## Quantum++ v0.8.4

Generated by Doxygen 1.8.9.1

Wed Oct 14 2015 08:59:16

## **Contents**

1	Qua	ntum++																	1
2	Nam	espace	Index																5
	2.1	Names	pace List						 			 		 					 5
3	Hier	archical	Index																7
	3.1	Class I	Hierarchy						 			 		 					 7
4	Clas	s Index																	9
	4.1	Class I	_ist						 			 		 					 9
5	File	Index																	11
_	5.1		t						 	 		 	 	 					 11
6	Nam	neenace	Documer	nta	tior	•													13
U	6.1		mespace																13
	0.1																		
		6.1.1	Detailed			•													24
		6.1.2	Typedef I																25
			6.1.2.1		•	t													25
			6.1.2.2																25
			6.1.2.3			t													25
			6.1.2.4	С	plx				 			 	 	 					 25
			6.1.2.5	d	lmat	t			 			 	 	 					 25
			6.1.2.6	d	lyn_	_col_	vec	t.	 			 		 					 25
			6.1.2.7	d	lyn_	mat			 			 	 	 					 25
			6.1.2.8	d	lyn_	row	_vec	ct	 			 	 	 					 25
			6.1.2.9	ic	xb				 			 		 					 25
			6.1.2.10	k	et				 			 	 	 					 26
			6.1.2.11	to	)_vc	oid			 			 	 	 					 26
			6.1.2.12	! u	bigi	nt			 	 		 	 	 					 26
		6.1.3	Function																26
			6.1.3.1			n													26
			6.1.3.2			<b>q</b>													26

iv CONTENTS

6.1.3.3	abssq	26
6.1.3.4	abssq	27
6.1.3.5	adjoint	28
6.1.3.6	anticomm	28
6.1.3.7	apply	28
6.1.3.8	apply	29
6.1.3.9	apply	29
6.1.3.10	apply	29
6.1.3.11	apply	30
6.1.3.12	applyCTRL	30
6.1.3.13	applyCTRL	31
6.1.3.14	avg	31
6.1.3.15	bloch2rho	31
6.1.3.16	choi2kraus	32
6.1.3.17	choi2super	32
6.1.3.18	comm	32
6.1.3.19	complement	32
6.1.3.20	compperm	33
6.1.3.21	concurrence	33
6.1.3.22	conjugate	33
6.1.3.23	contfrac2x	33
6.1.3.24	contfrac2x	34
6.1.3.25	cor	34
6.1.3.26	cosm	34
6.1.3.27	cov	34
6.1.3.28	cwise	35
6.1.3.29	det	35
6.1.3.30	dirsum	35
6.1.3.31	dirsum	36
6.1.3.32	dirsum	36
6.1.3.33	dirsum	36
6.1.3.34	dirsumpow	37
6.1.3.35	disp	37
6.1.3.36	disp	37
6.1.3.37	disp	37
6.1.3.38	disp	38
6.1.3.39	disp	38
6.1.3.40	eig	38
6.1.3.41	entanglement	39
6.1.3.42	entropy	39

CONTENTS

6.1.3.43	entropy	39
6.1.3.44	evals	40
6.1.3.45	evects	40
6.1.3.46	expm	40
6.1.3.47	factors	40
6.1.3.48	funm	41
6.1.3.49	gcd	41
6.1.3.50	gcd	41
6.1.3.51	gconcurrence	42
6.1.3.52	grams	42
6.1.3.53	grams	42
6.1.3.54	grams	42
6.1.3.55	heig	43
6.1.3.56	hevals	43
6.1.3.57	hevects	43
6.1.3.58	inverse	44
6.1.3.59	invperm	44
6.1.3.60	ip	44
6.1.3.61	ip	44
6.1.3.62	isprime	45
6.1.3.63	kraus2choi	45
6.1.3.64	kraus2super	45
6.1.3.65	kron	46
6.1.3.66	kron	46
6.1.3.67	kron	46
6.1.3.68	kron	47
6.1.3.69	kronpow	47
6.1.3.70	lcm	47
6.1.3.71	lcm	48
6.1.3.72	load	48
6.1.3.73	loadMATLABmatrix	48
6.1.3.74	loadMATLABmatrix	49
6.1.3.75	loadMATLABmatrix	49
6.1.3.76	logdet	49
6.1.3.77	logm	50
6.1.3.78	lognegativity	50
6.1.3.79	marginalX	50
6.1.3.80	marginalY	50
6.1.3.81	measure	51
6.1.3.82	measure	51

