AME 60621

Optimization-Based Robotics

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Office Hours: M: 12:30-2:00 (373 Fitzpatrick), F: 3:30-5:00 (Room TBD)

Class Meetings: TTh 12:30-1:45 PM, Debartolo Hall, Room 213
Live Schedule: `https://tinyurl.com/ydeageka`

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Learning Objectives: The objective of this course is for students to develop the ability to recognize, formulate, and solve optimization problems within the context of robotics applications. We will consider a range of classical problems, spanning dynamics, identification, control, and estimation, and show how they can be posed as constrained optimization problems. An emphasis will be placed on developing competency with state of the art optimization software (e.g., MATLAB optimization toolbox, CVX, YALMIP, SDPT3, CasADi, IPOPT) and on applications within legged robotics.

Free Online Resources:

- Springer Handbook of Robotics (Chapter 2, Dynamics) - Roy Featherstone and David Orin
  `https://link-springer-com.proxy.library.nd.edu/content/pdf/10.1007/978-3-540-30301-5_3.pdf`
- Rigid-Body Dynamics Algorithms - Roy Featherstone
  `https://link-springer-com.proxy.library.nd.edu/book/10.1007/978-1-4899-7560-7`
- Convex Optimization - Stephen P. Boyd and Lieven Vandenberghe
  `https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf`
- Numerical Optimization - Jorge Nocedal and Stephen Wright
  `https://link-springer-com.proxy.library.nd.edu/book/10.1007/978-0-387-40065-5`
- Calculus of Variations and Optimal Control Theory - Daniel Liberzon
  `http://liberzon.csl.illinois.edu/teaching/cvoc.pdf`
- Modern Robotics: Mechanics, Planning, and Control - Kevin M. Lynch and Frank C. Park
  `http://hades.mech.northwestern.edu/images/7/7f/MR.pdf`
- A Mathematical Introduction to Robotic Manipulation - Richard M. Murray, Zexiang Li, S. Shankar Sastry
  `https://www.cds.caltech.edu/~murray/books/MLS/pdf/mls94-complete.pdf`
- Optimal and Learning Control for Autonomous Robots - Jonas Buchli et al.
  `https://arxiv.org/pdf/1708.09342.pdf`
- Underactuated Robotics - Russ Tedrake
  `http://underactuated.csail.mit.edu/`

Topics:

- **Dynamics and Model Identification:** Rigid-body dynamics and spatial vector algebra, articulated-body algorithm, contact modeling, inertial parameter identification
- **Control:** Dynamic programming, LQR, trajectory optimization (shooting, direct collocation, and DDP), operational-space control, Poincaré analysis, model predictive control
- **Estimation:** State estimation and Kalman filtering, extended and unscented Kalman filters, sensor fusion
- **Locomotion:** Centroidal dynamics, templates and the IP family (SLIP, IP, LIP), the capture point, ZMP

Additional topics will be covered, as time allows, based on the shared interests of the instructor and class.

Prerequisites: This graduate course will assume mastery of introductory robotics content, an intuitive grasp of concepts from linear algebra, a strong understanding of linear ordinary differential equations, and a moderate level of mathematical maturity. If you are unsure if you meet this criteria, below are a few representative prerequisite topics that we will draw upon:

- Dynamics & Robotics: Newtonian dynamics in 3D, Lagrangian mechanics, Jacobians, general coordinate system gymnastics for expressing vectors in different frames.
- Linear algebra: Eigenvalues, eigenvectors, rank, nullspace, and positive definite matrices. If you need a refresher, I highly recommend Gilbert Strang's lectures on linear algebra (link).
- Systems: State space representation of linear systems, stability criteria for systems of linear ordinary differential equations.
- Multivariable Calculus: Multivariable chain rule, optimality criteria for unconstrained optimization, Lagrange multipliers for equality-constrained optimization.

