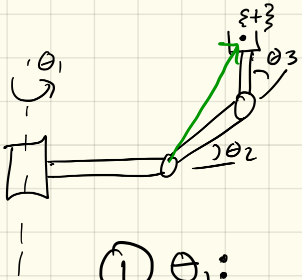
Midterm 2 Review

Inverse Kinematics : 3R Manipulator



$$
{}^{0}p_t = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} L_1 c_1 + L_2 c_1 c_2 + L_3 c_1 c_{23} \\ L_1 s_1 + L_2 s_1 c_2 + L_3 s_1 c_{23} \\ L_2 s_2 + L_3 s_{23} \end{bmatrix} = \begin{bmatrix} c_1 (L_1 + L_2 c_2 + L_3 c_{23}) \\ s_1 (L_1 + L_2 c_2 + L_3 c_{23}) \\ L_2 s_2 + L_3 s_{23} \end{bmatrix}
$$

① θ_1: $\theta_1 = \text{atan2}(\pm p_y, \pm p_x)$ 2 solutions

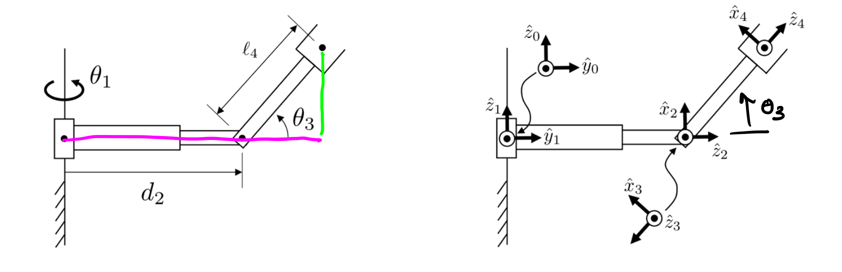
② θ_2: $\underbrace{c_1 p_x + s_1 p_y}_{:=\gamma} = L_1 + L_2 c_2 + L_3 c_{23} \Rightarrow \begin{cases} L_3 c_{23} = \gamma - L_1 - L_2 c_2 \\ L_3 s_{23} = p_z - L_2 s_2 \end{cases}$

Square & Add: $L_3^2 = (\gamma - L_1)^2 - 2L_2(\gamma - L_1)c_2 + L_2^2 + p_z^2 - 2L_2 p_z s_2$

$$
\underbrace{L_3^2 - (\gamma - L_1)^2 - p_z^2 - L_2^2}_{C} = \underbrace{-2L_2(\gamma - L_1)}_{A} c_2 \underbrace{- 2L_2 p_z}_{B} s_2
$$

$\theta_2 = \text{atan2}(B, A) \pm \text{atan2}(\sqrt{A^2 + B^2 - C^2}, C)$ 2 solutions

③ θ_3: $\theta_{23} = \text{atan2}(p_z - L_2 s_2, \gamma - L_1 - L_2 c_2)$ $\quad \theta_3 = \theta_{23} - \theta_2$



Compute the Jacobian ${}^4J_4$?

$$\begin{bmatrix} {}^4\omega_4 \\ {}^4v_4 \end{bmatrix} = {}^4J_4 \dot{\Theta}$$

$$ {}^4J_4 = \left[\begin{array}{c|c|c} c_3 & 0 & 0 \\ 0 & 0 & 1 \\ s_3 & 0 & 0 \\ \hline 0 & -s_3 & \ell_4 \\ -(d_2+\ell_4 c_3) & 0 & 0 \\ 0 & c_3 & 0 \end{array}\right]$$

<u>Third</u> <u>column:</u> ${}^4\omega_4$, ${}^4v_4$ when $\dot{\theta}_3 = 1$

$${}^4\omega_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad {}^4v_4 = \odot \times \vec{\ell_4} = \begin{bmatrix} \ell_4 \\ 0 \\ 0 \end{bmatrix}$$

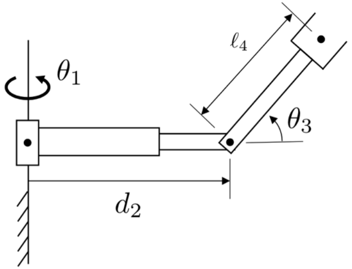
<u>Second</u> <u>column:</u> ${}^4\omega_4$, ${}^4v_4$, $\dot{d}_2 = 1$

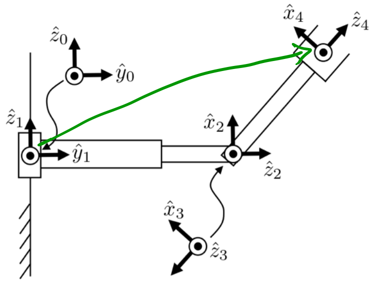
$${}^4\omega_4 = 0 \qquad {}^4v_4 = \begin{bmatrix} -s_3 \\ 0 \\ c_3 \end{bmatrix}$$

