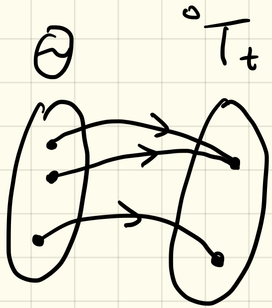
Lecture 10 - 9/12/2018 - IK Introduction

So far: Forward (direct) Kinematics



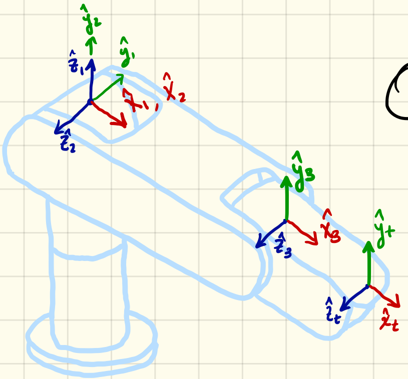
Given $\theta_1, \ldots, \theta_n$

$$(\theta_1, \ldots, \theta_n) \xrightarrow{f} ({}^0P_t, {}^0R_t)$$

Next: Inverse Kinematics

Given ${}^0P_t, {}^0R_t$ find all possible $\theta_1, \ldots, \theta_n$

s.t. $f(\theta_1, \ldots, \theta_n) = ({}^0P_t, {}^0R_t)$



① Numeric IK:

$L_3 = 5 \quad L_4 = 1$

$$\begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & L_3 c_1 c_2 + L_4 c_1 c_{23} \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & L_3 s_1 c_2 + L_4 s_1 c_{23} \\ s_{23} & c_{23} & 0 & L_3 s_2 + L_4 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .8 & 0 & .6 & 5 \\ .6 & 0 & -.8 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve $\theta_1, \theta_2, \theta_3$ $\quad$ [e.g. $\theta_1 = 0^\circ$, $\theta_2 = 36.9^\circ$, $\theta_3 = -36.9$]

② Symbolic IK:

$${}^0T_t = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & L_3 c_1 c_2 + L_4 c_1 c_{23} \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & L_3 s_1 c_2 + L_4 s_1 c_{23} \\ s_{23} & c_{23} & 0 & L_3 s_2 + L_4 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve $\theta_1, \theta_2, \theta_2$ $\quad$ [eg. $\theta_1 = \operatorname{atan}(r_{13} / -r_{23})$]

Given ${}^0P_t$ and ${}^0R_t$ find all possible $\theta_1, \dots, \theta_n$ such that

$$f(\theta_1, \dots, \theta_n) = ({}^0P_t, {}^0R_t)$$

Challenges & Features:

Wolfram alpha gives up

- System of nonlinear equations (AH!)
- 3 equations for position
- 9 equations for orientation (3 independent)
- 6 ind. equations for n variables

Solvability of Inverse Kinematics

Given ${}^0p_t$, ${}^0R_t$ is the IK problem solvable?

1. Do any solutions exist at all?
2. Are they unique?
3. Can I write the solutions in closed form?

Numerical IK

① Given ${}^0p_t$, ${}^0R_t$ Do Solutions Exist?

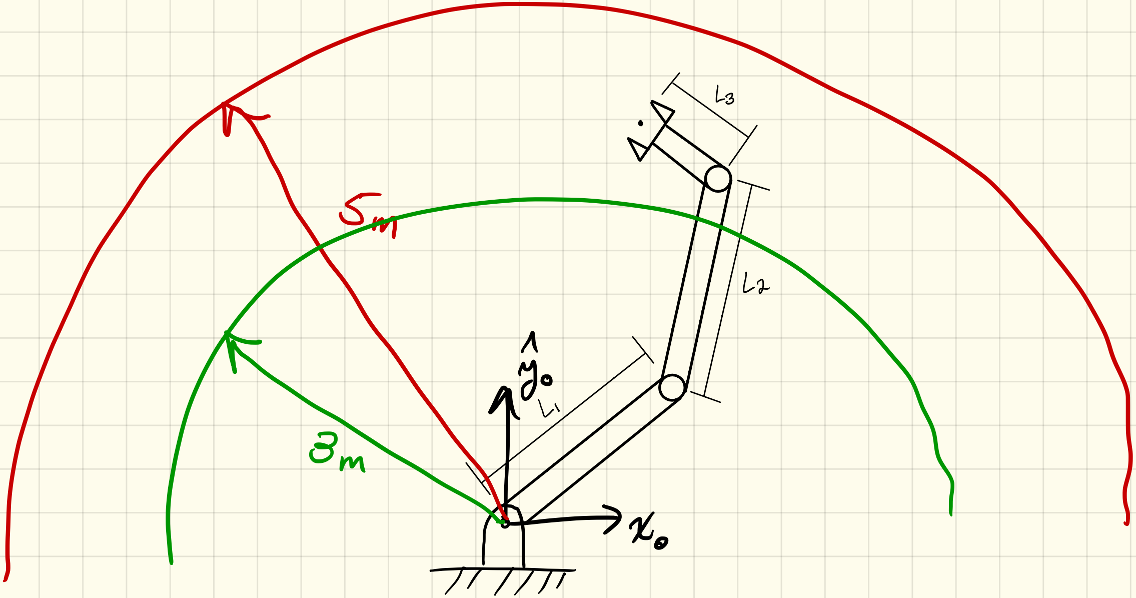
Only if ${}^0p_t$ is within "workspace" of the tool

