Lecture 15 - Velocity Kinematics - 9/25/17

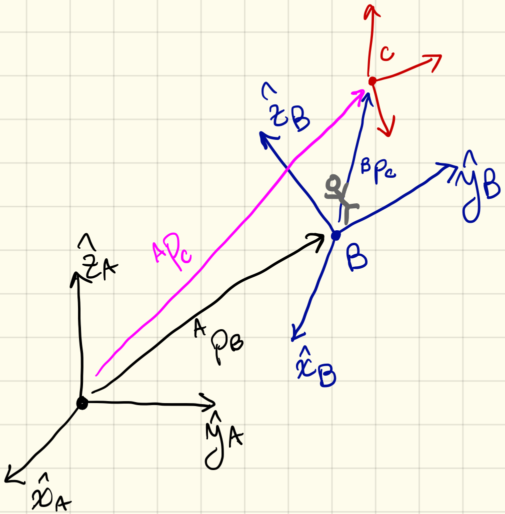
Goals For Today:

- Return & Review Exam 1
- Descriptions of Velocity (Linear & Angular)

Velocity Analysis

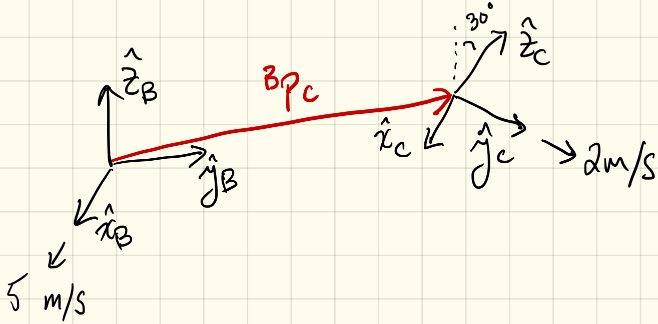
$${}^A P_C = {}^A P_B + {}^A R_B \, {}^B P_C$$

$${}^A\dot{P}_C = {}^A\dot{P}_B + {}^A\dot{R}_B \, {}^B P_C + {}^A R_B \, {}^B\dot{P}_C := {}^B V_C$$



${}^B V_C$: Velocity of frame {C} as observed in frame {B} (expressed w.r.t. $\hat{x}_B, \hat{y}_B, \hat{z}_B$)

Example: Imagine {B}, {C} attached to two different drones. (${}^B R_C = R_x(-30°)$)



$${}^B V_C = {}^B\dot{P}_C = \begin{bmatrix} -5 \text{ m/s} \\ \sqrt{3} \text{ m/s} \\ -1 \text{ m/s} \end{bmatrix}$$

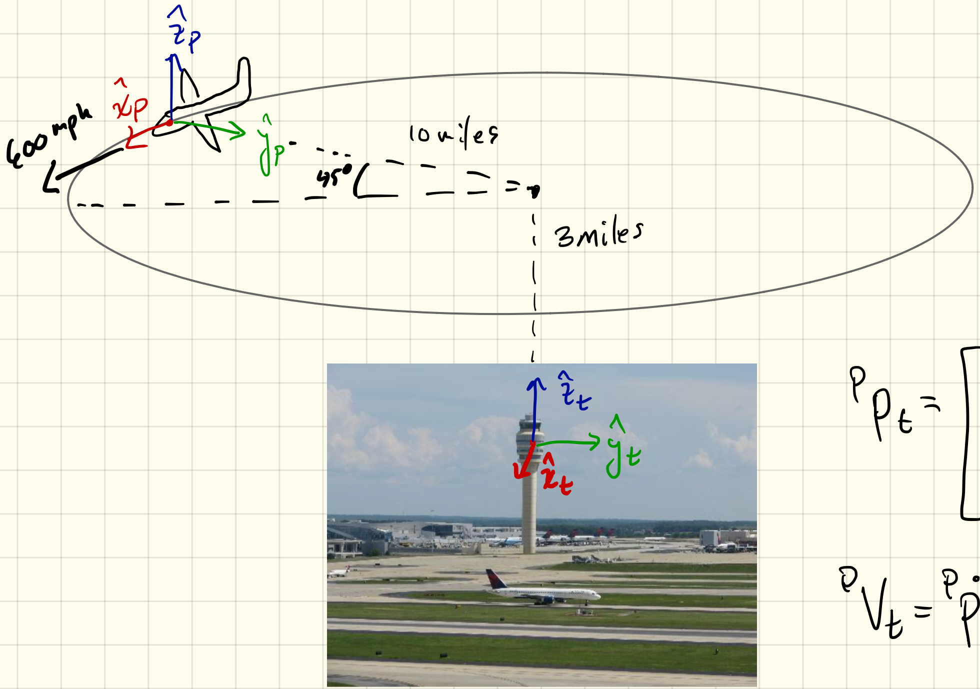
Example: Consider an airplane circling a control tower @ 600 mph. What is ${}^{p}V_t$?

Ⓐ $[600\text{ mph}, 0, 0]^T$

Ⓑ $[-600\text{ mph}, 0, 0]^T$

Ⓒ $\left[\frac{600}{\sqrt{2}}\text{mph}, -\frac{600}{\sqrt{2}}\text{mph}, 0\right]^T = {}^{t}V_p$

Ⓓ $[0, 0, 0]^T$

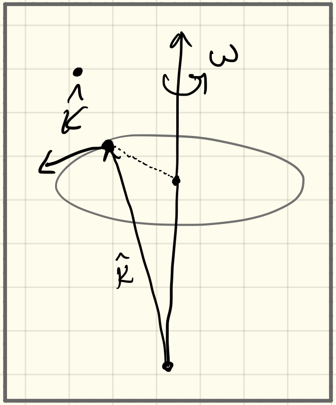


$${}^{p}p_t = \begin{bmatrix} 0 \\ 10\text{ miles} \\ -3\text{ miles} \end{bmatrix}$$

$${}^{p}V_t = {}^{p}\dot{p}_t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Velocity Analysis

$$^{A}V_C = {}^{A}V_B + {}^{A}R_B\,{}^{B}V_C + {}^{A}\dot{R}_B\,{}^{B}P_C$$



Consider a unit vector $\hat{K}$ spinning w/ ang. vel. ω

$$\dot{\hat{K}} = \omega \times \hat{K}$$

Define: $^{A}\Omega_B$ angular velocity $\{B\}$ relative to $\{A\}$ expressed w.r.t. $\hat{x}_A, \hat{y}_A, \hat{z}_A$

$$^{A}R_B = \begin{bmatrix} ^{A}\hat{x}_B & ^{A}\hat{y}_B & ^{A}\hat{z}_B \end{bmatrix} \quad {}^{A}\dot{R}_B = \begin{bmatrix} ^{A}\Omega_B \times {}^{A}x_B & ^{A}\Omega_B \times {}^{A}y_B & ^{A}\Omega_B \times {}^{A}\hat{z}_B \end{bmatrix}$$

$$\boxed{^{A}V_C = {}^{A}V_B + {}^{A}R_B\,{}^{B}V_C + {}^{A}\Omega_B \times {}^{A}R_B\,{}^{B}P_C} \quad ☆$$