

# Lecture 32 - Design Considerations

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## Announcements:

- Exam corrections due Weds
- Office Hours today in Fitz B22
- Robotics talk tomorrow: 1 PM in Fitz 258

## Today:

- Manipulator Design
- Workspace Analysis
- Metric for workspace usability (Manipulability)

# Recap: Fundamentals:

- Forward (direct) kinematics - position & 3D orientation
- Inverse kinematics - closed form & numerical
- Forward differential kinematics - velocity & acceleration
- Jacobian -  $v = J\dot{\theta}$      $\tau = J^T f$
- Dynamics  $\tau = M\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) + J^T f$ 
  - Recursive Newton Euler
  - Lagrangian

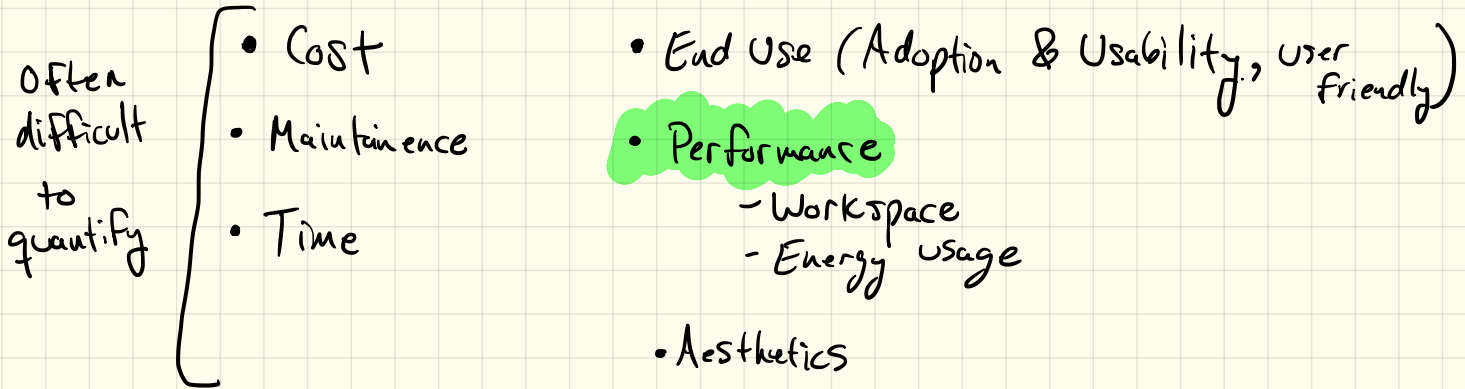
## What's Next:

- Design Considerations
  - Control
- [HW] - Trajectory generation [2 lectures]
- [HW, Proj] - Linear & Nonlinear Control [4 lectures]
- ↙ No HW, Short answers on the final

# Manipulator Design

Caveat: A bit Subjective

- Multi-Objective Optimization Problem



- All specific to task @ hand

(But at the same time we want robots to be versatile)

# Performance Considerations

- # DOFs (# actuators sets overall cost  $\geq \$1/2$  DoF)
  - Often don't need 6 DoF
- Workspace (influenced by prismatic vs. revolute, joint axis selection)
- Load capacity (Actuators & Structural elements)
- Speed
- Repeatability & Accuracy

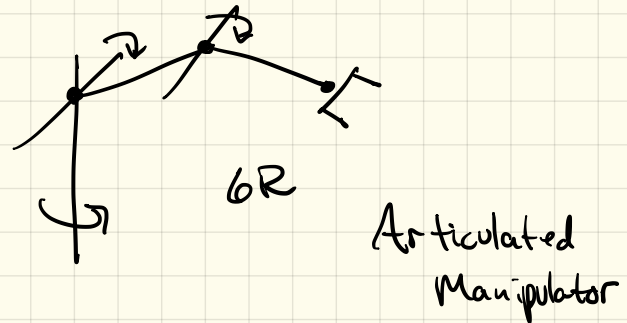
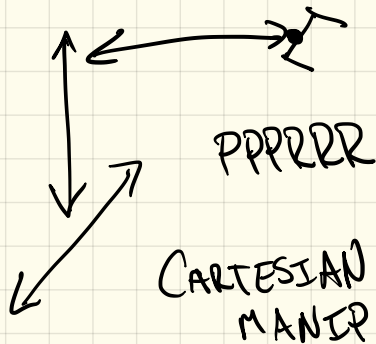
# Choosing How to Arrange Your DoFs

