TTK4190 Guidance and Control of Vehicles

Assignment 2

Aircraft Autopilot Design with State Estimation

Fall 2017

Deadline Part 1: Sunday the 8th of October at 23:59
Deadline Part 2: Sunday the 15th of October at 23:59

Objective

In this assignment you will apply linear theory and design an autopilot for course control for an aircraft. You will also learn about state estimation using a Kalman filter. You will mostly refer to the book and lecture notes from Beard and McLain (2012) [1], but the book by Fossen [2] is also very relevant.

Grading

This assignment accounts for 10% of your overall course grade (6 % for Part 1 and 4 % for Part 2). You will receive a score between 0 and 10, with 10 being the best possible score. Both your simulations and the written report will be evaluated. Both problems will be graded together even though they are delivered separately. **You are encouraged/supposed to work in groups of two people, but are allowed to do the assignment individually if that is preferred for some reason**. Note that the grading will be equally severe if you do the work individually. Both participants in the group will receive the same score. Because someone might want to change groups, a new set of groups for the second assignment should be used and are available on Blackboard. It is obviously allowed to cooperate with the same person as on the first assignment, but you still need to register again to get access to the delivery.

Deadline and Delivery Details

Part 1 must be handed in **by 23:59 on Sunday the 8th of October and part 2 by Sunday the 15th of October, respectively**. If you use Simulink to solve the simulations, they should be started via a m-file and not the Simulink window. Matlab code and Simulink models should not be included in the report. The code you hand in must work as is, so we should just run it once in order to obtain *all* the required results. Use degrees as the unit in all of your figures (degrees for angles and degrees/s for the rates). Further information regarding the Matlab code is given in the corresponding tasks. The report and the required code have to be handed in **via Blackboard**. You are strongly urged to write the report on your PC using your favorite editor (LaTeX, Word, Pages...). You can hand in a scanned version (not recommended), but in the end it has to be a **PDF** document. Paper versions are not accepted.

Aircraft Model

The linear lateral model of an aircraft controlled with ailerons is given as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned} \tag{1}$$

where

$$\mathbf{x} = \begin{bmatrix} \beta \text{ (rad)} \\ \phi \text{ (rad)} \\ p \text{ (rad/s)} \\ r \text{ (rad/s)} \\ \delta_a \text{ (rad)} \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \delta_a^c \text{ (rad)} \end{bmatrix}, \mathbf{y} = \begin{bmatrix} r \text{ (rad/s)} \\ p \text{ (rad/s)} \\ \beta \text{ (rad)} \\ \phi \text{ (rad)} \end{bmatrix} \tag{2}$$

The actuator dynamics of the aileron are augmented to the lateral equations of motion as a state δ_a, which is modeled as a low-pass filter ($H_l(s)$) with time constant $T_l = 1/10$ s. Moreover,

$$H_l(s) = \frac{1}{T_l s + 1} = \frac{10}{s + 10} \tag{3}$$

The control input for the augmented model is denoted δ_a^c. The aileron has a maximum deflection of ± 25 degrees. All of your simulations need to make sure that this constraint is satisfied.

During a coordinated turn the course angle χ satisfies the *bank-to-turn* equation:

$$\dot{\chi} = \frac{g}{V_g} \tan(\phi) \cos(\chi - \psi) \tag{4}$$

Assume that the wind speed is zero (no wind).

State-space matrices and trim condition

Airspeed $V_a = 637$ km/h

$$\mathbf{A} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & 0 & 1 & -0.001 & 0 \\ -10.6 & 0 & -2.87 & 0.46 & -0.65 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 0 & 0 & -10 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \tag{5}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Part 1 (6 % of final grade)

The first part of the assignment starts here and should be handed in by 23:59 on Sunday the 8th of October.

Problem 1 - Open-loop analysis

When investigating the modes in this problem, use the definitions from the book [1].

a) What is the ground speed of the aircraft (numerical value) in the absence of wind?

b) Write down two expressions for the sideslip (crab) angle β in the absence of wind. One expression should depend on aircraft velocity and the other on aircraft heading.

c) Compute the Dutch-roll natural frequency and relative damping ratio for the aircraft. Can you, very briefly with your own words, describe how the Dutch roll mode affect the yaw and roll motion? How would the motion change with increased relative damping ratio?

d) Compute the spiral-divergence mode. Is the mode unstable?

e) Compute the roll mode. Is the roll mode faster or slower than the spiral-divergence mode?

