Image Denoising using Spatial Domain Filters: A Quantitative Study

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***Abstract* – Image denoising is the first preprocessing step dealing with image processing. In image denoising an image is processed using certain restoration techniques to remove induced noise which may creep in the image during acquisition, transmission or compression process. Examples of noise in an image can be Additive White Gaussian Noise (AWGN), Impulse Noise, etc. The goal of restoration techniques is to obtain an image that is as close to the original input image as possible. In this paper objective evaluation methods are used to judge the efficiency of different types of spatial domain filters applied to different noise models, with a quantitative approach. Performance of each filter is compared as they are applied on images affected by a wide variety of noise models. Conclusions are drawn in the end, about which filter is best suited for a number of noise models individually induced in an image, according to the experimental data obtained.**

***Keywords* – Image denoising, Image restoration, Noise models, Denoising spatial filters.**

I. INTRODUCTION

Image Restoration is the operation in which a clean image is estimated and redrawn from a corrupt and degraded image. There are a variety of reasons that could cause degradation of an image, namely blur, motion and noise [4]. Denoising has been a long-standing and important issue in image processing since decades.[5]

Image Denoising is the operation performed on an image affected by unwanted noise which adds spurious and extraneous information. It is usually induced during the image acquisition, transmission or compression processes [6]. Noise leads to destruction of minute details in the image, and also hinders automated edge detection. Visually, a noisy image is undesirable for human perception as well as for machine tasks.

Mathematically, the degradation and restoration problem can be described as

$$\boldsymbol{g(x,y) = f(x,y) * h(x,y) + n(x,y)} \tag{1}$$

$$\boldsymbol{r(x,y) = T.g(x,y)} \tag{2}$$

where *g(x,y)* is the output degraded image, *f(x,y)* is the original image, *h(x,y)* is the degradation function and *n(x,y)* is the additive noise model induced in the image, '*' denotes convolution, and *r(x,y)* denotes the final output image obtained and estimated after applying inverse degradation technique T.

Filters are essentially, the inverse degradation models of the image which when applied to a degraded image, can estimate the original image. They are designed to be used for one particular type of degradation and noise model, but some filters can be applied for other types as well (Arithmetic Mean Filter can be used to treat Gaussian, uniform or Erlang Noise). Filters are essentially classified into two types – Spatial Domain Filters and Transform Domain Filters [6].

In this paper, various filters under the spatial domain category are discussed. No image restoration technique is universal[6], that is, for all types of noise models. Proper estimation of noise and knowledge of best available toolset at hand for a particular noise type is mandatory for efficient implementation of image restoration task. Through this paper, different filters are applied and checked on images with different types of noise models induced, and their performance was evaluated individually, thereby concluding what filter is best suited for a particular type of noise model according to the experimental data obtained.

This paper aims to fill the gap of required know how about various denoising filters, and their efficiency against some of the most common noise models affecting the images.

Paper is organized in sections as – **Section 2** discusses various popular noise models and their histograms. **Section 3** discusses various popular spatial domain filters presently in use. **Section 4** discusses various image similarity measures. **Section 5** presents experimental results along with comments and conclusion drawn according to the experimental data obtained is included in **Section 6.**

II. NOISE MODELS

A. Gaussian Noise

One of the most common type of noise found in images, also known as Additive White Gaussian Noise (AWGN).

Probability Density Function (PDF) of Gaussian Distribution is.[7]

$$P_g(z) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right) e^{-\frac{(z-\bar{z})^2}{2\sigma^2}} \qquad (3)$$

where z represents the gray level, $\bar{z}$ is the mean, σ being the Standard Deviation (σ^2 is the variance of z). The histogram plot of this PDF is shown below –

