Lecture 8 - Kinematics Examples (2D)

DH Homogeneous Transform:

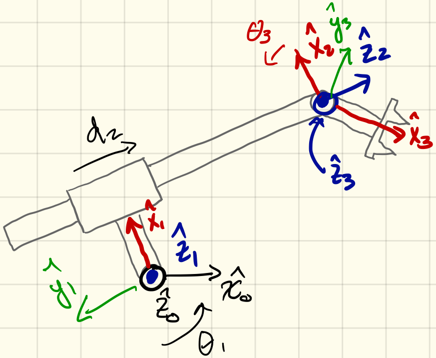
$$ {}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1} s\theta_i & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\alpha_{i-1} s\theta_i & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Goals For Today:

- Kinematics in 2D
  - Solution via homogeneous transforms
  - Solution via direct spatial reasoning

RPR: DH Fwd Kin.

	$\circlearrowleft \hat{x}_{i-1}$	$\uparrow \hat{x}_{i-1}$	$\uparrow \hat{z}_i$	$\circlearrowleft \hat{z}_i$
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	a_1	d_2	0
3	-90°	0	0	θ_3



DH Homogeneous Transform:

$$
{}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1} s\theta_i & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\alpha_{i-1} s\theta_i & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Goal: ${}^0T_3 = {}^0T_1 \, {}^1T_2 \, {}^2T_3$

$$
{}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_2 = {}^0T_1 \, {}^1T_2 = \begin{bmatrix} c_1 & 0 & s_1 & a_2 c_1 + d_2 s_1 \\ s_1 & 0 & -c_1 & a_2 s_1 - d_2 c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_2 \, {}^2T_3 = \begin{bmatrix} c_1 c_3 - s_1 s_3 & -c_1 s_3 - s_1 c_3 & 0 \\ s_1 c_3 + c_1 s_3 & -s_1 s_3 + c_1 c_3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
= \boxed{\begin{bmatrix} c_{13} & -s_{13} & 0 & a_1 c_1 + d_2 s_1 \\ s_{13} & c_{13} & 0 & a_1 s_1 - d_2 c_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
$$

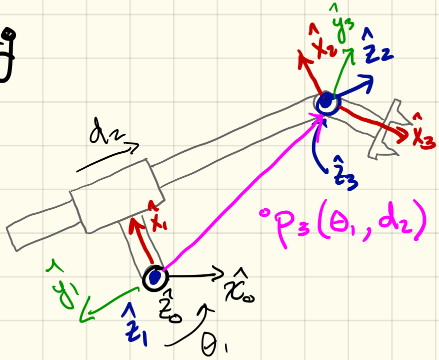
Trig Identities

$$c\alpha \, c\beta \pm s\alpha \, s\beta = \cos(\alpha \mp \beta)$$

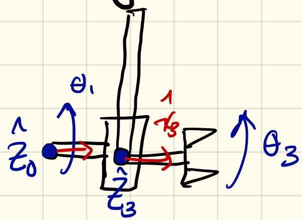
$$s\alpha \, c\beta \pm c\alpha \, s\beta = \sin(\alpha \pm \beta)$$

RPR: Spatial Reasoning

	$\circlearrowleft \hat{x}_{i-1}$	$\uparrow \hat{x}_{i-1}$	$\uparrow \hat{z}_i$	$\circlearrowleft \hat{z}_i$
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$90°$	a_1	d_2	0
3	$-90°$	0	0	θ_3



Draw The Mechanism in its zero configuration.

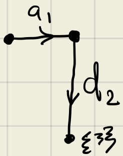


(since $^0p_3$ and axis of rotation originate @ the origin of $\{0\}$)

$$^0p_3 = c_1\, {}^0p_3(0°, d_2) + S_1\, {}^0p_3(90°, d_2)$$

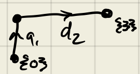
$$^0T_3 = \begin{bmatrix} {}^0R_3 & {}^0p_3 \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

① when $\theta_1 = 0°$



$$^0p_3 = \begin{bmatrix} a_1 \\ -d_2 \\ 0 \end{bmatrix}$$

② when $\theta_1 = 90°$



$$^0p_3 = \begin{bmatrix} d_2 \\ a_1 \\ 0 \end{bmatrix}$$

$$^0R_3 = R_z(\theta_1 + \theta_3)$$

$$= \begin{bmatrix} C_{13} & -S_{13} & 0 \\ S_{13} & C_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

③ Thus

$$^0p_3 = C_1 \begin{bmatrix} a_1 \\ -d_2 \\ 0 \end{bmatrix} + S_1 \begin{bmatrix} d_2 \\ a_1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1c_1 + d_2S_1 \\ -d_2c_1 + a_1S_1 \\ 0 \end{bmatrix}$$