Lecture 12 - IK Examples

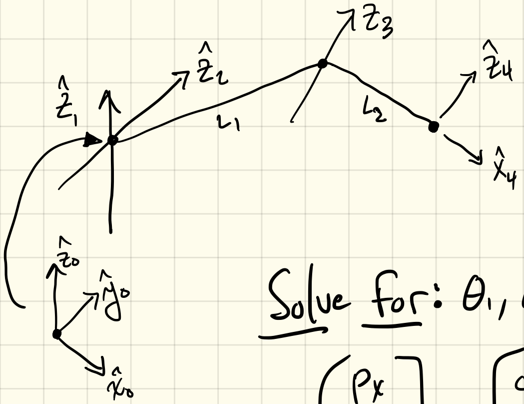
- Last time - Geometric Methods for solving IK problems

- Goals for Today:

    - Exam Logistics

    - Algebraic Methods for IK

Position IK: Simplified Puma



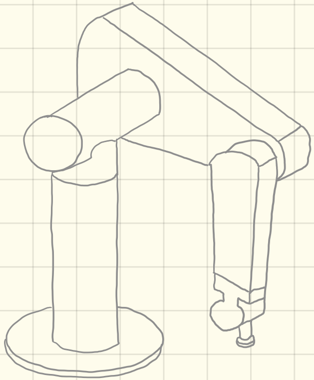
$$
{}^0T_4 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & c_1(c_{23}L_2 + c_2 L_1) \\ s_1 c_{23} & -s_1 s_{23} & c_1 & s_1(c_{23}L_2 + c_2 L_1) \\ -s_{23} & -c_{23} & 0 & -s_{23}L_2 - s_2 L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Solve for: $\theta_1, \theta_2, \theta_3$

$$
\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} c_1(c_{23}L_2 + c_2 L_1) \\ s_1(c_{23}L_2 + c_2 L_1) \\ -s_{23}L_2 - s_2 L_1 \end{bmatrix}
$$

Last Time: 4 solutions

- Elbow down vs. up
- Facing fwd. vs. backward



Position IK:

Solve for $\theta_1, \theta_2, \theta_3$

$$p_x = c_1(c_{23}l_2 + c_2 l_1)$$
$$p_y = s_1(c_{23}l_2 + c_2 l_1)$$
$$p_z = -s_{23}l_2 - s_2 l_1$$

① Solve for θ_1: $\theta_1 = \text{atan2}(\pm p_y, \pm p_x)$

(2 solution for θ_1, fwd. vs. backward)

② $\underbrace{c_1 p_x + s_1 p_y}_{:=\gamma} = c_1^2(c_{23}l_2 + c_2 l_1) + s_1^2(\cdots)$

$$\gamma = c_{23}l_2 + c_2 l_1$$
$$p_z = -s_{23}l_2 - s_2 l_1$$

Isolate c_{23} and s_{23}:

$$\left.\begin{aligned} c_{23}l_2 &= \gamma - c_2 l_1 \\ s_{23}l_2 &= -s_2 l_1 - p_z \end{aligned}\right\} \text{Square \& Add}$$

$$l_2^2 = \gamma^2 - 2\gamma l_1 c_2 + c_2^2 l_1^2 + s_2^2 l_1^2 + 2 p_z l_1 s_2 + p_z^2$$

$$\underbrace{l_2^2 - l_1^2 - \gamma^2 - p_z^2}_{C} = \underbrace{-2\gamma l_1}_{A} c_2 + \underbrace{2 p_z l_1}_{B} s_2$$

$$\theta_2 = \text{atan2}(B, A) \pm \text{atan2}\left(\sqrt{A^2 + B^2 - C^2}, C\right)$$

(2 solutions for θ_2 for each solution θ_1)

