CE394M: Critical State and Cam-Clay

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April 28, 2019

Overview

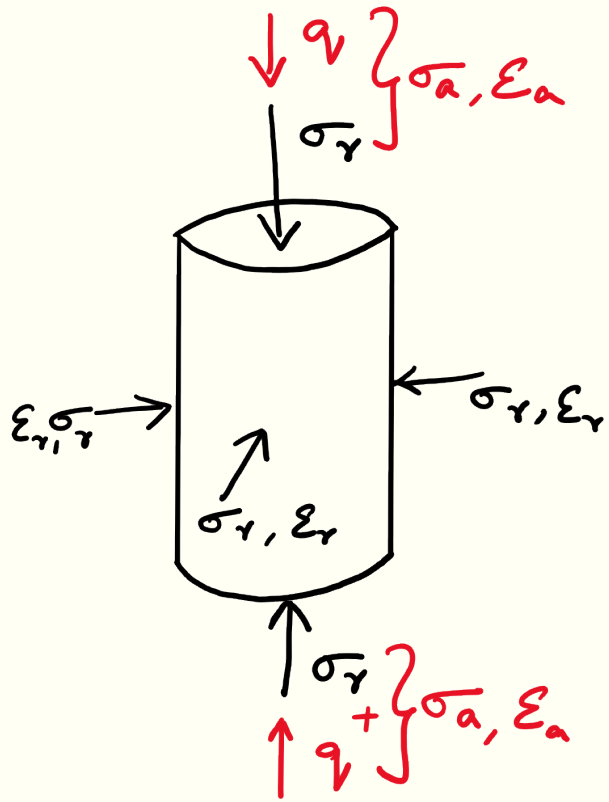
Critical State Soil Mechanics

Roscoe et al., (1958), Schofield & Worth (1968), Wood (1990):

- Provides a conceptual framework in which to interpret stress-strain-strength-volumetric strain response of soil.
- Started as a qualitative, rather than a mathematical model
- A unified framework of known or observed soil responses: drained / undrained / etc

Critical state variables

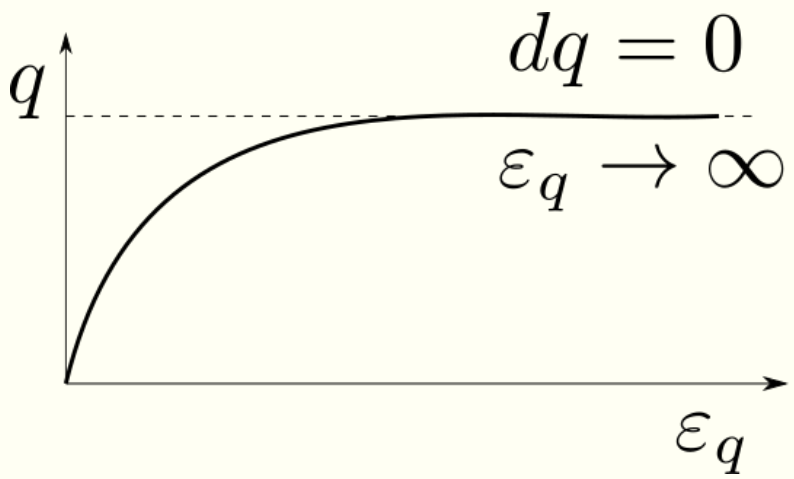
- Mean stress: $p' = \frac{\sigma'_a + 2\sigma'_r}{3} = p - u.$
- Deviatoric stress: $q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
- Specific volume: $v = \frac{V_T}{V_s} = \frac{V_s + V_v}{V_s} = 1 + e.$

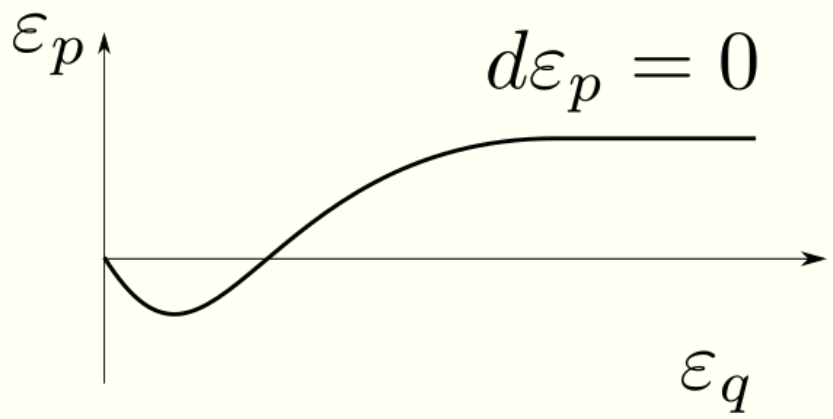


Critical State Hypothesis: I

Roscoe, Schofield & Worth (1958): **At shear-failure, soil exists at a unique state**

- $d\varepsilon_q >> 0$ unlimited shear strain potential.
- $dp' = dq = d\varepsilon_p = 0$ no change in p', q, ε_p.
- Critical state stress ratio: $\eta = q/p' = const = M$ at failure $q = Mp'$.