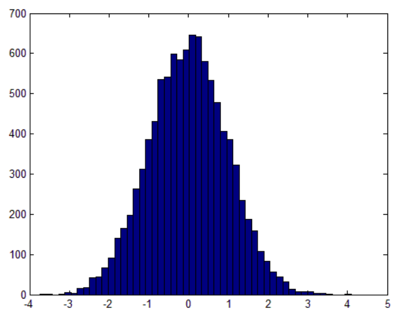

Figure 1. Gaussian Noise Histogram

B. Impulse Noise or Salt & Pepper Noise

Impulse noise or better known as Salt & Pepper Noise appears as black and/or white impulse of the image. Salt & Pepper noise is evident due to white and black spots which appear in gray scale images and are chaotically scattered along image area. The PDF for Salt & Pepper noise

$$P_{sp}(z) = \begin{cases} P_p & \text{for } z = p \\ P_s & \text{for } z = s \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

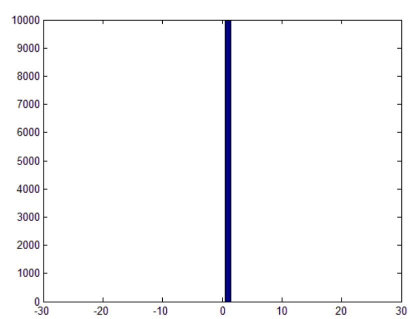

Figure 2. Impulse Noise Histogram

C. Uniform or Quantization Noise

Uniform noise is caused by quantization of pixels in an image in a number of discreet levels. Also known as Quantization Noise, the PDF is given by the equation –

$$P_u(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

where $a \ge 0$ and $0 < a < b$. The mean and variance of uniform noise is given by

$$\bar{z} = \frac{a+b}{2} \quad (6) \quad \text{and} \quad \sigma^2 = \frac{(b-a)^2}{12} \quad (7)$$

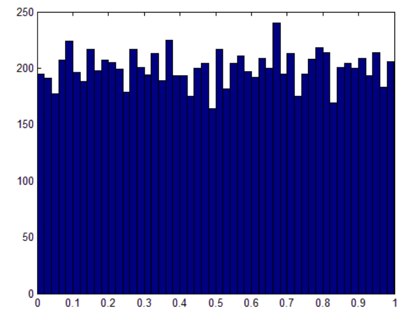

Figure 3. Quantization or Uniform Noise Histogram

D. Rayleigh Noise

Typically found in RADAR range and velocity images, this noise is modeled by Rayleigh Distribution. The histogram for this noise type is given by the PDF-

$$P_r(z) = \begin{cases} \left(\frac{z}{\sigma^2}\right) e^{-\frac{(z)^2}{2\sigma^2}} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases} \qquad (8)$$

where $a \ge 0$ and $0 < a < b$

Mean and Variance of Rayleigh PDF is given as –

$$\bar{z} = \sigma\sqrt{\frac{\pi}{2}} \quad (9) \quad \text{and} \quad \sigma_r^2 = \sigma^2\left(2 - \frac{\pi}{2}\right) \quad (10)$$

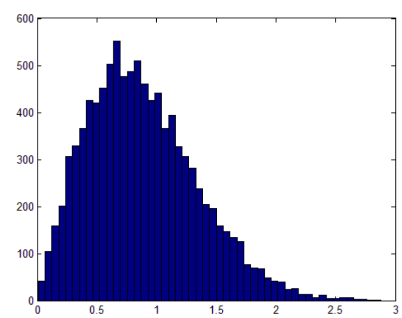

Figure 4. Rayleigh Noise Histogram

E. Gamma or Erlang Noise

Just like Gaussian, Gamma distribution has a distinct PDF –

$$P_e(z) = \begin{cases} \frac{(z^{b-1}a^{(b)})e^{-az}}{(b-1)!} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad (11)$$

where a > 0, b is a positive integer, and "!" denotes factorial. The mean and variance of gamma PDF is given by

$$\bar{z} = \frac{b}{a} \quad (12) \quad \text{and} \quad \sigma^2 = \frac{b}{a^2} \quad (13)$$