vi CONTENTS

6.1.3.83	measure	51
6.1.3.84	measure	52
6.1.3.85	measure	52
6.1.3.86	measure	53
6.1.3.87	measure	53
6.1.3.88	measure	54
6.1.3.89	measure	54
6.1.3.90	measure_seq	55
6.1.3.91	measure_seq	55
6.1.3.92	mket	55
6.1.3.93	mket	56
6.1.3.94	modpow	56
6.1.3.95	mprj	56
6.1.3.96	mprj	57
6.1.3.97	multiidx2n	58
6.1.3.98	n2multiidx	58
6.1.3.99	negativity	58
6.1.3.100	norm	59
6.1.3.101	omega	59
6.1.3.102	operator"""_i	59
6.1.3.103	operator"""_i	59
6.1.3.104	powm	59
6.1.3.105	prj	60
6.1.3.106	prod	60
6.1.3.107	prod	60
6.1.3.108	prod	60
6.1.3.109	ptrace	61
6.1.3.110	ptrace	61
6.1.3.111	ptrace1	61
6.1.3.112	ptrace2	63
6.1.3.113	ptranspose	63
6.1.3.114	ptranspose	63
6.1.3.115	qmutualinfo	64
6.1.3.116	qmutualinfo	64
6.1.3.117	rand	64
6.1.3.118	rand	65
6.1.3.119	rand	66
6.1.3.120	rand	66
6.1.3.121	rand	66
6.1.3.122	rand	67

CONTENTS vii

6.1.3.123 randH
6.1.3.124 randidx
6.1.3.125 randket
6.1.3.126 randkraus
6.1.3.127 randn
6.1.3.128 randn
6.1.3.129 randn
6.1.3.130 randn
6.1.3.131 randperm
6.1.3.132 randrho
6.1.3.133 randU
6.1.3.134 randV
6.1.3.135 renyi
6.1.3.136 renyi
6.1.3.137 reshape
6.1.3.138 rho2bloch
6.1.3.139 rho2pure
6.1.3.140 save
6.1.3.141 saveMATLABmatrix
6.1.3.142 saveMATLABmatrix
6.1.3.143 saveMATLABmatrix
6.1.3.144 schatten
6.1.3.145 schmidtA
6.1.3.146 schmidtB
6.1.3.147 schmidtcoeffs
6.1.3.148 schmidtprobs
6.1.3.149 sigma
6.1.3.150 sinm
6.1.3.151 spectralpowm
6.1.3.152 sqrtm
6.1.3.153 sum
6.1.3.154 sum
6.1.3.155 sum
6.1.3.156 super2choi
6.1.3.157 svals
6.1.3.158 svd
6.1.3.159 svdU
6.1.3.160 svdV
6.1.3.161 syspermute
6.1.3.162 syspermute

viii CONTENTS

		6.1.3.163 trace
		6.1.3.164 transpose
		6.1.3.165 tsallis
		6.1.3.166 tsallis
		6.1.3.167 uniform
		6.1.3.168 var
		6.1.3.169 x2contfrac
	6.1.4	Variable Documentation
		6.1.4.1 chop
		6.1.4.2 ee
		6.1.4.3 eps
		6.1.4.4 infty
		6.1.4.5 maxn
		6.1.4.6 pi
6.2	qpp::ex	perimental Namespace Reference
	6.2.1	Detailed Description
6.3	qpp::in	ernal Namespace Reference
	6.3.1	Detailed Description
	6.3.2	Function Documentation
		6.3.2.1 _check_cvector
		6.3.2.2check_dims
		6.3.2.3 _check_dims_match_cvect
		6.3.2.4 _check_dims_match_mat
		6.3.2.5 _check_dims_match_rvect
		6.3.2.6 _check_eq_dims
		6.3.2.7 _check_matching_sizes
		6.3.2.8 _check_nonzero_size
		6.3.2.9 _check_perm
		6.3.2.10 _check_qubit_cvector
		6.3.2.11 _check_qubit_matrix
		6.3.2.12 _check_qubit_rvector
		6.3.2.13 _check_qubit_vector
		6.3.2.14 _check_rvector
		6.3.2.15 _check_square_mat
		6.3.2.16 _check_subsys_match_dims
		6.3.2.17 _check_vector
		6.3.2.18 _dirsum2
		6.3.2.19 _kron2
		6.3.2.20 _multiidx2n
		6.3.2.21 _n2multiidx

