Exercises 4: Predicates

Exercises

1. Using the suggested notation, write each of the following statements in the language of predicate logic:

   (a) Bats are mammals: $B(x)$, $M(x)$.

   (b) Sparrows are not mammals: $S(x)$, $M(x)$.

   (c) Students are always hard-working: $S(x)$, $H(x)$.

   (d) No student ever gets up early: $S(x)$, $G(x)$.

   (e) Snake bites sometimes kill you: $S(x)$, $K(x)$.

   (f) A child was ill: $C(x)$, $I(x)$.

   (g) Not all children were ill: $C(x)$, $I(x)$.

   (h) All ginger cats are male: $G(x)$, $C(x)$, $M(x)$.

   (i) Some small dogs are vicious: $D(x)$, $S(x)$, $V(x)$.

   (j) No car is safe unless it has good brakes: $C(x)$, $S(x)$, $B(x)$.

   (k) Not all wealthy people are both educated and cultured: $P(x)$, $W(x)$, $E(x)$, $C(x)$.

   (l) Not all cheap instruments are either soft or breakable: $I(x)$, $C(x)$, $S(x)$, $B(x)$.

   (m) Some seeds are edible only if they are cooked: $S(x)$, $E(x)$, $C(x)$.

   (n) There is a printer that is networked to every computer : $P(x)$, $C(y)$, $N(x, y)$.

   (o) Every computer has some printer which is networked to it : $P(x)$, $C(y)$, $N(x, y)$.

2. Form the negation of each of the statements in the question above; write these both in the language of formal logic and in English. You should do this both by finding the negation in English, and by using the rules for negation of quantified expressions with your answers above; check that you get the same results.

3. Let L be an ordered list containing 100 entries, with $L(m)$ denoting the value of the entry in the mth position. Suppose that indexing starts at 1, so $L(1)$ is the first entry in the list and $L(100)$ is the last entry in the list.

The entry in the mth position is given by the rule $L(m) = 2m + 1$. Write the following in predicate logic notation. You may assume all variables used in the statements will be integers, but be careful to define appropriate ranges. (E.g., $m \leq 99$ or $n \geq 2$.) In each case, decide if each statement is true or false.

(a) All entries in L are positive.

(b) Every entry in L is a square number.

(c) The list L is in increasing order.

(d) Some entries in L are twice as much as other entries.

Solutions

1. (a) Bats are mammals: $\forall\ x\ \bullet\ B(x) \Rightarrow M(x)$.

   (b) Sparrows are not mammals: $\forall\ x\ \bullet\ S(x) \Rightarrow \neg\ M(x)$.

   (c) Students are always hard-working: $\forall\ x\ \bullet\ S(x) \Rightarrow H(x)$.

   (d) No student ever gets up early: $\forall\ x\ \bullet\ S(x) \Rightarrow \neg\ G(x)$.

   (e) Snake bites sometimes kill you: $\exists\ x\ \bullet\ S(x)\ \wedge\ K(x)$.

   (f) A child was ill: $\exists\ x\ \bullet\ C(x)\ \wedge\ I(x)$.

   (g) Not all children were ill: $\exists\ x\ \bullet\ C(x)\ \wedge\ \neg\ I(x)$.

   (h) All ginger cats are male: $\forall\ x\ \bullet\ C(x)\ \wedge\ G(x) \Rightarrow M(x)$.

   (i) Some small dogs are vicious: $\exists\ x\ \bullet\ D(x)\ \wedge\ S(x)\ \wedge\ V(x)$.

   (j) No car is safe unless it has good brakes: $\forall\ x \bullet C(x) \Rightarrow (S(x) \Rightarrow B(x))$ or $\forall\ x\ \bullet\ C(x)\ \wedge\ S(x) \Rightarrow B(x)$ or $\forall\ x\ \bullet\ C(x)\ \wedge\ \neg\ B(x) \Rightarrow \neg\ S(x)$.

   (k) Not all wealthy people are both educated and cultured: $\exists\, x \bullet P(x) \wedge W(x) \wedge (\neg\, (E(x) \wedge C(x)))$.

   (l) Not all cheap instruments are either soft or breakable: $\exists\, x \bullet I(x) \wedge C(x) \wedge \neg\, (S(x) \vee B(x))$.

