

Cryptography: Public Key Cryptography

Alex Corner

Sheffield Hallam University

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .
- ▶ She calculates $n = pq$ (the **modulus**) and then finds $\varphi(n) = \varphi(pq) = (p - 1)(q - 1)$.

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .
- ▶ She calculates $n = pq$ (the **modulus**) and then finds $\varphi(n) = \varphi(pq) = (p-1)(q-1)$.
- ▶ Alice also selects a value of e (the **encryption exponent**) such that $1 < e < \varphi(n)$ and $\text{hcf}(e, \varphi(n)) = 1$.

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .
- ▶ She calculates $n = pq$ (the **modulus**) and then finds $\varphi(n) = \varphi(pq) = (p - 1)(q - 1)$.
- ▶ Alice also selects a value of e (the **encryption exponent**) such that $1 < e < \varphi(n)$ and $\text{hcf}(e, \varphi(n)) = 1$.
- ▶ She also finds a value d (the **decryption exponent**) such that $de = 1 \bmod \varphi(n)$. (More on this later.)

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .
- ▶ She calculates $n = pq$ (the **modulus**) and then finds $\varphi(n) = \varphi(pq) = (p - 1)(q - 1)$.
- ▶ Alice also selects a value of e (the **encryption exponent**) such that $1 < e < \varphi(n)$ and $\text{hcf}(e, \varphi(n)) = 1$.
- ▶ She also finds a value d (the **decryption exponent**) such that $de = 1 \bmod \varphi(n)$. (More on this later.)
- ▶ Alice then publishes (n, e) publicly: this is her **public key**.

RSA: Implementation

- ▶ Alice chooses two 'large' primes p and q .
- ▶ She calculates $n = pq$ (the **modulus**) and then finds $\varphi(n) = \varphi(pq) = (p - 1)(q - 1)$.
- ▶ Alice also selects a value of e (the **encryption exponent**) such that $1 < e < \varphi(n)$ and $\text{hcf}(e, \varphi(n)) = 1$.
- ▶ She also finds a value d (the **decryption exponent**) such that $de = 1 \bmod \varphi(n)$. (More on this later.)
- ▶ Alice then publishes (n, e) publicly: this is her **public key**.
- ▶ She keeps the value of d secret: this is her **private key**.

RSA: Encryption

- ▶ Alice has published her public key (n, e) and kept her private key d secret.

RSA: Encryption

- ▶ Alice has published her public key (n, e) and kept her private key d secret.
- ▶ Bob can now encrypt messages to send to Alice. He selects a message value $0 \leq m < n$ and calculates $c = m^e \bmod n$. He sends this to Alice.

RSA: Encryption

- ▶ Alice has published her public key (n, e) and kept her private key d secret.
- ▶ Bob can now encrypt messages to send to Alice. He selects a message value $0 \leq m < n$ and calculates $c = m^e \bmod n$. He sends this to Alice.
- ▶ Alice can now decrypt this by finding $c^d = m^{de} = m \bmod n$.

RSA: Encryption

- ▶ Alice has published her public key (n, e) and kept her private key d secret.
- ▶ Bob can now encrypt messages to send to Alice. He selects a message value $0 \leq m < n$ and calculates $c = m^e \bmod n$. He sends this to Alice.
- ▶ Alice can now decrypt this by finding $c^d = m^{de} = m \bmod n$.
- ▶ That this works relies on a result in number theory called Euler's Theorem:
 - ▶ Let n be a non-negative integer and let a be an integer coprime to n . Then

$$a^{\varphi(n)} = 1 \bmod n.$$

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.
- ▶ She publishes the public key $(n, e) = (91, 5)$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.
- ▶ She publishes the public key $(n, e) = (91, 5)$.
- ▶ Bob wants to send the message 'Y' to Alice. He could convert this to ASCII (0101 1001 in binary, 89 in denary) so that the message value is $m = 89$.

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.
- ▶ She publishes the public key $(n, e) = (91, 5)$.
- ▶ Bob wants to send the message 'Y' to Alice. He could convert this to ASCII (0101 1001 in binary, 89 in denary) so that the message value is $m = 89$.
- ▶ Bob calculates $c = m^e = 89^5 \bmod 91$. He might do this by repeated squaring if the powers are large. (See previous notes on modular arithmetic.)

