Lecture 24 - Recursive Newton Euler

Announcements:

- Change: Project 1 due Friday @ 11:55 PM
- Reminder: Midterm evaluation closes tonight @ 11:55 PM

Goals For Today

- Review Dynamics of a Rigid Body
- High level overview of the Recursive Newton Euler Algorithm

  Given $\theta, \dot{\theta}, \ddot{\theta}$ Find τ

- First half of RNEA

Review:

${}^{i}f_i, {}^{i}n_i$: Force & Moment applied by Body i-1 onto Body i

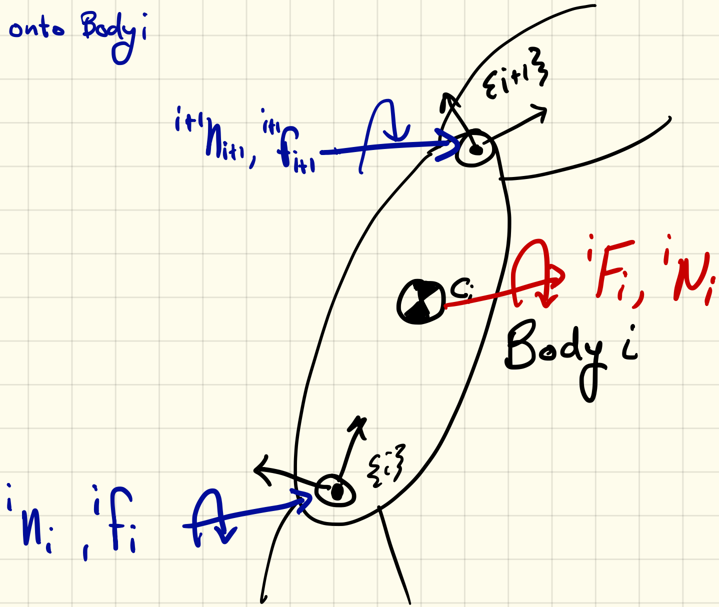
${}^{i}F_i$: Net Force on Body i

${}^{i}N_i$: Net Moment on Body i about C_i

Dynamics of Body i

$${}^{i}F_i = m_i \, {}^{i}\dot{v}_{C_i}$$

$${}^{i}N_i = {}^{C_i}I \, {}^{i}\dot{\omega}_i + {}^{i}\omega_i \times {}^{C_i}I \, {}^{i}\omega_i$$



Dynamic Force Balance

$${}^{i}f_i = {}^{i}R_{i+1} \, {}^{i+1}f_{i+1} + {}^{i}F_i$$

$${}^{i}n_i = {}^{i}N_i + {}^{i}P_{C_i} \times {}^{i}F_i + {}^{i}R_{i+1} \, {}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}R_{i+1} \, {}^{i+1}f_{i+1}$$

Inverse Dynamics Outline: Given $\theta, \dot\theta, \ddot\theta$, find τ

① Outward pass (Kinematics Propagation)

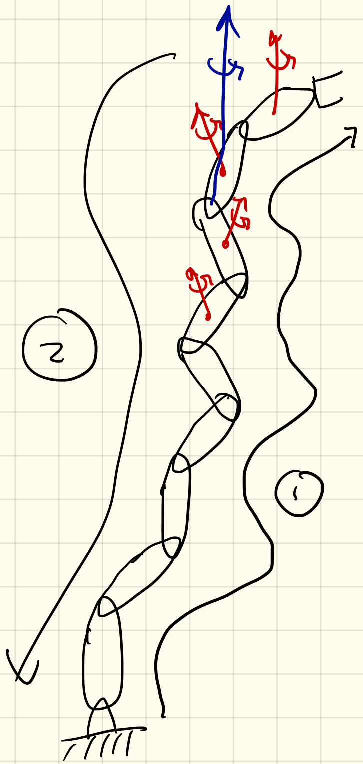
- ${}^i\omega_i$, ${}^i\dot\omega_i$, ${}^i\dot v_i$, ${}^i\dot v_{c_i}$
- Newton & Euler to get ${}^iF_i$, ${}^iN_i$

② Inward pass (Force Propagation)

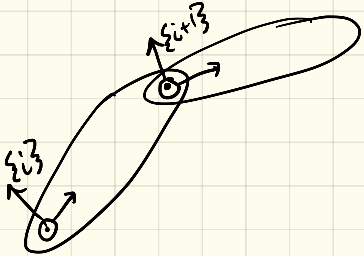
- Determine ${}^if_i$, ${}^in_i$
- Determine actuator efforts

