28. Definite Integral using Substitution.

Evaluate $\int_0^5 \frac{1}{(x+1)^3}dx$.

Step 1: Substitution. Let $u = x + 1$. Then $du = dx$.

Step 2: Change Limits.

- Lower: $x = 0 \implies u = 1$.
- Upper: $x = 5 \implies u = 6$.

Step 3: Evaluate.

$$\int_1^6 u^{-3}du = \left[\frac{u^{-2}}{-2}\right]_1^6 = \left[-\frac{1}{2u^2}\right]_1^6$$

$$= -\frac{1}{2(36)} - \left(-\frac{1}{2(1)}\right) = -\frac{1}{72} + \frac{1}{2} = \frac{36}{72} - \frac{1}{72}$$

$$\boxed{\frac{35}{72}}$$

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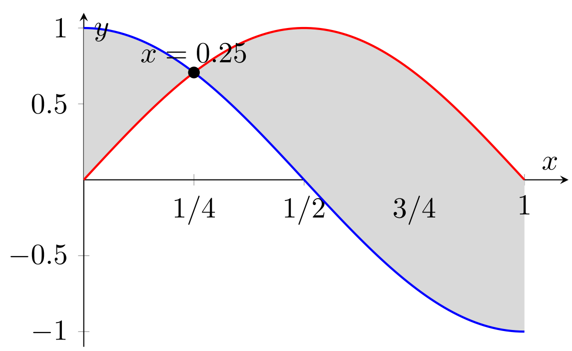
29. Area Between Curves.

Find the area between $y = \sin(\pi x)$ and $y = \cos(\pi x)$ for $0 \le x \le 1$.



Step 1: Find Intersection. $\sin(\pi x) = \cos(\pi x) \implies \tan(\pi x) = 1 \implies \pi x = \frac{\pi}{4} \implies x = \frac{1}{4} = 0.25$.

Step 2: Split Integral. The top curve switches at $x = 0.25$.

$$A = \int_0^{0.25} [\cos(\pi x) - \sin(\pi x)]dx + \int_{0.25}^1 [\sin(\pi x) - \cos(\pi x)]dx$$

Step 3: Evaluate. Antiderivative is $\frac{1}{\pi}[\sin(\pi x) + \cos(\pi x)]$ for the first part, and $\frac{1}{\pi}[-\cos(\pi x) - \sin(\pi x)]$ for the second.

$$= \frac{1}{\pi}\left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1)\right] + \frac{1}{\pi}\left[(-(-1) - 0) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right]$$

$$= \frac{1}{\pi}[\sqrt{2} - 1] + \frac{1}{\pi}[1 + \sqrt{2}] = \frac{2\sqrt{2}}{\pi}$$

$$\boxed{\frac{2\sqrt{2}}{\pi}}$$

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30. Net Change Theorem (Water Volume).
Rate $r(t) = 20 + 0.1t^2$ L/min over 10 minutes. Initial volume = 200 L.

$$V_{total} = V_{initial} + \int_0^{10} r(t)dt$$

$$V_{total} = 200 + \int_0^{10} \left(20 + \frac{1}{10}t^2\right) dt$$

$$= 200 + \left[20t + \frac{1}{30}t^3\right]_0^{10}$$

$$= 200 + \left(20(10) + \frac{1000}{3}\right) - 0$$

$$= 200 + 200 + 333.33 = \frac{1300}{3}$$

$$\boxed{\approx 433.33 \text{ L}}$$

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31. Basic Integrals Power Rule.

(a) $\int \frac{1}{x^3}dx = \int x^{-3}dx = \frac{x^{-2}}{-2} + C = \boxed{-\frac{1}{2x^2} + C}$

(b) $\int \frac{1}{\sqrt{x}}dx = \int x^{-1/2}dx = \frac{x^{1/2}}{1/2} + C = \boxed{2\sqrt{x} + C}$

(c) $\int \sqrt{x}dx = \int x^{1/2}dx = \frac{x^{3/2}}{3/2} + C = \boxed{\frac{2}{3}x^{3/2} + C}$

(d) $\int \frac{1}{x}dx = \int x^{-1}dx$. **Power rule does not apply here.**

$$\boxed{\ln|x| + C}$$

Note: The Power Rule $\int x^n dx = \frac{x^{n+1}}{n+1}$ works for all $n \neq -1$.

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32. Algebra Simplification before Integration.
Evaluate $\int \frac{x^3-2}{\sqrt{x}}dx$.
Step 1: Split the fraction.

$$\int \left(\frac{x^3}{x^{1/2}} - \frac{2}{x^{1/2}}\right) dx = \int (x^{5/2} - 2x^{-1/2})dx$$

Step 2: Integrate.

$$= \frac{x^{7/2}}{7/2} - 2\frac{x^{1/2}}{1/2} + C$$

$$\boxed{\frac{2}{7}x^{7/2} - 4\sqrt{x} + C}$$

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33. Properties of Definite Integrals.

Given $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$. Find $\int_0^9 [2f(x) + 3g(x)]dx$.

Using linearity properties:

$$= 2\int_0^9 f(x)dx + 3\int_0^9 g(x)dx$$

$$= 2(37) + 3(16)$$

$$= 74 + 48$$

$$\boxed{122}$$