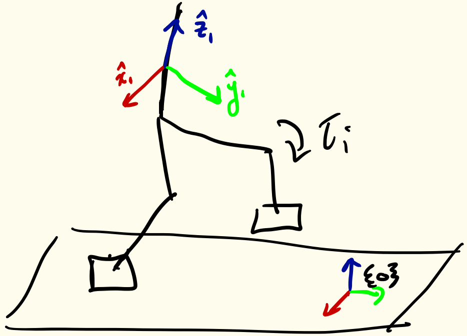
Lecture 18: Simple Models of Locomotion

- Floating Base Systems
  - ⇒ Configuration Space & Quaternions ✓
  - ⇒ Dynamics of Floating Base Systems
    - Features of the Dynamics ~
    - Floating Base Inverse Dynamics
- Simple Models of locomotion (a.k.a. Templates)
  - Inverted Pendulum / Linear Inverted Pendulum
  - Spring-Loaded Inverted Pendulum (SLIP)
  - Bipedal-SLIP

Configuration Manifold for Torso

$q_B = (\text{position } \{1\}, \text{ orientation } \{1\})$



Special-Orthogonal Group:

$$SO(n) = \{ X \in \mathbb{R}^{n \times n} \mid X^T X = I, \det(X) = 1 \}$$

$SO(3)$ is the set of all rotation matrices

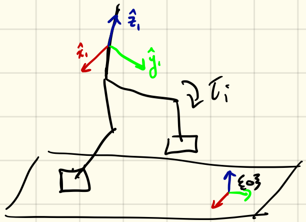
Special Euclidean Group:

$$SE(n) = \left\{ \begin{bmatrix} X & y \\ 0 & 1 \end{bmatrix} \;\middle|\; X \in SO(n), y \in \mathbb{R}^n \right\}$$

$SE(3)$ is the set of all Homogeneous Transforms

$$\boxed{q_B \in SE(3)}$$

Dynamics of Legged Systems:



- Attach a frame {1} to the body and let $q_B \in SE(3)$ denote the configuration of {1} relative to {0}

Stored on computer with ${}^0p_1$, ${}^0\varepsilon_1$

$$q = [q_B\,,\ q_J]$$

- With a slight liberty of notation

$$\dot{q} = [{}^1v_1^T,\ \dot{q}_J]$$

Dynamics

Top 6 rows indirectly actuated through contacts

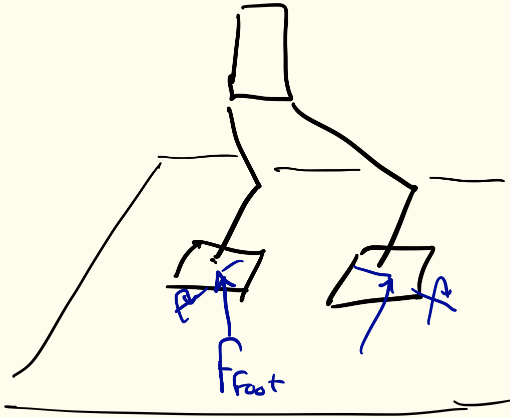
$$H(q)\ddot{q} + C\dot{q} + G(q) = \begin{bmatrix} 0 \\ \tau_J \end{bmatrix} + \sum_c J_c^T f_c$$

Bottom n rows directly actuated

$$\begin{bmatrix} H_{bb} & H_{bj} \\ H_{jb} & H_{jj} \end{bmatrix} \begin{bmatrix} {}^1\dot{v}_1 \\ \ddot{q}_J \end{bmatrix} + \begin{bmatrix} C_b + G_b \\ C_J + G_J \end{bmatrix} =$$

