

# CE394M: Stress paths and invariants

Krishna Kumar

University of Texas at Austin

*krishnak@utexas.edu*

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## 1 Stresses / strains in typical geotechnical lab tests

## 1D consolidation / simple shear

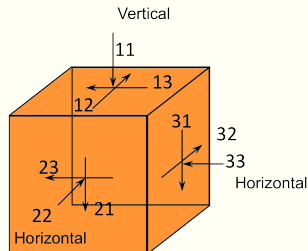
- Zero lateral strain ( $\epsilon_{22} = \epsilon_{33} = 0$ )
- Stresses:  $\sigma$  and  $\tau$
- Strains:  $\epsilon_{11} = \epsilon_v$  and  $\gamma$

## 2D plane strain

- Zero lateral strain ( $\epsilon_{22} = \gamma_{12} = \gamma_{23} = 0$ )
- Stresses:  $s$  and  $t$
- Strains:  $\epsilon_v$  and  $\epsilon_\gamma$

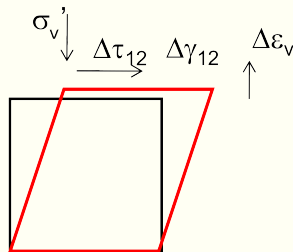
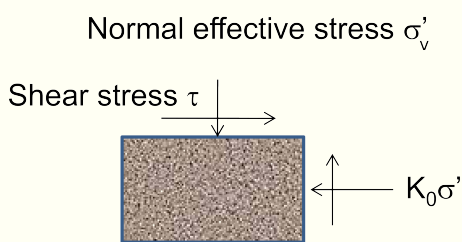
## 3D general (axi-symmetric as a special case)

- Stresses:  $p$  and  $q$
- Strains:  $\epsilon_v$  and  $\epsilon_s$



# 1D simple shear

- 1 No lateral strain
- 2 Constant normal effective stress  $\sigma'_v$
- 3 Increasing shear strain  $\gamma$
- 4 Measure shear resistance  $\tau$
- 5 Measure volumetric strain  $\varepsilon_v$  or void ratio  $e = e_0 - (1 + e_0)\varepsilon_v$
- 6 **No information for the lateral direction**



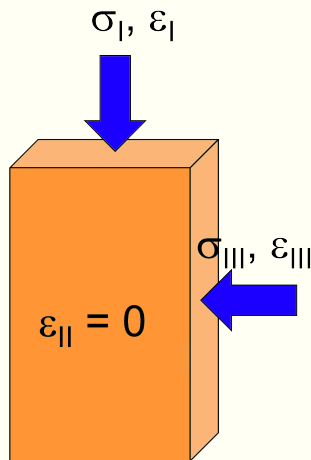
# 2D plane strain / Mohr-Coulomb model

## Stresses and strains: independent components

- 1 Mean stress:  $s = (\sigma_I + \sigma_{III})/2$  and  $s' = s - u$
- 2 Volumetric strain:  $\varepsilon_v = (\varepsilon_I + \varepsilon_{III})$
- 3 Deviatoric / shear stress:  $t = (\sigma_I - \sigma_{III})/2$  and  $t' = t$ .
- 4 Deviatoric / shear strain:  $\varepsilon_\gamma = (\varepsilon_I - \varepsilon_{III})$
- 5 Work increment:

$$\Delta W = \sigma'_I \Delta \varepsilon_I + \sigma'_{III} \Delta \varepsilon_{III} = s' \Delta \varepsilon_v + t \Delta \varepsilon_\gamma$$

- 6  $s$  and  $t$  are often used to derive parameters for Mohr-Coulomb model because it only considers  $\sigma_I$  and  $\sigma_{III}$  and not  $\sigma_{II}$ .



$$\sigma' = \sigma - u$$

## CE394M: Stresses - paths &amp; invariants

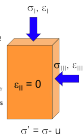
└ Stresses / strains in typical geotechnical lab tests

