Lecture 5: The Denavit Hartenberg Convention

<u>Goal For the Next Few Lectures</u>: $\{0\}$ = Earth-fixed, $\{t\}$ = Tool frame. Determine ${}^{0}T_t(\underbrace{\theta_1, \ldots, \theta_n}_{\text{joint variables}})$

<u>Goals For Today</u>:

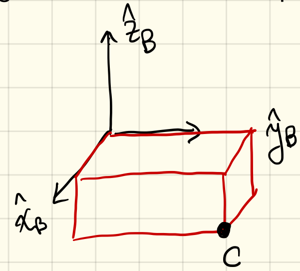
- Wrapping Up our Homogeneous T-form Example
- Attaching frames to links of robots
  - Sequence of transforms in body fixed Coordinates
- Denavit-Hartenberg Convention
  - Simplify kinematic analysis
  - Simplify robot software

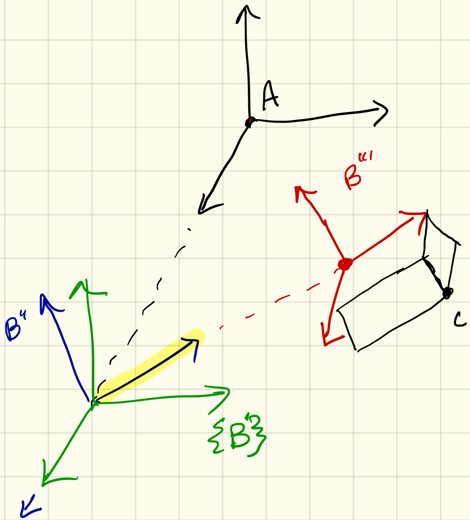
Consider a block with width 4m along $\hat{y}_B$, height 1m along $\hat{z}_B$, depth 2m along $\hat{x}_B$. (Shown below) Suppose the block frame {B} is initially aligned with a fixed frame {A}. Apply the following transformations to {B} in the order given below. Following these transformations, what are the homogeneous coordinates of point C on the block?

1) Translation along $\hat{x}_A$ by 3m

2) Rotation about $\hat{x}_B$ By 30 degrees

3) Translation along $\hat{y}_B$ By 2m





$$ {}^{A}T_{B'} = \begin{bmatrix} I & \begin{matrix} 3 \\ 0 \\ 0 \end{matrix} \\ 0 & 1 \end{bmatrix} $$

$$ {}^{B''}T_{B'''} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{B'}T_{B''} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c30^\circ & -s30^\circ & 0 \\ 0 & s30^\circ & c30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{B''}P_c = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} $$

$${}^{A}T_{B'} = \begin{bmatrix} I & \begin{matrix} 3 \\ 0 \\ 0 \end{matrix} \\ 0 & 1 \end{bmatrix}$$

$${}^{B'}T_{B''} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c30^\circ & -s30^\circ & 0 \\ 0 & s30^\circ & c30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{B''}T_{B'''} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{B'''}p_c = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{A}p_c \\ 1 \end{bmatrix} = {}^{A}T_{B'}\,{}^{B'}T_{B''}\,{}^{B''}T_{B'''} \begin{bmatrix} {}^{B'''}p_c \\ 1 \end{bmatrix}$$

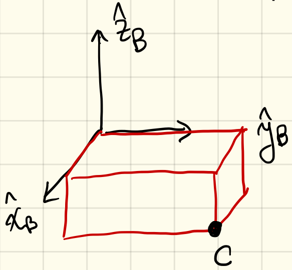
$$= \begin{bmatrix} 5 \\ 3\sqrt{3} - 1/2 \\ 3 + \sqrt{3}/2 \\ 1 \end{bmatrix}$$

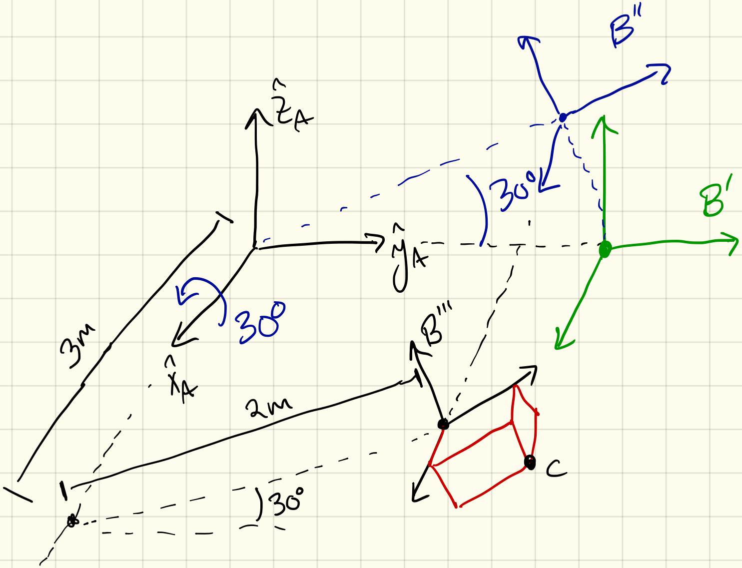
Consider a block with width 4m along $\hat{y}_B$, height 1m along $\hat{z}_B$, depth 2m along $\hat{x}_B$. (Shown below) Suppose the block frame {B} is initially aligned with a fixed frame {A}. Apply the following transformations to {B} in the order given below. Following these transformations, what are the homogeneous coordinates of point C on the block in {A}?

