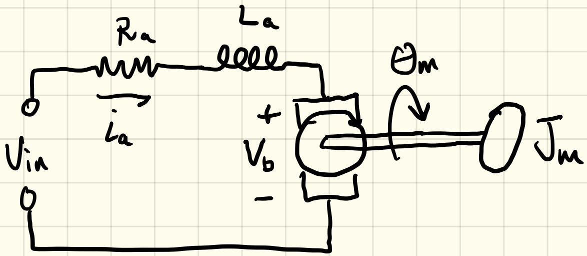
Lecture 37 - Motor Modelling & Control

- Introduce Motor Model
    - Electrical Dynamics
    - Mechanical Dynamics
    - Coupling
- Inclusion of Gearbox
    - Implications For Control

Control of a single DoF: DC Motor Model



L_a: Motor Inductance

R_a: Motor Resistance

V_b: Back "emf" Voltage

- As the motor spins it creates some voltage (Lenz's law, Faraday's law of induction)

$$V_b = K_b \dot{\theta}_m$$

"electromotive force"

K_b ← Back emf constant

Units: Vs/rad

- Voltage across inductor $= L_a \frac{d}{dt} i_a$

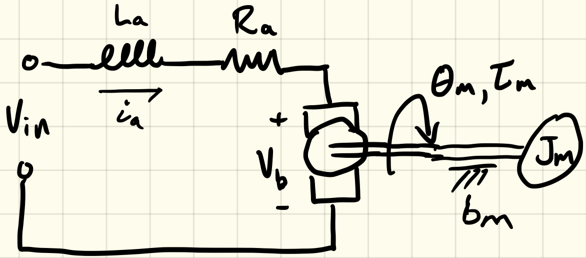
- Voltage around loop

$$V_{in} = R_a i_a + L_a \frac{d}{dt} i_a + K_b \dot{\theta}_m$$

Power in electrical Domain

$$P_m = i_a V_b = i_a (K_b \dot{\theta}_m)$$

DC Motor Model: Mechanical Side



$$\tau_m = b_m \dot{\theta}_m + J_m \ddot{\theta}_m$$

$$\tau_m = K_\tau \, i_a$$

Torque constant

Units: Nm/A

Power in Mechanical Domain: $P_m = \tau_m \dot{\theta}_m$

$$P_m = \underbrace{\tau_m \dot{\theta}_m = K_\tau \, i_a \dot{\theta}_m}_{\text{mechanical}} = \underbrace{V_b \, i_a = K_b \dot{\theta}_m \, i_a}_{\text{Electrical}}$$

Vs/rad   Nm/A

$$K_b = K_\tau \quad ?$$

How are these the same?

$$V = \frac{\text{"Potential Energy"}}{\text{"Charge"}} = \frac{J}{A \cdot s}$$

$$\frac{V \cdot s}{\text{rad}} = \frac{J}{A} = \frac{Nm}{A} \quad \checkmark$$

Coupling Domains:

Motor Torque: $K_\tau i_a = J_m \ddot{\theta}_m + b_m \dot{\theta}_m$

Input Voltage: $V_{in} = \cancel{L_a} \frac{d}{dt} i_a + R_a i_a + K_b \dot{\theta}_m$

small

$$V_{in} = \frac{R_a}{K_\tau} J_m \ddot{\theta}_m + \left(K_b + \frac{R_a b_m}{K_\tau}\right)\dot{\theta}_m$$

Laplace Transform:

Transfer Function

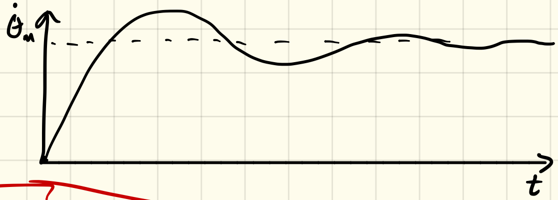
$$\frac{\Theta_m(s)}{V_{in}(s)} = \frac{K_\tau / R_a}{s\left[J_m s + \left(b_m + \frac{K_b K_\tau}{R_a}\right)\right]}$$

Physical Damping

damping due to back emf

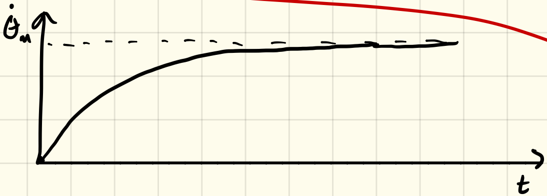
Suppose a constant Voltage is applied which are possible?

(a)



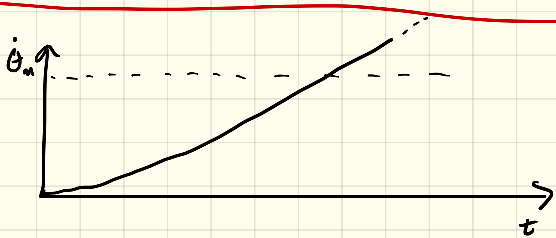
$$\frac{\Theta_m(s)}{V_{in}(s)} = \frac{K_T/R_a}{s\left[J_m s + \left(b_m + \frac{K_b K_T}{R_a}\right)\right]}$$

(b)



$$\frac{\dot{\Theta}_m(s)}{V_{in}(s)} = \frac{K_T/R_a}{\left[J_m s + \left(b_m + \frac{K_b K_T}{R_a}\right)\right]}$$

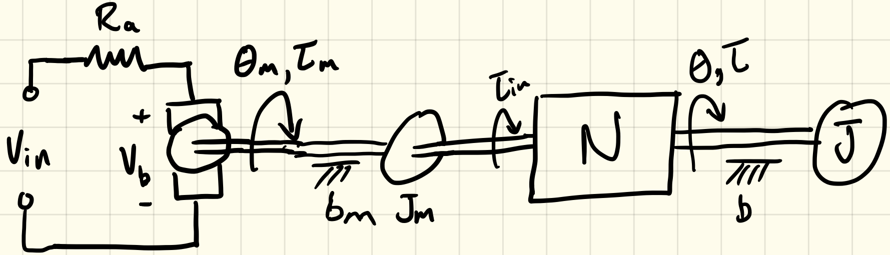
(c)



(d) (a) & (b)

(e) (a), (b), & (c)

DC Motor Model: Adding a Gearbox



$\tau = N \tau_{in}$

$\theta = \frac{1}{N} \theta_m$

Torque @ output of gearbox

$$\tau = N \tau_{in} = N\left[\tau_m - b_m \dot{\theta}_m - J_m \ddot{\theta}_m\right] = b\dot{\theta} + J\ddot{\theta}$$

$$N\tau_m = (b + N^2 b_m)\dot{\theta} + (J + N^2 J_m)\ddot{\theta}$$

inertia @ output

In terms of motor angles

$$\tau_m = \left(b_m + \frac{b}{N^2}\right)\dot{\theta}_m + \left(J_m + \frac{J}{N^2}\right)\ddot{\theta}_m$$

Recap:

- DC Electric Motor Models
  - Coupling between domains $\tau = K_\tau i$, $V_b = K_b \dot{\theta}_m$
  - Back EMF in electrical domain manifests as damping in the mechanical domain
- Common Applications of DC Electric motors
  - High gear ratio reductions to amplify motor torque
  - As a result motor dynamics dominate

    $\Rightarrow$ Linear Control is common