Exercises 2: Set Operations

Exercises

1. Using an appropriate Venn Diagram in each case indicate the following sets.

   (a) $(A \cap B) \cup B^c$
   (b) $A \setminus (A \setminus B)$
   (c) $A \cap (B \setminus A)$
   (d) $A \cup (B \cap C)$
   (e) $(A \cup B) \cap C$
   (f) $(A \cup B) \cap (A \cup C)$
   (g) $A^c \cup B^c \cup C^c$
   (h) $A^c \cap (B \setminus C^c)$

2. Write down the following sets by listing their elements:

   (a) $\mathbb{P}\{0, 1, 2\}$
   (b) $\{0, 1, 2\} \times \{1, 2\}$

3. A window on a computer screen has 480 rows of pixels, numbered from 0 (bottom) to 479 (top), and 640 pixels in each row, numbered from 0 (left) to 639 (right). How can the set of all possible pixel positions be represented as a Cartesian product? Write out, in the language of sets, the following parts of the screen:

   (a) the left-hand half of the window
   (b) the third row from the top
   (c) the right-hand column of the window
   (d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

4. If $A = \{\text{May}, \text{June}\}$, write down the following sets:

   (a) $\mathbb{P}(A)$
   (b) $A \cup \mathbb{P}(A)$
   (c) $A \cap \mathbb{P}(A)$
   (d) $\{A\} \cup \mathbb{P}(A)$
   (e) $\{A\} \cap \mathbb{P}(A)$

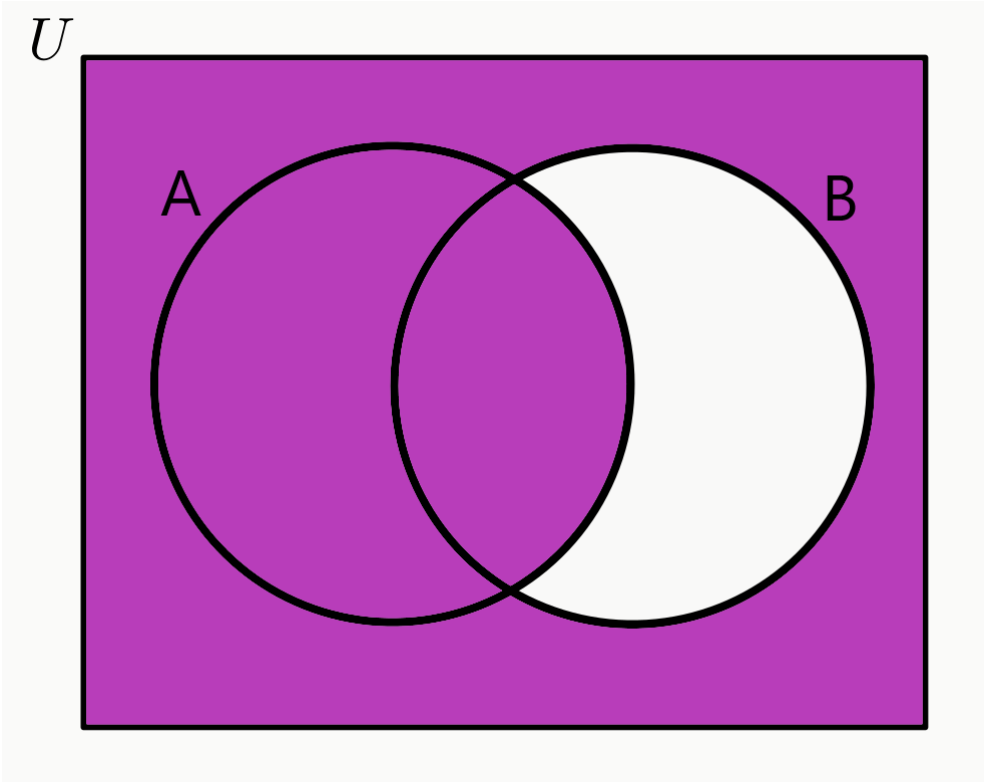
5. Give an example of three sets A, B and C such that $A \cap B \cap C = \emptyset$, but $A \cap B$, $B \cap C$ and $C \cap A$ are all non-empty.

