Exercises 2: Truth Tables

Exercises

1. Construct truth tables for the following formulae:

   (a) $\neg(\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
F	F				
F	T				
T	F				
T	T				

   (b) $\neg(\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
F	F				
F	T				
T	F				
T	T				

   (c) $p \wedge (p \vee q)$

p	q	$p \vee q$	$p \wedge (p \vee q)$
F	F		
F	T		
T	F		
T	T		

   (d) $p \wedge (\neg p \vee q)$

p	q	$\neg p$	$\neg p \vee q$	$p \wedge (\neg p \vee q)$
F	F			
F	T			
T	F			
T	T			

   (e) $p \vee (\neg p \vee q)$

p	q	$\neg p$	$\neg p \vee q$	$p \vee (\neg p \vee q)$
F	F			
F	T			
T	F			
T	T			

Do you notice anything interesting about your answers?

2. Construct a truth table for $p \wedge q \vee p \wedge \neg r$. Note: This table requires 8 rows; every possible combination of false and true for p, q and r must be included. You are advised to insert brackets in the correct places before you start. You can use the table below as a guide.

p	q	r	$p \wedge q$	$\neg r$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
F	F	F				
F	F	T				
F	T	F				
F	T	T				
T	F	F				
T	F	T				
T	T	F				
T	T	T				

3. Try to construct a truth table the final column of which is: $T\ F\ T\ F\ T\ F\ T\ T$, read from the top down with the values of p, q and r in the usual order.

4. Using truth tables check the following laws of boolean algebra:

   (a) $p \vee (p \wedge q) = p$

   (b) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

   (c) $p \vee (\neg p \wedge q) = p \vee q$

Solutions

1. Construct truth tables for the following formulae:

   (a) $\neg(\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
F	F	T	T	T	F
F	T	T	F	T	F
T	F	F	T	T	F
T	T	F	F	F	T

   sP $\neg\ (\neg\ p\ \vee\ \neg\ q) = p\ \wedge\ q$.

   (b) $\neg(\neg p \wedge \neg q)$

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p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
F	F	T	T	T	F
F	T	T	F	F	T
T	F	F	T	F	T
T	T	F	F	F	T

So $\neg (\neg p \wedge \neg q) = p \vee q$.

(c) $p \wedge (p \vee q)$

p	q	$p \vee q$	$p \wedge (p \vee q)$
F	F	F	F
F	T	T	F
T	F	T	T
T	T	T	T

So $p \wedge (p \vee q) = p$.

(d) $p \wedge (\neg p \vee q)$

p	q	$\neg p$	$\neg p \vee q$	$p \wedge (\neg p \vee q)$
F	F	T	T	F
F	T	T	T	F
T	F	F	F	F
T	T	F	T	T

Sp $p \wedge (\neg p \vee q) = p \wedge q$.

(e) $p \vee (\neg p \vee q)$

p	q	$\neg p$	$\neg p \vee q$	$p \vee (\neg p \vee q)$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	T
T	T	F	T	T

So $p \vee (\neg p \vee q) = T$, a tautology.

2. Construct a truth table for $p \wedge q \vee p \wedge \neg r$. Note: This table requires 8 rows; every possible combination of false and true for p, q and r must be included. You are advised to insert brackets in the correct places before you start. You can use the table below as a guide.

p	q	r	$p \wedge q$	$\neg r$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
F	F	F	F	T	F	F
F	F	T	F	F	F	F
F	T	F	F	T	F	F
F	T	T	F	F	F	F
T	F	F	F	T	T	T
T	F	T	F	F	F	F
T	T	F	T	T	T	T
T	T	T	T	F	F	T

3. There are many ways to attempt this, some of which are more systematic than others. To do it by hand, just look for a pattern and then slowly adjust it based on the inputs (T or F) that you are giving it from the different atomic statements (p, q, and r).

4. To verify these laws, draw up a truth table for each side of the equation. Then check that you get the same truth table for each part.

2.1   Selected Video Solutions

Video Visit the URL below to view a video:
`https://www.youtube.com/embed/9JHqTQE4zqs`