Random number generators (RNGs)

- Old-school : dice , coinflips , ...

- Hardware RNGs : Generates numbers based on some unpredictable feature of physical environment (e.g. thermal noise)

- Pseudo RNGs : Deterministic algorithms that generate numbers that are predetermined but appear random (unpredictable).

  ↑ What we will focus on !

  Initialised by the starting number (seed)

  ↳ Reproducible !

- Desired properties for a pseudo RNG :

  1) Produce numbers that are distributed uniformly on $[0,1]$, i.e. samples from $U(0,1)$

  2) Negligible correlations between numbers (Knowing a previous number shouldn't help you guess the next number — unless you know the algorithm of course...)

  3) The period before repetition should be as long as possible

  4) Computationally fast algorithm

⊗ End Nov 11

- Classic algorithm : Linear Congruential Generator (LCG)

(~ 1950s)

$$\boxed{N_{i+1} = (a N_i + c) \bmod (m)}$$

1) scale and shift current number

2) Use modulus operator

a : multiplier $0 < a < m$

c : increment $0 \leq c < m$

m : the modulus $0 < m$

N_0 : the seed $0 \leq N_0 < m$

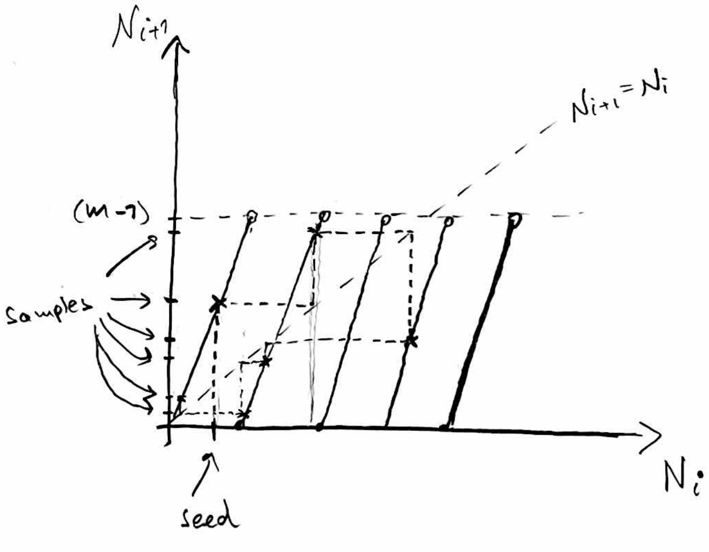
modulus operator :
"what's the remainder ?"

$13 \bmod (2) = 1$

$17 \bmod (5) = 2$

$8 \bmod (8) = 0$

$16 \bmod (17) = 16$



Other example :
12/24 hour clock

23:00 + 2hours → 01:00

$(23 + 2) \bmod (24) = 1$

- To get numbers on $[0,1)$: $X_i = \frac{N_i}{m}$

- How good the generator is depends critically on the choice of parameters : a, c, m (and potentially N_0)

  — There's a lot of research on such parameter choices for LCGs and other RNG algorithms !

- Examples :

$$\left.\begin{array}{l} m = 9 \\ a = 2 \\ c = 0 \\ N_0 = 1 \end{array}\right\} \quad \underbrace{1, 2, 4, 8, 7, 5,}_{\text{Period : 6}} \textcircled{1} \longrightarrow \text{repeats}$$

$$\left.\begin{array}{l} m = 9 \\ a = 2 \\ c = 0 \\ N_0 = 3 \end{array}\right\} \quad \underbrace{3, 6,}_{\underline{\text{Period : 2 !}}} \textcircled{3}$$

$$\left.\begin{array}{l} m = 9 \\ a = 4 \\ c = 1 \\ N_0 = 0 \end{array}\right\} \quad \underbrace{0, 1, 5, 3, 4, 8, 6, 7, 2,}_{\underline{\text{Period : 9}}} \textcircled{0}$$

- More realistic case :

$$m = 2^{32} = 4\,294\,967\,296 \quad (\text{more than } 4.2 \times 10^9)$$
$$a = 1664525$$
$$c = 1013904223$$

- Always look up period of a RNG !

  Famous RNGs have had surprisingly small periods

  ⇒ cannot trust results !

- RANDU (IBM, 1960s), famous worst-case example (samples in 3D would fall on distinct 2D planes ...)

- Period is not only concern!

  What happens for $a=1$, $c=1$, $m=$ large number ?

  Answer: get a "modulus counter": $N_0, N_0+1, N_0+2, N_0+3, \dots$

  – Long period, but does it look random? No!

- There are collection of statistical randomness tests used to test RNGs. (They all fail some ...)

[Diehard tests, TestU01]

[Donald Knuth (TeX inventor) wasthe first to propose a set of such tests...]

- Other RNG examples:

  – "Shift-register"

$$N_{i+1} = \left(aN_{i-j} + cN_{i-k}\right) \bmod (m)$$

Uses more than just the preceeding number!

  – A standard choice today: Mersenne Twister (MT19937)

  - Developed in 1997 [Matsumoto, Nishimura]
  - Available in <random> in C++11
  - Period of $2^{19937}-1$

- Pitfall when using RNGs with parallelitation :

  Make sure that RNGs on different threads get different seeds !!

  (Don't want different threads generating the exact same numbers)

- Can use thread number to modify a "base seed" such that all threds have a unique ceed.