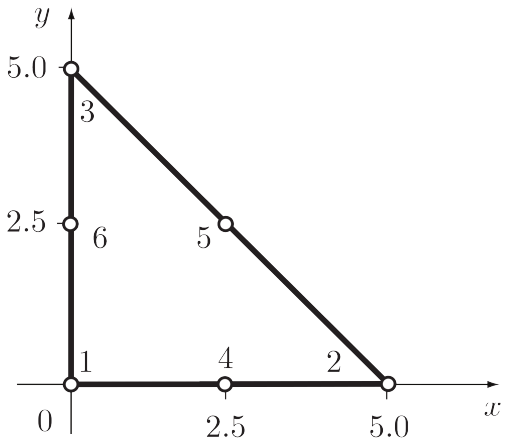
Assignment 4: FEM: Isoparametric elements, Gauss integration and solvers

Assigned: 25th February 2019

Due: 8th March 2019

1. For the six noded-triangle shown. The nodal temperatures are $T^e = \begin{bmatrix}300 & 0 & 0 & 0 & 340 & 0\end{bmatrix}^T$. Compute the temperature and its gradient at the point P with the coordinates $x = 1.5$ and $y = 2.0$.

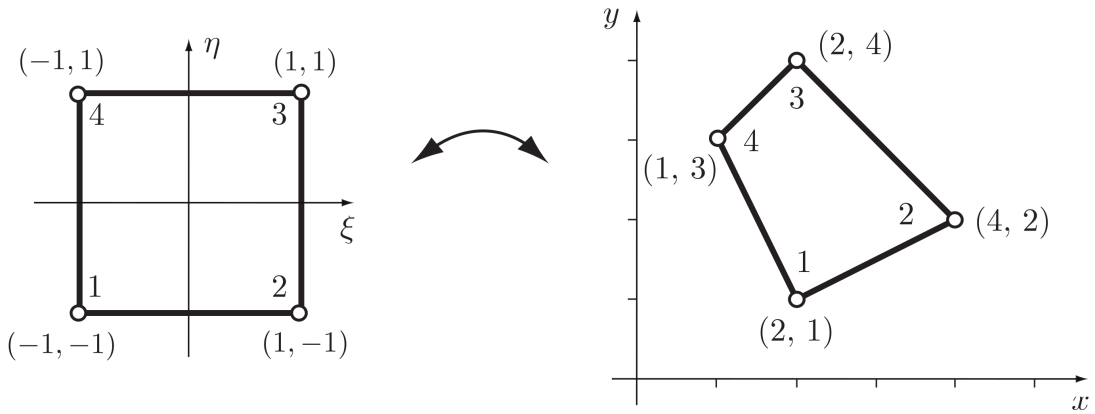


   Use the isoparametric shape functions provided in the data sheet.

2. For the isoparametric mapping shown below

   (a) Compute the x and y coordinates of the point $\xi = 0.5$, $\eta = 0.5$ in the physical domain.

   (b) Compute $\frac{\partial N_1}{\partial x}$ for the same point.



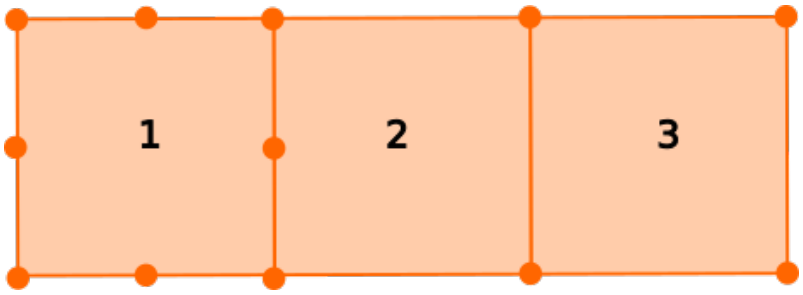
3. Evaluate the following three integrals using one, two and three-point Gauss integration. Compare your results with the results of analytic integration.

(a) $\int_{-1}^{+1}(3\xi^2 + 2\xi)d\xi$

(b) $\int_{-1}^{+1}\cos\xi d\xi$

(c) $\int_{0}^{3}(3x^2 + x)dx$

4. Consider the following quadrilateral mesh, sketch (1D is fine) the temperature distribution across the mesh assuming a hear source on the left boundary of the mesh and comment on the suitability of the mesh for finite element computation and the need for the transition element (element #2)?



5. Develop a Python script for the Newton Raphson method to solve non-linear force-displacement relationships. Test and comment on the accuracy and efficiency of the NR to solve: $f = -2u^2 + 2u$, where u denotes the displacement and f refers to the internal force. For the analytical solution plot the displacement u between 0 and 1.

Datasheet

The shape functions for a 6-noded triangle element are:

$$
\begin{aligned}
N_1 &= 2(1-\xi-\eta)^2 - (1-\xi-\eta) \\
N_2 &= 2\xi^2 - \xi \\
N_3 &= 2\eta^2 - \eta \\
N_4 &= 4\xi(1-\xi-\eta) \\
N_5 &= 4\xi\eta \\
N_6 &= 4\eta(1-\xi-\eta)
\end{aligned}
$$