24. Riemann Sums (Oil Leakage).

We are given a table of leakage rates $R(t)$ taken every 3 hours.

- Time points: $t = 0, 3, 6, 9, 12$.
- Rates: $114, 159, 235, 295, 404$ liters/h.
- Interval width (Δt): 3 hours.

(a) **Lower Estimate:** Notice that the leakage rate is **strictly increasing** (the numbers only go up). To get a lower estimate (underestimate), we use the leakage rate at the **beginning** of each 3-hour interval (Left Riemann Sum).

$$\text{Volume} \approx \Delta t \cdot [R(0) + R(3) + R(6) + R(9)]$$
$$\text{Volume} \approx 3 \cdot [114 + 159 + 235 + 295]$$
$$= 3 \cdot [803]$$
$$\boxed{2409 \text{ liters}}$$

(b) **Upper Estimate:** To get an upper estimate (overestimate) for an increasing function, we use the leakage rate at the **end** of each 3-hour interval (Right Riemann Sum).

$$\text{Volume} \approx \Delta t \cdot [R(3) + R(6) + R(9) + R(12)]$$
$$\text{Volume} \approx 3 \cdot [159 + 235 + 295 + 404]$$
$$= 3 \cdot [1093]$$
$$\boxed{3279 \text{ liters}}$$

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25. Definite Integral as Area.

Find the exact value of $\int_{-3}^{0}(1 + \sqrt{9 - x^2})dx$.

Instead of finding an antiderivative, we interpret this definite integral as the geometric area under the curve.

Step 1: Split the integral using linearity properties.

$$\int_{-3}^{0} 1\, dx + \int_{-3}^{0} \sqrt{9 - x^2}\, dx$$

Step 2: Analyze the first part (Rectangle). The function $y = 1$ is a horizontal line. Integrating from $x = -3$ to $x = 0$ creates a rectangle with width 3 and height 1.

$$\text{Area}_1 = \text{width} \times \text{height} = 3 \times 1 = 3$$

Step 3: Analyze the second part (Quarter Circle). The equation $y = \sqrt{9 - x^2}$ represents the upper half of a circle with radius $r = \sqrt{9} = 3$ (since $x^2 + y^2 = 9$). The limits of integration are from $x = -3$ (left edge) to $x = 0$ (center). This region represents the **top-left quarter** of the circle.

$$\text{Area}_2 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}$$

Step 4: Combine the areas.

$$\boxed{3 + \frac{9\pi}{4}}$$

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26. Riemann Sums (Car Travel).
We have velocity data tracked every 0.2 hours ($\Delta t = 0.2$).

(a) **Estimate Distance (Right Sum):** We calculate distance as Velocity $\times$ Time. Using a Right Sum means for each interval, we use the velocity recorded at the end of that interval.

Intervals end at $t = 0.2, 0.4, 0.6, 0.8, 1.0$. Corresponding velocities: $30, 30, 70, 90, 90$ km/hr.

$$\begin{aligned} \text{Total Distance} &\approx \Delta t[v(0.2) + v(0.4) + v(0.6) + v(0.8) + v(1.0)] \\ &= 0.2[30 + 30 + 70 + 90 + 90] \\ &= 0.2[310] \\ &\boxed{62 \text{ km}} \end{aligned}$$

(b) **Estimate Gas Consumption:** Gas consumption is based on efficiency (Liters per 100km) at specific speeds. We must calculate the gas used for *each* interval separately.

Formula: Gas $= \frac{\text{Distance (km)}}{100} \times$ Efficiency.

- **Interval 1 ($t = 0$ to 0.2):** End velocity is 30 km/h. Distance $= 0.2 \times 30 = 6$ km. Efficiency at 30 km/h is 15. Gas $= \frac{6}{100} \times 15 = 0.90$ L.
- **Interval 2 ($t = 0.2$ to 0.4):** End velocity is 30 km/h. Distance $= 0.2 \times 30 = 6$ km. Efficiency is 15. Gas $= \frac{6}{100} \times 15 = 0.90$ L.
- **Interval 3 ($t = 0.4$ to 0.6):** End velocity is 70 km/h. Distance $= 0.2 \times 70 = 14$ km. Efficiency at 70 km/h is 7. Gas $= \frac{14}{100} \times 7 = 0.98$ L.
- **Interval 4 ($t = 0.6$ to 0.8):** End velocity is 90 km/h. Distance $= 0.2 \times 90 = 18$ km. Efficiency at 90 km/h is 8. Gas $= \frac{18}{100} \times 8 = 1.44$ L.
- **Interval 5 ($t = 0.8$ to 1.0):** End velocity is 90 km/h. Distance $= 0.2 \times 90 = 18$ km. Efficiency is 8. Gas $= 1.44$ L.

$$\text{Total Gas} = 0.90 + 0.90 + 0.98 + 1.44 + 1.44 = \boxed{5.66 \text{ Liters}}$$

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27. Trapezoidal Rule.
Estimate the integral $I = \int_0^{\pi/2} \cos(x)dx$ using $n = 4$ intervals.
Step 1: Determine interval width (Δx).

$$\Delta x = \frac{b-a}{n} = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$$

Step 2: Identify the x-coordinates. We start at 0 and add $\pi/8$ each step:

$$x_0 = 0, \quad x_1 = \frac{\pi}{8}, \quad x_2 = \frac{\pi}{4}, \quad x_3 = \frac{3\pi}{8}, \quad x_4 = \frac{\pi}{2}$$

Step 3: Apply the Trapezoidal Formula. The formula takes the average of the outer points and the full value of interior points:

$$T_4 = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$$

$$T_4 = \frac{\pi/8}{2}\left[\cos(0) + 2\cos\left(\frac{\pi}{8}\right) + 2\cos\left(\frac{\pi}{4}\right) + 2\cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{\pi}{2}\right)\right]$$

Step 4: Compute values.

$$\cos(0) = 1, \quad \cos(\pi/2) = 0$$

$$T_4 = \frac{\pi}{16}\left[1 + 2(0.9239) + 2(0.7071) + 2(0.3827) + 0\right]$$

$$T_4 \approx \frac{\pi}{16}[1 + 1.8478 + 1.4142 + 0.7654]$$

$$T_4 \approx \frac{\pi}{16}[5.0274] \approx \boxed{0.987}$$

Step 5: Is this an Underestimate or Overestimate? We examine the concavity of the function $y = \cos(x)$ on the interval $[0, \pi/2]$. Since $f''(x) = -\cos(x)$, the second derivative is negative in this region. This means the graph is **concave down** (shaped like a frown). When you draw straight trapezoid lines under a concave down curve, the lines sit **below** the actual curve. Therefore, this is an $\boxed{\text{Underestimate}}$.