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.
- ▶ She publishes the public key $(n, e) = (91, 5)$.
- ▶ Bob wants to send the message 'Y' to Alice. He could convert this to ASCII (0101 1001 in binary, 89 in denary) so that the message value is $m = 89$.
- ▶ Bob calculates $c = m^e = 89^5 \bmod 91$. He might do this by repeated squaring if the powers are large. (See previous notes on modular arithmetic.)
- ▶ $c = m^e = 89^5 = (-2)^5 = -32 = 59 \bmod 91$

RSA: Encryption Example

- ▶ Suppose that Alice has chosen primes $p = 7$ and $q = 13$, and **encryption exponent** $e = 5$.
- ▶ Her **modulus** is $n = pq = 7 \times 13 = 91$.
- ▶ She also calculates $\varphi(n) = \varphi(91) = \varphi(7 \times 13) = 6 \times 12 = 72$.
- ▶ Alice finds $d = 29$, so that $de = 29 \times 5 = 145 = 1 \bmod 72$.
- ▶ She publishes the public key $(n, e) = (91, 5)$.
- ▶ Bob wants to send the message 'Y' to Alice. He could convert this to ASCII (0101 1001 in binary, 89 in denary) so that the message value is $m = 89$.
- ▶ Bob calculates $c = m^e = 89^5 \bmod 91$. He might do this by repeated squaring if the powers are large. (See previous notes on modular arithmetic.)
- ▶ $c = m^e = 89^5 = (-2)^5 = -32 = 59 \bmod 91$
- ▶ Alice decrypts this by calculating $c^d = 59^{29} \bmod 91$.

Quick Powers: Recap

- ▶ To decrypt Bob's encrypted message, Alice calculates $c^d = 59^{29} \bmod 91$.

Quick Powers: Recap

- ▶ To decrypt Bob's encrypted message, Alice calculates $c^d = 59^{29} \bmod 91$.
- ▶ The power is $29 = 16 + 8 + 4 + 1$. We write it as the sum of powers of 2, then $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1$.

Quick Powers: Recap

- ▶ To decrypt Bob's encrypted message, Alice calculates $c^d = 59^{29} \bmod 91$.
- ▶ The power is $29 = 16 + 8 + 4 + 1$. We write it as the sum of powers of 2, then $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1$.

$$59^1 = 59 \bmod 91$$

$$59^2 = 3481 = 23 \bmod 91$$

$$59^4 = 23^2 = 529 = 74 \bmod 91$$

$$59^8 = 74^2 = 5476 = 16 \bmod 91$$

$$59^{16} = 16^2 = 256 = 74 \bmod 91$$

Quick Powers: Recap

- ▶ To decrypt Bob's encrypted message, Alice calculates $c^d = 59^{29} \bmod 91$.
- ▶ The power is $29 = 16 + 8 + 4 + 1$. We write it as the sum of powers of 2, then $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1$.

$$59^1 = 59 \bmod 91$$

$$59^2 = 3481 = 23 \bmod 91$$

$$59^4 = 23^2 = 529 = 74 \bmod 91$$

$$59^8 = 74^2 = 5476 = 16 \bmod 91$$

$$59^{16} = 16^2 = 256 = 74 \bmod 91$$

- ▶ $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1 = 74 \times 16 \times 74 \times 59 = 89 \bmod 91$.

Quick Powers: Recap

- ▶ To decrypt Bob's encrypted message, Alice calculates $c^d = 59^{29} \bmod 91$.
- ▶ The power is $29 = 16 + 8 + 4 + 1$. We write it as the sum of powers of 2, then $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1$.

$$59^1 = 59 \bmod 91$$

$$59^2 = 3481 = 23 \bmod 91$$

$$59^4 = 23^2 = 529 = 74 \bmod 91$$

$$59^8 = 74^2 = 5476 = 16 \bmod 91$$

$$59^{16} = 16^2 = 256 = 74 \bmod 91$$

- ▶ $59^{29} = 59^{16} \times 59^8 \times 59^4 \times 59^1 = 74 \times 16 \times 74 \times 59 = 89 \bmod 91$.
- ▶ In practice we encrypt multiple letters at a time in blocks (255 letters), rather than just single letters. This prevents simple frequency analysis style attacks.

