Visualizing Geospatial Information Uncertainty: What We Know and What We Need to Know

Article *in* Cartography and Geographic Information Science · July 2005

DOI: 10.1559/1523040054738936

Visualizing Geospatial Information Uncertainty: What We Know and What We Need to Know

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ABSTRACT: Developing reliable methods for representing and managing information uncertainty remains a persistent and relevant challenge to GIScience. Information uncertainty is an intricate idea, and recent examinations of this concept have generated many perspectives on its representation and visualization, with perspectives emerging from a wide range of disciplines and application contexts. In this paper, we review and assess progress toward visual tools and methods to help analysts manage and understand information uncertainty. Specifically, we report on efforts to conceptualize uncertainty, decision making with uncertainty, frameworks for representing uncertainty, visual representation and user control of displays of information uncertainty, and evaluative efforts to assess the use and usability of visual displays of uncertainty. We conclude by identifying seven key research challenges in visualizing information uncertainty, particularly as it applies to decision making and analysis.

KEYWORDS: Uncertainty, geovisualization, representation, decision making, usability

Introduction

Information uncertainty is a complex concept with many interpretations across knowledge domains and application contexts. Efforts to develop visualization methods and tools that can help information analysts understand and cope with information uncertainty have been underway for more than a decade. Uncertainty in geospatial information has been given particular attention. Progress has been made, but that progress is reported in diverse outlets across many disciplines. As a result, we do not have a comprehensive understanding of the parameters that influence successful uncertainty visualization, nor is it easy to determine how close we are to achieving such an understanding. In turn, without this understanding, effective approaches to visualizing information uncertainty to support real-world geospatial information analysis remain elusive.

This paper integrates perspectives on visualizing uncertainty with those on how to cope with uncertainty in information analysis and decision making. Visualizing uncertainty, of course, requires that uncertainty be measured or otherwise assessed and encoded; and there is a substantial volume of literature in GIScience and related fields focused on this problem, including focused research on computing and propagating uncertainty (Veregin 1995; Worboys 1998) as well as comprehensive texts and edited collections (Foody and Atkinson 2002; Goodchild and Gopal 1989; Zhang and Goodchild 2002). We do not attempt in this single paper to review that literature comprehensively, although we draw upon it selectively to support our main goal. That goal is to characterize the status of geospatial uncertainty visualization science and practice, thus to review and assess knowledge about visual methods and tools that help analysts and decision makers cope with geospatial information uncertainty. Drawing upon this review and assessment, we conclude by identifying key research *challenges* in visualization of geospatial information uncertainty, particularly to support analysis and decision making.

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Status of Geospatial Uncertainty Visualization Science and Practice

This section of the paper will introduce five components of current understanding of information uncertainty and its visualization; emphasis is given to information uncertainty in a geospatial context. Specifically, we address efforts to conceptualize uncertainty generally, decision making with uncertainty and how it has been or might be supported visually, categories of and frameworks for representing information uncertainty, methods for visually representing and interacting with information uncertainty, and efforts to understand and assess the usability and utility of uncertainty visualization methods.

Conceptualizing Uncertainty

Good science requires statements of accuracy by which the reliability of results can be understood and communicated. When inaccuracy is known objectively, it can be expressed as *error*; when it is not known, the term *uncertainty* applies (Hunter and Goodchild 1993). Thus uncertainty covers a broader range of doubt or inconsistency than error alone and, in the context of this paper, includes the concepts of accuracy and error as components.

According to Buttenfield (1993) there are two general philosophies regarding uncertainty representation: that one can represent the good aspects of data certainty by reporting accuracy, and that one can represent the bad aspects of uncertainty by reporting error. Accuracy seems to be the preferred medium of communication, perhaps because it implies to the user that the data are reliable. Buttenfield (1993) cites three impediments to effective uncertainty representation. First, most discussion of uncertainty involves ambiguous terminology; uncertainty itself is an ill-defined concept, with distinction between it and related concepts such as data quality, reliability, accuracy, and error often remaining ambiguous. Thus, there is a need to explore and formalize the concepts and terms that underlie consideration of uncertainty as it relates to analysis and use of geospatial and other data. Second, we lack methods for measuring and representing many aspects of uncertainty in a GIS database. Third, we lack methods for depicting uncertainty simultaneously with data and interacting with those depictions in ways that are understandable, useful, and usable.

Many researchers have addressed specific aspects of the assessment and encoding of geospatial uncertainty information. In work focused explicitly on uncertainty visualization, Pang et al. (1997) delineated three types of uncertainty related to stages in a visualization pipeline: collection uncertainty due to measurements and models in the acquisition process, derived uncertainty arising from data transformations, and visualization uncertainty introduced during the process of data-to-display mapping. In complementary work, Plewe (2002), concentrating on uncertainty in temporal geospatial data, proposed a model that distinguishes between uncertainty resulting from the process of conceptualizing a phenomenon and that resulting from measurement based on that conceptualization. Lowell (1997) focused specifically on the process of generating and handling geospatial uncertainty information. His "outside-in" method involves creating an Uncertainty Library by cataloging multiple interpretations of phenomena, then using this to estimate uncertainty based on variance among the interpretations. The "inside-out" method first depicts the certain places, then uses interpolation methods to estimate uncertainty of other locations based on distance from those that are certain. A related idea offered by Couclelis (2003) is a conceptual "Encyclopedia of Ignorance." The core idea here is to catalog what is unknowable, so that attention is directed to the "tractable forms of not knowing."

Decision Making with Uncertainty

Information uncertainty affects the process and outcomes of information analysis and decision making. Uncertain situations often result in a bias toward initial potential solutions (Kobus et al. 2001), undervaluing negative evidence (Reece and Matthews 1993) and overvaluing past positive outcomes (Cohen and Wallsten 1992). Expert decision makers tend to respond to uncertainty by incorporating probabilities representing that uncertainty into mathematical equations that allow them to determine the "best" answer to a problem. Naïve decision makers, on the other hand, tend to rely on past experiences and stereotypes when dealing with an uncertain situation.

There is general agreement that uncertainty affects the decisions we make. The literature on decision making with uncertainty is extensive, and thus it is impractical to review comprehensively here. We focus on the role of uncertainty in geographic decisions and, more specifically, on the role of maps and uncertainty representation in that decision making. We first review concepts of expert and lay decision making, then explore the existing research on cartographically supported decision making under uncertainty. Some of the limited research available on map readers' use of cartographic representations of data uncertainty, reviewed below, suggests that inclusion of this information is helpful to decision makers (Evans et al. 1999; Leitner and Buttenfield 2000; St. John et al. 2000); however, there is little real-world verification for this tentative result. Instead, the research seems to take for granted that visual depictions of uncertainty are useful for decision making.

Particular attention has been given to the role of uncertainty and its visual representation in the domain of environmental decision making. Professional fields dealing with the natural environment routinely require their practitioners to make decisions using data that can include a range of uncertainties. For example, analyses in the field of wildlife conservation are often based on relatively small samples of data having large variability. Nevertheless, policy decisions must be made in spite of this uncertainty, and conservationists have developed a number of ways to accomplish this.

Most environmental policy decisions must be based on imperfect, thus uncertain, information, and multiple authors have focused on strategies for taking uncertainty into account (Dreschler 2004; Taylor. et al. 2000). In one specific example, Todd and Burgman (1998) described how methods for assigning species to categories of risk in Australia involve combining multiple criteria that include demographic, life history, and management variables relevant to each species. Assigning species to categories within each variable is a process that mixes multiple uncertainties, e.g., the task of determining how many individuals of a species exist in a particular area and their population trend are both typically based on samples that may have large confidence bounds. To address these uncertainties, Todd and Burgman (1998) proposed a fuzzy sets approach that can integrate multiple uncertainties related to each species. They also discussed the potential implications of including this information for management decisions about threatened species. Silviculturalists face similar challenges in dealing with uncertainty as it relates to decisions about managing and harvesting trees. They must, for example, include uncertain stochastic events such as wildfire, insect outbreaks, changing public values, and changing economic situations, into these decisions. In this context, Ducey (2001) promoted the use of the Dempster-Shafer theory of evidence as a method for handling imprecise probabilities of these events.

Analytical methods such as those outlined above are becoming common decision-making tools. However, humans are typically not adept at using statistical information in the process of making decisions. Tversky and Kahneman (1974) proposed and demonstrated that lay people usually depend on heuristics rather than statistics when making decisions under uncertainty. They focused specifically on situations in which the decision maker is not provided with all the information needed to make an accurate decision, which is a common situation for geographic analysts across a range of application domains from environmental management decisions, through crisis management, to intelligence analysis. They found that heuristics were frequently the basis of the decision-making process, e.g., subjects in their experiments used stereotyped representations as a basis of decision making when they were asked to identify an individual's occupation, even when the stereotype was unlikely to be right, based on the statistical information provided.

Tversky and Kahneman's (1974) research can be applied to understanding how map readers use uncertainty information in decision making, specifically because it suggests a conflict: some experts are dependent on statistical analyses to incorporate uncertainty into their decisions, but lay users tend to ignore or misinterpret statistical probabilities and instead rely on less accurate heuristics when making decisions. This divergence prompts several questions: (1) will experts revert to lay strategies of applying heuristics when statistical evidence is not available; (2) will providing information about data uncertainty in an explicit visual way help a lay or expert map reader make different decisions; and (3) if they do make different decisions, will provision of information about data uncertainty lead to better, more correct, decisions or simply cause analysts to discount the unreliable information, whether doing so is the best strategy or not. MacEachren (1992) raised a similar issue, wondering whether inclusion of uncertainty information and/or its mode of presentation may cause map readers to miss important patterns and relationships between data or see others that do not really exist.

As noted above, the limited research available on map readers' use of cartographic representation of data uncertainty suggests that inclusion of this information is helpful to decision makers (Evans et al. 1999; Leitner and Buttenfield 2000). Agumya and Hunter (2002), however, noted that the current focus of research involving representations of geographic data uncertainty seems to be centered on methods to create the representations, rather than on how those displays can aid a user in making better decisions. To address this deficiency, they applied formal risk management methods to develop an approach for managing the impact of geographical data uncertainty within the decision-making process. Specifically, their approach involves propagating data uncertainty into decision uncertainty measures, then conducting formal risk scenario identification and analysis that considers the risk in decisions that derive from uncertainty. Agumya and Hunter (2002) give specific examples of methods for reducing the risk of a decision that relate to the concept of insurance; these methods include the practice of self-insurance where an individual accepts the risk and sets aside resources to cope with it and the transfer of risk from one entity to another, either through a non-insurance contractual agreement such as a guarantee or through a purchased insurance agreement. They did not test their risk analysis approach to uncertainty with users, however. They also did not address the problem of how to signify risk to decision makers.