Grading: The following rough grade scale will be followed:

A : Thorough understanding of material. Can formulate and solve new problems with appropriate tools.
B : Good understanding of material. Can formulate and solve familiar robotics optimization problems.
C : Understanding of some material. Can solve basic problems.
D : Works hard to convince me that little was learned and little effort was expended.

Assessment will be based on the following breakdown:

Homeworks	50%	~ 5-7 10%
Midterm	25%	
Final Project	25%	- 5% Proposal
		- 15% Report
		- 5% Video

Homework: Homework assignments will be announced in class and posted on Sakai.

- Homeworks will be a mix of written work and MATLAB programming. Written work should be presented professionally and MATLAB code should be clearly commented. If your work cannot be clearly followed, deductions may apply.
- You are encouraged to collaborate on homework. However, submitted material must be your own work and accurately reflect your own understanding at the time of writing.
- Homework is due in class (usually on Tuesdays). Late homeworks will be docked 20% per day. You are allocated three free late days that can be used at your discretion before deductions are levied.

Midterm: The midterm will be a take-home exam with an open note policy. The exam will be an individual effort – providing or receiving aid from anyone is strictly prohibited. The date of the exam will be announced at least one week prior to its administration. It will take place roughly 2/3 of the way through the course.

Final Project: All students will conduct an open-ended final project on a topic of their choosing related to optimization-based robotics. If you are involved in research, the objectives of the project should be complementary to the objectives of your research work. You are encouraged to speak with the professor or TA about suitable topics. To receive formal feedback on your project, you will submit a one-page proposal in week 6. You may work in pairs, but every student will submit an individual proposal and final report. The final report will be typeset as an IEEE conference-style paper (6 pages max). The report will be accompanied by a video abstract (5 minutes max). Exceptional projects are expected to serve as the basis for conference paper submissions.

Honor Code: (http://honorcode.nd.edu/the-honor-code/) The university community has a shared commitment to respect and honor the intellectual and creative contributions of each individual. As a precondition for admission to the University, all students pledge:

"As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty."

Beyond this simple pledge, it is your responsibility to become familiar with the Academic Code of Honor, and to adhere to the responsibilities it outlines. Any suspected violation of this code will be reported through the procedure described in Section V.D of the Honor Code.

Accommodations: Any student who has a documented disability and is registered with Disability Services should speak with the professor as soon as possible regarding accommodations. Students who are not registered should contact the Office of Disability Services - http://disabilityservices.nd.edu.

Preliminary Schedule:

- Dynamic Modeling and Identification (3 weeks)
  - Spatial Vector Algebra and Recursive-Newton-Euler Dynamics
  - Lagrangian Dynamics
  - Contacts, Impacts, and Hybrid Dynamics
  - System Identification
- Optimal Control (5 weeks)
  - Dynamic Programming and the Hamilton-Jacobi-Bellman (HJB) Equation
  - Linear Quadratic Regulator (LQR) for Time-Invariant and Time-Varying Linear Systems
  - Pontryagin's Maximum Principle
  - Single and Multiple Shooting
  - Direct Collocation
  - Differential Dynamic Programming
  - Computational Considerations
- Legged Robots (5 Weeks)
  - Inverted Pendulum, Spring-Loaded Inverted Pendulum, and the Compass Gait Walker
  - Limit cycles and Poincaré analysis
  - The Linear Inverted Pendulum model and the capture point
  - The Zero Moment Point (ZMP) and ZMP planning
  - Model-predictive control of simple models
  - Floating-base dynamics, centroidal dynamics
  - Operational-space control
- State Estimation (2 weeks)
  - Multivariate Guassians
  - Kalman filtering
  - Unscented and Extended Kalman filters
  - Sensor fusion for state estimation in legged robots

Lecture 1: Math Review & Classification of Optimization Problems

Review

- Multivariable Calc & Linear Algebra
  (& a mashup of the two)

- Optimality criteria (first & second order)

New:

- Problem Classification
  - Static optimization (LPs, QPs, SOCPs, ...)
  - Dynamic optimization