$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^in_i \quad \text{Revolute}$$

$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^if_i \quad \text{Prismatic}$$



Outward Pass: Angular rates/accelerations



From velocity analysis

$$\boxed{{}^{i+1}\omega_{i+1} = {}^{i+1}R_i\,{}^{i}\omega_i + \begin{bmatrix}0\\0\\\dot\theta_{i+1}\end{bmatrix}}$$

Express it in frame {0}

$${}^{0}\omega_{i+1} = {}^{0}\omega_i + {}^{0}R_{i+1}\begin{bmatrix}0\\0\\\dot\theta_{i+1}\end{bmatrix}$$

Differentiate

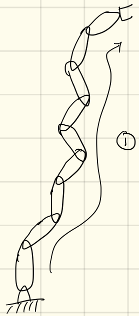
$${}^{0}\dot\omega_{i+1} = {}^{0}\dot\omega_i + {}^{0}R_{i+1}\begin{bmatrix}0\\0\\\ddot\theta_{i+1}\end{bmatrix} + {}^{0}\omega_{i+1} \times {}^{0}R_{i+1}\begin{bmatrix}0\\0\\\dot\theta_{i+1}\end{bmatrix}$$

Re-express in frame {i+1}:

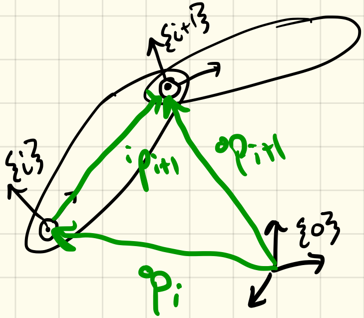
$$\boxed{{}^{i+1}\dot\omega_{i+1} = {}^{i+1}R_i\,{}^{i}\dot\omega_i + \begin{bmatrix}0\\0\\\ddot\theta_{i+1}\end{bmatrix} + {}^{i+1}\omega_{i+1} \times \begin{bmatrix}0\\0\\\dot\theta_{i+1}\end{bmatrix}}$$

Prismatic d_{i+1}:

$${}^{i+1}\dot\omega_{i+1} = {}^{i+1}R_i\,{}^{i}\dot\omega_i$$



Linear Acceleration: ${}^{i+1}\dot{v}_{i+1}$ from ${}^{i}\dot{v}_i$, ${}^0\omega_i$, ${}^i\dot{p}_{i+1}$, ${}^i\ddot{p}_{i+1}$



$${}^0p_{i+1} = {}^0p_i + {}^0R_i\,{}^ip_{i+1}$$

Differentiate:

$${}^0v_{i+1} = {}^0\dot{p}_{i+1} = {}^0v_i + \overbrace{{}^0\omega_i \times {}^0R_i}^{{}^0\dot{R}_i}\,{}^ip_{i+1} + {}^0R_i\,\overbrace{{}^i\dot{p}_{i+1}}^{={}^iv_{i+1}}$$

Helpful identity: $\frac{d}{dt}(a \times b) = \dot{a} \times b + a \times \dot{b}$

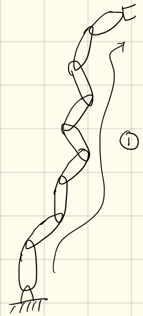
Differentiate Again:

$${}^0\dot{v}_{i+1} = {}^0\dot{v}_i + {}^0\dot{\omega}_i \times {}^0R_i\,{}^ip_{i+1} + {}^0\omega_i \times \left({}^0\omega_i \times {}^0R_i\,{}^ip_{i+1} + 2\,{}^0R_i\,{}^i\dot{p}_{i+1}\right) + {}^0R_i\,{}^i\ddot{p}_{i+1}$$

$$= {}^0\dot{v}_i + {}^0\dot{\omega}_i \times {}^0R_i\,{}^ip_{i+1} + {}^0\omega_i \times {}^0\omega_i \times {}^0R_i\,{}^ip_{i+1} + 2\,{}^0\omega_i \times {}^0R_i\,{}^i\dot{p}_{i+1} + {}^0R_i\,{}^i\ddot{p}_{i+1}$$

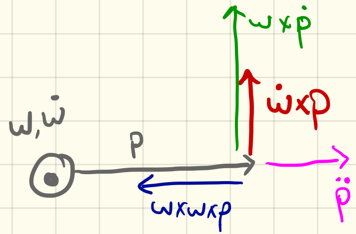
Re-express w.r.t. frame $\{i+1\}$:

$$\boxed{{}^{i+1}\dot{v}_{i+1} = {}^{i+1}R_i\left({}^i\dot{v}_i + {}^i\dot{\omega}_i \times {}^ip_{i+1} + {}^i\omega_i \times {}^i\omega_i \times {}^ip_{i+1}\right) + 2\left({}^{i+1}R_i\,{}^i\omega_i\right) \times \left({}^{i+1}R_i\,{}^i\dot{p}_{i+1}\right) + {}^{i+1}R_i\,{}^i\ddot{p}_{i+1}}$$



