),1,"New"))==pStruct
end
```

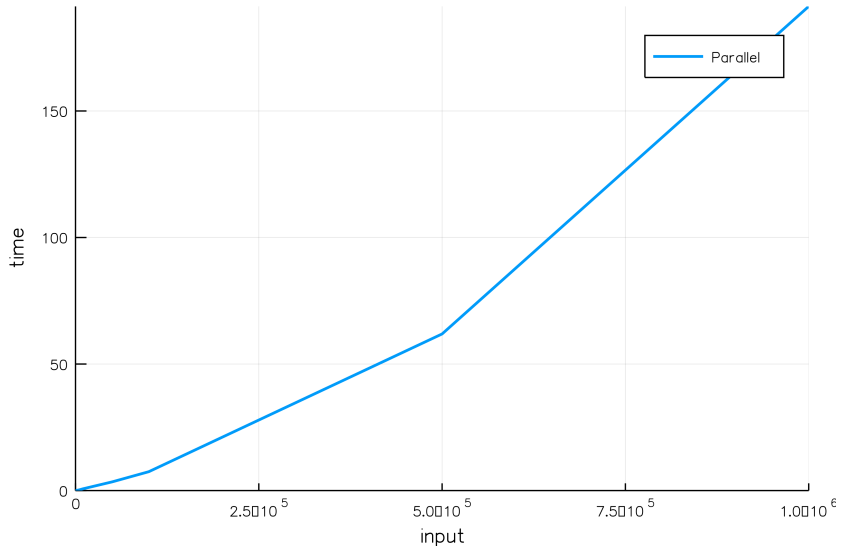
2.6.4 Results

```
input=[1,10,50,10^2,5*10^2,10^3,5*10^3,10^4,5*10^4,10^5,5*10^5,10^6]
function timeEmbedTraversal(model,input)
        t=Array{Float64}(length(input))
        pt=Array{Float64}(length(input))
        for i in range(1,length(input))
```

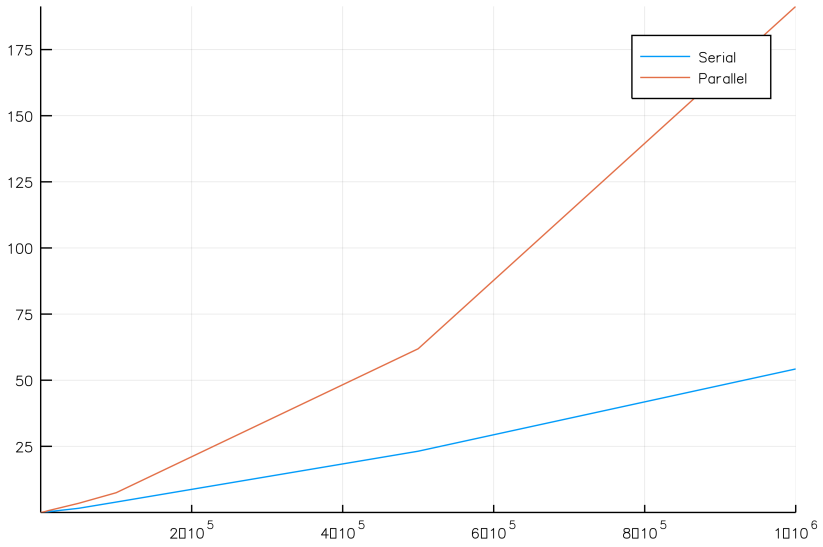
```
            structo=addn2D(input[i],model)
            pstructo=pStruct(structo.body)
            cloned=Struct()
            cloned.box=hcat((structo.box,[fill([0],10),fill([0],10)]))
            cloned.name=string(object_id(cloned))
            cloned.category=structo.category
            cloned.dim=structo.dim+10
            pcloned=pStruct()
            pcloned.box=hcat((pstructo.box,[fill([0],10),fill([0],10)]))
            pcloned.name=string(object_id(pcloned))
            pcloned.category=pstructo.category
            pcloned.dim=pstructo.dim+10
            embedTraversal(cloned,structo,10,"New")
            pembedTraversal(pcloned,pstructo,10,"New")
            t[i]=@elapsed embedTraversal(cloned,structo,10,"New")
            pt[i]=@elapsed pembedTraversal(pcloned,pstructo,10,"New")
        end
        return t,pt
end
y,yp=timeEmbedTraversal(square,input)
p=plot(input,y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```



```
pp=plot(input,yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```




```
yc=[y,yp]
pc=plot(input,yc,label=["Serial" "Parallel"])
```



2.7 embedStruct

2.7.1 Conversion

Python