└ 2D plane strain / Mohr-Coulomb model

## Stresses and strains: independent components

- Mean stress:  $s = (\sigma_I + \sigma_{III})/2$  and  $s' = s - u$
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- Deviatoric / shear stress:  $t = (\sigma_I - \sigma_{III})/2$  and  $t' = t$
- Deviatoric / shear strain:  $\varepsilon_v = (\varepsilon_I - \varepsilon_{III})$
- Work increment:  

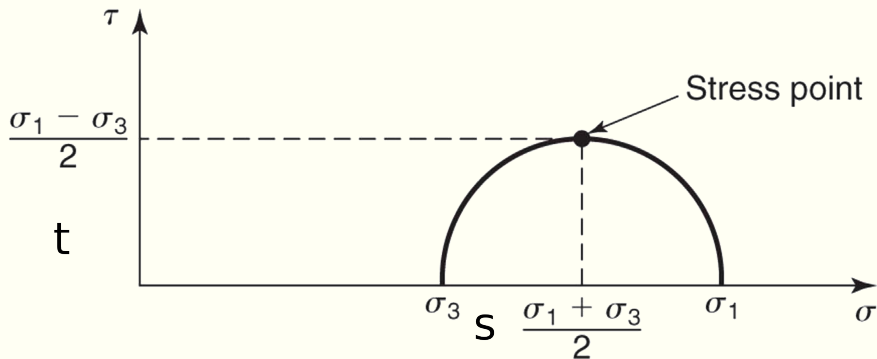
$$\Delta W = \sigma'_I \Delta \varepsilon_I + \sigma'_{III} \Delta \varepsilon_{III} = s' \Delta \varepsilon_v + t \Delta \varepsilon_v$$
- $s$  and  $t$  are often used to derive parameters for Mohr-Coulomb model because it only considers  $\sigma_I$  and  $\sigma_{III}$  and not  $\sigma_{II}$ .



Principal stresses  $\sigma_I > \sigma_{II} > \sigma_{III}$ . This equation holds good and is the definition of  $\sigma$  in principal stress notations, i.e.,  $I > II > III$ .

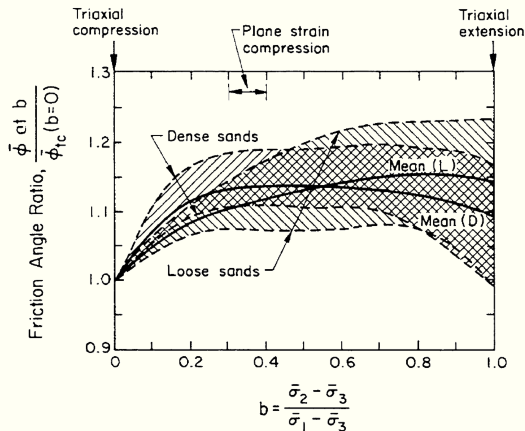
No shear stresses on these planes.

# 2D Mohr circle



# Effect of $\sigma_{II}$

Bishop (1966) defined **b-value**:  $b = (\sigma_{II} - \sigma_{III})/(\sigma_I - \sigma_{III})$ , where  $\sigma_I > \sigma_{II} > \sigma_{III}$ . Triaxial compression  $b = 0$ , triaxial extension  $b = 1$ . Typically  $\phi'_{ps} = (1.05 - 1.15)\phi'_{tx}$ .



Ladd et al., (1975)

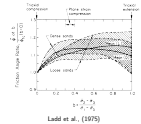


## CE394M: Stresses - paths &amp; invariants

└ Stresses / strains in typical geotechnical lab tests

└ Effect of  $\sigma_{II}$

Bishop (1956) defined **b-value**:  $b = (\sigma_{II} - \sigma_{III}) / (\sigma_I - \sigma_{III})$ , where  $\sigma_I > \sigma_{II} > \sigma_{III}$ . Triaxial compression  $b = 0$ , triaxial extension  $b = 1$ . Typically  $\phi_{\mu} = (1.05 - 1.15)\phi_{dc}$ .



$\sigma_{II}$  do have an effect on soil behavior. For example, the friction angle depends on the loading condition: triaxial compression, plane-strain, triaxial extension and others . . . .