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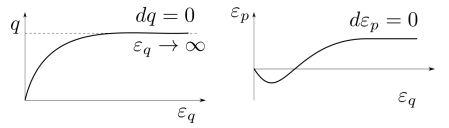
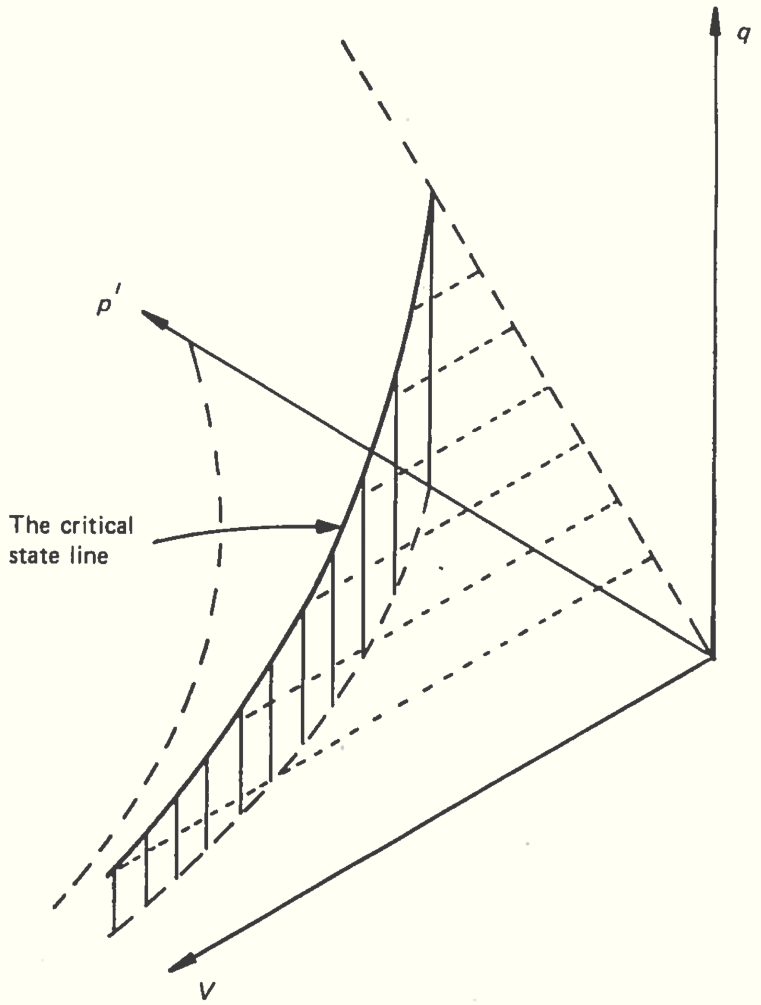


Soil is sheared to a point where stresses are stationary ($dq = dp' = 0$) with no futher change in volume ($d\varepsilon_p = 0$), unlimited shear strains ($d\varepsilon_q >> 0$) and q/p' has a fixed value: **critical state**.

M can be related to phi': $M = \frac{6 \sin \phi'}{3 - \sin \phi'}$.

Critical State Hypothesis: II

Critical state is a function of q, p', v.



The CSL (p', v, q) space is given by the intersection of two planes: $q = Mp'$ and a curved vertical plane $v = \Gamma - \lambda \ln p'$

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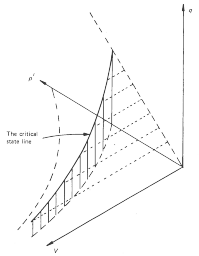

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Critical state curve connecting critical state points:

- Crticial state line
- Defined in 3D but we'll look at projections into $q - p'$ and $v - p'$ space