```
def embedStruct(n):
    def embedStruct0(struct,suffix="New"):
        if n==0:
            return struct, len(struct.box[0])
        cloned = Struct()
        cloned.box = hstack((struct.box, [n*[0],n*[0]])).tolist()
        cloned.name = str(id(cloned))  #struct.name+suffix
        cloned.category = struct.category
        cloned.dim = struct.dim + n
        cloned = embedTraversal(cloned,struct,n,suffix)
        return cloned
    return embedStruct0
```

Julia

```
function embedStruct(n)
    function embedStruct0(self,suffix="New")
        if n==0
            return self, length(self.box[1])
        end
        cloned=Struct()
        cloned.box=hcat((self.box,[fill([0],n),fill([0],n)]))
        cloned.name=string(object_id(cloned))
        cloned.category=self.category
        cloned.dim=self.dim+n
        cloned=embedTraversal(cloned,self,n,suffix)
        return cloned
    end
    return embedStruct0
end
```

2.7.2 Parallelization

```
@everywhere function pembedStruct(n)
    function pembedStruct0(self,suffix="New")
        if n==0
            return self, length(self.box[1])
        end
        cloned=pStruct()
```

```
        cloned.box=hcat((self.box,[fill([0],n),fill([0],n)]))
        cloned.name=string(object_id(cloned))
        cloned.category=self.category
        cloned.dim=self.dim+n
        cloned=pembedTraversal(cloned,self,n,suffix)
        return cloned
    end
    return pembedStruct0
end
```

2.7.3 Unit-Test

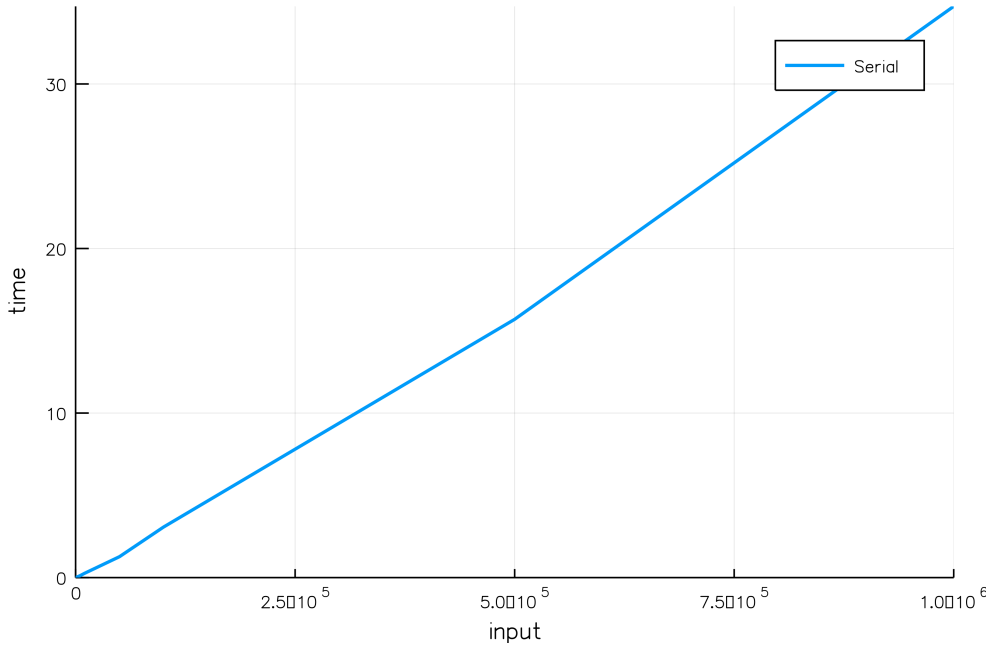
```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
x=Struct([square])
@testset "embedStruct Tests" begin
    @test length(embedStruct(1)(x).body[1][1][1])==length(x.body[1][1][1])+1
    #in this case n = 1, but generally:
    #length(embedStruct(n)(x).body[1][1][1])=length(x.body[1][1][1])+n
    @test length(embedStruct(3)(x).body[1][1][1])==length(x.body[1][1][1])+3
    @test typeof(embedStruct(1)(x))==Struct
end
```