Cliburn et al. (2002) addressed more specifically the idea of making decisions based on uncertain data with the help of uncertainty representations. As discussed further below, they designed a system to display the results and uncertainty in results of a global water balance model using data estimates from global circulation models as inputs. Their ultimate objective was a system that policy makers could use to make informed decisions. With this goal in mind, they developed and tested methods of simultaneously representing cartographic water balance data and uncertainty of the data. Domain and usability experts in their initial tests thought that the tool was generally helpful. However, they listed the depiction of uncertainty as a drawback, because policy makers typically want issues presented with no ambiguity—i.e., is the future global water balance a problem or not? One participant in their study suggested that a depiction of uncertainty could be used to discredit the models rather than having the intended effect of signaling unbiased results.

As in science and environmental policy, intelligence analysts in the field of national security deal with uncertainty in a wide variety of forms. Davis (2002) writes that "the central task of intelligence analysis is to help U.S. officials…deal more effectively with substantive uncertainty." Sources of uncertainty may include gaps or weaknesses in collection capabilities (Lowenthal 2003), processing transformation such as language translation, and credibility of the information source (Krizan 1999).

The intelligence analysis process can be characterized as a progression where the analyst's certainty grows over time until it reaches a level where the analyst is able to produce a warning or other report with a high degree of confidence (Graves 2000). Graves depicts this process as a graph, where the x-axis represents time and the y-axis represents certainty (Figure 1). A curve shows the growing certainty over time as an event approaches. Intelligence analysis works to raise the slope of the curve, while the opponents' efforts, characterized as ambiguity and deception, work to lower the slope. If the curve reaches the appropriate confidence level prior to an event (shown in the graph as the Recognition level), warning is given. If the opposing forces succeed in keeping certainty too low for a warning until after the event, then decision makers are surprised.

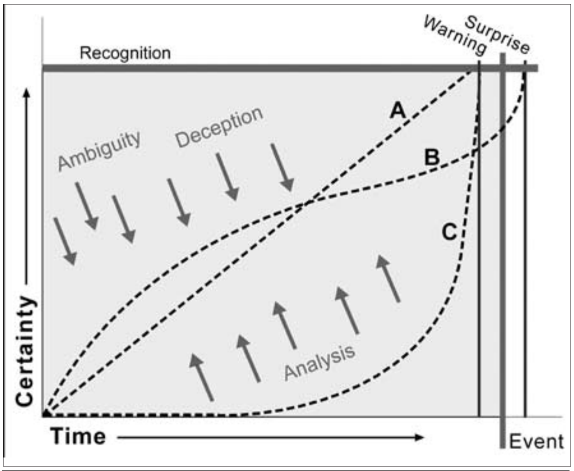

Figure 1. Impact of ambiguity and deception on success of intelligence analysis. [After Graves (2000); reproduced by permission.]

In related work, Kobus et al. (2001) looked simultaneously at the effects of uncertainty and experience on immediate decision making. The context was tactical military operations, where officers must make crucial decisions, often with new information arriving continuously. They characterized this kind of dynamic task context as involving "an intuitive decision-making strategy." They argued that this kind of intuitive decision making is characteristic of a naturalistic decision-making perspective rather than the analytical decision-making perspective focused on in military training. This distinction is relevant to consideration of kinds of uncertainty to depict as well as how to depict it; both of these are discussed in sections below.

Typology of Uncertainty

Uncertainty is a complex, multifaceted concept but this complexity has often been ignored in efforts to visualize uncertainty. The literature on decision making under uncertainty reviewed above provides a starting point for addressing the complexity of information uncertainty. This literature makes it clear that there are a variety of kinds of uncertainty that decision makers must face and that, to be useful, representations of uncertainty, visual or other, must address this variety. As a step in this direction, several efforts have been made to delineate the components of information uncertainty and relate them specifically to visual representation methods. These efforts have proceeded somewhat in parallel with limited sharing of ideas in the Geographic Visualization/Geographic Information Science (geovisualization/GIScience) and the Scientific Visualization/Information Visualization (SciVis/InfoVis) communities. Each will be discussed briefly, particularly as they relate to the development of a typology of geospatially referenced information uncertainty and its visualization that some authors of this paper participated in.

Geovisualization/GIScience

One of the earliest conceptual frameworks for geospatial uncertainty, recognizing the separate error components of *value*, *space*, *time*, *consistency* and *completeness,* was proposed by Sinton (1978) and later elaborated by Chrisman (1991). Uncertainty in geographic data has subsequently been described in a variety of alternative ways; such as those provided by Bedard (1987), Miller et al. (1989) and Veregin (1989). Although different, these approaches all have a number of aspects in common; including the observation that uncertainty itself occurs at different levels of abstraction.

Most of the efforts to formalize an approach to uncertainty visualization within geovisualization (and GIScience more generally) derive from long-term work on spatial data transfer standards (SDTS) (Fegeas et al. 1992; Moellering 1994; Morrison 1988). The focus of the initial SDTS effort was on specifying categories of "data quality" which were to be encoded as part of the metadata for cartographic data sets (later expanded to geographic data sets more generally), whether they were used to support mapping or not. The categories of data quality defined as part of the SDTS are:

- **Lineage**: a description of the source material from which the data were derived and the methods of derivation, including all transformations involved in producing the final digital files (USGS 1997, p. 15);
- **Positional accuracy**: must include the degree of compliance to the spatial registration standard; measures can include: deductive estimate, internal evidence, comparison to source, or independent source of higher authority (USGS, 1997, p. 15);
- **Attribute accuracy**: both measurement accuracy (for features measured on a continuous scale) and class assignment accuracy (for categorical features) are included here (USGS 1997, p. 16);
- **Logical consistency**: here, the objective is to describe the fidelity of relationships encoded in the data structure of the digital spatial data (USGS 1997, p. 16);
- **Completeness**: the goal here is to describe the relationship between the objects represented and the abstract universe of all such objects. Includes issues such as selection criteria (e.g., size thresholds for spatial features, frequency counts for attributes), definitions used, and other mapping/abstraction rules (USGS 1997, p. 17).

Buttenfield and Weibel (1988) were among the first to attempt a framework for categorizing components of "data quality" with a specific focus on their cartographic representation (Figure 2). Their approach matched the five categories of data quality from the SDTS with three data types: discrete, for point and line features; categorical, for area features assigned to categories through aggregation, and overlay or attributes assigned to classes through partitioning and enumeration; and continuous, for surfaces and volumes. For each cell in the resulting matrix, they focused on which "visual variables" were most appropriate to depict the category. MacEachren (1992; 1994) addressed related issues in his discussions of the representation of certainty and the quality of the resulting data representation. He added specific attention to information precision, as distinguished from accuracy, and focused on matching kinds of uncertainty to Sinton's (1978) distinction among location, attribute, and time components of data. Buttenfield and Beard (1994) also extended Buttenfield and Weibel's (1988) initial framework to include location, attribute, and time components (termed locational, thematic, and temporal). They dropped consideration of logical consistency and completeness, combined the two accuracy components into one, and added resolution as another component.

In related work, Gahegan and Ehlers (2000) focused on modeling uncertainty within the context of fusing activities between GIS and remote

Data Type / Data Quality	Positional Accuracy	Attribute Accuracy	Logical Consistency	Completeness	Lineage
Discrete Points and Lines	Size Shape (Error ellipses) (Epsilon bands)	Value Color Saturation (Feature code checks)	Color mixing Redundancy by overprinting Slivers by solid fills (Topological cleaning)	Mapping Technique Density traces Marginalia Generalization algorithm Mapping tolerance Buffer size	Mapping Technique Minimum Bounding Rectangles
Categorical Aggregation & Overlay (Tesselation, tiling, Areal coverages)	Texture Value (Certainty of boundary location)	Color mixing (Attribute code checks) (Topographic classifier)	lack error models	Mapping Technique Missing values Logical adjacency surface Marginalia Discrete model weights	
Partitioning & Enumeration (Metric class breaks)	not meaningful	Size = height (Blanket of error)	Size = height (Maximum likelihood prism maps)	Mapping Technique Missing values Missclassification matrix Classing scheme OAL/TAI	Marginalia Source of data Scale/Resolution Date Geometry
Continuous Interpolation (Surfaces and volumes)	no clear distinction b/w the two Value Color Saturation (Continuous tone vignettes) (Continuous tone isopleths)		Size = line wt Color Shape = compactness (TIN links)	not possible by definition Mapping Technique Surface of search attenuation Marginalia Interpolation algorithm	

← **Graphical Syntax** → ← **Graphical/Lexical Syntax** →

Figure 2. Buttenfield and Weibel's (1988) initial framework for matching types of uncertainty, kinds of data, and methods of representation. Characterization of representation methods focuses on matching visual variables to kinds of data/uncertainty. Forms of representation are also mentioned, but not systematically addressed (e.g., use of error ellipses, production of prism maps, addition of marginalia). [Modified from a version appearing in Buttenfield (1991); reproduced with author's permission.]

sensing. Their approach matched five types of uncertainty—data/value error/precision, space error/precision, time error/precision, consistency, and completeness—against four models of geographic space: field, image, thematic, and object, as shown in Table 1.

Thus, they merged the location, attribute, and time distinctions discussed above with types of uncertainty, while focusing on the implications of the different approaches to modeling geographic space. A particular emphasis in their work was on error and uncertainty propagation.

Scientific Visualization/Information Visualization

Visualization of uncertainty or reliability has been a topic of attention for several researchers within the SciVis (Cedilnik & Rheingans 2000; Johnson & Sanderson 2003; Lodha et al. 1996a) and InfoVis (Gershon 1992) communities, and some of this work has been explicitly geospatial (Wittenbrink 1995; Dungan et al. 2003). In spite of the wide array of visualization methods proposed there seems to have been less attention to formalizing approaches to uncertainty within these communities than there has been within the geovisualization/GIScience communities. Perhaps the specific attention within the latter communities to issues of database representation, metadata standards, and application of geospatial information is responsible for the increased attention to formalization. Within SciVis/InfoVis, however, there are at least two important contributions that focus on formalizing an approach to uncertainty visualization. Each is discussed below briefly.

