Cryptography: Advanced Encryption Standard (AES)

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The Story So Far

- In the 1970s the Data Encryption Standard is introduced.
- It is quickly deemed insecure, but is still in use up to 2023 in the form of 3DES (Triple-DES).
- Other developments are happening at the time (public key cryptography), but we'll visit these later.

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- In 1998 the National Institute of Standards and Technology held an open call for a new standard to replace DES.
- This was to be called **AES: A**dvanced **E**ncryption **S**tandard.
- Many of the world's top cryptographers submitted entries to this competition.

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- Speed was also a criterion.
- Fifteen candidates were considered and a shortlist of five were chosen as finalists:
  - MARS,
  - RC6,
  - Twofish,
  - Serpent,
  - **Rijndael**: the winner.
- All algorithms were up to three times faster than DES.

The Winner: Rijndael

- Designed by Belgian cryptographs Joan Daemen and Vincent Rijmen.
- Their algorithm became the official standard in 2002.
- It is fast, easy to implement, and requires little memory.
- **Not** a Feistel system like DES, as everything is considered as one block: no splitting.
- It is a substitution-permutation system.

Development of AES

- Like DES it is a block cipher.
- Keys are of length 128, 192, or 256 bits, compared to DES's 56-bit keys.
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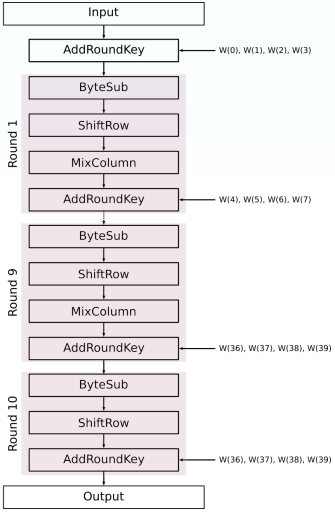
  possible keys: much more than even Triple DES.
- Each block is 16 bytes (128 bits) arranged as an array of 4 rows and 4 columns.
- AES uses 10 rounds (128-bit key), 12 rounds (192-bit key), or 14 rounds (256-bit key) using round keys based on the original key, plus a 0th round using the original key.

AES Scheme: 128 Bits



The Input

AES-128 is a block cipher, using blocks of 128 bits made up of 16 bytes. These are arranged in an array with 4 rows and 4 columns. E.g., if the plaintext is `to be or not to be that is the question...`, then the array is represented in hex as

$$\begin{bmatrix} t & o & t & e \\ o & r & t & t \\ b & n & o & h \\ e & o & b & a \end{bmatrix} = \begin{bmatrix} 74 & 6F & 74 & 65 \\ 6F & 72 & 74 & 74 \\ 62 & 6E & 6F & 68 \\ 65 & 6F & 62 & 61 \end{bmatrix}.$$

(Recall that each lower case letter is represented by an 8-bit binary number in ASCII. E.g., `t` $= 0111\,0100_2 = 74_{16}$.)

AddRoundKey

The round keys are derived from the original keys (more on that later). At this stage we perform a bitwise `XOR` with the plaintext, before beginning the first round. The 0th round key is the original key, where the columns are denoted by $W(0)$, $W(1)$, $W(2)$, and $W(3)$, respectively.

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$$K \oplus P = \begin{bmatrix} 0F & 47 & 0C & AF \\ 15 & D9 & B7 & 7F \\ 71 & E8 & AD & 67 \\ C9 & 59 & D6 & 98 \end{bmatrix} \oplus \begin{bmatrix} 74 & 6F & 74 & 65 \\ 6F & 72 & 74 & 74 \\ 62 & 6E & 6F & 68 \\ 65 & 6F & 62 & 61 \end{bmatrix} = \begin{bmatrix} 7B & 28 & 78 & CA \\ 7A & AB & C3 & 0B \\ 13 & 86 & C2 & 0F \\ AC & 36 & B4 & F9 \end{bmatrix}.$$

To see how this works for the first few entries:

- $0F = 0000\,1111$ and $74 = 0111\,0100$, so $0F \oplus 74 = 0111\,1011 = 7B$,
- $47 = 0100\,0111$ and $6F = 0110\,1111$, so $47 \oplus 6F = 0010\,1000 = 28$.

