Exercises 1: Elements and Subsets

Exercises

1. Let $enrolled = \{\text{Ann}, \text{Bob}, \text{Colin}, \text{Dan}\}$. Which of the following are true?

   (a) $\{\text{Ann}, \text{Dan}\} \subseteq enrolled$.
   (b) $\{\text{Ann}, \text{Dan}\} \subset enrolled$.
   (c) $\{\text{Ann}, \text{Dan}\} \subset \{\text{Ann}, \text{Dan}\}$.
   (d) $\{\text{Ann}, \text{Dan}\} \subseteq \{\text{Ann}, \text{Dan}\}$.
   (e) $\{\text{Ann}, \text{Dan}\} \in enrolled$.
   (f) $\text{Ann} \in enrolled$.
   (g) $\text{Ann} \subseteq enrolled$.
   (h) $\{\text{Ann}\} \subseteq enrolled$.
   (i) $\{\text{Ann}\} \in enrolled$.

2. Which of the following statements are true?

   (a) $\mathbb{Z} \subseteq \mathbb{N}$.
   (b) $\mathbb{N} \subseteq \mathbb{Z}$.
   (c) $\{1, 3, 7\} \subset \mathbb{N}$.
   (d) $\{1, 3, 7\} \subset \{1, 3, 7\}$.
   (e) $\{1, 3, 7\} \subseteq \{1, 3, 7\}$.
   (f) $1 \in \mathbb{Z}$.
   (g) $1 \subseteq \mathbb{Z}$.
   (h) $\{1\} \subseteq \mathbb{Z}$.
   (i) $\{1\} \in \mathbb{Z}$.
   (j) $\emptyset \subseteq \mathbb{Z}$.
   (k) $\{0\} \subseteq \emptyset$.
   (l) $\emptyset \in \{1, 2\}$.
   (m) $\{\emptyset\} \subseteq \emptyset$.
   (n) $\emptyset \subseteq \{\emptyset\}$.

3. Let $A = \{1, 2, 3, \ldots, 20\}$ (the set of all integers from 1 to 20) and $B = \{2, 4, 6, \ldots, 30\}$ (the set of all even integers from 2 to 30). Write down the following sets by listing their elements:

   (a) $\{x \mid x \in A \text{ and } x \text{ is a perfect square}\}$
   (b) $\{x \mid x \in B \text{ and } x \text{ is prime}\}$
   (c) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 \leq 25\}$
   (d) $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 2\}$
   (e) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = 2\}$
   (f) $\{x \mid x \in \mathbb{R} \text{ and } 6 < x < 3\}$

4. Say, with reasons, which, if any, of the following sets are equal:

   $A = \{1, 2, 3\}$,

$B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\}$,

$C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\}$,

$D = \emptyset$.

Solutions

1. (a) True
   (b) True
   (c) False
   (d) True
   (e) False
   (f) True
   (g) False
   (h) True
   (i) False

2. (a) False
   (b) True
   (c) True
   (d) False
   (e) True
   (f) True
   (g) False
   (h) True
   (i) False
   (j) True
   (k) False
   (l) False
   (m) False
   (n) True

   Video Visit the URL below to view a video:
   `https://www.youtube.com/embed/nqr6J0JU1Jk`

3. (a) This is the square numbers which are between 1 and 20: {1, 4, 9, 16}.
   (b) The set B only contains even numbers, so the only prime in that set is 2: $\{2\}$.
   (c) Integers which, when squared, are less than or equal to 25: $\{0, -1, 1, -2, 2, -3, 3, -4, 4, -5, 5\}$. It's easy to forget the negatives here!
   (d) Real numbers which, when squared, are equal to 2: $\{\sqrt{2}, -\sqrt{2}\}$.
   (e) Integers which, when squared, are equal to 2: $\varnothing$. There are no integers which square to 2.
   (f) Real numbers which are both more than 6, but less than 3: $\varnothing$. Think about it a bit to realise that there are no numbers which satisfy both of these conditions.

4. $A = \{1, 2, 3\}$,

   $B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\} = \{1, 2, 3\}$,

   $C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\} = \{0\}$,

   $D = \emptyset$.

   So only $A = B$ here.