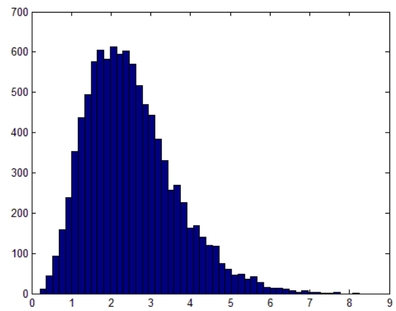

Figure 5. Gamma Noise Histogram

F. Exponential Noise

Exponential Noise is a special case of Gamma Noise, with b = 1. The PDF for Exponential Noise is given as

$$P_{exp}(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \tag{14}$$

The mean and variance of exponential PDF are –

$$\bar{z} = \frac{1}{a} \quad (15) \qquad \text{and} \qquad \sigma^2 = \frac{1}{a^2} \quad (16)$$

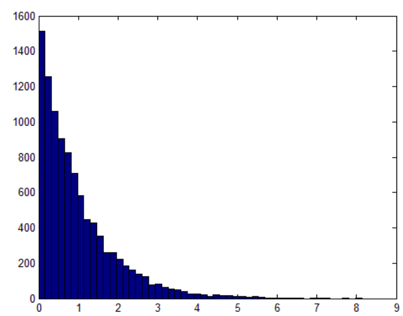

Figure 6. Exponential Noise Histogram

G. Poisson Noise

"The term arises because detection and recording processes involve random electron emission having a Poisson distribution with a mean response value."[7] The PDF[1] for this Noise is given by

$$P_p(z) = \frac{\mu^X}{X!} e^{-\mu} \tag{17}$$

where μ being the expectation value. Poisson noise greatly resembles Gaussian Noise as is evident from its histogram, but the difference becomes visible in excess dark regions.

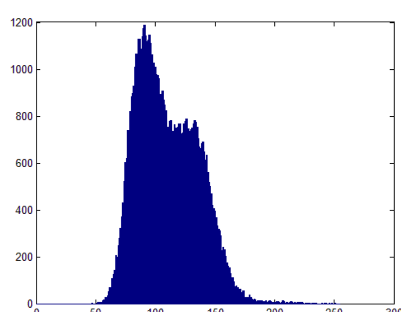

Figure 7. Poisson Noise Histogram

III. IMAGE DENOISING FILTERS IN SPATIAL DOMAIN

In the Spatial Domain, filters are broadly classified into three groups – Mean Filters, Order Statistics Filters and Adaptive Filters In this section, we will talk about the Mean and Order Statistics Filters.

A. Mean Filters

i) Arithmetic Mean *Filter*

A smoothing filter which reduces the intensity variations between adjacent pixels.[6]. Also called a Linear Filter or Averaging Filter, it operates on a m x n mask by averaging all pixel values within the window and replacing the center pixel value in the final image with the result. It also causes certain amount of blurring in the image.[2]

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(r,c)\in W} g(r, c) \tag{18}$$

ii) Geometric Mean Filter

Geometric Mean filter is a slight variation of the Arithmetic Mean Filter but it is known to retain higher image detail after application as compared to the arithmetic filter. The mathematical equation[1] for this filter is

$$\hat{f}(x, y) = \left(\prod_{(r,c)\in W} g(r, c)\right)^{\frac{1}{mn}} \tag{19}$$

iii) Harmonic Filter

This is yet another variation of the Arithmetic Mean Filter which effectively removes Salt noise or Gaussian Noise. Mathematically,

$$\hat{f}(x, y) = \frac{mn}{\sum_{(r,c)\in W} 1/g(r,c)} \tag{20}$$

iv) Contraharmonic Filter

Similar to Harmonic filter, it is capable of removing salt *or* pepper noise, but not both at a time. It can be used in collaboration with the Harmonic Filter to get desired results. It is represented as

$$\hat{f}(x, y) = \frac{\sum_{(r,c)\in W} g(r,c)^{(R+1)}}{\sum_{(r,c)\in W} g(r,c)^{R})} \tag{21}$$

