38. Work (Pumping Water).

Analysis: We need to calculate the work required to pump water out of a conical tank.

- **Radius ratio:** The radius r at height y is linear. given $\frac{r}{y} = \frac{2}{5} \implies r = \frac{2}{5}y$.
- **Volume of slice (dV):** A horizontal slice is a disk with area πr^2 and thickness dy.

$$dV = \pi \left(\frac{2}{5}y\right)^2 dy = \frac{4\pi}{25}y^2 dy$$

- **Mass of slice (dm):** $dm = \text{Density}(\rho) \times dV$.
- **Distance to lift:** The water at height y must be lifted to the outflow point. Based on the setup, this distance is $(y+7)$.

Integral Setup: Work is the integral of Force $\times$ Distance. Force is gravity acting on mass $(dm \cdot g)$. Limits are from the water surface levels: $y = 2$ to $y = 5$.

$$W = \int_2^5 \underbrace{(y+7)}_{\text{dist}} \cdot \underbrace{g}_{\text{gravity}} \cdot \underbrace{\rho \frac{4\pi}{25} y^2 dy}_{\text{mass}}$$

$$\boxed{W = \frac{4\pi\rho g}{25} \int_2^5 y^2(y+7)dy}$$

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39. Integration by Partial Fractions.

Evaluate $\int \frac{4x-2}{x^2-2x+1}dx$.

Step 1: Factor and Modify. Denominator is $(x-1)^2$. Split numerator to match $(x-1)$.

$$4x - 2 = 4(x-1) + 2$$

$$I = \int \frac{4(x-1)}{(x-1)^2}dx + \int \frac{2}{(x-1)^2}dx$$

Step 2: Integrate.

$$I = \int \frac{4}{x-1}dx + \int 2(x-1)^{-2}dx$$

$$\boxed{4\ln|x-1| - \frac{2}{x-1} + C}$$

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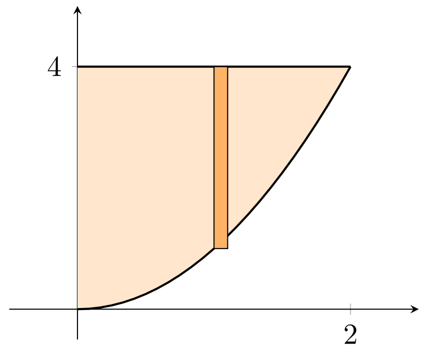
40. Volume of Revolution (Washer Method).

Region bounded by $y = x^2$ and $y = 4$. Rotated around x-axis ($y = 0$).



Setup:

- **Outer Radius (R):** Distance from axis ($y = 0$) to top ($y = 4$) $\implies R = 4$.
- **Inner Radius (r):** Distance from axis ($y = 0$) to bottom ($y = x^2$) $\implies r = x^2$.

$$V = \pi \int_0^2 (R^2 - r^2)dx = \boxed{\pi \int_0^2 (16 - x^4)dx}$$

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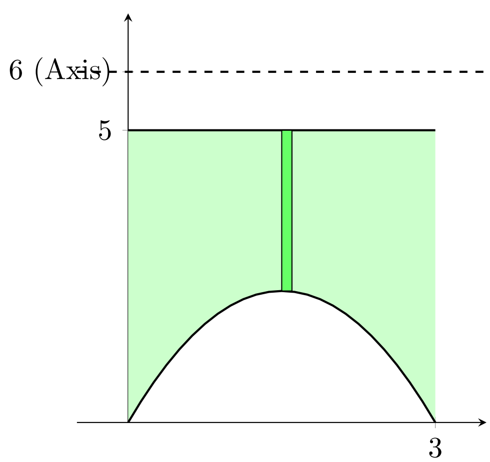
41. Volume of Revolution (Axis Shift).
Region bounded by $y = 3x - x^2$ and $y = 5$ ($x = 0$ to 3). Rotated around $y = 6$.



Setup: Axis of rotation is $y = 6$ (above the region).

- **Outer Radius (R):** Distance from axis ($y = 6$) to far curve ($y = 3x - x^2$).

$$R = 6 - (3x - x^2)$$

- **Inner Radius (r):** Distance from axis ($y = 6$) to near curve ($y = 5$).

$$r = 6 - 5 = 1$$

$$V = \pi \int_0^3 (R^2 - r^2)dx = \boxed{\pi \int_0^3 \left[(6 - 3x + x^2)^2 - 1\right] dx}$$