Assignment 1, COMP252, Winter 2024. Jan 9, 2024. Due Jan 18, 2024, 1 pm.

Exercise 1. A QUESTION RELATED TO FIBONACCI SEQUENCES. Consider a Fibonacci sequence started with $x_0 = 0$, $x_1 = 1$. For positive integers k and n, we would like to compute $x_n \bmod k$ and are using the RAM model of computation in which standard arithmetic operations, including "mod" take constant time. Describe how you would proceed in two cases: (1) $k = 627$, (2) $k = n$. In both cases, give your complexity in $O(\cdot)$ ("big oh") notation as a function of n.

Exercise 2. BIT COMPLEXITY OF AN ALGORITHM. In the bit model of computation, give an efficient algorithm for determining whether a given integer n is a perfect square, and determine its worst-case complexity in big oh notation as a function of n.

Exercise 3. DESIGN OF A DIVIDE-AND-CONQUER ALGORITHM. Assuming a RAM (uniform cost) model of computation, design a recursive divide-and-conquer style $O(n)$ worst-case time algorithm for the following problem. We are given an array $x[1], \ldots, x[n]$ of (possibly negative) integers, and are asked to find two indices $i \leq j$ such that $x[i] + \cdots + x[j]$ is maximal. Prove your claim.