7/

Systems of Particles

2D Rigid-Bodies

- Admin
  - HW 2 : Due ~~Wendesday~~ Friday 4PM to my mailbox (Fitz 365)
  - MT1 : Next Wednesday (Debartolo 140, Experiment: Open Book/MATLAB/Notes/Sakai)

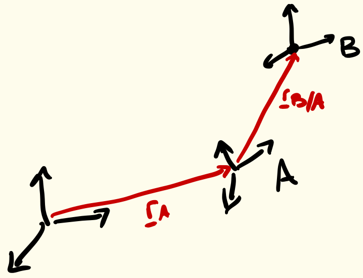
- Last time
  - Worked examples w/ Moving Frames

- Today
  - Examples w/ Moving frames — MATLAB
  - Review of 2D Rigid body Dynamics

Main Results From Last Time:

$$\underline{\omega}_{B/A} = x\,\hat{i}_A + y\,\hat{j}_A + z\,\hat{k}_A$$

$$\dot{\underline{\omega}}_{B/A} = \underbrace{\dot{x}\,\hat{i}_A + \dot{y}\,\hat{j}_A + \dot{z}\,\hat{k}_A}_{:=\,\dot{\underline{\omega}}_{B/A,rel}} + \underline{\omega}_A \times \underline{\omega}_{B/A}$$



Angular Acceleration

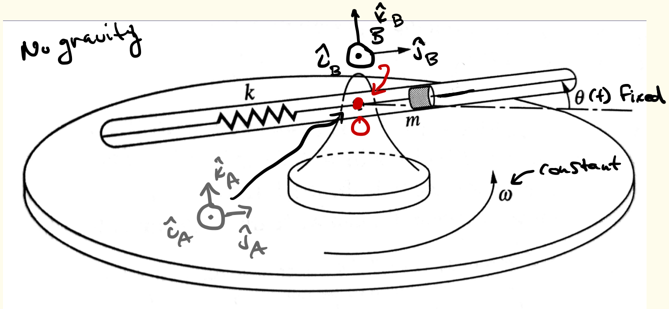
$$\dot{\underline{\omega}}_B = \dot{\underline{\omega}}_A + \underline{\omega}_A \times \underline{\omega}_{B/A} + \dot{\underline{\omega}}_{B/A,rel}$$

Linear Velocity & Acceleration

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_A \times \underline{r}_{B/A} + \underline{v}_{B/A,rel}$$

$$\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}}_A \times \underline{r}_{B/A} + \underline{\omega}_A \times (\underline{\omega}_A \times \underline{r}_{B/A}) + 2\,\underline{\omega}_A \times \underline{v}_{B/A,rel} + \underline{a}_{B/A,rel}$$

Find the equations of motion for the mass.



Let y its distance from $\hat{k}_B$ along $\hat{\imath}_B$

$\underline{\omega}_A = \omega \hat{k}_A \qquad \underline{r}_{m/A} = y\hat{\jmath}_B = y(\cos\theta\,\hat{\jmath}_A + \sin\theta\,\hat{k}_A)$

$${}^A\{\underline{r}_{m/A}\} = \begin{bmatrix} 0 \\ yc\theta \\ ys\theta \end{bmatrix} \qquad {}^A\{V_{m/A,rel}\} = \frac{d}{dt}\begin{bmatrix} 0 \\ yc\theta \\ ys\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -ys\theta\dot{\theta} + \dot{y}c\theta \\ yc\theta\dot{\theta} + \dot{y}s\theta \end{bmatrix} \qquad {}^A\{\underline{a}_{m/A,rel}\} = \frac{d}{dt}\,{}^A\{V_{m/A,rel}\}$$

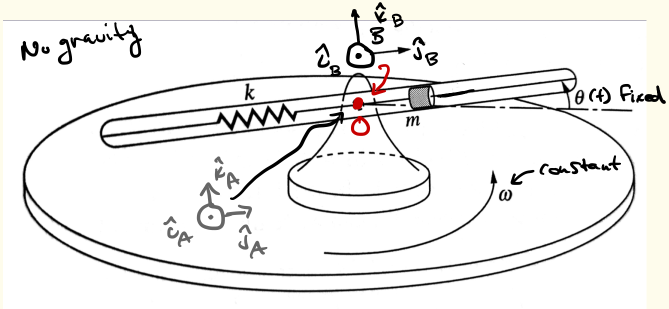
$$\underline{a}_m = \cancel{\underline{a}_A} + \cancel{\dot{\underline{\omega}}_A} \times \underline{r}_{m/A} + \underline{\omega}_A \times \underline{\omega}_A \times \underline{r}_{m/A} + 2\underline{\omega}_A \times \underline{V}_{m/A,rel} + {}^A\underline{a}_{m/A,rel}$$

MATLAB

$$m a_m \cdot \hat{\jmath}_B = -Ky$$

$$\boxed{m(y\omega^2 s_\theta^2 - y\omega^2 - y\dot{\theta}^2 + \ddot{y}) = -Ky}$$

Find the equations of motion for the mass.



