Lecture 16 - Velocity Kinematics - 9/26/18

Recap:

① $^{A}V_{B} = \frac{d}{dt}\, ^{A}P_{B}$ "Linear velocity of $\{B\}$ as observed in $\{A\}$"

② $\left(\frac{d}{dt}\, ^{A}R_{B}\right) = {}^{A}\Omega_{B} \times {}^{A}R_{B}$ "Anglar velocity of $\{B\}$ rel. to $\{A\}$"

③ $^{A}V_{C} = {}^{A}V_{B} + {}^{A}R_{B}\, ^{B}V_{C} + {}^{A}\Omega_{B} \times {}^{A}R_{B}\, ^{B}P_{C}$ Chaining rel. linear velocities

Today:

- Chaining Relative Angular Velocities
- Velocity Propagation Under DH

Sequences of Rotations:

- Chaining angular velocities is easier

$$\boxed{{}^{A}\Omega_{C} = {}^{A}\Omega_{B} + {}^{A}R_{B}\,{}^{B}\Omega_{C} \quad \bigstar\bigstar}$$

- We will often find it convenient to deal w/ absolute velocities (i.e. relative to an earth fixed frame)

$${}^{0}\Omega_{C} = {}^{0}\Omega_{B} + {}^{0}R_{B}\,{}^{B}\Omega_{C}$$

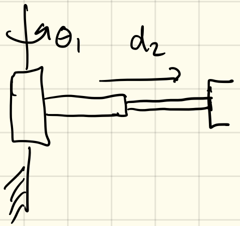
$${}^{0}\omega_{C}$$

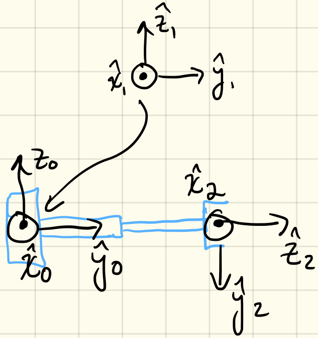
- We will use shorthand ${}^{B}\omega_{C}$ to denote the absolute angular velocity of $\{C\}$ expressed w.r.t. $\hat{x}_B, \hat{y}_B, \hat{z}_B$

${}^{B}\omega_{C}$: ${}^{B}R_{0}\,{}^{0}\Omega_{C}$ — Angular velocity of $\{C\}$ rel. to $\{0\}$ expressed in $\{B\}$

${}^{B}\Omega_{C}$: angular velocity of $\{C\}$ rel to $\{B\}$ expressed in $\{B\}$

Example: RP Manipulator





Let: $\theta_1 = 0°$, $\dot\theta_1 = 3\ \text{rad/s}$

$d_2 = 1\text{m}$, $\dot d_2 = 0.5\ \text{m/s}$

Find:

${}^1\Omega_2$, ${}^1\omega_2$, ${}^2\omega_2$

${}^1V_2$, ${}^0V_2$

$${}^1\Omega_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^1\omega_2 = {}^0R_1 \begin{bmatrix} 0 \\ 0 \\ 3\ \text{rad/s} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3\ \text{rad/s} \end{bmatrix}$$

$${}^2\omega_2 = \begin{bmatrix} 0 \\ -3\ \text{rad/s} \\ 0 \end{bmatrix}$$

$${}^1V_2 = \xrightarrow{\dot d_2} = \begin{bmatrix} 0 \\ 0.5\ \text{m/s} \\ 0 \end{bmatrix}$$

$${}^0V_2 = \cancel{{}^0V_1}^{\,0} + {}^0R_1\,{}^1V_2 + {}^0\Omega_1 \times {}^0R_1\,{}^1P_2$$

$$\xrightarrow{\dot d_2} + \uparrow\dot\theta_1 \times \xrightarrow{d_2}$$

$$= \xrightarrow{\dot d_2} + \otimes\,\dot\theta_1 d_2$$

$$= \begin{bmatrix} -3 \\ 0.5 \\ 0 \end{bmatrix} \text{m/s}$$

Velocity Propagation Under DH

★★ $\quad {}^{A}\Omega_C = {}^{A}\Omega_B + {}^{A}R_B \, {}^{B}\Omega_C$

Apply ★★ $\{A\} = \{0\}$, $\{B\} = \{i\}$, $\{C\} = \{i+1\}$:

$${}^{i+1}R_0 \left({}^{0}\omega_{i+1} = {}^{0}\omega_i + {}^{0}R_i \, {}^{i}\Omega_{i+1} \right)$$

Express w.r.t. $\{i+1\}$:

$${}^{i+1}\omega_{i+1} = {}^{i+1}R_i \, {}^{i}\omega_i + {}^{i+1}R_i \, {}^{i}\Omega_{i+1}$$

Relative ang. Velocity expressed in frame $i+1$

Under DH

$$\boxed{{}^{i+1}\omega_{i+1} = {}^{i+1}R_i \, {}^{i}\omega_i + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}}$$

Absolute Linear Velocity

$$^0\nu_c = {}^0V_c$$

$$^B\nu_c = {}^BR_0 \, {}^0\nu_c = {}^BR_0 \left[\frac{d}{dt} \, {}^0P_c \right]$$

derivative takes place in $\{0\}$

Note: Upper Case (V, Ω) denote relative velocities
Lower Case (ν, ω) denote absolute velocities

Velocity Propagation Under DH

$$(\star)\quad {}^{A}V_{C} = {}^{A}V_{B} + {}^{A}R_{B}\,{}^{B}V_{C} + {}^{A}\Omega_{B} \times {}^{A}R_{B}\,{}^{B}P_{C}$$

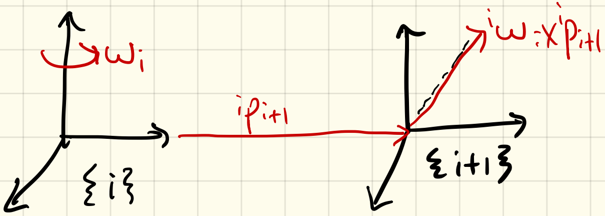
Apply $\star$ with $\{A\}=\{0\}$, $\{B\}=\{i\}$, $\{C\}=\{i+1\}$

$$ {}^{0}v_{i+1} = {}^{0}v_{i} + {}^{0}R_{i}\,{}^{i}V_{i+1} + \underbrace{{}^{0}\omega_{i} \times {}^{0}R_{i}\,{}^{i}P_{i+1}} $$

Re-express w.r.t. $\{i+1\}$ (Derive as exercise @home)

$$ {}^{i+1}v_{i+1} = {}^{i+1}R_{i}\left[{}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}\right] + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix} $$

Easily computed from DH table



Kinematics Propagation Under DH

$$ {}^0T_{i+1} = {}^0T_i \, {}^iT_{i+1}(\alpha_i, a_i, d_{i+1}, \theta_{i+1}) $$

Velocity Prop. Under DH

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}R_i \, {}^i\omega_i + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} $$

$$ {}^{i+1}v_{i+1} = {}^{i+1}R_i \left({}^iv_i + {}^i\omega_i \times {}^iP_{i+1} \right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix} $$