

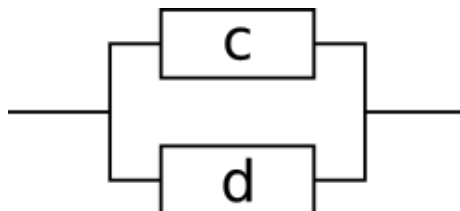
Exercises: Probability

1. A normal pack of 52 playing cards has 4 suits: clubs, diamonds, hearts, and spades, each with 13 cards. One card is drawn from the full pack at random, then replaced in the pack. Then a second card is drawn from the pack.
 - (a) Calculate the probability that both cards are clubs.
 - (b) Calculate the probability that at least one of the cards selected is a club.
 - (c) Calculate the probability that neither of the cards selected is a club.
 - (d) Use a different method to calculate the probability that at least one of the cards selected is a club.
2. A short network link consists of two sections, a and b , in series as shown below.



The link functions only if both a and b function. The two sections are independent in that the functionality of one does not affect the functionality of the other. The probability that a continues to be functional during a particular time period is 0.9. The probability that b continues to be functional during a particular time period is 0.95. Determine the probability that the link continues to function during a particular time period.

3. A short network link consists of two sections, c and d , in parallel as shown below.

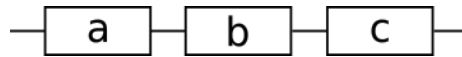


The link functions if either c or d (or both) function. The two sections are independent in that the functionality of one does not affect the functionality of the other. Consider the following statements.

- C : ' c continues to function during a particular time period'.
- D : ' d continues to function during a particular time period'.

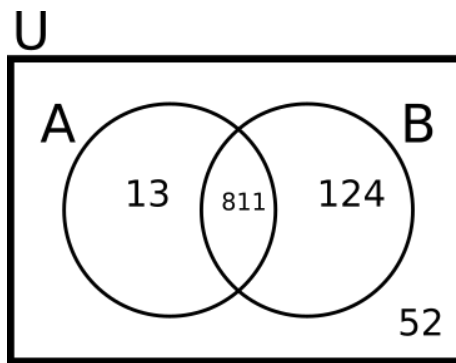
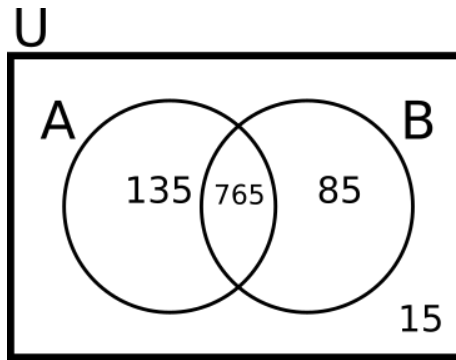
The probability that c continues to function during a particular time period is 0.95. The corresponding probability for d is 0.97.

- (a) Determine the probability that the link continues to function during a particular time period.
 - (b) Using the negation of each of the statements C and D define a statement which specifies the failure of the link.
 - (c) Determine the probability of link failure during a particular time period.
 - (d) Use your previous result to confirm the result of the first part.
4. A short network link consists of three sections, a , b , and c , in series as shown below.

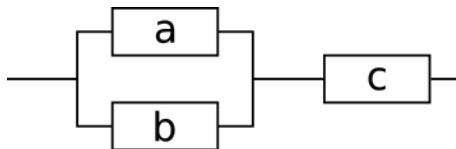


The link functions only if all three of a , b , and c function. The three sections are independent in that the functionality of one does not affect the functionality of any other. The probability that a continues to be functional during a particular time period is 0.95. The corresponding probabilities for b and c are 0.96 and 0.91, respectively. Determine the probability that the link continues to function during a particular time period.

5. The following Venn diagram represents the occurrence of two events A and B . The out rectangle is the universal set and the two circular regions represent A and B as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.
- (a) Determine $P(A)$ and $P(B)$.
 - (b) Determine $P(B|A)$ and $P(A|B)$.
 - (c) Using your results of the previous parts, state whether A and B are independent or not.
6. The following Venn diagram represents the occurrence of two events A and B . The out rectangle is the universal set and the two circular regions represent A and B as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.



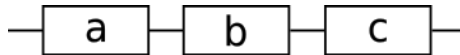
- (a) Determine $P(A)$ and $P(B)$.
 - (b) Determine $P(B|A)$ and $P(A|B)$.
 - (c) Using your results of the previous parts, state whether A and B are independent or not.
 - (d) Determine $P(\neg B|A)$ and compare it with $P(B|A)$.
 - (e) Determine $P(B|\neg A)$.
 - (f) Determine $P(\neg B|\neg A)$ and compare it with $P(B|\neg A)$.
 - (g) Determine $P(\neg A|\neg B)$.
7. A short network link consists of three sections a , b , and c as shown below.



The link functions if c functions and either a or b (or both) function. All sections are independent in the sense that the functionality of one does

not affect the functionality of any other. The probability that a continues to function during a particular time period is 0.94. The corresponding probability for b is 0.92 and for c is 0.96.