Linear Algebra Basics

Matrix Multiplication:

$A \in \mathbb{R}^{n \times m}$ (n×m matrix of Real numbers)

$B \in \mathbb{R}^{m \times p}$

$$C = AB \Rightarrow \underbrace{C_{ik}}_{\substack{\text{row } i \\ \text{column } k}} = \sum_{j=1}^{m} A_{ij} B_{jk}$$

Transpose: $A \in \mathbb{R}^{n \times m}$ $A^T \in \mathbb{R}^{m \times n}$

$$[A^T]_{ij} = [A]_{ji}$$

property: $(AB)^T = B^T A^T$

Eigenvalues: $A \in \mathbb{R}^{n \times n}$ eigenvalues of A are roots of $\det(\lambda I_{n \times n} - A) = 0$

- We then say $x \in \mathbb{R}^n$ is eig. vec. of A if $Ax = \lambda x$ for some eig val $\lambda \in \mathbb{C}$
- We say A is diagonizable if $\exists$ a basis of eig. vecs $v_1, \dots, v_n$

In this case $V = [v_1, \dots, v_n]$ gives $V^{-1} A V = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$

Multivariable Calculus

Jacobian: Consider $f(x): \mathbb{R}^n \to \mathbb{R}^m$. We define the Jacobian of f by

$$\frac{df}{dx} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

First-order approx: $x_0, \Delta x \in \mathbb{R}^n$

$$f(x_0 + \Delta x) = f(x_0) + \left[\frac{df}{dx}\Big|_{x=x_0} \right] \Delta x + \underbrace{o(\|\Delta x\|)}$$

denotes terms of higher than 1st order in Δx

Example: $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$ $f(x) = Ax$

$$\begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \\ A_{m1} & \cdots & A_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\left[\frac{df}{dx} \right]_{ij} = \frac{\partial f_i}{\partial x_j} = A_{ij} = [A]_{ij}$$

$$\boxed{\frac{df}{dx} = A}$$

Multivariable Calculus: Chain Rule

$$y = f(x) \qquad z = g(y) \qquad z = g(f(x)) \qquad y = [y_1, \dots, y_m]^T$$

$$f : \mathbb{R}^n \to \mathbb{R}^m \qquad g : \mathbb{R}^m \to \mathbb{R}^p \qquad g \circ f : \mathbb{R}^n \to \mathbb{R}^p \qquad x = [x_1, \dots, x_n]^T$$

$$\left[\frac{dz}{dx}\right]_{ik} = \frac{\partial z_i}{\partial x_k} = \sum_{j=1}^{m} \frac{\partial z_i}{\partial y_j} \frac{\partial y_j}{\partial x_k} = \sum_{j=1}^{m} \left[\frac{dz}{dy}\right]_{ij} \left[\frac{dy}{dx}\right]_{jk} = \left[\frac{dz}{dy}\frac{dy}{dx}\right]_{ik}$$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

What if had $z(x, y(x))$:

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{dy}{dx}$$

Quick comment:

$$f : \mathbb{R}^n \to \mathbb{R} \implies \frac{df}{dx} = \left[\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n}\right] = [\nabla_x f]^T$$

Second Order Taylor Approximation

Suppose $f: \mathbb{R}^n \to \mathbb{R}$ twice differentiable we denote Hessian of f as

$$\nabla_{xx} f := \begin{bmatrix} \partial^2 f/(\partial x_1)^2 & \cdots & \partial^2 f/\partial x_1 \partial x_n \\ \vdots & \ddots & \vdots \\ \partial^2 f/\partial x_n \partial x_1 & \cdots & \partial^2 f/(\partial x_n)^2 \end{bmatrix}$$

$$(\nabla_{xx} f)^T = \nabla_{xx} f$$

Hessian is symmetric

Let $x_0, \Delta x \in \mathbb{R}^n$

$$f(x_0 + \Delta x) = f(x_0) + \left[\nabla_x f\big|_{x=x_0}\right]^T \Delta x + \frac{1}{2} \underbrace{\Delta x^T \left[\nabla_{xx} f\big|_{x=x_0}\right] \Delta x}_{} + O(\|\Delta x\|^2)$$

In scalar case f concave up if $\nabla_{xx} f > 0$

How does this translate to Multi-D?

