Lecture 26 - Inward Pass of RNEA

Announcements

- HW7 Revised ⇒ Due Weds
- MT2 Next Friday. IK. Jacobians. RNEA.
- No Class Monday - Watch Video Lecture

Today:

- Inward Pass & Alogorithm Recap
- Accounting for gravity
- Example

Inverse Dynamics

Outline: Given $\theta, \dot{\theta}, \ddot{\theta}$, find τ

① Outward pass (Kinematics Propagation)

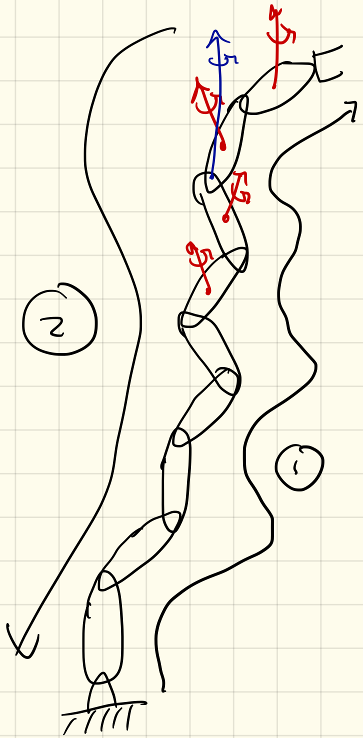
- ${}^{i}\omega_i,\ {}^{i}\dot{\omega}_i,\ {}^{i}\dot{v}_i,\ {}^{i}\dot{v}_{C_i}$
- Newton & Euler to get ${}^{i}F_i,\ {}^{i}N_i$

② Inward pass (Force Propagation)

- Determine ${}^{i}f_i,\ {}^{i}n_i$
- Determine actuator efforts

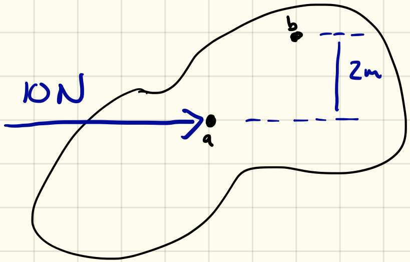
$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^{i}n_i \qquad \text{Revolute}$$

$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^{i}f_i \qquad \text{Prismatic}$$



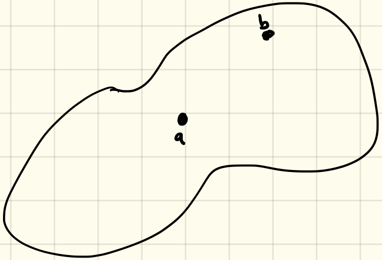
Equivalent Force Moment Systems:

1



$f_a, n_a = 0$

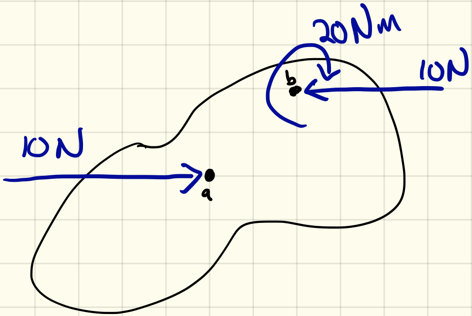
2



Equivalence of F
1 and 4

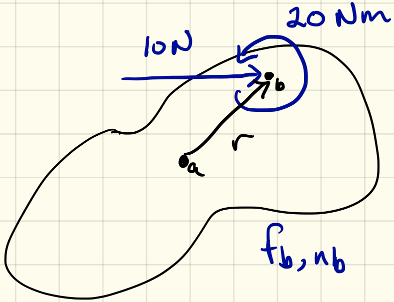
$$\boxed{\begin{aligned} f_a &= f_b \\ n_a &= n_b + r \times f_b \end{aligned}}$$