```
square=([[0,0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
x=pStruct([square])
@testset "pembedStruct Tests" begin
    @test length(pembedStruct(1)(x).body[1][1][1])==length(x.body[1][1][1])+1
    #in this case n = 1, but generally:
    #length(embedStruct(n)(x).body[1][1][1])=length(x.body[1][1][1])+n
    @test length(pembedStruct(3)(x).body[1][1][1])==length(x.body[1][1][1])+3
    @test typeof(pembedStruct(1)(x))==pStruct
end
```

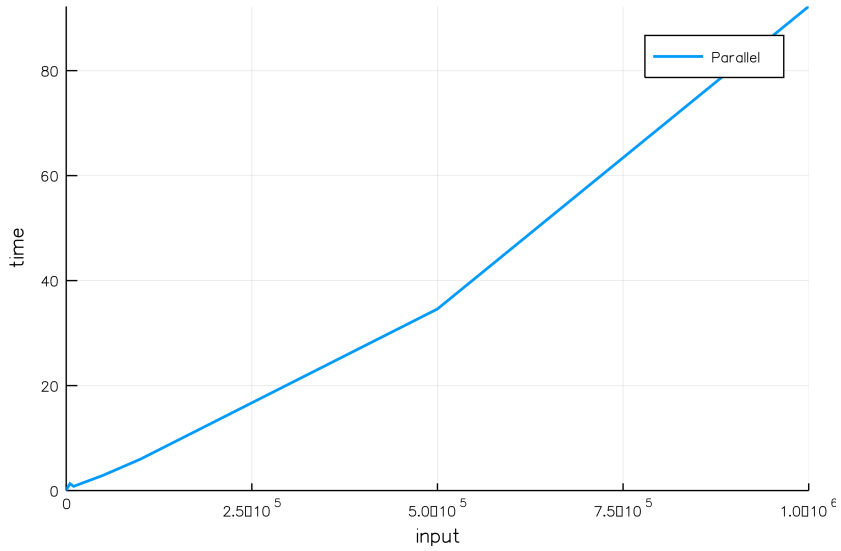
2.7.4 Results

```
function timeEmbedStruct(n,model,input)
        t=Array{Float64}(length(input))
        pt=Array{Float64}(length(input))
        for i in range(1,length(input))
            structo=addn2D(input[i],model)
             pstructo=pStruct(structo.body)
            embedStruct(n)(structo)
            pembedStruct(n)(pstructo)
             t[i]=@elapsed embedStruct(n)(structo)
            pt[i]=@elapsed pembedStruct(n)(pstructo)
```

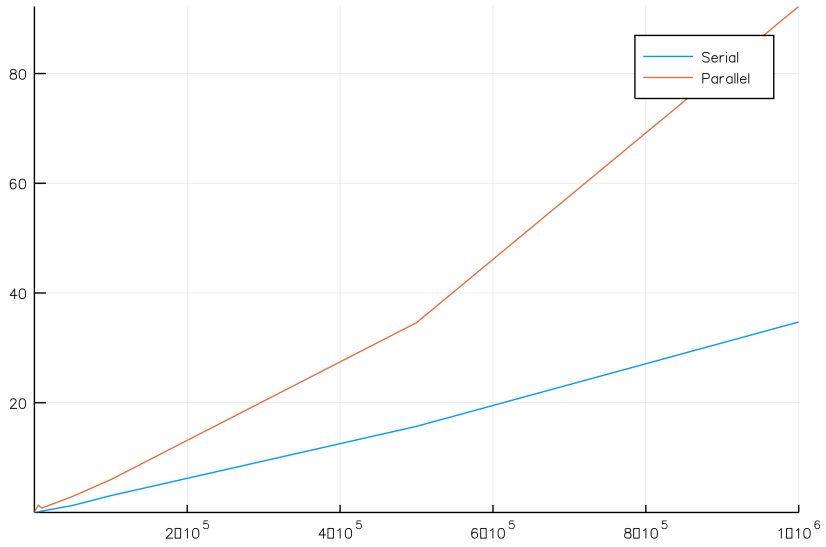
```
        end
        return t,pt
end

y,yp=timeEmbedStruct(10,square,input)
p=plot(input,y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```



```
pp=plot(input,yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
        ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```




```
yc=[y,yp]
pc=plot(input,yc,label=["Serial" "Parallel"])
```



2.8 removeDups

2.8.1 Conversion

Python

```
def removeDups (CW):
    CW = list(set(AA(tuple)(CW)))
    CWs = list(set(AA(tuple)  (AA(sorted)(CW))  ))
    no_duplicates = defaultdict(list)
    for f in CWs: no_duplicates[f] = [ ]
    for f in CW:
        no_duplicates[tuple(sorted(f))] += [f]
    CW = [f[0] for f in no_duplicates.values()]
    return CW
```

Julia

```
function removeDups(CW)
    CW=collect(Set(CW))
    CWs=collect(map(sort,CW))
    no_duplicates=Dict()
    for f in CWs
        no_duplicates[f] = [ ]
    end
    for f in CW
        no_duplicates[sort(f)]=[f]
    end
    CW=[f[1] for f in values(no_duplicates)]
    return CW
end
```

2.8.2 Parallelization

```
function premoveDups(CW)
    CW=collect(Set(CW))
    CWs=collect(@sync pmap(sort,CW))
    no_duplicates=Dict()
    @parallel for f in CWs
        no_duplicates[f] = [ ]
    end
    @parallel for f in CW
        no_duplicates[sort(f)]=[f]
    end
    @parallel for f in values(no_duplicates)
```

```
            append!(CW, f[1])
        end
    return CW
end
```

2.8.3 Unit-Test

