CE394M: Stress-strain-strength relationship of clay

Krishna Kumar

University of Texas at Austin

krishnak@utexas.edu

April 25, 2019

Overview

1. Stress-strain-strength relationship

2. Simple shear

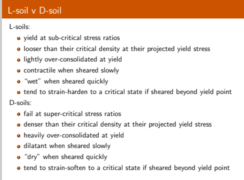
L-soil v D-soil

L-soils:

- yield at sub-critical stress ratios
- looser than their critical density at their projected yield stress
- lightly over-consolidated at yield
- contractile when sheared slowly
- "wet" when sheared quickly
- tend to strain-harden to a critical state if sheared beyond yield point

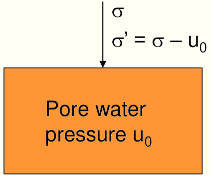
D-soils:

- fail at super-critical stress ratios
- denser than their critical density at their projected yield stress
- heavily over-consolidated at yield
- dilatant when sheared slowly
- "dry" when sheared quickly
- tend to strain-soften to a critical state if sheared beyond yield point



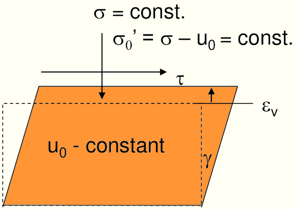
The terms "wet" and "dry" come from the idea of working a specimen of soil by hand. On the wet side of critical state the soil skeleton is too loosely compacted to support pressure stress—such stress, if applied (such as by squeezing the soil by hand) passes immediately into the pore water and thus causes this water to bleed out of the specimen and wet the hands. The opposite effect occurs when the soil is on the dry side of critical state.

Simple shear



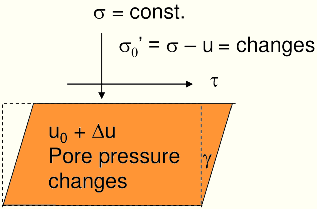
Initial consolidation
Void ratio e_0

Drained test



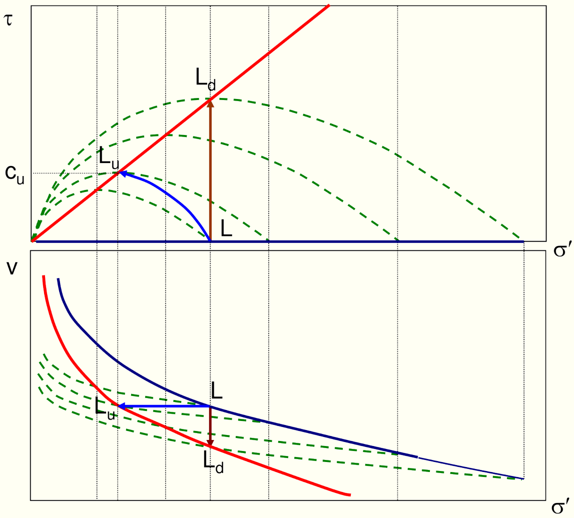
Change in void ratio
$\varepsilon_v = \Delta e/(1+e_0)$

