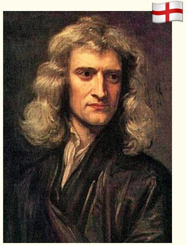
AME 40623/60623 Analytical Dynamics

Newtonian Dynamics

Analytical Dynamics

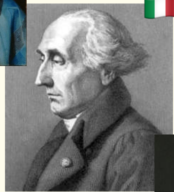
Current Applications



Newton
1687

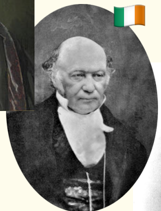
D'Alembert
1742

Euler
1765



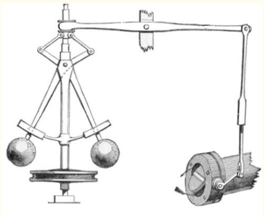
Lagrange
1778

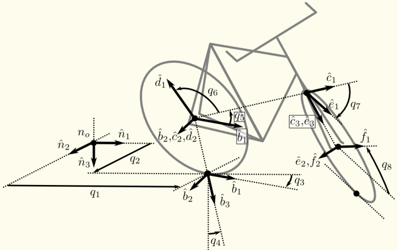
Gauss
1829

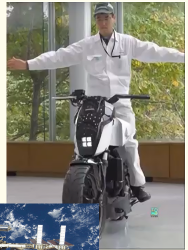


Hamilton
1833

Noether
1915







AME 40623/60623 - Analytical Dynamics

Fall 2019, University of Notre Dame

Instructor: Dr. Patrick Wensing, pwensing@nd.edu
Office: 373 Fitzpatrick Hall, **ph:** 631-2652
Office Hours: M 5:00 - 6:30 PM (DeBartolo Hall 232), Th 9:00 - 10:00 AM (Fitzpatrick Hall 373)

TA: Tan Chen, tchen8@nd.edu
Office Hours: Tu 3:30 - 5:00 PM, Fitzpatrick Hall 364

Class Meetings: MW 3:30 - 4:45 PM, DeBartolo Hall 208
Live Schedule: `https://tinyurl.com/y3ccpn3b`
Final Exam: Tuesday, December 17, 4:15 - 6:15 PM, DeBartolo Hall 208

Main Reference: Analytical Dynamics, H. Baruh, McGraw-Hill, 1999. *A good combination of rigor, insight, and accessibility. Unfortunately the book is no longer in print, so finding a hard copy may be difficult.*

Other References (Ranked in decreasing order of anticipated utility):

- Woodhouse, Introduction to Analytical Dynamics, 2nd Ed., Springer, 2009. *Available free online through Notre Dame. Not as accessible or thorough as Baruh, but still a good mix of rigor and clarity.* `https://link.springer.com/book/10.1007%2F978-1-84882-816-2`
- Greenwood, Principles of Dynamics, 2nd Edition, Prentice-Hall, 1987. *Dry but classical text.*
- Lanczos, The Variational Principles of Mechanics, 4th Edition, University of Toronto Press, 1970. *Mathematically rigorous, but full of insightful discussion of the theory. Very inexpensive.*
- Meirovitch, Methods of Analytical Dynamics, McGraw-Hill, 1970. *A bit more dry than Lanczos, but equally inexpensive and rigorous.*

Learning Objectives: This course will present fundamental principles and analytical methods for deriving and analyzing the dynamics of mechanical systems. Applications of these fundamentals span machine design, robot analysis, and spacecraft control. The primary learning objective is for students to be able to model multibody mechanical systems with complex constraints and derive their equations of motion (e.g., what differential equations describe the motion of a bicycle with no-slip conditions on the tires). A secondary objective of the course is for students to be able to apply state-of-the-art symbolic algebra packages (e.g., the Matlab symbolic math toolbox) to automate otherwise tedious and error-prone by-hand derivations. Following this course, students should be able to read and evaluate academic literature that emphasizes dynamic modelling of mechanisms and spacecraft, numerical methods for dynamics, and other associated topics.

