11. Circular Motion Analysis.

(a) **Find $\vec{r}(t)$:**

- Center $(0, 0)$, Radius 2. Starts at $(2, 0)$.
- Clockwise (CW) rotation $\implies$ y component is negative sine.
- Speed = 3 m/s.

$$\text{Angular velocity } \omega = \frac{v}{r} = \frac{3}{2}.$$

$$\boxed{\vec{r}(t) = \left\langle 2\cos\left(\frac{3}{2}t\right), -2\sin\left(\frac{3}{2}t\right)\right\rangle}$$

(b) **Verify Speed:**

$$\vec{v}(t) = \vec{r}'(t) = \left\langle -3\sin\left(\frac{3}{2}t\right), -3\cos\left(\frac{3}{2}t\right)\right\rangle$$

$$\text{Speed } = ||\vec{v}(t)|| = \sqrt{9\sin^2\left(\frac{3}{2}t\right) + 9\cos^2\left(\frac{3}{2}t\right)} = \sqrt{9(1)} = \boxed{3 \text{ m/s}}$$

(c) **Find time t parallel to $\langle -2, 2\rangle$:** Set $\vec{v}(t) = k\langle -2, 2\rangle$.

$$\left\langle -3\sin\left(\frac{3}{2}t\right), -3\cos\left(\frac{3}{2}t\right)\right\rangle = \langle -2k, 2k\rangle$$

Divide y-component by x-component:

$$\frac{-3\cos\left(\frac{3}{2}t\right)}{-3\sin\left(\frac{3}{2}t\right)} = \frac{2k}{-2k} = -1 \implies \cot\left(\frac{3}{2}t\right) = -1 \implies \tan\left(\frac{3}{2}t\right) = -1$$

One solution for tangent is $\frac{3\pi}{4}$:

$$\frac{3}{2}t = \frac{3\pi}{4} \implies t = \frac{3\pi}{4} \cdot \frac{2}{3} = \boxed{\frac{\pi}{2}}$$

Check: $\vec{v}(\pi/2) = \langle -3\sin(3\pi/4), -3\cos(3\pi/4)\rangle = \langle \frac{-3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\rangle$ (Parallel to $\langle -1, 1\rangle$).

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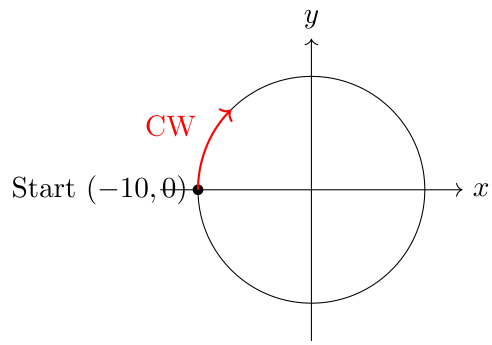
12. Design a Trajectory.

- Radius 10 circle, centered at origin.
- Starts at $(-10, 0)$.
- Moves Clockwise (CW) at 5 m/s.

Step 1: Find angular frequency.

$$\omega = \frac{v}{r} = \frac{5}{10} = \frac{1}{2}$$

Step 2: Determine signs for $(-10, 0)$ start and CW direction. Standard $\langle \cos, \sin\rangle$ starts at $(1, 0)$. To start at $(-10, 0)$, use negative cosine: $x = -10\cos(\dots)$. For CW from the left side (moving Up), y must become positive. Use positive sine: $y = 10\sin(\dots)$.



$$\boxed{\vec{r}(t) = \left\langle -10\cos\left(\frac{1}{2}t\right), 10\sin\left(\frac{1}{2}t\right) \right\rangle}$$

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13. Given $\vec{r}(t) = \langle t-2, t^2+1 \rangle$.

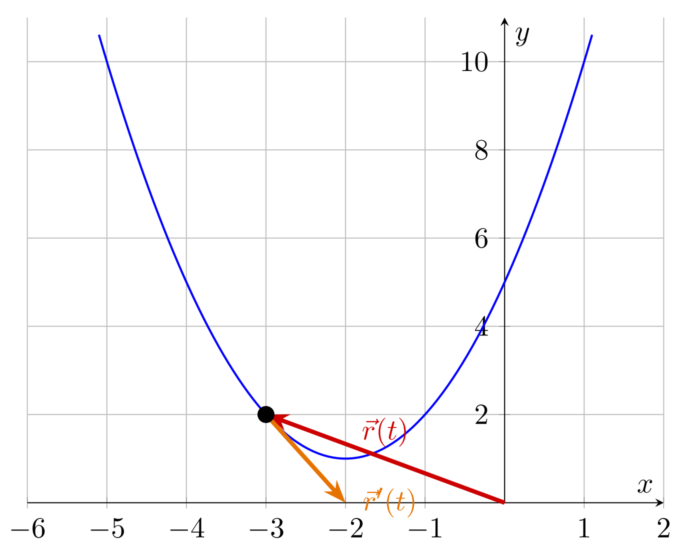
(a) **Table and Sketch:**

t	$\vec{r}(t)$	(x, y)
-3	$\langle -5, 10 \rangle$	$(-5, 10)$
-2	$\langle -4, 5 \rangle$	$(-4, 5)$
-1	$\langle -3, 2 \rangle$	$(-3, 2)$
0	$\langle -2, 1 \rangle$	$(-2, 1)$
1	$\langle -1, 2 \rangle$	$(-1, 2)$
2	$\langle 0, 5 \rangle$	$(0, 5)$
3	$\langle 1, 10 \rangle$	$(1, 10)$

(b) **Vectors at $t = -1$:**

$$\vec{r}(-1) = \langle -1-2, (-1)^2+1 \rangle = \boxed{\langle -3, 2 \rangle}$$

$$\vec{r}'(t) = \langle 1, 2t \rangle \implies \vec{r}'(-1) = \boxed{\langle 1, -2 \rangle}$$



14. Tenderheart Bear Problem.

- Position: $\vec{r}(t) = \langle t+1, t^2+2 \rangle$.
- Target: Funshine Bear at $(-1, 2)$.
- Find launch time a such that the tangent passes through target.

Step 1: Equation of tangent line at time a. Let hug reach target at time t.

$$\text{Line: } \vec{L}(t) = \vec{r}'(a)(t-a) + \vec{r}(a)$$

$$\langle -1, 2 \rangle = \langle 1, 2a \rangle (t-a) + \langle a+1, a^2+2 \rangle$$

Step 2: Solve components.

$$x : -1 = 1(t-a) + a + 1 \implies -1 = t + 1 \implies t = -2$$

The hug arrives at target at $t = -2$.

$$y : 2 = 2a(t-a) + a^2 + 2$$

Substitute $t = -2$:

$$0 = 2a(-2-a) + a^2$$

$$0 = -4a - 2a^2 + a^2 \implies -a^2 - 4a = 0$$

$$-a(a+4) = 0$$

Solutions: $a = 0$ or $a = -4$.

Step 3: Verify Causality. Launch time a must be before arrival time $t = -2$.

- If $a = 0$, $0 > -2$ (Impossible, arrival before launch).
- If $a = -4$, $-4 < -2$ (Valid).

$$\boxed{a = -4}$$