

Assignment 1: Stress equilibrium, compatibility and stiffness matrix
Assigned: 27th January 2019
Due: 8th February 2019

1. Prove that the following stress-field is a valid lower-bound solution.

$$\begin{aligned}\sigma_{xx} &= c_1 x^3 y - 2c_2 xy + c_3 y \\ \sigma_{yy} &= c_1 xy^3 - 2c_1 x^3 y \\ \sigma_{xy} &= -\frac{3}{2}c_1 x^2 y^2 + c_2 y^2 + \frac{1}{2}c_1 x^4 + c_4\end{aligned}$$

where c_1, c_2, c_3 and c_4 are constants.

2. Determine and describe the stress-state given by the following Airy stress functions:

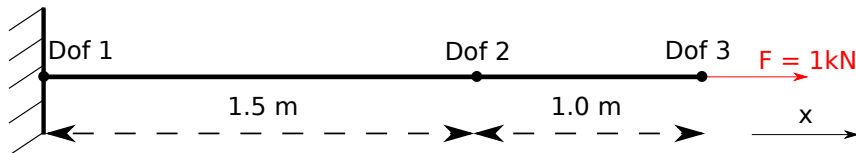
- (a) $\phi = Ay^2$
 (b) $\phi = Bxy$

Are these stresses valid for an isotropic elastic element? Note: please refer https://en.wikiversity.org/wiki/Airy_stress_function on how to determine cauchy stress components from an Airy function.

3. Determine whether the following strain fields are possible in a two-dimensional continuous body:

$$\begin{aligned}\text{(a)} \quad \varepsilon &= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^2) & c_1 xy \\ c_1 xy & c_2 y^2 \end{bmatrix} \\ \text{(b)} \quad \varepsilon &= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^3) & 3c_1 xy^2/2 \\ 3c_1 xy^2/2 & c_2 x^3 \end{bmatrix}\end{aligned}$$

4. Given the one-dimensional steel bar below, which is represented by two elements, determine the system's global stiffness matrix (\mathbf{K}). The bar has a cross sectional area (A) of $5 \times 10^{-4} \text{ m}^2$ and Young's modulus (E) of 20 GPa. Please note that the deformation of an elastic bar is governed as follows: $\delta = FL/AE$, where F is the applied force leading to a displacement of δ . Also, note that this bar can be characterized in one dimension only with just two degrees of freedom (i.e. displacement at each of the two nodes in the x direction only).



Assuming a horizontal force $F_x = 1 \text{ kN}$ at DOF 3, solve for displacements at all degrees of freedom. You are encouraged to use Jupyter notebooks to solve the linear system of equations.