Undrained test



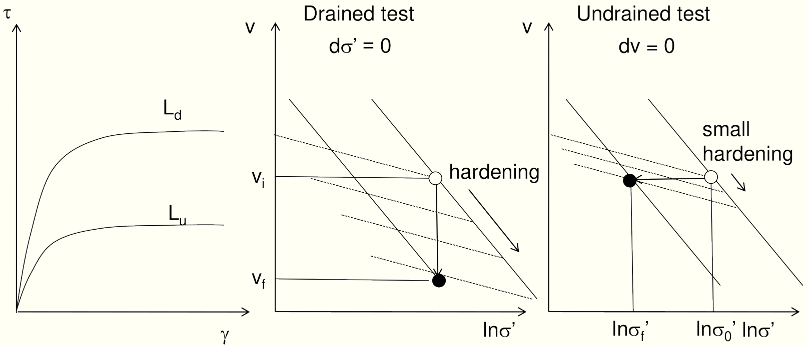
No volume change

Simple shear: Normally Consolidated Clay - L-soil

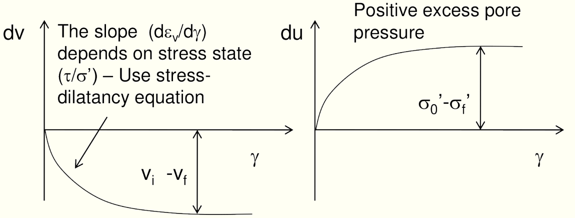


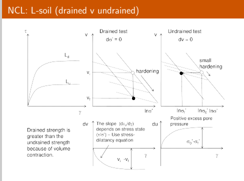
Drained shear: $L \rightarrow L_d$ and $\delta u = 0$. Undrained shear: $L \rightarrow L_u$ and $\delta v = 0$.

NCL: L-soil (drained v undrained)



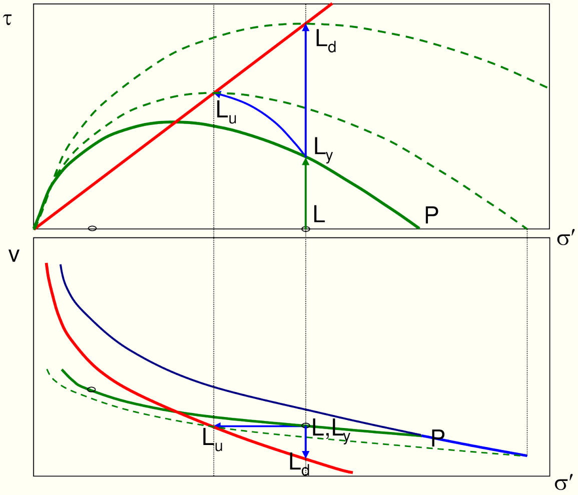
Drained strength is greater than the undrained strength because of volume contraction.



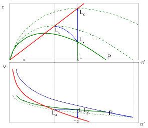


Note that the stack of $\kappa-$lines with their associated Cam Clay yield surfaces creates a 3D boundary surface in (τ, σ', v) space.
State paths starting inside the boundary surface begin as elastic until they hit the surface. Then they are forced to follow the boundary surface, obeying whatever test constraint is imposed (undrained v $=$ const, or drained $\sigma' =$ const) until the path reaches the critical state line. Then the soil shears at constant (τ, σ', v) state.
VCL: $d\varepsilon_p^e + d\varepsilon_p^p$ Elasto-plastic yielding RCL: $d\varepsilon_p^e$ only elastic
Isotropic loading along VCL ($q = 0$) can develop plastic strains: essentially hardening cap.
VCL: $N - \lambda \ln p' \quad \eta = 0$ RCL: $\Gamma - \lambda \ln p' \quad \eta = M \;\; N = \Gamma + (\lambda - \kappa)$

SS: LOC-soil (L-soils)



SS: LOC-soil (L-soils)

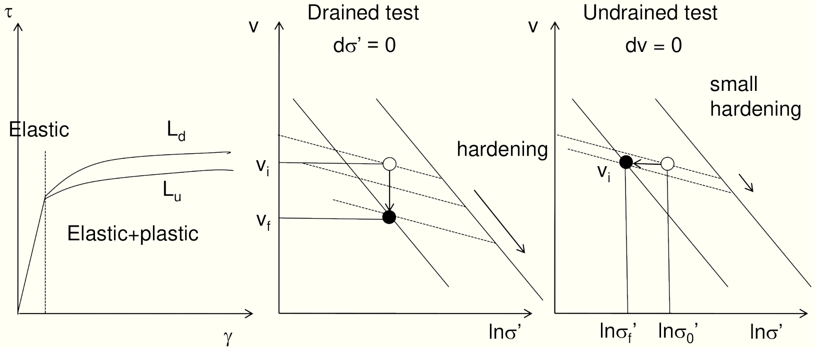


In undrained conditions, the effective stress path will be $d\sigma' = 0$ during elastic response.