Prerequisites: Newtonian dynamics in 2D, multivariable calculus (partial vs. total derivative), introductory linear algebra (including eigenvalue decomposition), differential equations, basic Matlab programming.

Topics: Particle dynamics, moving reference frames, systems of particles, variational calculus, variational principles of mechanics (via D'Alembert, Lagrange, Hamilton, Gauss, & Jourdain), holonomic and nonholonomic constraints, dynamics of rigid bodies in 3D.

40623 vs. 60623: Homeworks and exams for the 40623 offering will emphasize application of the tools learned in lectures to solve and analyze dynamics problems. By comparison, the graduate-level 60623 offering will emphasize engagement with more advanced and theoretical aspects of the course. Homeworks for 60623 will be targeted to require roughly 30% more time outside of class. As a rough rule of thumb, undergraduates that received A's in Mechanics II and Differential Equations I/II should consider signing up for 60623.

Important Dates:

Wed. Aug. 28	First class meeting	Fri. Nov. 1	Last day for discontinuance
Tues. Sept. 3	Last date for class changes	Fri. Nov. 6	Exam 2
Wed. Sept. 25	Exam 1	Wed. Dec. 11	Last class meeting
Mon. Oct. 21	Mid-term deficiency reports submitted	Tues. Dec. 17	Final Exam

Grading:

Breakdown:	
Exam 1	25%
Exam 2	25%
Final Exam	30%
HWs	20%

Grade Guarantees:			
A	93-100	B–	80-82.9
A–	90-92.9	C+	77-79.9
B+	87-89.9	C	73-76.9
B	83-86.9	C–	70-72.9

The class will most likely be curved. With the curve, your final grade is guaranteed to be no worse than the one earned according to the scale above. I may further add up to 5 percent to your final score if I do not believe your grade adequately reflects your mastery of the material. Approximate grade cutoffs will be communicated after each exam. If a curve is applied, 40623 and 60623 will be curved separately.

Exams: Exams will be closed book and you will be permitted to use your own notes sheets (one double-sided 8.5"×11" page for exam 1, two pages for exam 2, and three pages for the final). Providing or receiving aid from any students or unauthorized materials during an exam is strictly prohibited. If you have a legitimate reason to miss an exam, such as a documented illness or family emergency, contact me as soon as possible. In the case of an excused absence, a make-up exam will be administered for full credit. If the absence is unexcused, the opportunity for a make-up is left to the discretion of the instructor, but credit will not exceed 70%.

Homework:

- Homework is due at the beginning of class. *Late homework submissions will not be accepted for credit.* Late homeworks may still be submitted for feedback only. Your lowest homework score will be dropped.

- Homework deadline extensions may be granted at the instructor's discretion in the event of extenuating circumstances (e.g., a midterm in another course on the same day a HW is due). Extensions must be requested before the deadline.

- Homework should follow a number of technical guidelines:
  1. Homework should be done on 8.5 inch by 11 inch paper. Edges should be clean.
  2. The top of the first page should list your name and the homework number.
  3. Pages should be stabled together.
  4. The top right of each page should be numbered, with total pages indicated (e.g. Page 3 of 6 or 3/6)
  5. Derivations should be supplemented with text explaining the procedure. Symbols must be defined.
  6. Numerical calculations must always be preceded by the algebraic form of the equation.
  7. Final answer(s) should be identified with a surrounding box. Identify nothing else with a box.
  8. Assignments should be presented professionally, with writing clear and legible.

- Each homework will be graded out of 10 points, with 8 points allocated for grading the correctness of your solutions, and 2 points allocated for professional presentation of the solutions in accordance with the guidelines above.

- You are encouraged to collaborate on homework. However, submitted material must be your own work and accurately reflect your own understanding at the time of writing.

- Exact copying, wholly or in part, from another student or any outside resource is strictly prohibited.