Let y its distance from $\hat{k}_B$ along $\hat{\imath}_B$

$\underline{\omega}_A = \omega \hat{k}_A$ $\qquad$ $\underline{r}_{m/A} = y\,\hat{j}_B = y(\cos\theta\,\hat{j}_A + \sin\theta\,\hat{k}_A)$

$$^B\{r_{m/B}\} = \begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix} \qquad ^B\{v_{m/B,rel}\} = \begin{bmatrix} 0 \\ \dot{y} \\ 0 \end{bmatrix}$$

$$^B\{a_{m/B,rel}\} = \begin{bmatrix} 0 \\ \ddot{y} \\ 0 \end{bmatrix}$$

$$\underline{\omega}_B = \omega\hat{k}_A + \dot{\theta}\,\hat{\imath}_B$$

$$\dot{\underline{\omega}}_B = \ddot{\theta}\,\hat{\imath}_B + \dot{\theta}\,\underline{\omega}_B \times \hat{\imath}_B$$

$$\underline{a}_m = \cancelto{0}{\underline{a}_B} + \dot{\underline{\omega}}_B \times \underline{r}_{m/B} + \underline{\omega}_B \times \underline{\omega}_B \times \underline{r}_{m/B} + 2\,\underline{\omega}_B \times \underline{v}_{m/B,rel} + \underline{a}_{m/B,rel}$$

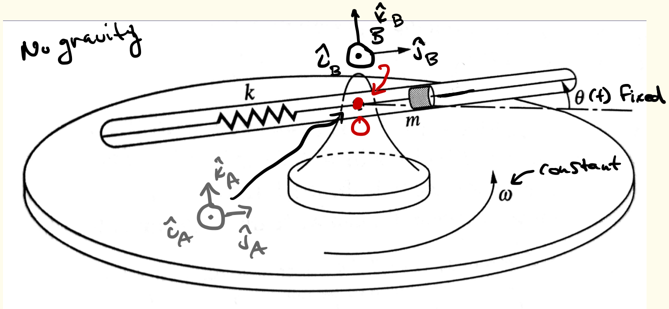
Still:

$$m\,a_m \cdot \hat{j}_B = -ky$$

$$\boxed{m(y\,\omega^2 s_\theta^2 - y\,\omega^2 - y\,\dot{\theta}^2 + \ddot{y}) = -ky}$$

Find the equations of motion for the mass.

Let y its distance from $\hat{K}_B$ along $\hat{\imath}_B$



$\underline{\omega}_A = \omega \hat{K}_A$ $\quad$ $\underline{r}_{m/A} = y\hat{\jmath}_B = y(\cos\theta\, \hat{\jmath}_A + \sin\theta\, \hat{K}_A)$

Another way of doing it: Use planar intuition for $\underline{V}_{m/A,rel}$ $\underline{a}_{m/A,rel}$

$$\underline{a}_m = \cancel{\underline{a}_A} + \cancel{\dot{\underline{\omega}}_A} \times \underline{r}_{m/A} + \underline{\omega}_A \times \underline{\omega}_A \times \underline{r}_{m/A} + 2\underline{\omega}_A \times \underline{V}_{m/A,rel} + \underline{a}_{m/A,rel}$$

$$= \uparrow\omega \times (\uparrow\omega \times y\nearrow) + 2\uparrow\omega \times (\dot{y}\nearrow + \uparrow y\dot{\theta}) + (\underbrace{\ddot{y}\nearrow_{\hat{\jmath}_B}}_{\text{relative}} + \underbrace{\uparrow 2\dot{y}\dot{\theta}}_{\text{coriolis}} + \underbrace{\uparrow y\ddot{\theta}}_{\text{tangential}} + \underbrace{\swarrow y\dot{\theta}^2}_{\text{Centripetal}})$$

$$= \underbrace{\uparrow\omega \times \otimes\omega y c_\theta}_{\leftarrow \omega^2 y c_\theta} + 2(\otimes\omega\dot{y}c_\theta + \odot\omega y\dot{\theta}s_\theta) + \text{Same}$$

