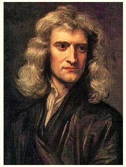
2/

Newtonian Particle Dynamics

Impulse, Momentum, Work, & Energy



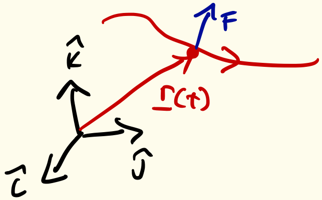
- Admin
  - HW 0: Will recap on Wednesday
  - Drop / Change Deadline: Tuesday
  - HW 1: Due next Wednesday
  - Office Hours: 5:00 - 6:30 DeBart 232

- Last time: Emphasis on differences in the approaches of Newton (vectorial) and those that followed (variational)

- Next few weeks: Newtonian particle dynamics building to planar dynamics of bodies. (More calculus focus to set the stage.)

- Today: Newtons "Laws", Momentum (linear and angular), Impulse, Work, & Energy

Newton's Laws for Particles (i.e., point masses)



① If no forces act on a particle its linear momentum is conserves

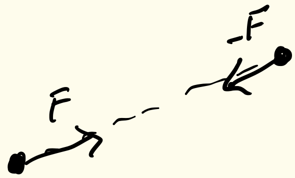
Linear momentum: $\underline{p}(t) = m\,\dot{\underline{r}}(t) = m\,\underline{v}(t)$

② The rate of change of linear momentum of a particle is equal to the sum of all forces applied to it

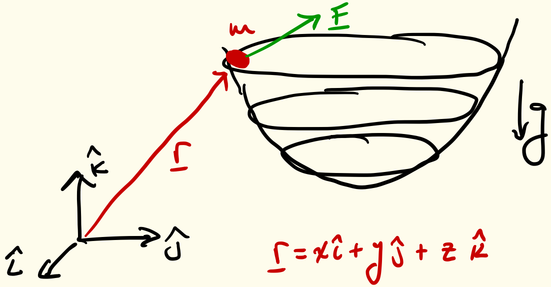
$$\dot{\underline{p}}(t) = \underline{F}(t) \qquad m\,\underline{a}(t) = \underline{F}(t)$$

Force: effect of one body on another

③ If two particles exert forces on one another they are equal and opposite, along the same line



Degrees of Freedom and Constraints:



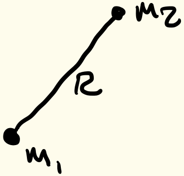
$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$z = h(x,y)$

Degrees of Freedom: How many #'s it takes to uniquely define the config of a system

$$\#\text{DoFs} = \underbrace{\#\text{coordinates}}_{\text{3 per particle, 6 per body}} - \#\text{Independent constraints}$$

Example: Two particles connected by a rod



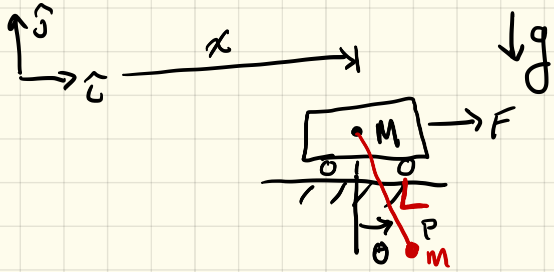
- 6 coordinates: $x_1, y_1, z_1, x_2, y_2, z_2$
- 1 constraint: $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = R^2$

Five Degrees of freedom

Key: Constraint forces perpendicular to constraint surface

Exercise @ home: Force $\underline{F}$ above in direction $-\frac{\partial h}{\partial x}\hat{i} - \frac{\partial h}{\partial y}\hat{j} + \hat{k}$

Example: Cart Pole



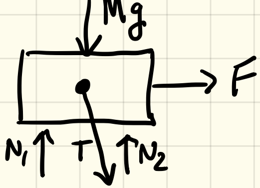
• Two degrees of freedom: x, θ

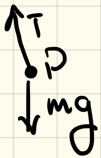
$$x_p = x + L\overbrace{\sin\theta}^{S_\theta} \qquad y_p = -L\overbrace{\cos\theta}^{C_\theta}$$

$$\dot{x}_p = \dot{x} + L C_\theta \dot{\theta} \qquad \dot{y}_p = L S_\theta \dot{\theta}$$

$$\ddot{x}_p = \ddot{x} + L C_\theta \ddot{\theta} - L S_\theta \dot{\theta}^2 \qquad \ddot{y}_p = L S_\theta \ddot{\theta} + L C_\theta \dot{\theta}^2$$