- (a) Write down statements, A , B , and C , defining the functionalities of a , b , and c respectively.
 - (b) Show that a statement which specifies the functionality of the link is $L = (A \vee B) \wedge C$.
 - (c) Calculate $P(A \vee B)$ and then determine $P((A \vee B) \wedge C)$.
 - (d) Use Boolean algebra to show that $L = (A \wedge C) \vee (B \wedge C)$ also specifies the functionality of the link.
 - (e) Calculate $P(A \wedge C)$ and $P(B \wedge C)$ and then determine $P((A \wedge C) \vee (B \wedge C))$.
8. A short network link consists of three sections a , b , and c as shown below.



The link functions only if all three of a , b , and c function. The functionality of section b depends upon that of a but is independent of c . Sections a and c are independent in that the functionality of one does not affect the functionality of any other. The probability that a continues to be functional during a particular time period is 0.95. The corresponding probabilities for b and c are 0.96 and 0.91. However, the probability that b **fails** during a particular time period, given that a has **already failed**, is 0.06.

- (a) Determine the probability that the link fails during a particular time period.
- (b) Use your previous result to determine the probability that the link continues to function during a particular time period.

Answers

1. A normal pack of 52 playing cards has 4 suits: clubs, diamonds, hearts, and spades, each with 13 cards. One card is drawn from the full pack at random, then replaced in the pack. Then a second card is drawn from the pack.
 - (a) Both are clubs: $\frac{13}{52} \times \frac{13}{52} = 0.00625$.
 - (b) At least one is a club: $(\frac{13}{52} \times \frac{39}{52}) + (\frac{39}{52} \times \frac{13}{52}) + (\frac{13}{52} \times \frac{13}{52}) = 0.4375$.
 - (c) Neither is a club: $1 - 0.4375 = 0.5625$.
 - (d) N/A
2. $P(A \wedge B) = 0.9 \times 0.95 = 0.855$

3. (a) $P(C \vee D) = P(C) + P(D) - P(C \wedge D) = 0.95 + 0.97 - 0.95 \times 0.97 = 0.9985$
- (b) The link failing corresponds to the following logical expression being true:
- $$\neg (C \vee D) = \neg C \wedge \neg D.$$
- (Or $\overline{C + D} = \overline{C} \cdot \overline{D}.$)
- (c) $P(\neg (C \vee D)) = 1 - P(C \vee D) = 1 - 0.9985 = 0.0015$
- (d) N/A
4. $P(A \wedge B \wedge C) = 0.95 \times 0.96 \times 0.91 = 0.83$
5. (a) $P(A) = \frac{135+765}{1000} = 0.9$, $P(B) = \frac{765+85}{1000} = 0.85$
- (b) For this we will need $P(A \wedge B) = \frac{765}{1000} = 0.765$. Then $P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{0.765}{0.9} = 0.85$ and $P(A|B) = \frac{0.765}{0.85} = 0.9$.
- (c) A and B are independent events if $P(A \wedge B) = P(A) \times P(B)$. Now $P(A \wedge B) = 0.765 = 0.9 \times 0.85 = P(A) \times P(B)$, so the events are independent.
6. The following Venn diagram represents the occurrence of two events A and B . The out rectangle is the universal set and the two circular regions represent A and B as shown. The values in the diagram indicate the number of outcomes (total 1000) in each case.
- (a) $P(A) = \frac{13+811}{1000} = 0.824$, $P(B) = \frac{811+124}{1000} = 0.935$
- (b) For this we will need $P(A \wedge B) = \frac{811}{1000} = 0.811$. Then $P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{0.811}{0.824} = 0.984$ and $P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.811}{0.935} = 0.867$.
- (c) A and B are independent if $P(A \wedge B) = P(A) \times P(B)$. Now $P(A \wedge B) = 0.811 = 0.77$, so the events are not independent.
- (d) $P(\neg B|A) = \frac{P(A \wedge \neg B)}{P(A)} = \frac{13}{13+811} = 0.0158$. Notice that this is $1 - P(B|A)$.
- (e) $P(B|\neg A) = \frac{P(\neg A \wedge B)}{P(\neg A)} = \frac{124}{124+52} = 0.705$
- (f) $P(\neg B|\neg A) = \frac{P(\neg A \wedge \neg B)}{P(\neg A)} = \frac{52}{52+124} = 0.295$, $P(B|\neg A) = 0.705$. These are complements again, in that $P(\neg B|\neg A) = 1 - P(B|\neg A)$.
- (g) $P(\neg A|\neg B) = \frac{52}{13+52} = 0.8$
7. (a)
 - A : 'a continues to function during a particular time period'.
 - B : 'b continues to function during a particular time period'.
 - C : 'c continues to function during a particular time period'.
- (b) There needs to be at least one route through the network link, either passing through a then c , or by passing through b then c . In order for this to be the case we need either, or both, of a and b to be functional at the same time that c is also function. Hence the logical expression of $(A \vee B) \wedge C$.

- (c) $P(A \vee B) = P(A) + P(B) - P(A \wedge B) = 0.94 + 0.92 - 0.94 \times 0.92 = 0.9952$. (We can use $P(A \wedge B) = P(A) \times P(B)$ since the events are independent.) Then $P((A \vee B) \wedge C) = P(A \vee B) \times P(C) = 0.9952 \times 0.96 = 0.955$.
- (d) The fact that $(A \wedge C) \vee (B \wedge C) = (A \vee B) \wedge C$ is a direct application of the distributivity law from Boolean algebra.
- (e) N/A
8. Given information: $P(A) = 0.95$, $P(B) = 0.96$, $P(C) = 0.91$, $P(\neg B | \neg A) = 0.06$.
- (a) To appear.