

# Exercises 2: Set Operations

## Exercises

1. Using an appropriate Venn Diagram in each case indicate the following sets.

(a)  $(A \cap B) \cup B^c$

(e)  $(A \cup B) \cap C$

(b)  $A \setminus (A \setminus B)$

(f)  $(A \cup B) \cap (A \cup C)$

(c)  $A \cap (B \setminus A)$

(g)  $A^c \cup B^c \cup C^c$

(d)  $A \cup (B \cap C)$

(h)  $A^c \cap (B \setminus C^c)$

2. Write down the following sets by listing their elements:

(a)  $\mathbb{P}\{0, 1, 2\}$

(b)  $\{0, 1, 2\} \times \{1, 2\}$

3. A window on a computer screen has 480 rows of pixels, numbered from 0 (bottom) to 479 (top), and 640 pixels in each row, numbered from 0 (left) to 639 (right). How can the set of all possible pixel positions be represented as a Cartesian product? Write out, in the language of sets, the following parts of the screen:

(a) the left-hand half of the window

(b) the third row from the top

(c) the right-hand column of the window

(d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

4. If  $A = \{\text{May}, \text{June}\}$ , write down the following sets:

(a)  $\mathbb{P}(A)$

(b)  $A \cup \mathbb{P}(A)$

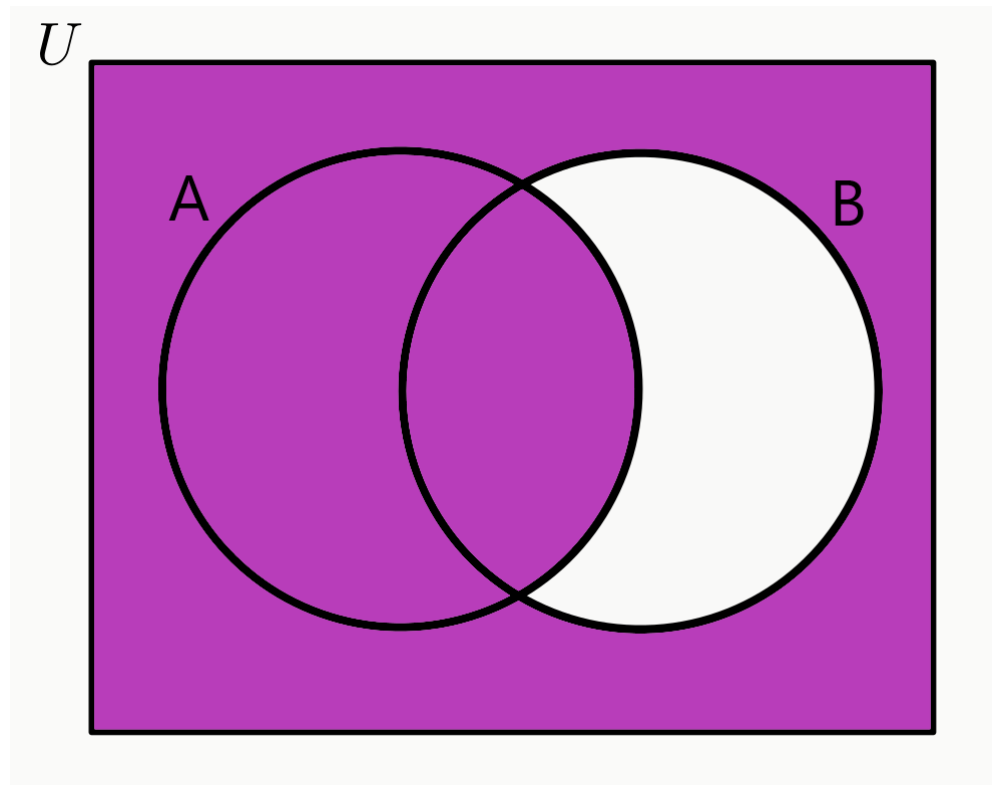
(c)  $A \cap \mathbb{P}(A)$

(d)  $\{A\} \cup \mathbb{P}(A)$

(e)  $\{A\} \cap \mathbb{P}(A)$

5. Give an example of three sets  $A$ ,  $B$  and  $C$  such that  $A \cap B \cap C = \emptyset$ , but  $A \cap B$ ,  $B \cap C$  and  $C \cap A$  are all non-empty.
6. If we wished to bring the operator  $A \setminus B$  into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)
  - (a)  $A \setminus (B \setminus C) = (A \setminus B) \setminus C$ .
  - (b)  $A \setminus \emptyset = A$ .
  - (c)  $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$ .
  - (d)  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ .
  - (e)  $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ .
  - (f)  $(A \cup B) \setminus C = (A \setminus C) \cup B$ .

## Solutions

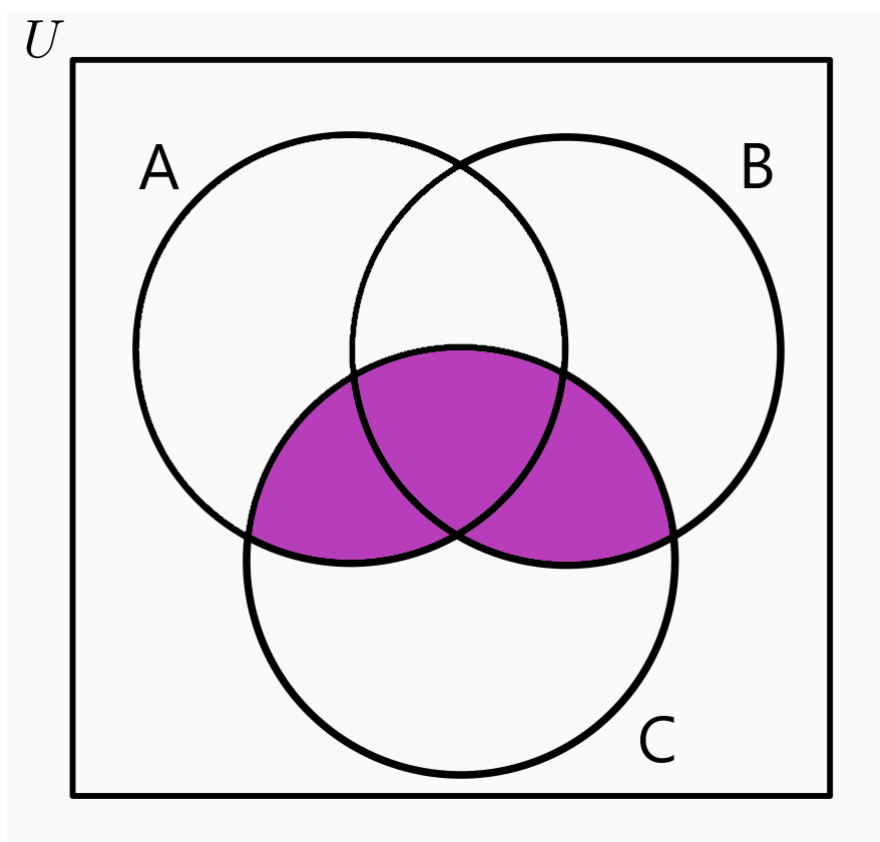


1. (a)

