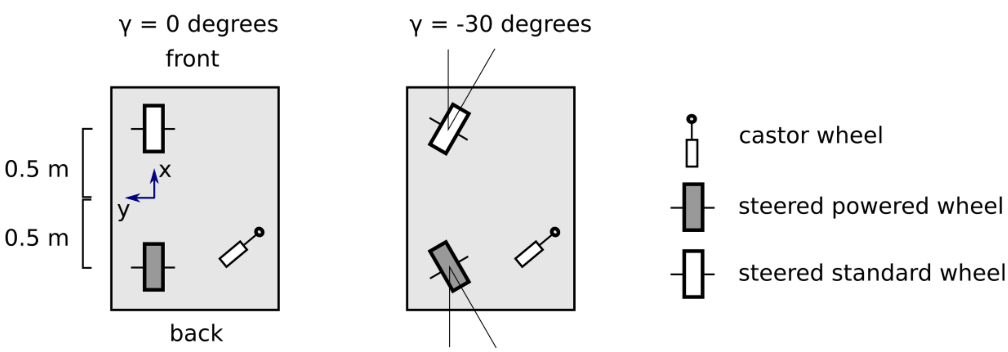
Robotics exercises for kinematics and control

March 11, 2025

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page or on the provided extra pages.
Write your name in the header of all pages. The handed-in material must be filled in pen, not pencil.
Clarity is part of the evaluation criteria. If you have doubts about the details of a question, make an assumption that you believe is convenient and reasonable and state it clearly. The exam is evaluated by a human being, not a robot.

Name, surname: ______________________________

1 Exercise (35/100 points)



Consider the 2D wheeled robot in the figure, where you control:

- the rotational speed of the back wheel $\dot{\varphi}_{\text{back}}$; the radius of the back wheel is 10 cm.
- the steering angle of both wheels at the same time. The wheel steering mechanisms are connected, so the two wheels steer in opposite directions, with a single controllable parameter γ representing the depicted angle. For $\gamma = 0$, both wheels are aligned with the x direction of the robot frame, and a positive value for $\dot{\varphi}_{\text{back}}$ will move the robot forwards. For $\gamma = +10\,\text{deg}$, the front wheel steers 10 degrees counterclockwise, and the back wheel steers 10 degrees clockwise.

1. (5 points) Assume we place the robot on an infinite floor without obstacles.

   If we limit $-30\,\text{deg} < \gamma < +30\,\text{deg}$: is the robot an holonomic robot? can the robot reach any pose (x, y, θ) on the plane?

   If we limit $+20\,\text{deg} < \gamma < +30\,\text{deg}$: is the robot an holonomic robot? can the robot reach any pose (x, y, θ) on the plane?

2. (5 points) Briefly comment on the kinematics of the robot for a steering angle $\gamma = 90\,\mathrm{deg}$

3. (15 points) From this point on, assume $-90\,\mathrm{deg} < \gamma < +90\,\mathrm{deg}$.

   Discuss how you could implement a function $f(\gamma, \dot{\varphi}_{\mathrm{back}}, \delta t)$ that returns a 3×3 homogeneous matrix representing the pose transformation that occurs when the robot moves for a time interval δt with a fixed rotational speed for the back wheel $\dot{\varphi}_{\mathrm{back}}$ and a fixed steering angle γ.

4. (5 points) From this point on, assume the function $f(\gamma, \dot{\varphi}_{\text{back}}, \delta t)$ is available and correct, so if you screwed up or didn't answer the previous question, that won't affect you.

   Consider the world reference frame O. The robot is initially placed at the origin, with the robot's x axis pointing towards the y axis of frame O. Relying on function $f(\cdots)$, write the expressions returning the pose of the robot ${}^{O}T_{R}$ in the world frame of reference O at each of the following times: $t = 0, 1, 5, 7$. Starting at time $t = 0$, the robot moves as follows:

   - for 2 seconds,
     $\dot{\varphi}_{\text{back}} = +10$ rad/s,
     $\gamma = +30\,\text{deg}$.
   - for 5 seconds,
     $\dot{\varphi}_{\text{back}} = -10$ rad/s,
     $\gamma = -45\,\text{deg}$.

5. (5 points) Approximately draw the trajectory the robot followed and its final pose

6. (Extra points) rewrite function $f(\cdots)$ in case that the robot reference frame was at the center of the robot chassis (the gray rectangle). Make the necessary reasonable assumptions.

2 Exercise (20/100 points)

We consider a wheeled robot that explores a 2D world looking for intruders; we define a frame of reference R for such robot in such a way that x points forwards. The robot is equipped with a special laser sensor that points forward; with this sensor, the robot can detect whether there is an intruder in front of the robot, i.e. at any point lying on the positive half of the robot frame's x axis; however, the sensor can't measure the distance of the intruder, nor it can see any intruders that are even just a bit to the side.

Let ${}^{O}T_R$ be a 3×3 homogeneous transformation matrix representing the pose of the robot with respect to the world frame O.

The robot's sensor is detecting an intruder. At the same time, a fixed radar sensor has detected *two* potential intruders. Let (x_1, y_1) and (x_2, y_2) be their positions expressed in the radar frame A. Let ${}^{O}T_A$ be a 3×3 homogeneous transformation matrix representing the 2D pose of the radar frame with respect to the world frame.

