Lecture 3 : Chaining Rotations

Announcements :

- HW 1 : Due Friday
- M: 9:30-11:00 , Fitz 373
- Project 0 Wrap-up

Goals for Today:

- Chaining rotations (Body Fixed vs. earth fixed)
- Representing orientation w/ 3 angles (Multiple conventions)

Reasons to take the class

- Robots are cool / general interest in robots ++++++++++++
- Interest in robots in industry / manufacturing / as a career +++++++++
- Formal instruction / foundation in robotics ++++++++++
- Robots cutting edge of industry / emerging applications ++++++
- I enjoyed Controls ++++
- I heard it was a good class +++
- Interest in robotics for grad school +++
- To tie together controls and design / practical applications of control ++
- Looking to talk intelligently about robotics
- Robot football club
- Enjoyed mechanics classes
- Robots in movies
- Replicating nature
- My Philo class got cancelled

Best ways of Learning

- Examples / activities in class +++++++++++++++
- Challenging / ramping homework ++++++++
- Relationship to real world applications ++++
- Straight lectures ++++
- Time to work problems in class / in groups +++
- Hands on learning +++
- Demonstrations / visuals +++
- Office Hours ++
- Homework solutions
- Video Lectures / Extra worked examples
- Key phases
- Projects
- Book
- Visual
- Quiz question at end of lecture

Favorite part of Notre Dame

- **Friends / People / Camaraderie ++++++++++++**
- **Dorm Life +++++++++++**
- **Football +++++**
- **Student groups / clubs ++++**
- **Study abroad +++**
- **Integration with Catholic faith +++**
- **Club sports / Dorm sports +++**
- **Traditions ++**
- **Marching Band**

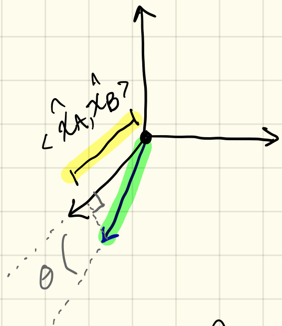
$${}^{A}R_{B} = \begin{bmatrix} {}^{A}\hat{x}_{B} & {}^{A}\hat{y}_{B} & {}^{A}\hat{z}_{B} \end{bmatrix}$$ "Rotation Matrix"

Coordinates of $\hat{x}_B$ in frame $\{A\}$

$$\hat{x}_B = \langle \hat{x}_A, \hat{x}_B \rangle \hat{x}_A + \langle \hat{y}_A, \hat{x}_B \rangle \hat{y}_A + \langle \hat{z}_A, \hat{x}_B \rangle \hat{z}_A$$

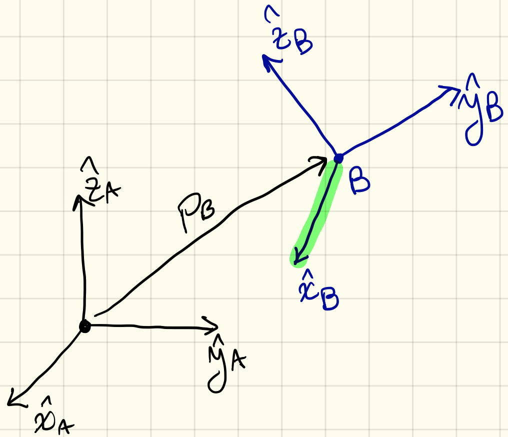
$$\Rightarrow {}^{A}\hat{x}_B = \begin{bmatrix} \langle \hat{x}_A, \hat{x}_B \rangle \\ \langle \hat{y}_A, \hat{x}_B \rangle \\ \langle \hat{z}_A, \hat{x}_B \rangle \end{bmatrix} \Rightarrow {}^{A}R_{B} = \begin{bmatrix} \langle \hat{x}_A, \hat{x}_B \rangle & \langle \hat{x}_A, \hat{y}_B \rangle & \langle \hat{x}_A, \hat{z}_B \rangle \\ \langle \hat{y}_A, \hat{x}_B \rangle & \langle \hat{y}_A, \hat{y}_B \rangle & \langle \hat{y}_A, \hat{z}_B \rangle \\ \langle \hat{z}_A, \hat{x}_B \rangle & \langle \hat{z}_A, \hat{y}_B \rangle & \langle \hat{z}_A, \hat{z}_B \rangle \end{bmatrix}$$

