Assignment 1: Stress equilibrium, compatibility and stiffness matrix
Assigned: 27th January 2019
Due: 8th February 2019

1. Prove that the following stress-field could be a valid lower-bound solution.

$$\sigma_{xx} = c_1 x^3 y - 2c_2 xy + c_3 y$$
$$\sigma_{yy} = c_1 xy^3 - 2c_1 x^3 y$$
$$\sigma_{xy} = -\frac{3}{2}c_1 x^2 y^2 + c_2 y^2 + \frac{1}{2}c_1 x^4 + c_4$$

   where c_1, c_2, c_3 and c_4 are constants.

2. Determine and describe the stress-state given by the following Airy stress functions:

   (a) $\phi = Ay^2$

   (b) $\phi = Bxy$

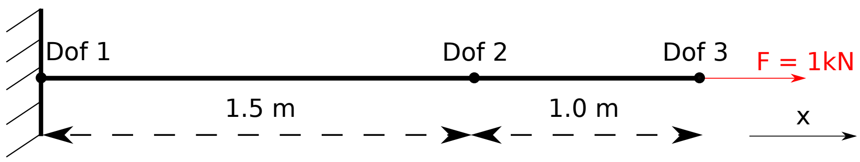
   Are these stresses valid for an isotropic elastic element? Note: please refer `https://en.wikiversity.org/wiki/Airy_stress_function` on how to determine cauchy stress components from an Airy function.

3. Determine whether the following strain fields are possible in a two-dimensional continous body:

   (a) $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2+y^2) & c_1 xy \\ c_1 xy & c_2 y^2 \end{bmatrix}$

   (b) $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2+y^3) & 3c_1 xy^2/2 \\ 3c_1 xy^2/2 & c_2 x^3 \end{bmatrix}$

4. Given the one-dimensional steel bar below, which is represented by two elements, determine the system's global stiffness matrix ($\mathbf{K}$). The bar has a cross sectional area (A) of 5×10^{-4} m and Young's modulus (E) of 20 GPa. Please note that the deformation of an elastic bar is governed as follows: $\delta = FL/AE$, where F is the applied force leading to a displacement of δ. Also, note that this bar can be characterized in one dimension only with just two degrees of freedom (i.e. displacement at each of the two nodes in the x direction only).



   Assuming a horizontal force F_x =1 kN at DOF 3, solve for displacements at all degrees of freedom. You are encouraged to use Jupyter notebooks to solve the linear system of equations.