   (m) Some seeds are edible only if they are cooked: $\exists\, x \bullet S(x) \wedge \neg\, C(x) \wedge \neg\, E(x)$.

   (n) There is a printer that is networked to every computer : $\exists\, x \bullet \forall\, y \bullet P(x) \wedge (C(y) \Rightarrow N(x, y))$.

   (o) Every computer has some printer which is networked to it : $\forall\, y \bullet \exists\, x \bullet C(y) \Rightarrow P(x)\ \wedge\ N(x, y)$.

2. (a) Some bats are not mammals: $\exists\ x\ \bullet\ B(x)\ \wedge\ M(x)$.

   (b) Some sparrows are mammals: $\exists\ x\ \bullet\ S(x)\ \wedge\ M(x)$.

   (c) Some students are not hard-working: $\exists\ x\ \bullet\ S(x)\ \wedge\ \neg\ H(x)$.

   (d) Some students get up early: $\exists\ x\ \bullet\ S(x)\ \wedge\ G(x)$.

   (e) No snake bite kills you: $\forall\ x\ \bullet\ S(x) \Rightarrow \neg\ K(x)$.

   (f) No child was ill: $\forall\ x\ \bullet\ C(x) \Rightarrow \neg\ I(x)$.

   (g) Every child was ill: $\forall\ x\ \bullet\ C(x) \Rightarrow I(x)$.

   (h) Some ginger cats are not male: $\exists\ x\ \bullet\ C(x)\ \wedge\ G(x)\ \wedge\ \neg\ M(x)$.

   (i) No small dogs are vicious: $\forall\ x\ \bullet\ D(x)\ \wedge\ S(x) \Rightarrow \neg\ V(x)$.

(j) Some car is safe but does not have good brakes: $\exists\, x \bullet C(x) \wedge S(x) \wedge \neg\, B(x)$.

(k) All people who are wealthy are both educated and cultured: $\forall\, x \bullet P(x) \wedge W(x) \Rightarrow (E(x)\ \wedge\ C(x))$.

(l) All cheap instruments are either soft or breakable: $\forall\, x \bullet (I(x) \wedge C(x)) \Rightarrow (S(x)\ \vee\ B(x))$.

(m) All seeds are edible raw: $\forall\ x\ \bullet\ S(x) \Rightarrow (\neg\, C(x) \Rightarrow E(x))$.

(n) Every printer has some computer not networked to it : $\forall\, x \bullet \exists\, y \bullet P(x) \Rightarrow C(y)\ \wedge\ \neg\, N(x, y)$.

(o) There is a computer which has no printer networked to it : $\exists\, y \bullet \forall\, x \bullet C(y) \wedge (P(x) \Rightarrow \neg\, N(x, y))$.

3. (a) $\forall m \bullet L(m) > 0$ (or $\neg\ (\exists m \bullet L(m) \leq 0)$): True - all values of m are greater than or equal to 1, so $L(m) = 2m + 1 \geq 2(1) + 1 = 3$.

(b) $\forall m \bullet \exists n \bullet L(m) = n^2$: False - $L(1) = 3$, which is not a square number.

(c) Suppose that $m \leq 99$: $\forall m \bullet L(m) < L(m + 1)$. True - $L(m) = 2m + 1 < 2m + 3 = 2(m + 1) + 1 = L(m + 1)$.

(d) $\exists m \bullet \exists n \bullet L(m) = 2L(n)$: False - If such values of m and n did exist, then we would have $L(m) = 2m + 1 = 2(2n + 1) = L(n)$, i.e., $2m + 1 = 4n + 2$, meaning that $2m - 4n = 2 - 1 = 1$, by rearranging. So $2(m - 2n) = 1$, which cannot happen for an values of m and n, since the left hand side is an even number (divisible by 2), but the right hand side is an odd number.

Tutorial: Week 3

These are recordings from 2020-21. They are similar to what we did in the tutorial classes in Week 3. **Video** Visit the URL below to view a video:

`https://www.youtube.com/embed/LyhiQVUagtU`

Video Visit the URL below to view a video:
`https://www.youtube.com/embed/RDj7IfrEZGs`

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