

7. Given $\vec{r}(t) = \langle \sqrt{t-1}, \frac{t}{2} \rangle$.

(a) **Eliminate the parameter:**

$$x = \sqrt{t-1} \implies x^2 = t-1 \implies t = x^2 + 1$$

Substitute t into the equation for y :

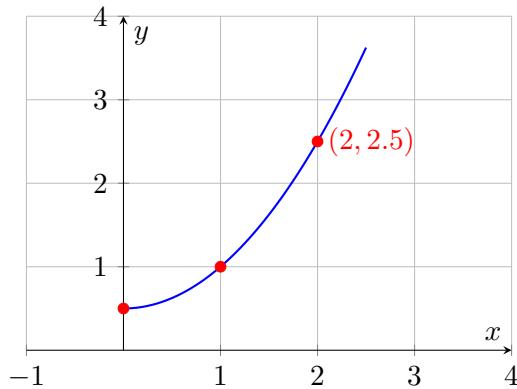
$$y = \frac{t}{2} = \frac{x^2 + 1}{2}$$

Since $x = \sqrt{t-1}$, we must have $x \geq 0$.

$$y = \frac{x^2 + 1}{2}, \quad x \geq 0$$

(b) **Sketch and Table:**

t	$y = t/2$	(x, y)
1	1/2	(0, 0.5)
2	1	(1, 1)
3	3/2	($\sqrt{2}$, 1.5)
5	5/2	(2, 2.5)



8. $\vec{r}_1(t) = \langle t-1, 3t-t^2 \rangle$ and $\vec{r}_2(t) = \langle -t-1, \frac{1}{4}t^2+2t \rangle$.

(a) **Find Collision (same t):** Set x -components equal:

$$t-1 = -t-1 \implies 2t = 0 \implies t = 0$$

Check y -components at $t = 0$:

$$y_1(0) = 3(0) - 0^2 = 0$$

$$y_2(0) = \frac{1}{4}(0)^2 + 2(0) = 0$$

Since $y_1 = y_2$, a collision occurs at $t = 0$.

Position: $\langle 0-1, 0 \rangle = \boxed{(-1, 0) \text{ at } t=0}$

(b) **Find Intersection (different times t_1, t_2):** Set $\vec{r}_1(t_1) = \vec{r}_2(t_2)$.

$$x\text{-eq: } t_1 - 1 = -t_2 - 1 \implies t_1 = -t_2$$

Substitute $t_1 = -t_2$ into y -equations:

$$\begin{aligned} 3t_1 - t_1^2 &= \frac{1}{4}t_2^2 + 2t_2 \\ 3(-t_2) - (-t_2)^2 &= \frac{1}{4}t_2^2 + 2t_2 \\ -3t_2 - t_2^2 &= \frac{1}{4}t_2^2 + 2t_2 \end{aligned}$$

Move terms to one side:

$$\frac{5}{4}t_2^2 + 5t_2 = 0 \implies 5t_2(\frac{1}{4}t_2 + 1) = 0$$

Solutions: $t_2 = 0$ (Collision found in part a) or $\frac{1}{4}t_2 = -1 \implies t_2 = -4$.

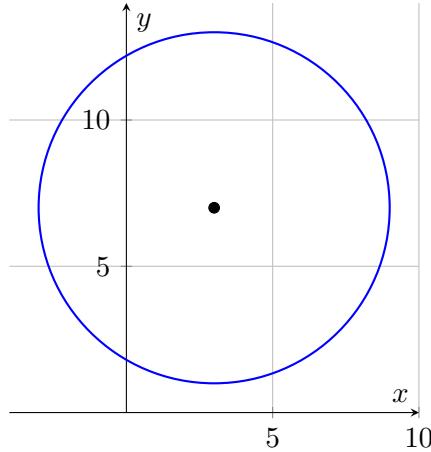
If $t_2 = -4$, then $t_1 = -(-4) = 4$.

Find point using $t_1 = 4$:

$$\vec{r}_1(4) = \langle 4 - 1, 3(4) - 4^2 \rangle = \langle 3, 12 - 16 \rangle$$

$(3, -4)$

9. Circle graph centered at $(3, 7)$ with radius $r = 6$ (from $y = 1$ to $y = 13$).



(a) **Cartesian Equation:** $(x - a)^2 + (y - b)^2 = r^2$

$$(x - 3)^2 + (y - 7)^2 = 36$$

(b) **Parametrization:** Use $x = r \cos t + a$ and $y = r \sin t + b$.

$$\vec{r}(t) = \langle 6 \cos t + 3, 6 \sin t + 7 \rangle$$

(c) **Verification:** Plug parametrized x, y into Cartesian equation:

$$((6 \cos t + 3) - 3)^2 + ((6 \sin t + 7) - 7)^2 = 36$$

$$(6 \cos t)^2 + (6 \sin t)^2 = 36$$

$$36 \cos^2 t + 36 \sin^2 t = 36(1) = 36 \quad \checkmark$$

10. Parametrization Practice

(a) Circle, center $(0, 0)$, radius 3.

$$\boxed{\langle 3 \cos t, 3 \sin t \rangle}$$

(b) Circle, center $(3, -2)$, radius 2.

$$\boxed{\langle 2 \cos t + 3, 2 \sin t - 2 \rangle}$$

(c) Motion starting at $(0, -1)$ moving to $(1, 0)$. We need $\vec{r}(0) = \langle 0, -1 \rangle$ and $\vec{r}(\pi/2) = \langle 1, 0 \rangle$.

Check values:

$$\sin(0) = 0, \quad -\cos(0) = -1$$

$$\sin(\pi/2) = 1, \quad -\cos(\pi/2) = 0$$

Therefore:

$$\boxed{\langle \sin t, -\cos t \rangle}$$