Lecture 35 – Trajectory Generation Pt. 2

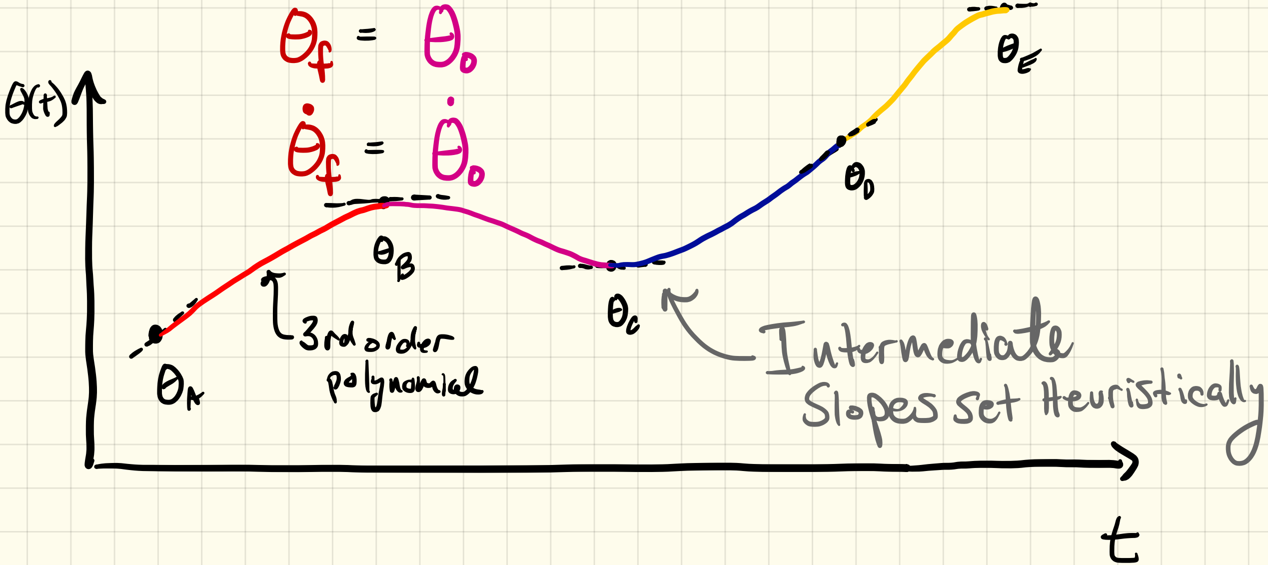
Announcements:

- HW 9 – online – Due Fri. after break
- Project 2 – online – Due Dec. 6th 11:55 PM

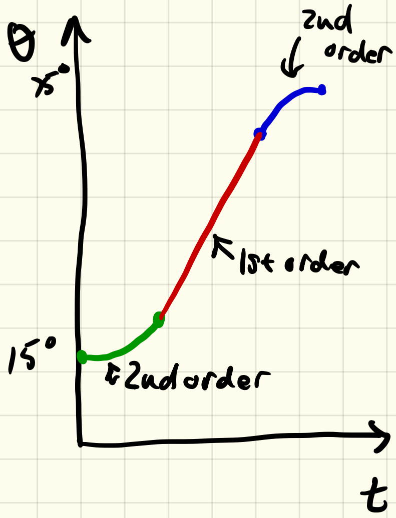
Roadmap:

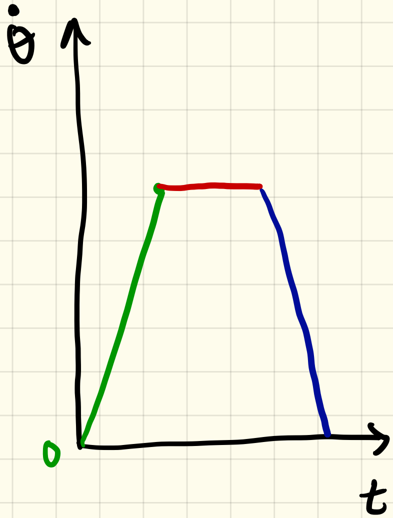
- Designing end-effector trajectories
  - Position & Orientation
  - Challenges
- Quick Discussion of Modern Approaches

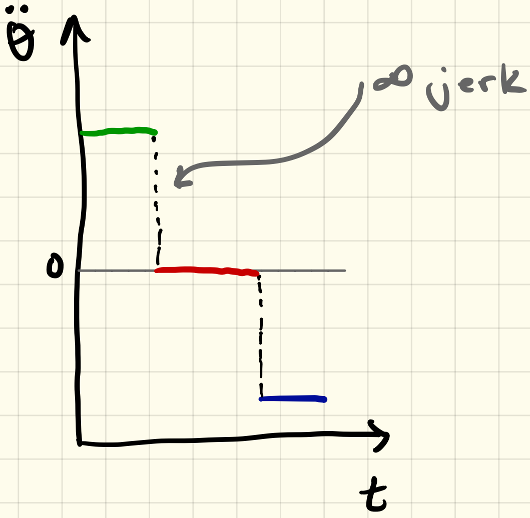
Via Points & combinations of cubic Splines



