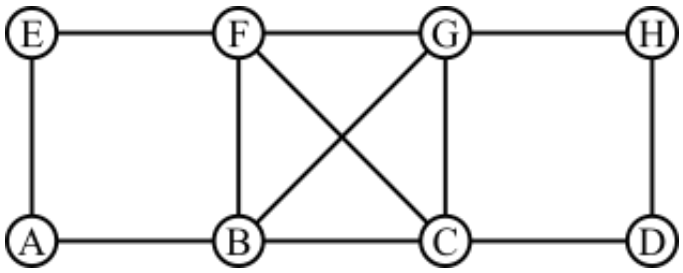
Exercises: Eulerian and Hamiltonian Graphs

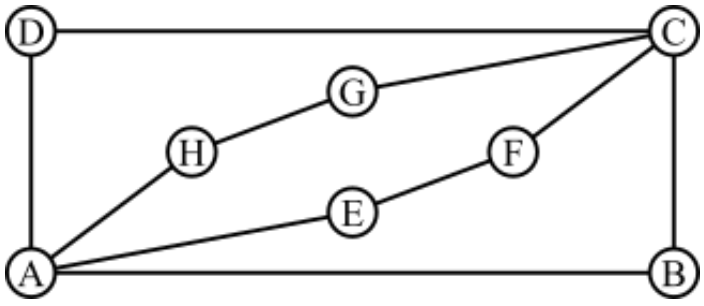
Exercises

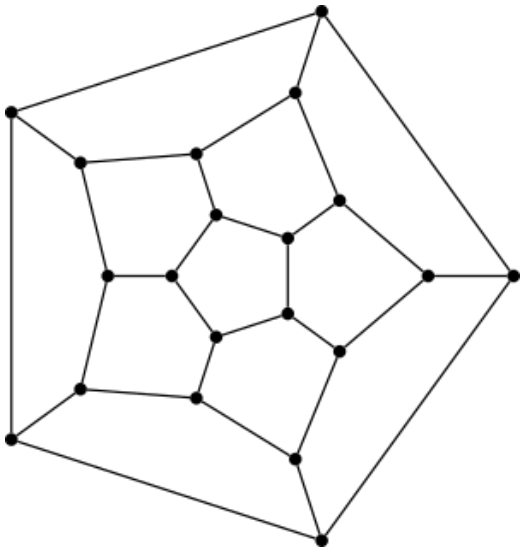
1. For each of the following graphs, decide if it is (i) Eulerian, semi-Eulerian, or non-Eulerian, and (ii) Hamiltonian, semi-Hamiltonian, or non-Hamiltonian. Find paths or cycles where appropriate to support your answers. If it is not Hamiltonian, then try to describe why this is.

(a)

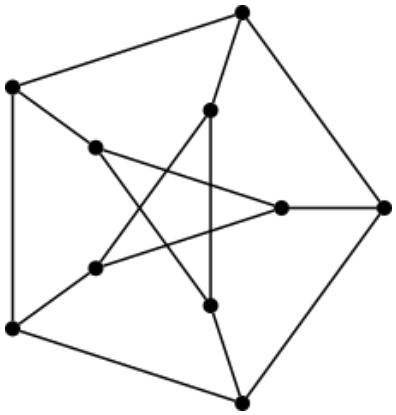


(b)

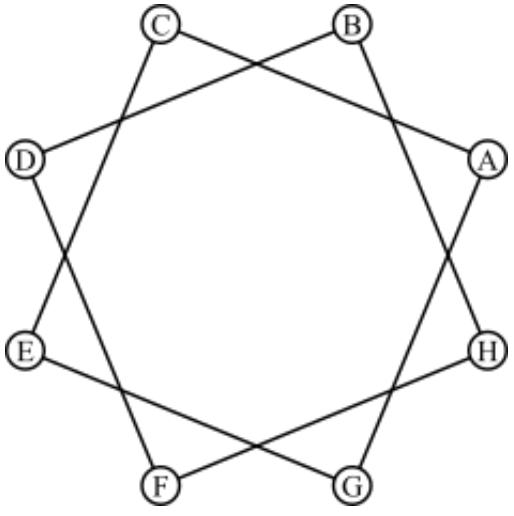




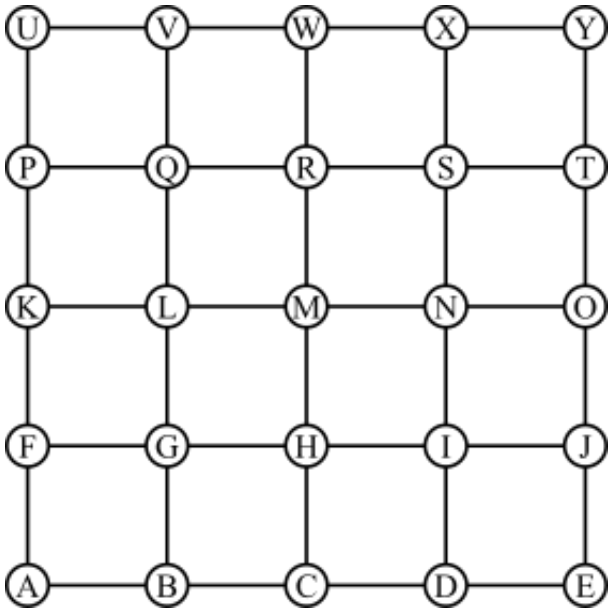
(c)



(d)



(e)

(f) 

2. You are asked to draw nine different graphs. Each must be connected and each must have exactly eight vertices. Use the table below as a guide to draw an appropriate graph according to the labels in the corresponding row and column. Pay attention to the following:

   - In the column labelled **Semi-Hamiltonian**, make sure that none of the graphs are actually **Hamiltonian**.
   - In the row labelled **Semi-Eulerian**, make sure that none of the graphs are actually **Eulerian**.
   - In the column labelled **Neither**, make sure that graphs are not **Hamiltonian** *and* not **Semi-Hamiltonian**.
   - In the row labelled **Neither**, make sure that graphs are not **Eulerian** *and* not **Semi-Eulerian**.

3. Find how many different trees there are with each of 1, 2, 3, 4, 5, and 6 vertices, and draw them. (Hint: There are 8 with 5 vertices or fewer.)

Solutions

1. (a) Eulerian and Hamiltonian
   (b) Eulerian but not Hamiltonian nor semi-Hamiltonian
   (c) Not Eulerian nor semi-Eulerian but it is Hamiltonian
   (d) Not Eulerian nor semi-Eulerian but it is semi-Hamiltonian
   (e) Not connected, so can't be either
   (f) Not Eulerian nor semi-Eulerian but it is semi-Hamiltonian

	Hamiltonian	**Semi-Hamiltonian**	**Neither**
Eulerian			
Semi-Eulerian			
Neither			

2. You should be able to check your answers for this. Use the theorems to check whether the graph is Eulirian or semi-Eulerian. For those that are Hamiltonian or semi-Hamiltonian, provide an example cycle or path. For those which are not Hamiltonian or semi-Hamiltonian, try to provide an argument as to why.

3. There are 1, 1, 1, 2, 3, and 6 for trees with 1, 2, 3, 4, 5, and 6 nodes, respectively. See this sequence of numbers. This exercise is not quite the same, but is related to, the problem in Good Will Hunting. **Video** Visit

   the URL below to view a video:
   `https://www.youtube.com/embed/iW_LkYiuTKE`