```
CW1=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],[1,3,5,7],
      [4,5,6,7],[8,9,10,11],[4,5,8,9],[6,7,10,11],[4,6,8,10],[5,7,9,11]]
CW2=[[0,1,2,3],[4,5,6,7],[8,9,10,11],[12,13,14,15],[16,17,18,19]]

@testset "removeDups Tests" begin
    @testset "removeDups 3D" begin
        @test length(removeDups(CW1))<= length(CW1)
        @test typeof(removeDups(CW1))==Array{Array{Int64,1},1}
    end
    @testset "removeDups 2D" begin
        @test length(removeDups(CW2))<= length(CW2)
        @test typeof(removeDups(CW2))==Array{Array{Int64,1},1}
    end
end

CW1=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],[1,3,5,7],
      [4,5,6,7],[8,9,10,11],[4,5,8,9],[6,7,10,11],[4,6,8,10],[5,7,9,11]]
CW2=[[0,1,2,3],[4,5,6,7],[8,9,10,11],[12,13,14,15],[16,17,18,19]]

@testset "premoveDups Tests" begin
    @testset "premoveDups 3D" begin
        @test length(premoveDups(CW1))<= length(CW1)
        @test typeof(premoveDups(CW1))==Array{Array{Int64,1},1}
    end
    @testset "premoveDups 2D" begin
        @test length(premoveDups(CW2))<= length(CW2)
        @test typeof(premoveDups(CW2))==Array{Array{Int64,1},1}
     end
end
```

2.9 struct2lar

2.9.1 Conversion

Python

```
def prepKey (args): return "["+",␣".join(args)+"]"

def fixedPrec(PRECISION):
    def fixedPrec0(value):
        out = round(value*10**(PRECISION))/10**(PRECISION)
        if out == -0.0: out = 0.0
        return str(out)
    return fixedPrec0

def vcode (PRECISION=4):
    def vcode0 (vect):
        return prepKey(AA(fixedPrec(PRECISION))(vect))
    return vcode0
```

This is not the original function of the larlib module. The original function did not run and this is the modified function that runs:

```
def struct2lar(structure,metric=ID):
    listOfModels = evalStruct(structure)
    vertDict = dict()
    index,defaultValue,CW,W,FW = -1,-1,[],[],[]
    for model in listOfModels:
        if isinstance(model,Model):
            V= model.verts
            FV=model.cells
        elif (isinstance(model,tuple) or isinstance(model,list)):
            if len(model)==2:
                print(len(model))
                V,FV=model
                dim=len(model)
            elif len(model)==3:
                V,FV,EV,dim=model,len(model)
                for k,incell in enumerate(EV):
                    outcell = []
                    for v in incell:
                        key = vcode(4)(A[v])
                        if vertDict.get(key,defaultValue) == defaultValue:
                            index += 1
                            vertDict[key] = index
                            outcell += [index]
```

```
                W+= [eval(key)]
            else:
                outcell += [vertDict[key]]
        FW+= [outcell]
    for k,incell in enumerate(FV):
        outcell = []
        for v in incell:
            key = vcode(4)(V[v])
            if vertDict.get(key,defaultValue) == defaultValue:
                index += 1
                vertDict[key] = index
                outcell += [index]
                W+= [eval(key)]
            else:
                outcell += [vertDict[key]]
            CW+= [outcell]
    if ((isinstance(model,tuple) or isinstance(model,list))
            and len(model)==2) or ((isinstance(model,Model)
                    and model.n==2)):
        if len(CW[0])==2:
            CW = list(set(AA(tuple)(AA(sorted)(CW))))
        else: CW = removeDups(CW)
        return metric(W),CW
    if ((isinstance(model,tuple) or isinstance(model,list))
            and len(model)==3):
        FW = list(set(AA(tuple)(AA(sorted)(FW))))
        CW = removeDups(CW)
        return metric(W),CW,FW
```

Julia

```
function fixedPrec(PRECISION)
    function fixedPrec0(value)
        out=round.(value,PRECISION)
        if out==-0.0
            out=0.0
        end
        return string(out)
    end
    return fixedPrec0
end

function vcode(PRECISION=4)
    function vcode0(vect)
```

```
        return fixedPrec(PRECISION)(vect)
    end
    return vcode0
end

function struct2lar(structure)
    listOfModels=evalStruct(structure)
    vertDict= Dict()
    index,defaultValue,CW,W,FW = -1,-1,[],[],[]
    for model in listOfModels
        if  length(model)==2
            V,FV=model
        elseif lenght(model)==3
            V,FV,EV=model
        end
        for (k,incell) in enumerate(FV)
            outcell=[]
            for v in incell
                key=vcode(4)(V[v+1])
                if get(vertDict,key,defaultValue)==defaultValue
                    index =index+1
                    vertDict[key]=index
                    append!(outcell,index)
                    append!(W,[eval(parse(key))])
                else
                    append!(outcell,vertDict[key])
                end
            end
            append!(CW,[outcell])
        end
        if length(model)==3
            for (k,incell) in enumerate(FV)
                outcell=[]
                for v in incell
                    key=vcode(4)(V[v+1])
                    if get(vertDict,key,defaultValue)==defaultValue
                        index =index+1
                        vertDict[key]=index
                        append!(outcell,[index])
                        append!(W,[eval(parse(key))])
                    else
                        append!(outcell,vertDict[key])
                    end
```

