

# Board questions set 4

## Problem 1: Variances

- (a) Prove that if  $X \sim \text{Bernoulli}(p)$ , then  $\text{Var}(X) = p(1 - p)$ .
- (b) Prove that if  $X \sim \text{Bin}(n, p)$ , then  $\text{Var}(X) = np(1 - p)$ .
- (c) Suppose  $X_1, X_2, \dots, X_n$  are independent and all have the same standard deviation  $\sigma = 2$ . Let  $\bar{X}$  be the average of  $X_1, X_2, \dots, X_n$ . What is the standard deviation of  $\bar{X}$ ?

## Problem 2: Covariance

Flip a fair coin 3 times. Let  $X$  be the number of heads in the first 2 flips and let  $Y$  be the number of heads in the last 2 flips. Compute  $\text{Cov}(X, Y)$ .

## Problem 3: More covariance

Toss a fair coin  $2n + t$  times. Let  $X$  be number of heads in the first  $n + t$  flips and let  $Y$  be number of heads in the last  $n + t$  flips. Compute  $\text{Cov}(X, Y)$  and  $\text{Cor}(X, Y)$ .