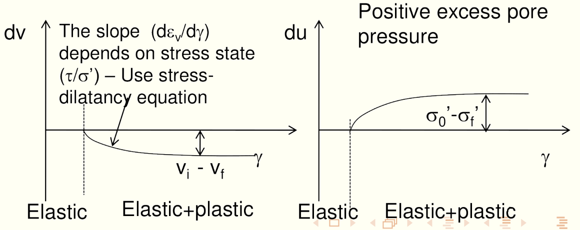
$d\sigma' = (vp'/\kappa)d\varepsilon_v$.

$d\varepsilon_v = 0$ hence $d\sigma' = 0$.

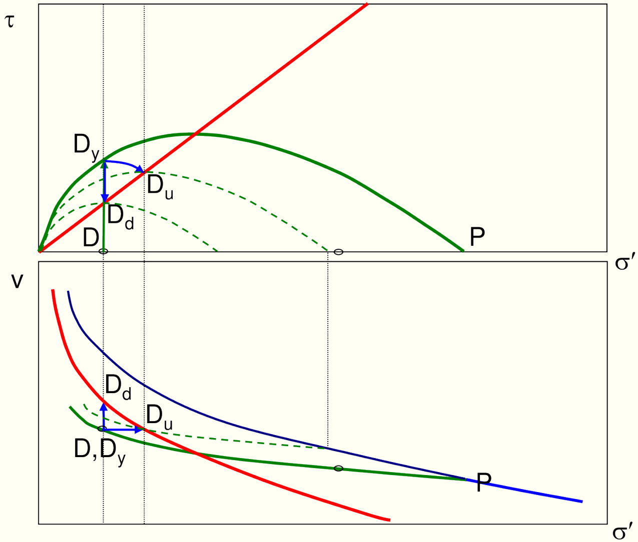
SS: LOC-soil (L-soils) (drained v undrained)



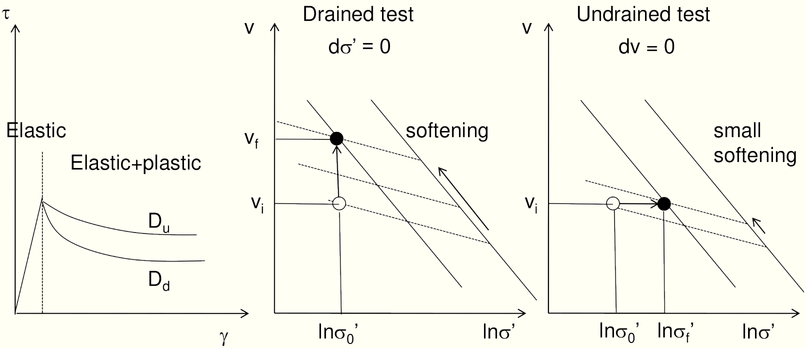
Drained strength is slightly greater than the undrained strength because of volume contraction.



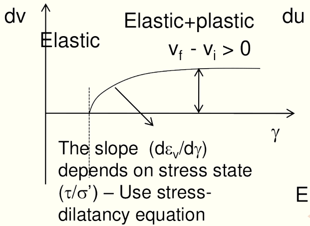
SS: HOC-soil (D-soils)

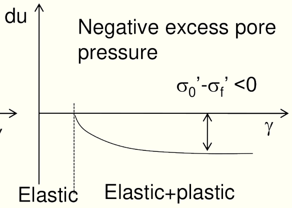


SS: HOC-soil (D-soils) (drained v undrained)