```
                end
                append!(FW,[outcell])
            end
        end
    end
    if length(listOfModels[end])==2
        if length(CW[1])==2
            CW=map(Tuple,map(sort,CW))
        else
            CW=removeDups(CW)
        end
        return W,CW
    end
    if length(listOfModels[end])==3
        FW=map(Tuple,map(sort,FW))
        CW=removeDups(CW)
        return W,CW,FW
    end
end
```

2.9.2 Parallelization

```
@everywhere function pfixedPrec(PRECISION)
    function pfixedPrec0(value)
        out=round.(value,PRECISION)
        if out==-0.0
            out=0.0
        end
        return string(out)
    end
    return pfixedPrec0
end

function pvcode(PRECISION=4)
    function pvcode0(vect)
        return pfixedPrec(PRECISION)(vect)
    end
    return pvcode0
end

@everywhere function pstruct2lar(structure)
    listOfModels=pevalStruct(structure)
    vertDict= Dict()
```

```
index,defaultValue,CW,W,FW = -1,-1,[ ],[ ],[ ]
for model in listOfModels
    if  length(model)==2
        V,FV=model
    elseif lenght(model)==3
        V,FV,EV=model
    end
    @sync begin
        for (k,incell) in enumerate(FV)
            outcell=[ ]
            @async begin
                for v in incell
                    key=pvcode(4)(V[v+1])
                    if get(vertDict,key,defaultValue)==defaultValue
                        index =index+1
                        vertDict[key]=index
                        append!(outcell,index)
                        append!(W,[eval(parse(key))])
                    else
                        append!(outcell,vertDict[key])
                    end
                end
            end
            append!(CW,[outcell])
        end
    end
    if length(model)==3
        @sync begin
            for (k,incell) in enumerate(FV)
                outcell=[ ]
                @async begin
                    for v in incell
                        key=pvcode(4)(V[v+1])
                        if get(vertDict,key,defaultValue)==defaultValue
                            index =index+1
                            vertDict[key]=index
                            append!(outcell,[index])
                            append!(W,[eval(parse(key))])
                        else
                            append!(outcell,vertDict[key])
                        end
                    end
                end
```

```
                append!(FW,[outcell])
            end
        end
    end
    if length(listOfModels[end])==2
        if length(CW[1])==2
            CW=pmap(Tuple,pmap(sort,CW))
        else
            CW=premoveDups(CW)
        end
        return W,CW
    end
    if length(listOfModels[end])==3
        FW=pmap(Tuple,pmap(sort,FW))
        CW=premoveDups(CW)
        return W,CW,FW
    end
end
```

2.9.3 Unit-Test

```
@testset "struct2lar" begin
    @testset "struct2lar 2D" begin
        square=([[0, 0],[0,1],[1,0],[1,1]],[[0,1,2,3]])
        table=larApply(t(-0.5,-0.5))(square)
        structure=Struct([repeat([table,r(pi/2)],outer=2)...])
        @test typeof(struct2lar(structure))==Tuple{Array{Any,1},
        Array{Array{Any,1},1}}
        @test length(struct2lar(structure)[1][1])==2
    end
    @testset "struct2lar 3D" begin
         BV=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],[1,3,5,7]]
        V=[[0 0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],
            [1,0,1],[1,1,0],[1,1,1]]
        block=[V,BV]
        structure=Struct(repeat([block,t(1,0,0)],outer=2));
        @test typeof(struct2lar(structure))==Tuple{Array{Any,1},
        Array{Array{Any,1},1}}
        @test length(struct2lar(structure)[1][1])==3
     end
end