RSA: Decryption Exponent and Euclidean Algorithm

When Alice sets up her RSA scheme, she chooses large primes p and q . Using these she calculates $n = pq$ and $\varphi(n) = (p - 1)(q - 1)$, but she also requires a value e such that $1 < e < \varphi(n)$ with $\text{hcf}(e, \varphi(n)) = 1$ and a value d such that $de = 1 \bmod \varphi(n)$. This can be hard to find by trial and error, so we can use the **Euclidean Algorithm**.

RSA: Decryption Exponent and Euclidean Algorithm

When Alice sets up her RSA scheme, she chooses large primes p and q . Using these she calculates $n = pq$ and $\varphi(n) = (p - 1)(q - 1)$, but she also requires a value e such that $1 < e < \varphi(n)$ with $\text{hcf}(e, \varphi(n)) = 1$ and a value d such that $de = 1 \bmod \varphi(n)$. This can be hard to find by trial and error, so we can use the **Euclidean Algorithm**.

We can easily check that the highest common factor of $e = 5$ and $\varphi(n) = 72$ is actually equal to 1.

RSA: Decryption Exponent and Euclidean Algorithm

When Alice sets up her RSA scheme, she chooses large primes p and q . Using these she calculates $n = pq$ and $\varphi(n) = (p - 1)(q - 1)$, but she also requires a value e such that $1 < e < \varphi(n)$ with $\text{hcf}(e, \varphi(n)) = 1$ and a value d such that $de = 1 \bmod \varphi(n)$. This can be hard to find by trial and error, so we can use the **Euclidean Algorithm**.

We can easily check that the highest common factor of $e = 5$ and $\varphi(n) = 72$ is actually equal to 1.

$$72 = 14 \times 5 + 2$$

$$5 = 2 \times 2 + 1$$

$$2 = 2 \times 1 + 0$$

RSA: Decryption Exponent and Euclidean Algorithm

When Alice sets up her RSA scheme, she chooses large primes p and q . Using these she calculates $n = pq$ and $\varphi(n) = (p - 1)(q - 1)$, but she also requires a value e such that $1 < e < \varphi(n)$ with $\text{hcf}(e, \varphi(n)) = 1$ and a value d such that $de = 1 \bmod \varphi(n)$. This can be hard to find by trial and error, so we can use the **Euclidean Algorithm**.

We can easily check that the highest common factor of $e = 5$ and $\varphi(n) = 72$ is actually equal to 1.

$$72 = 14 \times 5 + 2$$

$$5 = 2 \times 2 + 1$$

$$2 = 2 \times 1 + 0$$

The least number before 0 is the highest common factor. Here this is 1, so e and $\varphi(n)$ are coprime.

Extended Euclidean Algorithm

The RSA example used the value of $d = 29$, but didn't explain where this came from.

Extended Euclidean Algorithm

The RSA example used the value of $d = 29$, but didn't explain where this came from.

	$\varphi(n)$	e	
(1, 0)	72	5	(0, 1)
		$\times 14$	
(0, 14)	70	4	(2, -28)
		$\times 2$	
<hr/> (1, -14)	<hr/> 2	1	(-2, 29)
		$\times 2$	
(-4, 58)	2		
<hr/> (5, -72)	<hr/> 0		

Extended Euclidean Algorithm

The RSA example used the value of $d = 29$, but didn't explain where this came from.

	$\varphi(n)$	e	
(1, 0)	72	5	(0, 1)
		$\times 14$	
(0, 14)	70	4	(2, -28)
		$\times 2$	
<hr/> (1, -14)	<hr/> 2	1	(-2, 29)
		$\times 2$	
(-4, 58)	2		
<hr/> (5, -72)	<hr/> 0		

At each stage, the pair of coordinates (a, b) corresponding to a number c in the table tells us that $72a + 5b = c$. E.g., the pair $(2, -28)$ corresponds to the number 4 in the table, so we know that $72 \times 2 + 5 \times (-28) = 4$.

Extended Euclidean Algorithm

The RSA example used the value of $d = 29$, but didn't explain where this came from.