Looking @ First 6 Rows:

$$ {}^{foot}v_{foot} = {}^{foot}J_{foot}\,\dot q = {}^{foot}X_1\,{}^1v_1 + {}^{foot}J_{leg}\,\dot q_{leg} = \underbrace{\begin{bmatrix} {}^{foot}X_1 & J_{leg} \end{bmatrix}}_{{}^{foot}J_{foot} \in \mathbb{R}^{6\times(n+6)}} \dot q $$



$$ {}^{foot}J_{foot}^T f_{foot} = \begin{bmatrix} {}^{foot}X_1^T f_{foot} \\ J_{leg}^T f_{foot} \end{bmatrix} = \begin{bmatrix} {}^1X^*_{foot} f_{foot} \\ J_{leg}^T f_{foot} \end{bmatrix} $$

Inverse Dynamics For a Floating Base System: given $q, \dot q, \ddot q$ find τ_J and $\{f_c\}$

$$ H(q)\ddot q + C\dot q + G(q) = \begin{bmatrix} 0 \\ \tau_J \end{bmatrix} + \sum_c J_c^T f_c = \begin{bmatrix} \sum_c {}^1X_c^* f_c \\ \text{don't care} \end{bmatrix} $$

HW5

- One Foot (6D contact): Unique solution
- Two Feet (6D contact): multiple Solutions (Force Distribution Problem)
- One foot (point contact): may not have a solution

What about when you have no contacts?

we say: a constraint of the form

$$F(q, \dot{q}, t) = 0$$

is holonomic if it can be written as

$$g(q, t) = 0$$

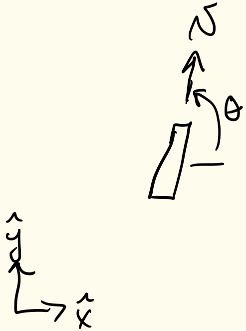
o/w: we say $F(q, \dot{q}, t)$ is a nonholonomic constraint.

Example 1: Conservation of linear momentum is a holonomic constraint

$$m \dot{p}_{com} = l_G \Rightarrow \dot{p}_{com} = \frac{l_G}{m} \qquad p_{com}(t) = \frac{l_G}{m} t + p_{com,0}$$

Example 2: No slip constraint for a tire is nonholonomic

$$\dot{y} \cos\theta - \dot{x} \sin\theta = 0$$



$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Why? Via parallel parking you can reach any state.

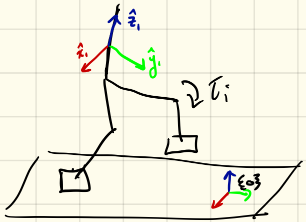
What about when you have no contacts?

Example: Conservation of angular momentum (in general) is a nonholonomic constraint.

Implications:

- To locomote translationally you must use contacts
- To "locomote rotationally"
  - Use contacts to modulate angular momentum
    $\Rightarrow$ reorientation w/ little shape change
  - Perform parallel parking sequences
    $\Rightarrow$ reorientation w/ major shape change.

Dynamics of Legged Systems:



- Attach a frame $\{1\}$ to the body and let $q_B \in SE(3)$ denote the configuration of $\{1\}$ relative to $\{0\}$

Stored on computer as ${}^0\varepsilon_1$, ${}^0p_1$

$$q = [\, q_B \,,\; q_J \,]$$ w/ the spatial velocity of $\{1\}$

- With a slight abuse of notation, denote

$$\dot{q} = [\, {}^1v_1^T \,,\; \dot{q}_J^T \,]^T$$

- Dynamics

$$H(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q)$$

$$= \begin{bmatrix} H_{bb} & H_{bj} \\ H_{jb} & H_{jj} \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \ddot{q}_J \end{bmatrix} + \begin{bmatrix} C_b \\ C_j \end{bmatrix} \dot{q} + \begin{bmatrix} G_b \\ G_j \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_J \end{bmatrix} + J^T(q)\, f$$