CONTENTS

			6.3.2.22 variadic_vector_emplace	32
			6.3.2.23 variadic_vector_emplace	32
7	Clas	s Docu	mentation 8	33
	7.1	qpp::C	odes Class Reference	33
		7.1.1	Detailed Description	34
		7.1.2	Member Enumeration Documentation	34
			7.1.2.1 Type	34
		7.1.3	Constructor & Destructor Documentation	34
			7.1.3.1 Codes	34
			7.1.3.2 ~Codes	34
		7.1.4	Member Function Documentation	35
			7.1.4.1 codeword	35
		7.1.5	Friends And Related Function Documentation	35
			7.1.5.1 internal::Singleton < const Codes >	35
	7.2	qpp::E	cception Class Reference	35
		7.2.1	Detailed Description	37
		7.2.2	Member Enumeration Documentation	37
			7.2.2.1 Type	37
		7.2.3	Constructor & Destructor Documentation	38
			7.2.3.1 Exception	38
			7.2.3.2 Exception	38
		7.2.4	Member Function Documentation	38
			7.2.4.1 _construct_exception_msg	38
			7.2.4.2 what	39
		7.2.5	Member Data Documentation	39
			7.2.5.1 _custom	39
			7.2.5.2 _msg	39
			7.2.5.3 _type	39
			7.2.5.4 _where	39
	7.3	qpp::G	ates Class Reference	39
		7.3.1	Detailed Description	91
		7.3.2	Constructor & Destructor Documentation	91
			7.3.2.1 Gates	91
			7.3.2.2 ~Gates	91
		7.3.3	Member Function Documentation	91
			7.3.3.1 CTRL	91
			7.3.3.2 expandout	92
			7.3.3.3 Fd	92
			7.3.3.4 ld	93

CONTENTS

		7.3.3.5 Rn
		7.3.3.6 Xd
		7.3.3.7 Zd
	7.3.4	Friends And Related Function Documentation
		7.3.4.1 internal::Singleton < const Gates >
	7.3.5	Member Data Documentation
		7.3.5.1 CNOT
		7.3.5.2 CNOTba
		7.3.5.3 CZ
		7.3.5.4 FRED
		7.3.5.5 H
		7.3.5.6 ld2
		7.3.5.7 S
		7.3.5.8 SWAP
		7.3.5.9 T
		7.3.5.10 TOF
		7.3.5.11 X
		7.3.5.12 Y
		7.3.5.13 Z
7.4	qpp::ID	isplay Class Reference
	7.4.1	Detailed Description
	7.4.2	Constructor & Destructor Documentation
		7.4.2.1 IDisplay
		7.4.2.2 IDisplay
		7.4.2.3 IDisplay
		7.4.2.4 ~IDisplay
	7.4.3	Member Function Documentation
		7.4.3.1 display
		7.4.3.2 operator=
		7.4.3.3 operator=
	7.4.4	Friends And Related Function Documentation
		7.4.4.1 operator <<
7.5	qpp::ln	it Class Reference
	7.5.1	Detailed Description
	7.5.2	Constructor & Destructor Documentation
		7.5.2.1 Init
		7.5.2.2 ~Init
	7.5.3	Friends And Related Function Documentation
		7.5.3.1 internal::Singleton < const Init >
7.6	qpp::in	ternal::IOManipEigen Class Reference

CONTENTS xi

	7.6.1	Constructor & Destructor Documentation
		7.6.1.1 IOManipEigen
		7.6.1.2 IOManipEigen
	7.6.2	Member Function Documentation
		7.6.2.1 display
	7.6.3	Member Data Documentation
		7.6.3.1 _A
		7.6.3.2 _chop
7.7	qpp::int	ernal::IOManipPointer< PointerType > Class Template Reference
	7.7.1	Constructor & Destructor Documentation
		7.7.1.1 IOManipPointer
		7.7.1.2 IOManipPointer
	7.7.2	Member Function Documentation
		7.7.2.1 display
		7.7.2.2 operator=
	7.7.3	Member Data Documentation
		7.7.3.1 _end
		7.7.3.2 _n
		7.7.3.3 _p
		7.7.3.4 _separator
		7.7.3.5 _start
7.8	qpp::int	ernal::IOManipRange< InputIterator > Class Template Reference
	7.8.1	Constructor & Destructor Documentation
		7.8.1.1 IOManipRange
		7.8.1.2 IOManipRange
	7.8.2	Member Function Documentation
		7.8.2.1 display
		7.8.2.2 operator=
	7.8.3	Member Data Documentation
		7.8.3.1 _end
		7.8.3.2 _first
		7.8.3.3 _last
		7.8.3.4 _separator
		7.8.3.5 _start
7.9	qpp::is_	_complex< T > Struct Template Reference
	7.9.1	Detailed Description
7.10	qpp::is_	_complex< std::complex< T > > Struct Template Reference
	7.10.1	Detailed Description
7.11	qpp::is_	_iterable< T, typename > Struct Template Reference
	7.11.1	Detailed Description