Quadratic Forms: $x \in \mathbb{R}^n$ $Q \in \mathbb{R}^{n \times n}$ $Q = Q^T$

$$f(x) = x^T Q x$$

- every eigvec choosen to have unit magnitude
- every pair of vec is $\perp$

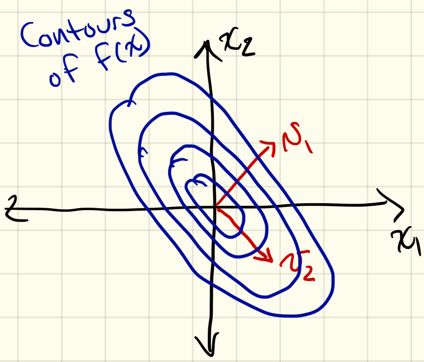
☆ Property: Every symmetric matrix can be diagonalized by an orthonormal set of eigen vectors w/ real eig vals!

$V = [v_1, \ldots, v_n]$ $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$

$z = [z_1, \ldots, z_n]^T$ $x = Vz$

You'll show in HW that $f(x) = f(Vz) = \sum_{i=1}^{n} \lambda_i z_i^2$

Example: $Q = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $v_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\lambda_2 = 2$ $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}}$ $\lambda_1 = 4$

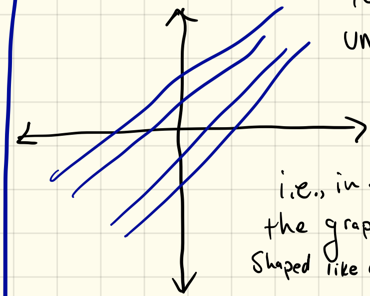


$$x = v_1 z_1 + v_2 z_2$$

$$f(x) = f(Vz) = 4 z_1^2 + 2 z_2^2$$

Contours if: $\lambda_1 = 0$ $\lambda_2 = 2$

v_1, v_2 unchanged



i.e., in 3D the graph is shaped like a taco.

Quadratic Forms:

① We say a matrix $Q = Q^T \in \mathbb{R}^{n \times n}$ is positive definite if $x^T Q x > 0 \ \forall x \in \mathbb{R}^n, x \neq 0$

- Equivalently Q has eig vals. $\lambda_i > 0 \quad i = 1, \ldots, n$
- We denote a PD matrix as $Q \succ 0$

② We'll say " " is positive semi definite (PSD) if $x^T Q x \geq 0 \ \forall x$

- $\lambda_i \geq 0$
- $Q \succeq 0$

③ " " negative definite if $x^T Q x < 0 \ \forall x \in \mathbb{R}^n \ x \neq 0$

- $\lambda_i < 0$
- $Q \prec 0$

Optimality Criteria: $f(x) : \mathbb{R}^n \to \mathbb{R}$

- Suppose x^* is a local minimizer of f.

(a) $\left.\dfrac{df}{dx}\right|_{x=x^*} = 0$ Multi dim analog $\left.\nabla_x f\right|_{x=x^*} = 0$

(b) $\left.\dfrac{d^2f}{dx^2}\right|_{x=x^*} \geq 0$ $\left.\nabla_{xx} f\right|_{x=x^*} \succeq 0$

necessary conditions for optimality

- $x_c \in \mathbb{R}^n$

(a) $\left.\nabla_x f\right|_{x=x_c} = 0$

(b) $\left.\nabla_{xx} f\right|_{x=x_c} \succ 0$ then x_c is a local minimizer

Sufficient conditions for local optimality

Sufficient conditions for global optimality?