Free Body Diagrams:





$$M\ddot{x} = F + T\sin\theta$$

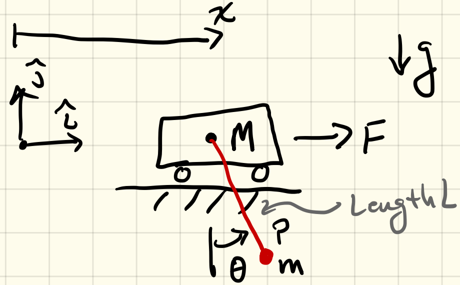
$$-m\ddot{x}_p = +T\sin\theta$$

$$M\ddot{x} + m\ddot{x}_p = F$$

$$\boxed{(m+M)\ddot{x} + mL\left(C_\theta\ddot{\theta} - S_\theta\dot{\theta}^2\right) = F \quad (1)}$$

SEE SAKAI FOR MATLAB CODE

Example: Cart Pole

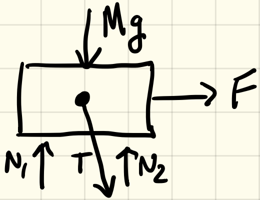


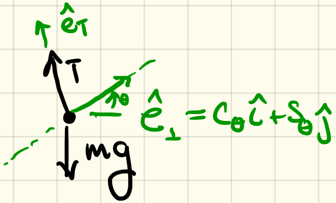
Two degrees of freedom: x, θ

$$\ddot{x}_p = \ddot{x} + L c_\theta \ddot{\theta} - L s_\theta \dot{\theta}^2 \qquad \ddot{y}_p = L s_\theta \ddot{\theta} + L c_\theta \dot{\theta}^2$$

Newton's laws for m: in direction $\hat{e}_\perp$

$$\left[m\left(\ddot{x}_p \hat{\imath} + \ddot{y}_p \hat{\jmath}\right) = T\hat{e}_T - mg\hat{\jmath} \right] \cdot \hat{e}_\perp$$





$$m\ddot{x}_p c_\theta + m\ddot{y}_p s_\theta = -mg s_\theta$$

$$\boxed{\begin{aligned} & mL\ddot{\theta} + m\ddot{x} c_\theta + mg s_\theta = 0 && (2) \\ & (m+M)\ddot{x} + mL\left(c_\theta \ddot{\theta} - s_\theta \dot{\theta}^2\right) = F && (1) \end{aligned}}$$

System of two second-order diffeqs in (x, θ)

No closed form solution

SEE SAKAI FOR MATLAB CODE

Impulse - Momentum Theorem

2nd Law: $\underline{F}(t) = \dot{\underline{p}}(t)$

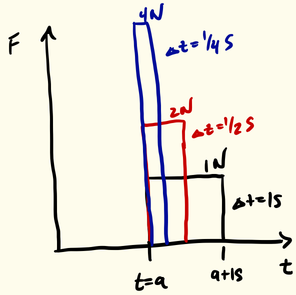
We integrate over time to obtain momentum

$$\int_{t_1}^{t_2} \underline{F}(t) = \int_{t_1}^{t_2} \dot{\underline{p}}(t)\,dt = \underline{p}(t_2) - \underline{p}(t_1) = \Delta p$$

$$\underline{p}(t_2) = \underline{p}(t_1) + \underbrace{\int_{t_1}^{t_2} \underline{F}(t)\,dt}_{\substack{\text{Impulse} \\ \text{of } \underline{F}(t) \text{ over} \\ [t_1, t_2]}}$$

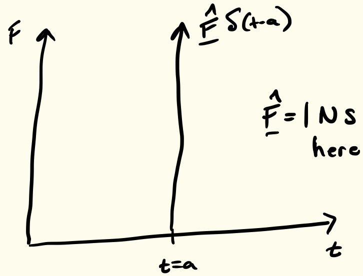
Impulsive Loading

- Impulses can also be used to approximate large forces over short periods



$$\int_a^{a+\Delta t} \underline{F}\, dt = 1\ N\cdot s$$

Even as $\Delta t \to 0$



- Approximate F(t) for small Δt via

$\underline{F}(t) = \hat{\underline{F}}\,\delta(t-a)$ where $\delta(t-a)$ is the Dirac Delta

$$\boxed{\begin{array}{l} \delta(t) = 0 \text{ if } t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \end{array}}$$

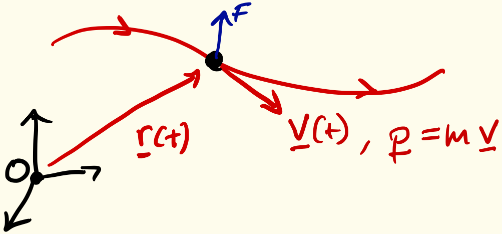
- Effect: Instantaneous change in velocity

$$m \Delta \underline{v} = \hat{\underline{F}}$$

without change in position.