6. If we wished to bring the operator $A \setminus B$ into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)

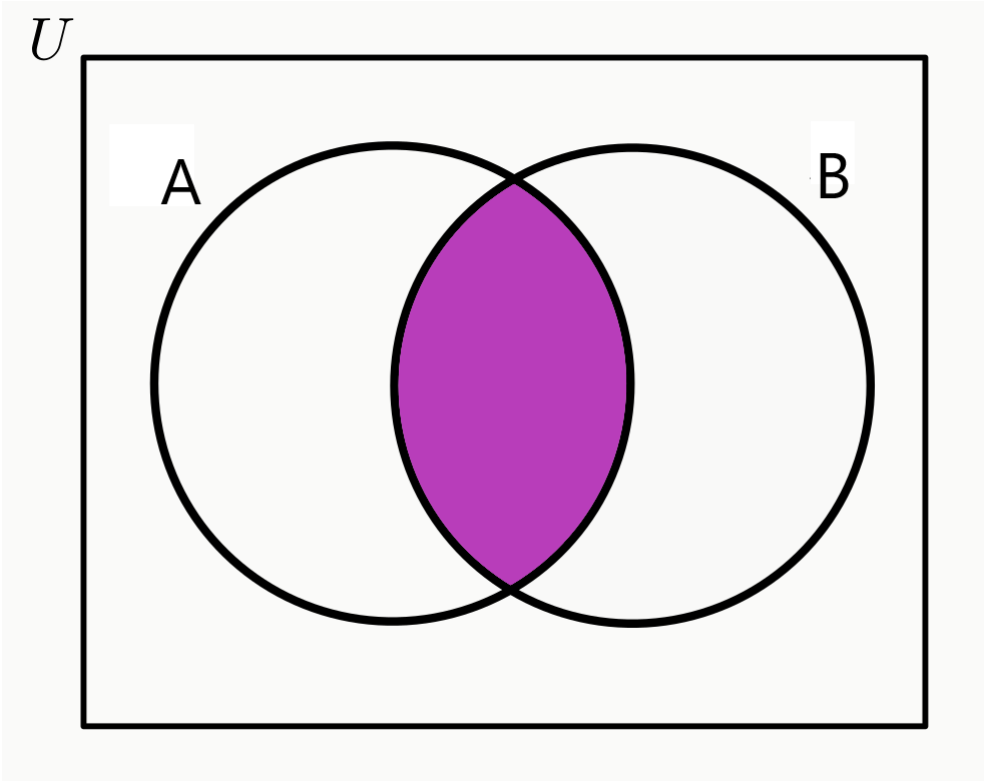
   (a) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$.
   (b) $A \setminus \emptyset = A$.
   (c) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$.
   (d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
   (e) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.
   (f) $(A \cup B) \setminus C = (A \setminus C) \cup B$.