B. Order Statistics or Rank Filters

i*) Median Filter*

One of the most popular Order Statistic or Rank Filter due to it's edge preserving nature[12]. It works by selecting the middle pixel value from the ordered set of values within the m x n neighborhood (W) around the reference pixel. [2]

$$\hat{f}(x, y) = median\{g(r, c) | (r, c) \in W\} \tag{22}$$

ii) The Min and Max Filters

Instead of replacing the reference pixel with the median value of the ordered set, in median filter, the *min* filter replaces the reference pixel with the lowest value. Similarly the *max* replaces the reference pixel with the highest value.[1]

$$\hat{f}(x,y) = \min\{g(r,c) \mid (r,c) \in W\} \quad (23)$$

$$\hat{f}(x,y) = \max\{g(r,c) \mid (r,c) \in W\} \quad (24)$$

iii) Midpoint Filter

This filter uses the average value of the highest and lowest pixel values in the window, thereby combining rank filter and averaging into one filter. [2]

$$\hat{f}(x,y) = \frac{1}{2}[min\{g(r,c)|(r,c) \in W\} + max\{g(r,c)|(r,c) \in W\}] \quad (25)$$

iv) Alpha-Trimmed Mean Filter

"This filter also combines the order statistics and averaging, which in this case, an average of the pixel values closest to the median, after D lowest and D highest values in an ordered set have been excluded."[2] This filters allows the user to control its behavior by specifying a parameter 'D'. Mathematical Formulation for this filter is

$$f(x,y) = \frac{1}{mn-2D}\sum_{(r,c)\in W} g(r,c) \quad (26)$$

IV. IMAGE SIMILARITY MEASURES

The criterion used for devising what filter is best suited for a particular type of noise model is discussed here. Two similarity measures, Peak Signal to Noise Ratio (PSNR) and 2D Cross Correlation Value were used.

A. Peak Signal to Noise Ratio

One of the most famous and commonly used similarity measure in both Digital Image and Digital Signal Processing. The mathematical expression for the same is –

$$\text{PSNR} = 10\log_{10} \frac{B^2}{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(I_1(i,j) - I_2(i,j)\right)^2} \quad (27)$$

where B is the largest possible value of signal (either 255 or 1), $I_1(i,j)$ and $I_2(i,j)$ denote the pixel values of the restored image and original image respectively. $\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(I_1(i,j) - I_2(i,j)\right)^2$ is the mean square difference between two images. PSNR Values are given in decibel units (dB). A ten fold decrease in RMS difference is obtained corresponding to 20dB rise in PSNR value. Results according to PSNR are presented in Table 1.

B. 2D Cross Correlation

This is yet another image similarity measure which compares the original image to the processed or restored image. Consider two images x_i and y_i where i=1,2 ,n are its pixels. The 2D cross correlation is represented by equation –

$$\rho = \frac{\sum_{i=1}^{i=n}[(x_i - m_x)*(y_i - m_y)]}{\sqrt{\sum_{i=1}^{i=n}(x_i - m_x)^2}\sqrt{\sum_{i=1}^{i=n}(y_i - m_y)^2}} \quad (28)$$

where m_x and m_y are the means of the corresponding image. Table II shows the obtained 2D Cross Correlation Values for different applied filters.

V. EXPERIMENTAL RESULTS

The following tables and graphs show the experimental results obtained after the tests were done on noise affected images. Minimum and maximum filters in the case of Salt & Pepper noise was used in conjunction. First minimum filter was applied, followed by maximum filter[1].

