6/

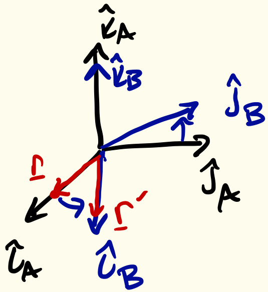
Reference Frames

Examples with Moving Frames

- Admin
  - HW 1 : Solutions online
  - HW 2 : Due ~~Wendesday~~ Friday 4PM to my mailbox (Fitz 365)
  - MT 1 : Next Wednesday (Debartolo 140, Experiment: Open Everything Except peers)
- Last time
  - Moving frames & angular velocity
  - Relative velocities and accelerations
- Today
  - HW 1 Revisited
  - Examples w/ Moving frames
  - Review of 2D Rigid body Dynamics (Maybe)

SEE Panopto for HW1 Review

HW2 Hint: 3 uses of Rotation Matrices



① ${}^A R_B$ describes orientation of frame B

$${}^A R_B = \left[{}^A\{\hat{i}\} \ {}^A\{\hat{j}\} \ {}^A\{\hat{k}\} \right]$$

② ${}^A R_B$ allows us to change coordinates used to express a fixed vector

$${}^A\{\underline{r}\} = {}^A R_B \, {}^B\{\underline{r}\} \qquad (\underline{r} \text{ doesn't move})$$

③ ${}^A R_B$ can be used as an operator that actively rotates vectors. It applies the same operation as the one needed to transform frame A to frame B.

Let $R = {}^A R_B$

$$\underbrace{{}^A\{r'\}}_{\text{After rotation}} = R \, \underbrace{{}^A\{r\}}_{\text{Before rotation}}$$

6/

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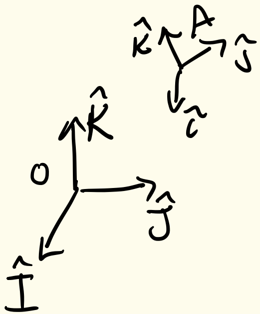
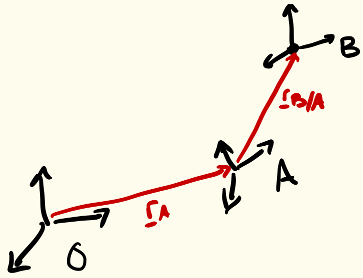


Main Results From Last Time:



$$\underline{\omega}_B = \underline{\omega}_A + \underline{\omega}_{B/A}$$

$$\underline{\omega}_{B/A} = x\,\hat{i}_A + y\,\hat{j}_A + z\,\hat{k}_A$$

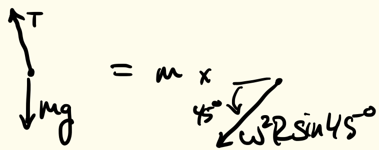
$$\dot{\underline{\omega}}_{B/A} = \underbrace{\dot{x}\,\hat{i}_A + \dot{y}\,\hat{j}_A + \dot{z}\,\hat{k}_A}_{\coloneqq\, \dot{\underline{\omega}}_{B/A,rel}} + \underline{\omega}_A \times \underline{\omega}_{B/A}$$

$$\underline{V}_B = \underline{V}_A + \underline{V}_{B/A,rel} + \underline{\omega}_A \times \underline{r}_{B/A}$$

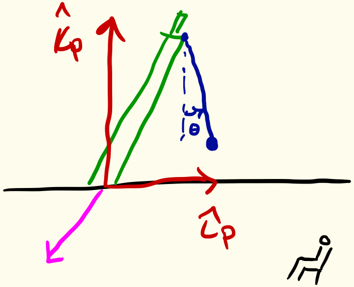
$$\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}}_A \times \underline{r}_{B/A} + \underline{\omega}_A \times (\underline{\omega}_A \times \underline{r}_{B/A}) + 2\,\underline{\omega}_A \times \underline{V}_{B/A,rel} + \underline{a}_{B/A,rel}$$

