$$M[i][j] = \begin{cases} 0 & \text{se } i = j \\ \min\limits_{i \le k < j} \left(M[i][k] + M[k+1][j] + P_{i-1} P_k P_j \right) \end{cases}$$

$$\begin{array}{ccccccc} P_0 & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \end{array}$$
$$\langle 3, 4, 2, 3, 5, 2, 3 \rangle$$

$M[1][2] = 0 + 0 + 24 = 24$
$M[2][3] = 0 + 0 + 24 = 24$
$M[3][4] = 0 + 0 + 30 = 30$
$M[4][5] = 0 + 0 + 30 = 30$
$M[5][6] = 0 + 0 + 30 = 30$

$$M[1][3] = \min_{i \le k < j} \begin{pmatrix} k=1 & M[1][1] + M[2][3] + (3 \cdot 4 \cdot 3) \longrightarrow 0 + 24 + 36 = 60 \\ k=2 & M[1][2] + M[3][3] + (3 \cdot 2 \cdot 3) \longrightarrow 24 + 0 + 18 = 42 \end{pmatrix} = 42$$

$$M[2][4] = \min_{i \le k < j} \begin{pmatrix} k=2 & M[2][2] + M[3][4] + (4 \cdot 2 \cdot 5) = 0 + 30 + 40 = 70 \\ k=3 & M[2][3] + M[4][4] + (4 \cdot 3 \cdot 5) = 24 + 0 + 60 = 84 \end{pmatrix} = 70$$

	1	2	3	4	5	6
1	0	24	42	84	78	96
2		0	24	70	58	78
3			0	30	42	54
4				0	30	48
5					0	30
6						0

COSTO OTTIMO

$$M[3][5] = \min_{i \le k < j} \begin{pmatrix} k=3 & \boxed{\boxed{M[3][3]} + \boxed{M[4][5]} + (2 \cdot 3 \cdot 2) = 0 + 30 + 12 = 42} \\ k=4 & M[3][4] + M[5][5] + (2 \cdot 5 \cdot 2) = 30 + 0 + 20 = 50 \end{pmatrix} = 42$$

$(A\ B)((C\ (D\ E))\ F)$

$$M[4][6] = \min_{i \le k < j} \begin{pmatrix} k=4 & M[4][4] + M[5][6] + (3 \cdot 5 \cdot 3) = 0 + 30 + 45 = 75 \\ k=5 & M[4][5] + M[6][6] + (3 \cdot 2 \cdot 3) = 30 + 0 + 18 = 48 \end{pmatrix} = 48$$

$$M[1][4] = \min_{i \le k < j} \begin{pmatrix} k=1 & M[1][1] + M[2][4] + (3 \cdot 4 \cdot 5) = 0 + 70 + 60 = 130 \\ k=2 & M[1][2] + M[3][4] + (3 \cdot 2 \cdot 5) = 24 + 30 + 30 = 84 \\ k=3 & M[1][3] + M[4][4] + (3 \cdot 3 \cdot 5) = 42 + 0 + 45 = 87 \end{pmatrix} = 84$$

$$M[2][5] = \min_{i \le k < j} \begin{pmatrix} k=2 & M[2][2] + M[3][5] + (4 \cdot 2 \cdot 2) = 0 + 42 + 16 = 58 \\ k=3 & M[2][3] + M[4][5] + (4 \cdot 3 \cdot 2) = 24 + 30 + 24 = 78 \\ k=4 & M[2][4] + M[5][5] + (4 \cdot 5 \cdot 2) = 70 + 0 + 40 = 110 \end{pmatrix} = 58$$

$$M[3][6] = \min_{i \le k < j} \begin{pmatrix} k=3 & M[3][3] + M[4][6] + (2 \cdot 3 \cdot 3) = 0 + 48 + 18 = 66 \\ k=4 & M[3][4] + M[5][6] + (2 \cdot 5 \cdot 3) = 30 + 30 + 30 = 90 \\ k=5 & \boxed{\boxed{M[3][5]} + \boxed{M[6][6]} + (2 \cdot 2 \cdot 3) = 42 + 0 + 12 = 54} \end{pmatrix} = 54$$

$(A\ B)((C\ D\ E)\ F)$

Per l'ordine ottimale di parentesizzazione si parte dal costo ottimo, si prendono gli intervalli usati per il calcolo del minimo, si posizionano le parentesi di conseguenza e si ripete questo procedimento sugli intervalli appena trovati finché non si arriva ad intervalli di sole 2 matrici oppure celle della diagonale

$$M[1][5] = \min_{i \le k < j} \begin{pmatrix} k=1 & M[1][1] + M[2][5] + (3 \cdot 4 \cdot 2) = 0 + 58 + 24 = 82 \\ k=2 & M[1][2] + M[3][5] + (3 \cdot 2 \cdot 2) = 24 + 42 + 12 = 78 \\ k=3 & M[1][3] + M[4][5] + (3 \cdot 3 \cdot 2) = 42 + 30 + 18 = 90 \\ k=4 & M[1][4] + M[5][5] + (3 \cdot 5 \cdot 2) = 84 + 0 + 30 = 114 \end{pmatrix} = 78$$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ (A & B) & (C & D & E & F) \end{array}$$

$$M[2][6] = \min_{i \le k < j} \begin{pmatrix} k=2 & M[2][2] + M[3][6] + (4 \cdot 2 \cdot 3) = 0 + 54 + 24 = 78 \\ k=3 & M[2][3] + M[4][6] + (4 \cdot 3 \cdot 3) = 24 + 48 + 36 = 108 \\ k=4 & M[2][4] + M[5][6] + (4 \cdot 5 \cdot 3) = 70 + 30 + 60 = 160 \\ k=5 & M[2][5] + M[6][6] + (4 \cdot 2 \cdot 3) = 58 + 0 + 24 = 82 \end{pmatrix} = 78$$

$$M[1][6] = \min_{i \le k < j} \begin{pmatrix} k=1 & M[1][1] + M[2][6] + (3 \cdot 4 \cdot 3) = 0 + 78 + 36 = 114 \\ k=2 & \boxed{\boxed{M[1][2]} + \boxed{M[3][6]} + (3 \cdot 2 \cdot 3) = 24 + 54 + 18 = 96} \\ k=3 & M[1][3] + M[4][6] + (3 \cdot 3 \cdot 3) = 42 + 48 + 27 = 117 \\ k=4 & M[1][4] + M[5][6] + (3 \cdot 5 \cdot 3) = 84 + 30 + 45 = 159 \\ k=5 & M[1][5] + M[6][6] + (3 \cdot 2 \cdot 3) = 78 + 0 + 18 = 96 \end{pmatrix} = 96$$