Solutions

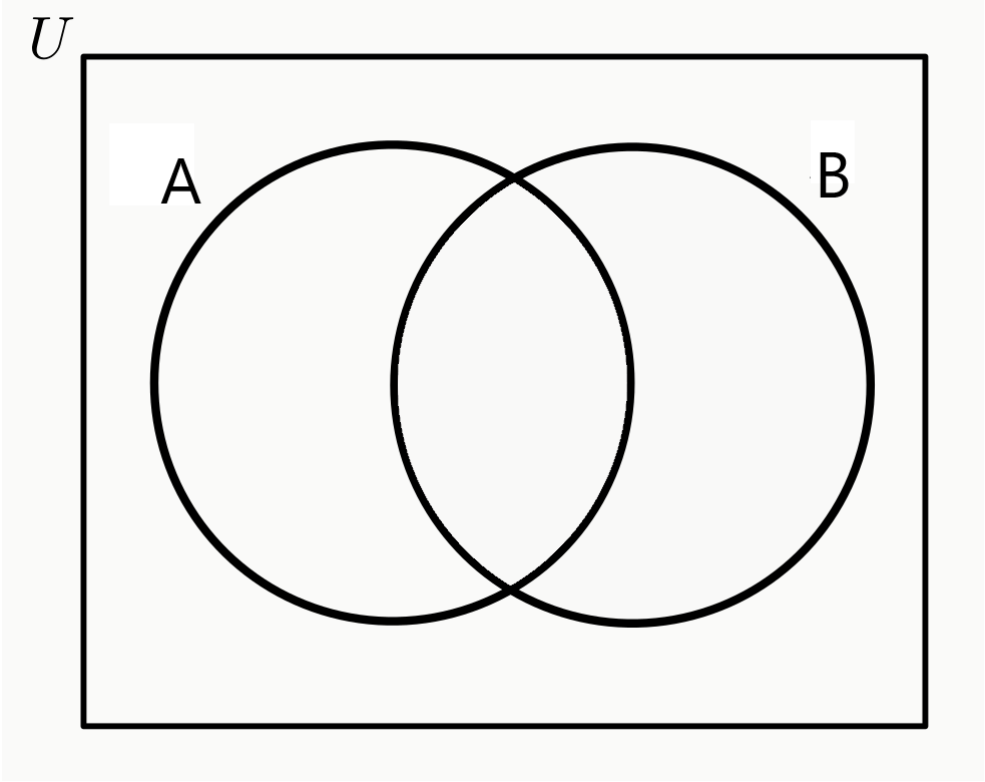
1. (a)



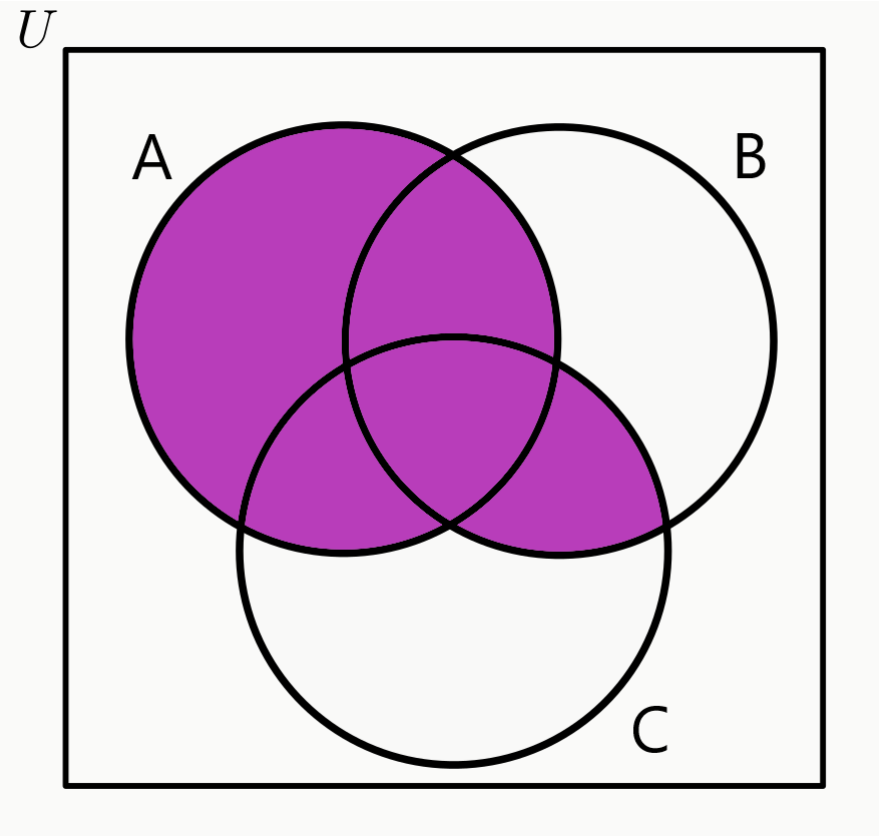
(b)