1) Translation along $\hat{y}_A$ by 2m

2) Rotation about $\hat{x}_A$ By 30 degrees

3) Translation along $\hat{x}_A$ By 3m





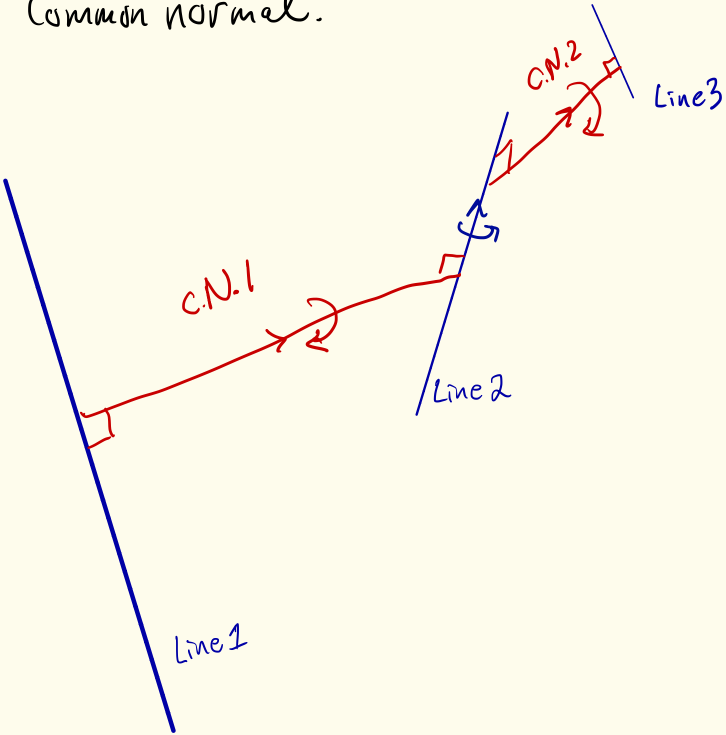
Reasoning Spatially:

Same Answer as previous Question!

More generally: A sequence of transformations applied w.r.t. earth-fixed axes is equivalent to the opposite sequence of transformations applied w.r.t. body-fixed (moving) axes.

Sequence of Lines: Axes of Revolute (R) or Prismatic (P) Joints

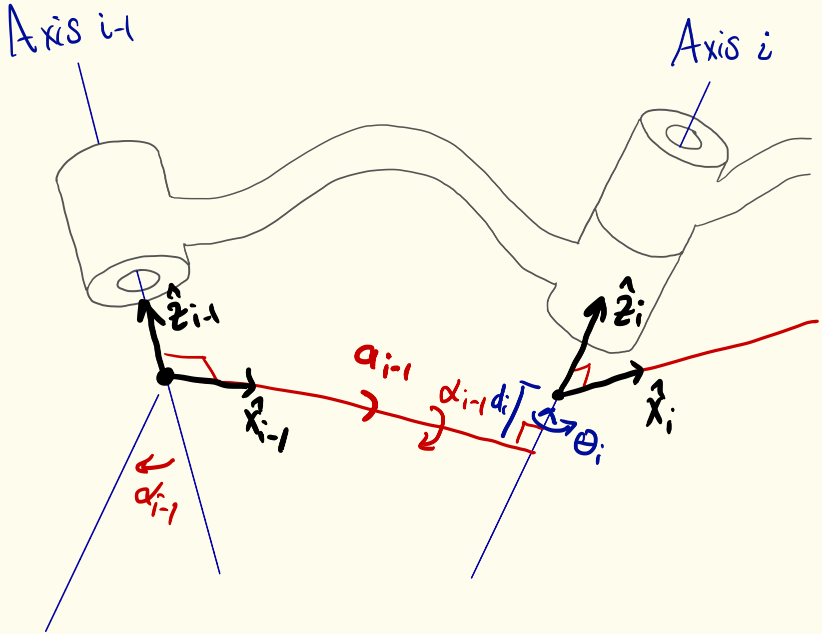
- Axial Screw: Rotation & Translation along an axis
- Any line can be transformed into another by an axial screw along their common normal.