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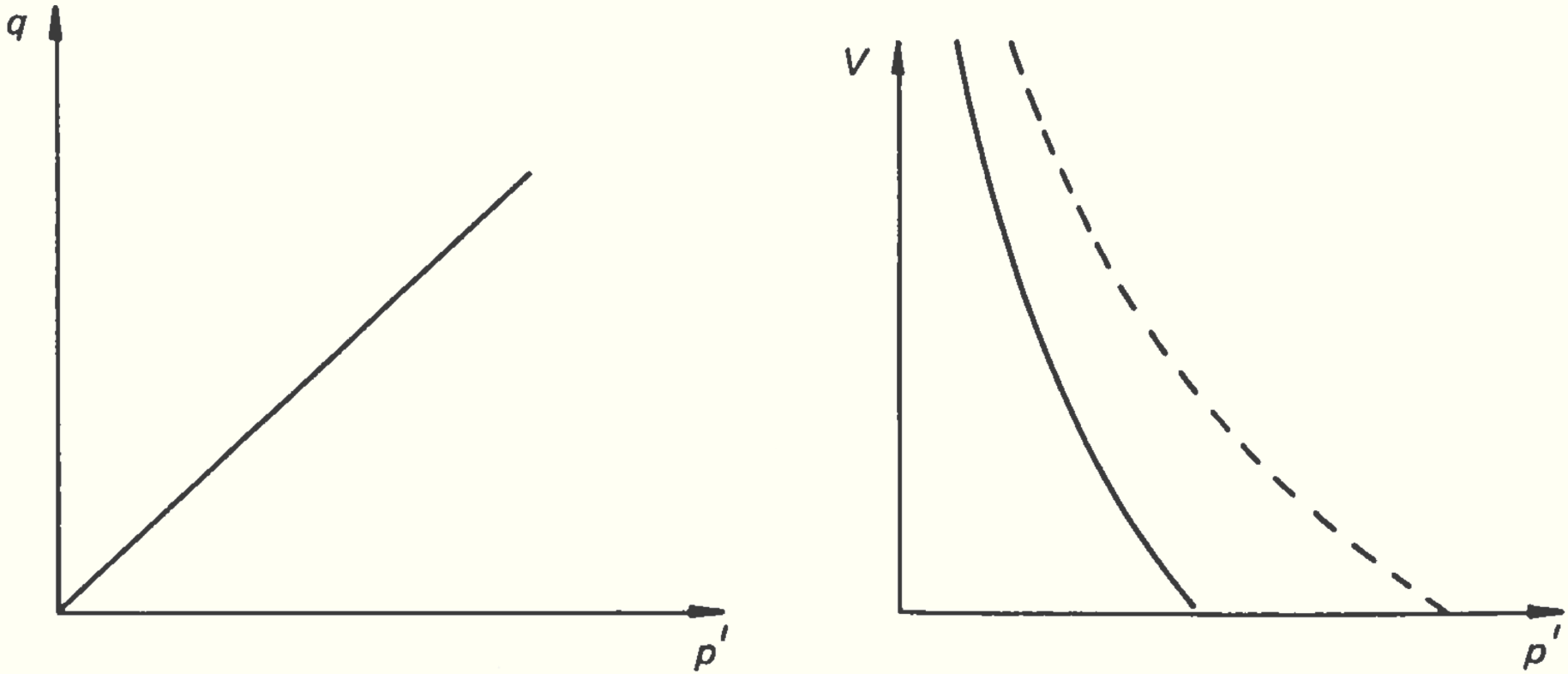

The CSL in (a) (p', q) plot and (b) (p', v) plot (isotropic normal compression line is shown in dashed)

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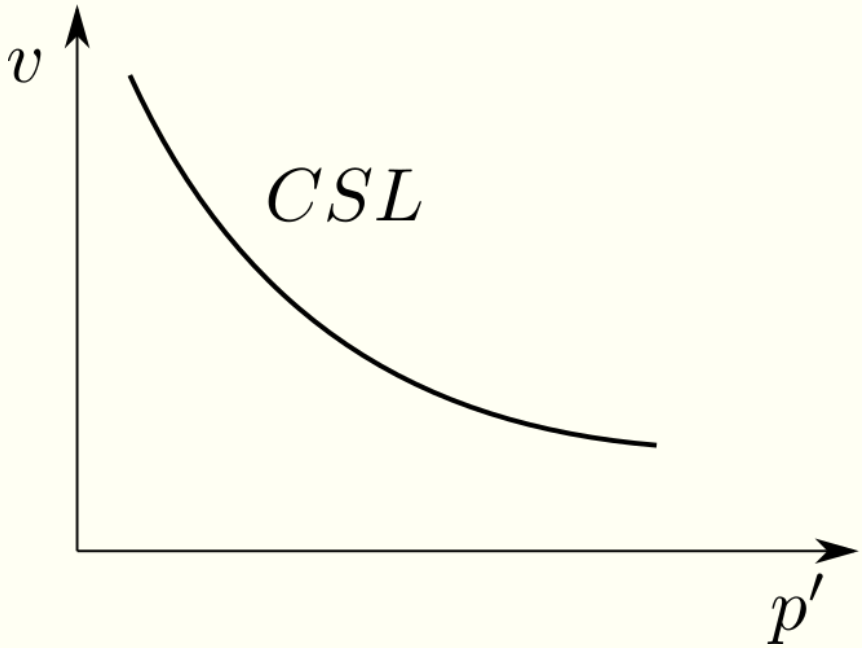