<u>First</u> <u>column:</u> $\dot{\theta}_1 = 1$

$${}^4\omega_4 = \uparrow_{\dot{\theta}_1} = \begin{bmatrix} c_3 \\ 0 \\ s_3 \end{bmatrix}$$

$${}^4v_4 = \uparrow_{\dot{\theta}_1} \times \xrightarrow{d_2+\ell_4 c_3} = \otimes_{d_2+\ell_4 c_3} = \begin{bmatrix} 0 \\ -(d_2+\ell_4 c_3) \\ 0 \end{bmatrix}$$





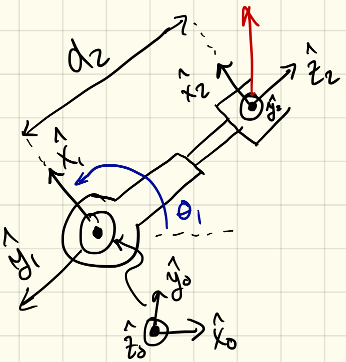
Compute 1st column w/ Geometric Method.

$$^1T_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & d_2 + \ell_4 c_3 \\ c_3 & 0 & s_3 & \ell_4 s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4\omega_4 = {}^4R_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -s_3 & c_3 \\ 1 & 0 & 0 \\ 0 & c_3 & s_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_3 \\ 0 \\ s_3 \end{bmatrix}$$

$${}^4v_4 = {}^4R_1 \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ d_2 + \ell_4 c_3 \\ \ell_4 s_3 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ -(d_2 + \ell_4 c_3) \\ 0 \end{bmatrix}$$

Static Force Analysis



- Suppose we want to apply

$$ {}^2f = \begin{bmatrix} 10\,N \\ 0\,N \\ 5\,N \end{bmatrix} $$

to the world.

- What are required torques forces in static equilibrium when $d_2 = 0.3m$?

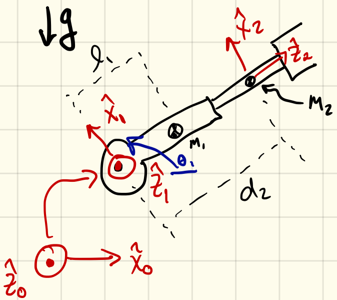
$$ \tau = \left({}^2J_2^N \right)^T \begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix} $$

$$ {}^2J_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ \hline d_2 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} $$

$$ \tau = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ \hline 0.3m & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10N \\ 0 \\ 5N \end{bmatrix} = \begin{bmatrix} 3Nm \\ 5N \end{bmatrix} $$

- What would you do if given ${}^0f$ instead?

RNEA:



$${}^1p_{c_1} = [0\ \ -l_1\ \ 0]^T \qquad {}^1p_2 = [0\ \ -d_2\ \ 0]^T$$

$${}^1p_c^2 = [0,0,0]^T$$

$${}^0R_1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^{C_1}I_1 = \begin{bmatrix} I_{xx_1} & 0 & 0 \\ 0 & I_{yy_1} & 0 \\ 0 & 0 & I_{zz_1} \end{bmatrix} \qquad {}^{C_2}I = \begin{bmatrix} I_{xx_2} & 0 & 0 \\ 0 & I_{yy_2} & 0 \\ 0 & 0 & I_{zz_2} \end{bmatrix}$$

Outward Pass:

$${}^0\omega_0 = {}^0\dot{\omega}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^0\dot{v}_0 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$$

Body 1:

$${}^1\omega_1 = {}^1R_0\, \cancel{{}^0\omega_0}^{\to 0} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \qquad {}^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}$$

$${}^1\dot{v}_1 = {}^1R_0\, {}^0\dot{v}_0 = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix} = \begin{bmatrix} g s_1 \\ g c_1 \\ 0 \end{bmatrix}$$

$${}^1\dot{v}_{c_1} = {}^1\dot{v}_1 + \underbrace{{}^1p_{c_1} \times {}^1\dot{\omega}_1}_{\nearrow l_1 \times \odot\ddot{\theta}_1 \;\to\; \uparrow l_1\ddot{\theta}_1} + \underbrace{{}^1\omega_1 \times {}^1\omega_1 \times {}^1p_{c_1}}_{\odot\dot{\theta}_1\ \odot\dot{\theta}_1 \times \nearrow l \;\to\; \swarrow l_1\dot{\theta}_1^2} = \begin{bmatrix} g s_1 + l_1\ddot{\theta}_1 \\ g c_1 + l_1\dot{\theta}_1^2 \\ 0 + 0 \end{bmatrix}$$

$${}^1F_1 = \begin{bmatrix} m_1(g s_1 + l_1\ddot{\theta}_1) \\ m_1(g c_1 + l_1\dot{\theta}_1^2) \\ 0 \end{bmatrix}$$

$${}^1N_1 = \underbrace{{}^{c_1}I\, {}^1\dot{\omega}_1}_{\odot} + \underbrace{{}^1\omega_1}_{\odot} \times \underbrace{{}^{c_1}I\, {}^1\omega_1}_{\odot} = \begin{bmatrix} 0 \\ 0 \\ I_{zz_1}\ddot{\theta}_1 \end{bmatrix}$$

