

7. Given  $\vec{r}(t) = \langle \sqrt{t-1}, \frac{t}{2} \rangle$ .

(a) **Eliminate the parameter:**

$$x = \sqrt{t-1} \implies x^2 = t-1 \implies t = x^2 + 1$$

Substitute  $t$  into the equation for  $y$ :

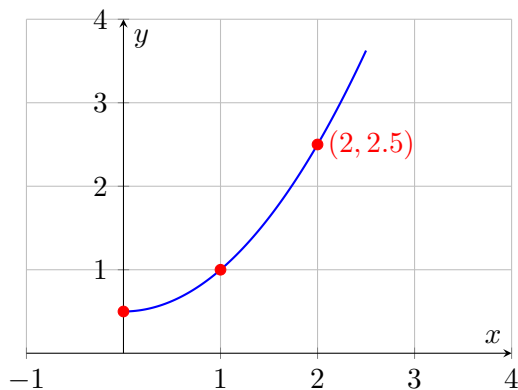
$$y = \frac{t}{2} = \frac{x^2 + 1}{2}$$

Since  $x = \sqrt{t-1}$ , we must have  $x \geq 0$ .

$$y = \frac{x^2 + 1}{2}, \quad x \geq 0$$

(b) **Sketch and Table:**

| $t$ | $y = t/2$ | $(x, y)$            |
|-----|-----------|---------------------|
| 1   | 1/2       | (0, 0.5)            |
| 2   | 1         | (1, 1)              |
| 3   | 3/2       | ( $\sqrt{2}$ , 1.5) |
| 5   | 5/2       | (2, 2.5)            |



8.  $\vec{r}_1(t) = \langle t-1, 3t-t^2 \rangle$  and  $\vec{r}_2(t) = \langle -t-1, \frac{1}{4}t^2 + 2t \rangle$ .

(a) **Find Collision (same  $t$ ):** Set  $x$ -components equal:

$$t-1 = -t-1 \implies 2t = 0 \implies t = 0$$

Check  $y$ -components at  $t = 0$ :

$$y_1(0) = 3(0) - 0^2 = 0$$

$$y_2(0) = \frac{1}{4}(0)^2 + 2(0) = 0$$

Since  $y_1 = y_2$ , a collision occurs at  $t = 0$ .

$$\text{Position: } \langle 0-1, 0 \rangle = \boxed{\langle -1, 0 \rangle \text{ at } t = 0}$$

(b) **Find Intersection (different times  $t_1, t_2$ ):** Set  $\vec{r}_1(t_1) = \vec{r}_2(t_2)$ .

$$x\text{-eq: } t_1 - 1 = -t_2 - 1 \implies t_1 = -t_2$$

Substitute  $t_1 = -t_2$  into  $y$ -equations:

$$3t_1 - t_1^2 = \frac{1}{4}t_2^2 + 2t_2$$

$$3(-t_2) - (-t_2)^2 = \frac{1}{4}t_2^2 + 2t_2$$

$$-3t_2 - t_2^2 = \frac{1}{4}t_2^2 + 2t_2$$

Move terms to one side:

$$\frac{5}{4}t_2^2 + 5t_2 = 0 \implies 5t_2\left(\frac{1}{4}t_2 + 1\right) = 0$$

Solutions:  $t_2 = 0$  (Collision found in part a) or  $\frac{1}{4}t_2 = -1 \implies t_2 = -4$ .

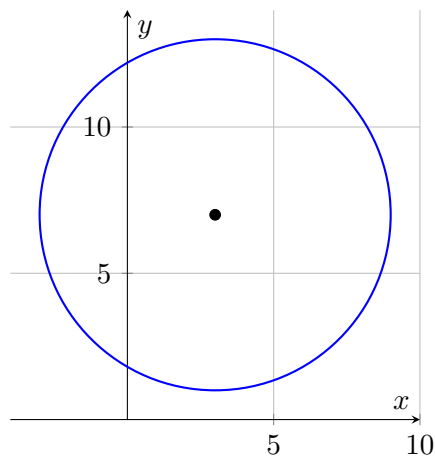
If  $t_2 = -4$ , then  $t_1 = -(-4) = 4$ .

Find point using  $t_1 = 4$ :

$$\vec{r}_1(4) = \langle 4 - 1, 3(4) - 4^2 \rangle = \langle 3, 12 - 16 \rangle$$

$$\boxed{(3, -4)}$$

9. Circle graph centered at  $(3, 7)$  with radius  $r = 6$  (from  $y = 1$  to  $y = 13$ ).



(a) **Cartesian Equation:**  $(x - a)^2 + (y - b)^2 = r^2$

$$\boxed{(x - 3)^2 + (y - 7)^2 = 36}$$

(b) **Parametrization:** Use  $x = r \cos t + a$  and  $y = r \sin t + b$ .

$$\boxed{\vec{r}(t) = \langle 6 \cos t + 3, 6 \sin t + 7 \rangle}$$

(c) **Verification:** Plug parametrized  $x, y$  into Cartesian equation:

$$((6 \cos t + 3) - 3)^2 + ((6 \sin t + 7) - 7)^2 = 36$$

$$(6 \cos t)^2 + (6 \sin t)^2 = 36$$

$$36 \cos^2 t + 36 \sin^2 t = 36(1) = 36 \quad \checkmark$$

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### 10. Parametrization Practice

(a) Circle, center  $(0, 0)$ , radius 3.

$$\langle 3 \cos t, 3 \sin t \rangle$$

(b) Circle, center  $(3, -2)$ , radius 2.

$$\langle 2 \cos t + 3, 2 \sin t - 2 \rangle$$

(c) Motion starting at  $(0, -1)$  moving to  $(1, 0)$ . We need  $\vec{r}(0) = \langle 0, -1 \rangle$  and  $\vec{r}(\pi/2) = \langle 1, 0 \rangle$ .

Check values:

$$\sin(0) = 0, \quad -\cos(0) = -1$$

$$\sin(\pi/2) = 1, \quad -\cos(\pi/2) = 0$$

Therefore:

$$\langle \sin t, -\cos t \rangle$$