Top 6 rows only indirectly actuated through contacts

Bottom n rows directly actuated

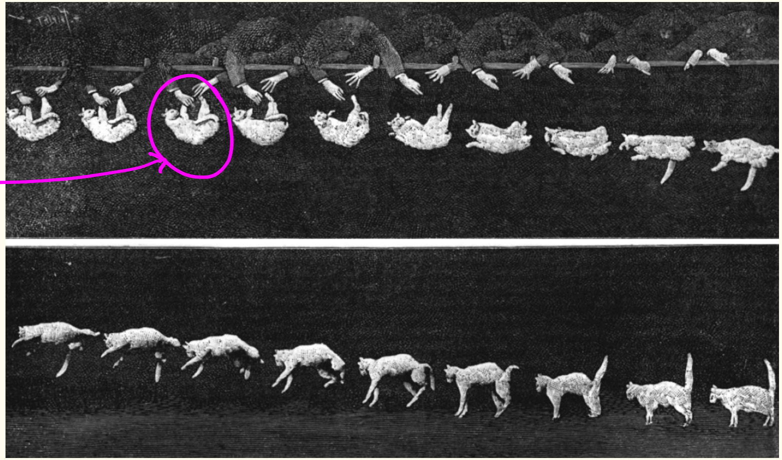
Subject to $J\dot{q} \equiv 0$

Assertion: The challenge of locomotion is choosing and exploiting contacts. The rest is easy.

Etienne-Jules Marey: Chrono photography

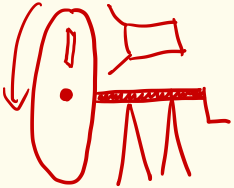
Is the cat pushing off the hands?

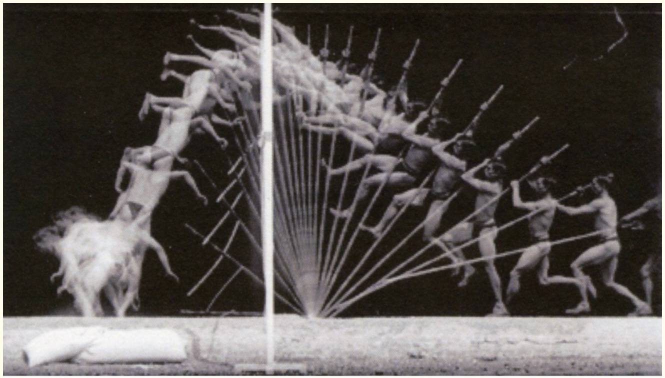
.... It doesn't need to since conseration of angular momentum is a nonholonomic constraint



