10. Projectile Motion Analysis. Ball launched from $(0, 10)$, speed 12 m/s, angle θ.

(a) **Intuition:** The angle for maximum range is typically 45° (assuming level ground and start/end at same height).

(b) **Find $\vec{r}(t)$:** Acceleration $\vec{a}(t) = \langle 0, -9.8 \rangle$. Integrate to find velocity:

$$\vec{v}(t) = \langle 12\cos\theta, 12\sin\theta - 9.8t \rangle$$

Integrate to find position (starting at $\langle 0, 10 \rangle$):

$$\vec{r}(t) = \langle 12\cos\theta \cdot t, 10 + 12\sin\theta \cdot t - 4.9t^2 \rangle$$

(c) **Compare Horizontal Distance ($\theta = 45°$ vs 30°):** Find impact time when $y(t) = 0$.

Case 1: $\theta = 45°$

$$10 + 12\sin(45°)t - 4.9t^2 = 0 \implies 10 + 6\sqrt{2}t - 4.9t^2 = 0$$

Using quadratic formula: $t \approx 2.54$ s.

$$x(2.54) = 12\cos(45°)(2.54) = 6\sqrt{2}(2.54) \approx \boxed{21.6 \text{ m}}$$

Case 2: $\theta = 30°$

$$10 + 12\sin(30°)t - 4.9t^2 = 0 \implies 10 + 6t - 4.9t^2 = 0$$

Using quadratic formula: $t \approx 2.17$ s.

$$x(2.17) = 12\cos(30°)(2.17) = 6\sqrt{3}(2.17) \approx \boxed{22.5 \text{ m}}$$

(d) **Conclusion:** The ball travels further at 30°. The intuition of 45° is only strictly true when the launch and landing heights are equal. Since we start at 10 m and land at 0 m, a lower angle helps maintain horizontal velocity.

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14. Exact Values (Triangle Method).

c) **Calculate $\cos(\arcsin(1/2))$:** Let $\theta = \arcsin(1/2)$. Then $\sin\theta = 1/2$ and $\theta = \pi/6$.

$$\cos(\pi/6) = \boxed{\frac{\sqrt{3}}{2}}$$

d) **Calculate $\sin(\arctan(7/2))$:** Let $\theta = \arctan(7/2)$. This corresponds to a right triangle with opposite $= 7$ and adjacent $= 2$.

$$\text{Hypotenuse} = \sqrt{7^2 + 2^2} = \sqrt{49 + 4} = \sqrt{53}$$

$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \boxed{\frac{7}{\sqrt{53}}}$$

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16. Exact Values (Quadrants).

(a) **Evaluate** $\arcsin(\sin(13\pi/12))$**:** $13\pi/12$ is in Quadrant 3 (sine is negative). Range of arcsin is $[-\pi/2, \pi/2]$ (Quadrants 1 and 4). Find equivalent angle in Q4: Reference angle is $\pi/12$.

$$\boxed{-\frac{\pi}{12}}$$

(b) **Evaluate** $\arccos(\cos(-5\pi/8))$**:** $-5\pi/8$ is in Quadrant 3 (cosine is negative). Range of arccos is $[0, \pi]$ (Quadrants 1 and 2). Find equivalent angle in Q2 that shares the same cosine value. $\cos(-x) = \cos(x)$, so $\cos(-5\pi/8) = \cos(5\pi/8)$. Since $5\pi/8$ is in Q2 (within range), the answer is:

$$\boxed{\frac{5\pi}{8}}$$

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17. Derivatives.

(a) **Function:** $f(s) = \frac{\arctan(s)}{4^s} = \arctan(s) \cdot 4^{-s}$.

Product Rule:

$$f'(s) = \frac{d}{ds}(\arctan s) \cdot 4^{-s} + \arctan(s) \cdot \frac{d}{ds}(4^{-s})$$

$$f'(s) = \frac{1}{1+s^2}4^{-s} + \arctan(s)(4^{-s}\ln 4 \cdot (-1))$$

$$\boxed{f'(s) = 4^{-s}\left(\frac{1}{1+s^2} - \arctan(s)\ln 4\right)}$$

(b) **Vector Function:** $\vec{r}(t) = \left\langle \frac{\arccos(t)}{e^{2t}}, 10^{\sqrt[3]{t}} \right\rangle$.

x-component (Product/Quotient Rule):

$$\frac{d}{dt}(\arccos(t)e^{-2t}) = \frac{-1}{\sqrt{1-t^2}}e^{-2t} + \arccos(t)e^{-2t}(-2)$$

y-component (Chain Rule):

$$\frac{d}{dt}(10^{t^{1/3}}) = 10^{t^{1/3}}\ln(10) \cdot \frac{1}{3}t^{-2/3}$$

$$\boxed{\vec{r}'(t) = \left\langle e^{-2t}\left(\frac{-1}{\sqrt{1-t^2}} - 2\arccos(t)\right), \frac{\ln 10}{3}t^{-2/3}10^{\sqrt[3]{t}} \right\rangle}$$

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18. Vector Kinematics.

(a) **Find Position Vector $\vec{r}(t)$:** Given $\vec{v}(t) = \vec{r}'(t) = \langle -9t + 10, -9.8t + 6 \rangle$ and $\vec{r}(0) = \langle 0, 0 \rangle$. Integrate each component:

$$x(t) = \int (-9t + 10)dt = -\frac{9}{2}t^2 + 10t + C_1 \implies -4.5t^2 + 10t$$

$$y(t) = \int (-9.8t + 6)dt = -\frac{9.8}{2}t^2 + 6t + C_2 \implies -4.9t^2 + 6t$$

$$\boxed{\vec{r}(t) = \langle 10t - 4.5t^2, 6t - 4.9t^2 \rangle}$$

(b) **Hit the Ground:** Set $y(t) = 0$:

$$6t - 4.9t^2 = 0 \implies t(6 - 4.9t) = 0$$

$t = 0$ is the start. The ball hits the ground at:

$$t = \frac{6}{4.9} \approx \boxed{1.22 \text{ s}}$$

(c) **Acceleration Vector:** Differentiate velocity:

$$\vec{a}(t) = \vec{v}'(t) = \boxed{\langle -9, -9.8 \rangle}$$

(d) **Magnitude of Gravity:**

$$||\vec{a}|| = \sqrt{(-9)^2 + (-9.8)^2} = \sqrt{81 + 96.04} = \sqrt{177.04}$$

$$\boxed{13.3 \text{ m/s}^2}$$

(Note: This is significantly higher than Earth's gravity, roughly matching Neptune).