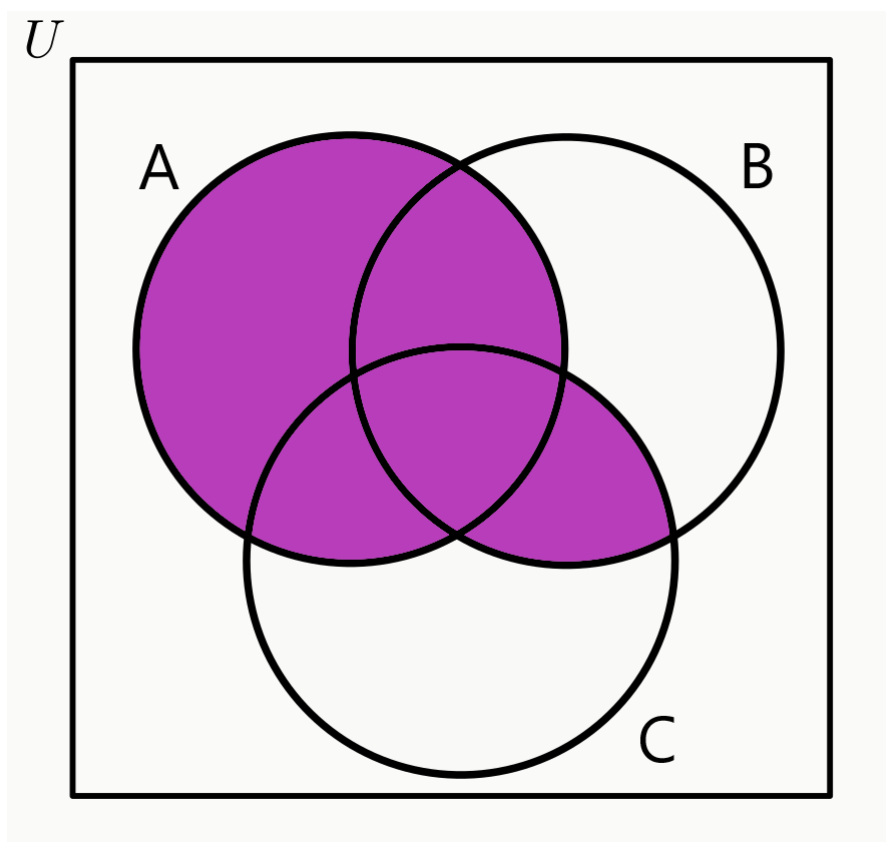




(d)



(e)



(f)



(g)



(h)

2. Write down the following sets by listing their elements:

(a)  $\mathbb{P}\{0, 1, 2\} = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$

(b)  $\{0, 1, 2\} \times \{1, 2\} = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

3. Let  $W = \{0, 1, 2, \dots, 639\} \times \{0, 1, 2, \dots, 479\}$  be the set representing the pairs of pixel in the window area.

(a) the left-hand half of the window

$$L = \{(x, y) \in W \mid x \leq 319\}$$

(b) the third row from the top

$$R_{477} = \{(x, y) \in W \mid y = 477\}$$

(c) the right-hand column of the window

$$C_{639} = \{(x, y) \in W \mid x = 639\}$$

(d) a rectangular area going from the sixth row from the bottom to the eighteenth, a quarter of the width of the window and centrally placed.

$$A = \{(x, y) \in W \mid 5 \leq y \leq 17 \wedge 239 \leq x \leq 399\}$$



4. If  $A = \{\text{May}, \text{June}\}$ , write down the following sets:
- (a)  $\mathbb{P}(A) = \{\emptyset, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$
  - (b)  $A \cup \mathbb{P}(A) = \{\text{May}, \text{June}, \emptyset, \{\text{May}\}, \{\text{June}\}, \{\text{May}, \text{June}\}\}$
  - (c)  $A \cap \mathbb{P}(A) = \emptyset$
  - (d)  $\{A\} \cup \mathbb{P}(A) = \mathbb{P}(A)$
  - (e)  $\{A\} \cap \mathbb{P}(A) = A$
5.  $A = \{1, 2\}$ ,  $B = \{1, 3\}$ ,  $C = \{2, 3\}$  is but one example.
6. If we wished to bring the operator  $A \setminus B$  into the laws of the algebra of sets, which of the following candidates would be acceptable as laws? (You may find it helpful to draw Venn diagrams.)
- (a)  $A \setminus (B \setminus C) = (A \setminus B) \setminus C$  is False.
  - (b)  $A \setminus \emptyset = A$  is True.
  - (c)  $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$  is False.
  - (d)  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$  is True.
  - (e)  $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$  is True.
  - (f)  $(A \cup B) \setminus C = (A \setminus C) \cup B$  is False.