Pang et al. (1997) took a systematic approach to methods for visualizing uncertainty; while they provide geospatial examples, their focus covers a wide range of data types. They produced a classification of methods for uncertainty visualization that matches data type (scalar, multivariate, vector,

	Field	Image	Thematic	Object
Data or value	Measurement error and precision	Quantization of value in terms of spectral bands and dynamic range	Labeling uncertainty (classification error)	Identity error (incorrect assignment of object type), object definition uncertainty
Space	Locational error and precision	Registration error, sampling precision	Combination effects when data represented by different spatial properties are combined	Object shape error, topological inconsistency, 'split and merge' errors
Time	Temporal error and precision	(Temporal error and precision are usually negligible for image data)	Combination effects when data representing different times are combined	Combination effects when data representing different times are combined
Consistency	Samples / readings collected or measured in an identical manner	Image is captured identically for each pixel, but medium between satellite and ground is not consistent; inconsistent sensing, light falloff; shadows	Classifier strategies are usually consistent in their treatment of a dataset	Methods for object formation may be consistent, but often are not. Depends on extraction strategy
Completeness	Sampling strategy covers space, time and attribute domains adequately	Image is complete, but parts of ground may be obscured	Completeness depends on the classification strategy. (Is all the dataset classified or are only some classes extracted?)	Depends on extraction strategy. Spatial and topological inconsistencies may arise as a result of object formation

Table 1. Types of uncertainty in four models of geographic space (Source: Gahegan and Ehlers, 2000)

and tensor) to visualization form (discrete and continuous). For each of the eight cells in the resulting matrix (e.g., continuous scalar data, discrete multivariate data), they proposed some logical representation methods, including both static and dynamic representation forms.

From an InfoVis rather than SciVis perspective, Gershon (1998) took a very different approach than Pang, focusing on kinds of "imperfection" in the information about which an analyst or decision maker might need to know. His argument is that imperfect information, while involving uncertainty, is more complex than typically considered from the viewpoint of uncertainty alone. Figure 3 depicts Gershon's (1998, p. 43) "high-level taxonomy of causes for imperfect knowledge of the information state." Two important points are that (1) uncertainty is considered to be just one of six inputs to interpretation of, and decision making with, imperfect information; and that (2) the quality of the presentation is a critical factor, a point also made by MacEachren (1994; 1995). The argument being made in point number two above is that standard graphic and cartographic guidelines for effective and logical symbolization, i.e., data to display mappings, and for clear graphic design apply to uncertainty visualization. Thus, with poor symbolization or design choices, our uncertainty visualization could lead to more rather than less uncertainty about the data depicted.

Typology of Geospatial Intelligence Information Uncertainty

Building on the typology efforts above, three of the current authors and two additional colleagues propose a typology of uncertainty relevant to geospatial information visualization in the context of intelligence analysis (Thomson et al. 2004). The typology integrates key elements from those outlined above and extends them by adding three categories of uncertainty that are particularly important in the context of intelligence information assessment, credibility, subjectivity, and interrelatedness. The result is

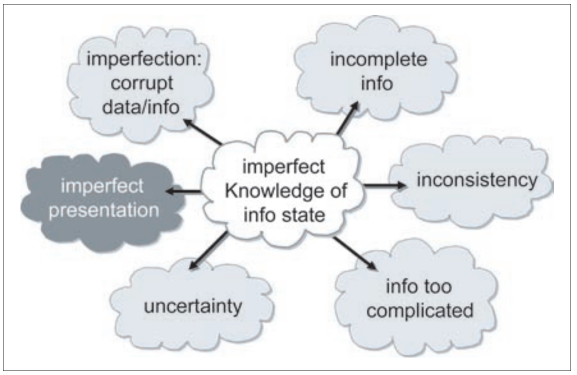

Figure 3. Gershon's taxonomy of imperfect knowledge of the information state. [Modified from Gershon (1998); reproduced by permission from IEEE © 1998.]

a characterization of uncertainty as having the following components extended from those in Thomson et al. (2004).

- **Accuracy/error:** difference between observation and reality, usually estimated based on knowledge of the measurement/estimation device and of phenomena in the work.
- **Precision:** exactness of measurement/estimate, derived from parameters of the measurement, estimation device, and/or procedure.
- **Completeness:** extent to which information is comprehensive.
- **Consistency:** extent to which information components agree. This is a more general definition than that found in formal standards for spatial data.
- **Lineage:** conduit through which information has passed. This is a complex category that has at least the following subcomponents: number of individuals, organizations, processes through which information moves; specification of which individuals, organizations, or processes.
- **Currency:** time span from occurrence through information collection/processing to use. The certainty that information is "current" will be a function of both time span and context, e.g., year-old data about vehicles parked in a factory loading bay is less certain to be current than year-old data about location of the factory.
- **Credibility:** combination of factors such as reliability of information source. Certainty may be based on past experience, e.g., the analyst is correct 85 percent of the time, or on categorization of the source, e.g., U.S. analyst versus a non-U.S. informant; motivation, experience, or other factors.
- **Subjectivity:** the extent to which human interpretation or judgment is involved in information construction. This component of uncertainty is, of course, difficult to assess—and that assessment will have some level of subjectivity.
- **Interrelatedness:** source independence from other information. This is a common standard used in the news media to assess certainty that a story is authentic.

Thomson et al. (2004) matched these categories of uncertainty to the space, time, and attribute components of data to produce the typology of uncertainty depicted in Table 2, with examples indicated for each category.

Visualizing Uncertainty

Much of the research focused on visualizing uncertainty has been directed to the fundamental signification problem of deciding which components of a sign-vehicle, or symbol, should depict the data and which components should depict data uncertainty. A common strategy is to start with Bertin's (1983) visual variables and their static, dynamic, and sonic extensions (MacEachren 1992; McGranaghan 1993; van der Wel et al. 1994). A second focus has been on the development of interfaces that are designed to allow users to access or suppress uncertainty as needed, control how uncertainty is depicted when displayed, adjust the visual dominance of the depiction, and manipulate the display in other ways. In spite of past efforts to categorize uncertainty, most approaches to uncertainty visualization have treated uncertainty as a single attribute of data; thus there is a mismatch between efforts to conceptualize and to represent uncertainty. Slocum et al. (2004), in their cartographic text, provide a detailed discussion of many of the issues and highlight several specific solutions offered by selected authors. Our consideration of research on visualizing uncertainty complements Slocum's overview by emphasizing and comparing approaches from different disciplines. In keeping with the focus of the text, Slocum et al. (2004) emphasize cartographic solutions, citing only one non-geographic research team.

Below we divide discussion of visualization methods into two sections. First, fundamental aspects of visual representation are considered, with the focus on application of the visual variables and their combinations. Second, the potential of dynamic representation for uncertainty visualization is reviewed, with dynamic interpreted to include animated, sonic, and interactive representations.

Visual Signification of Uncertain Information

The most basic methods of visually representing uncertainty are available through direct application of Bertin's (1983) visual variables, following guidelines already used in traditional cartography. The original set of variables includes location, size, color value, grain (often mislabeled as texture), color hue, orientation, and shape. In work focusing specifically on uncertainty visualization, Davis and Keller (1997) asserted that

Category	Attribute Examples	Location Examples	Time Examples
Accuracy/error	counts, magnitudes	coordinates, buildings	+/- 1 day
Precision	nearest 1000	1 degree	once per day
Completeness	75% of people reporting	20% of photos flown	2004 daily/12 missing
Consistency	multiple classifiers	from / for a place	5 say Mon; 2 say Tues
Lineage	transformations	#/quality of input sources	# of steps
Currency	census data	age of maps	$C = T_{present} - T_{info}$
Credibility	U.S. analyst interpretation of financial records <…> informant report of financial transaction	direct observation of training camp <…> e-mail interception with reference to training camp	time series air photos indicating event time <…> anonymous call predicting event time
Subjectivity	fact <…> guess	local <…> outsider	expert <…> trainee
Interrelatedness	all info from same author	source proximity	time proximity

Table 2. Typology of uncertainty of geospatial information, (Adapted from (Thomson et al., 2004).

using color hue, color value, and "texture" are the "best candidates" for representing uncertain information using static methods. Others have also emphasized the creative usage of color attributes to signify uncertainty. Jiang et al. (1995) described a technique in which hue, lightness, and saturation are manipulated to depict fuzzy datasets. According to their approach, hue is used to assign nominal categories, saturation can confer data values, and lightness or darkness is changed to show uncertainty. More recently, Hengl (2003) outlined a similar method using the hue, saturation, and intensity color model to visually depict uncertainty. Hengl's method resulted in uncertain data appearing increasingly white or "pale," depending on the magnitude of uncertainty. This approach relies on manipulation of both saturation and value for depicting uncertainty.

MacEachren (1992) argued for an extended set of visual variables to depict uncertainty. Specifically, he addressed the potential of four methods to signify uncertainty. These were color saturation; crispness, split into contour crispness and fill clarity; transparency, initially termed fog; and resolution of raster images and of vector line work. With color saturation (Figure 4), MacEachren proposed that map elements with a high level of certainty should use pure hues, while those with less certain information should use a correspondingly less saturated color, thereby graying out uncertain areas making their color hue "uncertain." MacEachren also suggests applying a general metaphor of "focusing"—using "out of focus" depictions for uncertain information and "in focus" depictions for certain information. As noted, this general metaphor was initially split into two components—contour or edge crispness and fill clarity—but both apply the strategy of making the edge of sign-vehicle elements fuzzy to depict uncertainty. The idea of crispness, specifically fill clarity, is essentially the same as that used by Gershon (1992) who created an application that animated through increasingly blurred versions of data to signify fuzzy sea-surface temperature data (see dynamic representation section below).

MacEachren's initial application of transparency to uncertainty depiction was based on a metaphor of fog obscuring our view. From this perspective, a transparent atmosphere tells the reader that the objects they see depicted on the display, through that clear atmosphere, are fairly certain while a cloudy atmosphere, through which it is hard to see the data representation, indicates uncertain information. Drecki (2002), in contrast, proposed an "opacity" display for classified, remotely sensed images in which integral methods of symbolization were used, rather than the visually separable fog floating above a data depiction. In this case, he argued that it was most logical to consider opaque objects to be the certain ones. This alternative metaphor may also be appropriate for use of transparency with discrete sign-vehicles. With a point symbol, a highly transparent object might be considered to be an uncertain one, a figment of one's imagination, while a relatively opaque object might be considered to be fairly certain, thus real. Thus, in Figure 4c, the bottom sign-vehicle may be interpreted as the most certain place if the first metaphor is assumed. The background data are not obscured by the foreground sign-vehicle, thus they are certain, while the top sign-vehicle may be interpreted as the most certain if the second metaphor is assumed and attention is directed to the foreground sign-vehicle, which is distinct at the top and almost invisible at the bottom.

In complementary research, Pang (2001) described the use of glyphs, which are compound point symbols, as an alternative method of visually representing data plus uncertainty. Glyphs are graphical objects through which multiple visual variables can be manipulated in order to summarize a wide array of data aspects simultaneously. Glyphs have been used as representations of data themselves, and also as ways to understand temporal series. Pang suggests that glyphs are useful for representing uncertainty, especially multiple types of uncertainty (Figure 5). However, he also cautions that glyphs can become visually overwhelming.

