

Exercises 2: Set Operations

Exercises

1. Using an appropriate Venn Diagram in each case indicate the following sets.

(a) $(A \cap B) \cup B^c$

(e) $(A \cup B) \cap C$

(b) $A \setminus (A \setminus B)$

(f) $(A \cup B) \cap (A \cup C)$

(c) $A \cap (B \setminus A)$

(g) $A^c \cup B^c \cup C^c$

(d) $A \cup (B \cap C)$

(h) $A^c \cap (B \setminus C^c)$

2. Write down the following sets by listing their elements:

(a) $\mathbb{P}\{0, 1, 2\}$

(b) $\{0, 1, 2\} \times \{1, 2\}$

3. A window on a computer screen has 480 rows of pixels, numbered from 0 (bottom) to 479 (top), and 640 pixels in each row, numbered from 0 (left) to 639 (right). How can the set of all possible pixel positions be represented as a Cartesian product? Write out, in the language of sets, the following parts of the screen:

(a) the left-hand half of the window

(b) the third row from the top

(c) the right-hand column of the window

(d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

4. If $A = \{\text{May}, \text{June}\}$, write down the following sets:

(a) $\mathbb{P}(A)$

(b) $A \cup \mathbb{P}(A)$

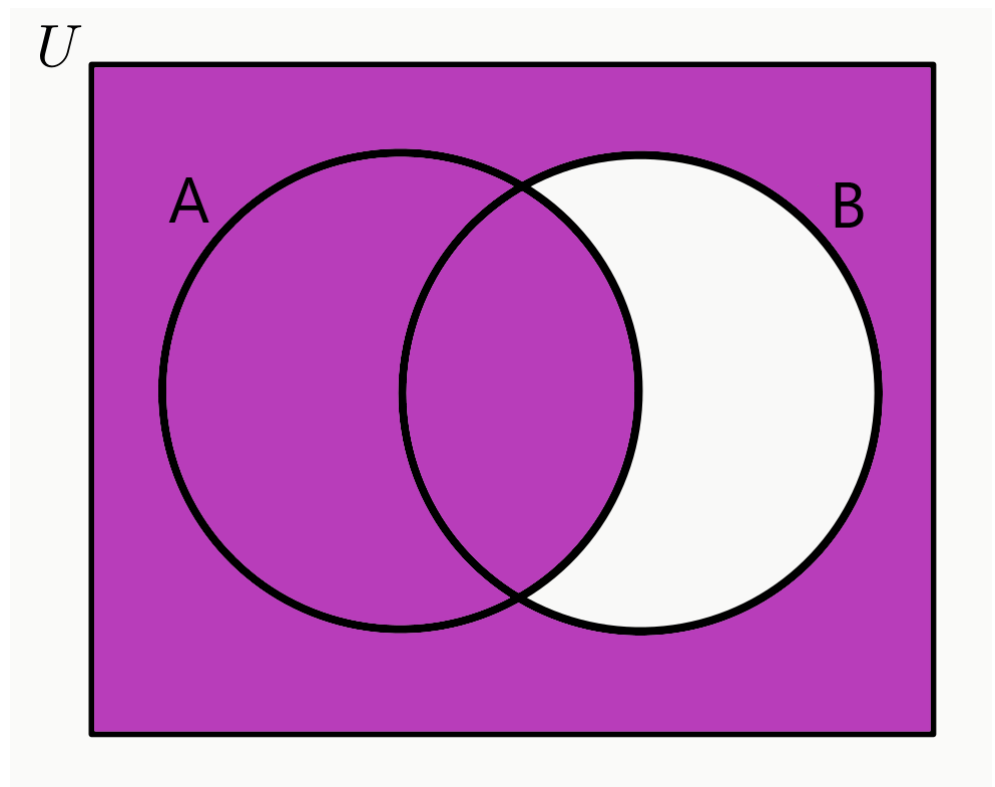
(c) $A \cap \mathbb{P}(A)$

(d) $\{A\} \cup \mathbb{P}(A)$

(e) $\{A\} \cap \mathbb{P}(A)$

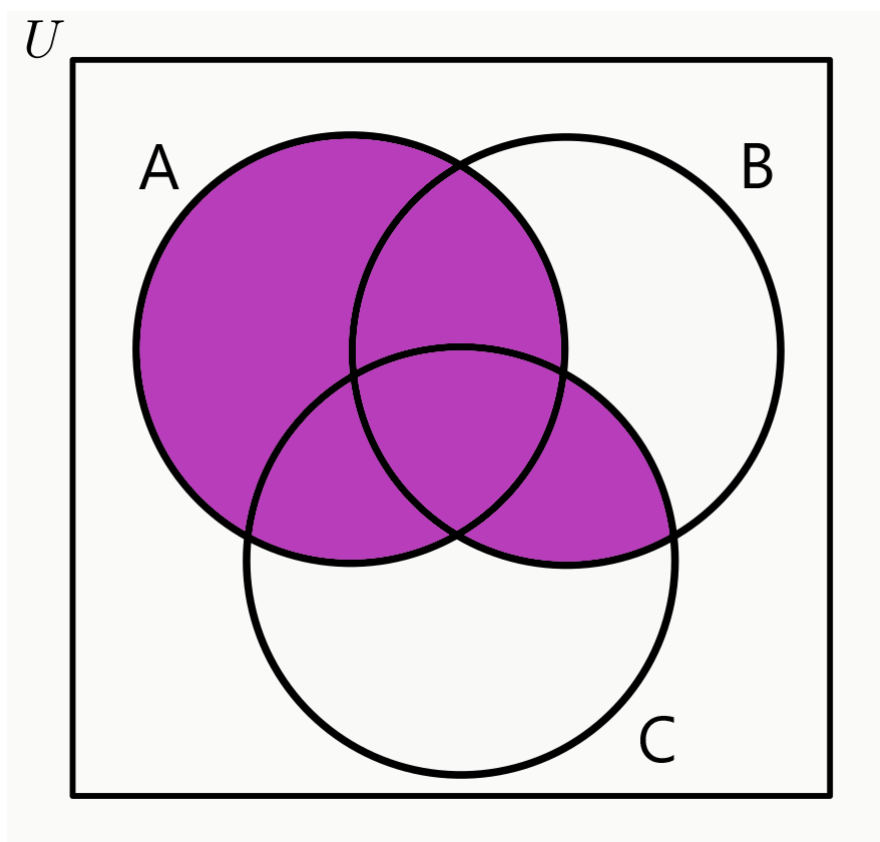
5. Give an example of three sets A , B and C such that $A \cap B \cap C = \emptyset$, but $A \cap B$, $B \cap C$ and $C \cap A$ are all non-empty.
6. If we wished to bring the operator $A \setminus B$ into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)
 - (a) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$.
 - (b) $A \setminus \emptyset = A$.
 - (c) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$.
 - (d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
 - (e) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.