Problem 2 - Autopilot for course hold using aileron and successive loop closure

Figure 1 shows the block diagram for a lateral autopilot using successive loop closure. The autopilot is based on a simplified version of the coordinated turn equation and a linearization of the roll dynamics from the book [1]. This represents an autopilot for course hold using aileron δ_a as control input. The disturbance d in Figure 1 models a bias in the coordinated-turn equation, and should have a constant value of 2 degrees.

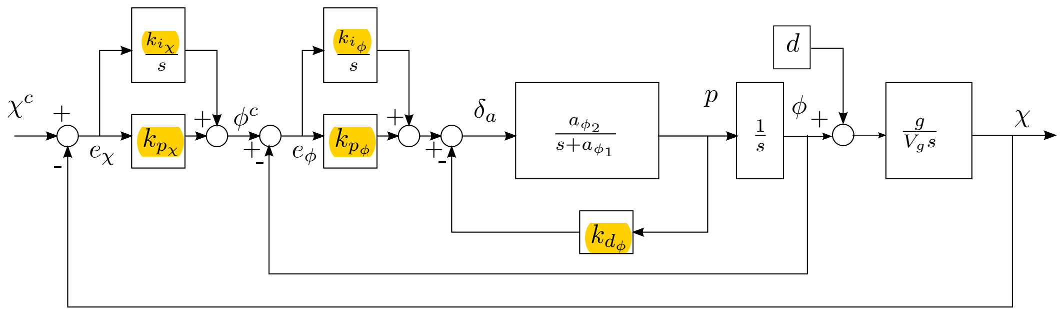

Figure 1: Successive loop closure for autopilot design.

a) Find numerical values for a_{ϕ_1} and a_{ϕ_2} based on the state-space model (1).

b) Find numerical values for the five gains in Figure 1 associated with the lateral autopilot using successive loop closure. Use root-locus analysis to propose an interval where the integral gain in the roll loop (k_{i_ϕ}) should be chosen within in order to maintain system stability. Include a figure of the root-locus analysis in your report and remember to specify the input range for the gain somewhere in the figure (use both positive and negative gain values). Justify how you choose the design parameters in the course loop.

Hint: when designing the roll loop, choose $\delta_a^{max} = 25°$, $e_\phi^{max} = 15°$, $\zeta_\phi = 0.707$

c) Can you come up with a reason for why it in some cases may be beneficial to remove the integral action from the roll loop? You are free to remove the integral action from the roll loop in the rest of the assignment if you think that is appropriate, but write why if you choose to do so.

d) Find the open-loop transfer functions from ϕ^c to ϕ ($\frac{\phi}{\phi^c}(s)$) and χ^c to χ ($\frac{\chi}{\chi^c}(s)$) (it can be achieved both on paper and in Matlab). The transfer functions should be based on Figure 1 and you cannot simplify the inner loop as a gain of 1 in this case. You can, however, neglect the disturbance. Present the transfer functions together in a bode plot and attach a figure in your report. Comment on the stability of the system. What happens with the stability of the system if the natural frequency of the course loop is designed with the same frequency as the roll loop? Attach a figure of the bode plot in this case as well.

Hint 1: The transfer function from χ^c to χ should have a denominator of order 4 or 5 depending on your choice of k_{i_ϕ}. It can be clever to use the "minreal" function in Matlab to find the transfer function with the lowest possible order.
Hint 2: A system is stable if the phase is > -180 degrees at ω_c and if the amplitude is $< 1 = 0$ dB at ω_{180}. ω_c is the frequency where the amplitude crosses the 0 dB line (from above) and ω_{180} is the frequency where the phase crosses the -180 degrees line (from above). Check the book in TTK 4105 ("Reguleringsteknikk") if you want to know more.

e) Present simulation results for the system in Figure 1 with course changing maneuvers: choose a series of steps (you may choose yourself the desired values). Comment on the results and remember that you are simulating a simplification of the system dynamics (1). You should not have larger steps than ±15 degrees since we are working with a linearized model. Include figures of the course and the aileron input in the report.

Code hand-in: *Hand in an m-file named* `run_task_2e.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_2e.m`. *Place all Matlab files in a folder entitled "task_2e".*

f) Present simulation results for course changing maneuvers (choose the same desired values as in Problem 2e) with the complete state-space model (1). Keep your control gains from Problem 2e). The aircraft model must be the complete state-space model, including the actuator dynamics, and the simplified version of the coordinated turn equation must be replaced by the real coordinated turn equation (4). Moreover, the system dynamics (1) should replace the simplified roll dynamics in Figure 1. Remember to add the bias d at the appropriate place. Compare the results with the results obtained in Problem 2e). Would you say that the simplified model in Problem 2e) accurately reproduce the true dynamics? Include figures of the course and aileron input in the report.