Common Option: Linear Trajectory w/ parabolic Blends

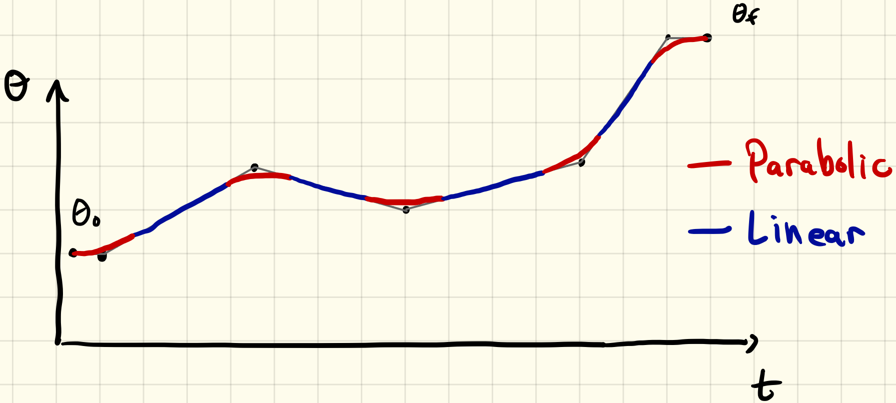






"Via" Points w/ Trapezoidal Velocity Profiles

- Similar to cubic splines these profiles may be combined for multi segment paths.
- However intermediate points are not reached exactly. Depending on reqd. precision this may be acceptable. (spray painting vs. welding)



Planning Cartesian Paths: ${}^{0}T_e(t)$

$${}^{0}T_e(t) = \begin{bmatrix} {}^{0}R_e(t) & {}^{0}p_e(t) \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- Planning ${}^{0}p_e(t)$: cubic splines, trap. vel. profiles, etc.
- Planning ${}^{0}R_e(t)$: Cannot interpolated on R directly
  - 9 entries, $R^T R = I \Rightarrow$ 6 constraints, 3 DoF
  - Common to parameterize ${}^{0}R_e$ by 3 numbers
    - Z-Y-X Euler angles
    - Z-Y-Z
    - Angle Axis rotation (interpolate the angle)
  - Then use the common methods (cubic splines, trap. vel. ...)

Analog of Linear Interpolation For Rotations

- Linear interpolation for positions: traslation along fixed axis
- Linear interpolation For orientations: rotation about a fixed axis
    - Initial ${}^0R_e^0$
    - Final ${}^0R_e^f$
    - Angle-axis error ${}^0\hat{k}, \theta$ such that

$$ {}^0R_e^f = e^{S({}^0\hat{k})\theta} \, {}^0R_e^0 \qquad \text{(just like project 1)} $$

- Analog of linear interp

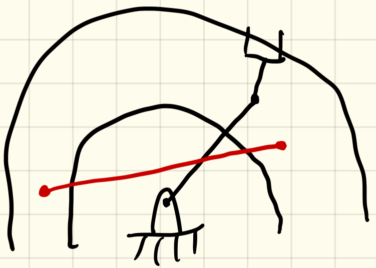
$$ R(t) = e^{S({}^0\hat{k})\theta t/t_f} \cdot {}^0R_e^0 $$

Challenges w/ Designing Cartesian Paths

- Workspace issues: Even if intitial and final pose in workspace intermediate points may not be reachable

- Often IK has multiple (of ∞ solutions). Sometimes smooth end effector trajectory gives discontinuous joint trajectory

- If trajectory passes near singularity, joint rates may become high



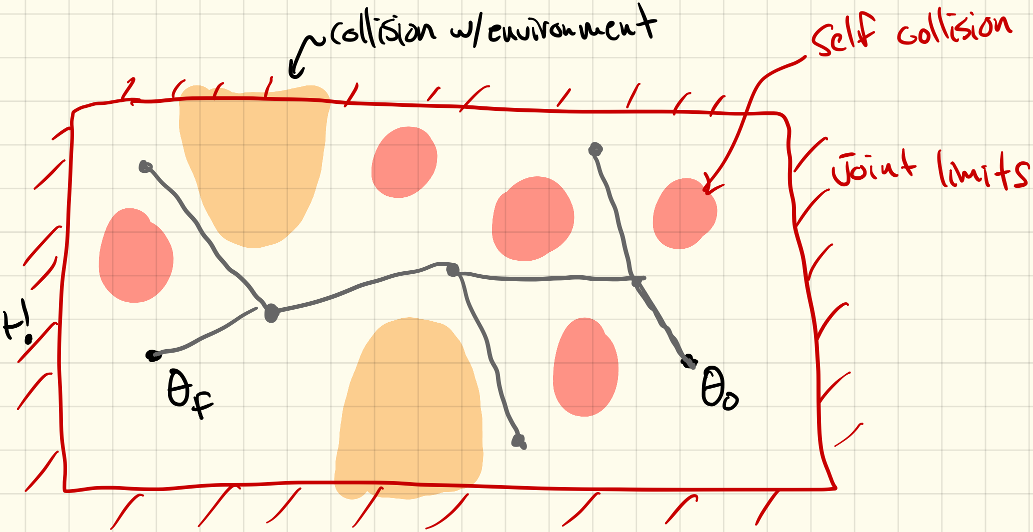
Advanced Trajectory Design Methods: Probalistic Road Map

- Modern methods are often random in nature
- Steps: ① Randomly sample possible way points
  - if in collision throw away

  ② Try to connect neighbors
  - if in collision throw away

  ③ If path ✓ else go back to ①

If a path exists it will find it!



Recap:

Designing Trajectories in end-effector space

- Largely the same methods as last time

- Orientation Trajectory planning requires selection of some compact representation (Euler angles or angle axis)
    - Angle-Axis provides shortest path

- Workspace & Singularities can complicate design
    - Modern Planners address this issue w/exploration to find quality paths

After break: Control (Linear & Nonlinear)