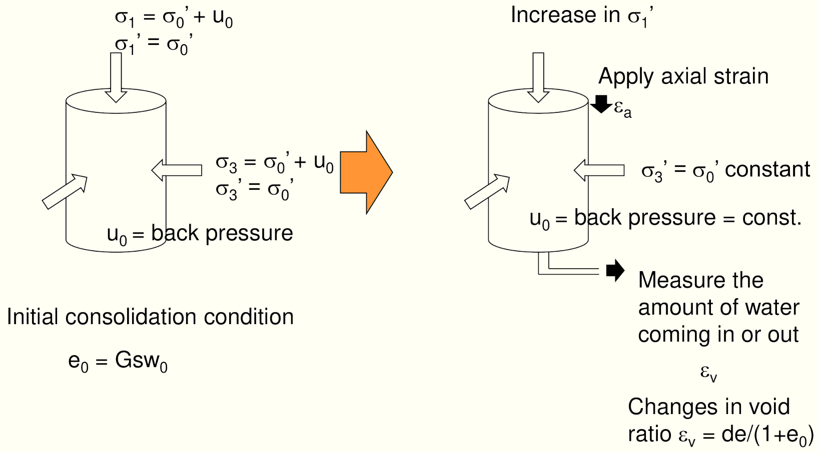


Drained strength is smaller than the undrained strength because of volume dilation.

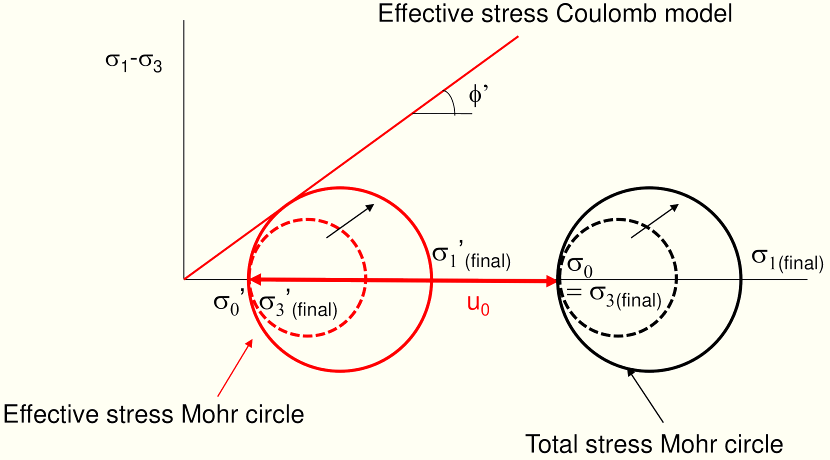




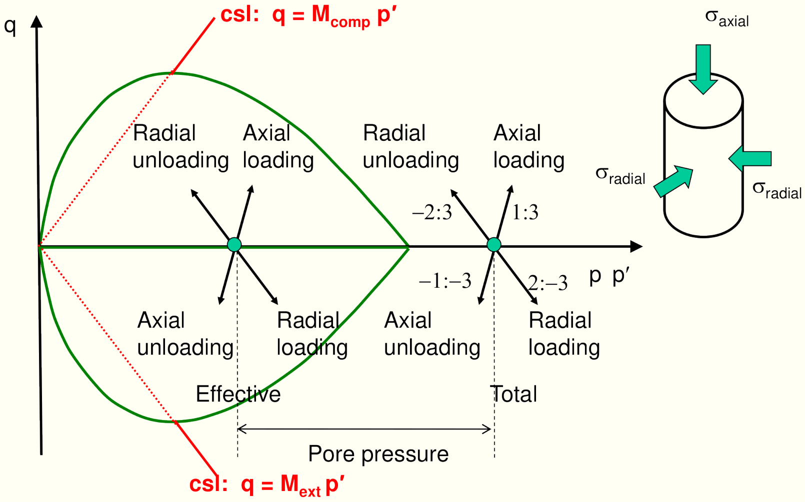
TXC: Drained strength and volume at failure using CS



