Lecture 25 - RNEA (Outward Pass Example)

Announcements

- HW7 Revised ⇒ Due Weds
- MT2 Next Friday. IK. Jacobians. RNEA.

Today:
- Finish our example
- Equivalent Force Moment Systems

Inverse Dynamics

Outline: Given $\theta, \dot{\theta}, \ddot{\theta}$, find τ

① Outward pass (Kinematics Propagation)

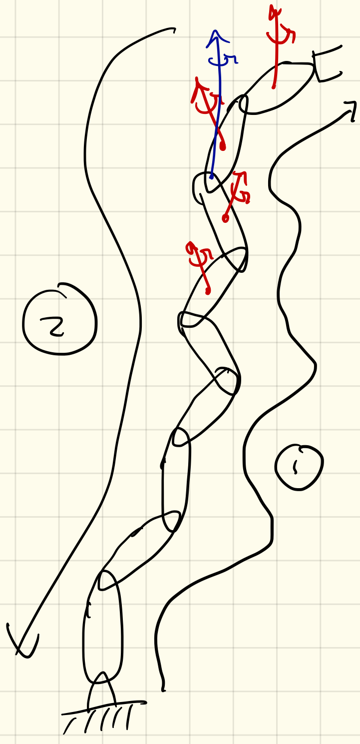
- ${}^{i}\omega_i$, ${}^{i}\dot{\omega}_i$, ${}^{i}\dot{v}_i$, ${}^{i}\dot{v}_{C_i}$
- Newton & Euler to get ${}^{i}F_i$, ${}^{i}N_i$

② Inward pass (Force Propagation)

- Determine ${}^{i}f_i$, ${}^{i}n_i$
- Determine actuator efforts

$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^{i}n_i \quad \text{Revolute}$$

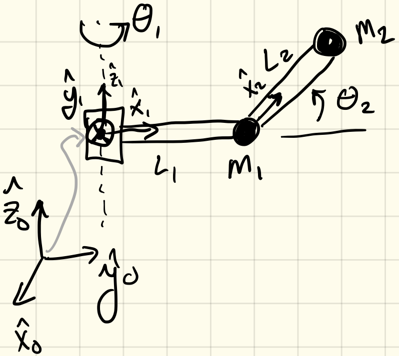
$$\tau_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^{i}f_i \quad \text{Prismatic}$$



Example: 2R Manip

Goal: Complete the outward pass of RNEA

- ${}^i\omega_i,\ {}^i\dot{\omega}_i,\ {}^i\dot{v}_i,\ {}^i\dot{v}_{c_i},\ {}^iF_i,\ {}^iN_i \quad i=1,2$



Body 1

$${}^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \qquad {}^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} \qquad {}^1\dot{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^1\dot{v}_{c_1} = \cancel{{}^1\dot{v}_1}^{\,0} + {}^1\dot{\omega}_1 \times {}^1p_{c_1} + {}^1\omega_1 \times {}^1\omega_1 \times {}^1p_{c_1}$$

$$\uparrow_{\ddot{\theta}_1} \times \xrightarrow{L_1} \; + \; \uparrow_{\dot{\theta}_1} \times \left[\uparrow_{\dot{\theta}_1} \times \xrightarrow{L_1} \right]$$

$${}^1I_{zz} = m_1 L_1^2$$

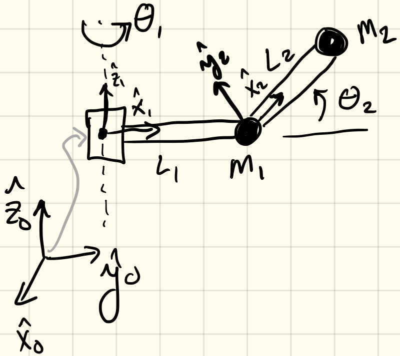
$$= \otimes\, L_1\ddot{\theta}_1 \; + \; \xleftarrow{\dot{\theta}_1^2 L_1} \; = \begin{bmatrix} -L_1\dot{\theta}_1^2 \\ L_1\ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$${}^1F_1 = m_1 \begin{bmatrix} -L_1\dot{\theta}_1^2 \\ L_1\ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$${}^1N_1 = \cancel{{}^{c_1}I}^{\,0}\, {}^1\dot{\omega}_1 + {}^1\omega_1 \times \cancel{{}^{c_1}I}^{\,0}\, {}^1\omega_1 = 0$$