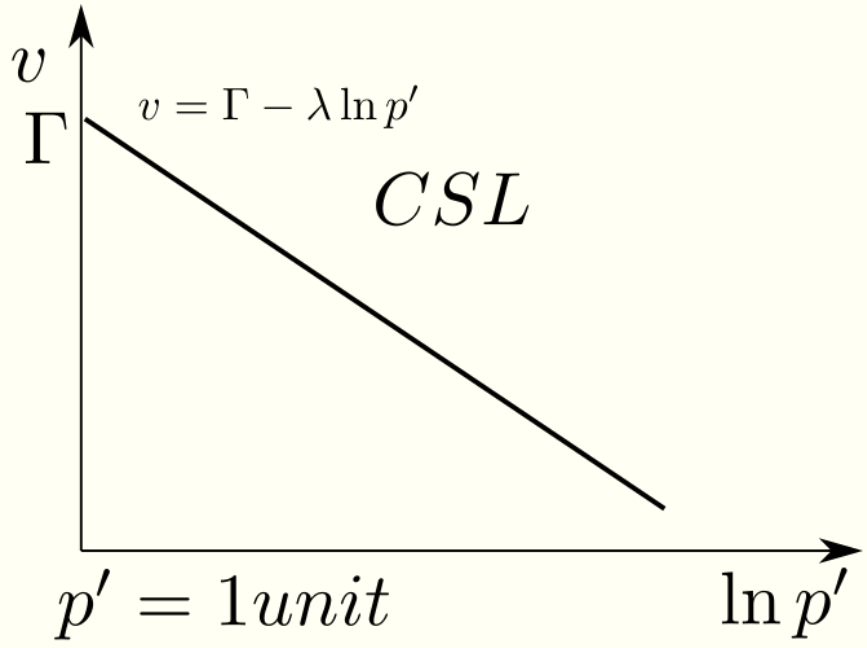

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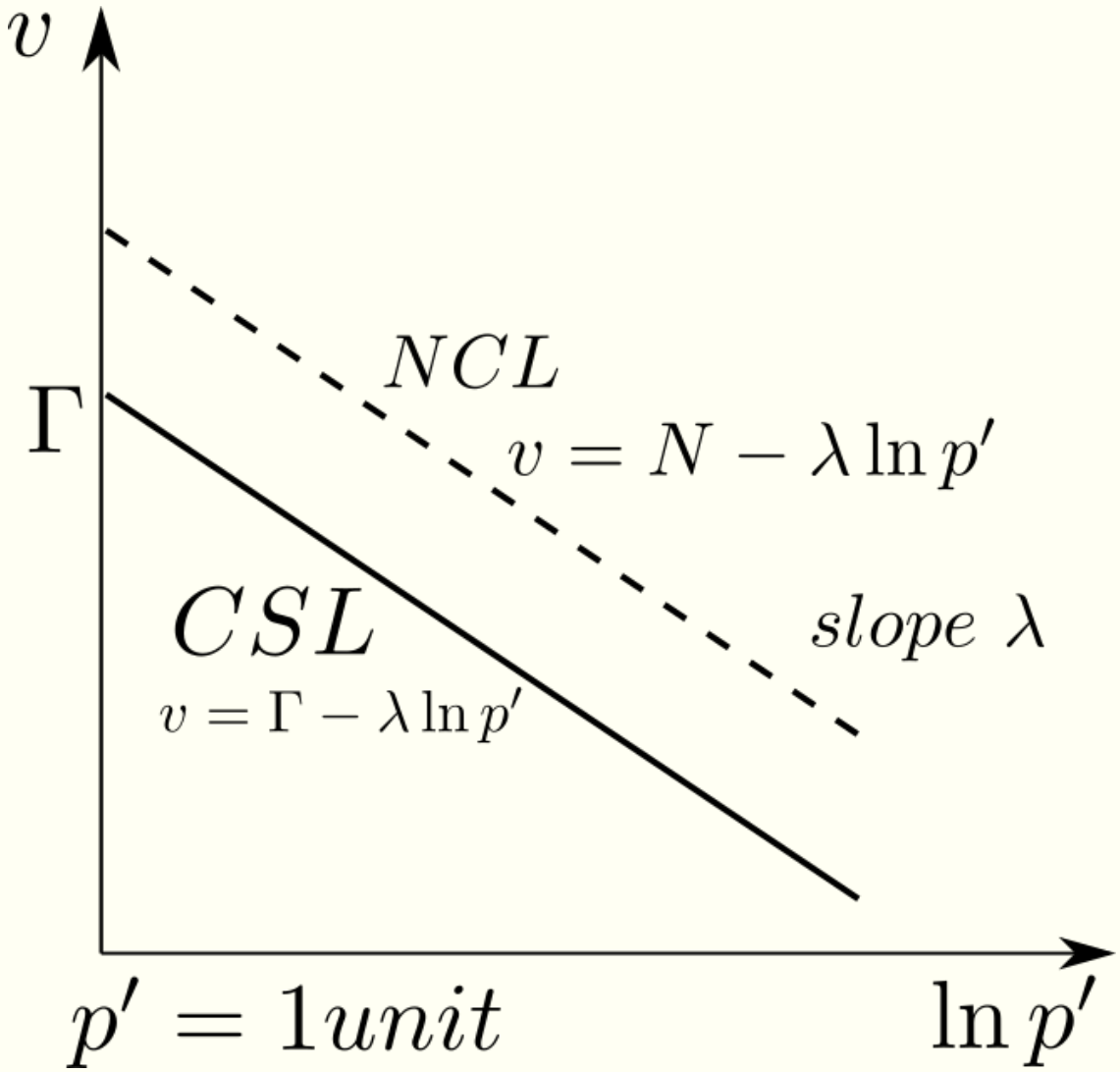

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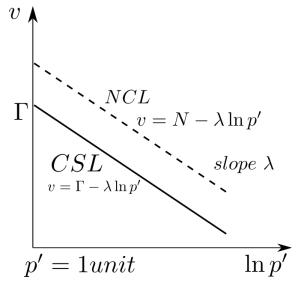

Isotropic virgin compression line (VCL) $\eta = 0$. NCL is parallel to CSL. VCL is $\eta = 0$, while CSL $\eta = M$. Oedometer falls between VCL and CSL at a constant η: $0 < \eta < M$.