xii CONTENTS

7.12		_iterable< T, to_void< decltype(std::declval< T >().begin()), decltype(std::declval< T	107
		d()), typename T::value_type > > Struct Template Reference	
7.40			
7.13		_matrix_expression< Derived > Struct Template Reference	
		Detailed Description	
7.14		_matrix_expression< typename Eigen::MatrixBase< Derived >> Struct Template Reference 1	
		Detailed Description	
7.15		andomDevices Class Reference	
		Detailed Description	
	7.15.2	Constructor & Destructor Documentation	
		7.15.2.1 RandomDevices	
		7.15.2.2 ~RandomDevices	
	7.15.3	Friends And Related Function Documentation	12
		7.15.3.1 internal::Singleton < RandomDevices >	
	7.15.4	Member Data Documentation	12
		7.15.4.1 _rd	12
		7.15.4.2 _rng	12
7.16	qpp::int	rernal::Singleton< T > Class Template Reference	13
	7.16.1	Detailed Description	13
	7.16.2	Constructor & Destructor Documentation	14
		7.16.2.1 Singleton	14
		7.16.2.2 Singleton	14
		7.16.2.3 ~Singleton	14
	7.16.3	Member Function Documentation	14
		7.16.3.1 get_instance	14
		7.16.3.2 get_thread_local_instance	14
		7.16.3.3 operator=	14
7.17	qpp::St	ates Class Reference	14
	7.17.1	Detailed Description	16
	7.17.2	Constructor & Destructor Documentation	16
		7.17.2.1 States	16
		7.17.2.2 ~States	16
	7.17.3	Friends And Related Function Documentation	16
		7.17.3.1 internal::Singleton < const States >	16
	7.17.4	Member Data Documentation	16
		7.17.4.1 b00	16
		7.17.4.2 b01	17
		7.17.4.3 b10	17
		7.17.4.4 b11	17
		7.17.4.5 GHZ	17

CONTENTS xiii

	7.17.4.6 pb00
	7.17.4.7 pb01
	7.17.4.8 pb10
	7.17.4.9 pb11
	7.17.4.10 pGHZ
	7.17.4.11 pW
	7.17.4.12 px0
	7.17.4.13 px1
	7.17.4.14 py0
	7.17.4.15 py1
	7.17.4.16 pz0
	7.17.4.17 pz1
	7.17.4.18 W
	7.17.4.19 x0
	7.17.4.20 x1
	7.17.4.21 y0
	7.17.4.22 y1
	7.17.4.23 z0
	7.17.4.24 z1
7.18 qpp::T	imer< T, CLOCK_T > Class Template Reference
7.18.1	Detailed Description
7.18.2	Constructor & Destructor Documentation
	7.18.2.1 Timer
	7.18.2.2 Timer
	7.18.2.3 Timer
	7.18.2.4 ~Timer
7.18.3	Member Function Documentation
	7.18.3.1 display
	7.18.3.2 get_duration
	7.18.3.3 operator=
	7.18.3.4 operator=
	7.18.3.5 tic
	7.18.3.6 tics
	7.18.3.7 toc
7.18.4	Member Data Documentation
	7.18.4.1 _end
	7.18.4.2 _start
File Docum	
8.1 classe	s/codes.h File Reference

8

XIV

	8.1.1 Detailed Description	123
8.2	classes/exception.h File Reference	123
	8.2.1 Detailed Description	124
8.3	classes/gates.h File Reference	124
	8.3.1 Detailed Description	125
8.4	classes/idisplay.h File Reference	125
	8.4.1 Detailed Description	125
8.5	classes/init.h File Reference	126
	8.5.1 Detailed Description	126
8.6	classes/random_devices.h File Reference	126
	8.6.1 Detailed Description	127
8.7	classes/states.h File Reference	127
	8.7.1 Detailed Description	127
8.8	classes/timer.h File Reference	128
	8.8.1 Detailed Description	128
8.9	constants.h File Reference	128
	8.9.1 Detailed Description	129
8.10	entanglement.h File Reference	129
	8.10.1 Detailed Description	130
8.11	entropies.h File Reference	130
	8.11.1 Detailed Description	131
8.12	experimental/experimental.h File Reference	132
	8.12.1 Detailed Description	132
8.13	functions.h File Reference	132
	8.13.1 Detailed Description	136
8.14	input_output.h File Reference	136
	8.14.1 Detailed Description	137
8.15	instruments.h File Reference	137
	8.15.1 Detailed Description	139
8.16	internal/classes/iomanip.h File Reference	139
	8.16.1 Detailed Description	140
8.17	internal/classes/singleton.h File Reference	140
	8.17.1 Detailed Description	141
8.18	internal/util.h File Reference	
	8.18.1 Detailed Description	142
8.19	macros.h File Reference	142
	The state of the s	143
		143
	8.19.2.1 ERROR	
	8.19.2.2 ERRORLN	143

CONTENTS xv

	8.19.2.3 PRINT	143
	8.19.2.4 PRINTLN	143
8.20	MATLAB/matlab.h File Reference	143
	8.20.1 Detailed Description	144
8.21	number_theory.h File Reference	144
	8.21.1 Detailed Description	145
8.22	operations.h File Reference	145
	8.22.1 Detailed Description	147
8.23	qpp.h File Reference	147
	8.23.1 Detailed Description	149
8.24	random.h File Reference	149
	8.24.1 Detailed Description	150
8.25	statistics.h File Reference	150
	8.25.1 Detailed Description	151
8.26	traits.h File Reference	151
	8.26.1 Detailed Description	152
8.27	types.h File Reference	152
	8.27.1 Detailed Description	153
Index		155
illuex		100

## Chapter 1

## Quantum++

### Version 0.8.5 - development

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. Quantum++ is written in standard C++11 and has very low external dependencies, using only the Eigen 3 linear algebra header-only template library and, if available, the OpenMP multi-processing library.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance.

