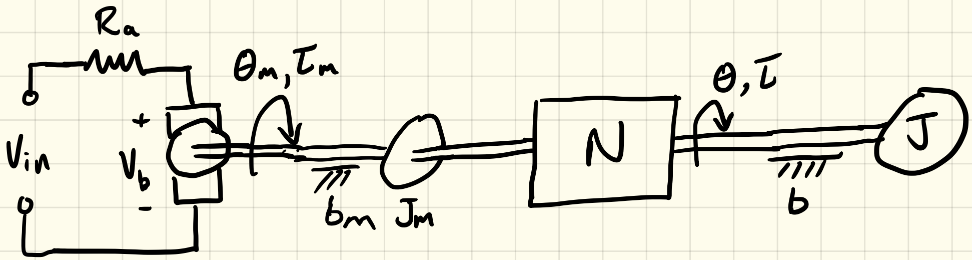
Lecture 38 - Nonlinear Control

- **Announcements:**
  - HW 10: Online, never due. Turn into my mailbox by Thursday ⇒ I'll replace your 2nd lowest HW w/ a 10
  - Final Review: Friday 7th, 4-6 PM, Fitz 356

Today:

- Wrap up: Motor modelling
- Introductory Nonlinear Control
  - ⇒ Computed Torque
  - ⇒ PD+ control

DC Motor Model: Adding a Gearbox



Torque @ output of Gearbox:

$$\tau = \left[\tau_m - \dot{\theta}_m b_m - J_m \ddot{\theta}_m\right] N = b\dot{\theta} + J\ddot{\theta}$$

$$N\theta = \theta_m$$

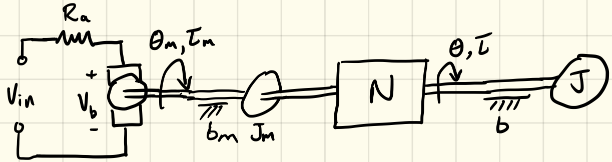
$$N\tau_m = N\dot{\theta}_m b_m + N J_m \ddot{\theta}_m + b\dot{\theta} + J\ddot{\theta}$$

$$= (b + N^2 b_m)\dot{\theta} + (J + N^2 J_m)\ddot{\theta}$$

inertia felt @ output

Or in terms of motor angles:

$$\tau_m = \left(b_m + \frac{b}{N^2}\right)\dot{\theta}_m + \left(J_m + \frac{J}{N^2}\right)\ddot{\theta}_m$$



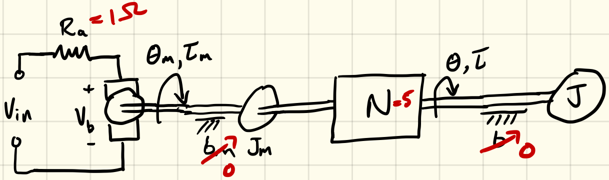
$$\tau_m = \underbrace{\left(b_m + \frac{b}{N^2}\right)}_{b_m'}\dot{\theta}_m + \underbrace{\left(J_m + \frac{J}{N^2}\right)}_{J_m'}\ddot{\theta}_m$$

Comments:

- In a multijoint robot J is not fixed
- When N is large motor components dominate dynamics
- Industrial robots $N > 100$ not uncommon
- Design a critically damped control for maximum J
  $\Rightarrow$ Then at least critically damped all the time

Aside:

- $a\ddot{x} + b\dot{x} + cx = 0$ Overdamped when $b^2 > 4ac$
- if $a_{min} \le a \le a_{max}$ System overdamped if $b^2 > 4a_{max}c$



$N = 5$

$b = b_m = 0$

$R = 1\ \Omega$

$K_b = \frac{1}{2}\ Vs/rad$

$J_m = 2\ Kg\ cm^2$

If J ranges from 100 to 300 $Kg\ cm^2$, and $V_{in} = -K_p\,\theta_m$ what is maximum K_p such that the system is always critically or over damped?

$$\frac{R_a}{K_\tau} J_m' \ddot{\theta}_m + \left(K_b + \frac{R_a b_m'}{K_\tau}\right)\dot{\theta}_m + K_p \theta_m = 0$$

$$\left(K_b + \cancel{\frac{R_a b_m}{K_\tau}}^{\,0}\right)^2 > 4 \frac{R_a}{K_\tau} J_m' K_p$$

$$K_p < \frac{K_b^2 K_\tau}{4 R_a J_m'} = \frac{\left(\frac{1}{2}\ Vs/rad\right)^2 \left(\frac{1}{2}\ Nm/A\right)}{4\Omega\ \left(2\times10^{-4} + \frac{300}{25}\times10^{-4}\right)\ kg\ m^2} = 22.3\ V/rad$$

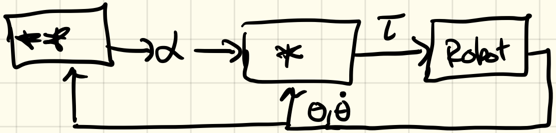
A simple Nonlinear Control law: Goal $\theta \to 0$ as $t \to \infty$

Dynamics: $\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$

- Nonlinear dynamics are difficult to work with
- Let's design a feedback controller to turn this system into a linear one
  "Feedback linearization"

$$\tau = M(\theta)\alpha + V(\theta, \dot{\theta}) + G(\theta) \quad (*)$$

$$\cancel{M}\alpha + \cancel{V} + \cancel{G} = \cancel{M}\ddot{\theta} + \cancel{V} + \cancel{G} \Rightarrow \ddot{\theta} = \alpha$$



- Try PD control: $\alpha = -K_p\theta - K_D\dot{\theta} \quad (**)$
- Closed loop: $\ddot{\theta} + K_d\dot{\theta} + K_p\theta = 0$

Computed Torque Tracking Control

- Desired Trajectory $\theta_d(t)$
- $\tau = M\alpha + V + G$
- $\alpha = \ddot{\theta}_d + K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$
- Consider $e = \theta_d - \theta$

$$\ddot{e} + K_p e + K_d \dot{e} = 0$$

Stable if and only if $K_p > 0$ and $K_d > 0$

(Project 2)