Djurcilov et al. (2002) applied a range of methods including transparency, glyphs, and other methods to representation of uncertainty in 3-dimensional renderings of model-derived ocean data. They used Monte-Carlo simulations to generate the potential variance for each point in the data field. The modeled ocean data and these variance values were then rendered together in 3-dimensional data volumes using alternative depictions of uncertainty that range from showing only those data below a threshold of certainty, an idea also implemented by Howard and MacEachren (1996), to creating visual effects such as textures and noisy spots for those areas in the data which are less certain. Also, areas can be rendered more or less opaquely, depending on the level of data quality, thus creating semi-transparent areas of the data field that appear intuitively to be less understood than the rest. This strategy is essentially the same as proposed by Drecki (2002), but it is applied to 3D views. Although their signification and interaction methods were flexible, Djurcilov et al. (2002) paid little attention to how users can or might interact with their rendering method, and thus far, no follow-up study has been performed to analyze the usability of their approach.

Rather than represent the most likely interpretation of data and the uncertainty associated with that interpretation, an alternative considered by several authors is to generate multiple realizations based on different processing and/or interpretations of data, then use a comparison of the realizations to signify uncertainty (Goodchild et al. 1994; Juang 2004; Unwin 1995). Bastin et al. (2002) applied this approach to visualization of Landsat Thematic Mapper data quality, specifically using a "fuzzy membership" function in order to create multiple possible surfaces.

Most of the uncertainty visualization research discussed above includes an implicit assumption that users of uncertainty information are homogeneous. In contrast, Beard and Mackaness (1993) proposed

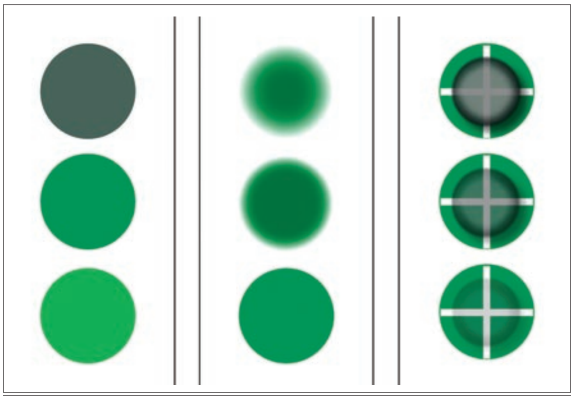

Figure 4. Point symbol sets depicting uncertainty with variation in (a) saturation, i.e., colors vary from saturated green, bottom, to unsaturated—top; (b) crispness of symbol edge—middle; and (c) transparency of symbol—right. In (c), transparency is applied to the smaller symbol in the foreground.

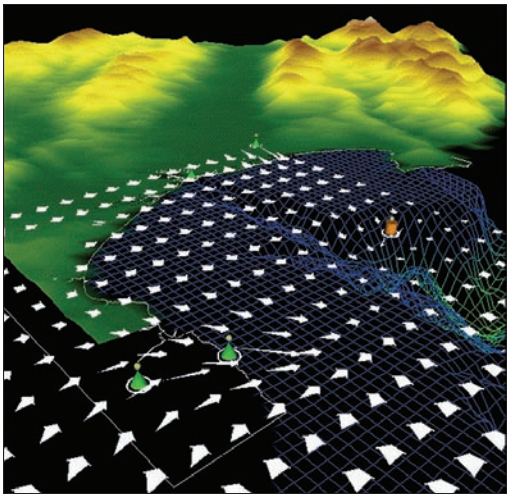

Figure 5. Glyphs indicating wind direction, magnitude and uncertainty.[Figure provided by Alex Pang (2001; reproduced by permission.]

a system that involves three levels of uncertainty indicators, based upon the experience and needs of the user. Their first level was simply a notification of poor data quality, with "poor" defined on the basis of a predetermined threshold. The second level adds detail about characteristics of the uncertainty, specifically the location and type of quality conflict. The third level focuses on giving users methods for investigating the reasons for uncertainty. Another important point brought up in this paper is that it is difficult to keep the display of data certainty from getting in the way of the data themselves, an inherent conflict that is a source of frustration for researchers.

Dynamic Representations for Uncertain Information

Among the uncertainty visualization methods addressed in the literature, an idea repeated frequently is that users need control over depictions of uncertainty. Howard and MacEachren (1996) described an interactive environment, called R-VIS for reliability visualization, which was developed to support exploration of uncertainty in interpolated surfaces and volumes derived from sample data. They proposed two fundamental strategies for interactive visualization of uncertainty. One is to use bivariate representations that depict data and uncertainty together, treating uncertainty as a second variable. These representations are most useful when users are allowed to control which component—data or certainty—is visually dominant in ways similar to the direct manipulation approach that Rheingans (1992) proposed for exploring bivariate maps of two data variables. The bivariate representations Howard and MacEachren proposed include visually integral methods using bivariate color schemes (as shown in Figure 6, left panel) and visually separable methods in which data and uncertainty are mapped to symbols of different dimensionality—for example, data to a smooth or abruptly changing color fill and uncertainty to positional or linear features layered on top of the data representation.

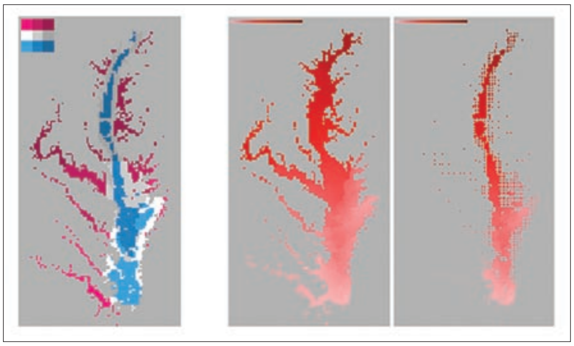

Figure 6. Alternative depictions of data (inorganic nitrogen in Chesapeake Bay) and uncertainty of data interpolated from sparse point samples. Left view shows bivariate depiction in which dark=more nitrogen and certainty is depicted with a diverging color scheme (blue = most certain and red = most uncertain). The right view depicts data in both panels (dark = more nitrogen), with the right side of this view showing the results of interactive focusing on the most certain data.

The second strategy that Howard and MacEachren described is to depict only data, but allow users to control an uncertainty threshold above which the data are not represented or are represented less clearly (Figure 6, right panel). Faiz and Boursier (1996) and Drecki (1997) implemented similar interactive representations, although the uncertainty levels were stored as metadata or pre-created images, rather than being dynamically controlled on the fly. More recently, Lucieer and Kraak (2004) developed and implemented a set of linked tools to enable the exploration of uncertainty in the classification of remotely sensed imagery. Their environment incorporates dynamic linking and brushing to support flexible exploration, the goal of which is to "improve insight into classification and uncertainty."

Howard and MacEachren (1996) made a distinction between intrinsic and extrinsic methods for visualizing uncertainty; this distinction is similar to one proposed by (Gershon 1998). Intrinsic approaches are those that change appearance of an object, while extrinsic approaches use additional symbols to provide information about an object. Thus, Howard and MacEachren's use of multiple, overlaid symbol dimensions, i.e., area symbols for the data and point symbols for data uncertainty, is a kind of extrinsic representation (not shown here), while color bivariate maps and maps that employ user interaction to control how much of the data is shown based on their uncertainty are both intrinsic.

In a more recent visualization environment designed to support decisions about large-scale water resource issues, Cliburn et al. (2002) used manipulable glyphs to depict uncertainty, thus, an extrinsic representation. Specifically, they depicted data values, i.e., water balance change predictions, with height on a 2.5D mesh surface and the uncertainty in those values, based on the range of predictions from different models, with vertical bars through the nodes of the mesh (Figure 7). The bars are colored purple above the predicted surface and orange below, allowing users to quickly see regions in which the prediction is more likely to be an under- or over-estimate. Cliburn et al. (2002) suggested that this type of display can become visually confusing and overwhelming, particularly when extended to depict more than the magnitude of spread in estimates above and below the value specified. One solution Cliburn et al. (2002) proposed for helping users cope with display complexity is to provide interactivity. They allow users to click on a single area, or outline a region of the data, and see a subset of uncertainty measures for that place or set of places.

In addition to allowing users to dynamically interact with the uncertainty component of a display, several authors have explored the potential of dynamic signification in the form of animation to highlight aspects of uncertainty. Fisher (1993) was among the first to apply animation to uncertainty representation. His focus was on uncertainty in multivariate classification of the sort encountered when classifying soils and land cover. His approach is to map the certainty in soils or land cover classification for a grid cell, point, or region to the dynamic variable of *duration*. Color was used to represent the category of each grid cell making up a map display in the dynamic representation. The proportion of time that each cell is represented by a specific color signifies the likelihood that the cell is in that particular category. The result is that certain parts of the map have a stable color indicating relatively certain classification, while uncertain regions change continuously. Fisher did not report on the success of his method with map users, and instead he called for testing of his model in further experimentation.

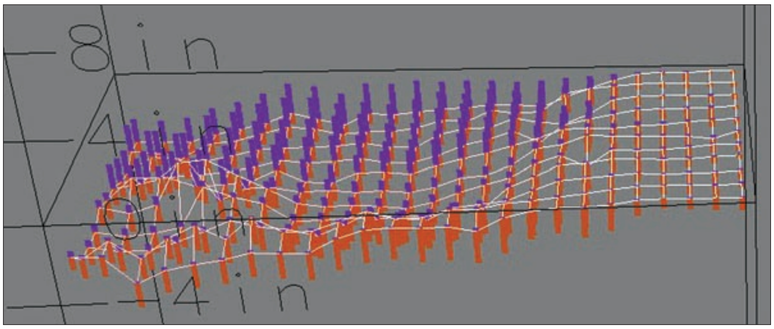

Figure 7. Representation of estimated water balance surplus/deficit (using a mesh surface) and uncertainty in the estimates (using bars above and below the surface). The bars depict the range of a set of model predictions, with predictions above the mean shown in purple and those below the mean in orange. [After Cliburn et al. (2002); figure supplied by Terry Slocum. Reprinted with permission from Elsevier © 2002.]

While Fisher's use of animation focused on direct representation of uncertainty—long duration in one color = high certainty of classification—others have emphasized indirect representation of uncertainty through animated sequences of different potential realizations. Gershon (1992), in a study mentioned above, focused on uncertainty in gridded surfaces that depict continuous distributions derived from numerical data samples at points; specifically, he considered sea surface temperature surfaces. He proposed (and demonstrated) an animated sequence of realizations that are systematically blurred, spatially, by applying an increasingly large filter to the grid to dampen the impact of local variation. Gershon found that the method enhanced key structures in the sea surface temperature data, drawing user attention to them.

Several other authors have proposed complementary uses of animation to present sequences of alternative realizations, including Ehlschlaeger et al. (1997), Davis and Keller (1997), and Bastin et al. (2002). Bastin et al. focused on visualization to depict fuzzy classification of categorical data. Specifically, they proposed generating animated series of realizations for specific map/image categories, e.g., land cover types, with a systematically varied "alpha function." Thus, they generated slices across the fuzzy membership function, resulting in a dynamic map that builds from a depiction of the category core to a depiction that includes all locations with any likelihood of being in the category.