Reachable Workspace: ${}^0p_t$ is in reachable workspace if IK is soluable for some ${}^0R_t$

Dexterous Workspace: ${}^0p_t$ is in the dexterous workspace if IK soluable for all ${}^0R_t$

Workspace Example

$L_1 = 2m,\ L_2 = 2m,\ L_3 = 1m$



RW: $\{ (x, y, z)$ such that $z = 0$ and $\sqrt{x^2+y^2} \leq 5 \}$

DW: $\{ (x, y, z)$ such that $z = 0$ and $\sqrt{x^2+y^2} \leq 3 \}$

② Multiple Solutions?

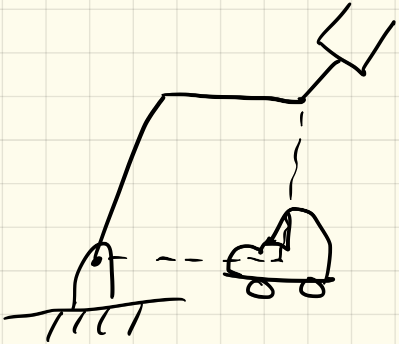
n degrees of freedom
↳ 6 equations

- $n > 6$, if one solution $\Rightarrow \infty$ solutions
  (mechanism still has $n-6$ DoFs with tool fixed)

- when $n \leq 6$ can we have multiple solutions
  Yes! (discrete solutions)

Options are a good thing:

- Avoid joint limits
- Avoid obstacles
- Avoid collisions w/ yourself



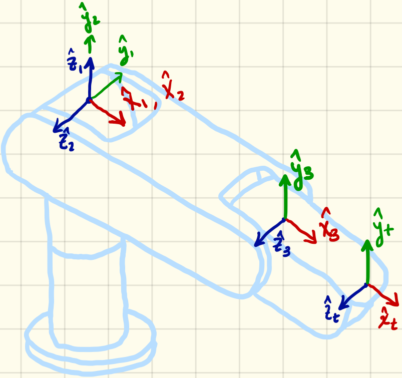
(3) Can I write Solutions in closed form?

i.e. answer comes in the form of an analytical expression

$$\theta_1 = \cos^{-1}\left(\frac{p_x}{l_1}\right)$$

- In this class (i.e. on HW & Tests) yes!
- In general only in special cases
  - Intersecting joint axes
  - $\alpha_i \approx 0, 90°, 180°$
  - Most older Manipulators
- Benefits: Closed-form answers are fast to compute!
- Now: Computers are fast enough that this isn't such a concern.

Project 1



Full IK:

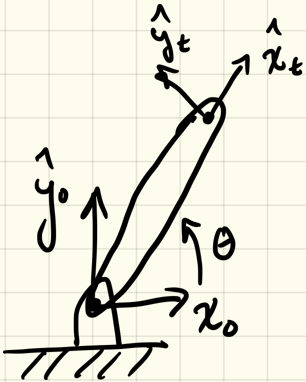
$$
{}^{0}T_t = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & L_3 c_1 c_2 + L_4 c_1 c_{23} \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & L_3 s_1 c_2 + L_4 s_1 c_{23} \\ s_{23} & c_{23} & 0 & L_3 s_2 + L_4 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

solve for $\theta_1, \theta_2, \theta_3$

Position IK:

$$
\begin{bmatrix} L_3 c_1 c_2 + L_4 c_1 c_{23} \\ L_3 s_1 c_2 + L_4 s_1 c_{23} \\ L_3 s_2 + L_4 s_{23} \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}
$$

The Simplest IK Problem: The R manipulator



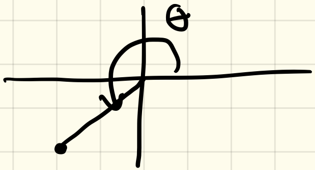
$$ {}^0P_t = \begin{bmatrix} l\,c_\theta \\ l\,s_\theta \\ 0 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} $$

Solve for θ

$$\cancel{0 \le \theta = \arccos(p_x/l) \le \pi}$$

$$\cancel{-\pi/2 < \theta = \arctan(p_y/p_x) < \pi/2}$$

$$\cancel{-\pi/2 \le \theta = \arcsin(p_y/l) \le \pi/2}$$



$$\boxed{\begin{array}{c}\text{4-Quadrant arc tan} \\ \theta = \operatorname{atan2}(p_y, p_x)\end{array}}$$