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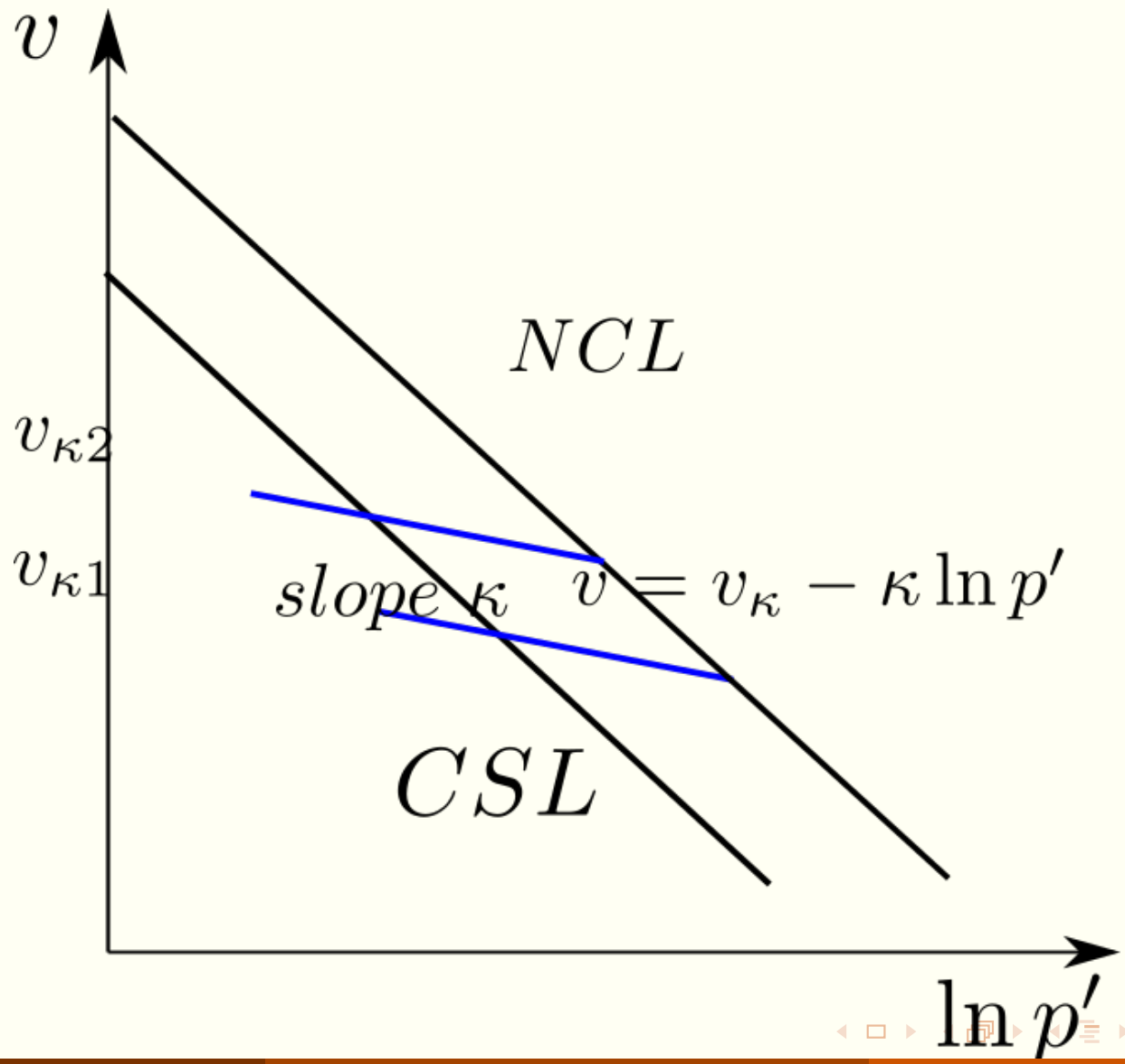

