

**Assignment 1: Stress equilibrium, compatibility and stiffness matrix**  
**Assigned: 27th January 2019**  
**Due: 8th February 2019**

1. Prove that the following stress-field is a valid lower-bound solution.

$$\begin{aligned}\sigma_{xx} &= c_1x^3y - 2c_2xy + c_3y \\ \sigma_{yy} &= c_1xy^3 - 2c_1x^3y \\ \sigma_{xy} &= -\frac{3}{2}c_1x^2y^2 + c_2y^2 + \frac{1}{2}c_1x^4 + c_4\end{aligned}$$

where  $c_1$ ,  $c_2$ ,  $c_3$  and  $c_4$  are constants.

2. Determine and describe the stress-state given by the following Airy stress functions:

(a)  $\phi = Ay^2$

(b)  $\phi = Bxy$

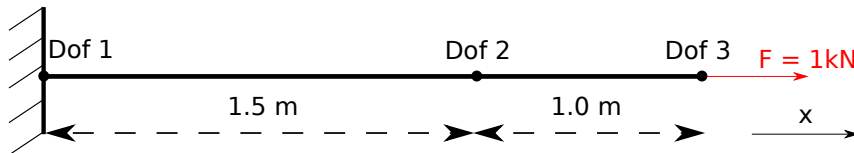
Are these stresses valid for a linear elastic material?

3. Determine whether the following strain fields are possible in a two-dimensional continuous body:

(a)  $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^2) & c_1xy \\ c_1xy & c_2y^2 \end{bmatrix}$

(b)  $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^3) & 3c_1xy^2/2 \\ 3c_1xy^2/2 & c_2x^3 \end{bmatrix}$

4. Given the one-dimensional steel bar below, which is represented by two elements, determine the system's global stiffness matrix ( $\mathbf{K}$ ). The bar has a cross sectional area ( $A$ ) of  $5 \times 10^{-4} \text{ m}^2$  and Young's modulus ( $E$ ) of 20 GPa. Please note that the deformation of an elastic bar is governed as follows:  $\delta = FL/AE$ , where  $F$  is the applied force leading to a displacement of  $\delta$ . Also, note that this bar can be characterized in one dimension only with just two degrees of freedom (i.e. displacement at each of the two nodes in the  $x$  direction only).



Assuming a horizontal force  $F_x = 1 \text{ kN}$  at DOF 3, solve for displacements at all degrees of freedom. You are encouraged to use Jupyter notebooks to solve the linear system of equations.