Geometry:



"projection of $\hat{x}_B$ onto $\hat{x}_A$"

$$\langle \hat{x}_A, \hat{x}_B \rangle = \|\hat{x}_A\| \|\hat{x}_B\| \cos\theta$$
$$= \cos\theta$$

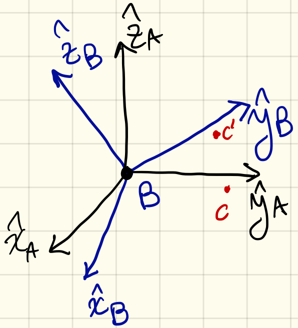


Rotation Matrix $\Leftrightarrow$ Direction cosine matrix

Two uses of Rotation Matricies:

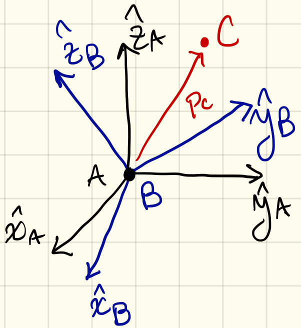
① A transformation operator that rotates all points in space

$$R := {}^A R_B$$



$${}^A p_{c'} = R\,{}^A p_c$$

② Change of Basis (point doesn't move)



$${}^B p_c \Rightarrow {}^A p_c\,?$$

$${}^A p_c = \begin{bmatrix} {}^B\hat{x}_A^T\,{}^B p_c \\ {}^B\hat{y}_A^T\,{}^B p_c \\ {}^B\hat{z}_A^T\,{}^B p_c \end{bmatrix} = {}^B R_A^T\,{}^B p_c$$

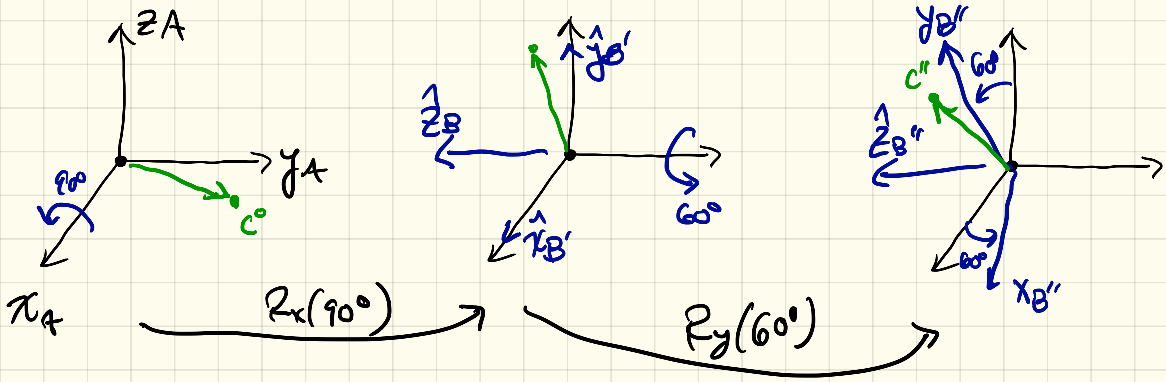
$$= {}^A R_{\cancel{B}}\,{}^{\cancel{B}} p_c$$

<u>Example:</u> Consider a point C fixed in $\{B\}$, and $\{B\}$ initially aligned with $\{A\}$.