Code hand-in: *Hand in an m-file named* `run_task_2f.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_2f.m`. *Place all Matlab files in a folder entitled "task_2f".*

g) The integral action in the course loop compensates for the disturbance d. However, if the aileron input is saturated, integrator windup can occur. Would you claim that integrator windup is a problem in your simulations? If that is the case, can you propose a simple solution that would limit the the problem? Show simulation results (course changing maneuvers) with the system from Problem 2f) with your suggested action to avoid integrator windup. Attach a Figure of the course and input in the report if you choose to do something.

Part 2 (4 % of final grade)

The second part of the assignment starts here and should be handed in by 23:59 on Sunday the 15th of October.

Problem 3 - State Estimation using a Kalman filter

In Problem 2f)/2g) it was assumed that all states could be measured perfectly and were available for feedback. In practice, accurate measurements of all states are usually not available and state feedback cannot be used directly. Therefore, it may be necessary to design a state estimator that can be used for feedback in the control system. In this problem, you will implement a Kalman filter to estimate the states based on measurements affected by Gaussian white noise. The bias d affecting the coordinated-turn equation should still be a part of the system.

a) Write down the system equations for a Kalman filter that estimates sideslip, roll, roll rate and yaw rate (β, ϕ, p and r). Measurements contaminated by Gaussian white noise are available for ϕ, p and r (sideslip cannot be measured). The set-point of the aileron input is obviously also available, but the aileron dynamics should be neglected and considered to be unknown in the Kalman filter. In other words: write down the expression for the system matrices $\mathbf{A}, \mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. Moreover, describe the purpose of the matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{P}$ in the Kalman filter and state their dimension.

b) Measurements of roll, roll rate and yaw rate are available. What type of sensors would you use to measure these states and how? What kind of noise would your sensors be affected by in practice? Can you come up with an example of a situation where the white noise assumption may be problematic?

c) Under which conditions are the Kalman filter the optimal linear state estimator (unbiased and minimizes the variance)? Are all of these conditions met in this case?

d) Design a Matlab function that implements a Kalman filter. If you want to use Simulink, the Kalman filter should still be implemented as a Matlab function. You cannot use the Kalman filter block in Simulink when you do simulations later. Add the Kalman filter to your simulation setup from Problem 2g)/2f). The Kalman filter should estimate β, ϕ, p and r and be based on the matrices and measurements stated in Problem 3a).

e) Simulate the course loop with the Kalman filter included. Use the desired course from Problem 2. Moreover, use noise-contaminated measurements of roll, roll rate and yaw rate for feedback to the control loop. A measurement of the sideslip is not available so use the estimated sideslip in the coordinated turn equation. In other words, you need to generate the noise-contaminated measurements at the output of the state-space model by adding Gaussian white noise to the true output for ϕ, p and r. The Gaussian white noise should have the following variance (the measurements are uncorrelated):

Measurement	Variance
Roll (ϕ)	$(2 \text{ deg})^2$
Roll rate (p)	$(0.5 \text{ deg/s})^2$
Yaw rate (r)	$(0.2 \text{ deg/s})^2$

Use zero as initial values for the estimates in the Kalman filter and choose an appropriate initial covariance ($\mathbf{P}(0)$). You can assume that the cross-variance is zero. $\mathbf{Q}$ can be chosen as a diagonal matrix with a value of 10^{-6} on the diagonal, but you are free to choose other values.

Comment on the results and attach the following figures in the report:

- Course together with desired course.
- Aileron input.

- The noise-contaminated measurements of the roll angle together with the estimated state and the true value (given as the output from the state-space).
- The noise-contaminated measurements of the roll rate together with the estimated state and the true value.
- The noise-contaminated measurements of the yaw rate together with the estimated state and the true value.

Code hand-in: *Hand in an m-file named* `run_task_3e.m` *that will initialize the simulation, run the simulation, and generate the necessary plots. Include any other files necessary to execute* `run_task_3e.m`. *Place all Matlab files in a folder entitled "task_3e".*

f) In the previous problem, we used the noise-contaminated measurements directly for feedback. In this problem, you will replace these measurements with the corresponding estimates from the Kalman filter. Simulate the system from Problem 3e), but replace the feedback from the measurements of roll, roll rate and yaw rate with feedback from the Kalman filter. Comment on the performance of the course control compared to the control system in Problem 3e)? Would you say that the course control is better than in the previous task? In theory, how should the bandwidth of the estimator be compared to the bandwidth of the control loop?

   Attach the same set of figures as in Problem 3e).

References

[1] R. W. Beard and T. W. McLain, *Small unmanned aircraft: Theory and practice.* Princeton university press, 2012.

[2] T. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, 2011.