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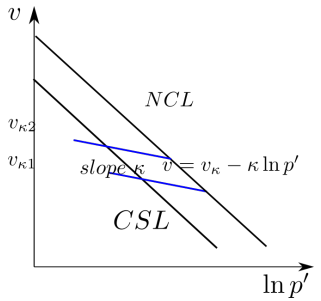
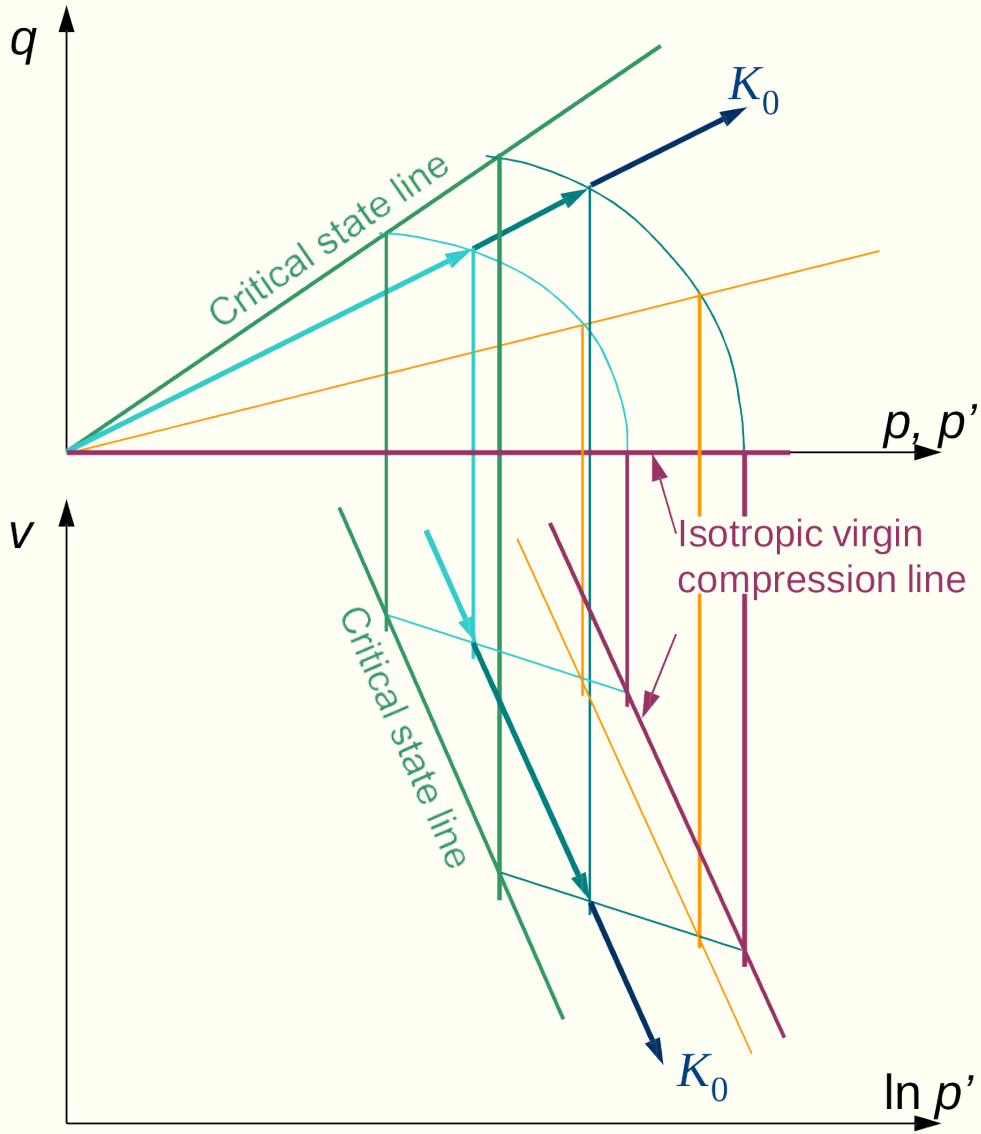
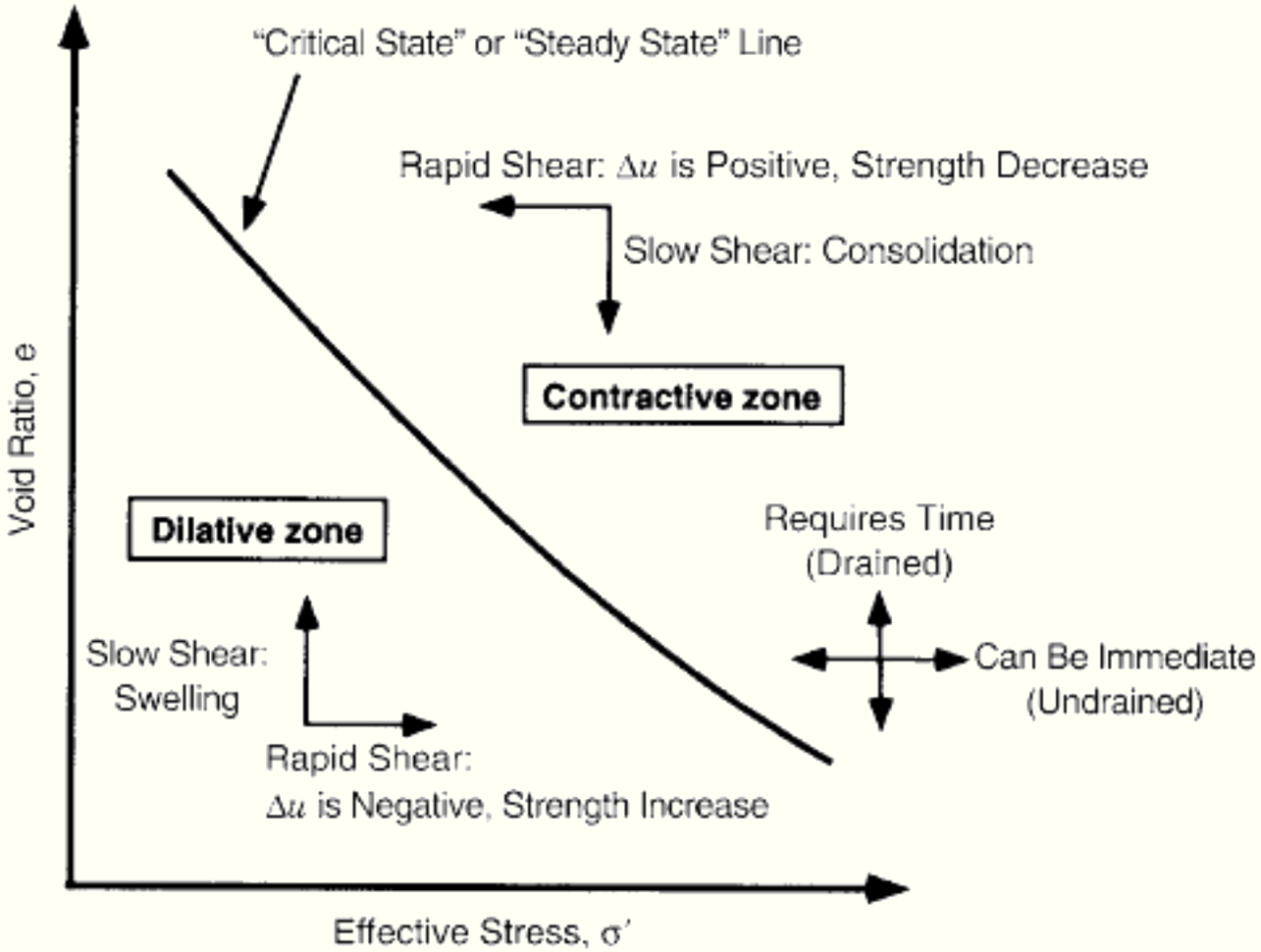


