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Exam Review

Reminder

- Exam Wednesday DeBart 140

Tasks

1. Go over exam format
2. Concepts
3. Practice Problems

Concepts Review: Conservative Forces:

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

- Depends on position of particle only
  - In the form

$$\underline{F}(\underline{r}) = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$ for some potential function V

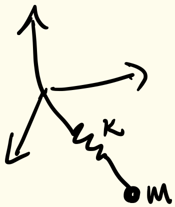
- If only conservative forces

  $E = T + V$ is conserved

- Be comfortable going from $V \Leftrightarrow \underline{F}$
- If a particle has degrees of freedom $\theta = [\theta_1, \theta_2]^T$

Let $V'(\theta) = V(\underline{r}(\theta)) \Rightarrow \frac{\partial V'}{\partial \theta_i} = 0$ @ equilibrium

Example: Spring Force



- Consider a Hookean spring w/ rest Length L_0
- What is the force on the mass $\underline{F}(\underline{r})$ from the spring?

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\underline{F}(\underline{r}) = -k\left(\sqrt{x^2+y^2+z^2} - L_0\right)\cdot\hat{s} \qquad \hat{s} = \frac{\underline{r}}{|\underline{r}|}$$

$$\underline{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

Alternate approach:

$$V(\underline{r}) = \frac{1}{2}k\left(\overbrace{\sqrt{x^2+y^2+z^2}}^{\ell} - L_0\right)^2$$

$$F_x = -\frac{\partial V}{\partial x} = -k(\ell - L_0)\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x}$$

$$= -k(\ell - L_0)\frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}}\cdot\cancel{2}x$$

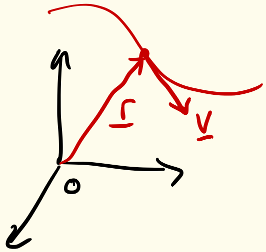
$$= -k(\ell - L_0)\frac{x}{\ell}$$

$$\underline{F} = -k(\ell - L_0)\frac{(x\hat{i} + y\hat{j} + z\hat{k})}{\ell}$$

Concept Review:

- Linear momentum $\underline{p} = m\underline{v}$ $\qquad \dot{\underline{p}} = \underline{F}_{net}$

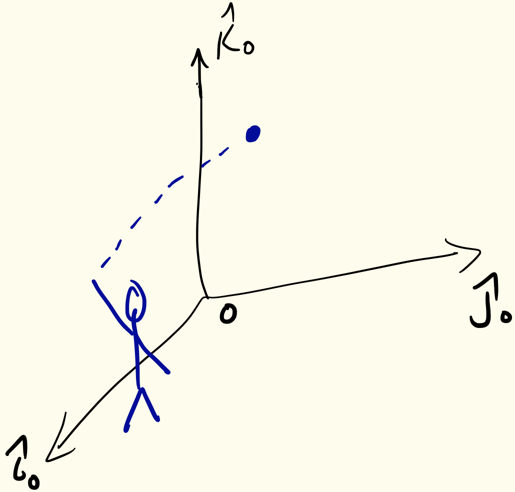
- Angular momentum $\underline{H}_0 = \underline{r} \times \underline{p}$ $\qquad \dot{\underline{H}}_0 = \underline{M}_0 = \underline{r} \times \underline{F}_{net}$



- $T = \frac{1}{2} m \underline{v}^T \underline{v}$

- Integral Motion
  - Conserved quantity

- Example: projectile motion. What are the integrals of motion?
  - Total energy ✓
  - Linear momentum along $\hat{i}_0, \hat{j}_0$
  - Angular momentum about $\hat{k}_0$

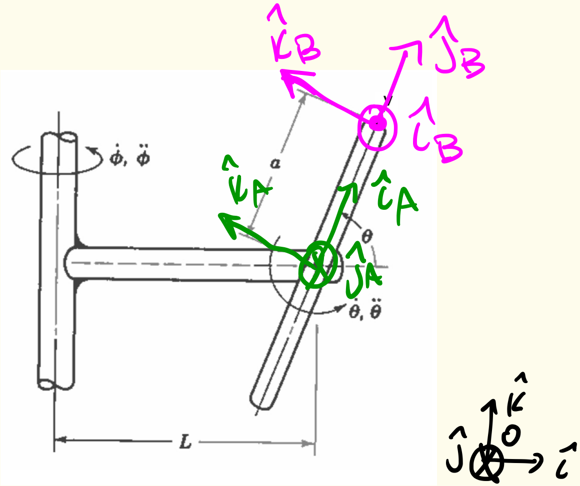


(33 pts) In the figure, a particle is ejected from the tip of the bar of length $2a$ with a constant velocity of v relative to and parallel to the bar in the position shown.

(a) Give ${}^{O}R_A$ and ${}^{A}R_O$ at the instant shown:

$$
{}^{O}R_A = \left[\, {}^{O}\{\hat{i}_A\} \;\; {}^{O}\{\hat{j}_A\} \;\; {}^{O}\{\hat{k}_A\} \,\right] = \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix}
$$

$$
{}^{A}R_O = {}^{O}R_A^{T} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}
$$

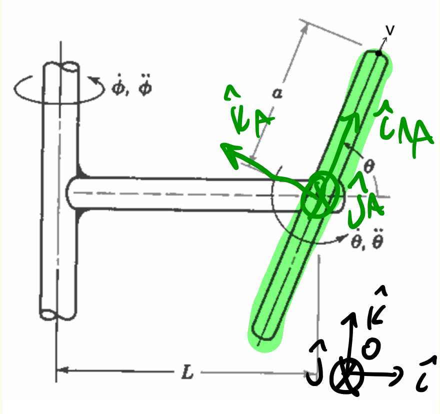


(b) Suppose ${}^{A}R_B$ given. How would you compute ${}^{O}R_B$?

$$
{}^{O}R_B = {}^{O}R_{\cancel{A}}\,{}^{\cancel{A}}R_B
$$

(33 pts) In the figure, a particle is ejected from the tip of the bar of length $2a$ with a constant velocity of v relative to and parallel to the bar in the position shown.