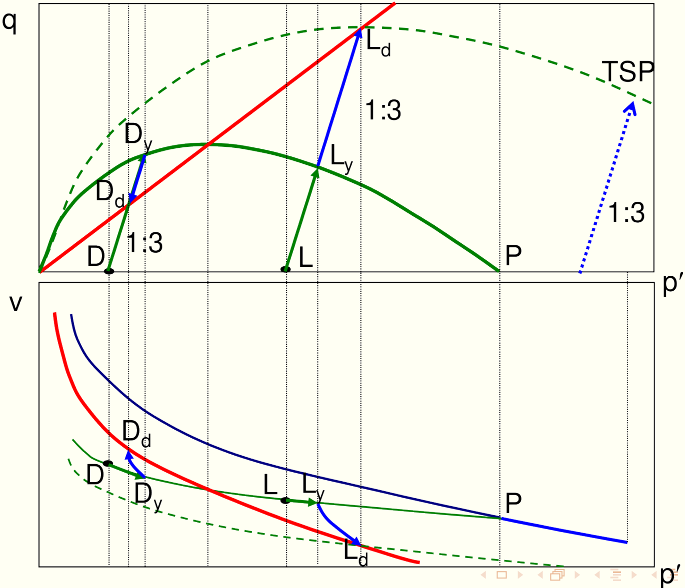
TXC: Drained (Mohr-Coulomb ESA)



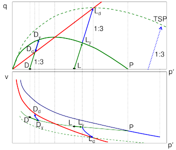
TXC: Drained Cam-Clay yield and failure



TXC Drained (axial loading)

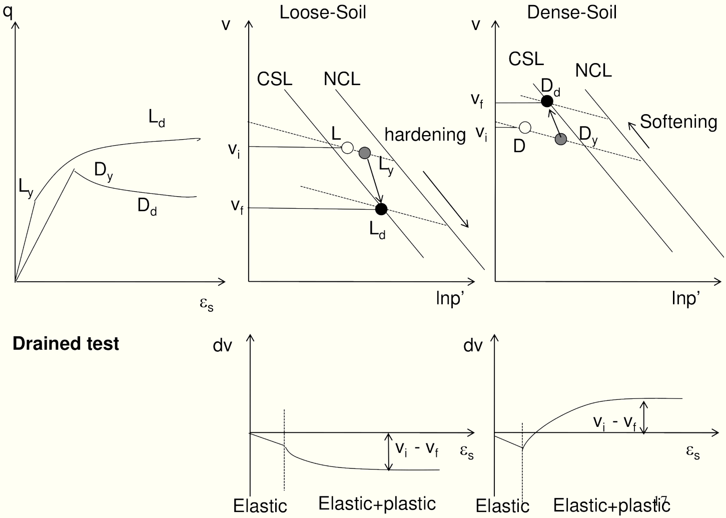


TXC Drained (axial loading)

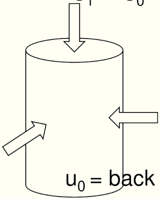


The total stress path of triaxial compression has a slope of 1:3. In drained test, the pore pressure is kept constant. Hence, the effective stress path will have a slope of 1:3.

TXC Drained (axial loading)



$\sigma_1 = \sigma_0' + u_0$
$\sigma_1' = \sigma_0'$



$\sigma_3 = \sigma_0' + u_0$
$\sigma_3' = \sigma_0'$

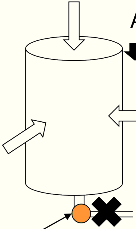
u_0 = back pressure

Initial consolidation condition

$e_0 = Gsw_0$

Increase in σ_1 and $\sigma_1' = \sigma_1 - u$

Apply axial strain



ε_a

σ_3 = constant
$\sigma_3' = \sigma_3 - u$
$= \sigma_0' + u_0 - u$
$= \sigma_0' + \Delta u$