Linear Acceleration: ${}^{i+1}\dot{v}_{i+1}$ (Derivation in posted notes)

$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}R_i\Big({}^i\dot{v}_i + \underbrace{{}^i\dot{\omega}_i \times {}^ip_{i+1}}_{\text{tangential}} + \underbrace{{}^i\omega_i \times \left[{}^i\omega_i \times {}^ip_{i+1}\right]\Big)}_{\text{Centripetal}} + \underbrace{2\left({}^{i+1}R_i\,{}^i\omega_i\right) \times \left({}^{i+1}R_i\,{}^i\dot{p}_{i+1}\right)}_{\text{Coriolis}} + \underbrace{{}^{i+1}R_i\,{}^i\ddot{p}_{i+1}}_{\text{radial}} $$



Revolute θ_{i+1}

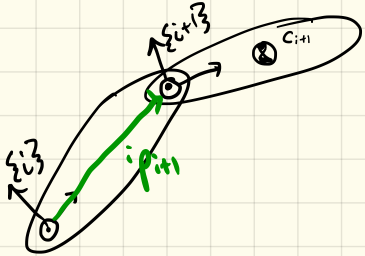
$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}R_i\left({}^i\dot{v}_i + {}^i\dot{\omega}_i \times {}^ip_{i+1} + {}^i\omega_i \times \left[{}^i\omega_i \times {}^ip_{i+1}\right]\right) $$

Prismatic:

$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}R_i\left({}^i\dot{v}_i + {}^i\dot{\omega}_i \times {}^ip_{i+1} + {}^i\omega_i \times \left[{}^i\omega_i \times {}^ip_{i+1}\right]\right) + 2\,{}^{i+1}\omega_{i+1} \times \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d}_{i+1} \end{bmatrix} $$

Bonus!

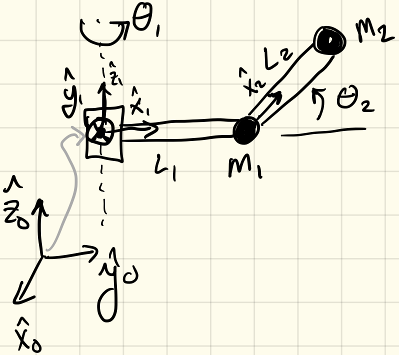
$$ {}^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{v}_{i+1} + {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times {}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_i} $$



Example: 2R Manip

Goal: Complete the outward pass of RNEA

- ${}^i\omega_i,\ {}^i\dot\omega_i,\ {}^i\dot v_i,\ {}^i\dot v_{c_i},\ {}^iF_i,\ {}^iN_i \quad i=1,2$



Body 1

$$ {}^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \qquad {}^1\dot\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot\theta_1 \end{bmatrix} \qquad {}^1\dot v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^1\dot v_{c_1} = \cancel{{}^1\dot v_1}^{\,0} + {}^1\dot\omega_1 \times {}^1p_{c_1} + {}^1\omega_1 \times {}^1\omega_1 \times {}^1p_{c_1} $$

$$ \uparrow_{\ddot\theta_1} \times \xrightarrow[L_1]{} + \uparrow_{\dot\theta_1} \times \left[\uparrow_{\dot\theta_1} \times \xrightarrow[L_1]{} \right] $$

$$ = \otimes L_1\ddot\theta_1 + \xleftarrow{\dot\theta_1^2 L_1} = \begin{bmatrix} -L_1\dot\theta_1^2 \\ L_1\ddot\theta_1 \\ 0 \end{bmatrix} $$

TO BE CONTINUED...

Recap:

- Given velocity & accelerations, Newton and Euler's laws provide net force & moment

$$ {}^{i}\omega_i,\ {}^{i}\dot{\omega}_i,\ {}^{i}\dot{v}_{c_i} \longrightarrow {}^{i}F_i,\ {}^{i}N_i $$

- Recursive Newton Euler:

  Given joint angles, rates and accerations

  ① Find Velocity and accelerations of each body (Today)

  ② Sum up inertial Forces to determine required torques (Next Time)

$$ \theta, \dot{\theta}, \ddot{\theta} \rightarrow \tau $$