

Exercises 1: Elements and Subsets

Exercises

1. Let $enrolled = \{\text{Ann}, \text{Bob}, \text{Colin}, \text{Dan}\}$. Which of the following are true?

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|---|---|
| (a) $\{\text{Ann}, \text{Dan}\} \subseteq enrolled$. | (f) $\text{Ann} \in enrolled$. |
| (b) $\{\text{Ann}, \text{Dan}\} \subset enrolled$. | (g) $\text{Ann} \subseteq enrolled$. |
| (c) $\{\text{Ann}, \text{Dan}\} \subset \{\text{Ann}, \text{Dan}\}$. | (h) $\{\text{Ann}\} \subseteq enrolled$. |
| (d) $\{\text{Ann}, \text{Dan}\} \subseteq \{\text{Ann}, \text{Dan}\}$. | (i) $\{\text{Ann}\} \in enrolled$. |
| (e) $\{\text{Ann}, \text{Dan}\} \in enrolled$. | |

2. Which of the following statements are true?

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|---|---|
| (a) $\mathbb{Z} \subseteq \mathbb{N}$. | (h) $\{1\} \subseteq \mathbb{Z}$. |
| (b) $\mathbb{N} \subseteq \mathbb{Z}$. | (i) $\{1\} \in \mathbb{Z}$. |
| (c) $\{1, 3, 7\} \subset \mathbb{N}$. | (j) $\emptyset \subseteq \mathbb{Z}$. |
| (d) $\{1, 3, 7\} \subset \{1, 3, 7\}$. | (k) $\{0\} \subseteq \emptyset$. |
| (e) $\{1, 3, 7\} \subseteq \{1, 3, 7\}$. | (l) $\emptyset \in \{1, 2\}$. |
| (f) $1 \in \mathbb{Z}$. | (m) $\{\emptyset\} \subseteq \emptyset$. |
| (g) $1 \subseteq \mathbb{Z}$. | (n) $\emptyset \subseteq \{\emptyset\}$. |

3. Let $A = \{1, 2, 3, \dots, 20\}$ (the set of all integers from 1 to 20) and $B = \{2, 4, 6, \dots, 30\}$ (the set of all even integers from 2 to 30). Write down the following sets by listing their elements:

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| (a) $\{x \mid x \in A \text{ and } x \text{ is a perfect square}\}$ | (d) $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 2\}$ |
| (b) $\{x \mid x \in B \text{ and } x \text{ is prime}\}$ | (e) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = 2\}$ |
| (c) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 \leq 25\}$ | (f) $\{x \mid x \in \mathbb{R} \text{ and } 6 < x < 3\}$ |

4. Say, with reasons, which, if any, of the following sets are equal:

$$A = \{1, 2, 3\},$$

$$B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\},$$

$$C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\},$$

$$D = \emptyset.$$

Solutions

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|--------------|-----------|
| 1. (a) True | (f) True |
| (b) True | (g) False |
| (c) False | (h) True |
| (d) True | (i) False |
| (e) False | |
| 2. (a) False | (h) True |
| (b) True | (i) False |
| (c) True | (j) True |
| (d) False | (k) False |
| (e) True | (l) False |
| (f) True | (m) False |
| (g) False | (n) True |

Video Visit the URL below to view a video:

<https://www.youtube.com/embed/nqr6J0JU1Jk>

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|--|---|
| 3. (a) This is the square numbers which are between 1 and 20: $\{1, 4, 9, 16\}$. | squared, are equal to 2: $\{\sqrt{2}, -\sqrt{2}\}$. |
| (b) The set B only contains even numbers, so the only prime in that set is 2: $\{2\}$. | (e) Integers which, when squared, are equal to 2: \emptyset . There are no integers which square to 2. |
| (c) Integers which, when squared, are less than or equal to 25: $\{0, -1, 1, -2, 2, -3, 3, -4, 4, -5, 5\}$. It's easy to forget the negatives here! | (f) Real numbers which are both more than 6, but less than 3: \emptyset . Think about it a bit to realise that there are no numbers which satisfy both of these conditions. |
| (d) Real numbers which, when | |

4. $A = \{1, 2, 3\},$

$$B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\} = \{1, 2, 3\},$$

$$C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\} = \{0\},$$

$$D = \emptyset.$$

So only $A = B$ here.