Lecture 9 - Kinematics Examples (3D)

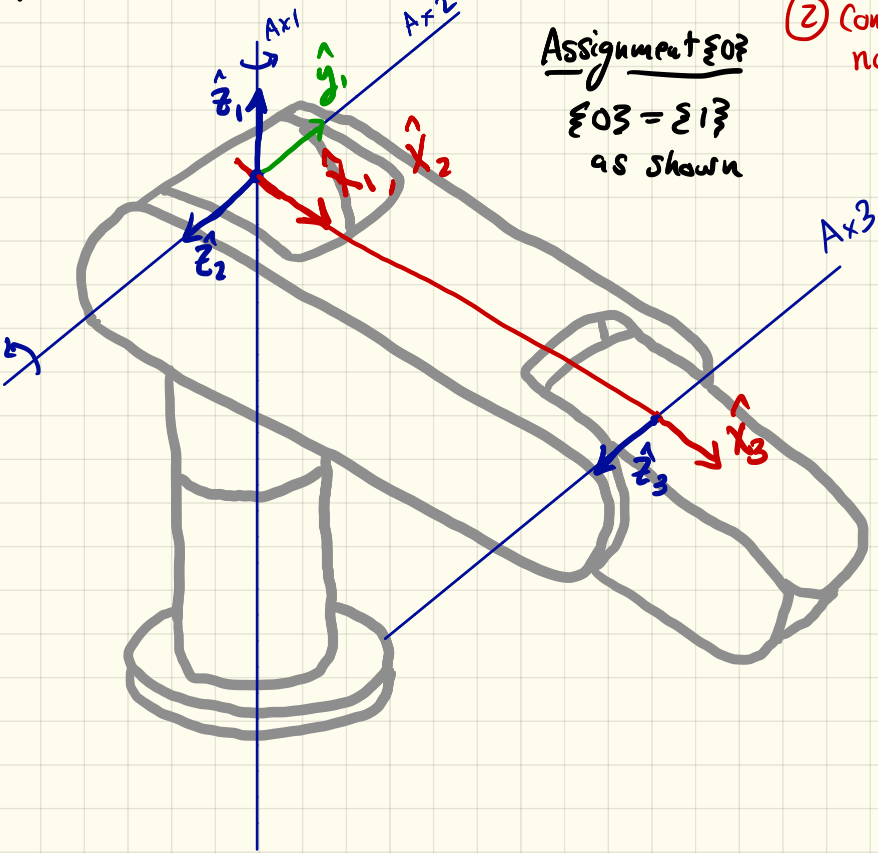
Goals For Today:

- Kinematics in 3D
  - Solution via homogeneous transforms
  - Solution via direct spatial reasoning

Example: RRR Manipulator (In the zero Config as Shown)

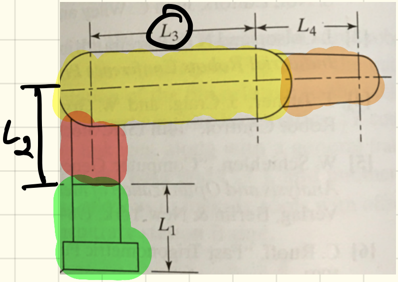
① Axes
② Common normals

	↺ $\hat{x}_{i-1}$	↑ $\hat{x}_{i-1}$	↑ $\hat{z}_i$	↺ $\hat{z}_i$
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	0	θ_2
3	0	L_3	0	θ_3

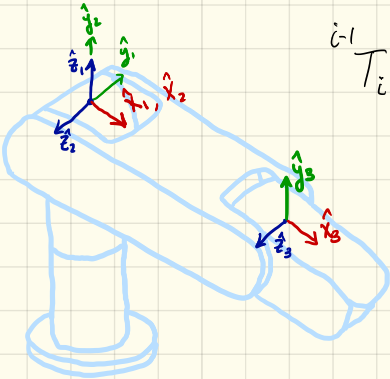


Assignment {0}

$\{0\} = \{1\}$ as shown



	↺ $\hat{x}_{i-1}$	↑ $\hat{x}_{i-1}$	↑ $\hat{z}_i$	↺ $\hat{z}_i$
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$90°$	0	0	θ_2
3	0	L_3	0	θ_3



$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1}s\theta_i & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\alpha_{i-1}s\theta_i & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S(\alpha \pm \beta) = S_\alpha C_\beta \pm C_\alpha S_\beta$$

$$C(\alpha \pm \beta) = C_\alpha C_\beta \mp S_\alpha S_\beta$$

$$^0T_3 = {}^0T_1 \, {}^1T_2 \, {}^2T_3$$

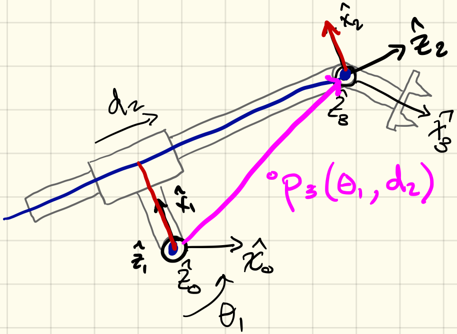
$$^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_3 = \begin{bmatrix} c_2c_3 - s_2s_3 & -c_2s_3 - s_2c_3 & 0 & L_3c_2 \\ 0 & 0 & -1 & 0 \\ s_2c_3 + c_2s_3 & c_2c_2 - s_2s_3 & 0 & L_3s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{23} & -s_{23} & 0 & L_3c_2 \\ 0 & 0 & -1 & 0 \\ s_{23} & c_{23} & 0 & L_3s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