Example: 2R Manip

$${}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1P_{c_1} = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} \qquad {}^2P_{c_2} = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$



Outward pass for Body 2

$$\omega_1 = \uparrow \dot{\theta}_1$$

$${}^2\omega_2 = {}^2R_1\, {}^1\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

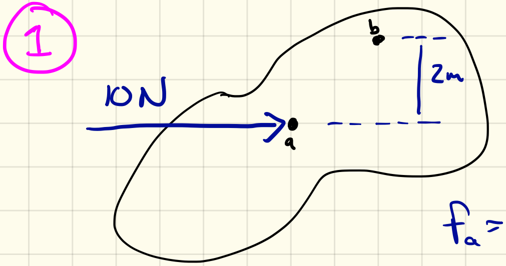
$${}^2\dot{\omega}_2 = {}^2R_1\, {}^1\dot{\omega}_1 + {}^2\omega_2 \times \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} s_2\ddot{\theta}_1 + c_2\dot{\theta}_1\dot{\theta}_2 \\ c_2\ddot{\theta}_1 - s_2\dot{\theta}_1\dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \frac{d}{dt}\left[{}^2\omega_2\right]$$

$${}^2\dot{v}_2 = {}^2R_1\, {}^1\dot{v}_{c_1} = \otimes L_1\ddot{\theta}_1 + \leftarrow L_1\dot{\theta}_1^2 = \begin{bmatrix} -c_2L_1\dot{\theta}_1^2 \\ s_2L_1\dot{\theta}_1^2 \\ -L_1\ddot{\theta}_1 \end{bmatrix}$$

$${}^2\dot{v}_{c_2} = {}^2\dot{v}_2 + {}^2\dot{\omega}_2 \times {}^2p_{c_2} + {}^2\omega_2 \times \left({}^2\omega_2 \times {}^2p_{c_2}\right) = \begin{bmatrix} -(L_1+L_2c_2)c_2\dot{\theta}_1^2 - L_2\dot{\theta}_1^2 \\ (L_1+L_2c_2)s_2\dot{\theta}_1^2 + L_2\ddot{\theta}_2 \\ -(L_1+L_2c_2)\ddot{\theta}_1 + 2L_2s_2\dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$${}^2F_2 = m_2\, {}^2\dot{v}_{c_2} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad {}^2N_2 = 0$$

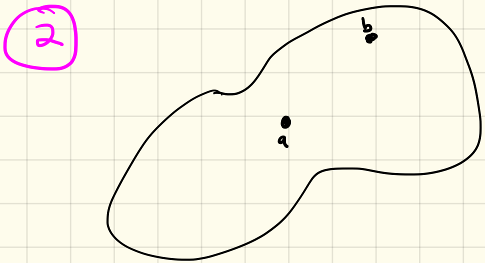
Equivalent Force Moment Systems:

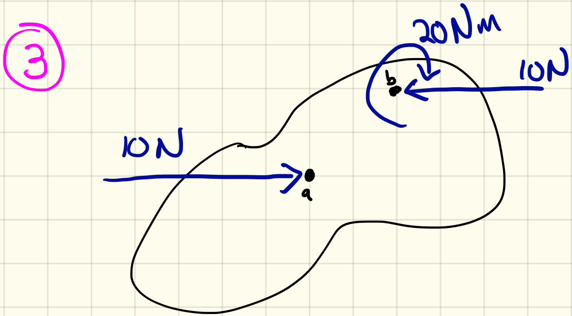


$f_a, n_a = 0$

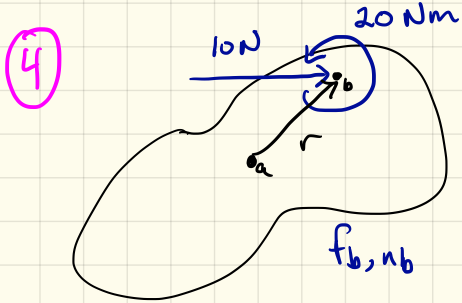
$f_a = f_b$

$n_a = n_b + r \times f_b$





f_a, n_a



f_b, n_b