- The use of outside resources (other text books, websites, academic papers, etc.) is allowed. However, previous homework solutions, solution manuals, or graded homeworks from a previous course are not permitted as outside resources. Any result/equation/etc. obtained from an approved outside resource must be cited in your homework submission. You do not need to cite lecture notes from this course.

Honor Code: (http://honorcode.nd.edu/the-honor-code/) The university community has a shared commitment to respect and honor the intellectual and creative contributions of each individual. As a precondition for admission to the University, all students pledge:

> *"As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty."*

Beyond this simple pledge, it is your responsibility to become familiar with the Academic Code of Honor, and to adhere to the responsibilities it outlines. Any suspected violation of this code, including but not limited to violation of integrity requirements detailed in this syllabus, will be reported through the procedure described in Section V.D of the Honor Code.

Regrade Requests: If there is an error in the grading of an exam or homework, students may submit a written regrade request. Requests must adhere to the following guidelines.

1. Regrade requests will *not* be accepted in the first 24 hours after an assignment/exam is returned.
2. Following this initial waiting period, regrade requests will be accepted for two business days.
3. Requests should make a clear case for where and why a regrade is being sought.
4. Requests should be placed in my mailbox within the AME main office, 365 Fitzpatrick.

Accommodations: Any student who has a documented disability and is registered with Disability Services should speak with the professor as soon as possible regarding accommodations. Students who are not registered should contact the Office of Disability Services - http://disabilityservices.nd.edu.

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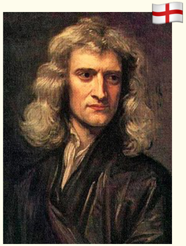
Current Applications



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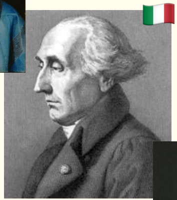

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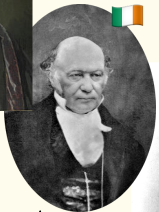

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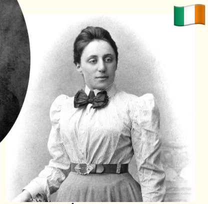

Noether
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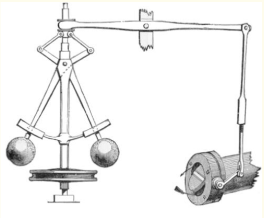
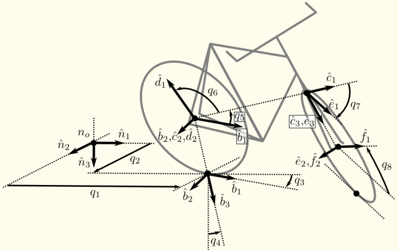
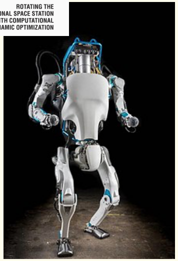






Analytical Dynamics: Scope

Kinematics: the study of movement w/o regard for the forces that cause it

Dynamics: the study of the relationship between movement & forces

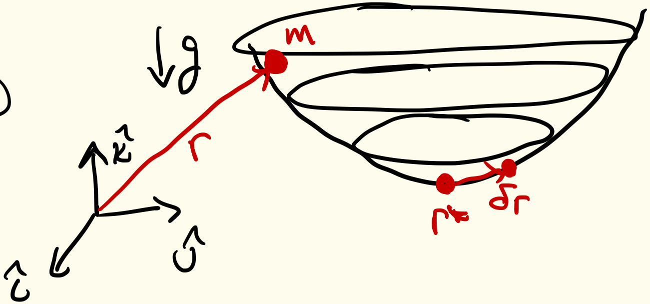
Statics: Special case of dynamics when bodies @ rest

Newton (classical) Dynamics: Consider vector quantities of motion (velocities, forces, momenta) and their relationship. Reaction forces and free body diagrams help treat the interconnection of bodies.

