

# Exercises 1: Elements and Subsets

## Exercises

1. Let  $enrolled = \{Ann, Bob, Colin, Dan\}$ . Which of the following are true?

- |   |                                    |
|---|------------------------------------|
| (a) $\{Ann, Dan\} \subseteq enrolled$ .     | (f) $Ann \in enrolled$ .           |
| (b) $\{Ann, Dan\} \subset enrolled$ .       | (g) $Ann \subseteq enrolled$ .     |
| (c) $\{Ann, Dan\} \subset \{Ann, Dan\}$ .   | (h) $\{Ann\} \subseteq enrolled$ . |
| (d) $\{Ann, Dan\} \subseteq \{Ann, Dan\}$ . | (i) $\{Ann\} \in enrolled$ .       |
| (e) $\{Ann, Dan\} \in enrolled$ .           |                                    |

2. Which of the following statements are true?

- |   |   |
|---|---|
| (a) $\mathbb{Z} \subseteq \mathbb{N}$ .   | (h) $\{1\} \subseteq \mathbb{Z}$ .        |
| (b) $\mathbb{N} \subseteq \mathbb{Z}$ .   | (i) $\{1\} \in \mathbb{Z}$ .              |
| (c) $\{1, 3, 7\} \subset \mathbb{N}$ .    | (j) $\emptyset \subseteq \mathbb{Z}$ .    |
| (d) $\{1, 3, 7\} \subset \{1, 3, 7\}$ .   | (k) $\{0\} \subseteq \emptyset$ .         |
| (e) $\{1, 3, 7\} \subseteq \{1, 3, 7\}$ . | (l) $\emptyset \in \{1, 2\}$ .            |
| (f) $1 \in \mathbb{Z}$ .                  | (m) $\{\emptyset\} \subseteq \emptyset$ . |
| (g) $1 \subseteq \mathbb{Z}$ .            | (n) $\emptyset \subseteq \{\emptyset\}$ . |

3. Let  $A = \{1, 2, 3, \dots, 20\}$  (the set of all integers from 1 to 20) and  $B = \{2, 4, 6, \dots, 30\}$  (the set of all even integers from 2 to 30). Write down the following sets by listing their elements:

- |   |  |
|---|--|
| (a) $\{x \mid x \in A \text{ and } x \text{ is a perfect square}\}$ | (d) $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 2\}$   |
| (b) $\{x \mid x \in B \text{ and } x \text{ is prime}\}$            | (e) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = 2\}$   |
| (c) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 \leq 25\}$          | (f) $\{x \mid x \in \mathbb{R} \text{ and } 6 < x < 3\}$ |

4. Say, with reasons, which, if any, of the following sets are equal:

$$A = \{1, 2, 3\},$$

$$B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\},$$

$$C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\},$$

$$D = \emptyset.$$

## Solutions

- |              |           |
|--------------|-----------|
| 1. (a) True  | (f) True  |
| (b) True     | (g) False |
| (c) False    | (h) True  |
| (d) True     | (i) False |
| (e) False    |           |
| 2. (a) False | (h) True  |
| (b) True     | (i) False |
| (c) True     | (j) True  |
| (d) False    | (k) False |
| (e) True     | (l) False |
| (f) True     | (m) False |
| (g) False    | (n) True  |

**Video** Visit the URL below to view a video:

<https://www.youtube.com/embed/nqr6J0JU1Jk>

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|--|---|
| 3. (a) This is the square numbers which are between 1 and 20: $\{1, 4, 9, 16\}$ .  | squared, are equal to 2: $\{\sqrt{2}, -\sqrt{2}\}$ .  |
| (b) The set $B$ only contains even numbers, so the only prime in that set is 2: $\{2\}$ .  | (e) Integers which, when squared, are equal to 2: $\emptyset$ . There are no integers which square to 2.  |
| (c) Integers which, when squared, are less than or equal to 25: $\{0, -1, 1, -2, 2, -3, 3, -4, 4, -5, 5\}$ . It's easy to forget the negatives here! | (f) Real numbers which are both more than 6, but less than 3: $\emptyset$ . Think about it a bit to realise that there are no numbers which satisfy both of these conditions. |
| (d) Real numbers which, when   |   |

4.  $A = \{1, 2, 3\},$

$$B = \{n \mid n \in \mathbb{N} \text{ and } n > 0 \text{ and } n^2 < 10\} = \{1, 2, 3\},$$

$$C = \{n \mid n \in \mathbb{N} \text{ and } n^2 < 1\} = \{0\},$$

$$D = \emptyset.$$

So only  $A = B$  here.