1. (3 points) Draw the scenario and label the frames.

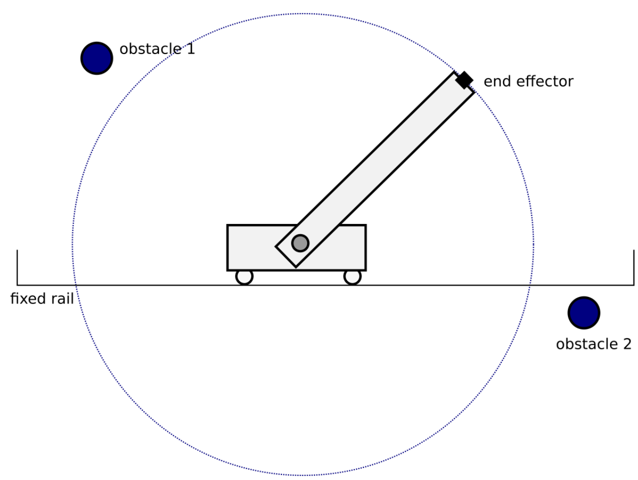
2. (10 points) We want to determine which of the two potential intruders detected by the radar was detected by the robot. Describe how you would implement a function to do that.

3. (7 points) Assume we determine that the intruder position is at the following coordinates of the world frame: $(x, y) = (30, 40)$ meters.

   Let ${}^{O}T_{K}$ be the pose (3×3 matrix in homogeneous coordinates) of another intruder-detecting robot K. Describe how we can determine by which angle the robot K must turn in place in order to detect the intruder.

3 Exercise (23/100 points)

Consider the robot arm in the figure:



1. (2 points) Draw the workspace, ignoring the obstacles.
2. (4 points) Mark the parts of the workspace not reachable because of the obstacles (no part of the robot can hit any of the obstacles).
3. (2 points) Consider the whole robot as a manipulator with one prismatic joint and one revolute joint. Define two reasonable variables that can describe the configuration of the manipulator.

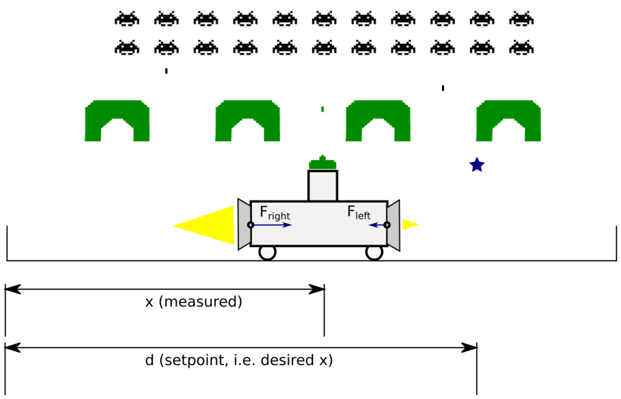
4. (3 points) Ignoring the obstacles, draw the configuration space and label the axes and their ranges appropriately. Discuss the topology of the configuration space.

5. (7 points) Approximately draw the two obstacles in the configuration space.

6. (3 points) Draw two different points on the configuration space for which the end-effector position is the same.
7. (2 points) Mark one of these points and draw the robot in that configuration.

4 Exercise (22/100 points)

The space-invaders tank can freely move on an horizontal rail with very little friction. Few people know that it is in fact actuated by two powerful thrusters, one pushing towards the right with a force $F_{\text{right}} \geq 0$, and one pushing towards the left with a force $F_{\text{left}} \geq 0$.



You have been asked to implement a software function to control this cart in a closed-loop way. The cart position along the rail is known as x. The task is to reach and stay at a position d, which is defined by the Great Commander. Your function is run 10 times per second and must produce a value for F_{left} and a value for F_{right}, which are instantaneously implemented by the thrusters.

1. (3 points) Your function has two inputs (x and d) and two outputs (F_{right} and F_{left}). Inside your function, you want to use one PID controller, which needs a single input (the position error) and produces a single output (the total force acting on the cart). Describe: how you compute the error from the function inputs, and; how you compute the function outputs from the PID output. Note that fuel is expensive and scarce in times of war, so it would be really stupid to fire both rockets at the same time.

2. (3 points) According to your definition of the error and output, elaborate on the sign of the proportional gain (K_p) of the controller.

3. (3 points) Do you expect that using a derivative gain $K_d \neq 0$ will be useful? Briefly motivate.

4. (3 points) Do you expect that using an integral gain $K_i \neq 0$ will be useful? Briefly motivate.

5. (5 points) A colleague insists that there is a setting which would work very well using $K_p = K_i = 0$, and some magic value for K_d. Write a single sentence to convince them that they are wrong.

6. (5 points) Briefly explain why a bang-bang controller would not be a good idea in place of the PID controller.

7. (Extra points) Sketch a hierarchical control architecture with one PID controller to set a desired velocity and one PID controller to set the force.

8. (Extra points) It turns out we don't have enough memory to implement two PID controllers. We have to substitute one with a Bang-Bang controller. Which one do you choose and why?

Extra page 1

Extra page 2