③ Finding θ_3:

$$c_{23} L_2 = \gamma - c_2 L_1$$
$$s_{23} L_2 = - s_2 L_1 - p_z$$

$$\Rightarrow \theta_{23} = \operatorname{atan2}(-s_2 L_1 - p_z, \gamma - c_2 l_1)$$

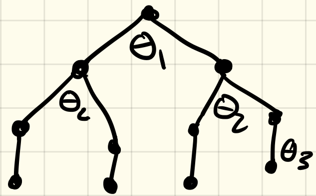
$$\theta_3 = \operatorname{atan2}(-s_2 L_1 - p_z, \gamma - c_2 l_1) - \theta_2$$

(one solution for each θ_1, θ_2)

$$\theta_1 = \operatorname{atan2}(\pm p_y, \pm p_x)$$ (2 solutions) (Fwd. vs. back)

$$\theta_2 = \operatorname{atan2}(B, A) \pm \operatorname{atan2}(\sqrt{A^2 + B^2 - C^2}, C)$$ (2 solutions for each θ_1) (elbow up vs. down)

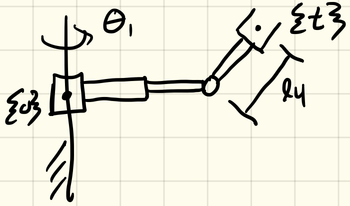
$$\theta_3 = \operatorname{atan2}(-s_2 L_1 - p_z, \gamma - c_2 l_1) - \theta_2$$



Another Example: RPR

solve for: θ_1, d_2, θ_3

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} -s_1(d_2 - l_4 s_3) \\ c_1(d_2 - l_4 s_3) \\ l_4 c_3 \end{bmatrix}$$



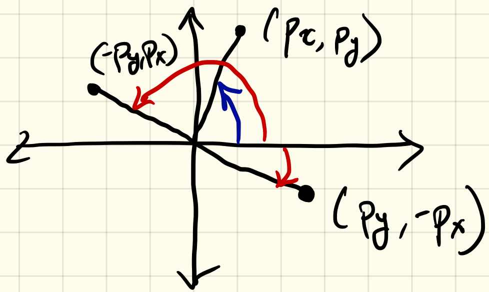
Solve for θ_1:

$$\theta_1 = \text{atan2}(\mp p_x, \pm p_y) \quad (2 \text{ solutions})$$

Alternate approach:

$$c_1 \underbrace{p_x}_{A} + s_1 \underbrace{p_y}_{B} = \underbrace{0}_{C} \Rightarrow \theta_1 = \text{atan2}(B, A) \pm \text{atan2}(\sqrt{A^2 + B^2 - C^2}, C)$$

$$= \text{atan2}(p_y, p_x) \pm 90^\circ$$



Another Example: RPR

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} -s_1(d_2 - l_4 s_3) \\ c_1(d_2 - l_4 s_3) \\ l_4 c_3 \end{bmatrix}$$

Solve for d_2:

① remove dependence on θ_1

$$\underbrace{-s_1 p_x + c_1 p_y}_{\text{define as } \gamma} = (-s_1)^2 (d_2 - l_4 s_3) + (c_1)^2 (d_2 - l_4 s_3)$$

$$\Rightarrow \gamma = d_2 - l_4 s_3$$

② Isolate s_2 and c_2

$$\left.\begin{aligned} l_4 s_3 &= d_2 - \gamma \\ l_4 c_3 &= p_z \end{aligned}\right] \text{ Square \& Add}$$

$$l_4^2 = d_2^2 - 2\gamma d_2 + \gamma^2 + p_z^2$$

$$0 = d_2^2 \underbrace{- 2\gamma}_{b} d_2 + \underbrace{(\gamma^2 + p_z^2 - l_4^2)}_{c}$$

$$\boxed{d_2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2}}$$

(Quadratic formula)

Solve for θ_3:

$$\theta_3 = \operatorname{atan2}\left(\frac{d_2 - \gamma}{l_4}, \frac{p_z}{l_4}\right)$$