Exercises: Euclidean Algorithm

Spreadsheet Exercise: Extended Euclidean Algorithm

In this part we will get Excel to run the Extended Euclidean Algorithm, so that next week we can use it to calculate decryption exponents for RSA. To begin with, download the Excel spreadsheet: `05-RSA.xlsx`.

1. First, use the extended Euclidean algorithm and, by hand, show that the highest common factor of 17 and 60 is 1. Show that the algorithm also gives you that $2 \times 60 - 7 \times 17 = 1$.

2. First make sure you are in the worksheet named '`RSA Decryption Exponent`'. Ignore the values in the range H1:H5 for now, these will come into play next week for RSA.

3. First we will calculate how many times the second number goes into the first. In Cell D3, use the INT function to find the value of C1 divided by C2, rounded down.

4. In Cells A3:C3, we now want to take the second number (in Row 2), multiplied by the value in Cell D3, from the first number (in Row 1). E.g., in Cell A3 we would use a formula like `=A1-D3*A2`, except you will need to put in appropriate absolute referencing so that you can copy it across to Cells B3 and C3.

5. Now drag the formulas from A3:D3 down for about ten or twenty rows. Don't worry about `#DIV/0!` errors. If you have done this correctly you should find that the number 1 is in Cell C5 - this is the highest common factor of 60 and 17. Try putting in 112 into Cell C1 and 986 into Cell C2 to see that the highest common factor is 2.

Exercises

1. Use the Euclidean algorithm to find the highest common factor of:

   (a) 24 and 36,

   (b) 56 and 36,

(c) 1200 and 256,

(d) 12345 and 54321.

2. Use the extended Euclidean algorithm to find integer solutions to the following equations:

   (a) $24x + 36y = 12$,

   (b) $24x + 36y = 0$,

   (c) $56x + 36y = 4$,

   (d) $56x + 36y = 8$,

   (e) $1200x + 256y = 80$,

   (f) $1200x + 256y = 32$,

   (g) $12345x + 54321y = 3$.

3. You have 2kg of flour. Your biscuit recipe requires 15g of flour per biscuit and your cupcake recipe requires 25 of flour per cupcake. We will solve the equation $15b + 25c = 2000$ (b, $c \in \mathbb{Z}$) to find the possible numbers of biscuits and cupcakes we can make with 2kg of flour.

   (a) Use the extended Euclidean algorithm to find the highest common factor of 15 and 20, and to find a solution to the equation $15b+25c = 5$.

   (b) Use your previous solution to find a solution to the equation $15b + 25c = 2000$.

   (c) Find a solution to the equation $15b + 25c = 0$.

   (d) Suppose that you want to make at least 50 biscuits and at least 40 cupcakes. Can you use your previous solutions (and some technology) to find the possible combinations of biscuits and cupcakes that you can make with 2kg of flour?

   (e) If you are able to sell biscuits at £2.00 and cupcakes at £3.00, what is your best option to make the most profit?