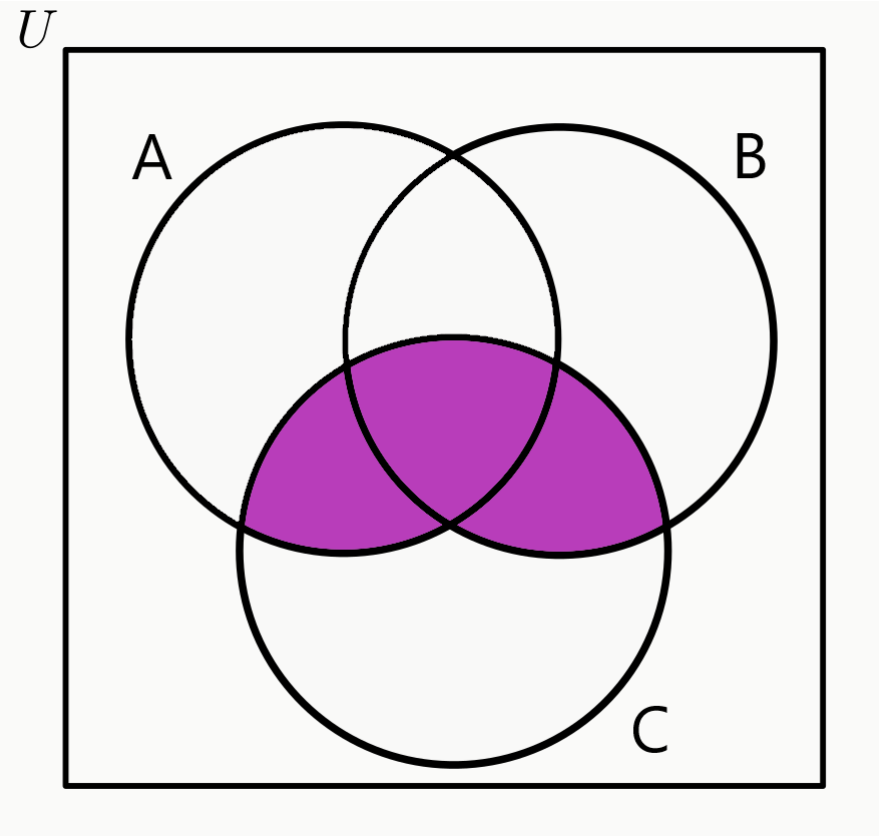
(c)



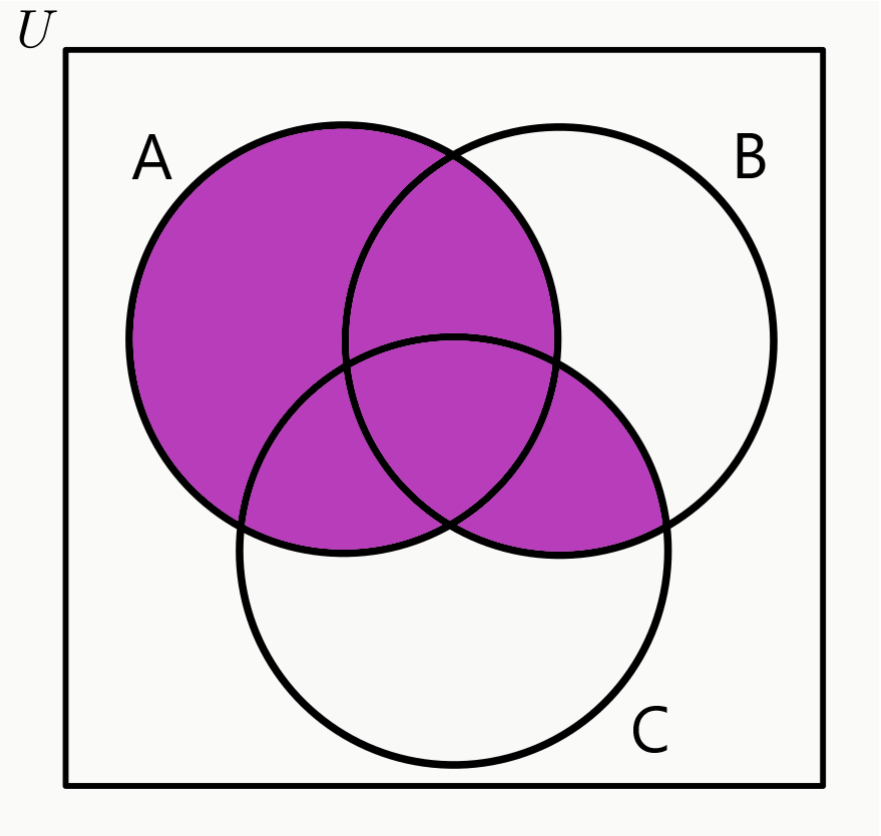
(d)



