

# Exercises 1: Probability

1. A standard pack of 52 playing cards has 4 suits: clubs, diamonds, hearts, and spades, each with 13 cards. One card is drawn from the full pack at random, then replaced in the pack. Then a second card is drawn from the pack.
  - (a) Calculate the probability that both cards are clubs.
  - (b) Calculate the probability that at least one of the cards selected is a club.
  - (c) Calculate the probability that neither of the cards selected is a club.
2. A short network link consists of two sections,  $a$  and  $b$ , in series as shown below.



The link functions only if both  $a$  and  $b$  function. The two sections are independent in that the functionality of one does not affect the functionality of the other. The probability that  $a$  continues to be functional during a particular time period is 0.9. The probability that  $b$  continues to be functional during a particular time period is 0.95. Determine the probability that the link continues to function during a particular time period.

3. A short network link consists of two sections,  $c$  and  $d$ , in parallel as shown below.



The link functions if either  $c$  or  $d$  (or both) function. The two sections are independent in that the functionality of one does not affect the functionality of the other. Consider the following statements.

- $C$ : ' $c$  continues to function during a particular time period'.
- $D$ : ' $d$  continues to function during a particular time period'.

The probability that  $c$  continues to function during a particular time period is 0.95. The corresponding probability for  $d$  is 0.97.

- (a) Determine the probability that the link continues to function during a particular time period.
  - (b) Using the negation of each of the statements  $C$  and  $D$  define a logical statement which specifies the failure of the link.
  - (c) Determine the probability of link failure during a particular time period.
4. A short network link consists of three sections,  $a$ ,  $b$ , and  $c$ , in series as shown below.



The link functions only if all three of  $a$ ,  $b$ , and  $c$  function. The three sections are independent in that the functionality of one does not affect the functionality of any other. The probability that  $a$  continues to be functional during a particular time period is 0.95. The corresponding probabilities for  $b$  and  $c$  are 0.96 and 0.91, respectively. Determine the probability that the link continues to function during a particular time period.

5. For more questions like the network link questions above, try the following NUMBAS test: **Test Yourself** Visit the URL below to try a numbas

exam:

<https://numbas.mathcentre.ac.uk/question/168927/link-probabilities/>

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6. The following Venn diagram represents the occurrence of two events  $A$  and  $B$ . The outer rectangle is the universal set and the two circular regions represent  $A$  and  $B$  as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.

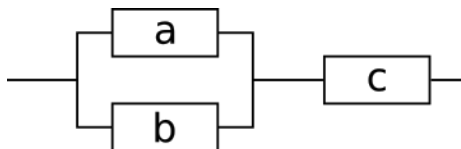


- (a) Determine  $P(A)$  and  $P(B)$ .
- (b) Determine  $P(B|A)$  and  $P(A|B)$ .
7. The following Venn diagram represents the occurrence of two events  $A$  and  $B$ . The outer rectangle is the universal set and the two circular regions represent  $A$  and  $B$  as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.



- (a) Determine  $P(A)$  and  $P(B)$ .
- (b) Determine  $P(B|A)$  and  $P(A|B)$ .
- (c) Determine  $P(\neg B|A)$  and compare it with  $P(B|A)$ .
- (d) Determine  $P(B|\neg A)$ .
- (e) Determine  $P(\neg B|\neg A)$  and compare it with  $P(B|\neg A)$ .
- (f) Determine  $P(\neg A|\neg B)$ .
8. A short network link consists of three sections  $a$ ,  $b$ , and  $c$  as shown below.

The link functions if  $c$  functions and either  $a$  or  $b$  (or both) function. All sections are independent in the sense that the functionality of one does



not affect the functionality of any other. The probability that  $a$  continues to function during a particular time period is 0.94. The corresponding probability for  $b$  is 0.92 and for  $c$  is 0.96. Statements for the functionality of each part of the link are given below:

- $A$ : ' $a$  continues to function during a particular time period'.
- $B$ : ' $b$  continues to function during a particular time period'.
- $C$ : ' $c$  continues to function during a particular time period'.

The link will function if  $(A \vee B) \wedge C = (A + B) \cdot C$  is **True**.

(a) Calculate  $P(A \vee B)$  and then determine  $P((A \vee B) \wedge C)$ .

