Big Data Hw1

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1 Questions

Let y = Ax + b be a random vector. show that expectation is linear:
E[y] = E[Ax + b] = AE[x] + b

Also show that:
cov[y] = cov[Ax + b] = Acov[x]A = A Σ A

a) There are two seperate solutions. If x is continous, we will use the second solution.

For dicsrete values:
E[y] = Σ (Ax + b) P(x) Definition of E
= Σ Ax * P(x) + Σ bP(x)
= A Σ x * P(x) + b Σ P(x)
= AE[x] + b

For continous values:

$$E[y] = \int (Ax + b)P(x)$$

By subsitution similar to the above:

$$E[y] = \int (Ax)P(x) + \int bP(x)$$

=

$$A \int xP(x) + b \int P(x)$$

= AE[x] + b

b) Prove cov[y] = Acov[X]A^T

cov[y]= E[(y - E[y])(y - E[y]T)]

= E [(Ax + b - E[Ax +b])(Ax + b - E[Ax + b])T]

= E[(Ax + b - AE[x +b]) (Ax + b - AE[x +b] T]

= E[(Ax - AE[x])(Ax - Ax - AE[x])T]

= E[A (x- E[x]) (X$^T - E[x]$T)AT]

= A E[(x - E[x])(XT - E[x]T) AT]

= A Σ AT

= Acov[x]A^T

2a) Find least squares using Cramer's rule

y = Ax

$$A \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}$$

In order to have determinants for Cramer's rule be squared, we will multiply both sides by X^T

$X^T * y = X^T * Ax$

Cramer's Rule:

$X0 = X^T * X = \begin{pmatrix} 4 & 9 \\ 9 & 29 \end{pmatrix}$

$X = X^T * X = \begin{pmatrix} 18 & 9 \\ 56 & 29 \end{pmatrix}$

$X2 = X^T * X = \begin{pmatrix} 4 & 18 \\ 9 & 56 \end{pmatrix}$

Det X1 = 18

Det X2 = 62

Det x0 = 35

m = 18/35

b = 62/35

y = (18/35)X + 62/35

2 Part B: Normal Equation for linear Regression

$$\theta = (\mathrm{X}^T * X)^-1(X^T) * y$$

$$= \begin{pmatrix} 4 & 9 \\ 9 & 29 \end{pmatrix}^{-1} * \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} .828 & -.25 \\ -.25 & .114 \end{pmatrix}^{-1} * \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} .514 \\ 1.77 \end{pmatrix}$$

This is the decimal equivalent of the solution found in part A.