Transf. of random variables

- **Recap**

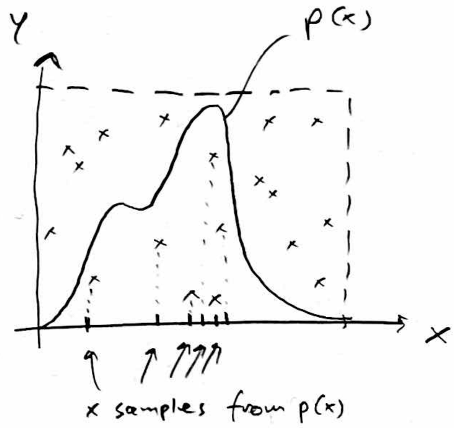
  1) If pdf of one variable is $U(0,1)$, what is pdf of some related variable?

  2) If we can <u>generate samples</u> from $U(0,1)$, how can we use this to generate samples from some other pdf?

  Our focus

- Two methods:

  1) Rejection sampling

  - Sample $U(x_{min}, x_{max})$ and $U(y_{min}, y_{max}) \Rightarrow (x, y)$
  - Keep x samples where Y sample satisfies $Y < p(x)$
  - Improve w/ importance sampling ...

  2) Inverse transformation sampling

Inverse transform sampling

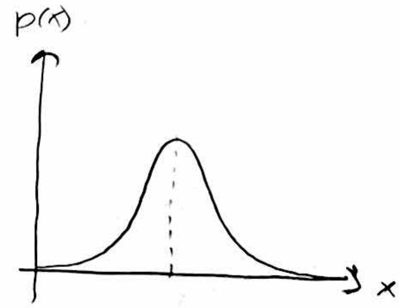
- Recap: The cumulative distr. function (cdf) is defined by

$$F(x) = \text{Prob}(X \leq x) = \int_{-\infty}^{x} p(x')\,dx'$$

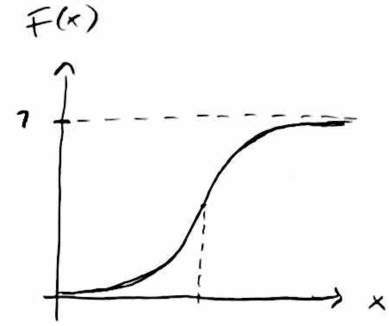
End lect. Nov 12

- Example:

pdf:

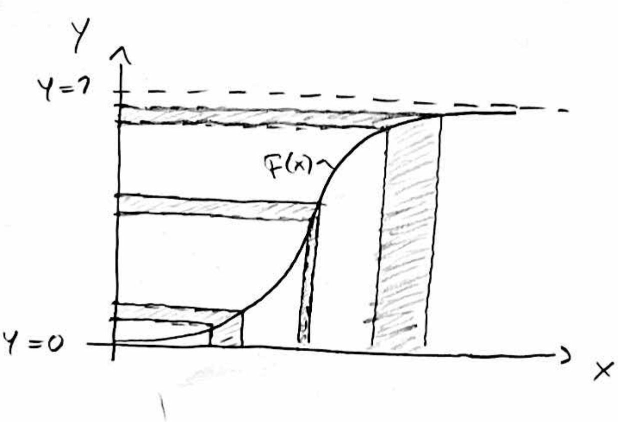


cdf:



$$\boxed{\frac{dF}{dx} = p(x)}$$

- Idea for inverse transform sampling:
  - Assume we know $y = F(x)$ and can find inverse function F^{-1}
  - Can use this to generate x samples from $p(x)$



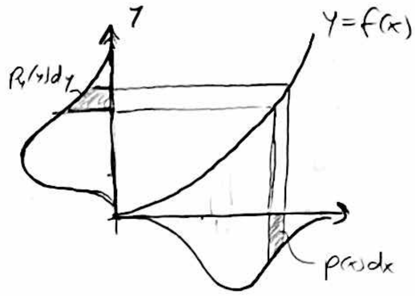
Algo:

1) Sample y from $U(0,1)$
2) Compute $x = F^{-1}(y)$

Repeat for as many samples as needed

- Explanation:
  - In general, if $x \sim p_x(x)$
    and y is some function $y = f(x)$
    we can find $p_y(y)$ from requirement

$$p_y(y)\,dy = p_x(x)\,dx$$



  - Now look at special case
    where $y = F(x)$ = cdf for x
  - Recall that $p_x(x) = \dfrac{dF}{dx}$

$$p_y(y)\,dy = p_x(x)\,dx \qquad (1)$$

$$\begin{aligned} p_y(y)\,dy &= p_y(y)\,\frac{dy}{dx}\,dx \\ &= p_y(y)\,\frac{dF}{dx}\,dx \\ &= p_y(y)\,p_x(x)\,dx \qquad (2) \end{aligned}$$

$$(1),(2) \Rightarrow \quad p_y(y)\,p_x(x)\,dx = p_x(x)\,dx$$

$$\Rightarrow \quad p_y(y) = 1 = U(0,1)$$

- So if we sample $y \sim U(0,1)$
  and compute $x = F^{-1}(y)$
  we get an $x \sim p_x(x)$, which was our goal

- Pro: No rejection sampling needed (efficiency)
- Con: Requires knowledge of <u>inverse cdf</u>

- Similar structure to <random>
  1) Use a <u>RNG</u> to sample from $U(0,1)$
  2) Use a <u>transformation</u> to obtain sample from $p(x)$