## Common Approach

Positioning Structure - 3 inboard joints set position of wrist in a volume of workspace

Orienting Structure - 3 outboard joints orient wrist

↳ successive revolute joints w/ common intersection point called concurrency point



# Maximizing workspace Volume

- Maximize Volume in which concurrency point can be placed for a given "length" of manipulator structure. (Sphere)

3R

Joint 1: Axis 1 vertical

Joint 2: Axis 2 intersects Axis 1 and is  $\perp$  to axis 1

Joint 3: Halfway between Axis 1 and concurrency point

Evaluating Workspace:

$$L = \sum a_i + d_i \quad \text{"Structural length sum"}$$

Normalized  
Volume index:

$$N = \frac{V}{\frac{4}{3}\pi L^3}$$

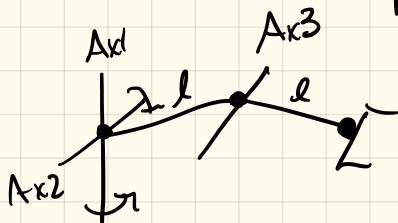
Volume of workspace

(Large  $N$  is good)

Structural  
Length  
index

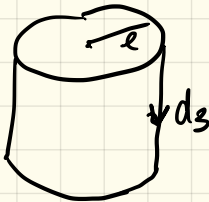
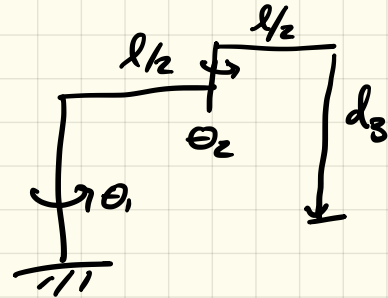
$$L / \sqrt[3]{V} = Q_L$$

(Small  $Q_L$  is good)



## Example: SCARA Manipulator

- What value for  $d_3$  maximizes Normalized Volume index?
- What is the workspace shape?



$$\text{Volume } V = \pi l^2 d_3$$

$$L = \frac{l}{2} + \frac{l}{2} + d_3 = l + d_3$$

$$N = \frac{\pi l^2 d_3}{\frac{4}{3} \pi L^3} = \frac{3}{4} l^2 \frac{d_3}{(l + d_3)^3}$$

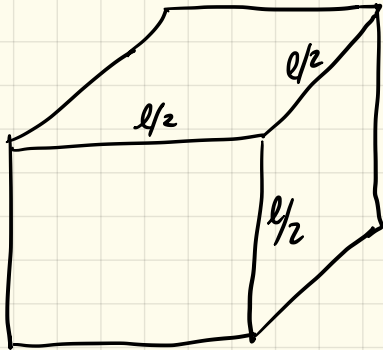
$$\frac{dN}{dd_3} = \frac{(l + d_3)^{\cancel{3}} - 3(l + d_3)^{\cancel{2}} d_3}{(l + d_3)^{\cancel{4}}} = 0$$

$$l + d_3 - 3d_3 = 0 \Rightarrow \boxed{d_3 = \frac{l}{2}} \quad \text{Symmetry!}$$

## Design Comparison:

$$N = \frac{3}{4} l^2 \frac{d_3}{(l + d_3)^3} \bigg|_{d_3 = \frac{l}{2}} = \frac{1}{9}$$

VS. Cartesian Manipulator?



$$N = \frac{\left(\frac{l}{2}\right)^3}{\frac{4}{3} \pi \left(\frac{3l}{2}\right)^3} = \frac{1}{36\pi}$$