TABLE I
COMPARISON OF FILTER PERFORMANCE ON THE BASIS OF PSNR (dB)

	Arithmetic Mean	**Geometric Mean**	**Harmonic mean**	**Contraharmonic Mean**	**Median**	**Min**	**Max**	**Midpoint**	**Alpha Trimmed**	**Best Filter**
Gaussian	24.87	24.84	24.48	24.53	**25.06**	17.60	18.05	23.88	23.07	**Median**
Salt & Pepper	23.75	24.47	14.41	21.04	**25.25**	19.32[1]	---	17.44	23.07	**Median**
Uniform	15.97	16.10	16.36	16.05	15.95	**18.85**	12.07	15.83	15.72	**Minimum**
Rayleigh	17.21	17.55	18.04	17.57	17.37	**20.53**	11.34	16.57	17.06	**Minimum**
Gamma	19.86	22.76	23.97	22.30	**24.74**	18.59	9.90	13.12	22.84	**Median**
Exponential	18.54	19.26	**20.07**	19.33	19.60	20.07	10.25	16.00	18.96	**Harmonic Mean**
Poisson	22.44	24.12	24.53	22.67	**25.32**	18.41	11.81	14.75	23.10	**Median**

TABLE II
COMPARISON OF FILTER PERFORMANCE ON THE BASIS OF 2D CROSS CORRELATION

	Arithmetic Mean	Geometric Mean	Harmonic mean	Contra-harmonic Mean	Median	Min	Max	Midpoint	Alpha-Trimmed
Gaussian	0.9639	0.9639	0.9622	0.9626	**0.9654**	0.922	0.9220	0.9544	0.9447
Salt & Pepper	0.9531	0.9607	0.7025	0.9139	**0.9669**	0.8931[1]	---	0.7994	0.9448
Uniform	0.9624	0.9648	0.9645	0.9310	0.9652	**0.9655**	0.9196	0.9522	0.9425
Rayleigh	0.9585	0.9606	0.9601	0.9308	0.9576	**0.9608**	0.9107	0.9465	0.9410
Gamma	0.9122	0.9500	0.9575	0.9367	**0.9639**	0.8962	0.6034	0.6998	0.9432
Exponential	0.9517	0.9561	**0.9569**	0.9312	0.9519	0.9035	0.8346	0.9165	0.9395
Poisson	0.9405	0.9574	0.9609	0.9407	**0.9674**	0.8943	0.6313	0.7336	0.9451