Angular Momentum

- Consider the moment of the linear momentum about O



$$\underline{H}_o = \underline{r} \times m\underline{v}$$

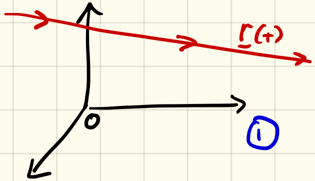
- Let's see how it evolves

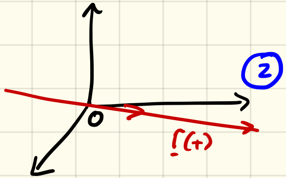
$$\underline{\dot{H}}_o = \cancel{\underline{\dot{r}} \times m\underline{v}} + \underline{r} \times m\underline{\dot{v}} = \underline{r} \times m\underline{a} = \underline{r} \times \underline{F}$$

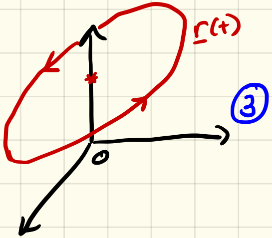
- Moment of the force $\underline{F}$ about O: $\underline{M}_o = \underline{r} \times \underline{F}$

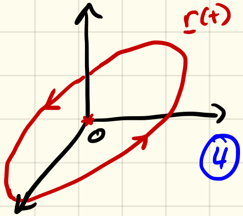
$$\Rightarrow \underline{\dot{H}}_o = \underline{M}_o$$

$\Rightarrow$ Angular Impulse momentum Thm: $\underline{H}_o(t_2) = \underline{H}_o(t_1) + \int_{t_1}^{t_2} M_o(t)\,dt$









1 Only when velocity is constant

2 Always: r and F in same direction

3 Never. Force is in plane of motion, always creates moment about O!

4 Only when $|\dot{r}(t)|$ is constant (no tangential acceleration)

In which cases is angular momentum about O constant?

a) Only sometimes for all cases

b) Always in 2, 3, and 4, but never in 1

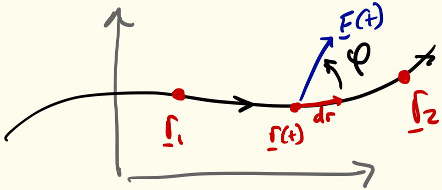
c) Always for 2 and 4, sometimes for 1 and 3

d) Always for 2, sometimes for 1 and 4, never for 3

e) None of the above.

Work & Energy:

- Incremental work done over an incremental distance $d\underline{r}$



$$dW = \underline{F} \cdot d\underline{r} = |\underline{F}||d\underline{r}| \cos\varphi$$

$$= m\underline{a} \cdot d\underline{r}$$

$m|\underline{v}|^2 =$ "vis viva"
life force

- Kinetic Energy of Particle: $T = \frac{1}{2} m |\underline{v}(t)|^2 = \frac{1}{2} m \underline{v} \cdot \underline{v}$

$$dW = \quad \ldots \quad = dT$$

Next Time

Summary

	Linear	Angular
Momentum	$\underline{p} = m\underline{v}$	$\underline{H}_o = \underline{r} \times \underline{p}$
Rate	$\dot{\underline{p}} = \underline{F} = m\underline{a}$	$\dot{\underline{H}}_o = \underline{M}_o = \underline{r} \times \underline{F}$
Impulse/ Momentum Principles	$\underline{p}(t_2) = \underline{p}(t_1) + \int_{t_1}^{t_2} \underline{F}(t)\, dt$	$\underline{H}_o(t_2) = \underline{H}_o(t_1) + \int_{t_1}^{t_2} \underline{M}_o(t)\, dt$

- These vectorial relationships can be specialized into certain directions.
- All derived (so far) from newton's second law.