Exercises 1: Propositions and Connectives

Exercises

1. State which of the following sentences are propositions:
   (a) The cow jumped over the moon.
   (b) Don't go away.
   (c) If you read all the notes, try all the exercises, and do the assignments, then you'll learn all the maths you need.
   (d) What are you doing?
2. Let p = "Jill is rich", q = "Jill is brave" and r = "Jill is honest". Write the following statements in symbolic form, using p, q, r, and using $\wedge$ = AND, $\vee$ = OR and $\neg$ = NOT:
   (a) Jill is rich and cowardly.
   (b) Jill is neither brave nor honest.
   (c) Jill is either rich or a dishonest coward (or both).
   (d) Jill is either rich or brave (but not both).
3. Suppose that `year` is an integer variable holding a year number (e.g., 2008). The condition for an integer a to be divisible by q is written $a \bmod q = 0$. So, for example, $(2008 \bmod 4 = 0)$ is a true statement.
   (a) A number from 2001 to 2399 represents a leap year if it is divisible by 4 but not by 100. Write this condition in symbolic form.
   (b) Any number from 1800 onwards represents a leap year if it is divisible by 4 but not by 100, except that numbers divisible by 400 (such as 2000) do represent leap years. Write this condition in symbolic form.
   (c) Check your answer by working out your expressions for the years 2000, 2008,2009 and 2100.

Solutions

1. A proposition must either be true (T) or false (F). When deciding if a statement is a proposition we do not need to say whether it *is* true or false, just that it must be one or the other.

(a) 'The cow jumped over the moon.' This is a proposition since the cow either did or did not jump over the moon.

(b) 'Don't go away.' This is an instruction. You could comply with it or you could ignore it. But it isn't true or false.

(c) 'If you read all the notes, try all the exercises, and do the assignments, then you'll learn all the maths you need.' This is a proposition. Either you will learn everything you need, or you won't. Only time will tell!

(d) 'What are you doing?' This is a question. You could respond or you could ignore it. But again, it is neither true nor false.

2. (a) 'Jill is rich and cowardly.' So

$$\text{'Jill is cowardly'} \;=\; \text{NOT 'Jill is brave.'}.$$

Or we can write $\neg\, q$. To have both rich `AND` cowardly we can then put

$$\text{'Jill is rich and cowardly'} \;= p \;\wedge\; (\neg\, q).$$

(b) 'Jill is neither brave nor honest.' This means that Jill is not brave nor honest. This would mean that q is false and r is false, so $q \vee r$ is false. Hence $\neg\,(q \vee r)$ would be true. (Aside: That $\neg\, q \wedge \neg\, r = \neg\,(q \vee r)$ is an instance of the rule called *de Morgan's law*.) It is tempting to think that $\neg\,(q \wedge r)$ would work here. But this means that Jill is not (brave `AND` honest). She could still be one of them but we want her to be neither.

(c) 'Jill is either rich or a dishonest coward (or both).' The '(or both)' tells us that we need an incluse `OR` ($\vee$). A 'dishonest coward' is someone who is 'neither brave nor honest', so the answer from the previous part works: $\neg\, q \;\wedge\; \neg\, r$. Altogether, we get

$$p \;\vee\; (\neg\, q \;\wedge\; \neg\, r).$$

(d) 'Jill is either rich or brave (but not both).' The '(but not both)' tells us that we need more than just the usual `OR`. What we need is an exclusive `OR`. (Or `XOR`.) We get this as follows:

$$(p \;\wedge\; \neg\, q) \;\vee\; (\neg\, p \;\wedge\; q).$$

With this we can have p or q being true but not both.

3. (a) $(\texttt{year} \bmod 4 = 0) \wedge (\texttt{year} \bmod 100 \neq 0)$

(b) $(\texttt{year} \bmod 400 = 0) \vee (\texttt{year} \bmod 4 = 0) \wedge (\texttt{year} \bmod 100 \neq 0)$ or, alternatively, $(\texttt{year} \bmod 4 = 0) \wedge ((\texttt{year} \bmod 400 = 0) \vee (\texttt{year} \bmod 100 \neq 0))$

(c) If $\texttt{year} = 2000$, then $\texttt{year} \bmod 4 = 0$ is true, $\texttt{year} \bmod 400 = 0$ is true, and $\texttt{year} \bmod 100 \neq 0$ is false.

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