3



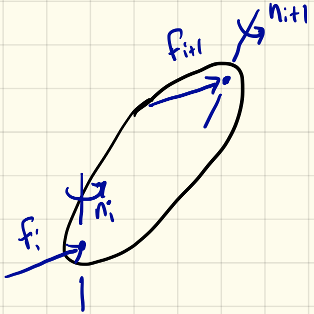
f_a, n_a

4



f_b, n_b

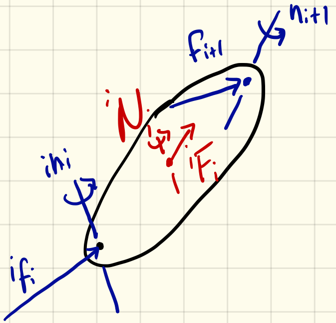
Static Case: zero net force and moment



$$ {}^{i}F_i = {}^{i}R_{i+1}\, {}^{i+1}f_{i+1} $$

$$ {}^{i}n_i = {}^{i}R_{i+1}\, {}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}R_{i+1}\, {}^{i+1}f_{i+1} $$

Dynamic Case:



can be computed $\theta, \dot{\theta}, \ddot{\theta}$

$$ {}^{i}f_i = {}^{i}F_i + {}^{i}R_{i+1}\, {}^{i+1}f_{i+1} $$

Known if you start @ tips and move backwards

$$ {}^{i}n_i = {}^{i}N_i + {}^{i}P_{c_i} \times {}^{i}F_i + {}^{i}R_{i+1}\, {}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}R_{i+1}\, {}^{i+1}f_{i+1} $$

Summary

$$ {}^0\omega_0 = {}^0\dot{\omega}_0 = {}^0\dot{v}_0 = 0 $$

$i = 0:5$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}R_i \, {}^i\omega_i + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} $$

$$ {}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}R_i \, {}^i\dot{\omega}_i + {}^{i+1}\omega_{i+1} \times \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_{i+1} \end{bmatrix} $$

$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}R_i \left({}^i\dot{v}_i + {}^i\dot{\omega}_i \times {}^iP_{i+1} + {}^i\omega_i \times {}^i\omega_i \times {}^iP_{i+1} \right) + 2\, {}^{i+1}\omega_{i+1} \times \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d}_{i+1} \end{bmatrix} $$

$$ {}^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{v}_{i+1} + {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times {}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}} $$

$$ {}^{i+1}F_{i+1} = m_{i+1} \, {}^{i+1}\dot{v}_{C_{i+1}} $$

$$ {}^{i+1}N_{i+1} = {}^{C_{i+1}}I \, {}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \, {}^{C_{i+1}}\underline{I} \, {}^{i+1}\omega_{i+1} $$

end

$i = 6:1$

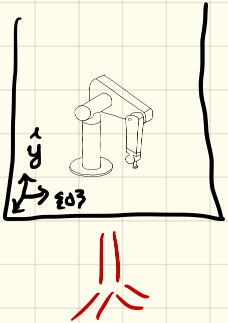
$$ {}^if_i = {}^iR_{i+1} \, {}^{i+1}f_{i+1} + {}^iF_i $$

$$ {}^in_i = {}^iR_{i+1} \, {}^{i+1}n_{i+1} + {}^iN_i + {}^iP_{i+1} \times {}^iR_{i+1} \, {}^{i+1}f_{i+1} + {}^iP_{C_i} \times {}^iF_i $$

$$ \tau_i = [0\ 0\ 1] \, {}^in_i $$ or $$ \tau_i = [0\ 0\ 1] \, {}^if_i $$

end

What about Gravity: It's all relative.



On Earth: $\{0\}$ accelerate downwards @ $9.81\ m/s^2$. Relationship between τ & $\ddot{\theta}$ same as if no gravity.

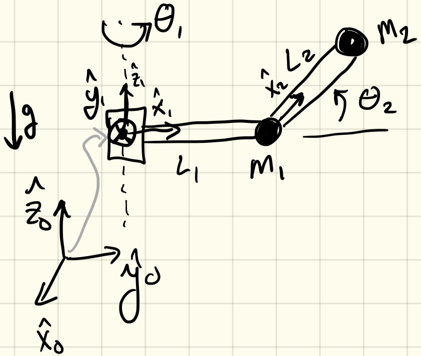
In zero-g: $\{0\}$ accelerates upwards @ $9.81\ m/s^2$. Relationship between $\ddot{\theta}$, τ same as back on earth.