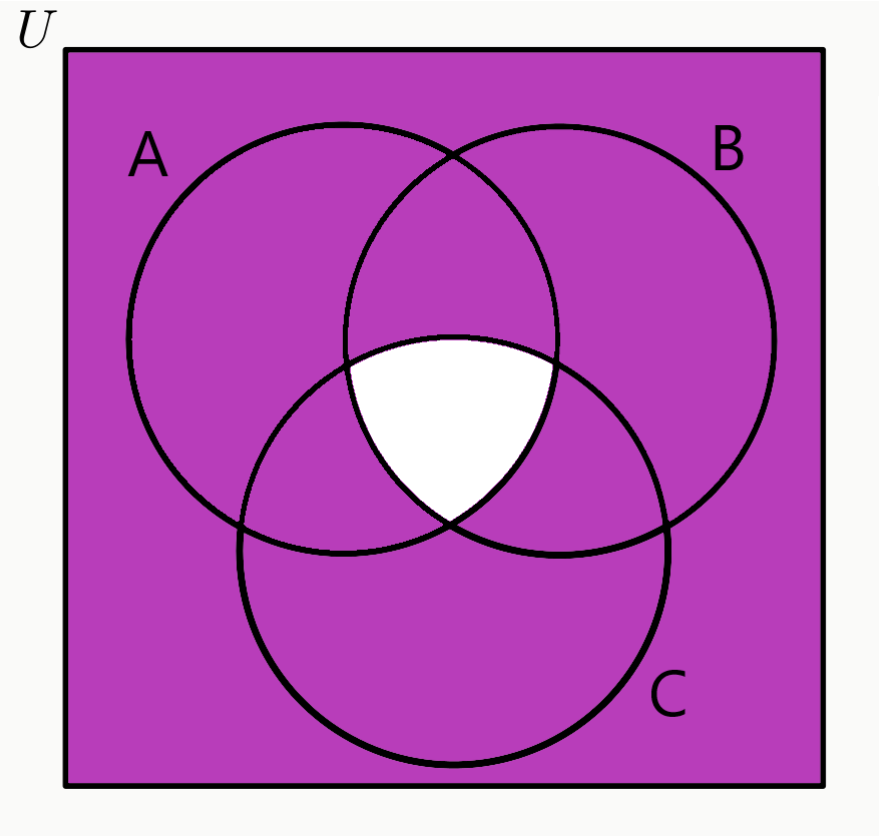
(e)