Related uses of animation were proposed by both Davis and Keller (1997) in a study focused on landforms, soils, and site-stability analysis and Ehlschlaeger et al. (1997) in a study focused on visualizing the impacts of elevation certainty on optimal path calculations. Both studies advocated use of interactive animation for presenting sequences of complete realizations (rather than animated sequences directed to one category at a time). Figure 8 shows two frames from one of Davis and Keller's animations constructed to depict possible slope stability and its certainty. In related work, Ehlschlaeger et al. (1997) sought to illustrate the impact of varied quality of Digital Elevation Model (DEM) data on the generation of potential cost surfaces associated with planning a new highway. They generated an array of 250 possible DEM configurations for a study area in southern California and calculated optimal paths across each. Then, they developed a method for logically ordering the realizations, generated additional intermediate representations using interpolation between adjacent scenes, and constructed an animation that highlights the possible paths for a highway and the relative certainty that each path is optimal.

In addition to its use in representation of uncertainty in attributes of places, animation has been employed as a method for understanding uncertainty in space–time processes. For example, Fauerbach et al. (1996) applied animation

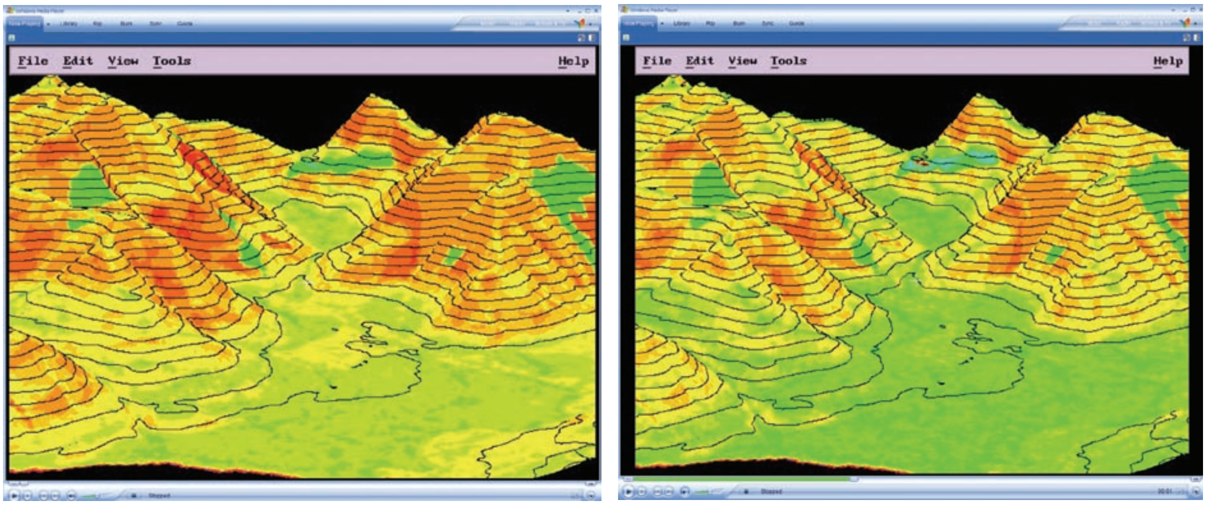

Figure 8. Two frames from an animation that depicts an ordered set of realizations of a maximum likelihood model for slope stability, based on variance data. A color scheme is applied that ranges from red (representing unstable slope) through yellow to green (representing stable slopes). The realization at left suggested a substantially greater soil stability problem on the steep slopes. [Reprinted from David and Keller (1997); reprinted with permission from Elsevier © 1997.

to representations of uncertainty in time series outputs from meteorological models. Specifically, they generated uncertainty surfaces for multiple time steps based on differences in predictions among multiple models, then combined these in animations with data about the real world weather that the models were trying to predict. In their animation, the predicted pressure surface was represented by isolines, and areas of disagreement between models were represented as filled isolines behind the data depiction (Figure 9; and http://www.geovista.psu.edu/sites/icavis/icavis/febm/sdhbivar.html). These areas of disagreement change dramatically through the animation as it progresses over time. As illustrated in Figure 9, the application that Fauerbach and colleagues developed was web-based, incorporating simple VCR-style controls. The interaction, although limited, allowed users to explore the relationships in space and time between particular weather events and the uncertainty in their prediction. The method was found to be particularly useful for helping analysts separate spatial and temporal uncertainties in predictions. For the data explored, the models had similar spatial predictions for the storm trajectory but there was considerable disagreement, thus uncertainty, about when the storm system would be at each location along that trajectory.

Dynamic representation of uncertainty is not restricted to only visual channels; several authors have proposed sonification as a way to encode uncertainty along with data. Fisher (1994) produced gridded representations in which users could "scan" a variably sized box around an image and hear the uncertainty for individual pixels or sets of pixels. Uncertainty in this system could be mapped to any of the ordered sonic variables identified by Krygier (1994). Lodha et al. (1996b) built on this idea in a software package called LISTEN that allows users to control the sonification of data quality. LISTEN provided users with the ability to select which measures of uncertainty are represented by which types of sounds. In order to facilitate exploration of data sonically, Lodha et al. advocated using a stepping process, from points of low to high uncertainty, or vice versa. Also, Lodha et al. (1996b) described using multiple sounds for multiple types of uncertainty, e.g., pitch for different instruments to represent different uncertainty components, enabling more than one measure to be aurally signified at once. Lodha et al. (1996b) claimed their design was successful in communicating uncertainty and removing some of the visual overload inherent in many visualizations of data quality, but they did not present results of any user studies in support of this assertion.

Testing Use and Usability

Most research directed to uncertainty visualization has focused on developing representation methods or software applications for the display of uncertainty, or on developing theory about what may work. Much less has been done to empirically evaluate whether the proposed applications work, or whether the theoretical perspectives lead to supportable hypotheses. Key empirical contributions made thus far are highlighted below. They provide a starting point for a more systematic approach to understanding: (a) the use of information uncertainty in information analysis and decision making, and (b) the usability of uncertainty representation methods and manipulable interfaces for using those representations. Here, we will highlight key approaches and findings from these past empirical studies. We divide discussion into that focused on the form of representation and that focused on software environments to enable access to and application of uncertainty visualization tools.

Understanding and Assessing Uncertainty Representation Methods

Several studies have addressed the effectiveness of specific uncertainty visualization methods, with the earliest studies focused on the core visual variables. In one of these, Schweizer and Goodchild (1992) examined the effectiveness of color value and saturation in representing uncertainty on choropleth maps. The experiment tested continuous tone, unclassed bivariate maps, comparing maps in which saturation signified data attributes and value signified data quality to the reverse. Darkness or color value rather than grayness, i.e., color saturation, was found to be more consistently associated with data quality.

In subsequent research, MacEachren et al. (1998) tested three methods of representing reliability, i.e., certainty, of health data on choropleth maps and again found that color saturation, counter to their prediction, was less effective for signifying uncertainty than the alternatives tested. Their study focused on a distinction between visually separable and visually integral representation methods. Three specific methods were developed for combining a choropleth map of health data with associated uncertainty. Two resulted in visually separable depictions of data and data uncertainty: (a) an adjacent display of data and binary reliability maps, and (b) a single display showing data with a sequential or diverging color scheme and uncertainty with a texture overlay of parallel black and white lines. These were compared to a visually integral display similar to that used by Schweizer and Goodchild (1992). The display depicted data with a sequential or diverging color scheme and uncertainty with a shift in saturation. Results, based on user performance on tasks ranging from simple value look-up to overall map comparisons, indicated that reliability information can be added successfully to choropleth maps without inhibiting users' map-reading ability. The visually separable, but coincident display using texture was found to be most successful. Readers were less able to identify clusters using the visually integral scheme with color saturation depicting uncertainty.

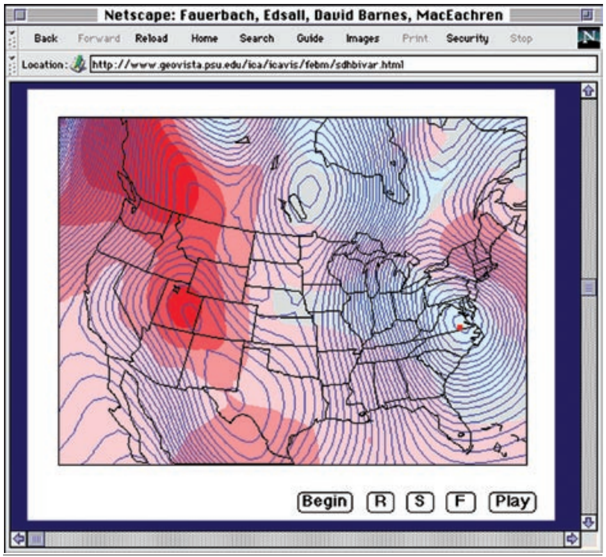

Figure 9. Screen capture from weather model uncertainty animation. [For details, see Fauerbach et al. (1996).]

Drecki (2002) also examined methods applied to areal data, but his focus was on the representation of uncertainty in land-cover classifications. He evaluated five methods for depicting a classification result and its certainty in the same representation: color saturation; opacity, the inverse of transparency; "squares" (a method in which size of a square glyph assigned to each location represents certainty while color fill represents data); blinking, modeled on Fisher's animation method discussed above; and 3D reliability surfaces. Drecki's (2002) empirical comparison of these methods, based on 50, mostly student users, found the squares method to be most effective, followed by opacity, blinking, 3D reliability surfaces, and color saturation, in descending order. Thus, Drecki's results agree with MacEachren et al.' (1998) in relation to color saturation. Interestingly, user preference did not match user success—while users agreed that the squares method was good, they also had a strong preference for the use of color saturation.

In another study focused on thematic maps depicting numerical data, Edwards and Nelson (2001) evaluated the effectiveness of four methods for signifying uncertainty on graduated circle maps on which circle area depicted the data. Methods to depict uncertainty were: verbal statement in the legend, certainty diagram in the legend, bivariate method using value of circle outline, e. g., lighter = less certain, and bivariate method using value of circle fill, e.g., light = less certain. Eighty geography students completed three kinds of tasks; they marked areas of highest data value and highest certainty, answered multiple choice questions on the variation in data value and certainty, and rated their own confidence in responses. Key results were that the two bivariate methods resulted in more accurate and more confident interpretations of certainty, and that the ability to interpret the data depicted by circle size was not influenced by the uncertainty signification method.

Most empirical efforts to assess different uncertainty representation strategies for maps focus on whether map users can interpret the uncertainty representation correctly and the extent to which different visual variables support this task. Leitner and Buttenfield (2000) went beyond this to consider the impact of different representation methods on map interpretation for decision making. Their research complements previous research in its focus on the relative appropriateness and effectiveness of specific visual variables, directing attention to color value, color saturation, and texture, and on the level of map detail, comparable to MacEachren's "resolution."