	$\varphi(n)$		e	
$(1, 0)$	72		5	$(0, 1)$
		$\times 14$		
$(0, 14)$	70		4	$(2, -28)$
		$\times 2$		
<hr/> $(1, -14)$	2		1	$(-2, 29)$
		$\times 2$		
$(-4, 58)$	2			
<hr/> $(5, -72)$	0			

At each stage, the pair of coordinates (a, b) corresponding to a number c in the table tells us that $72a + 5b = c$. E.g., the pair $(2, -28)$ corresponds to the number 4 in the table, so we know that $72 \times 2 + 5 \times (-28) = 4$. Similarly, we know that $72 \times (-2) + 5 \times 29 = 1$.

Extended Euclidean Algorithm

The RSA example used the value of $d = 29$, but didn't explain where this came from.

	$\varphi(n)$		e	
$(1, 0)$	72		5	$(0, 1)$
		$\times 14$		
$(0, 14)$	70		4	$(2, -28)$
		$\times 2$		
<hr/> $(1, -14)$	<hr/> 2		1	$(-2, 29)$
		$\times 2$		
$(-4, 58)$	2			
<hr/> $(5, -72)$	<hr/> 0			

At each stage, the pair of coordinates (a, b) corresponding to a number c in the table tells us that $72a + 5b = c$. E.g., the pair $(2, -28)$ corresponds to the number 4 in the table, so we know that $72 \times 2 + 5 \times (-28) = 4$. Similarly, we know that $72 \times (-2) + 5 \times 29 = 1$.

We need a value of d such that $de = 1 \pmod{\varphi(n)}$, so in our case such that $5d = 1 \pmod{72}$. Now we know that $72 \times (-2) + 5 \times 29 = 1$, so $5 \times 29 = 1 \pmod{72}$. Hence $d = 29$.

RSA: Example

Alice changes her RSA scheme to use $p = 11$, $q = 19$, and $e = 7$.

RSA: Example

Alice changes her RSA scheme to use $p = 11$, $q = 19$, and $e = 7$. She calculates $n = pq = 11 \times 19 = 209$ and $\varphi(n) = (p - 1)(q - 1) = 10 \times 18 = 180$. She publishes $(n, e) = (209, 7)$ and calculates d using the extended Euclidean algorithm.

RSA: Example

Alice changes her RSA scheme to use $p = 11$, $q = 19$, and $e = 7$. She calculates $n = pq = 11 \times 19 = 209$ and $\varphi(n) = (p - 1)(q - 1) = 10 \times 18 = 180$. She publishes $(n, e) = (209, 7)$ and calculates d using the extended Euclidean algorithm.

	$\varphi(n)$		e	
(1, 0)	180		7	(0, 1)
		$\times 25$		
(0, 25)	175		5	(1, -25)
		$\times 1$		
<hr/> (1, -25)	<hr/> 5		2	(-1, 26)
		$\times 2$		
(-2, 52)	4			
<hr/> (3, -77)	<hr/> 1			

RSA: Example

Alice changes her RSA scheme to use $p = 11$, $q = 19$, and $e = 7$. She calculates $n = pq = 11 \times 19 = 209$ and $\varphi(n) = (p - 1)(q - 1) = 10 \times 18 = 180$. She publishes $(n, e) = (209, 7)$ and calculates d using the extended Euclidean algorithm.

	$\varphi(n)$		e	
	(1, 0)	180		7 (0, 1)
			$\times 25$	
	(0, 25)	175		5 (1, -25)
			$\times 1$	
	<hr/>	<hr/>		
	(1, -25)	5		2 (-1, 26)
			$\times 2$	
	(-2, 52)	4		
	<hr/>	<hr/>		
	(3, -77)	1		

The algorithm suggests that $d = -77$, but we want d to be positive. Since we want $de = 1 \bmod \varphi(n)$ and $\varphi(n) = 180$, we can add 180 to -77 to find that

$$d = -77 = -77 + 180 = 103 \bmod 180.$$

RSA in Practice

- ▶ The main idea behind RSA is that your encryption key (n, e) can be made **public**: anybody can know it without any security issues. But, only you have access to the decryption key (n, d) .

RSA in Practice

- ▶ The main idea behind RSA is that your encryption key (n, e) can be made **public**: anybody can know it without any security issues. But, only you have access to the decryption key (n, d) .
- ▶ Theoretically, an attacker could factor the modulus n and use the primes to calculate the decryption exponent d . The security of RSA depends on this being a very difficult problem.