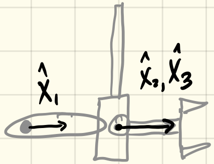
$$^0T_3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & L_3c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & L_3s_1c_2 \\ s_{23} & c_{23} & 0 & L_3s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_3^2c_1^2c_2^2 + L_3^2s_1^2c_2^2 + L_3^2s_2^2 = L_3^2$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
	$\circlearrowleft \hat{x}_{i-1}$	$\uparrow \hat{x}_{i-1}$	$\uparrow \hat{z}_i$	$\circlearrowleft \hat{z}_i$
1	0	0	0	θ_1
2	90°	a_1	d_2	0
3	-90°	0	0	θ_3



Zero Configuration:
$\theta_1=0,\ d_2=0,\ \theta_3=0$



Since each $\theta_i=0$ all $\hat{x}_i$'s are in the same direction

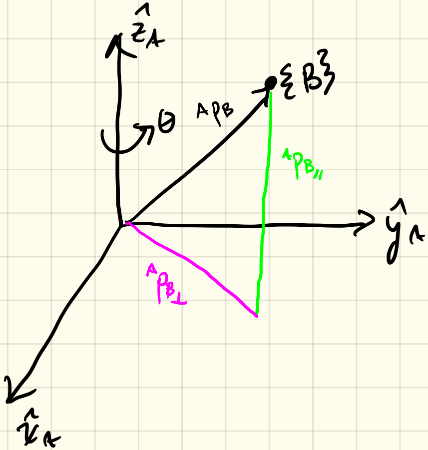
(Since ${}^0p_3$ and axis of rotation originate @ the origin of $\{0\}$)

$$\underbrace{{}^0p_3 = c_1\,{}^0p_3(0^\circ, d_2) + s_1\,{}^0p_3(90^\circ, d_2)}$$

This was a special case

Since ${}^0p_3$ was perpendicular to the axis of rotation for θ_1.

Consider Rotation of a general vector:



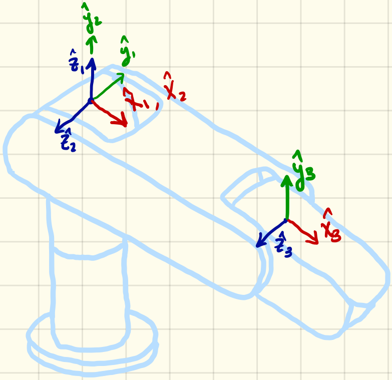
Decompose $^A p_B$ into parts parallel to and perpendicular to the axis of rotation:

$$^A p_B = {}^A p_{B_\parallel} + {}^A p_{B_\perp}$$

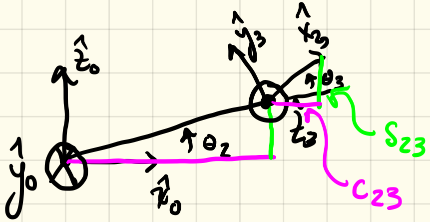
$$R_z(\theta)\,{}^A p_B = \underbrace{{}^A p_{B_\parallel}} + C(\theta)\,\underbrace{R_z(0^\circ)\,{}^A p_{B_\perp}} + S(\theta)\,\underbrace{R_z(90^\circ)\,{}^A p_{B_\perp}}$$

Apply previous "trick" to express the result of rotating $^A p_{B_\perp}$

Parallel portion is not affected by the rotation

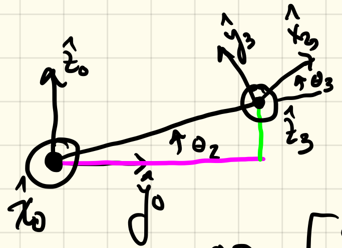


when $\theta_1 = 0°$



$${}^0P_3 = \begin{bmatrix} L_3 C_2 \\ 0 \\ L_3 S_2 \end{bmatrix}$$

when $\theta_1 = 90°$:



$${}^0P_3 = \begin{bmatrix} 0 \\ L_3 C_2 \\ L_3 S_2 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} {}^0\hat{x}_3 & {}^0\hat{y}_3 & {}^0\hat{z}_3 \end{array} \\ {}^0R_3 = & \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 \\ S_{23} & C_{23} & 0 \end{bmatrix} \end{array}$$

$${}^0P_3 = \begin{bmatrix} C_1 C_2 L_3 \\ S_1 C_2 L_3 \\ L_3 S_2 \end{bmatrix}$$