① Rotate $\{B\}$ by $90°$ about $\hat{x}_A$

② Then rotate $\{B\}$ by $60°$ about $\hat{y}_A$

What is ${}^A P_{C''}$?



① ${}^A P_{C'} = R_x(90°)\, {}^A P_{C°}$

② ${}^A P_{C''} = R_y(60°)\, {}^A P_{C'}$

$= R_y(60°)\, R_x(90°)\, {}^A P_{C°}$

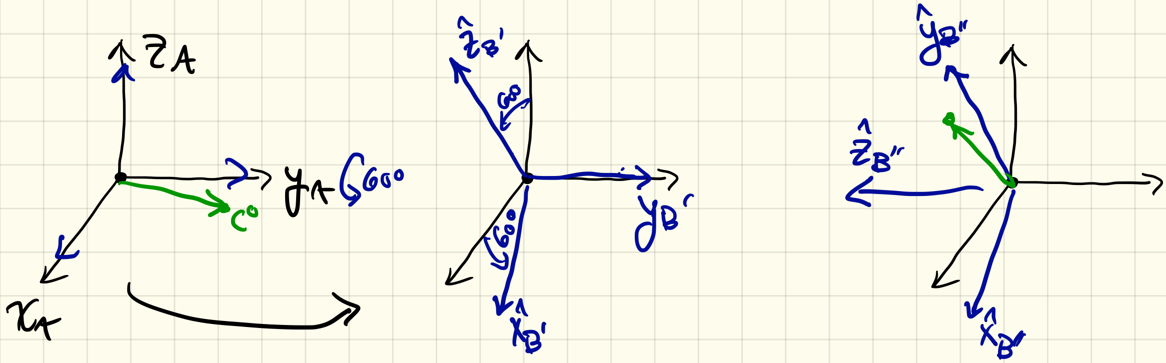
$= R_y(60°)\, R_x(90°)\, {}^B P_{C°}$

Example: Consider a point C fixed in $\{B\}$, and $\{B\}$ initially aligned with $\{A\}$.

① Rotate $\{B\}$ by $\hat{y}_A$ by $60°$

② Rotate $\{B\}$ by its once rotated $\hat{x}_B$ axis by $90°$

what is ${}^{A}P_{C''}$?



① ${}^{A}R_{B'} = R_y(60°)$

② ${}^{B'}R_{B''} = R_x(90°)$

③ ${}^{A}P_{C''} = {}^{A}R_{\cancel{B'}}\,{}^{\cancel{B'}}R_{\cancel{B''}}\,{}^{\cancel{B''}}P_{C''}$

$= R_y(60°)\, R_x(90°)\, {}^{B}P_c$

Sequence		Sequence
① $90°$ about $\hat{x}_A$	SAME	① $60°$ abt $\hat{y}_A$
② $60°$ about $\hat{y}_A$		② $90°$ about $\hat{x}_{B'}$

$$\boxed{\text{A sequence of rotations taken about earth-fixed axis is equivalent to the opposite sequence taken about body-fixed axes.}}$$

$${}^{A}R_{B''} = R_y(60^\circ)\, R_x(90^\circ)$$

sequence of rotations
about axes in Earth fixed coordinates

$${}^{A}p_{C''} = \underrightarrow{\overleftarrow{R_y(60^\circ)\, R_x(90^\circ)}}\ {}^{A}p_{C^{\circ}}$$

Sequence of rotations
about axes in body-fixed coordinates

Left vs. Right Multiplication of Rotations:

Let $\{A\}$, $\{B\}$ two frames ${}^{A}R_{B}$ the rotation between them. $R_z(\theta)$

${}^{A}R_{B'} = R_z(\theta)\,{}^{A}R_{B}$: Rotation applied to $\{B\}$ about $\hat{z}_A$

${}^{A}R_{B'} = {}^{A}R_{B}\,R_z(\theta)$: Rotation applied to $\{B\}$ about $\hat{z}_B$