Body 2:

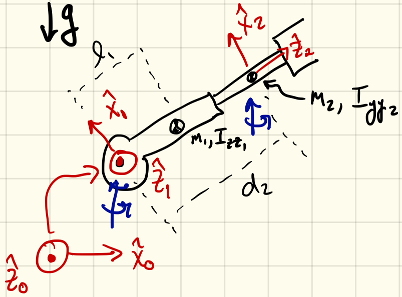
$${}^2\omega_2 = {}^2R_1\, {}^1\omega_1 = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} \qquad {}^2\dot{\omega}_2 = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$${}^2\dot{v}_2 = {}^2\dot{v}_{c_2} = {}^2R_1 \Big[{}^1\dot{v}_1 + \underbrace{{}^1\dot{\omega}_1 \times {}^1p_2}_{d_2\ddot{\theta}_1} + \underbrace{{}^1\omega_1 \times {}^1\omega_1 \times {}^1p_2}_{d_2\dot{\theta}_1^2} \Big] + \underbrace{2\, {}^2\omega_2 \times \begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix}}_{2\dot{\theta}_1\dot{d}_2} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d}_2 \end{bmatrix}$$

$${}^2\dot{v}_{c_2} = \begin{bmatrix} s_1 g + d_2\ddot{\theta}_1 + 2\dot{\theta}_1\dot{d}_2 \\ 0 \\ -c_1 g - d_2\dot{\theta}_1^2 + \ddot{d}_2 \end{bmatrix}$$

$${}^2F_2 = m_2\, {}^2\dot{v}_{c_2} \qquad {}^2N_2 = \begin{bmatrix} 0 \\ I_{yy_2}\ddot{\theta}_1 \\ 0 \end{bmatrix}$$

RNEA:



$${}^1p_{c_1} = [0 \ -l_1 \ 0]^T \qquad {}^1p_2 = [0 \ -d_2 \ 0]^T$$

$${}^2p_{c_2} = [0, 0, 0]^T$$

$${}^0R_1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^1F_1 = m_1 \begin{bmatrix} s_1 g + l_1 \ddot{\theta}_1 \\ c_1 g + l_1 \dot{\theta}_1^2 \\ 0 \end{bmatrix} \qquad {}^1N_1 = \begin{bmatrix} 0 \\ 0 \\ I_{zz_1} \ddot{\theta}_1 \end{bmatrix}$$

$${}^2F_2 = m_2 \begin{bmatrix} s_1 g + d_2 \ddot{\theta}_1 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ -c_1 g - d_2 \dot{\theta}_1^2 + \ddot{d}_2 \end{bmatrix} \qquad {}^2N_2 = \begin{bmatrix} 0 \\ I_{yy_2} \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

Body 2: ${}^2f_2 = {}^2F_2$ $\qquad \tau_2 = [0 \ 0 \ 1]\, {}^2f_2 = m_2(-c_1 g - d_2 \dot{\theta}_1^2 + \ddot{d}_2)$ $\;|\;$ ${}^2n_2 = {}^2N_2 + \cancel{{}^2p_{c_2}}^{\,0} \times {}^2F_2$

Body 1: ${}^1f_1 = {}^1F_1 + {}^1R_2\, {}^2f_2$ $\qquad$ (Doesn't influence τ_1)

$${}^1n_1 = {}^1N_1 + {}^1R_2\, {}^2n_2 + {}^1p_{c_1} \times {}^1F_1 + {}^1p_2 \times {}^1R_2\, {}^2f_2$$

$$= \begin{bmatrix} 0 \\ 0 \\ I_{zz_1}\ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ I_{yy_2}\ddot{\theta}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} m_1(s_1 g + l_1 \ddot{\theta}_1) \\ * \\ * \end{bmatrix} + \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} m_2(s_1 g + d_2 \ddot{\theta}_1 + 2\dot{d}_2 \dot{\theta}_1) \\ * \\ * \end{bmatrix}$$

$$= \begin{bmatrix} * \\ * \\ I_{zz_1}\ddot{\theta}_1 + I_{yy_2}\ddot{\theta}_1 + m_1 l_1 (s_1 g + l_1 \ddot{\theta}_1) + m_2 d_2 (s_1 g + d_2 \ddot{\theta}_1 + 2\dot{\theta}_1 \dot{d}_2) \end{bmatrix}$$

$$\tau_1 = (I_{zz_1} + I_{yy_2} + m_1 l_1^2 + m_2 d_2^2)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + s_1 g (m_1 l_1 + m_2 d_2)$$