In a controlled experiment with a regional planning scenario, they asked 68 student participants to use a map to make site decisions for a park (easy decision) and an airport (difficult decision), while considering the impact of the development on surrounding wetlands. Eight test maps of four pairs were used that varied in detail within pairs and in the method by which uncertainty is signified among pairs. Map pairs included a one-category map and a three-category map. One pair did not depict certainty and the other three pairs each relied on a different visual variable to depict certainty, color value, color saturation, and texture. Certain data were represented with darker value, finer texture, and greater saturation, respectively. Map detail was found to have limited impact on results, but the maps that depicted uncertainty led to significantly more correct location decisions than those that did not. Response times were similar with and without uncertainty representation, from which the authors conclude that representation of uncertainty acts to clarify mapped information rather than to make the map cluttered or complex. Maps relying on a color value to depict uncertainty were used most effectively; results were best with a lighter value representing more certain information and a darker value representing less certain information. The authors also conclude that, if value cannot be used to depict certainty, finer texture, followed by higher saturation, should be used for more certainty.

While the impact of uncertainty representation on decision making has received some attention, as clear from the examples above, the focus has been on decision outcomes. The *process* of decision making has been given less attention (see the Challenges section below for further discussion).

Among the few studies that have focused on the role of uncertainty in the process of decision making is one by Kobus et al. (2001), geared toward understanding and improving military tactical decision making. In this empirical study, the authors used a map-based command post simulation exercise to control the amount and kinds of information provided to decision makers, measure the promptness of decisions, and collect performance data. The authors focused specifically on differences in decision-making processes and outcomes between experts and novices, as those differences are influenced by information certainty. Certainty was not directly visualized. Instead, it was implicit in the kinds and range of information provided to participants (who were split into two groups that started the exercise at different time points in the simulation), with substantially more information that was more certain available to those starting at the later time point. Among the study results, one that is particularly relevant here is that experienced officers were faster than inexperienced officers at executing a course of action under conditions of low certainty but not under conditions of high certainty. A factor in this difference related to the decision-making process was that the experienced officers were better than novices at developing situation awareness when information certainty was low. The authors interpret their findings to suggest that improving displays to enhance the process of acquiring situation awareness will improve user performance.

Dynamic Tools to Enable Access to and Understanding of Information Uncertainty

As the potential of dynamic representations has been applied to uncertainty visualization there have been many calls for empirical research focused on understanding the factors that influence the relative success of different dynamic approaches. However, only a small number of studies has been conducted, some focused on answering specific questions about dynamic representation, e.g., is the mapping of uncertainty to the dynamic variables of duration or frequency understood and effective, and some taking a usability approach intended to improve decision support environments in which visualization of information uncertainty is a factor. Key examples of each are highlighted below.

In one of the first studies focused on dynamic visualization of uncertainty, Evans (1997) conducted an empirical study that compared the use of color saturation to animated flickering as methods to depict uncertainty on land cover maps. Evans designed three different depictions of the reliability of land use/land cover classification: (a) a map where only pixels with high classification certainty (at least 95 percent certain) were shown; (b) a "static" map where all pixels were shown, but with those having high classification certainty depicted with highly saturated colors; and (c) an animated map that "flickered" back and forth between a standard map displaying all classified pixels and a map showing only highly certain pixels. Users were also given a map that did not provide information about data uncertainty. Evans assessed the success of the map forms with both expert and novice users across a range of task types. A majority of the users tested were able to understand and apply the information about map certainty, regardless of the method of graphic presentation, task type, or level of expertise used. Most users also judged both the "static" and the "flickering" maps to be helpful, although some users found flickering to be annoying.

Evans' (1997) study is one of several to focus attention on methods for visualizing land-cover classification uncertainty. Blenkinsop et al. (2000) extended this work and that by van der Wel et al. (1998) to explore methods for visualizing uncertainty that are grounded in fuzzy classification of satellite imagery data. Their focus complements Bastin et al.'s (2002) study discussed above. Using the FLIERS software developed to communicate fuzzy set membership of satellite imagery to general users and decision makers, Blenkinsop et al. compared three methods for representing uncertainty in classified images. These were grayscale images where white areas represent more certainty, random animation of possible outcomes of image classification, and serial animation of the grayscale image with user control. A series of map-reading questions about classification and certainty were posed, along with qualitative questions assessing confidence and effectiveness. Results showed that subjects were effective in extracting uncertainty from pixels with the grayscale and random animations techniques but unsuccessful in the serial animation technique. Random animation was determined to be best for showing the overall uncertainty of an image, while grayscale was better for extracting specific pixel uncertainty information. Adding linked graphic information was also found to enhance all three visualization techniques.

In common with several other authors, Aerts et al. (2003) focused on the visual variable of the color value, which they term lightness, as a method for representing uncertainty. Their empirical study focused on uncertainty representation in a spatial decision support context. Their approaches to uncertainty visualization were implemented within SLUETH, an urban growth model designed to aid planners, and applied to a case study of growth in Santa Barbara, California. Two visualization methods were used: a side-by-side static comparison of a model and its associated uncertainty, similar to that tested by MacEachren et al. (1998), and a toggling animation showing model results and uncertainty in an alternating four-frame-per-second sequence, as in Evans (1997). Uncertainty was shown in this animation using a color value, with a higher value, lighter, meaning less certain. Both methods enhanced the efficiency of spatial decision making, and participants recognized uncertainty with both techniques; however, 72 percent preferred the static technique and in contrast to Evans, those using the static method were significantly more accurate than those using toggling animation.

The above studies focused on assessing specific aspects of dynamic signification of uncertainty, and, in some cases, on comparing those dynamic representations to static ones. Cliburn et al. (2002) were among the first to apply a broader usability-engineering approach, designed to support development of a decision support system that incorporates uncertainty visualization. The system and its use of multivariate, manipulable glyphs to depict uncertainty was discussed above. As part of their approach to developing a useful and usable system, Cliburn et al. conducted a user task analysis and an assessment of their initial prototype decision support tool. One finding from this assessment was a general dissatisfaction with the clutter that multivariate glyphs depicting data and uncertainty generated, despite a flexible, interactive environment that allows users to select individual subsections of the display. Some users did find the ability to zoom in useful, but most had trouble getting past the initial overall display in order to decide which area they might want to examine separately.

Slocum et al. (2003) report on a follow-up to the Cliburn et al. (2002) study cited above, in which the focus is on usability of their 3D, large-screen display system for enabling decision makers to visualize uncertainty of the future global water balance. Intrinsic and extrinsic methods were used to visualize uncertainty in the modeled results. An intrinsic method used in this experiment integrated both data depiction and its certainty into the color assigned to each place. Three aspects of uncertainty are represented by the components of RGB, with data magnitudes assigned to color values. The extrinsic method tested used the height of a glyph, i.e., vertical bars, to show uncertainty of the surface and bar color to represent four input parameters of the model. Usability testing of these two signification methods showed that water balance experts preferred the extrinsic method, while decision makers preferred the intrinsic method. In addition, intrinsic methods were found to be best for communicating the "big picture" of uncertainty, while extrinsic methods were better for extracting specific locational uncertainty information.

Discussion

Uncertainty in geospatial information is a fundamental issue that cuts across many disciplines. It continues to attract attention as a research challenge, as evidenced by a recent National Research Council, Board on Mathematical Sciences and Their Applications Workshop: *Toward Improved Visualization of Uncertain Information* [http://www7.nationalacademies.org/bms/visualization_title_page.html].

Visualization of uncertainty is considered to be an important part of a broad strategy to enable analysts, decision makers, and others to cope with uncertain information. A wide variety of strategies has been proposed for uncertainty visualization, and there is a growing body of empirical research that is providing insights concerning which methods are effective in different use contexts and for which types of tasks. Still, we have only scratched the surface of the problem. For example, we cannot yet say definitively whether decisions are better if uncertainty is visualized or suppressed, or under what conditions they are better; nor do we understand the impact of uncertainty visualization on the process of analysis or decision making.

In relation to decision making with uncertain information, empirical results thus far provide mixed results in relation to the role of visualization. It is well known that experts and lay people make decisions differently. For example, for map-based decision making in orienteering, Crampton (1992) has demonstrated substantial strategy differences between expert and novice map use. Studies such as Cliburn et al. (2002) and Slocum et al. (2003), however, have yielded mixed results on expert–novice differences with different uncertainty visualization methods. It does appear that experts and novices may incorporate uncertainty into their decision-making processes differently. However, beyond this, existing research provides conflicting reports as to whether or not inclusion of uncertainty information is generally helpful and whether its helpfulness differs between expert and novice users. When included, there is little agreement in the literature about the best way to represent uncertainty. Vastly different methods have been suggested by many people. A great number of these methods seem to have potential for displaying attribute certainty on static and dynamic data representations, but only a few of them have been empirically assessed and the results have not been studied in depth, e.g., with the exception of color saturation and color value, most methods for depicting uncertainty visually have been tested in only a single narrow study, if at all.

There is considerable potential, we believe, to build upon the multidisciplinary base of knowledge represented by the research reviewed here. Progress toward an effective use of visualization for coping with uncertainty in geospatial information will be enabled by a clear articulation of important challenges. In the next section, we offer our perspective on these challenges.

Challenges

Visualization of uncertainty, even when the focus is on geospatial information and decision making, is an interdisciplinary problem. Based on our review and synthesis of literature on uncertainty visualization and related issues, we identify seven core challenges that will require interdisciplinary efforts to meet. These challenges relate to users and their information needs, as well as to methods and tools for making information uncertainty accessible and useful. Each challenge is outlined below, briefly.

1. *Understanding the components of uncertainty and their relationships to domains, users, and information needs*—Although there have been attempts to develop typologies of uncertainty, we know little about the range in conceptualization of uncertainty across domains of practice, across

data analysis and decision-making tasks, or across individuals. Most of the reported work has focused on more concrete uncertainties, such as uncertainty about the accuracy of location or time specifications, while limited attention has been given to abstract forms of uncertainty, such as those dealing with consistency and completeness. It would also be useful to study the prevalence and impact of different components by domain, understanding that these may be very different. A component that is very rare may still be highly impactful when it occurs. In addition, these components have interrelationships that may again differ by domain. For example, credibility of evidence is qualitatively different in the domains of intelligence analysis, where credibility may be a subjective decision about an individual informant, and cancer epidemiology, where credibility may be based on defined, quantitative standards for rating state cancer registries.

