

# Exercises 2: Truth Tables

## Exercises

- Construct truth tables for the following formulae:

(a)  $\neg(\neg p \vee \neg q)$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
$F$	$F$				
$F$	$T$				
$T$	$F$				
$T$	$T$				

(b)  $\neg(\neg p \wedge \neg q)$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
$F$	$F$				
$F$	$T$				
$T$	$F$				
$T$	$T$				

(c)  $p \wedge (p \vee q)$

$p$	$q$	$p \vee q$	$p \wedge (p \vee q)$
$F$	$F$		
$F$	$T$		
$T$	$F$		
$T$	$T$		

(d)  $p \wedge (\neg p \vee q)$

$p$	$q$	$\neg p$	$\neg p \vee q$	$p \wedge (\neg p \vee q)$
$F$	$F$			
$F$	$T$			
$T$	$F$			
$T$	$T$			

(e)  $p \vee (\neg p \vee q)$

$p$	$q$	$\neg p$	$\neg p \vee q$	$p \vee (\neg p \vee q)$
$F$	$F$			
$F$	$T$			
$T$	$F$			
$T$	$T$			

Do you notice anything interesting about your answers?

2. Construct a truth table for  $p \wedge q \vee p \wedge \neg r$ . Note: This table requires 8 rows; every possible combination of false and true for  $p$ ,  $q$  and  $r$  must be included. You are advised to insert brackets in the correct places before you start. You can use the table below as a guide.

$p$	$q$	$r$	$p \wedge q$	$\neg r$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
$F$	$F$	$F$				
$F$	$F$	$T$				
$F$	$T$	$F$				
$F$	$T$	$T$				
$T$	$F$	$F$				
$T$	$F$	$T$				
$T$	$T$	$F$				
$T$	$T$	$T$				

3. Try to construct a truth table the final column of which is:  $T F T F T F T T$ , read from the top down with the values of  $p$ ,  $q$  and  $r$  in the usual order.
4. Using truth tables check the following laws of boolean algebra:
- (a)  $p \vee (p \wedge q) = p$
- (b)  $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
- (c)  $p \vee (\neg p \wedge q) = p \vee q$

## Solutions

1. Construct truth tables for the following formulae:

(a)  $\neg(\neg p \vee \neg q)$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
$F$	$F$	$T$	$T$	$T$	$F$
$F$	$T$	$T$	$F$	$T$	$F$
$T$	$F$	$F$	$T$	$T$	$F$
$T$	$T$	$F$	$F$	$F$	$T$

$\text{sP } \neg(\neg p \vee \neg q) = p \wedge q.$

(b)  $\neg(\neg p \wedge \neg q)$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
$F$	$F$	$T$	$T$	$T$	$F$
$F$	$T$	$T$	$F$	$F$	$T$
$T$	$F$	$F$	$T$	$F$	$T$
$T$	$T$	$F$	$F$	$F$	$T$

So  $\neg(\neg p \wedge \neg q) = p \vee q$ .

(c)  $p \wedge (p \vee q)$

$p$	$q$	$p \vee q$	$p \wedge (p \vee q)$
$F$	$F$	$F$	$F$
$F$	$T$	$T$	$F$
$T$	$F$	$T$	$T$
$T$	$T$	$T$	$T$

So  $p \wedge (p \vee q) = p$ .

(d)  $p \wedge (\neg p \vee q)$

$p$	$q$	$\neg p$	$\neg p \vee q$	$p \wedge (\neg p \vee q)$
$F$	$F$	$T$	$T$	$F$
$F$	$T$	$T$	$T$	$F$
$T$	$F$	$F$	$F$	$F$
$T$	$T$	$F$	$T$	$T$

So  $p \wedge (\neg p \vee q) = p \wedge q$ .

(e)  $p \vee (\neg p \vee q)$

$p$	$q$	$\neg p$	$\neg p \vee q$	$p \vee (\neg p \vee q)$
$F$	$F$	$T$	$T$	$T$
$F$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$T$
$T$	$T$	$F$	$T$	$T$

So  $p \vee (\neg p \vee q) = T$ , a tautology.

2. Construct a truth table for  $p \wedge q \vee p \wedge \neg r$ . Note: This table requires 8 rows; every possible combination of false and true for  $p$ ,  $q$  and  $r$  must be included. You are advised to insert brackets in the correct places before you start. You can use the table below as a guide.

$p$	$q$	$r$	$p \wedge q$	$\neg r$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
$F$	$F$	$F$	$F$	$T$	$F$	$F$
$F$	$F$	$T$	$F$	$F$	$F$	$F$
$F$	$T$	$F$	$F$	$T$	$F$	$F$
$F$	$T$	$T$	$F$	$F$	$F$	$F$
$T$	$F$	$F$	$F$	$T$	$T$	$T$
$T$	$F$	$T$	$F$	$F$	$F$	$F$
$T$	$T$	$F$	$T$	$T$	$T$	$T$
$T$	$T$	$T$	$T$	$F$	$F$	$T$

3. There are many ways to attempt this, some of which are more systematic than others. To do it by hand, just look for a pattern and then slowly adjust it based on the inputs ( $T$  or  $F$ ) that you are giving it from the different atomic statements ( $p$ ,  $q$ , and  $r$ ).
4. To verify these laws, draw up a truth table for each side of the equation. Then check that you get the same truth table for each part.

## 2.1 Selected Video Solutions

**Video** Visit the URL below to view a video:  
<https://www.youtube.com/embed/9JHqTQE4zqs>