If you are interesting in contributing, please contact me. To contribute, you need to have a good knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and working experience with Eigen 3.

For additional Eigen 3 documentation see http://eigen.tuxfamily.org/dox/. For a simple Eigen 3 quick ASCII reference see http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt.

Copyright (c) 2013 - 2016 Vlad Gheorghiu, vgheorgh AT gmail DOT com.

Quantum++ is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation, either version 3 of the License, or (at your option) any later version.

Quantum++ is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

You should have received a copy of the GNU General Public License along with Quantum++. If not, see http-://www.gnu.org/licenses/.

### **Building instructions**

#### Configuration:

- Compiler: g++ version 4.8 or later (for good C++11 support)
- Eigen 3 library located in \$HOME/eigen
- Quantum++ library located in \$HOME/qpp
- MATLAB compiler include header files: /Applications/MATLAB\_R2014b.app/extern/include
- MATLAB compiler shared library files: /Applications/MATLAB\_R2014b.app/bin/maci64

2 Quantum++

#### Building without a build system

- Example file: \$HOME/qpp/examples/minimal.cpp
- Output executable: \$HOME/qpp/examples/minimal
- Must run the commands below from inside the directory \$HOME/qpp/examples

## Release version (without ${\tt MATLAB}$ support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

#### Debug version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

#### Release version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2014b.app/extern/include \
    -L/Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

#### Debug version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2014b.app/extern/include \
    -L /Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

#### Building using cmake

The current version of the repository has a ./CMakeLists.txt configuration file for building examples using cmake. To build an example using cmake, I recommend an out-of-source build, i.e., from the root of the project (where ./include is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the relase version (default) executable qpp, from the source file ./examples/minimal.cpp, without MATLAB support (default), inside the directory ./build. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build  \begin{tabular}{ll} $\sf cd ./build \\ $\sf rm -rf * \\ $\sf cmake -DCMAKE\_BUILD\_TYPE=Debug -DWITH\_MATLAB=ON .. \\ $\sf make \\ \end{tabular}
```

#### Or, to disable OpenMP support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the ./CMakeLists.txt file. See also ./CMakeLists.txt for additional options. Do not forget to remove everything from the ./build directory before a fresh build!

#### **Additional remarks**

- The C++ compiler must be C++11 compliant.
- If using Windows, I recommend compiling under cygwin via cmake and g++. See also http←://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2 for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file <cmath> using the provided patch./cmath\_cygwin.patch.
- If your compiler does not support OpenMP (as it is the case e.g with clang++), disable OpenMP in your build, as otherwise the linker may not find the gomp library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD\_L← IBRARY\_PATH is set to point to the MATLAB compiler library location, see the run\_OSX\_MATLAB script. Otherwise, you will get a runtime error like dyld: Library not loaded: @rpath/libmat.← dylib.

```
* I recommend running via a script, as otherwise setting the 
'DYLD_LIBRARY_PATH' globally may interfere with 
[macports] (https://www.macports.org/)' [cmake] (http://www.cmake.org/) 
installation (in case you use [cmake] (http://www.cmake.org/) from 
[macports] (https://www.macports.org/)). If you use a script, 
then the environment variable is local to the script and 
does not interfere with the rest of the system.

* Example of running script, run from inside the directory where 
the executable 'qpp' is located:

