

# Mt. Hood Precalculus

## Elementary Functions



# Mt. Hood Precalculus

## Elementary Functions

Jack Green  
Mt. Hood Community College

Nick Chura  
Mt. Hood Community College

March 2, 2016



# Contents



# Chapter 1

## Functions

### 1.1 Student Learning Outcomes

- Identify a function from a table of values, a graph or an equation.
- Recognize, apply, interpret, evaluate and solve equations using function notation.
- Define a relation and define a function.
- Perform calculations using function notation including Average Rate of Change.
- Determine if a variable is dependent or independent.
- Determine the intervals over which a function is increasing or decreasing or constant based on a numerical, graphical or algebraic model.
- Demonstrate appropriate use of inequality notation and interval notation.

### 1.2 Gist

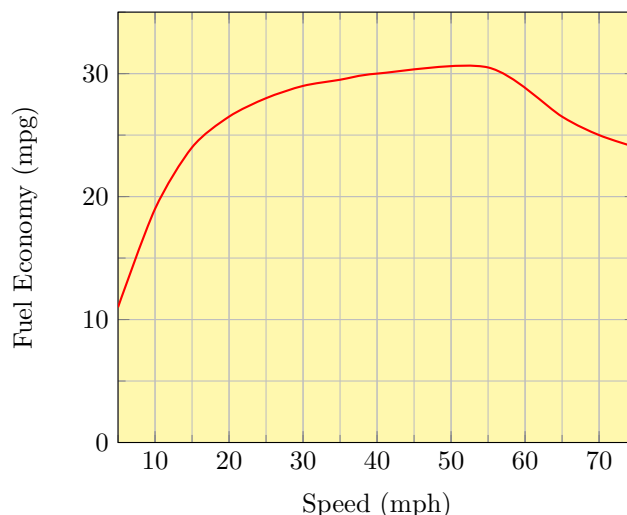
A function is a rule that be in the form of a graph or a table of values or a formula. It may even be a sentence or a set of instructions. A function takes an input value and uses the rule to create an output value.

Function notation looks like this:  $f(\text{input}) = \text{output}$

But, instead of writing the words “input” and “output” we usually use variables, like  $x$  and  $y$ . Then we define in words what the variables actually mean or represent. Most often the notation will look something like this:  $f(x) = y$

Inside the parentheses is the **Independent Variable**, like  $x$ , and outside the parentheses, on the other side of the equals sign, is the **Dependent Variable**, like  $y$ .

**Example 1.2.1.** Let  $E$  be the fuel efficiency, in miles per gallon, of a car traveling at  $s$  miles per hour. For speeds between 5 and 75 miles per hour, the function in ?? tells us the efficiency of the car.



**Figure 1.2.2:** Fuel Economy versus Speed

Therefore, the *efficiency is a function of the speed*. Using function notation we could write an equation saying the same thing:  $E = f(s)$

The input is the speed,  $s$ , and the output is the efficiency  $E$ . The function  $f(s)$  is the graph itself.

The efficiency of the car is given by the height of the graph. Therefore  $f(35) = 28$  means, “The efficiency of the car traveling at 35 mph is 28 mpg”.

### 1.2.1 Inputs have *unique* outputs

An important aspect of functions is that each input can only have one output. For instance the car can only have one efficiency at a given speed. The cannot get 10 mpg and 20 mpg simultaneously.

However, it is possible there are multiple inputs that give you the same output. From the graph we see that a fuel efficiency of 25 mpg can be attained by traveling at a speed of about 20 mph and also at about 70 mph.

### 1.2.2 Describing functions with intervals

A function may have many different characteristics. The function may increase or decrease, it may curve up or curve down, it may have positive outputs or negative outputs. When we describe a function, we make our observations relative to the intervals (sections) of the input on which the observation takes place.

For instance, using our efficiency example, the efficiency may be the lowest at about 5 mph and highest at about 55 mph. The efficiency increases (goes up) between 5 and 55 mph then decreases (goes down) for speeds between 55 and 75 mph.

Any portion of the speed (horizontal) axis or the efficiency (vertical) axis, is called an interval. An interval represents a section of the input axis where something special occurs on the function.

We can use inequalities to describe when the efficiency is increasing,  $5 < s < 55$ . This means that fuel efficiency increases for speeds between 5 and 55 mph.

Using inequalities we can say the efficiency graph is

- **concave down** (bends downwards) for  $5 < s < 60$ .
- **concave up** (bends downwards) for  $60 < s < 75$ .



### 1.2.3 Evaluate vs. Solve

**Evaluate** means to use a known input value to find the output value. Evaluating a function looks like this:

$$f(25)$$

It means, “Find the result of choosing 25 mph”. The symbol  $f(25)$  represents an **output**.

From the graph we see that  $f(25) \approx 27$ .

**Solve** means the output is already known and we are trying to find all the possible inputs that give us the desired result. Solving an equation will look like this:

$$\text{Solve } f(s) = 25$$

In other words it means, “Find the input or inputs that give us an output of 25”.

Using our efficiency graph, we can solve the equation  $f(s) = 25$ .

At the speeds 20 mph and 70 mph the car will have an efficiency of 25 mpg.

### 1.2.4 Exercises

1. (Equation for a Secant Line)

**Problem.** Find a formula for the line passing through the function  $f(x) = \frac{10}{x^2+1}$  at  $x = 1$  and  $x = 3$ .

**Solution.** Since  $f(1) = 5$ , and  $f(3) = 1$ , then the slope of this line is  $\frac{1-5}{3-1} = 2$ . Using the point-slope form, we can write the line's equation as  $y = 2(x - 1) + 5$ .



## Chapter 2

# Domain, Range, and Piecewise-Defined Functions



## Chapter 3

# Exponential Functions



## Chapter 4

# Logarithmic Functions





## Chapter 5

# Vertical and Horizontal Translations



## Chapter 6

# Reflections and Vertical Stretches



## Chapter 7

# Composition and Inverse Functions



## Chapter 8

# Combinations of Functions





## Chapter 9

# Power Functions and Polynomials