CE394M: Constitutive Modeling

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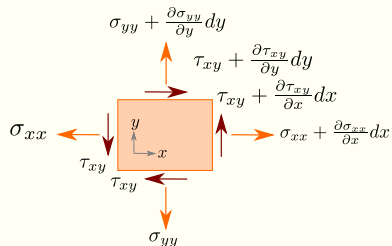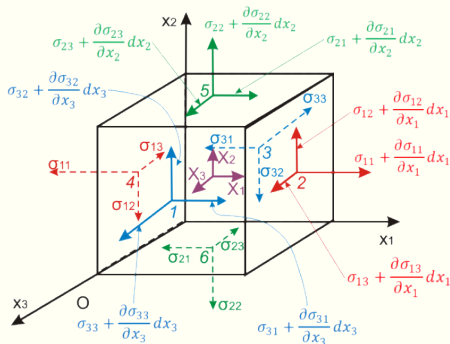March 10, 2019

Overview

1. Definition of stress and strain tensors

The objective of constitutive modelling is the determination of stiffness tensor **C**, a relation between stress and strain tensors.

Stresses

- Stress state at a point is defined by σ_{ij} in a frame of reference.
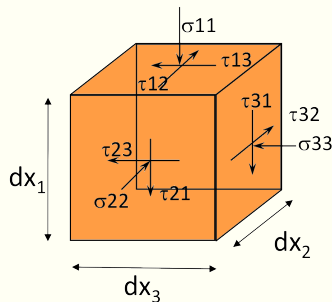- Equilibrium of moments require $sigma_{xz} = \sigma_{zx}, \ldots$ etc.,

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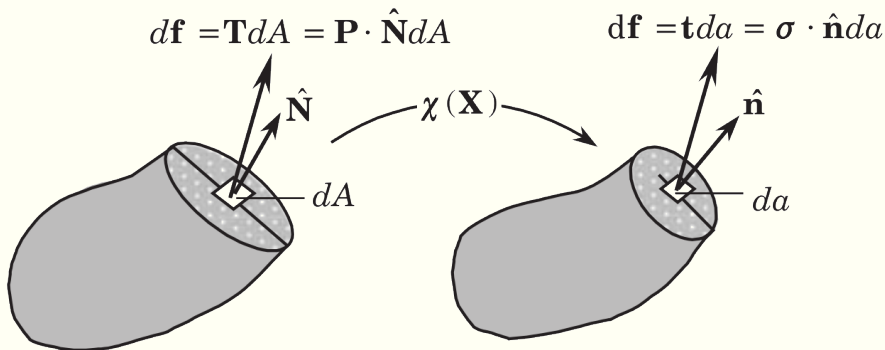- Equilibrium of moments require $sigma_{xz} = \sigma_{zx}, \dots$ etc.,
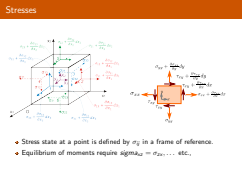
- σ_{xz} stress acting on plane perpendicular to axis x and in the direction of z

- σ_{xx} stress acting on plane perpendicular to axis x and in the direction of x

Stresses

- 9 components of the stress tensor.
- 6 stresses: $\sigma_{11}, \sigma_{22}, \sigma_{33}, \tau_{12}, \tau_{23}, \tau_{31}$.
- $\tau_{21} = -\tau_{12}, \tau_{32} = -\tau_{23}, \tau_{13} = -\tau_{31}$
- Compression is positive
- Shear stress, anti-clockwise is positive
- In order to write the components in a more concise way we can use the indices notation: σ_{ij} (use $i = 1, 2, 3$ and $j = 1, 2, 3$)
- Correspondence from x, y, z to $1, 2, 3$ (e.g., $\sigma_{11} = \sigma_{xx}, \sigma_{12} = \sigma_{xy}$)

Cauchy stress vs Piola-Kirchoff stress

$$d\mathbf{f} = \mathbf{T} dA = \mathbf{P} \cdot \hat{\mathbf{N}} dA$$

$$d\mathbf{f} = \mathbf{t} da = \boldsymbol{\sigma} \cdot \hat{\mathbf{n}} da$$

$\hat{\mathbf{N}}$ $\chi(\mathbf{X})$ $\hat{\mathbf{n}}$

dA da

An introduction to continuum mechanics - J. N. Reddy (2008)

- The first Piola–Kirchhoff stress tensor, also referred to as the *nominal stress tensor*, or *Lagrangian stress tensor*, gives the current force per unit undeformed area.