Analytical Dynamics: Considers scalar quantities that address system as a whole (Energy, power, work). Uses a branch of math known as variational calculus to back out dynamics.

- Equivalent to classical Dynamics (but no free body diagrams)

Regular Calculus in Statics:



- Consider a ball in a bowl (point mass)
- Position of ball

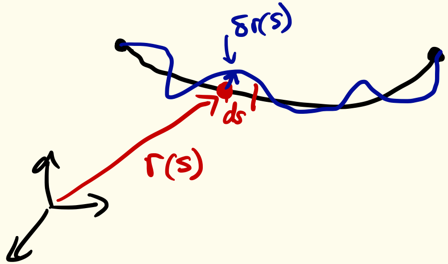
  $r = x\hat{i} + y\hat{j} + z\hat{k}$

- Assuming the ball does not leave surface we describe its height based on x,y position $z = h(x,y)$

Ball has 2 DoF

- Potential Energy: $V(x,y) = mg\,h(x,y)$
- In equilibrium (x^*, y^*)
  - Potential energy minimized
  - Gradient of potential energy is zero $\nabla V(x^*,y^*) = 0$
  - $V(x^*+\delta x, y^*+\delta y) > V(x^*,y^*)$ if $\delta x, \delta y \neq 0$

Variational Calculus in Statics



- Consider a thin rope 1m in length, 1kg of total mass. Uniform density. Endpoints Fixed.

- We specify the rope by the Function

$$r(s) = x(s)\hat{\imath} + y(s)\hat{\jmath} + z(s)\hat{k}$$

s: arclength parameter $\in [0, 1]$

- Potential energy of rope? (Let $\rho = {}^{1\,kg}/_{1\,m}$)

$$V(r(\cdot)) = \int_0^1 \rho\, g\, z(s)\, ds = \int_0^1 \rho\, g\, \hat{k} \cdot r(s)\, ds$$

"Functional: Function of a function"

missing in lecture.

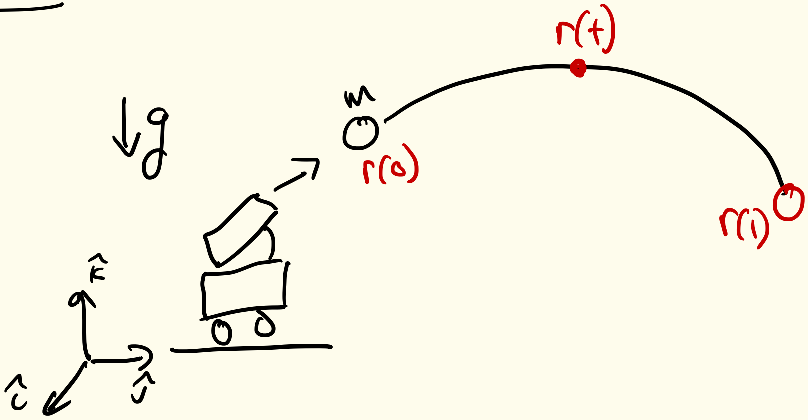
- In equilibrium $r^*(s)$
  - Potential energy minimized
  - Gradient? More on that later.
  - $V(r^* + \delta r) > V(r^*)$ for any variation $\delta r(s)$ s.t. $\delta r(0) = \delta r(1) = 0$

Variational Calculus in Dynamics

Newtonian Dynamics:

$$\ddot{r} = -g\hat{k}$$

local information



Analytical Dynamics:

- Kinetic energy $T(\dot{r}) = \frac{1}{2}m|\dot{r}|^2$
- Potential energy $V(r) = mg\,(r \cdot \hat{k})$

Out of all the ways the projectile could have traveled from $r(0)$ to $r(1)$

The path taken mimimizes

$$\int_0^1 \left(T(\dot{r}(t)) - V(r(t))\right) dt$$

global information in time

Why?! We'll see after MT1.

