# CE394M: Critical State and Cam-Clay

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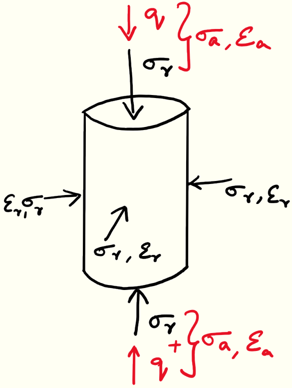
# Overview

# Critical State Soil Mechanics

Roscoe et al., (1958), Schofield & Worth (1968), Wood (1990):

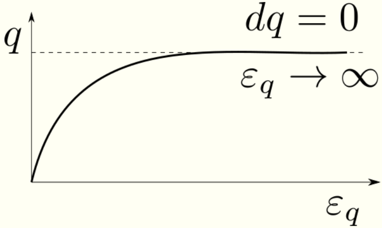
# Critical state variables

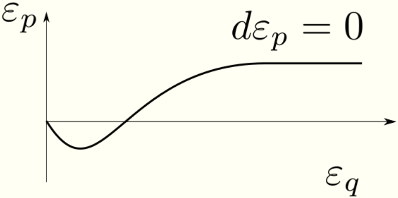
- Mean stress: $p' = \frac{\sigma'_a + 2\sigma'_r}{3} = p - u$.
- Deviatoric stress: $q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$



# Critical State Hypothesis: I

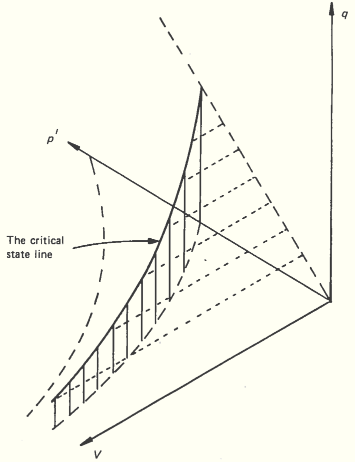
Roscoe, Schofield & Worth (1958): **At shear-failure, soil exists at a unique state**



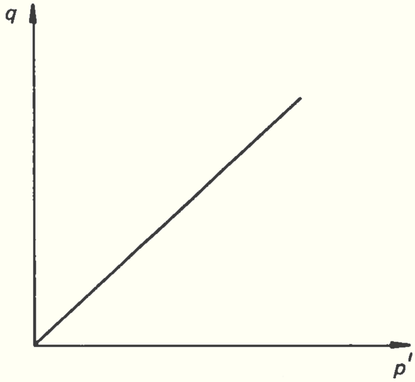
# Critical State Hypothesis: II

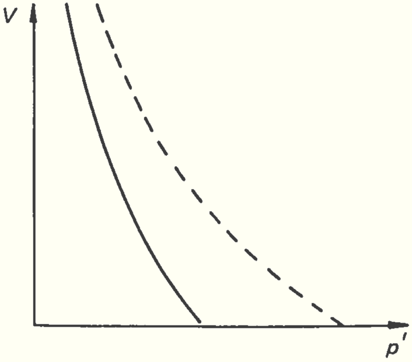
**Critical state is a function of $q, p', v$.**




The CSL $(p', v, q)$ space is given by the intersection of two planes: $q = Mp'$ and a cruved vertical plane $v = \Gamma - \lambda \ln p'$

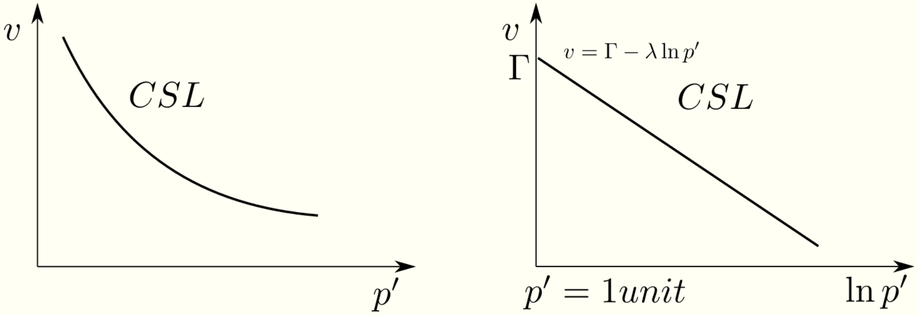
**Critical state is a function of $q, p', v$.**



