Lecture 7 - Kinematics Under DH

Goals For Today:

- Finish PUMA 560 Example
- Intro to Kinematics w/ DH

Fuzziest Point:
* Using homogeneous transforms (4)
* Common normals (3)
* Earth-fixed and body-fixed rotations (3)
* Normals/axis direction (2)
* DH Tables (2)
* Chaining homogeneous transforms (2)
* Drawing frames / spatial geometry (2)
* Euler angles (2)
* DH Convention
* Transformations between coordinate systems
* Notation
* What defines the zero configuration?

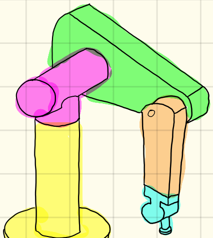
Application for for forward kin

Supplemental Videos

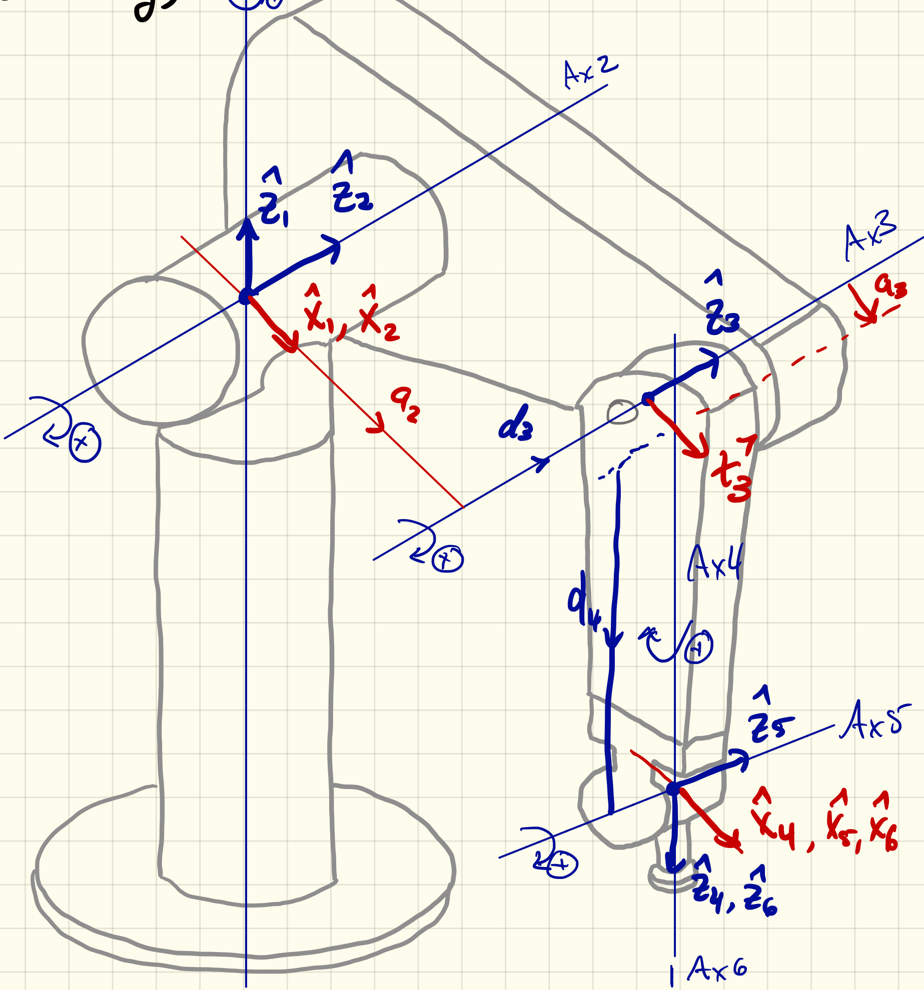
Feedback:
* Spend less time on recap & foreshadowing what's to come.

PUMA 560 (Shown in the zero config)

