Lecture 36 - Linear Control

Today:

- Basics of Linear control for 2nd order systems
- Two Different Approaches to PD Control

On Deck

- Linear Control of a DC Motor + Gearbox
- Nonlinear Control of a Manipulator
- Force & Impedance Control

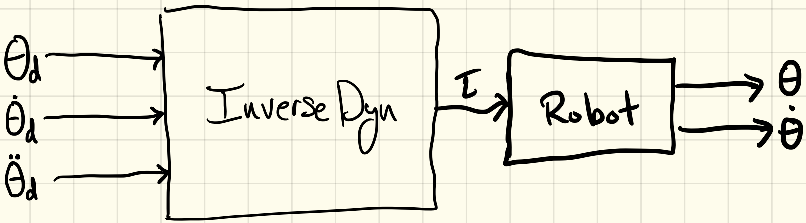
Open-Loop Control:

- Output from Trajectory Generation $\Rightarrow \theta_d, \dot{\theta}_d, \ddot{\theta}_d$

$$\tau = M(\theta_d)\ddot{\theta}_d + V(\theta_d, \dot{\theta}_d) + G(\theta_d) = \text{InverseDyn}(\theta_d, \dot{\theta}_d, \ddot{\theta}_d)$$

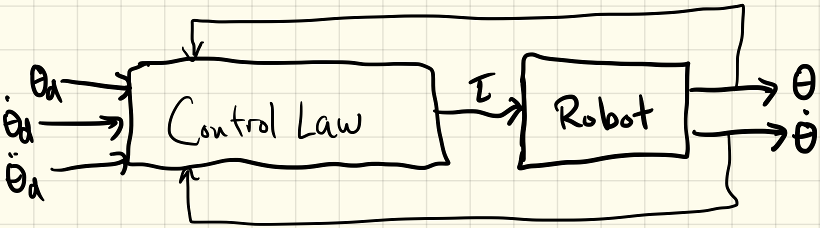
- Whats the problem?
    - Disturbances may occur
    - Model will be wrong

- Solution: Add Feedback

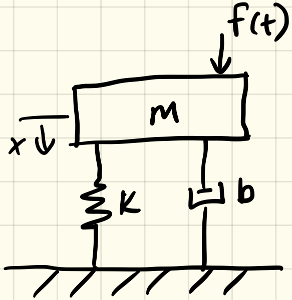


Feedback Control:

- Source: Position/Velocity sensors that give you $\theta, \dot{\theta}$
- Specifications:
    - Robustness
    - Stability: Signals stay bounded
    - Fregency Domain
        - Closed-loop bandwidth: Highest frequency sinusoid that you can track
    - Time Domain
        - Rise time, overshoot, etc. for step response



Review: Mass / Spring / Damper



$$m\ddot{x} = f(t) - kx - b\dot{x} \Rightarrow m\ddot{x} + b\dot{x} + kx = f(t)$$

- Consider $f(t) = 0$ (no control)

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

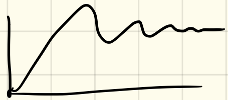
$\lambda_{1,2}$ are roots characteristic eqn. $ms^2 + bs + k = 0$

$\lambda_{1,2}$ are also the poles $\frac{X(s)}{F(s)}$

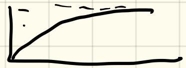
- Roots @ $s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$

Three Cases

1. $b^2 < 4mk$ "Under damped" complex conjugate pair of roots

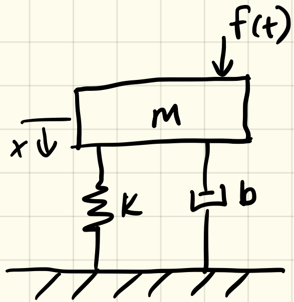


2. $b^2 = 4mk$ "critically damped" two real roots $\lambda_1 = \lambda_2$



3. $b^2 > 4mk$ overdamped two real poles $\lambda_1 \neq \lambda_2$

First Approach to PD Control: Supplementing natural dynamics



$$f = -K_p x - K_d \dot{x} \Rightarrow m\ddot{x} + b\dot{x} + kx = -K_p x - K_d \dot{x}$$

$$\Rightarrow 0 = m\ddot{x} + (K_d + b)\dot{x} + (K_p + K)x$$

- This is the same as increasing spring constant by K_p and increasing damping by K_d!

- When $K_p < 0$ its like you are softening the spring
  $K_d < 0$ removing damping

- If $\underbrace{K_p + K}_{K_{eff}} > 0$ and $\underbrace{K_d + b}_{b_{eff}} > 0$ then stable

Example:

- Consider a mass/spring/damper with $m = 5\,kg$, $k = 100\,N/m$, $b = 200\,N/ms^{-1}$
- Which PD control law ($f(t) = -K_p x - K_d \dot{x}$) sets the closed-loop poles @ -10 rad/sec

ⓐ $K_p = 500\,N/m$, $K_d = 100\,N/ms^{-1}$

ⓑ $K_p = 500\,N/m$, $K_d = -100\,N/ms^{-1}$

ⓒ $K_p = 400\,N/m$, $K_d = -100\,N/ms^{-1}$

ⓓ $K_p = -400\,N/m$, $K_d = -100\,N/ms^{-1}$

$$s = \frac{-b_{eff} \pm \cancel{\sqrt{b_{eff}^2 - 4mK_{eff}}}^{\to 0}}{2m}$$

- $\frac{b_{eff}}{2m} = 10$ $\quad b_{eff} = 100\,N/ms^{-1}$

$\Rightarrow K_D = -100\,N/ms^{-1}$

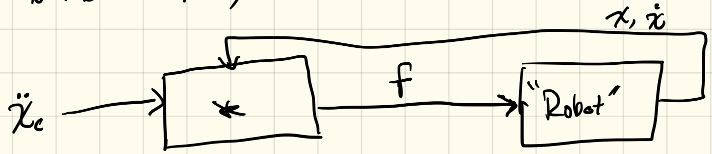
- $b_{eff}^2 = 4mK_{eff} \Rightarrow K_{eff} = 500\,N/m$

$K_p = 400\,N/m$

Second Approach to PD Control: Cancelling the Natural Dynamics

① Cancelling the stiffness and damping

$$m\ddot{x} + b\dot{x} + kx = f(t) \qquad\qquad f(t) = kx + b\dot{x} + m\ddot{x}_c \;\; *$$



- Equivalent closed-loop dynamics $\ddot{x} = \ddot{x}_c$

$$\ddot{x}_c \to \boxed{\int} \to \boxed{\int} \to x$$

<u>Note</u>: All of the complexities have disappeared!
We'll see this again soon for nonlinear control

- Desired trajectory: $x_d, \dot{x}_d, \ddot{x}_d$ Let the error: $e = x_d - x$

② Set commanded acceleration

$$\ddot{x}_c = \ddot{x}_d + K_p e + K_d \dot{e} \;\Rightarrow\; \ddot{e} + K_d \dot{e} + K_p e = 0 \qquad \left(\text{Stable if } \begin{array}{l} K_p > 0 \\ K_d > 0 \end{array}\right)$$

Disturbance Analysis:

Recap:

- Introduced control design for simple linear systems

    ⓐ one solution considered effective stiffness and damping of the closed-loop system

    ⓑ the other first cancelled all physical properties via a preliminary control design and left a double integrator system to control

- Next lecture: Look @ a motor model to justify why linear control is often enough

- After that we'll look @ advanced methods to replicate ⓐ and ⓑ for nonlinear control