So. To handle gravity ${}^0\dot{v}_0$ = "Opposite gravity"

$$= \begin{bmatrix} 0 \\ 9.81\ m/s^2 \\ 0 \end{bmatrix}$$

Example: 2R Manip

UPDATES TO THE OUTWARD PASS



Body 1

$$^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \qquad ^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} \qquad ^1\dot{v}_1 = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$^1\dot{v}_{c_1} = \cancelto{0}{^1\dot{v}_1} + {}^1\dot{\omega}_1 \times {}^1p_{c_1} + {}^1\omega_1 \times {}^1\omega_1 \times {}^1p_{c_1}$$

$$\uparrow \ddot{\theta}_1 \times \xrightarrow{L_1} + \uparrow \dot{\theta}_1 \times \left[\uparrow \dot{\theta}_1 \times \xrightarrow{L_1} \right]$$

$$= \otimes L_1 \ddot{\theta}_1 + \xleftarrow{\dot{\theta}_1^2 L_1} = \begin{bmatrix} -L_1 \dot{\theta}_1^2 \\ L_1 \ddot{\theta}_1 \\ g \end{bmatrix}$$

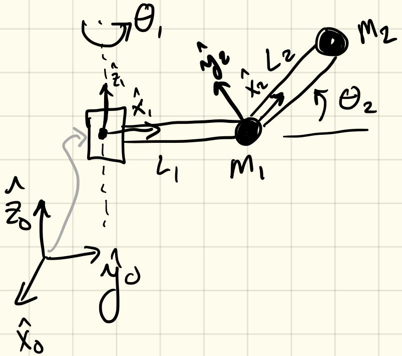
$$^1I_{zz} = m_1 L_1^2$$

$$^1F_1 = m_1 \begin{bmatrix} -L_1 \dot{\theta}_1^2 \\ L_1 \ddot{\theta}_1 \\ g \end{bmatrix}$$

$$^1N_1 = \cancelto{0}{^{c_1}I}\, {}^1\dot{\omega}_1 + {}^1\omega_1 \times \cancelto{0}{^{c_1}I}\, {}^1\omega_1 = 0$$

Example: 2R Manip

$${}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1P_{c_1} = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \qquad {}^2P_{c_2} = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$



Outward pass for Body 2

$$\omega_1 = \uparrow \dot{\theta}_1$$

$${}^2\omega_2 = {}^2R_1\,{}^1\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$${}^2\dot{\omega}_2 = {}^2R_1\,{}^1\dot{\omega}_1 + {}^2\omega_2 \times \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} s_2\ddot{\theta}_1 + c_2\dot{\theta}_1\dot{\theta}_2 \\ c_2\ddot{\theta}_1 - s_2\dot{\theta}_1\dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \frac{d}{dt}\left[{}^2\omega_2\right]$$

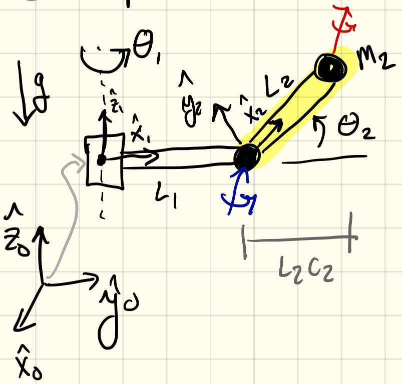
$${}^2\dot{v}_2 = {}^2R_1\,{}^1\dot{v}_{c_1} = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -L_1\dot{\theta}_1^2 \\ L_1\ddot{\theta}_1 \\ g \end{bmatrix} = \begin{bmatrix} -L_1 c_2\dot{\theta}_1^2 + s_2 g \\ L_1 s_2\dot{\theta}_1^2 + c_2 g \\ -L_1\ddot{\theta}_1 \end{bmatrix}$$

$${}^2\dot{v}_{c_2} = {}^2\dot{v}_2 + {}^2\dot{\omega}_2 \times {}^2p_{c_2} + {}^2\omega_2 \times \left({}^2\omega_2 \times {}^2p_{c_2}\right) = \begin{bmatrix} -(L_1 + L_2c_2)c_2\dot{\theta}_1^2 - L_2\dot{\theta}_1^2 + s_2 g \\ (L_1 + L_2c_2)s_2\dot{\theta}_1^2 + L_2\ddot{\theta}_2 + c_2 g \\ -(L_1 + L_2c_2)\ddot{\theta}_1 + 2L_2s_2\dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$${}^2F_2 = m_2\,{}^2\dot{v}_{c_2} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad {}^2N_2 = 0$$

