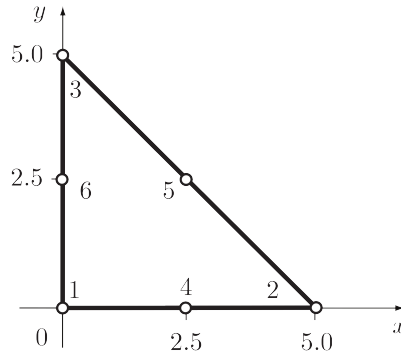


Assignment 4: FEM: Isoparametric elements, Gauss integration, solvers and errors (20 points)

Assigned: 25th February 2019

Due: 13th March 2019

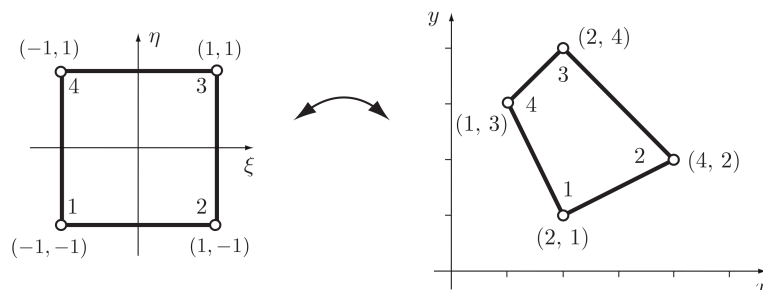
1. For the six noded-triangle shown. The nodal temperatures are $T^e = [300 \ 0 \ 0 \ 0 \ 340 \ 0]^T$. Compute the temperature and its gradient at the point P with the coordinates $x = 1.5$ and $y = 2.0$.



The shape functions for a 6-noded triangle element are:

$$\begin{aligned} N_1 &= 2(1 - \xi - \eta)^2 - (1 - \xi - \eta) & N_2 &= 2\xi^2 - \xi \\ N_3 &= 2\eta^2 - \eta & N_4 &= 4\xi(1 - \xi - \eta) \\ N_5 &= 4\xi\eta & N_6 &= 4\eta(1 - \xi - \eta) \end{aligned}$$

2. For the isoparametric mapping shown below
 - (a) Compute the x and y coordinates of the point $\xi = 0.5$, $\eta = 0.5$ in the physical domain.
 - (b) Compute $\frac{\partial N_1}{\partial x}$ for the same point.



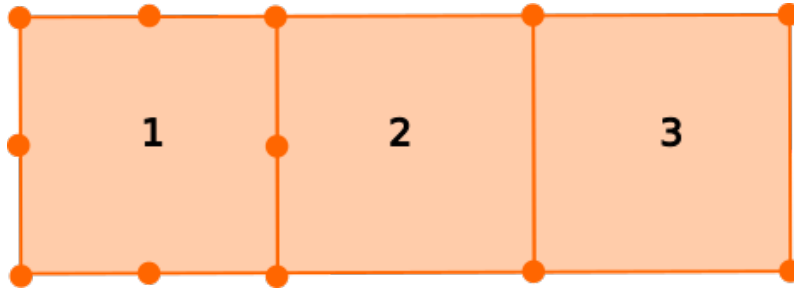
3. Evaluate the following three integrals using one, two and three-point Gauss integration. Compare your results with the results of analytic integration.

(a) $\int_{-1}^{+1} (3\xi^2 + 2\xi) d\xi$

(b) $\int_{-1}^{+1} \cos \xi d\xi$

(c) $\int_0^3 (3x^2 + x) dx$

4. Consider the following quadrilateral mesh, sketch (1D is fine) the temperature distribution across the mesh assuming a heat source on the left boundary of the mesh and comment on the suitability of the mesh for finite element computation and the need for the transition element (element #2)?



5. Develop a Python script for the Newton Raphson method to solve non-linear force-displacement relationships. Test and comment on the accuracy and efficiency of the NR to solve: $f = -2u^2 + 2u$, where u denotes the displacement and f refers to the internal force. For the analytical solution plot the displacement u between 0 and 1.
6. Two FE simulations of a 3D foundation settlement are performed using a structured mesh of tri-linear (8-noded) hexahedral elements. Assume the geometry of the mesh is cubic and all the elements are hexahedron and of the same size. Case A uses n elements and Case B uses $2n$ elements, where n is large.
- How many non-zero entries would you expect in most rows of the global stiffness matrix for Case A and Case B?
 - Compute the approximate increase in memory required to build the global stiffness matrix for Case B compared to Case A?
 - Cases A and B are solved using an LU solver. Based on the theoretical complexity of the LU solver estimate the increase in solver time in going from Case A to Case B.
 - If the error in L^2 norm of the solution is proportional to Ch^2 , where h is the length of an element edge and C is a problem constant that does not depend on h , estimate the reduction in the error for Case B compared to Case A.