CE394M: Advanced Analysis in Geotechnical Engineering

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Overview

1. Geotechnical modeling
   - Complexity in Geotechnical modeling

2. Numerical methods for differential equations
   - Direct method

Geotechnical modeling of the complex world

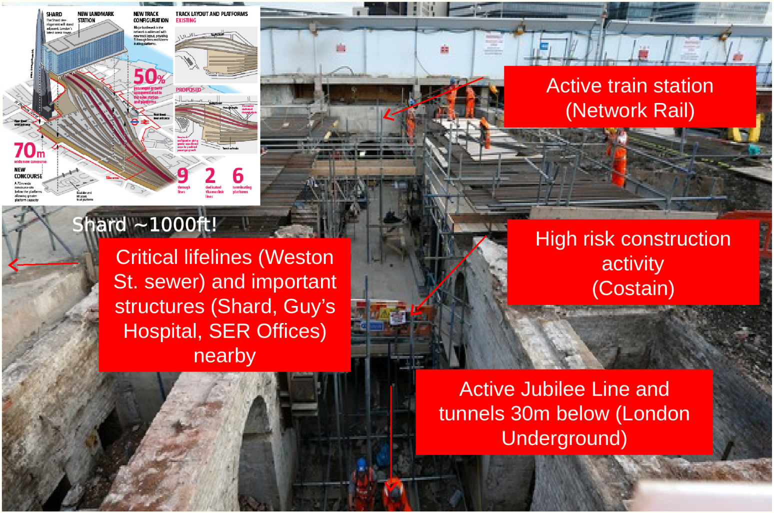
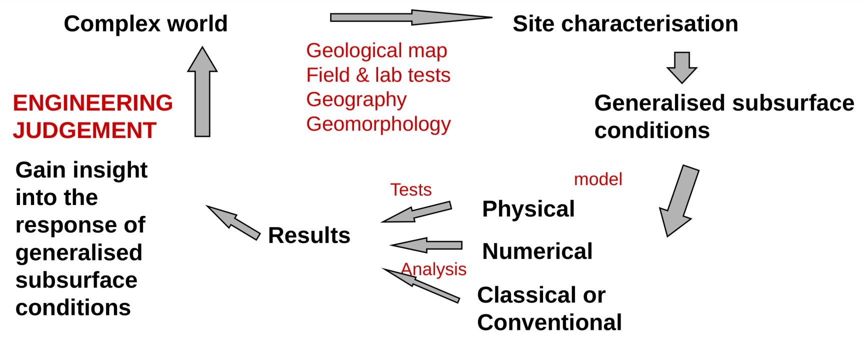


Fig. London Bridge Station, London, UK

Geotechnical modeling of the complex world

Fig. London Victoria station upgrade, London, UK

Geotechnical modeling



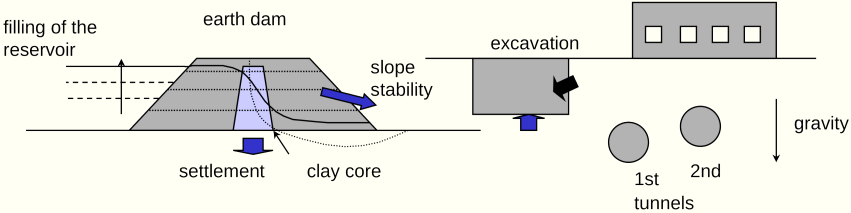
Soil behavior

- nonhomogeneous,
- anisotropic,
- non-linear,
- initial stress conditions,
- stress history
- Geometry - very complex

Soil Mechanics in practice - largely empirical

Geotechnical modeling: What should be modeled?

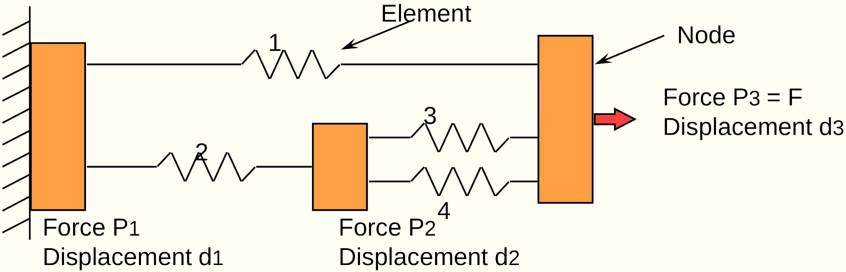
- Self weight effect of soils (This is why soil moves)
- Construction sequence (Complex geometry)
- Water movement (undrained, consolidation, drained)
- Insitu stresses (stiffness/strength depends on current stresses and stress history)
- Predict the ability of a design to withstand extreme loading conditions (you only have one chance)



Analysis of engineering problems

- Conceptualize the system
  - Geometry
  - Properties
  - Processes
- Describe it mathematically
  - Select the relevant differential equations
- Solve the equations (numerically)
  - Discretise the system
- Settle for approximation (numerical techniques)
  - Interpret the results

Matrix analysis of structures



- What are the known variables? $d_1 = 0, P_2 = 0, P_3 = F(\text{constant})$
- What are the unknowns? P_1, d_2, d_3
- What do we know? Force or distortion relations at an element level.

Matrix analysis of structures: Equilibrium

A structure is considered to be in equilibrium if, initially at rest, it remains at rest when subjected to a system of forces and couples. If a structure is in equilibrium, then all of its members and joints must also be in equilibrium.

- $P_1 = -S_1 - S_2$
- What are the unknowns? P_1, d_2, d_3
- What do we know? Force or distortion relations at an element level.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ s_4 \end{bmatrix}$$

$$\mathbf{P} = \mathbf{A^T S}$$

Matrix analysis of structures: Compatibility

- compatibility relates the deformations of a structure so that its various parts (members, joints, and supports) fit together without any gaps or overlaps.
- ensure that the deformed shape of the structure is continuous (except at the locations of any internal hinges or rollers), and is consistent with the support conditions.

Matrix analysis of structures: Compatibility

v = internal spring distortion δ = nodal displacement

- $v_1 = d_3 - d_1$
- $v_2 = d_2 - d_1$
- $v_3 = d_3 - d_2$
- $v_4 = d_3 - d_2$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\mathbf{v} = \mathbf{Ad}$$

Matrix analysis of structures: Physical condition

Force-distance relationship: spring constant

spring #	1	2	3	4
stiffness ($F.L^{-1}$)	3	2	1	2

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$$\mathbf{s} = \mathbf{D}\mathbf{v}$$

Matrix analysis of structures: Direct Method

Combine all the equations: $\mathbf{P} = \mathbf{A^T S} = \mathbf{A^T D v} = \mathbf{A^T D A d} = \mathbf{K d}$
where $\mathbf{K} = A^T DA$ (Global stiffness matrix)

$$K = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & -3 \\ -2 & 5 & -3 \\ -3 & -3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -3 \\ -2 & 5 & -3 \\ -3 & -3 & 6 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Apply Boundary conditions $d_1 = 0$, $P_2 = 0$ and $P_3 = F$ and solve P_1, d_2 and d_3

Matrix analysis of structures

known displacements and forces

↓

model at element level

↓

assemble global stiffness matrix

↓

calculate unknown displacements and forces