9. The following NUMBAS test will allow you to practice more questions about conditional probability: **Test Yourself** Visit the URL below to

try a numbas exam:

<https://numbas.mathcentre.ac.uk/question/169052/conditional-probability/>

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## Answers

1. A normal pack of 52 playing cards has 4 suits: clubs, diamonds, hearts, and spades, each with 13 cards. One card is drawn from the full pack at random, then replaced in the pack. Then a second card is drawn from the pack. Let  $C$  denote the event that a club is drawn.

(a) Both are clubs:  $P(C \wedge C) = \frac{13}{52} \times \frac{13}{52} = 0.00625$ .

(b) To calculate the probability that at least one of the cards is a club, we can find the following probabilities:

- First card is a club, second card is a club:  $P(C \wedge C) = \frac{13}{52} \times \frac{13}{52} = 0.00625$ ;
- First card is not a club, second card is a club:  $P(\neg C \wedge C) = \frac{39}{52} \times \frac{13}{52} = 0.1875$ ;
- First card is a club, second card is not a club:  $P(C \wedge \neg C) = \frac{13}{52} \times \frac{39}{52} = 0.1875$ .

Then we can add them together:

$$\left(\frac{13}{52} \times \frac{39}{52}\right) + \left(\frac{39}{52} \times \frac{13}{52}\right) + \left(\frac{13}{52} \times \frac{13}{52}\right) = 0.4375.$$

- (c) Neither is a club:  $P(\neg C \wedge \neg D) = 1 - 0.4375 = 0.5625 = \frac{39}{52} \times \frac{39}{52}$ .
2.  $P(A \wedge B) = 0.9 \times 0.95 = 0.855$
3. (a)  $P(C \vee D) = P(C) + P(D) - P(C \wedge D) = 0.95 + 0.97 - 0.95 \times 0.97 = 0.9985$
- (b) The link failing corresponds to the following logical expression being true:
- $$\neg(C \vee D) = \neg C \wedge \neg D.$$
- (Or  $\overline{C+D} = \overline{C} \cdot \overline{D}$ .)
- (c)  $P(\neg(C \vee D)) = 1 - P(C \vee D) = 1 - 0.9985 = 0.0015$ . Or  $P(\neg C \wedge \neg D) = P(\neg C) \times P(\neg D) = 0.05 \times 0.3 = 0.015$ .
4. The events are independent, so  $P(A \wedge B \wedge C) = P(A) \times P(B) \times P(C) = 0.95 \times 0.96 \times 0.91 = 0.83$ .
5. The NUMBAS test provides a worked solution for each question in this part.
6. (a)  $P(A) = \frac{135+765}{1000} = 0.9$ ,  $P(B) = \frac{765+85}{1000} = 0.85$
- (b) For this we will need  $P(A \wedge B) = \frac{765}{1000} = 0.765$ . Then  $P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{0.765}{0.9} = 0.85$  and  $P(A|B) = \frac{0.765}{0.85} = 0.9$ .
7. The following Venn diagram represents the occurrence of two events  $A$  and  $B$ . The outer rectangle is the universal set and the two circular regions represent  $A$  and  $B$  as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.
- (a)  $P(A) = \frac{13+811}{1000} = 0.824$ ,  $P(B) = \frac{811+124}{1000} = 0.935$
- (b) For this we will need  $P(A \wedge B) = \frac{811}{1000} = 0.811$ . Then  $P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{0.811}{0.824} = 0.984$  and  $P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.811}{0.935} = 0.867$ .
- (c)  $P(\neg B|A) = \frac{P(A \wedge \neg B)}{P(A)} = \frac{13}{13+811} = 0.0158$ . Notice that this is  $1 - P(B|A)$ .
- (d)  $P(B|\neg A) = \frac{P(\neg A \wedge B)}{P(\neg A)} = \frac{124}{124+52} = 0.705$
- (e)  $P(\neg B|\neg A) = \frac{P(\neg A \wedge \neg B)}{P(\neg A)} = \frac{52}{52+124} = 0.295$ ,  $P(B|\neg A) = 0.705$ . These are complements again, in that  $P(\neg B|\neg A) = 1 - P(B|\neg A)$ .
- (f)  $P(\neg A|\neg B) = \frac{52}{13+52} = 0.8$
8. (a)  $P(A \vee B) = P(A) + P(B) - P(A \wedge B) = 0.94 + 0.92 - 0.94 \times 0.92 = 0.9952$ . (We can use  $P(A \wedge B) = P(A) \times P(B)$  since the events are independent.) Then  $P((A \vee B) \wedge C) = P(A \vee B) \times P(C) = 0.9952 \times 0.96 = 0.955$ .
9. The NUMBAS test provides a worked solution for each question in this part.