Lagrangian HW Common Mistake: Partial vs. Total Derivative

$$\mathcal{L} = \left[\frac{1}{2} m_3 \, l^2 \dot{\theta}_1^2 S_2 \qquad \right]$$

$$\tau_1 = \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right] - \cancelto{0}{\frac{\partial \mathcal{L}}{\partial \theta_1}}$$

$$= \frac{d}{dt}\left[m_3 l^2 \dot{\theta}_1 S_2 \right] = \frac{d}{dt}\left[m_3 l^2 \dot{\theta}_1(t) \sin(\theta_2(t)) \right]$$

$$= m_3 l^2 \ddot{\theta}_1 S_2 + m_3 l^2 \dot{\theta}_1 C_2 \dot{\theta}_2$$

Lecture 39 - Force Control

Home Stretch

- Final Review Videos on Sakai
- List of topics that are fair game for the final: on Sakai as well.
- Final Review Session: Friday 4PM-6PM, Fitz 356
- Project 2 Due Thursday 11:55PM on Sakai
- HW 10: Turn it into my mailbox by 5PM Thusday ⇒ 2nd lowest HW gets 10/10

Today

- Wrap up Nonlinear Control
- Force Control for Physical Interaction

Suppose you want your robot to stay in place, supporting is own weight. The robot has total mass m and dynamics

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) \qquad \text{[dynamics]}$$

$\dot{\theta}_i^2$ or $\dot{\theta}_i \dot{\theta}_j$

What torque should you use?

$$\tau = G(\theta) \qquad \text{[control law]}$$

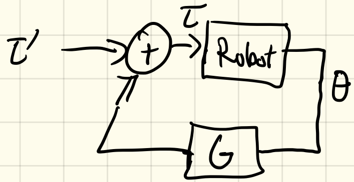
Closed loop:

$$0 = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta})$$

Common Approach: "PD +" Gravity Compensation

$\theta \to 0$ as $t \to \infty$

$$\tau = M\ddot{\theta} + V + G$$



- When moving slowly $\tau \approx G$
- Candidate control $\tau = \tau' + G$

  $\Rightarrow$ closed loop dynamics $\tau' = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta})$

  $\Rightarrow$ Same dynamics as if you turned off gravity

- Try PD control for τ' (always first thing to try)

$$\tau' = -K_p\theta - K_D\dot{\theta}$$

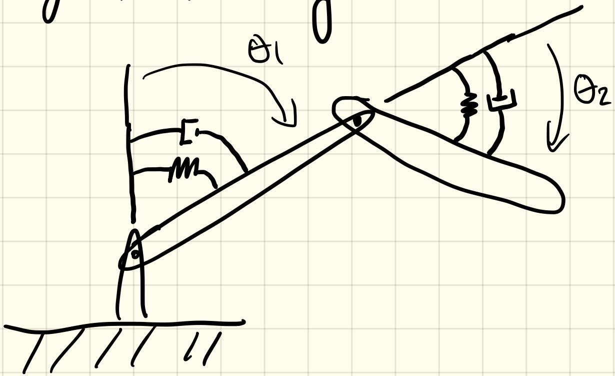
$\Rightarrow$ Closed loop dynamics

$$0 = M\ddot{\theta} + V + K_p\theta + K_D\dot{\theta}$$

$\Rightarrow$ Horribly Nonlinear!

Consider: Completely Passive System

$E = K + U_{spring}$



No Gravity

- Whenever it is moving its losing energy: so $\dot{\theta} \to 0$ as $t \to \infty$
- If $\theta \to \theta_f$ $\theta_f \neq 0$? can't happen since system would start moving again $\theta \to 0$ as $t \to \infty$
- This system behaves the same as PD+ controller

$$0 = M\ddot{\theta} + V(\theta, \dot{\theta}) + K_p \theta + K_D \dot{\theta}$$

- PD+ is asymptotically stable!

All control so far : Position Control

- Good for many assembly like tasks

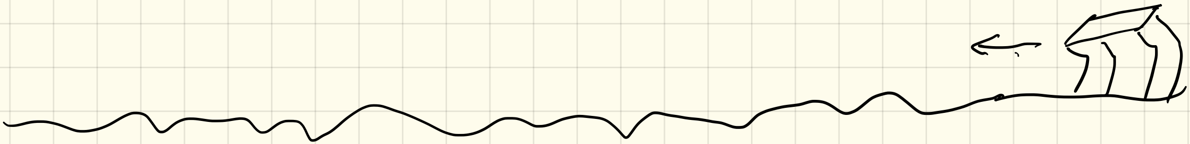
Next frontier applications of robots:

- Wearable robots
- Mobile Robot (Unexpected contacts
- Assitive Robots

Physical Interaction
- force cognizant control

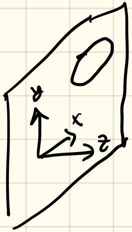
Force Control:

- In position control: We want to control position (no matter what force is applied)
- In force control: We would want to control force (no matter what position is achieved)
- In any direction you have to pick
  Do I control motion or force?



Hybrid Motion/Force Control

- often task @ hand dictates certain selections
- Consider: Erasing a frictionless board



Environment imposes

- $v_z = 0$
- $\omega_x = 0$
- $\omega_y = 0$
- $f_x = 0$
- $f_y = 0$
- $n_z = 0$

Natural Constraints

Control can impose desired

- f_z
- n_x
- n_y

3 controlled forces/moment

- v_x
- v_y
- ω_z

3 position/orientation DoF

Artificial constraints

- In general if you control K position/orientation DoFs you can control 6-K Force/moment components

Hybrid Position/Force Control: A simple approach

- Consider error as in your numeric IK problem

$$e = \begin{bmatrix} e_\theta \\ e_p \end{bmatrix} \begin{matrix} \leftarrow \text{angle-axis error} \\ \leftarrow \text{position error} \end{matrix}$$

- Let S a 6x6 matrix that "selects" position control components: For eraser example

$$S = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} \qquad (\text{control of } \omega_z, v_x, v_y)$$

- Then $I - S = \bar{S}$ gives force control components

$$\tau = J(\theta)^T \left[\bar{S} f_d + S \left(K_p e + K_d \dot{e} \right) \right] + G(\theta)$$

Summary:

- When it comes to force or position control you have to pick
  - position only **OR**
  - force only
  - Control k position/orientation DoFs ⇒ 6-k directions of forces/moments can be controlled

- Hybrid force/position control
  - Allows you to control positions in some directions and forces in other directions
  - Often motivated by environmental considerations