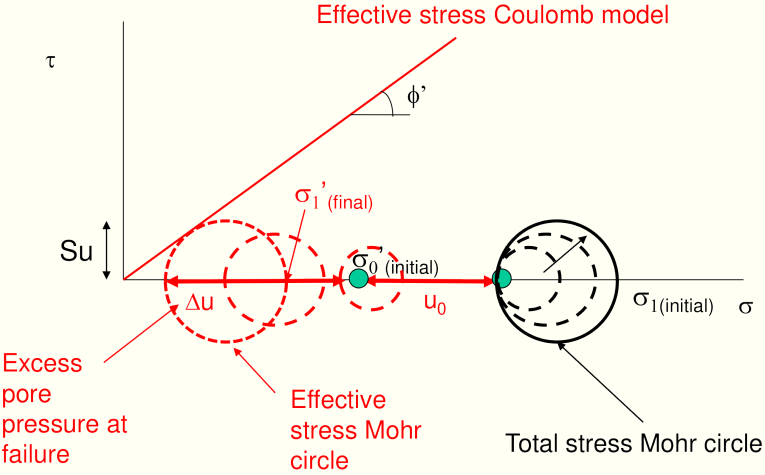
No drainage - undrained

Measure pore pressure (u) by a pressure transducer

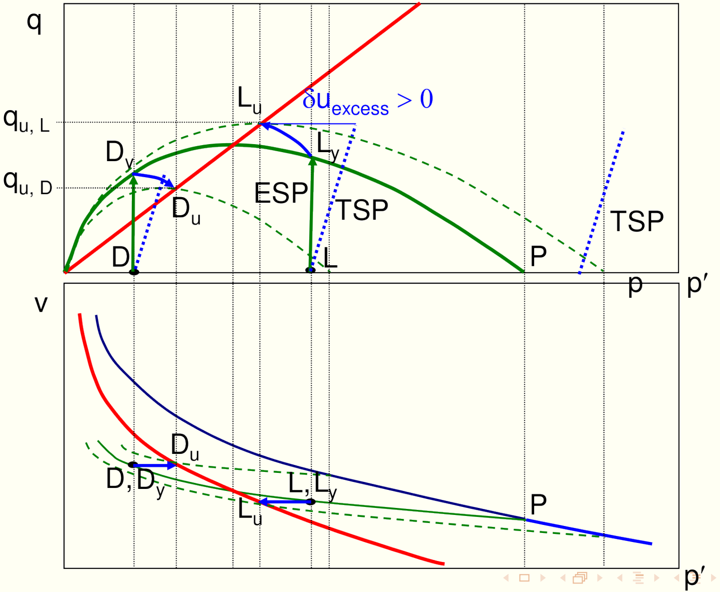
Excess pore pressure $\Delta u = u - u_0$

e_0 or w_0 is kept constant

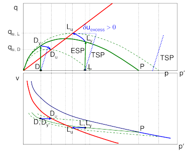
TXC: Undrained (Mohr-Coulomb ESA)



TXC Undrained (axial loading)



TXC Undrained (axial loading)



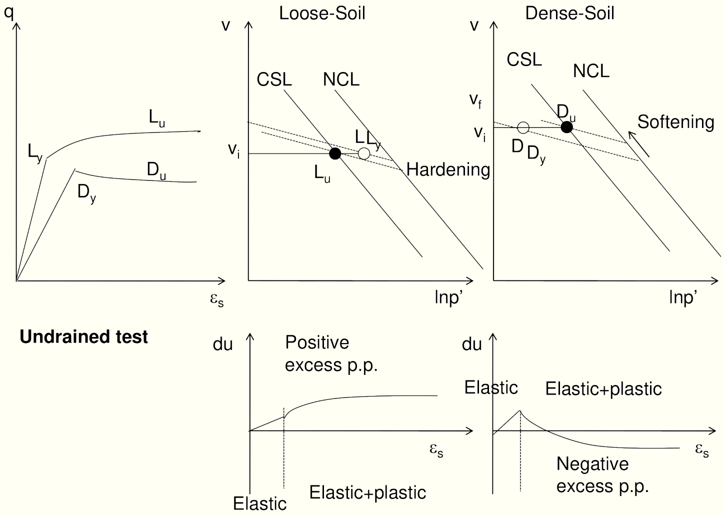
Elastic deformation: $dp' = Kd\varepsilon_v$ and $dq = 3Gd\varepsilon_s$.

$$K = \frac{vp'}{\kappa}$$
$$G = \frac{3K(1-2\nu)}{2(1+\nu)}$$

In undrained conditions, the effective stress path will be $dp' = 0$ during elastic response.