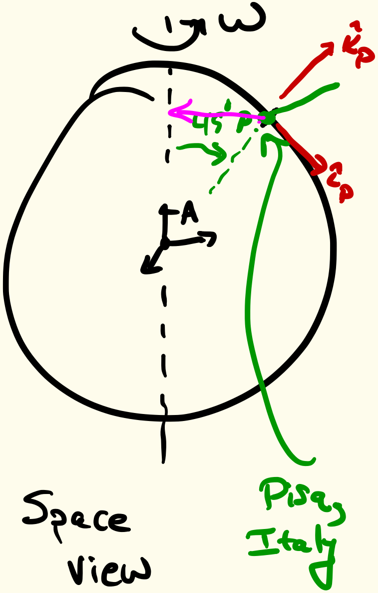
Example: Frame A @ earth's core. You head to Pisa, hang a plumbob from leaning tower. What angle does it make w/ the line $\overrightarrow{AP}$?

Free Body Diagram (Pisa View)



Pisa View





$$m\underline{a}_P = -mg\hat{k} + T\hat{e}_T$$

$$a_P = \cancel{a_A}^{\,0} + \cancel{\dot{\omega}_A}^{\,0} \times r_{P/A} + \omega_A \times \omega_A \times r_{P/A} + 2\omega_A \times \cancel{v_{P/A,rel}} + \cancel{a_{P/A,rel}}^{\,0}$$

$$\uparrow\omega \times (\uparrow\omega \times \nearrow R)$$

$$\uparrow\omega \times \otimes\, \omega R \sin 45^\circ$$

$$\leftarrow \omega^2 R \sin 45^\circ$$

Drawn in Space View

$$\theta = 1.7 \text{ millirad}$$

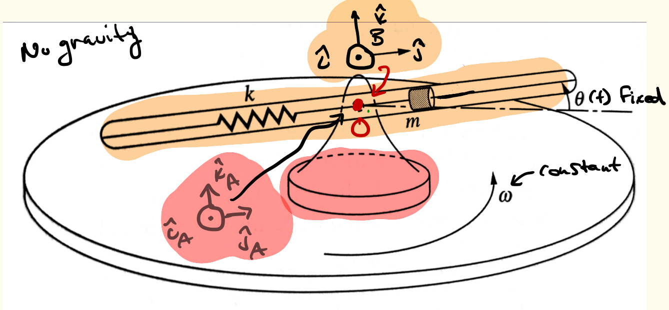
$$T\cos\theta = mg - \sin^2 45^\circ\, m\omega^2 R$$

$$T\sin\theta = m\omega^2 R \sin^2 45^\circ$$

$$\tan\theta = \frac{\omega^2 R \sin^2 45^\circ}{g - \cancel{\omega^2 R \sin^2 45^\circ}} = \frac{\omega^2 R \sin^2 45^\circ}{g}$$

Find the equations of motion for the mass.

Let y its distance from $\hat{k}$ along $\hat{\imath}$



$$\underline{\omega}_A = \omega\,\hat{k}_A \qquad \underline{r}_{m/A} = y\hat{\jmath} = y c_\theta\,\hat{\jmath}_A + y s_\theta\,\hat{k}_A$$

$$^A\{\underline{v}_{m/A,rel}\} = \frac{d}{dt}\,^A\{\underline{r}_{m/A}\}$$

$$^A\{a_{m/A,rel}\} = \frac{d^2}{dt^2}\,^A\{r_{m/A}\}$$

$$\underline{a}_m = \underline{a}_A + \dot{\underline{\omega}}_A \times \underline{r}_{m/A} + \underline{\omega}_A \times \underline{\omega}_A \times \underline{r}_{m/A} + 2\underline{\omega}_A \times \underline{v}_{m/A,rel} + \underline{a}_{m/A,rel}$$

TO BE CONTINUED