CE394M: Tresca and Mohr-Coulomb

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Overview

Stress invariants

- The magnitudes of the component of the stress vector depend on the chosen direction of the coordinate axes (in 3D: 6 variables).
- Principal stresses always act on the same planes and have the same magnitude (invariant to the coordinate axes), but still need to define the corresponding orientations (in 3D: 6 variables).
- For isotropic materials, it is very convenient to work with alternative invariant quantities which are combinations of principal stresses.

Stress invariants

- Mean effective stress $p = \frac{1}{3}(\sigma_I + \sigma_I I + \sigma_I II)$
- Deviatoric stress: $J = \frac{1}{\sqrt{6}}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}$
- Lode's angle $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_{II} - \sigma_{III})}{\sigma_I - \sigma_{III}} - 1\right)\right]$

Principal stresses can be expressed in terms of invariants:

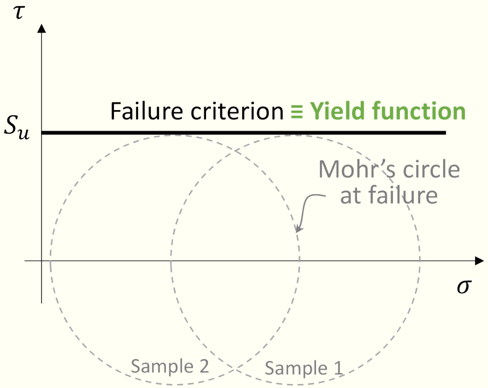
$$\begin{bmatrix} \sigma_I \\ \sigma_{II} \\ \sigma_{III} \end{bmatrix} = p \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{2}{\sqrt{3}} J \begin{bmatrix} \sin\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right) \end{bmatrix}$$

Tresca model

Simulation of undrained behavior of saturated clay
Failure criteria:

$$\tau_f = s_u$$

where τ_f is the shear stress at failure. s_u is the undrained strength.



Tresca model

Yield function

$$F(\sigma, W_p) = \sigma_I - \sigma_{III} - 2s_u = 0$$
$$= J\cos\theta - s_u = 0$$