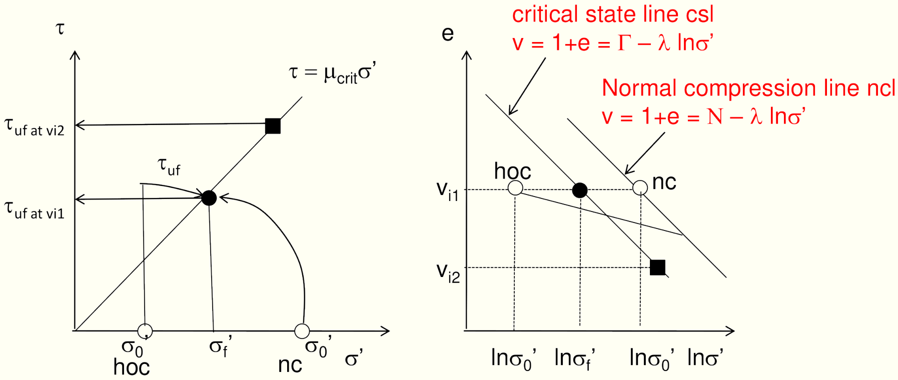
$$d\varepsilon_v = 0 \quad dp' = 0$$

TXC Undrained (axial loading)

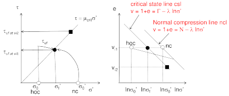


Undrained test

Critical state concept



Critical state concept



The critical state concept shows that undrained shear strength is purely a function of initial void ratio (or water content).

Critical state concept

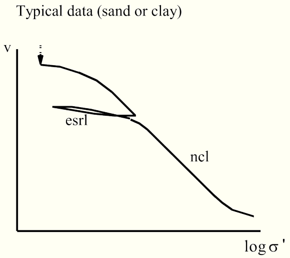
Interchangeable parameters for stress at yield and $d\varepsilon^p$.

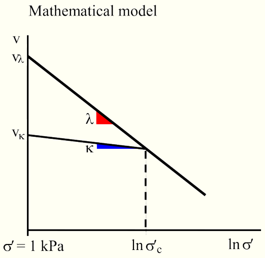
System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan \phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin \phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

Plastic work and dissipation: $\sigma^* \partial \varepsilon^* + \tau^* \partial \gamma^* = \mu^*_{crit} \sigma^* \partial \gamma^*$.

General yield surface: $\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} \ln \left[\frac{\sigma^*_c}{\sigma} \right]$

Critical state concept: 1D compression



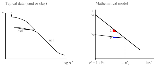


Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c = 1\text{kPa}$.
Plastic compression (normal compression line): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$.
Elastic swelling and recompression line ($\kappa-$line): $v = v_c + \kappa(\ln \sigma'_c - \ln \sigma'_v)$.

2019-04-25

CE394M: Stress-strain-strength
└─Simple shear
  └─Critical state concept: 1D compression

Critical state concept: 1D compression



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c = 1$kPa.
Plastic compression (normal compression line): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$.
Elastic swelling and recompression line ($\kappa-$line): $v = v_c + \kappa(\ln \sigma'_c - \ln \sigma'_v)$.

Equivalent parameters for log 10 stress scale:
Terzaghi's compression index: $C_c = \lambda \log e = \lambda \times 2.3026$
Terzaghi's swelling index: $C_s / C_r = \kappa \log e = \kappa \times 2.3026$