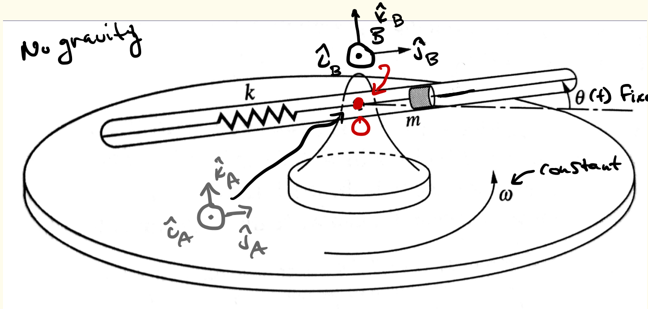
$$= \underset{\omega^2 y c_\theta}{\leftarrow} + \odot(2\omega y\dot{\theta}s_\theta - \omega\dot{y}c_\theta) + \nearrow_{\hat{\jmath}_B}(\ddot{y} - y\dot{\theta}^2) + \nwarrow 2\dot{y}\dot{\theta} + y\ddot{\theta}$$

$$m\underline{a}_m \cdot \hat{\jmath}_B = \boxed{m\left[-\omega^2 y c_\theta^2 + \ddot{y} - y\dot{\theta}^2\right] = -ky}$$

Find the equations of motion for the mass.

Let y its distance from $\hat{k}_B$ along $\hat{\imath}_B$

$$\underline{\omega}_A = \omega \hat{K}_A \qquad \underline{r}_{m/A} = y\,\hat{\jmath}_B = y(\cos\theta\,\hat{J}_A + \sin\theta\,\hat{K}_A)$$



Take 4: Using frame B as moving frame

$$\underline{\omega}_B = \omega\hat{K}_A + \dot\theta\,\hat{\imath}_B = \uparrow\omega + \odot\dot\theta$$

$$\dot{\underline{\omega}}_B = \ddot\theta\,\hat{\imath}_B + \dot\theta\,\underline{\omega}_B\times\hat{\imath}_B = \ddot\theta\,\hat{\imath}_B + \omega\dot\theta\,\hat{J}_A = \odot\ddot\theta + \overset{\omega\dot\theta}{\rightarrow}$$

$$\underline{a}_m = \cancel{\underline{a}_B}^{\,0} + \dot{\underline{\omega}}_B\times\underline{r}_{m/B} + \underline{\omega}_B\times\underline{\omega}_B\times\underline{r}_{m/B} + 2\,\underline{\omega}_B\times\underline{V}_{m/B,rel} + \underline{a}_{m/B,rel}$$

$$= \left(\odot\ddot\theta + \overset{\omega\dot\theta}{\rightarrow}\right)\times \overset{y}{\underset{\uparrow\theta}{\nearrow}} + (\uparrow\omega+\odot\dot\theta)\times(\uparrow\omega+\odot\dot\theta)\times\overset{y}{\nearrow} + 2\,(\uparrow\omega+\odot\dot\theta)\times\overset{\dot y}{\nearrow} + \overset{\ddot y}{\nearrow}$$

$$= \left(\nwarrow\ddot\theta y + \odot\omega\dot\theta y s_\theta\right) + \underbrace{(\uparrow\omega+\odot\dot\theta)\times\left(\otimes\omega y c_\theta + \nwarrow\dot\theta y\right)}_{\overset{\leftarrow}{\omega^2 y c_\theta}\; + \;\odot\omega\dot\theta y s_\theta \;+\; \swarrow y\dot\theta^2} + 2\left(\otimes\omega\dot y c_\theta + \nwarrow\dot y\dot\theta\right) + \overset{\ddot y}{\nearrow}$$

$$= \overset{\theta}{\nwarrow}\left(\ddot\theta y + 2\dot y\dot\theta\right) + \underset{\uparrow\theta}{\nearrow}\left(\ddot y - y\dot\theta^2\right) + \odot\left[2\omega\dot\theta y s_\theta - 2\omega\dot y c_\theta\right] + \leftarrow \omega^2 y c_\theta$$