 - (f) $(A \cup B) \setminus C = (A \setminus C) \cup B$.

Solutions



1. (a)





(d)



(e)



(f)



(g)



(h)

2. Write down the following sets by listing their elements:

(a) $\mathbb{P}\{0, 1, 2\} = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$

(b) $\{0, 1, 2\} \times \{1, 2\} = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

3. Let $W = \{0, 1, 2, \dots, 639\} \times \{0, 1, 2, \dots, 479\}$ be the set representing the pairs of pixel in the window area.

(a) the left-hand half of the window

$$L = \{(x, y) \in W | x \leq 319\}$$

(b) the third row from the top

$$R_{477} = \{(x, y) \in W | y = 477\}$$

(c) the right-hand column of the window

$$C_{639} = \{(x, y) \in W | x = 639\}$$

(d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

$$A = \{(x, y) \in W | 5 \leq y \leq 17 \wedge 239 \leq x \leq 399\}$$

4. If $A = \{\text{May}, \text{June}\}$, write down the following sets:
- (a) $\mathbb{P}(A) = \{\emptyset, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$
 - (b) $A \cup \mathbb{P}(A) = \{\text{May}, \text{June}, \emptyset, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$
 - (c) $A \cap \mathbb{P}(A) = \emptyset$
 - (d) $\{A\} \cup \mathbb{P}(A) = \mathbb{P}(A)$
 - (e) $\{A\} \cap \mathbb{P}(A) = A$
5. $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{2, 3\}$ is but one example.
6. If we wished to bring the operator $A \setminus B$ into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)
- (a) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$ is False.
 - (b) $A \setminus \emptyset = A$ is True.
 - (c) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$ is False.
 - (d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ is True.
 - (e) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ is True.
 - (f) $(A \cup B) \setminus C = (A \setminus C) \cup B$ is False.