

# A PRICING FORMULA FOR THE SHARE DISPENSER

## 1. LINEAR PRICING

Assume that a total of  $N$  shares are made available for sale, we can number them  $1, \dots, N$ . The first share will be sold at price  $p_{min}$ , the last one at  $p_{max}$  with linear interpolation inbetween. This means that the  $k$ -th share will be sold at price

$$P(k) = p_{min} + \frac{k(p_{max} - p_{min})}{N}.$$

The share dispenser smart contract needs to be able to quickly compute the cumulated price for buying shares  $m$  through  $l$ , which is easily possible as follows:

$$\begin{aligned} P(m, l) &= \sum_{k=m}^l P(k) \\ &= (l - m + 1)p_{min} + \frac{k(p_{max} - p_{min})}{N} \sum_{k=m}^l k \\ &= (l - m + 1)p_{min} + \frac{p_{max} - p_{min}}{2N} (l(l + 1) - m(m - 1)) \end{aligned}$$

This expression is quadratic in  $l$  which means that after fixing  $m$  as well as the cumulated price we can solve for  $l$ . This gives us the maximum number of shares that can be purchased starting from share number  $m$  given a fixed price.

In Solidity, the above can be implemented as follows (where  $m$  and  $l$  are called `first` and `last` respectively):

```
1 function helper(uint256 first, uint256 last) internal view returns
  (uint256) {
2   uint256 tempa = last.sub(first).add(1).mul(minPriceInXCHF);
3   uint256 tempb = maxPriceInXCHF.sub(minPriceInXCHF).div(
    initialNumberOfShares).div(2);
4   uint256 tempc = last.mul(last).add(last).add(first).sub(first.
    mul(first));
5   return tempb.mul(tempc).add(tempa);
6 }
```

Assuming a more complex price dependency a closed inverse formula is highly unlikely and therefore it might be more efficient to use a binary search type algorithm to find  $l$ .

## 2. THE AVAILABLE SUPPLY CAN EXCEED $N$

Furthermore, if a company decides to also provide XCHF to the share dispenser, a situation can occur where more than  $N$  share are available for purchase. No share should be sold below the minimum price. Assume there are a total of  $N + i$  shares available, then shares  $N + i$  through  $N + 1$  will be sold at price  $p_{min}$  and starting at share  $N$  the regular pricing formula takes over.

In Solidity, this can be implemented as follows (note that in the code  $N$  is called `initialNumberOfShares`):

```
1 function getCumulatedPrice(uint256 amount, uint256 supply) public
  view returns (uint256){
2   uint256 cumulatedPrice = 0;
3   if (supply <= initialNumberOfShares) {
4     uint256 first = initialNumberOfShares.sub(supply);
5     uint256 last = first.add(amount).sub(1);
6     cumulatedPrice = helper(first, last);
7   }
8
9   else if (supply.sub(amount) >= initialNumberOfShares) {
10    cumulatedPrice = minPriceInXCHF.mul(amount);
11  }
12
13  else {
14    cumulatedPrice = supply.sub(initialNumberOfShares).mul(
      minPriceInXCHF);
15    uint256 first = 0;
16    uint256 last = amount.sub(supply.sub(initialNumberOfShares)).
      add(1));
17    cumulatedPrice = cumulatedPrice.add(helper(first, last));
18  }
19
20  return cumulatedPrice;
21 }
```