2. *Understanding how knowledge of information uncertainty influences information analysis, decision making, and decision outcomes*—In spite of considerable attention given to uncertainty visualization there has been little effort to assess the impact of visual depictions of uncertainty on work with information. Even the basic question of whether decision outcomes change in the face of an explicit depiction of uncertainty remains largely unanswered. A starting point here is to build upon the large body of work done on the process of decision making generally, and decision making under uncertainty specifically, in order to develop formal and testable models of the role of visual, external display of uncertainty in the decision-making process. Recent theoretical and empirical work in distributed/external cognition is likely to be relevant here (Hutchins 1996; Scaife and Rogers 1996).
3. *Understanding how (or whether) uncertainty visualization aids exploratory analysis*—Most of the empirical evaluations reported in this paper address visualization used to present information for a particular decision, such as the location of an airport. However, visualization is commonly used for exploratory analysis where there is no single question to be answered; rather the user is seeking to glean insights from the data. A core question here is how the portrayal and interaction with uncertainty can help the user better find and assess these insights. Related questions are when uncertainty should be considered during exploratory analysis, how representation of uncertainty influences the process of data exploration, and whether the outcomes of exploration are different and better when data uncertainty is explicitly visualized.
4. *Developing methods for capturing and encoding analysts' or decision makers' uncertainty*—When uncertainty is associated with parameters of measurement (e.g., error estimates for contour lines or feature position on topographic maps), uncertainty estimates are typically a bi-product of data processing and are usually numerical. When uncertainty is based on human judgment, e.g., as is typical for intelligence analysis or crisis management, then a fundamental problem that underlies incorporation of uncertainty in analysis and decision making is its capture and encoding as part of the information compilation and integration process. How can we estimate uncertainties without either undue burden on the user or skewed results due to uneven data entry?
5. *Developing representation methods for depicting multiple kinds of uncertainty*—Although most research dealing with uncertainty visualization has focused on representation, comprehensive guidelines for representing uncertainty do not yet exist. Some recommendations for encoding have been proposed (e.g., Buttenfield 1993), although little has been done to address the challenge of depicting multiple forms of uncertainty in the same display. As discussed in the section on typologies, uncertainty comes in many forms and applies to all kinds of data, e.g., to spatial, temporal, or attribute data. The categories of uncertainty are often interdependent, and the category boundaries are often hard to delineate. These factors exacerbate the already difficult challenge of signifying uncertainty in ways that do not prevent users from understanding the data of interest.
6. *Developing methods and tools for interacting with uncertainty depictions*—In spite of growing attention to interactivity as a fundamental component of visualization environments, limited attention in research on uncertainty visualization has been directed to capitalizing upon recent advances in direct manipulation interfaces (e.g., brushing, linking, dynamic query, conditioning). Similarly, the implications of user control over uncertainty representation have typically not been considered. This is an issue relevant to visualization in gen-

eral, since we lack the conceptual frameworks for understanding interactive representations that we have developed over the years for data representations. While cartographic and visualization textbooks increasingly include guidelines for matching symbolization types to data, few similar guidelines have been developed for matching interaction types to tasks.

7. *Assessing the usability and utility of uncertainty capture, representation, and interaction methods and tools*—Making progress toward the challenges identified above requires research on utility and usability of uncertainty representations. To accomplish this, we need progress toward two additional challenges: new empirical methods are needed to study the use of highly interactive display forms; and new empirical methods are needed for studying the role of visual representation as an input to strategies for addressing ill-structured problems. Additionally, it is worth focusing as much attention on the integrated and situated usage of uncertainty in real world analysis as has been traditionally placed on identifying effective techniques in and of themselves. Empirical usability evaluation of systems is described in several sources (Shneiderman 1998). However, current methods were developed for assessing performance on relatively well defined task (e.g., information retrieval), rather than for evaluating success of tools in support of ill-defined tasks, e.g., interactive visualization applied to analysis or decision making. Further, the commonly recommended metrics, such as error rates and time-to-task completion are less helpful for evaluating the utility of a visualization approach as opposed to its usability. In one example, recent work by the National Institute of Standards and Technology (NIST) has proposed candidate measures for assessing the impact of tools on complex information analysis tasks (Morse et al. 2005).

Uncertainty is a fundamental issue for information analysis and decision making within contexts ranging from personal decisions about optimal travel routes, through business decisions about facility locations, to strategic decisions about sending U.S. troops overseas. The cost of poor decisions is often high, thus efforts to develop strategies for visualizing uncertainty in information and in analytic outcomes have been identified as research priorities multiple times over the past decade or more. However, while we have made progress, routine use of uncertainty visualization remains rare. Achieving solutions that work in real-world applications will require new strategies for formalizing the kinds of uncertainty that matter in decision making, as well as greater attention to the processes of analytical reasoning and how those processes cope with uncertainty.

ACKNOWLEDGEMENT

Ideas reported here are based upon work supported by the Advanced Research and Development Activity and by the National Science Foundation under Grant No. 9983451.

REFERENCES

Aerts, J. C. J. H., K. C. Clarke, and A. D. Keuper. 2003. Testing popular visualization techniques for representing model uncertainty. *Cartography and Geographic Information Science* 30(3): 249-61.

Agumya, A., and G. J. Hunter. 2002. Responding to the consequences of uncertainty in geographical data. *International Journal of Geographical Information Science* 16(5): 405-17.

Bastin, L., P. F. Fisher, and J. Wood. 2002. Visualizing uncertainty in multi-spectral remotely sensed imagery. *Computers & Geosciences*, 28: 337-50.

Beard, K., and W. Mackaness. 1993. Visual access to data quality in geographic information systems. *Cartographica* 30(2): 37-45.

Bedard, Y. 1987. Uncertainties in land information databases. *In: Proceedings, Auto-Carto 8*. Baltimore, ACSM. pp. 175-84.

Bertin, J. 1983. *Semiology of graphics: Diagrams, networks, maps*. (Translation from a French 1967 edition by W. Berg, editor). Madison, WI: University of Wisconsin Press.

Blenkinsop, S., P. Fisher, L. Bastin, and J. Wood. 2000. Evaluating the perception of uncertainty in alternative visualization strategies. *Cartographica* 37(1): 1-13.

Buttenfield, B. P. 1991. Visualizing cartographic matadata. Working paper prepared for the NCGIA Specialist Meeting: Initiative 7, Visualization of the Data Quality, Orono, Maine.

Buttenfield, B. P. 1993. Representing data quality. *Cartographica* (special content, Mapping Data Quality) 30(2&3): 1-7.

Buttenfield, B. P., and M. K. Beard. 1994. Graphical and geographical components of data quality. In: D. Unwin and H. Hearnshaw (eds), *Visualization in geographic Information Systems*. London, U.K.: John Wiley & Sons. pp. 150-7.

Buttenfield, B. P., and R. Weibel. 1988. Visualizing the quality of cartographic data. Presented at Third International Geographic Information Systems Symposium (GIS/LIS 88), San Antonio, Texas.

Cedilnik, A., and P. Rheingans. 2000. Procedural annotation of uncertain information. In: *Visualization 2000 Proceedings*. IEEE, New York, New York. pp. 77-84, 542.

Chrisman, N. R. 1991. The error component in spatial data. In: D. J. Maguire, M. F. Goodchild, and D. W. Rhind (eds), *Geographical information systems*. London, U.K.: Longman. Vol. 1, Chapter 12, pp. 165-74.

Cliburn, D. C., J. J. Feddema, J. R. Miller, and T. A. Slocum. 2002. Design and evaluation of a decision support system in a water balance application. *Computers & Graphics* (UK) 26(6): 931-49.

Cohen, B. L., and T. S. Wallsten. 1992. The effect of constant outcome value on judgments and decision making given linguistic probabilities. *Journal of Behavioral Decision Making* 1(5): 53-72.

Couclelis, H. 2003. The certainty of uncertainty: GIS and the limits of geographic knowledge. *Transactions in GIS* 7(2): 165-75.

Crampton, J. 1992. A cognitive analysis of wayfinding expertise. *Cartographica* 29(3/4): 46-65.

Davis, J. 2002. Improving CIA Analytic Performance: Strategic Warning: Sherman Kent Center for Intelligence Analysis. [http://www.au.af.mil/au/awc/awcgate/cia/strategic_warning_kent.htm].

Davis, T. J., and C. P. Keller. 1997. Modelling and visualizing multiple spatial uncertainties. *Computers & Geosciences* 23(4): 397-408.

Djurcilov, S., K. Kim, P. Lermusiaux, and A. Pang. 2002. Visualizing scalar volumetric data with uncertainty. *Computers and Graphics* 26: 239-48.

Drecki, I. 1997, *Visualisation of uncertainty in spatial data*. Unpublished MSc thesis, The University of Auckland, Auckland.

Drecki, I. 2002. Visualisation of uncertainty in geographical data. In: W. Shi, P. Fisher, and M. Goodchild (eds), *Spatial data quality*. London: Taylor & Francis. pp. 140-159, color plates: 141-143.

Dreschler, M. 2004. Model-based conservation decision aiding in the presence of goal conflicts and uncertainty. *Biodiversity and Conservation* 13: 141-64.

Ducey, M. J. 2001. Representing uncertainty in silvicultural decisions: An application of the Dempster-Shafer theory of evidence. *Forest Ecology and Management* 150: 199-211.

Dungan, J.L., D.L. Kao, and A. Pang. 2003. Modeling and visualizing uncertainty in continuous variables predicted using remotely sensed data. In: *International Geoscience and Remote Sensing Symposium, 2003*. pp. 3017-19.

Edwards, L. D., and E. S. Nelson. 2001. Visualizing data certainty: A case study using graduated circle maps. *Cartographic Perspectives* 38:19-36.

Ehlschlaeger, C. R., A.M. Shortridge, and M. F. Goodchild. 1997. Visualizing spatial data uncertainty using animation. *Computers & Geosciences* 23(4): 387-96.

Evans, A., R. Kingston, S. Carver, and I. Turton. 1999. Web-based GIS to enhance public democratic involvement. In: *GeoComputation '99*, July 24-28, 1999, Mary Washington College, Virginia, USA.

Evans, B. J. 1997. Dynamic display of spatial data-reliability: Does it benefit the user? *Computers & Geosciences* (special issue on Exploratory Cartographic Visualization) 23(4): 409-22.

Faiz, S., and P. Boursier. 1996. Geographic data quality: From assessment to exploitation. *Cartographica* 33(1): 33-40.

Fauerbach, E., R. Edsall, D. Barnes, and A. MacEachren. 1996. Visualization of uncertainty in meteorological forecast models. *Proceedings of the International Symposium on Spatial Data Handling*, Delft, The Netherlands, August 12-16, 1996. New York, New York: Taylor & Francis. pp. 465-76.

Fegeas, R. G., J. L. Cascio, and R. A. Lazar. 1992. An overview of FIPS 173: The spatial data transfer standard. *Cartography and Geographic Information Systems* 19(5): 278-93.

Fisher, P. 1993. Visualizing uncertainty in soil maps by animation. *Cartographica* 30(2+3): 20-27.