ByteSub

In the ByteSub stage we replace each entry in the array using the Rijndael S-Box. We determine the row number using the first hex digit (first byte) and the column by the second hex digit (second byte).

$$\begin{bmatrix} 7B & 28 & 78 & CA \\ 7A & AB & C3 & 0B \\ 13 & 86 & C2 & 0F \\ AC & 36 & B4 & F9 \end{bmatrix}$$

E.g., the first entry is $7B$, which corresponds to the 7th row and column B, the 11th column.

Rijndael S-Box

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**A**	**B**	**C**	**D**	**E**	**F**
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
A	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
B	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
C	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
D	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
E	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
F	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

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1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	**21**	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
A	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
B	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
C	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
D	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
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E.g., the first entry is $7B$, which corresponds to the 7th row and column B, the 11th column. This gives 21. Doing this for each entry gives the following array.

$$\begin{bmatrix} 21 & 34 & BC & 74 \\ DA & 62 & 2E & 2B \\ 7D & 44 & 25 & 76 \\ 91 & 05 & 8D & 99 \end{bmatrix}$$

ShiftRow

The ShiftRow stage is quite simple. Each row is shifted by a certain amount:

- The first row doesn't shift.
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MixColumn

We won't fully investigate what happens in the MixColum stage, as the mathematics gets rather complicated. It depends on an area of mathematics called Galois Fields. In particular, the Galois field $GF(2^8)$. In practice, this is achieved by shifting bits and applying `XOR`, but the mathematical underpinnings ensure that the scheme works out nicely.

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Each column of the array is replaced by a combination of the bytes in the column, where $\oplus$ means the usual bitwise `XOR`, but $2a$, $3a$, $2b$, etc. are **not** by simple multiplication.

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \mapsto \begin{bmatrix} 2a \oplus 3b \oplus c \oplus d \\ a \oplus 2b \oplus 3c \oplus d \\ a \oplus b \oplus 2c \oplus 3d \\ 3a \oplus b \oplus c \oplus 2d \end{bmatrix}$$

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  2. ShiftRow Transformation,
  3. AddRoundKey.
- Round 10 has no MixColumns Transformation, for decryption purposes.

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  3. InverseShiftRow shifts rows to the right, instead of left,
  4. InvMixColumn is more complicated than the encryption process, but relies on a similar mathematical manipulation.

Modes of Operation

Five standard ways of implementing AES:

- Mode 1: Electronic Code Book - not recommended due to plaintext attacks
- Mode 2: Cipher Block Chaining
- Mode 3: Cipher Feedback Mode
- Mode 4: Output Feedback Mode
- Mode 5: Counter Mode - NIST recommended mode (2001)

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- Compression Tools: WinZip, RAR, 7-Zip.
- Wireless Connections: WPA2.
- VPNs: IPsec.
- Password Managers: LastPass.
- Communication: Signal Protocol in WhatsApp, Signal, Facebook Messenger, etc.
- Data Storage: iCloud (unless you live in the UK...)

The common Cipher Block Chaining mode of operation is processor intensive, so some chip designs include partial AES instructions to improve efficiency:

- x86-64 architecture processors.

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- SECRET: 128, 192, or 256-bit keys.
- TOP SECRET: 192 or 256-bit keys.
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With that in mind, there is some concern that quantum computation will allow various techniques to be brought to bear on the problem of cracking AES. But the NIST doesn't seem too worried for now:

'*Taking these mitigating factors into account, it is quite likely that Grover's algorithm will provide little or no advantage in attacking AES, and AES 128 will remain secure for decades to come.*'

Tutorials

In the tutorial this week we will:

- Create a spreadsheet to perform the first round of AES encryption. (Compare with Q3 on the coursework.)
- Use any remaining time to carry on with the coursework.