@testset "pstruct2lar" begin
```

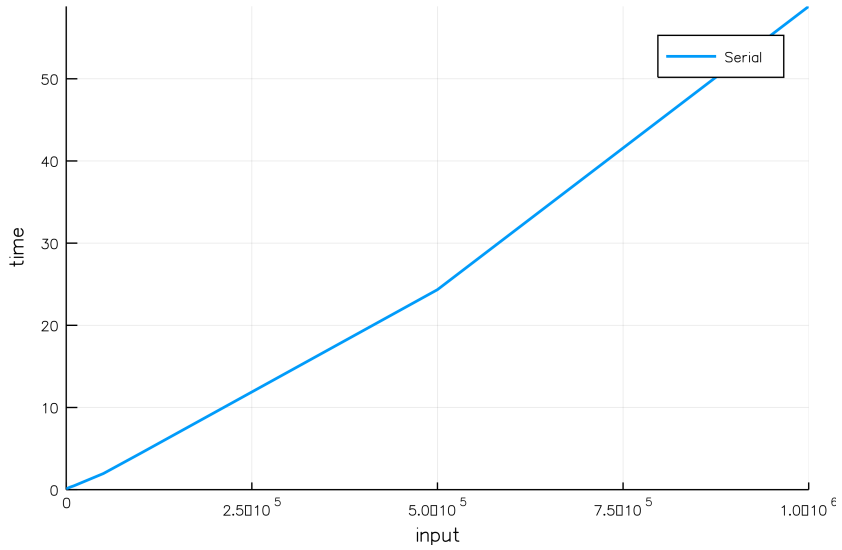
```
    @testset "pstruct2lar 2D" begin
        square=([[0, 0], [0, 1], [1, 0], [1, 1]], [[0, 1, 2, 3]])
        table=plarApply(t(-0.5,-0.5))(square)
        structure=pStruct([repeat([table,r(pi/2)],outer=2)...])
        @test typeof(pstruct2lar(structure))==Tuple{Array{Any,1},Array{Any,1}}
        @test length(pstruct2lar(structure)[1][1])==2
    end
    @testset "pstruct2lar 3D" begin
         BV=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],[0,2,4,6],[1,3,5,7]]
        V=[[0  0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],
            [1,0,1],[1,1,0],[1,1,1]]
        block=[V,BV]
        structure=pStruct(repeat([block,t(1,0,0)],outer=2));
        @test typeof(pstruct2lar(structure))==Tuple{Array{Any,1},Array{Any,1}}
        @test length(pstruct2lar(structure)[1][1])==3
    end
end
```

2.9.4 Results

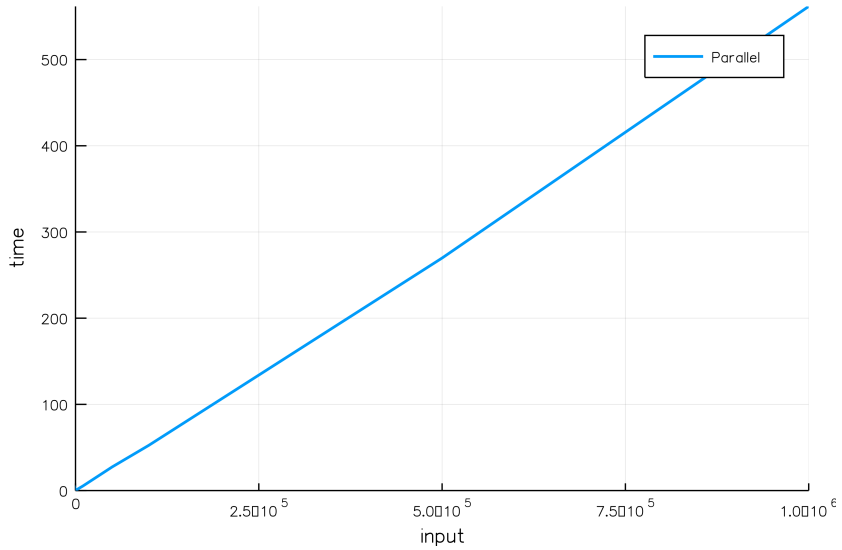
Tesla

```
input=[1,10,50,10^2,5*10^2,10^3,5*10^3,10^4,5*10^4,10^5,5*10^5,10^6]
function timeFstruct(f::Function,pf::Function,model,input)
    t=Array{Float64}(length(input))
    pt=Array{Float64}(length(input))
    for i in range(1,length(input))
        structo=addn2D(input[i],model)
        pstructo=pStruct(structo.body)
        f(structo)
        pf(pstructo)
        t[i]=@elapsed f(structo)
        pt[i]=@elapsed pf(pstructo)
    end
    return t,pt
end
```

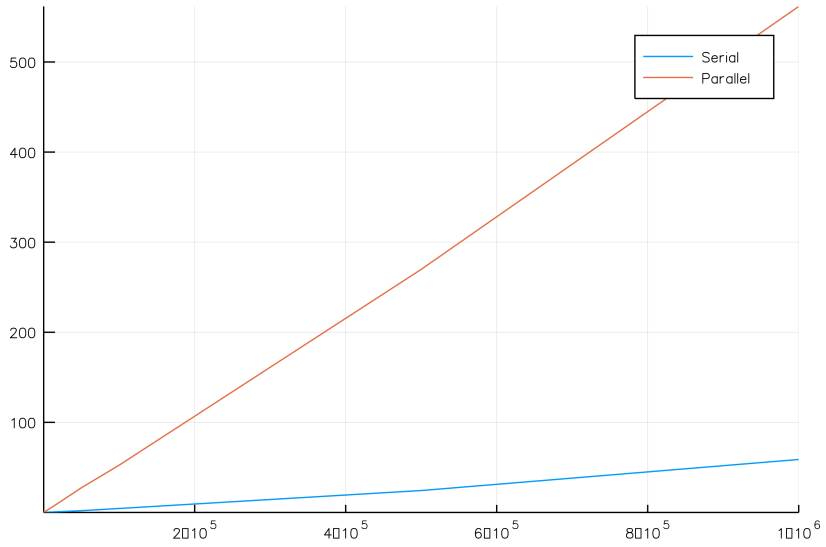
```
y,yp=timeFstruct(struct2lar,pstruct2lar,square,input)
p=plot(input,y,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
     ylims=(0,maximum(y)+0.5), label=[''Serial''],lw=2)
```