$$\boxed{\text{Lines} \equiv \text{Joint Axes} \qquad \text{Common Normals} \equiv \text{Links}}$$

Any many w/ R & P Joints:
Kinematics described by successive axial screws about $\perp$ axes

Attaching Coordinate Frames



$\circlearrowleft \hat{x}_{i-1}$, α_{i-1}: Angle between $\hat{z}_{i-1}$ and $\hat{z}_i$

$\uparrow \hat{x}_{i-1}$, a_{i-1}: Distance between $\hat{z}_{i-1}$ and $\hat{z}_i$

$\circlearrowleft \hat{z}_i$, θ_i: Angle between $\hat{x}_{i-1}$ and $\hat{x}_i$

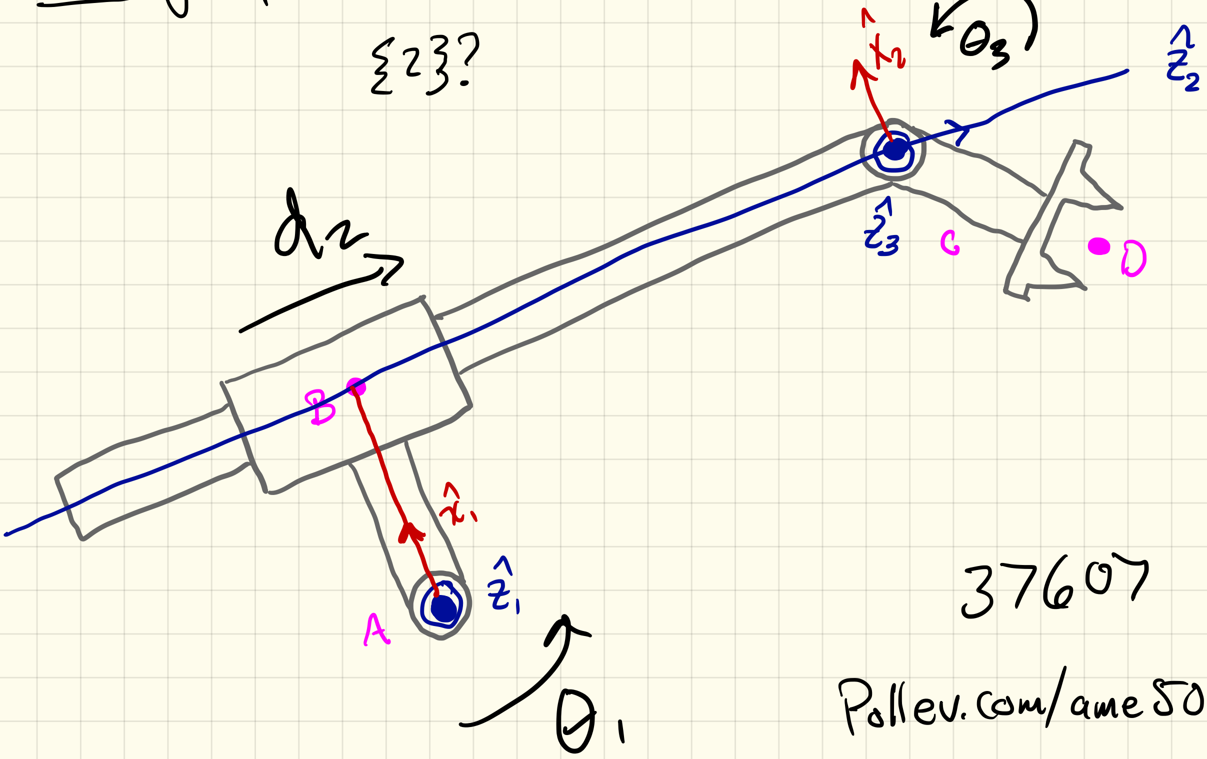
$\uparrow z_i$, d_i: Distance between $\hat{x}_{i-1}$ and $\hat{x}_i$

Rules:

1. $\hat{z}_{i-1}$ along joint axis i-1
2. $\hat{x}_{i-1}$ along common normal of axis i-1 and axis i
3. $\hat{y}_{i-1}$ according to right-hand rule

Where should you place $\{1\}$?

$\{2\}$?



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Summary:

Goal:

1. ^0T_t as a function of joint variables

2. Solve in pieces by attaching frames to each body

3. $\hat{z}_i$ are joint axes, $\hat{x}_i$ axis along c.n. of $\hat{z}_i$ and $\hat{z}_{i+1}$

   DH convention T matrices expressed w/ 4 params.