RSA in Practice

- ▶ The main idea behind RSA is that your encryption key (n, e) can be made **public**: anybody can know it without any security issues. But, only you have access to the decryption key (n, d) .
- ▶ Theoretically, an attacker could factor the modulus n and use the primes to calculate the decryption exponent d . The security of RSA depends on this being a very difficult problem.
- ▶ Our examples only include small primes so that we can understand how RSA works. In practice, the modulus will be 2048 or 3072 bits as recommended by NIST in 2015. This means that the modulus will be of the order of 600+ decimal digits.

RSA in Practice

- ▶ The main idea behind RSA is that your encryption key (n, e) can be made **public**: anybody can know it without any security issues. But, only you have access to the decryption key (n, d) .
- ▶ Theoretically, an attacker could factor the modulus n and use the primes to calculate the decryption exponent d . The security of RSA depends on this being a very difficult problem.
- ▶ Our examples only include small primes so that we can understand how RSA works. In practice, the modulus will be 2048 or 3072 bits as recommended by NIST in 2015. This means that the modulus will be of the order of 600+ decimal digits.
- ▶ In general, RSA is not recommended for encrypting whole messages as it is computationally impractical. Instead it finds most of its use in key distribution (alongside Diffie-Hellman key exchange) and digital verification/authentication schemes.

RSA Public Key: shu.ac.uk

- ▶ shu.ac.uk uses the PKCS #1 RSA Encryption specification.

RSA Public Key: shu.ac.uk

► shu.ac.uk uses the PKCS #1 RSA Encryption specification.

► It uses public key

30 82 01 0a 02 82 01 01 00 ad ef f1 f6 86 63 6a 58 60 b7 d9 16 ad ed 3b
8a 76 f5 47 30 aa a2 b2 b7 14 4c 57 72 c8 8f 11 9b 7d 62 2f 15 15 45 0e
c8 4c 66 97 00 d1 9e f3 9a 59 9a 03 e3 93 93 a6 38 d9 16 5d ce 1e f6 02
f1 07 f3 ba 3e f1 a0 97 b6 d5 95 8c 5a c2 ad 88 04 15 86 15 08 45 a9 18
81 01 dd 75 ba 7e 31 07 78 3a f6 05 93 3e 37 90 fc 99 61 c2 ad 5f 69 a4
f3 59 b5 b5 90 f0 8e 14 97 be 31 6e c4 75 5b bf eb 11 4a 1f 87 c1 8f 97
ae ae 80 2f 8c 77 6a f8 c7 5c a2 d9 0b dd 3e 2f e4 03 d0 b4 d5 be 4f 50
8a d0 f7 c7 ea 7c 8c a5 69 94 7e dd 4b 72 e5 e0 fa 7a f9 be 38 f9 d9 90
af 34 d5 02 92 a2 99 3c 16 fb 23 e9 2d 0a 74 a1 38 f6 71 e2 8b 7a e1 ff
0b b1 b1 85 61 8f dc 21 9f e7 81 92 4c f3 24 01 df e1 c7 d2 66 94 f7 9b
8a 70 25 71 cf db a9 c4 94 a7 b0 e9 69 a6 2f ff a0 a5 bc f6 5f dc 23 6e
cf 02 03 01 00 01

► The modulus is the part of the key in black. The encryption exponent is the last three bytes (in green). The rest is to do with Distinguished Encoding Rules.

RSA Public Key: shu.ac.uk

► shu.ac.uk uses the PKCS #1 RSA Encryption specification.

► It uses public key

30 82 01 0a 02 82 01 01 00 ad ef f1 f6 86 63 6a 58 60 b7 d9 16 ad ed 3b
8a 76 f5 47 30 aa a2 b2 b7 14 4c 57 72 c8 8f 11 9b 7d 62 2f 15 15 45 0e
c8 4c 66 97 00 d1 9e f3 9a 59 9a 03 e3 93 93 a6 38 d9 16 5d ce 1e f6 02
f1 07 f3 ba 3e f1 a0 97 b6 d5 95 8c 5a c2 ad 88 04 15 86 15 08 45 a9 18
81 01 dd 75 ba 7e 31 07 78 3a f6 05 93 3e 37 90 fc 99 61 c2 ad 5f 69 a4
f3 59 b5 b5 90 f0 8e 14 97 be 31 6e c4 75 5b bf eb 11 4a 1f 87 c1 8f 97
ae ae 80 2f 8c 77 6a f8 c7 5c a2 d9 0b dd 3e 2f e4 03 d0 b4 d5 be 4f 50
8a d0 f7 c7 ea 7c 8c a5 69 94 7e dd 4b 72 e5 e0 fa 7a f9 be 38 f9 d9 90
af 34 d5 02 92 a2 99 3c 16 fb 23 e9 2d 0a 74 a1 38 f6 71 e2 8b 7a e1 ff
0b b1 b1 85 61 8f dc 21 9f e7 81 92 4c f3 24 01 df e1 c7 d2 66 94 f7 9b
8a 70 25 71 cf db a9 c4 94 a7 b0 e9 69 a6 2f ff a0 a5 bc f6 5f dc 23 6e
cf 02 03 01 00 01