```
pp=plot(input,yp,xaxis=''input'',yaxis=''time'',xlims=(0,length(input)+1),
       ylims=(0,maximum(y)+0.5),label=[''Parallel''],lw=2)
```

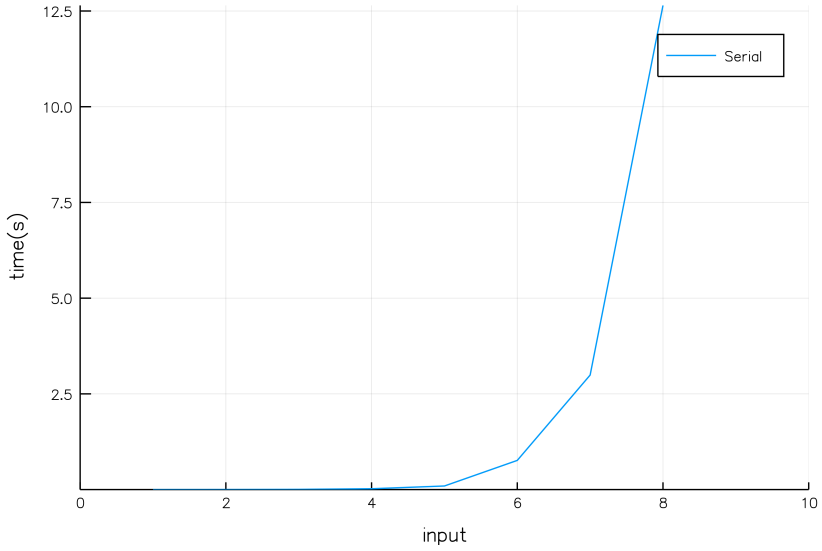


```
yc=[y,yp]
pc=plot(input,yc,label=["Serial" "Parallel"])
```

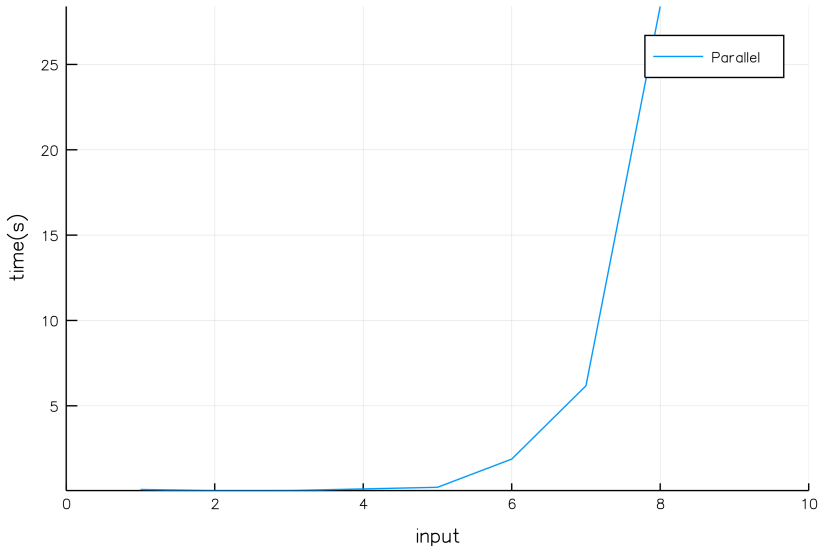


PC

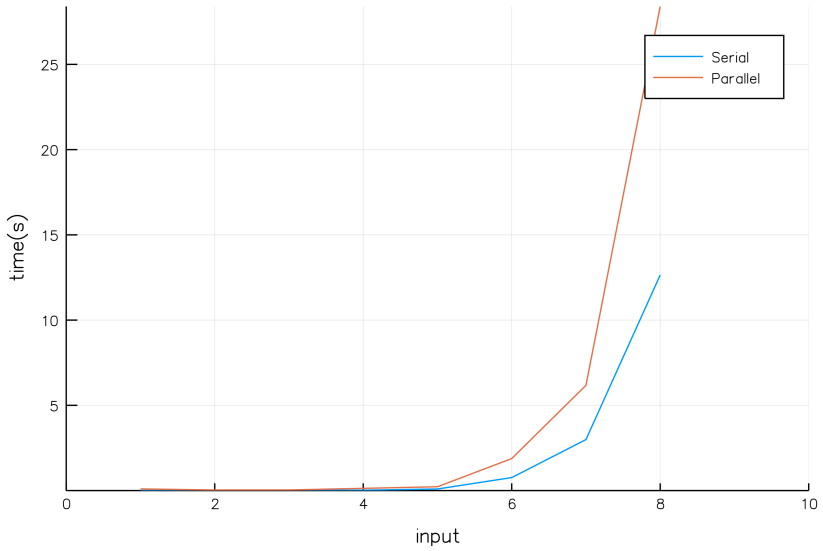
```
times=[ ]
ptimes=[ ]
input=[ ]
for i in range(1,length(l)-1)
    push!(input,Struct([repeat([l[i]],outer=i)...]))
end
for i in range(1,length(input))
    append!(times,Time(struct2lar,input[i]))
end
for i in range(1,length(input))
    append!(ptimes,Time(pstruct2lar,input[i]))
end