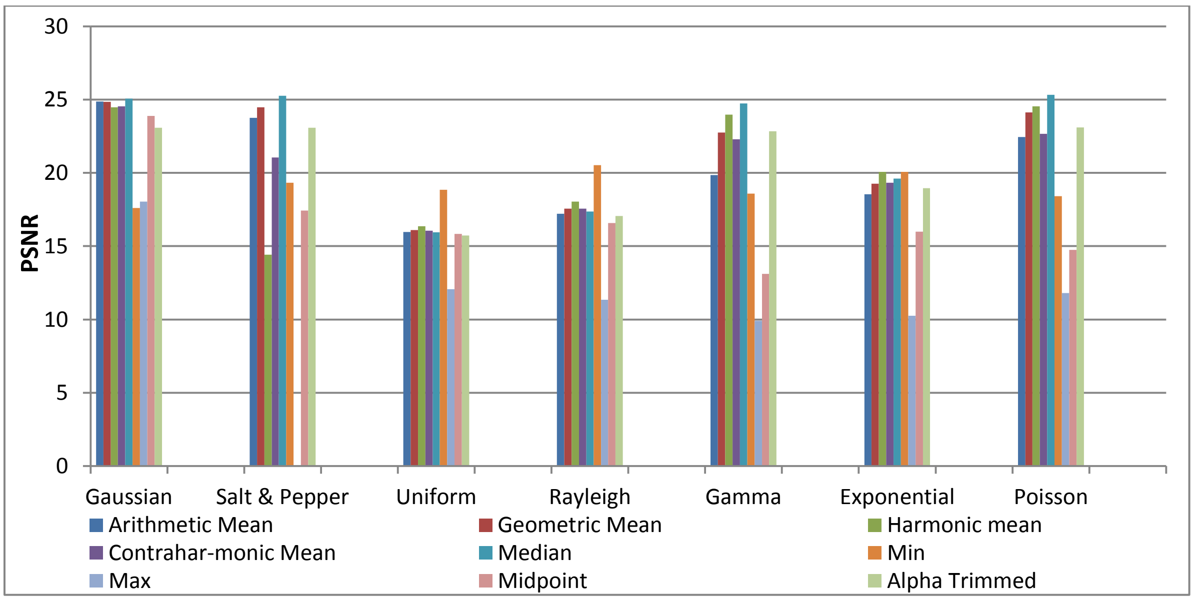

Figure 8. Filter performance analysis using PSNR

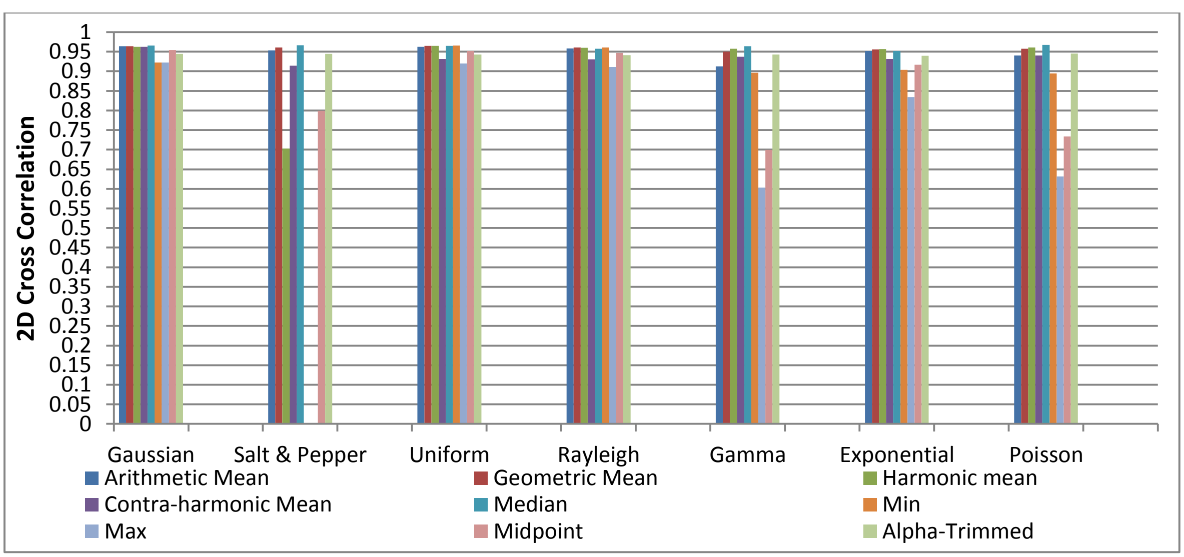

Figure 9. Filter performance analysis using 2d cross correlation

VI. CONCLUSION

Denoising techniques in Spatial Domain are fairly developed at the moment, but to adopt them for their proper application and their performance measure is still a matter of ongoing research. In this paper, a quantitative analysis of spatial domain filters was carried out. The best filters according to the experimental results on the famous benchmark image Lena, are as follows –

For Gaussian, Salt & Pepper, Poisson and Gamma Noise best filter was Median Filter, Uniform noise and Rayleigh Noise was corrected best by Minimum filter and for Exponential Noise, best filter was found to be Harmonic Mean Filter.

The tests were conducted on several other benchmark images to validate experimental conclusions.

VII. ACKNOWLEDGMENT

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VII. REFERENCES

[1] Rafael C. Gonzalez, Digital Image Processing, Pearson Education India, 2009.

[2] Oge Marques, Practical Image and Video Processing Using MATLAB, Wiley, New York, September 21, 2011.

[3] Azriel Rosenfeld, Picture Processing by Computer, New York: Academic Press, 1969.

