CE394M: 1D-Finite Element Method

Krishna Kumar

University of Texas at Austin

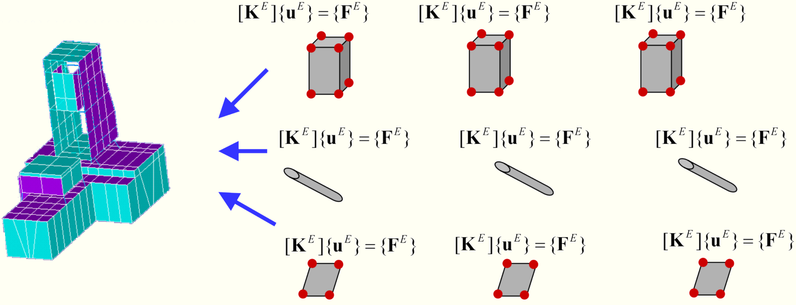
krishnak@utexas.edu

January 20, 2019

Overview

1. FEM workflow
2. 1D FEM

Finite Element Analysis



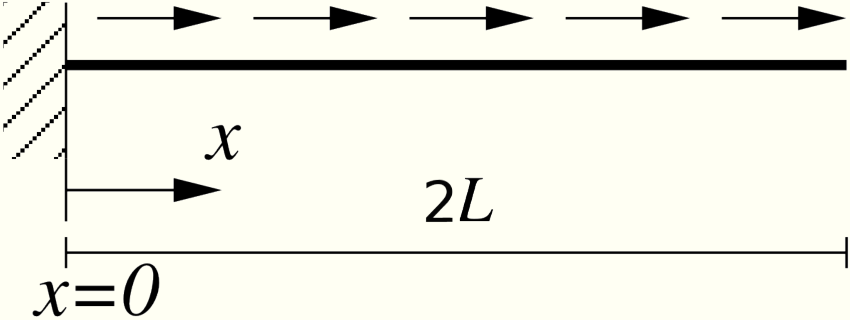
$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\} \longrightarrow \{\mathbf{u}\} = [\mathbf{K}]^{-1}\{\mathbf{F}\}$$

Finite Element Analysis

FEM is a systematic procedure for approximating differential equations. For any problem in any spatial dimension it follows the same steps:

1. Identify the equation of interest
2. Cast the equation of interest in a weak form
3. Select a finite element type
4. Construct the element matrix and vector
5. Assemble the global matrix and vector and apply boundary conditions
6. Solve the system of linear equations

1D Finite Element Analysis of a cantilever beam



1D cantilever beam

Assume L as unit length $L = 1$. Unit force $f = 1$.

What element should be used?

Linear elements

Qudrilateral element

1D discretization of a cantilever beam

1D FEM: Shape functions and derivatives

Shape function **N**:

$$\mathbf{N} = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} 1 - x & x \end{bmatrix}$$

B is the derivatives of the shape functions:

$$\mathbf{B} = \begin{bmatrix} \frac{dN_1(x)}{dx} & \frac{dN_2(x)}{dx} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

In matrix format: $u_h = \mathbf{N}\mathbf{a_e}$ and $\epsilon_h = \mathbf{B}\mathbf{a_e}$.

1D FEM: Stiffness and force

Element stiffness k_e:

$$k_e = \int \mathbf{B}^T EA\, B \,\mathrm{d}x = \int_0^L \begin{bmatrix} \frac{dN_1}{dx} \\ \frac{dN_2}{dx} \end{bmatrix} EA \begin{bmatrix} \frac{dN_1}{dx} & \frac{dN_2}{dx} \end{bmatrix} dx$$

$$k_e = EA \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Right-hand side vector b_e is:

$$b_e = \int N^T f \,\mathrm{d}x = \int_0^L \begin{bmatrix} -\frac{x}{L} + 1 \\ \frac{x}{L} \end{bmatrix}$$

$$b_e = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$