#!/bin/sh # Run Quantum++ under OS X with MATLAB support

export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:"/Applications/MATLAB_R2014b.app/bin/maci64"
./qpp
```

• If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See http://stackoverflow.← com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks for more details about this problem.

Quantum++

# Chapter 2

# Namespace Index

## 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

qpp	
Quantum++ main namespace	13
qpp::experimental	
Experimental/test functions/classes, do not use or modify	80
qpp::internal	
Internal utility functions, do not use/modify	80

6 Namespace Index

# **Chapter 3**

## **Hierarchical Index**

## 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

sta::exception
qpp::Exception
false_type
qpp::is_complex< T >
qpp::is_iterable < T, typename >
qpp::is_matrix_expression< Derived >
qpp::IDisplay
qpp::internal::IOManipEigen
qpp::internal::IOManipPointer< PointerType >
qpp::internal::IOManipRange< InputIterator >
qpp::Timer< T, CLOCK_T >
$qpp::internal::Singleton < T > \dots \dots$
$qpp::internal::Singleton < const \ Codes > \dots $
qpp::Codes
qpp::internal::Singleton < const Gates >
qpp::Gates
qpp::internal::Singleton < const Init >
qpp::Init
app::internal::Singleton < const States >
gpp::States
qpp::internal::Singleton< RandomDevices >
gpp::RandomDevices
true type
qpp::is_complex< std::complex< T >>
qpp::is_iterable < T, to_void < decltype(std::declval < T >().begin()), decltype(std::declval < T
>().end()), typename T::value_type >>
qpp::is_matrix_expression< typename Eigen::MatrixBase< Derived >>

8 **Hierarchical Index** 

# **Chapter 4**

## **Class Index**

## 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

qpp::Codes	
Const Singleton class that defines quantum error correcting codes	83
qpp::Exception	
Generates custom exceptions, used when validating function parameters	85
qpp::Gates	
Const Singleton class that implements most commonly used gates	89
qpp::IDisplay	
Abstract class (interface) that mandates the definition of virtual std::ostream& display(std←	
::ostream& os) const	95
qpp::Init	
Const Singleton class that performs additional initializations/cleanups	97
qpp::internal::IOManipEigen	99
qpp::internal::IOManipPointer< PointerType >	100
qpp::internal::IOManipRange< InputIterator >	102
qpp::is_complex< T >	
Checks whether the type is a complex type	104
qpp::is_complex< std::complex< T > >	
Checks whether the type is a complex number type, specialization for complex types	105
qpp::is_iterable < T, typename >	
Checks whether $T$ is compatible with an STL-like iterable container $\dots$	106
$qpp::is\_iterable < T, \ to\_void < \ decltype(std::declval < T > ().begin()), \ decltype(std::declval < T > ().end()), \ decltype(std::decltype$	
typename T::value_type > >	
Checks whether $T$ is compatible with an STL-like iterable container, specialization for STL-like	
iterable containers	107
qpp::is_matrix_expression< Derived >	
Checks whether the type is an Eigen matrix expression	109
qpp::is_matrix_expression< typename Eigen::MatrixBase< Derived >>	
Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expres-	
sions	110
qpp::RandomDevices	
Singeleton class that manages the source of randomness in the library	111
qpp::internal::Singleton< T >	
Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously	
recurring template pattern)	113
qpp::States	
Const Singleton class that implements most commonly used states	114
qpp::Timer< T, CLOCK_T >	
Chronometer	119

10 Class Index

# **Chapter 5**

## File Index

## 5.1 File List

Here is a list of all files with brief descriptions:

constants.h
Constants
entanglement.h
Entanglement functions
entropies.h
Entropy functions
functions.h
Generic quantum computing functions
input_output.h
Input/output functions
instruments.h
Measurement functions
macros.h
Preprocessor macros
number_theory.h
Number theory functions
operations.h
Quantum operation functions
qpp.h
Quantum++ main header file, includes all other necessary headers
random.h
Randomness-related functions
statistics.h
Statistics functions
traits.h
Type traits
types.h
Type aliases
classes/codes.h
Quantum error correcting codes
classes/exception.h
Exceptions
classes/gates.h
Quantum gates
classes/idisplay.h
Display interface via the non-virtual interface (NVI)
classes/init.h
Initialization

12 File Index

classes/random_devices.h
Random devices
classes/states.h
Quantum states
classes/timer.h
Timing
experimental/experimental.h
Experimental/test functions/classes
internal/util.h
Internal utility functions
internal/classes/iomanip.h
Input/output manipulators
internal/classes/singleton.h
Singleton pattern via CRTP
MATLAB/matlab.h
Input/output interfacing with MATLAB

## **Chapter 6**

## **Namespace Documentation**

## 6.1 qpp Namespace Reference

Quantum++ main namespace.

### **Namespaces**

· experimental

Experimental/test functions/classes, do not use or modify.

internal

Internal utility functions, do not use/modify.

### Classes

class Codes

const Singleton class that defines quantum error correcting codes

class Exception

Generates custom exceptions, used when validating function parameters.

• class Gates

const Singleton class that implements most commonly used gates

class IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

class Init

const Singleton class that performs additional initializations/cleanups

· struct is\_complex

Checks whether the type is a complex type.

struct is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

• struct is\_iterable

Checks whether T is compatible with an STL-like iterable container.

struct is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().end()), typename T::value\_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

· struct is matrix expression

Checks whether the type is an Eigen matrix expression.

struct is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

· class RandomDevices

Singeleton class that manages the source of randomness in the library.

· class States

const Singleton class that implements most commonly used states

· class Timer

Chronometer.

### **Typedefs**

```
template<typename... >
using to_void = void
```

Alias template that implements the proposal for void\_t.

• using idx = std::size\_t

Non-negative integer index.

• using bigint = long long int

Big integer.

using ubigint = unsigned long long int

Non-negative big integer.

• using cplx = std::complex< double >

Complex number in double precision.

• using ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

using bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$ 

```
using dyn mat = Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic >
```

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

```
using dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
```

Dynamic Eigen column vector over the field specified by Scalar.

template<typename Scalar >

```
using dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
```

Dynamic Eigen row vector over the field specified by Scalar.

#### **Functions**

constexpr cplx operator"" i (unsigned long long int x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

constexpr cplx operator""\_i (long double x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

• cplx omega (idx D)

D-th root of unity.

• template<typename Derived >

dyn\_col\_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
&dims)

Schmidt coefficients of the bi-partite pure state A.