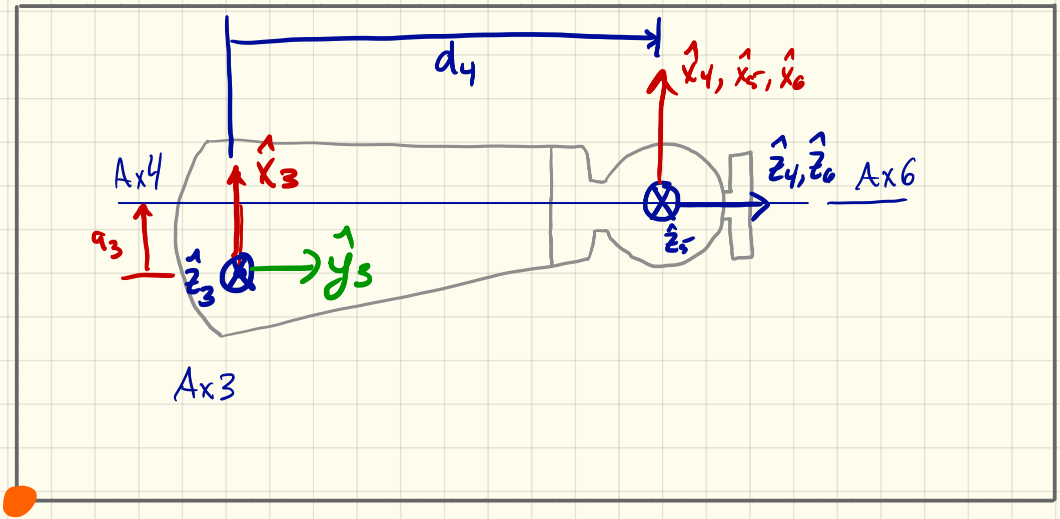
	↺ $\hat{x}_{i-1}$	↑ $\hat{x}_{i-1}$	↑ $\hat{z}_i$	↺ $\hat{z}_i$
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

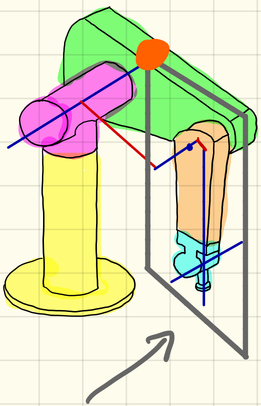


① Axes



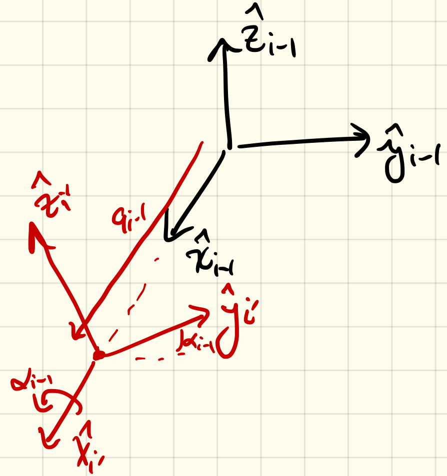
Wrist Kinematics





$$ {}^3R_6 \quad \underbrace{(\theta_4, \theta_5, \theta_6)}_{Y-Z-Y \text{ Euler angles}} $$

DH: Homogeneous Transforms



$$ {}^{i-1}T_i = \underbrace{{}^{i-1}T_{i'}}_{\substack{\text{axial screw} \\ \hat{x}_{i-1}}} \; \underbrace{{}^{i'}T_{i''}}_{\substack{\text{axial screw} \\ \hat{z}_i}} $$

$$ {}^{i-1}T_{i'} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & a_{i-1} \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{i'}T_{i''} = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

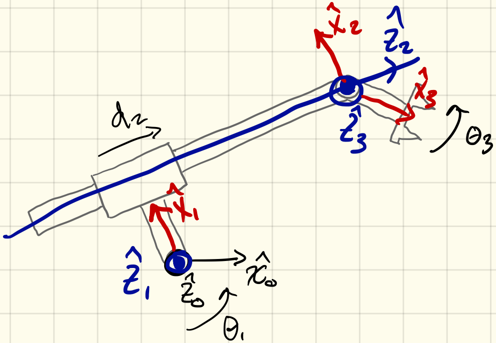
$$ {}^{i-1}T_i(\alpha_{i-1}, a_{i-1}, d_i, \theta_i) = \begin{bmatrix} c\theta & -s\theta & 0 & a \\ c\alpha s\theta & c\theta c\alpha & -s\alpha & -s\alpha d \\ s\alpha s\theta & c\theta s\alpha & c\alpha & c\alpha d \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_n = {}^{0}T_1 \, {}^{1}T_2 \cdots {}^{n-1}T_n $$

given by formula above

DH turns analyzing spatial geometry into matrix multiplication

i	α_{i-1}	a_{i-1}	d_i	θ_i
	$\circlearrowleft \hat{x}_{i-1}$	$\uparrow \hat{x}_{i-1}$	$\uparrow \hat{z}_i$	$\circlearrowleft \hat{z}_i$
1	0	0	0	θ_1
2	$90°$	a_1	d_2	0
3	$-90°$	0	0	θ_3



DH Homogeneous Transform:

$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1} s\theta_i & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\alpha_{i-1} s\theta_i & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Goal: $ {}^{0}T_3 = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 $

i	α_{i-1} ($\circlearrowleft \hat{x}_{i-1}$)	a_{i-1} ($\uparrow \hat{x}_{i-1}$)	d_i ($\uparrow \hat{z}_i$)	θ_i ($\circlearrowleft \hat{z}_i$)
1	0	0	0	θ_1
2	90°	a_1	d_2	0
3	-90°	0	0	θ_3