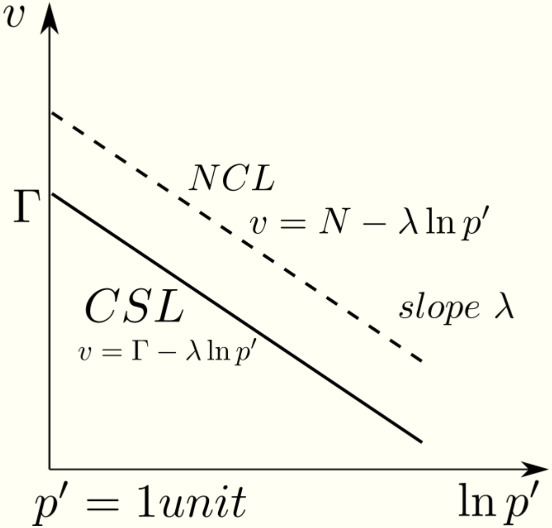




The CSL in (a) $(p', q)$ plot and (b) $(p', v)$ plot (isotropic normal compression line is shown in dashed)

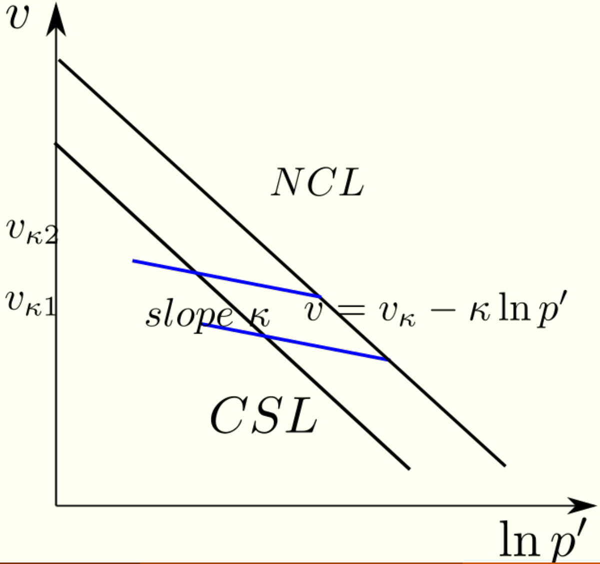
**Critical state is a function of $q, p', v$.**



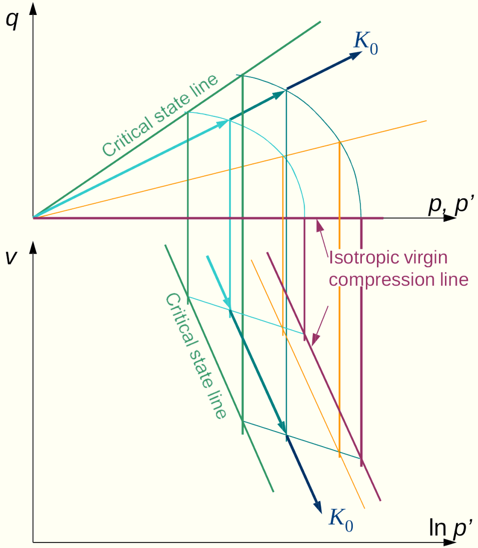
**Critical state is a function of $q, p', v$.**



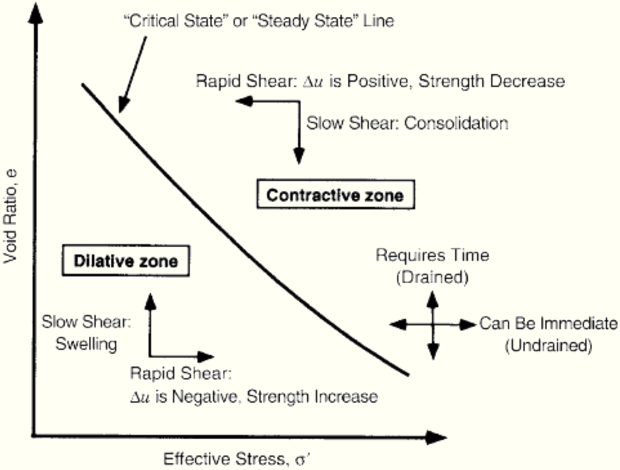
**Critical state is a function of $q, p', v$.**



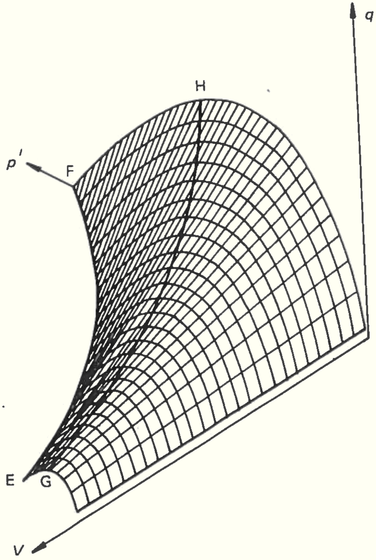
# Stress paths $\sigma'_3/\sigma'_1 = K_c = const$