```
• template<typename Derived >
  cmat schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Schmidt basis on Alice side.

    template<typename Derived >

  cmat schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Schmidt basis on Bob side.
• template<typename Derived >
  std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
  &dims)
     Schmidt probabilities of the bi-partite pure state A.

    template<typename Derived >

  double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Entanglement of the bi-partite pure state A.

    template<typename Derived >

  double gconcurrence (const Eigen::MatrixBase< Derived > &A)
      G-concurrence of the bi-partite pure state A.

    template<typename Derived >

  double negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Negativity of the bi-partite mixed state A.

    template<typename Derived >

  double lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Logarithmic negativity of the bi-partite mixed state A.
• template<typename Derived >
  double concurrence (const Eigen::MatrixBase< Derived > &A)
      Wootters concurrence of the bi-partite qubit mixed state A.
• template<typename Derived >
  double entropy (const Eigen::MatrixBase< Derived > &A)
      von-Neumann entropy of the density matrix A

    double entropy (const std::vector< double > &prob)

     Shannon entropy of the probability distribution prob.

    template<typename Derived >

  double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)
      Renyi- \alpha entropy of the density matrix A, for \alpha \geq 0.

    double renyi (const std::vector< double > &prob, double alpha)

     Renyi- \alpha entropy of the probability distribution prob, for \alpha \geq 0.

    template < typename Derived >

  double tsallis (const Eigen::MatrixBase< Derived > &A, double q)
      Tsallis- q entropy of the density matrix A, for q \ge 0.

    double tsallis (const std::vector< double > &prob, double q)

      Tsallis- q entropy of the probability distribution prob, for q \geq 0.
• template<typename Derived >
  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, const std::vector< idx > &dims)
      Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  double gmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector < idx > &subsysB, idx d=2)
      Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
```

Left singular vectors.

```
Complex conjugate.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)
     Adjoint.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar det (const Eigen::MatrixBase < Derived > &A)
     Determinant.

    template<typename Derived >

  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum of A.

    template<typename Derived >

  Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)
     Element-wise product of A.

    template<typename Derived >

  double norm (const Eigen::MatrixBase< Derived > &A)
     Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition.

    template<typename Derived >

  dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)
     Eigenvalues.
• template<typename Derived >
  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.

    template<typename Derived >

  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn_col_vect< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.

    template<typename Derived >

  dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.
• template<typename Derived >
  cmat svdU (const Eigen::MatrixBase< Derived > &A)
```

```
• template<typename Derived >
  cmat svdV (const Eigen::MatrixBase< Derived > &A)
     Right singular vectors.

    template<typename Derived >

  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
• template<typename Derived >
  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.

    template<typename Derived >

  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template < typename Derived >

  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.
• template<typename Derived >
  cmat sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.

    template<typename Derived >

  cmat cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.
• template<typename Derived >
  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.
ullet template<typename Derived >
  double schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn_mat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const type-
  name Derived::Scalar &))
     Functor.

    template<typename T >

  dyn mat< typename T::Scalar > kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn mat< typename T::Scalar > kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > kron (const std::vector< Derived > &As)
     Kronecker product.
template<typename Derived >
  dyn_mat< typename Derived::Scalar > kron (const std::initializer_list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
```

```
• template<typename T >
  dyn_mat< typename T::Scalar > dirsum (const T &head)
• template<typename T , typename... Args>
  dyn mat< typename T::Scalar > dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsum (const std::vector< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsum (const std::initializer_list< Derived > &As)
      Direct sum.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
      Direct sum power.
\bullet \ \ {\it template}{<} {\it typename Derived} >
  dyn_mat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx
  cols)
     Reshape.
 template<typename Derived1 , typename Derived2 >
  dyn mat< typename Derived1::Scalar > comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::↔
  MatrixBase< Derived2 > &B)
      Commutator.
ullet template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > grams (const std::initializer list< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)
      Gram-Schmidt orthogonalization.

    std::vector< idx > n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx multiidx2n (const std::vector < idx > &midx, const std::vector < idx > &dims)

     Multi-index to non-negative integer index.

    ket mket (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Multi-partite qudit ket.

