

42. Complex Number Arithmetic Polar Form.

Given: $z_1 = 1 + \sqrt{3}i$ and $z_2 = -1 - i$.

(a) **Draw vectors:** See graph below.

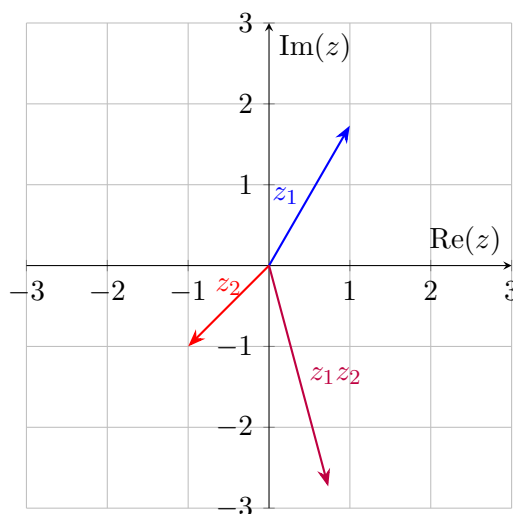
(b) **Direct Product:**

$$z_1 z_2 = (1 + \sqrt{3}i)(-1 - i) = -1 - i - \sqrt{3}i - \sqrt{3}i^2$$

$$= (-1 + \sqrt{3}) + i(-1 - \sqrt{3})$$

$$\boxed{(\sqrt{3} - 1) - (1 + \sqrt{3})i} \approx 0.732 - 2.732i$$

(c) **Graph of z_1, z_2 and Product:**



(d) **Convert to Polar Form:**

$$\bullet \ z_1: |z_1| = 2, \theta_1 = \pi/3 \implies \boxed{2e^{i\pi/3}}$$

$$\bullet \ z_2: |z_2| = \sqrt{2}, \theta_2 = 5\pi/4 \implies \boxed{\sqrt{2}e^{i5\pi/4}}$$

(e) **Product in Polar Form:**

$$z_1 z_2 = (2\sqrt{2})e^{i(\pi/3+5\pi/4)} = \boxed{2\sqrt{2}e^{i19\pi/12}}$$

(f) **Convert back:** Matches part (b).

43. Complex Conjugates.

(a) $\bar{z} = \boxed{2 + 3i}$

(b) $z + \bar{z} = 4$ (Purely Real).

(c) $z \cdot \bar{z} = 13$ (Purely Real, $|z|^2$).

(d) Geometric Relationship: Reflection across the Real axis.

44. Powers of Complex Numbers.

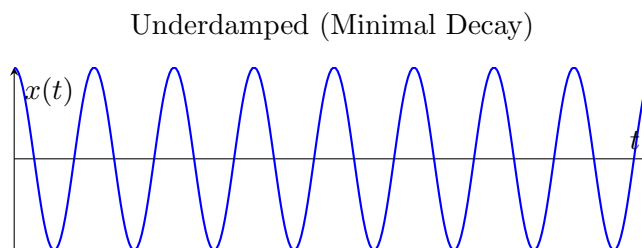
(a) $i^{95} = i^{92} \cdot i^3 = (1) \cdot (-i) = \boxed{-i}$

(b) $(1 + i)^{100} = (\sqrt{2}e^{i\pi/4})^{100} = 2^{50}e^{i25\pi} = 2^{50}(\cos \pi + i \sin \pi) = \boxed{-2^{50}}$

45. Spring/Mass/Damper System.

(a) **Equation of Motion:** $ma = -kx - cx' \implies \boxed{mx'' + cx' + kx = 0}$

(b) **Small c (Underdamped): Meaning:** Very weak friction. The mass oscillates with minimal decay (steady oscillation).



(c) **Large c (Heavy Damping/Overdamped): Meaning:** Strong friction. Drops fast, briefly overshoots equilibrium, then settles to zero.

Heavy Damping (Fast Drop with Overshoot)