[4] Charu Khare and Kapil Kumar Nagwanshi, "Image Restoration Technique with Non Linear Filter", International Journal of Advanced Science and Technology, vol. 39, February 2012, pp. 67-44.

[5] H J Seo, et al, "A comparison of some state of the art image denoising methods", Conference Record of the Forty-First Asilomar Conference on Signals, Systems and Computers, 2007, ACSSC 2007, pp. 518 – 522.

[6] Jappreet Kaur, et al, "Comparative Analysis of Image Denoising Techniques" International Journal of Emerging Technology and Advanced Engineering, vol. 2, June, 2012.

[7] A. Aboshosha , et al, "Image denoising based on spatial filters, an analytical study." International Conference on Computer Engineering & Systems, 2009, ICCES 2009, IEEE, 2009, pp. 245-250.

[8] W. K. Pratt, "Digital Image Processing: PIKS Scientific Inside" Communications of the ACM, vol. 27.3, 1984, pp. 236-239.

[9] Klaus E Timmermann, Robert D. Nowak, "Multiscale modeling and estimation of Poisson processes with application to photon-limited imaging." IEEE Transactions on Information Theory, 1999, pp. 846-862.

[10] Alan C Bovik, Handbook of image and video processing, Access Online via Elsevier, 2010.

[11]Leipo Yan, L.P. Wang and Kim Hui Yap, "A noisy chaotic neural network approach to image denoising," IEEE International Conference on Image Processing (ICIP 2004), Singapore, October, 2004.

[12] Raymond H. Chan, Chung-Wa Ho, and Mila Nikolova,"Salt-and-Pepper Noise Removal by Median-Type Noise Detectors and Detail-Preserving Regularization", IEEE Transactions On Image Processing, Vol. 14, No. 10,October 2005.

[13] S.J.Ko and Y.H Lee, "Center Weighted Median Filters and their Application to Image Enhancement", IEEE Transactions on Circuits Systems., Vol 38, no.9, Sept.1991 .

[14]G. Motta, et al, "The iDUDE framework for grayscale image denoising," IEEE Transactions on Image Processing, Vol.20, pp.1-21, 2011.

[15]R.Gayathri, R.S.Sabeenian, "A Survey on Image Denoising Algorithms (IDA)" International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering Vol. 1, Issue 5, November 2012.

[16] Scott E Umbaugh, Computer Vision and Image Processing: A Practical Approach Using Cviptools with Cdrom, Prentice Hall PTR, 1997.

[17] Joseph W Goodman, "Some fundamental properties of speckle" Journal of the Optical Society of America, Vol 66.11, 1976, pp. 1145-1150.

[18]Leipo Yan and L.P. Wang, "Image denoising using noisy chaotic neural networks", Proceedings of the 2nd International Conference on Cognitive Neurodynamics (ICCN), 2010.

[19] C. Kervrann, J. Bourlanger, "Optimal spatial adapation for patchbased image denoising" IEEE Transactions on Image Processing, Vol. 15, no. 30, October 2006, pp. 2866-2878.

[20] Roni Chakre, Pooja Thapa, Rispana Siddharth, Udayan Baruah, "An Experimental Study on Image De-Noising Filters", International Journal of Emerging Technology and Advanced Engineering, Volume 3, Issue 2, February 2013.

[21] A. Mahmoud, S. EL Rabaie, T. E. Taha, O. Zahran, F. E. Abd El-Samie, "Comparative study between different denoising filters for speckle noise reduction in ultrasonic b-mode images" 2012 8th International Computer Engineering Conference (ICENCO), 29-30 Dec. 2012.

[22] Subhojit Sarker, Shalini Chowdhury, Samanwita Laha and DebikaDey, "Use of non-local means filter to denoise image corrupted by salt and pepper noise", Signal & Image Processing : An International Journal (SIPIJ) Vol.3, No.2, April 2012.