# Clay behavior

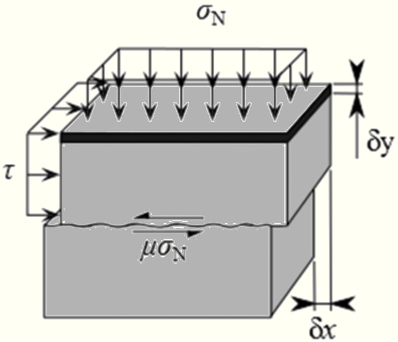


# Critical state boundary surface



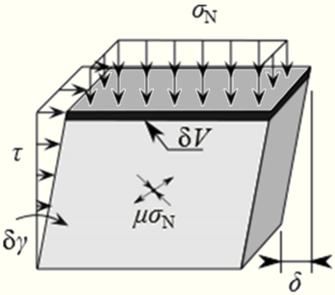
# Summary of critical state behavior

# Stress - dilatancy theory (Taylor, 1948)



# Formulation of elasto-plastic Cam-Clay (OCC): Yield function

Derived from work consideration:



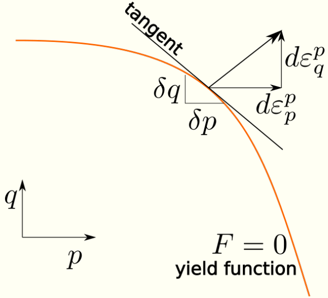
# Cam-Clay (OCC): Stress dilatancy relation

$$p' d\varepsilon_p^p + q d\varepsilon_q^p = M p' d\varepsilon_q^p$$

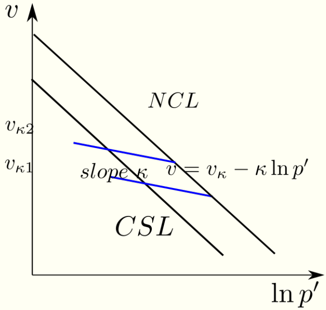
Rearranging the terms (divide by $p' d\varepsilon_q^p$):

# Cam-Clay (OCC): flow-rule

The original idea was very simple. The yield locus must be such that each associated flow rule $(\delta\varepsilon_{\boldsymbol{p}}, \delta\varepsilon_{\boldsymbol{q}})$ would be orthogonal to the tangent to the yield locus.



# Cam-Clay (OCC): Elastic properties



# Cam-Clay (OCC): Hardening law

We need to define how the yield surface hardens as plastic work is being performed. Only "*memory*" parameter in our yield surface is the size: $p'_c$. From the isotropic NCL:

$$d\varepsilon_p = \frac{-dv}{v} = \frac{-de}{1+e} = \frac{+\lambda}{v}\frac{dp'_c}{p'_c}$$

But the increment in elastic volumetric strain is:

$$d\varepsilon_p^e = \left(\frac{-dv}{v}\right)^{elastic} = +\frac{\kappa}{v}\left(\frac{dp'_c}{p'_c}\right)$$

Therefore the increment of $p_c$ can be related to the increment of plastic volumetric strain:

$$d\varepsilon_p^p = d\varepsilon_p - d\varepsilon_p^e = (\lambda - \kappa)\left(\frac{dp'_c}{p'_c}\right) \rightarrow \quad dp'_c = \left(\frac{v \cdot p'_c}{(\lambda - \kappa)}\right) \cdot d\varepsilon_p^p$$

# Cam-Clay (OCC): Hardening law

We have seen that the hardening law:

$$H = -\left(\frac{\partial F}{\partial Wp}\right)\left(\frac{\partial Wp}{\partial \varepsilon^p}\right)^T \cdot \frac{\partial G}{\partial \sigma}$$

$W_p$ is the vector of memory parameters. In our case, the CC model has only one parameter:

# Cam-Clay (OCC): Hardening law

We know:

$$\frac{\partial F}{\partial p'_c} = -Mp'/p'_c$$

$$\frac{\partial p'_c}{\partial \varepsilon^p} = \frac{v}{(\lambda - \kappa)} p'_c$$

$$\frac{\partial G}{\partial \sigma} = P_p = Q_p = M - \eta$$

$$\boxed{H = -\left(\frac{\partial F}{\partial p'_c}\right)\left(\frac{\partial p'_c}{\partial \varepsilon^p}\right)^T \cdot \frac{\partial G}{\partial \sigma} = M\frac{(M-\eta)}{(\lambda-\kappa)} \cdot (1+e) \cdot p'}$$

# Limitations of original Cam-Clay