► The modulus is the part of the key in black. The encryption exponent is the last three bytes (in green). The rest is to do with Distinguished Encoding Rules.

RSA Public Key: shu.ac.uk

- ▶ The actual modulus in denary is:

1439009874334747912175708580696616782489633913392424042420786858
8493385513036561327933889300201125505036733509500125430474349113
8392109631132209102846208024434773766208990866664115880472151837
2314480955340551600970256190940881256277567983112709841760074703
7746991422383533253603181640828388090653889268036267798634609551
9367579321471090171158258338355210007966141574788860342131168459
2859805329240698372132726428980674548023372409653959742420976883
5330296670239424922714039292364401817591460156344469804908668602
9534570294078337741486136447231027678544026989306624260832616420
6560609424012939961934155724915138865916740099.

RSA Public Key: shu.ac.uk

- ▶ The actual modulus in denary is:

1439009874334747912175708580696616782489633913392424042420786858
8493385513036561327933889300201125505036733509500125430474349113
8392109631132209102846208024434773766208990866664115880472151837
2314480955340551600970256190940881256277567983112709841760074703
7746991422383533253603181640828388090653889268036267798634609551
9367579321471090171158258338355210007966141574788860342131168459
2859805329240698372132726428980674548023372409653959742420976883
5330296670239424922714039292364401817591460156344469804908668602
9534570294078337741486136447231027678544026989306624260832616420
6560609424012939961934155724915138865916740099.

- ▶ The encryption exponent in denary is: 65537.

RSA Public Key: shu.ac.uk

- ▶ The actual modulus in denary is:

1439009874334747912175708580696616782489633913392424042420786858
8493385513036561327933889300201125505036733509500125430474349113
8392109631132209102846208024434773766208990866664115880472151837
2314480955340551600970256190940881256277567983112709841760074703
7746991422383533253603181640828388090653889268036267798634609551
9367579321471090171158258338355210007966141574788860342131168459
2859805329240698372132726428980674548023372409653959742420976883
5330296670239424922714039292364401817591460156344469804908668602
9534570294078337741486136447231027678544026989306624260832616420
6560609424012939961934155724915138865916740099.

- ▶ The encryption exponent in denary is: 65537.
- ▶ Encryption exponents are often primes of the form $2^n + 1$, as this speeds up computation (repeated squaring). E.g., $2^1 + 1 = 3$, $2^4 + 1 = 17$, or $2^{16} + 1 = 65537$.

RSA in Practice

- ▶ Part of PKCS #1 is how to **pad** the message. This is a way of adding random noise to the message to avoid sending the same message in the same way multiple times (certain attacks can exploit this).

RSA in Practice

- ▶ Part of PKCS #1 is how to **pad** the message. This is a way of adding random noise to the message to avoid sending the same message in the same way multiple times (certain attacks can exploit this).
- ▶ Another common padding scheme is OAEP.

RSA in Practice

- ▶ Part of PKCS #1 is how to **pad** the message. This is a way of adding random noise to the message to avoid sending the same message in the same way multiple times (certain attacks can exploit this).
- ▶ Another common padding scheme is OAEP.
- ▶ Investigating possible attacks on RSA is a very common topic: there are plenty of articles about this in 2024 alone.

Tutorials

In the tutorial this week we will:

- ▶ Create a spreadsheet to perform simple RSA encryption and allow us to handle larger modular powers.
- ▶ Use the spreadsheet from last week to calculate decryption exponents for RSA.
- ▶ Practice implementing RSA.