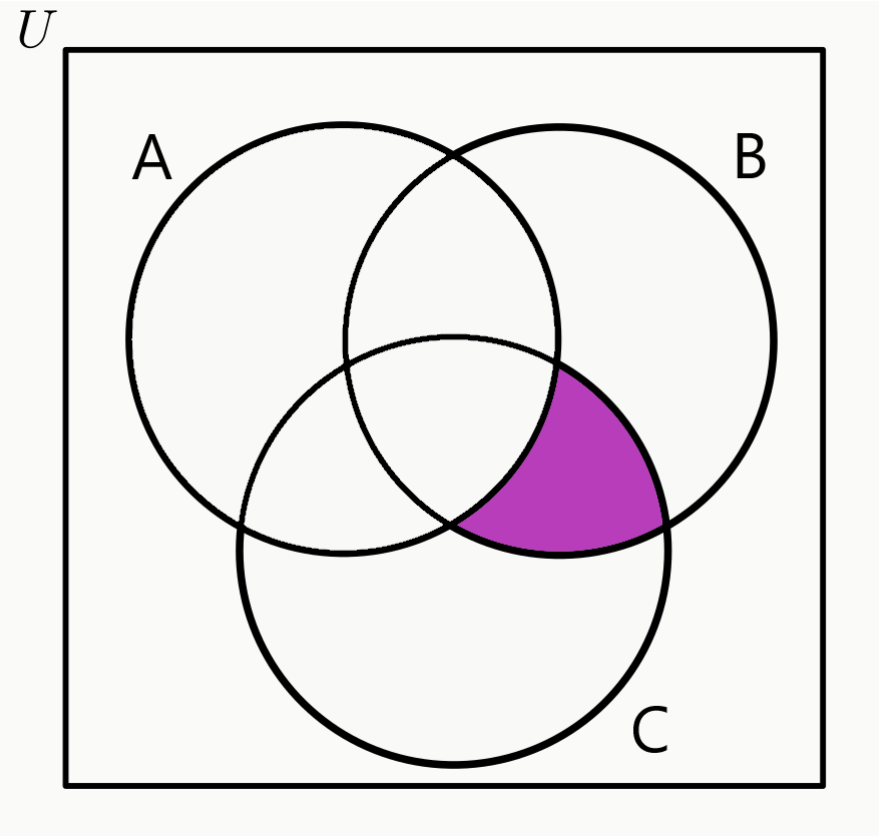
(f)



(g)



(h)



2. Write down the following sets by listing their elements:

   (a) $\mathbb{P}\{0,1,2\} = \{\varnothing, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$

   (b) $\{0,1,2\} \times \{1,2\} = \{(0,1), (0,2), (1,1), (1,2), (2,1), (2,2)\}$

3. Let $W = \{0,1,2,\ldots,479\} \times \{0,1,2,\ldots,639\}$ be the set representing the pairs of pixel in the window area.

   (a) the left-hand half of the window

   $$L = \{(x,y) \in W | x \leq 239\}$$

   (b) the third row from the top

   $$R_4 77 = \{(x,y) \in W | x = 477\}$$

   (c) the right-hand column of the window

   $$C_{639} = \{(x,y) \in W | y = 639\}$$

   (d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

   $$A = \{(x,y) \in W | 5 \leq x \leq 17 \wedge 239 \leq y \leq 399\}$$

4. If $A = \{\text{May}, \text{June}\}$, write down the following sets:

   (a) $\mathbb{P}(A) = \{\varnothing, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$

   (b) $A \cup \mathbb{P}(A) = \{\text{May}, \text{June}, \varnothing, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$

   (c) $A \cap \mathbb{P}(A) = \varnothing$

   (d) $\{A\} \cup \mathbb{P}(A) = \mathbb{P}(A)$

   (e) $\{A\} \cap \mathbb{P}(A) = A$

5. $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{2, 3\}$ is but one example.

6. If we wished to bring the operator $A \setminus B$ into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)

   (a) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$ is False.

   (b) $A \setminus \emptyset = A$ is True.

   (c) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$ is False.

   (d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ is True.

   (e) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ is True.

   (f) $(A \cup B) \setminus C = (A \setminus C) \cup B$ is False.