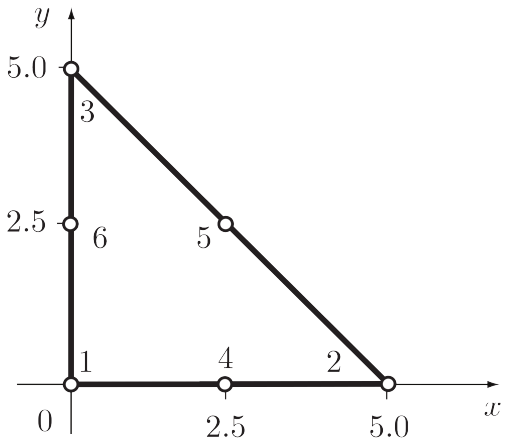
Assignment 4: FEM: Isoparametric elements, Gauss integration and solvers

Assigned: 25th February 2019

Due: 8th March 2019

1. For the six noded-triangle shown. The nodal temperatures are $T^e = \begin{bmatrix}300 & 0 & 0 & 0 & 340 & 0\end{bmatrix}^T$. Compute the temperature and its gradient at the point P with the coordinates $x = 1.5$ and $y = 2.0$.

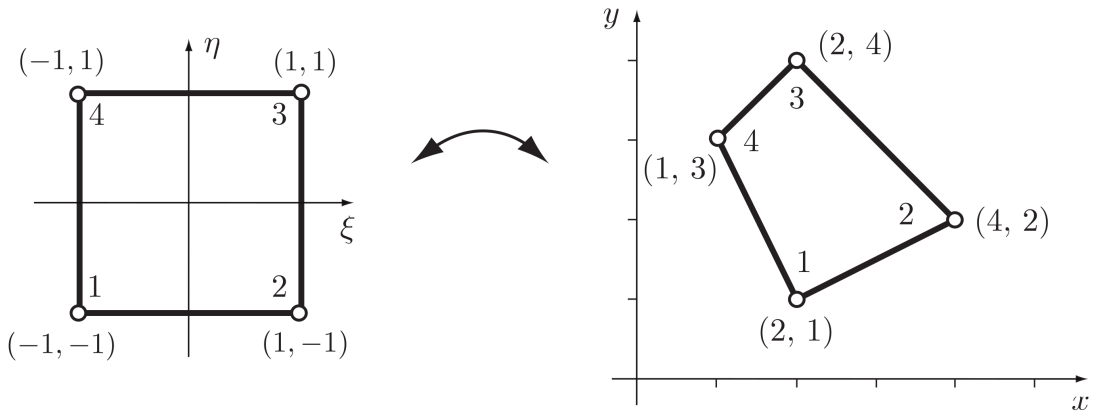


   Use the isoparametric shape functions provided in the data sheet.

2. For the isoparametric mapping shown below

   (a) Compute the x and y coordinates of the point $\xi = 0.5$, $\eta = 0.5$ in the physical domain.

   (b) Compute $\frac{\partial N_1}{\partial x}$ for the same point.



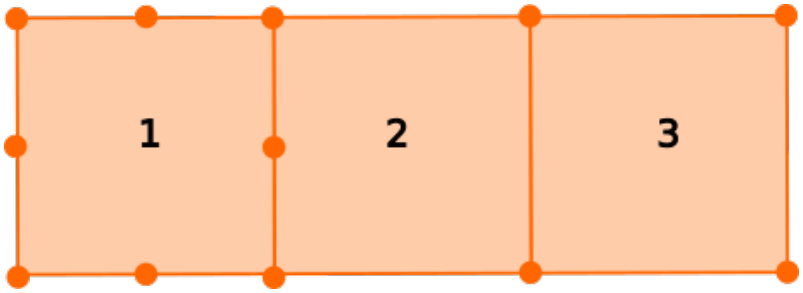
3. Evaluate the following three integrals using one, two and three-point Gauss integration. Compare your results with the results of analytic integration.

(a) $\int_{-1}^{+1}(3\xi^2 + 2\xi)d\xi$

(b) $\int_{-1}^{+1}\cos\xi d\xi$

(c) $\int_{0}^{3}(3x^2 + x)dx$

4. Consider the following quadrilateral mesh, sketch (1D is fine) the temperature distribution across the mesh and comment on the suitability of the mesh for finite element computation and where should they be used?



5. Develop a Python script for the Newton Raphson method to solve non-linear force-displacement relationships. Test and comment on the accuracy and efficiency of the NR to solve: $f = -2u^2 + 2u$, where u denotes the displacement and f refers to the internal force. *Hint: Use displacement control with u varying from 0 to 1.*

Datasheet

The shape functions for a 6-noded triangle element are:

$$
\begin{aligned}
N_1 &= 2(1-\xi-\eta)^2 - (1-\xi-\eta) \\
N_2 &= 2\xi^2 - \xi \\
N_3 &= 2\eta^2 - \eta \\
N_4 &= 4\xi(1-\xi-\eta) \\
N_5 &= 4\xi\eta \\
N_6 &= 4\eta(1-\xi-\eta)
\end{aligned}
$$