Schedule

Class	Day	Date	Topic	Subtopic	Text
1	W	8/28	**Particle Dynamics**	Equations of Motion, Impulse and Momentum	B1.4, 1.6, W2.1
2	M	9/2		Work, Energy, and Integrals of motion	B1.7-8, B1.12-13
3	W	9/4	**Reference Frames**	Rotations	B2.1-5, W1.3-5
4	M	9/9		Velocity and acceleration	B2.6-7, W1.6-7
5	W	9/11		Moving frames	B2.8, W1.8
6	M	9/16	**Systems of Particles**	Equations of Motion	B3.1-5
7	W	9/18		Rigid bodies in 2D	B3.10-12, W1.9
8	M	9/23	**Analytical Mechanics of Particles**	Generalized coordinates, holonomic constraints, virtual work	B4.1-4, W2.4,3.7-8
9	W	9/25		**Exam 1 (Lectures 1-8)**	
10	M	9/30		The calculus of variations	W2.7-8
11	W	10/2		Generalized forces and D'Alembert's principle	B4.5-7, W3.5
12	M	10/7		Hamilton's variational principle	B4.8, W2.6,9, 3.9
13	W	10/9		Lagrange's equations	B4.9-10, W2.5,3.6
14	M	10/14		Equilibrium	B5.1-2
15	W	10/16		Linearized Systems	B5.5-6
				Midterm Break	
16	M	10/28		First Integrals, Routhian Reduction, and Noether's Theorem	B5.8-9, W4.1
17	W	10/30		Generalized momentum, Hamilton's equations	B5.10-11, W7.1-2
18	M	11/4	**Analytical Mechanics of Bodies**	Mass moments and rigid bodies in 3D	B6.1-6,7.1-7, W5.1-3
19	W	11/6		**Exam 2 (Lectures 9-18)**	
20	M	11/11		Euler's equations and angular dynamics	B8.5-6, W5.5
21	W	11/13		Impulse-momentum relationships	B8.7-9
22	M	11/18		Lagrange's equations for rigid bodies	B8.10, W5.6
23	W	11/20		Nonholonomic constraints	B8.10, W5.7
24	M	11/25		D'Alembert's principle for rigid bodies	B8.11-12
				Thanksgiving Break	
25	M	12/2		Nonholonomic constraints, generalized speeds	B9.5-6
26	W	12/4		Gauss principle and Gibbs-Appell equations	B5.13, 9.7
27	M	12/9		Jourdain's principle and Kane's equations	B5.13, 9.8
28	W	12/11		Semester Review	
	Tues	12/17		**Final (Cumulative)**	

Newtonian (50%-80% review)

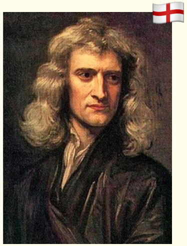
Variational

Newtonian in 3D

Variational

Newtonian

Why So Many Approaches?



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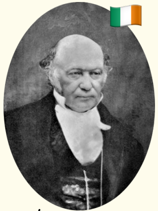

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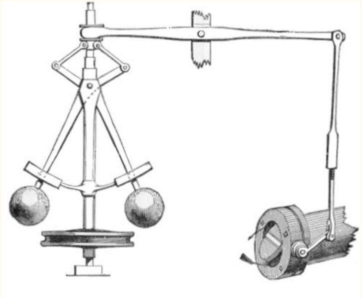
Different ways of viewing the problem

- Easiest / most inuitive strategy problem dependent

Applications

- Newtonian Mechanics – Understanding interactions
  Intuition for design

- Advances in computing means we can derive & compute dynamics for systems we couldn't before.



1788

Spacecraft, Vehicles, Robots

Spacecraft ↓ Conservation of momentum

Vehicles ↓ Rolling constraints

Robots ↓ Many DoFs

ISS

Self Balancing Bike

Humanoid Robot