Fisher, P. 1994. Randomization and sound for the visualization of uncertain spatial information. I: D. Unwin and H. Hearnshaw (eds), *Visualization in geographic information systems*. London, U.K.: John Wiley & Sons. pp. 181-5.

Foody, G. M., and P. M. Atkinson (eds). 2002. *Uncertainty in remote sensing and GIS*. Hoboken, New Jersey: Jossey-Bass, an imprint of Wiley. 326 pp.

Gahegan, M., and M. Ehlers. 2000. A framework for the modelling of uncertainty between remote sensing and geographic information. *ISPRS Journal of Photogrammetry and Remote Sensing* 55(3):176-88.

Gershon, N. D. 1992. Visualization of fuzzy data using generalized animation. In: *Proceedings, Visualization '92*, October 19-23, Boston, Massachusetts, October 19-23, 1992. IEEE Computer Society Technical Committee on Computer Graphics. pp. 268-73.

Gershon, N. D. 1998. Visualization of an imperfect world. *Computer Graphics and Applications* (IEEE) 18(4): 43-5.

Goodchild, M., L. Chih-Chang, and Y. Leung. 1994. Visualizing fuzzy maps. In: H. Hearnshaw and D. Unwin (eds), *Visualization in geographical information systems*. London, U.K.: John Wiley & Sons. pp. 158-67.

Goodchild, M. F. & S. Gopal. 1989. *Accuracy of Spatial Databases*. London: Taylor & Francis.

Graves, E. 2000. *Depicting strategic warning: A new model for the age of information operations*. Joint Military Intelligence College.

Hengl, T. 2003. Visualisation of uncertainty using the HIS colour model: Computations with colours. In: *Proceedings of the 7th International Conference on GeoComputation,* Southampton, United Kingdom. CD-Rom, pp. 8-17.

Howard, D., and A. M. MacEachren. 1996. Interface design for geographic visualization: Tools for representing reliability. *Cartography and Geographic Information Systems* 23(2): 59-77.

Hunter, G. J., and M. F. Goodchild. 1993. Managing uncertainty in spatial databases: Putting theory into practice. In: *Proceedings, URISA*, Atlanta, July 25–29, 1993. pp. 1–14.

Hutchins, E. 1996. *Cognition in the wild*. Cambridge, Massachusetts: MIT Press. 408 pp.

Jiang, B., F. Ormeling, and W. Kainz. 1995. Visualization support for fuzzy spatial analysis. *Auto Carto 12*, Charlotte, North Carolina. pp. 291-300.

Johnson, C., and A. Sanderson. 2003. A next step: Visualizing errors and uncertainty. *IEEE Computer Graphics and Applications* 23(5): 6-10.

Juang, K.-W. 2004. Using sequential indicator simulation to assess the uncertainty of delineating heavy-metal contaminated soils. *Environmental Pollution* 127: 229-38.

Kobus, D., S. Proctor, and S. Holste. 2001. Effects of experience and uncertainty during dynamic decision making. *International Journal of Industrial Ergonomics* 28: 275-90.

Krizan, L. 1999. *Intelligence essentials for everyone*: Joint Military Intelligence College.

Krygier, J. 1994. Sound and geographic visualization. In: A. M. MacEachren and D. R. F. Taylor (eds), *Visualization in modern cartography*. Oxford, UK: Pergamon. pp. 149-66.

Leitner, M., and B. P. Buttenfield. 2000. Guidelines for the display of attribute certainty. *Cartography and Geographic Information Science* 27(1): 3-14.

Lodha, S. K., A. Pang, R. E. Sheehan, and C. M. Wittenbrink. 1996a. UFLOW: Visualizing uncertainty in fluid flow. *Visualization '96*, San Francisco, California. IEEE. pp. 249-54.

Lodha, S. K., C. M. Wilson, and R. E. Sheehan. 1996b. LISTEN: Sounding uncertainty visualization. *Visualization '96*, San Francisco, California. IEEE. pp. 189-95.

Lowell, K. 1997. Outside-in, inside-out: Two methods of generating spatial certainty maps. In: Proceedings, Second Annual Conference of GeoComputation, Dunedin, New Zealand, 26-29 August 1997. University of Otago, Dunedin, New Zealand.

Lowenthal, M. M. 2003. *Intelligence, from secrets to policy*, 2nd ed. Washington, D.C.: CQ Press. 274p.

Lucieer, A., and M.-J. Kraak. 2004. Interactive and visual fuzzy classification of remotely sensed imagery for exploration of uncertainty. *International Journal of Geographical Information Science* 18(5): 491-512.

MacEachren, A. M. 1992. Visualizing uncertain information. *Cartographic Perspectives* (13): 10-19.

MacEachren, A. M. 1994. *SOME truth with maps: A primer on symbolization and design*. Association of American Geographers, Washington, D. C.

MacEachren, A. M. 1995. *How maps work: Representation, visualization and design*. New York, New York: Guilford Press.

MacEachren, A. M., C. A. Brewer, and L. W. Pickle. 1998. Visualizing georeferenced data: Representing reliability of health statistics. *Environment and Planning: A* 30: 1547-61.

McGranaghan, M. 1993. A cartographic view of spatial data quality. *Cartographica* 30(2 & 3): 8-19.

Miller, R., H. Karimi, and M. Feuchtwanger. 1989. Uncertainty and its management in geographical information systems. In: *Proceedings, CISM'89*. Canadian Institute of Surveying and Mapping, Ottawa, Canada. pp. 252-9.

Moellering, H. 1994. Continuing research needs resulting from the SDTS. *Cartography and Geographic Information Systems* 21(3): 180-9.

Morrison, J. (ed.). 1988. *NCDCDS* (National Committee for Digital Cartographic Data Standards): The proposed standard for digital cartographic data. American Congress on Surveying and Mapping, Washington, D.C.

Morse, E., M. P. Steves, and J. Scholtz. 2005. Metrics and methodologies for evaluating technologies for intelligence analysts. In: *Proceedings, International Conference on Intelligence Analysis*, McLean Virginia, May 2-4, 2005.

Pang, A. 2001. Visualizing uncertainty in geo-spatial data. In: *Proceedings of the Workshop on the Intersections between Geospatial Information and Information Technology*. National Academies Committee of the Computer Science and Telecommunications Board, Washington, D.C.

Pang, A. T., C. M. Wittenbrink, and S. K. Lodha. 1997. Approaches to uncertainty visualization. *Visual Computer* 13: 370-90.

Plewe, B. 2002. The nature of uncertainty in historical geographic information. *Transactions in GIS* 6(4): 431-56.

Reece, W., and L. Matthews. 1993. Evidence and uncertainty in subjective prediction: Influences on optimistic judgment. *Psychological Reports* 2(72): 435-9.

Rheingans, P. 1992. Color, change, and control for quantitative data display. In: *Proceedings, Visualization '92*, October 19-23, Boston, Massachusetts. IEEE Computer Society Technical Committee on Computer Graphics. pp. 252-9.

Scaife, M., and Y. Rogers. 1996. External cognition: How do graphical representations work? *International Journal of Human-Computer Studies* 45: 185-213.

Schweizer, D. M., and M. F. Goodchild. 1992. Data quality and choropleth maps: An experiment with the use of color. In: *Proceedings, GIS/LIS '92*, San Jose, California. ACSM and ASPRS, Washington, D.C. pp. 686-99.

Shneiderman, B. 1998. *Designing the user interface—Strategies for effective human-computer interaction*, 3rd ed. Reading, U.K. : Addison-Wesley Longman, Inc.

Sinton, D. 1978. The inherent structure of information as a constraint to analysis. In: D. Dutton (ed.), *Harvard papers on geographic information systems*. Reading, U.K.: Addison Wesley.

Slocum, T., R. McMaster, F. Kessler, and H. H. Howard. 2004. *Thematic cartography and visualization*, 2nd ed. Upper Saddle River, New Jersey: Prentice Hall. 528 pp.

Slocum, T. A., D. C. Cliburn, J. J. Feddema, and J. R. Miller. 2003. Evaluating the usability of a tool for visualizing the uncertainty of the future global water balance. *Cartography and Geographic Information Science* 30(4): 299-317.

St. John, M., J. Callan, S. Proctor, and S. Holste. 2000. *Tactical decision-making under uncertainty: Experiments*

I and II. San Diego, California: Space and Naval Warfare Systems Center. 22 pp.

Taylor., B. L., P. R. Wade, D. P. D. Master, and J. Barlow. 2000. Incorporating uncertainty into management models for marine mammals. *Conservation Biology* 14(5): 1243-52.

Thomson, J., B. Hetzler, A. MacEachren, M. Gahegan, and M. Pavel. 2005. Typology for visualizing uncertainty. In: Proceedings, *Conference on Visualization and Data Analysis 2005 (part of the IS&T/SPIE Symposium on Electronic Imaging 2005)*, San Jose, California, 16-20 January 2005. 12 pp.

Todd, C. R., and M. A. Burgman. 1998. Assessment of threat and conservation priorities under realistic levels of uncertainty and reliability. *Conservation Biology* 12(5): 966-74.

Tversky, A., and D. Kahneman. 1974. Judgement under uncertainty: Heuristics and biases. *Science* 185: 1124-31.

Unwin, D. J. 1995. Geographical information systems and the problem of 'error and uncertainty'. *Progress in Human Geography* 19(4): 549-58.

USGS. 1997. *Spatial Data Transfer Standard (SDTS): Logical specifications*. U.S. Geological Survey, Virginia.

van der Wel, F. J. M., R.M. Hootsman, and F. Ormeling. 1994. Visualization of data quality. In: A. M. MacEachren and D. R. F. Taylor (eds), *Visualization in modern cartography*. London, U.K.: Pergamon. pp. 313-332.

van der Wel, F. J. M., L. C. Van der Gaag, and B. G. H. Gorte. 1998. Visual exploration of uncertainty in remote-sensing classification. *Computers & Geosciences* 24(4): 335-43.

Veregin, H. 1989. Error modeling for the map overlay operation. In: M. Goodchild and S. Gopal (eds), *Accuracy of spatial databases*. New York, New York: Taylor and Francis. pp. 3-18.

Veregin, H. 1995. Developing and testing of an error propagation model for GIS overlay operations. *International Journal of Geographical Information Systems* 9(6): 595-619.

Wittenbrink, C.M., E. Saxon, J.J. Furman, A. Pang, and S. Lodha. 1995. Glyphs for visualizing uncertainty in environmental vector fields. IEEE Translations on Visualization and Computer Graphics 2(3): 266-79.

Worboys, M. 1998. Computation with imprecise geospatial data. *Computers, Environment and Urban Systems* 22(2): 85-106.

Zhang, J., and M. F. Goodchild (eds). 2002. *Uncertainty in geographical information*. New York, New York: Taylor & Francis.