

# Matrix Basics

## Definitions

**Video** Visit the URL below to view a video:  
<https://www.youtube.com/embed/goIznLLAW34>

A **matrix** is a rectangular array of numbers or other elements. Upper case letters are often used to denote matrices. For example,

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \end{pmatrix}.$$

The matrix  $A$  has 2 rows (going across) and 3 columns; it is a  $2 \times 3$  matrix. This is its **dimension** or **size**. In general, a rectangular matrix has dimension  $m \times n$ , meaning it has  $m$  rows and  $n$  columns.

A **square** matrix has  $m = n$ . E.g.,

$$B = \begin{pmatrix} 7 & 1 \\ -3 & 2 \end{pmatrix}$$

is a  $2 \times 2$  square matrix.

A **row** matrix (or row vector) has  $m = 1$ , i.e., it has only one row. E.g.,

$$C = (5 \quad 1)$$

is a  $1 \times 2$  row matrix.

A **column** matrix (or column vector) has  $n = 1$ , i.e., it has only one column. E.g.,

$$\begin{pmatrix} -4 \\ 9 \end{pmatrix}$$

is a  $2 \times 1$  column matrix.

Two matrices are **equal** if they have the same size and the corresponding elements are identical. E.g., if

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$$

then we must have  $a = x$ ,  $b = y$ ,  $c = z$ , and  $d = w$ .

Given a matrix  $A$ , we may want to refer to the entry in the  $i$ th row and  $j$ th column. In this case we can use the notation or  $A_{i,j}$ . For example, the 7th row and 11th column would be  $i = 7$  and  $j = 11$  and so we would refer to the this entry as  $A_{7,11}$ .

	Customer 1	Customer 2
Product <i>A</i>	20	10
Product <i>B</i>	30	20
Product <i>C</i>	5	20

## Examples of Matrices

Matrices can be useful as a concise way of storing information, similar to a table.

### Customer and Product Information

The following table shows the volume of sales to two customers. We can represent this data in a  $3 \times 2$  matrix.

$$\begin{pmatrix} 20 & 10 \\ 30 & 20 \\ 5 & 20 \end{pmatrix}$$

There are many other uses for matrices. They are key to describing geometrical transformations such as rotation, shearing, reflection, and scaling. We will see how they can be used to solve systems of equations, which is a common problem in many different areas, including genetic analysis, signal processing, and electrical engineering, to name but a few.