$\{O\}$ inertial. $\{A\}$ rotates w/ green bar.



① Give $\underline{\omega}_A$ at the current instant using $\hat{i},\hat{j},\hat{k}$

$$\boxed{\underline{\omega}_A = \dot\phi\,\hat{k} - \dot\theta\,\hat{j}} \qquad {}^O\{\underline{\omega}_A\} = \begin{bmatrix} 0 \\ -\dot\theta \\ \dot\phi \end{bmatrix}$$

② Give $\underline{\omega}_A$ at the current instant using $\hat{i}_A, \hat{j}_A, \hat{k}_A$

$${}^A\{\underline{\omega}_A\} = {}^A R_O\, {}^O\{\underline{\omega}_A\} = \begin{bmatrix} \dot\phi S_\theta \\ -\dot\theta \\ \dot\phi C_\theta \end{bmatrix}$$

$$\Rightarrow \underline{\omega}_A = \dot\phi S_\theta\,\hat{i}_A - \dot\theta\,\hat{j}_A + \dot\phi C_\theta\,\hat{k}_A$$

③ Give $\underline{\dot\omega}_A$ at the current instant using $\hat{i},\hat{j},\hat{k}$

$$\underline{\omega}_A = \dot\phi\,\hat{k} - \dot\theta\,\hat{j}_A$$

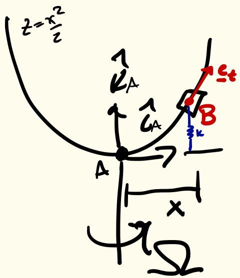
Using $\hat{j}_A$ makes $\underline{\omega}_A$ valid for all times. Not just the current instant.

$$\underline{\dot\omega}_A = \ddot\phi\,\hat{k} - \ddot\theta\,\hat{j}_A - \dot\theta\,\overbrace{\underbrace{\underline{\omega}_A \times \hat{j}_A}_{(\uparrow\dot\phi + \odot\dot\theta)\times\otimes}}^{\dot{\hat{j}}_A}$$

$$\leftarrow\dot\phi + 0$$

$$= \ddot\phi\,\hat{k} - \ddot\theta\,\hat{j} + \hat{i}\,\dot\theta\dot\phi$$

$\hat{i}_A, \hat{j}_A, \hat{k}_A$ rotating @ $\Omega \hat{K}$ with wire: Find the EOM



$$r_{B/A} = x\hat{i}_A + \frac{x^2}{2}\hat{k}_A$$

$$\underline{v}_B = \cancel{\underline{v}_A}^{\,0} + \underline{\omega}_A \times r_{B/A} + v_{B/A,rel}$$

$$= \uparrow\Omega \times \nearrow_{x}^{z} + \dot{x}\hat{i}_A + x\dot{x}\hat{k}_A$$

$$= \otimes\Omega x + \dot{x}\hat{i}_A + x\dot{x}\hat{k}_A$$

$$= \dot{x}\hat{i}_A + \Omega x\hat{j}_A + x\dot{x}\hat{k}_A$$

Derivatives taken from A frame

$$\underline{v}_{B/A,rel} = \dot{x}\hat{i}_A + x\dot{x}\hat{k}_A$$

$$\underline{a}_{B/A,rel} = \ddot{x}\hat{i}_A + (\dot{x}^2 + x\ddot{x})\hat{k}_A$$

$$\underline{a}_B = \cancel{a_A} + \cancel{\dot{\omega}_A} \times r_{B/A} + \omega_A \times \omega_A \times r_{B/A} + 2\omega_A \times v_{B/A,rel} + a_{B/A,rel}$$

$$= \uparrow\Omega \times \underbrace{\uparrow\Omega \times \nearrow_{x}^{z}}_{} + 2\uparrow\Omega \times \nearrow_{\dot{x}}^{x\dot{x}} + \nearrow_{\ddot{x}}^{\dot{x}^2 + x\ddot{x}}$$

$$\uparrow\Omega \times \otimes\Omega x + \otimes 2\Omega\dot{x} + \nearrow_{\ddot{x}}^{x\ddot{x}}$$

$$= -\Omega^2 x\hat{i}_A + 2\Omega\dot{x}\hat{j}_A + \ddot{x}\hat{i}_A + x\ddot{x}\hat{k}_A$$

$$e_t = \hat{i}_A + \overset{\frac{dz}{dx}}{x}\hat{k}_A$$

Newton's Eqn: in direction e_t

$$m\,\underline{a}_B \cdot e_t = \left(-mg\hat{k}_A - \frac{Kx^2}{2}\hat{k}_A\right) \cdot e_t$$

$$\boxed{m(\ddot{x} - \Omega^2 x + x^2\ddot{x}) = -mgx - \frac{Kx^3}{2}}$$