Lecture 20: Exam Wrap Up

- Rest of Today: Poincare Stability analysis of limit cycles

- On Deck:
    - Zero Moment Point and the contact wrench cone
    - Planning with the LIP Model

Stability of an Orbit: Poincare Section

- Define a surface S transversal to the orbit
- If $x \in \mathbb{R}^n$ the S can be defined by

$$S = \{x \mid g(x) = 0\} \qquad g: \mathbb{R}^n \to \mathbb{R}$$

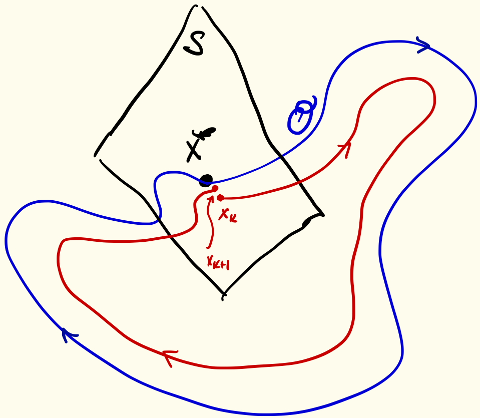
w/ Requirement $\left.\frac{\partial g}{\partial x}\right|_{x^*} f(x^*) \neq 0$

- States Starting near $x^* \in S$ will return to S. So we define the return map

$$P(x): S \to S$$

$\Rightarrow P(x^*) = x^*$

$\Rightarrow x_{k+1} = P(x_k)$ gives a discrete-time sys



- The orbit $\mathcal{O}$ is stable/(asy stable)/(exp stable) if and only if x^* is a stable/(asy stable)/(exp stable) equilibrium of $x_{k+1} = P(x_k)$

Stability of Orbits:

① If $\frac{\partial P}{\partial x}\Big|_{x^*}$ has eigenvalues in the unit circle then the orbit is exponentially stable (n-1 eigenvalues)

② Let $X(0) = I$ $\quad \dot{X} = \left[\frac{\partial f}{\partial x}\Big|_{x(t;x^*)}\right] \cdot X \qquad X \in \mathbb{R}^{n \times n}$

X is called the "monodromy" matrix. Same as forward sensitivity analysis

$$\delta x_T = X(T)\, \delta x_0$$

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$\Rightarrow$ X has an eigval of 1 (perturbations along the orbit persist)

$\Rightarrow$ If the remaining eigvals are in the unit circle

$\Rightarrow$ exponentially stable