Example: 2R Manip

$${}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1P_{c_1} = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \qquad {}^2P_{c_2} = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$



Outward Pass: Body 2

$${}^2\dot{v}_{c_2} = {}^2\dot{v}_2 + {}^2\dot{\omega}_2 \times {}^2P_{c_2} + {}^2\omega_2 \times \left({}^2\omega_2 \times {}^2P_{c_2}\right)$$

$$= \begin{bmatrix} -(L_1 + L_2c_2)c_2\dot{\theta}_1^2 - L_2\dot{\theta}_2^2 + g s_2 \\ (L_1 + L_2c_2)s_2\dot{\theta}_1^2 + L_2\ddot{\theta}_2 + g c_2 \\ -(L_1 + L_2c_2)\ddot{\theta}_1 + 2L_2 s_2 \dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$${}^2F_2 = m_2\, {}^2\dot{v}_{c_2} =: \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

$${}^2N_2 = 0$$

Inward Pass: Body 2

$${}^2f_2 = {}^2F_2$$

$${}^2n_2 = \cancel{{}^2N_2}^{\,0} + {}^2P_{c_2} \times {}^2F_2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ -l_2F_z \\ l_2F_y \end{bmatrix}$$

$$\tau_2 = l_2 m_2\left[(l_1 + l_2c_2)s_2\dot{\theta}_1^2 + l_2\ddot{\theta}_2 + g c_2\right]$$

From Before:

$$ {}^2\dot{\nu}_{c_2} = \begin{bmatrix} -(L_1+L_2c_2)c_2\dot{\theta}_1^2 - L_2\dot{\theta}_2^2 + g s_2 \\ (L_1+L_2c_2)s_2\dot{\theta}_1^2 + L_2\ddot{\theta}_2 + g c_2 \\ -(L_1+L_2c_2)\ddot{\theta}_1 + 2L_2 s_2\dot{\theta}_1\dot{\theta}_2 \end{bmatrix} $$

$$ {}^2f_2 = {}^2F_2 = m_2\,{}^2\dot{\nu}_{c_2} := \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} $$

$$ {}^1P_{c_1} = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^2n_2 = {}^2P_{c2} \times {}^2F_2 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ -L_2F_z \\ L_2F_y \end{bmatrix} $$

$$ {}^1F_1 = m_1\,{}^1\dot{\nu}_{c_1} = \begin{bmatrix} -m_1L_1\dot{\theta}_1^2 \\ m_1L_1\ddot{\theta}_1 \\ m_1 g \end{bmatrix} $$

$$ {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Inward Pass: Body 1 (${}^1F_1$, ${}^1n_1$, τ_1)

$$ {}^1f_1 = {}^1F_1 + {}^1R_2\,{}^2f_2 \quad \otimes $$

not needed to find τ_1

$$ {}^1n_1 = \cancel{{}^1N_1}^{\to 0} + {}^1P_{c_1}\times{}^1F_1 + {}^1R_2\,{}^2n_2 + {}^1P_2\times{}^1R_2\,{}^2f_2 $$

$$ = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} * \\ m_1L_1\ddot{\theta}_1 \\ * \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & * \\ s_2 & c_2 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ -L_2F_z \\ * \end{bmatrix} + \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} x & x & x \\ 0 & 0 & -1 \\ x & x & x \end{bmatrix}\begin{bmatrix} * \\ * \\ F_z \end{bmatrix} $$

$$ \tau_1 = m_1L_1^2\ddot{\theta}_1 - c_2L_2F_z - L_1F_z = m_1L_1^2 - (L_1+L_2c_2)F_z $$

$$ \boxed{\tau_1 = \left[m_1L_1^2 + m_2(L_1+L_2c_2)^2\right]\ddot{\theta}_1 - 2(L_1+L_2c_2)m_2L_2s_2\dot{\theta}_1\dot{\theta}_2} $$

Final Answer

$$\tau_1 = \left[m_1 L_1^2 + m_2 (L_1 + L_2 c_2)^2\right] \ddot{\theta}_1 - 2 (L_1 + L_2 c_2) m_2 L_2 s_2 \dot{\theta}_1 \dot{\theta}_2$$

$$\tau_2 = m_2 L_2 \left[(L_1 + L_2 c_2) s_2 \dot{\theta}_1^2 + L_2 \ddot{\theta}_2 + g c_2 \right]$$