v_κ depends on which κ line you are on. $\kappa \neq c_r$ and $\lambda \neq C_c$

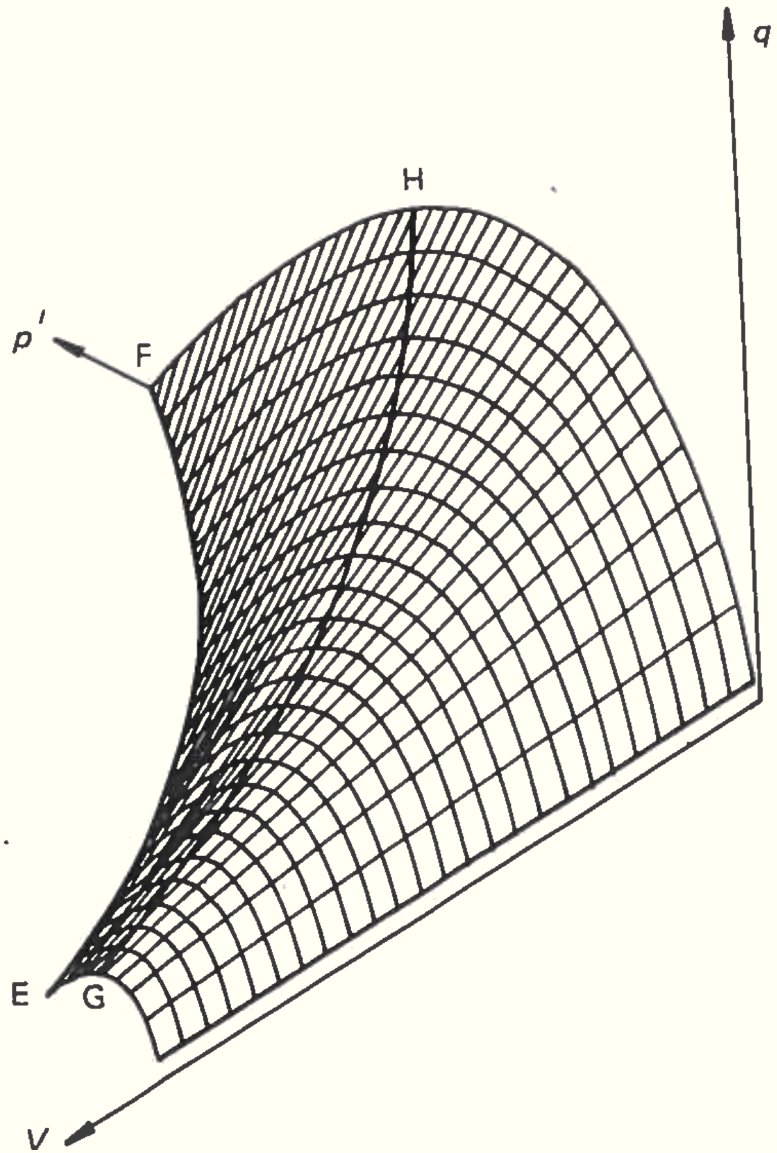
Stress paths $\sigma_3'/\sigma_1' = K_c = const$



Clay behavior



Critical state boundary surface



Summary of critical state behavior

- Can only traverse NCL in one direction
- Can traverse RCL (κ-line) in both directions
- To move from one κ-line to another must move along NCL. Hence, plastic volumetric strains must occur.
- Critical state line is **NOT** a yield surface. It's where it's going but a lot of plastic straining is needed to get there. (if $CSL = F = 0$) then with associative flow rule $d\varepsilon_p^p \neq 0$ at critical state. Real F is horizontal at critical state.