plot(times,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Serial"])
```




```
plot(ptimes,xlabel="input",xlims=(0,length(times)+2),
        ylabel="time(s)",label=["Parallel"])
```



```
plot([times,ptimes],xlabel="input",xlims=(0,length(times)+2),
    ylabel="time(s)",label=["Serial","Parallel"])
```



2.10 larRemoveVertices

2.10.1 Conversion

Python

```
def larRemoveVertices(V,FV):
    vertDict = dict()
    index,defaultValue,FW,W = -1,-1,[],[]
    for k,incell in enumerate(FV):
        outcell = []
        for v in incell:
            key = vcode(4)(V[v])
            if vertDict.get(key,defaultValue) == defaultValue:
                index += 1
                vertDict[key] = index
                outcell += [index]
                W+= [eval(key)]
            else:
                outcell += [vertDict[key]]
        FW+= [outcell]
    return W,FW
```

Julia

```
function larRemoveVertices(V,FV)
    vertDict= Dict()
    index,defaultValue,CW,W,FW = -1,-1,[],[],[]
    for (k,incell) in enumerate(FV)
        outcell=[]
        for v in incell
            key=vcode(4)(V[v+1])
            if get(vertDict,key,defaultValue)==defaultValue
                index =index+1
                vertDict[key]=index
                append!(outcell,index)
                append!(W,[eval(parse(key))])
            else
                append!(outcell,vertDict[key])
            end
        end
        append!(FW,[outcell])
    end
    return W,FW
end
```

2.10.2 Parallelization

```
@everywhere function plarRemoveVertices(V,FV)
    vertDict= Dict()
    index,defaultValue,CW,W,FW = -1,-1,[],[],[]
    @async begin
        for (k,incell) in enumerate(FV)
            outcell=[]
            @sync begin
                for v in incell
                    key=pvcode(4)(V[v+1])
                    if get(vertDict,key,defaultValue)==defaultValue
                        index =index+1
                        vertDict[key]=index
                        append!(outcell,index)
                        append!(W,[eval(parse(key))])
                    else
                        append!(outcell,vertDict[key])
                    end
                end
            end
            append!(FW,[outcell])
        end
    end
    return W,FW
end
```

2.10.3 Unit-Test

```
V=[[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],
    [1, 1, 0], [1, 1, 1]]
FV=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],
[0,2,4,6],[1,3,5,7]]

@testset "larRemoveVertices Tests" begin
    @test typeof(larRemoveVertices(V,FV))==Tuple{Array{Any,1},Array{Any,1}}
    @test length(larRemoveVertices(V,FV)[1])<= length(V)
end

V=[[0,0,0],[0,0,1],[0,1,0],[0,1,1],[1,0,0],[1,0,1],
    [1, 1, 0], [1, 1, 1]]
FV=[[0,1,2,3],[4,5,6,7],[0,1,4,5],[2,3,6,7],
[0,2,4,6],[1,3,5,7]]
```

```
@testset "plarRemoveVertices Tests" begin
        @test typeof(plarRemoveVertices(V,FV))==Tuple{Array{Any,1},Array{Any,1}}
        @test length(plarRemoveVertices(V,FV)[1])<= length(V)
end
```

3 Conclusion

It can be seen from the results of the graphs that parallelization generally slows down execution times. The execution times also depend on how the input is increased, in fact the execution times calculated on Tesla also increase for the sequential. Performing a parallelization on native language objects such as Tuple or Array is faster compared to one carried out on a Struct type object.

References

[1] A.Paoluzzi *Hierarchical structures with LAR*. March 29,2016.

[2] *The Julia language*, https://docs.julialang.org/en/stable/ .