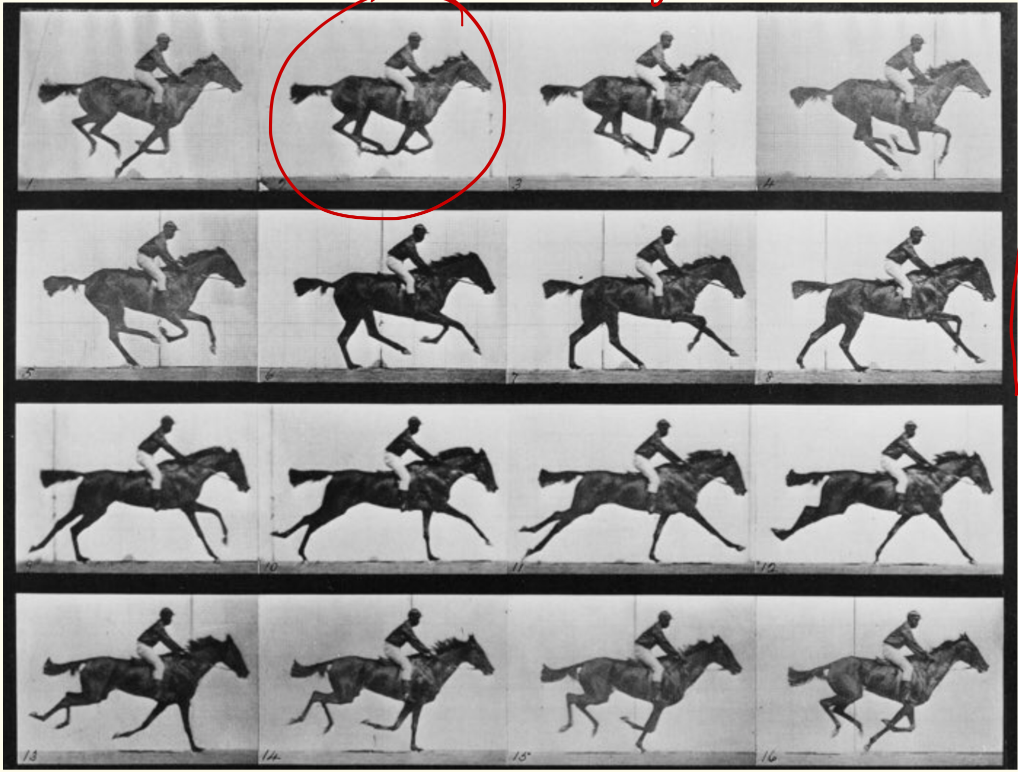
1894

Rotating Slotted wheel in front of camera





"gathered" Flight Phase



At higher Speeds

"Extended" flight phase

- Commissioned by Leland Stanford (founder of Stanford)
- Led to early version of a movie projector

Eadward Muybridge
1878

Simple Models of Locomotion: (a.k.a. templates)

- Abstract away complexities of system as a whole
- Focus on our ability to propel CoM
    - Relationship CoM, CoP, and ground forces

- Studied both:
    - In biomechanics: Understand human/animal locomotion
    - In robos: For controlling robot motion

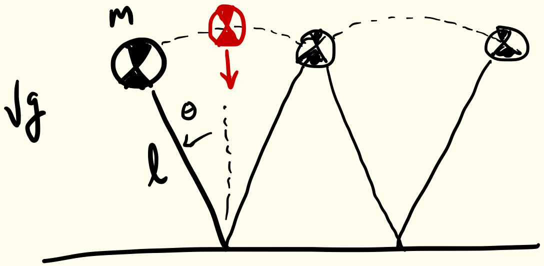
Common Strategy

1. Choose a suitable simple model for the locomotion task of interest

2. Solve control problems for the simple model

3. Control your complex robot to mimic the behavior of the simple model

The Inverted Pendulum Model of Walking



When: $\theta = 0$

- Tangential Accel $= 0$
- Centripetal Accel $= \frac{v^2}{l}$
- Max Accel $= g$ $\quad \frac{v^2}{l} \leq g$

$$v \leq \sqrt{lg}$$

- Single Support

$$ml^2\ddot{\theta} = mgl\sin\theta$$

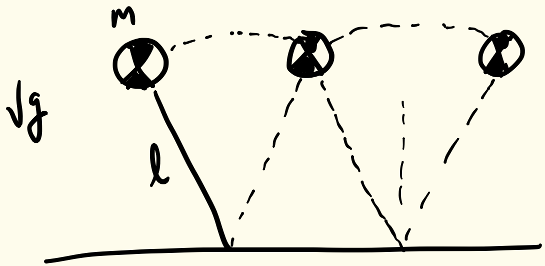
- Impact when $\theta = \theta_{TD}$

$$\dot{\theta}^+ = \dot{\theta}^-$$

$$\theta^+ = -\theta_{TD}$$

- Exchange of Kinetic & Potential En.
- Decent physiological explanation of the dynamics of single support (Vaulting over stiff leg)

The Inverted Pendulum Model of Walking



- **Def:** Froude Number $= \frac{v^2}{lg}$ Units $= \frac{m^2/s^2}{m \cdot m/s^2}$ = unitless

- Max walking speed $Fr = 1$
- Froude number is unitless (nondimensional)

  - Cats, Rats, Dogs, Sheep, Rhinos

    $\Rightarrow$ Walk to Trot @ $Fr \approx .8$

    $\Rightarrow$ Trot to gallop @ $Fr \approx 2.5$