    ket mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
```

std::vector< double > abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of an STL-like range of complex numbers.

• template<typename Container >

std::vector< double > abssq (const Container &c, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Computes the absolute values squared of an STL-like container.

template<typename Derived >

std::vector< double > abssq (const Eigen::MatrixBase< Derived > &A)

Computes the absolute values squared of an Eigen expression.

template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

template<typename Container >

Container::value\_type sum (const Container &c)

Element-wise sum of the elements of an STL-like container.

template<typename InputIterator >

std::iterator\_traits < InputIterator >::value\_type prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

template<typename Container >

Container::value\_type prod (const Container &c)

Element-wise product of the elements of an STL-like container.

• template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

template<typename T >

std::vector< T > complement (std::vector< T > subsys, idx N)

Constructs the complement of a subsystem vector.

template<typename Derived >

std::vector< double > rho2bloch (const Eigen::MatrixBase< Derived > &A)

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

template<typename Derived >

internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

Eigen expression ostream manipulator.

• internal::IOManipEigen disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

• template<typename InputIterator >

internal::IOManipRange < InputIterator > disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

• template<typename Container >

internal::IOManipRange< typename Container::const\_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

• template<typename PointerType >

internal::IOManipPointer< PointerType > disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

template<typename Derived >

void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

• template<typename Derived >

 $\label{lem:dyn_col_vect} $$ \down_col_vect< typename\ Derived::Scalar > ip\ (const\ Eigen::MatrixBase< Derived > &phi,\ const\ Eigen::$$ \down_col_vect< \dwown_col_vect< \down_col_vect< \d$ 

Generalized inner product.

• template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::← MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

template<typename Derived >

std::tuple < idx, std::vector < double >, std::vector < cmat > > measure (const Eigen::MatrixBase < Derived > &A, const std::vector < cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

• template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure\_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, std::vector < idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

 $std::tuple < std::vector < idx >, double, cmat > measure\_seq (const Eigen::MatrixBase < Derived > \&A, std::vector < idx > subsys, idx d=2)$ 

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

• template<typename Derived >

Derived loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<</li>

dmat loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template

cmat loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

• template<typename Derived >

void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std
::string &var name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

template<</li>

void saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

• template<>

void saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

std::vector< int > x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

double contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

ubigint gcd (ubigint m, ubigint n)

Greatest common divisor of two non-negative integers.

ubigint gcd (const std::vector< ubigint > &ns)

Greatest common divisor of a list of non-negative integers.

• ubigint lcm (ubigint m, ubigint n)

Least common multiple of two positive integers.

ubigint lcm (const std::vector< ubigint > &ns)

Least common multiple of a list of positive integers.

std::vector< idx > invperm (const std::vector< idx > &perm)

Inverse permutation.

std::vector< idx > compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< ubigint > factors (ubigint n)

Prime factor decomposition.

bool isprime (ubigint n)

Primality test.

ubigint modpow (ubigint a, ubigint n, ubigint p)

Integer power modulo p.

template<typename Derived1 , typename Derived2 >

 $\frac{dyn\_mat}{dyn\_mat} < typename \ Derived1::Scalar > \frac{dyn\_mat}{dyn\_mat} < typename \ Derived1::Scalar > \frac{dyn\_mat}{dyn\_mat} < typename \ Derived1 > &state, \ const \ Eigen::MatrixBase < Derived2 > &A, \ const \ std::vector < \frac{idx}{idx} > &subsys, \ const \$ 

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
 Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx
 d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template < typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)
```

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
idx > &subsys, const std::vector< idx > &dims)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
idx > &subsys, idx d=2)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat kraus2choi (const std::vector< cmat > &Ks)

Choi matrix

std::vector < cmat > choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

```
dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &dims)
```

Partial trace.

• template<typename Derived >

```
dyn_mat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std↔ ::vector< idx > &dims)
```

Partial trace.

• template<typename Derived >

Partial trace.

 $\bullet \ \ \text{template}{<} \text{typename Derived} >$ 

```
dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector
idx > &subsys, idx d=2)
```

Partial trace.

• template<typename Derived >

Partial transpose.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial transpose.

template<typename Derived >

Subsystem permutation.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std ← ::vector < idx > &perm, idx d=2)

Subsystem permutation.

double rand (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

bigint rand (bigint a=std::numeric limits < bigint >::min(), bigint b=std::numeric limits < bigint >::max())

Generates a random big integer uniformly distributed in the interval [a, b].

• ubigint rand (ubigint a=std::numeric\_limits< ubigint >::min(), ubigint b=std::numeric\_limits< ubigint >↔ ::max())

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

idx randidx (idx a=std::numeric\_limits < idx >::min(), idx b=std::numeric\_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

• template<typename Derived >

Derived rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

template<typename Derived >

Derived randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

• template<>

cmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat randU (idx D)

Generates a random unitary matrix.

• cmat randV (idx Din, idx Dout)

Generates a random isometry matrix.

```
    std::vector< cmat > randkraus (idx N, idx D)

          Generates a set of random Kraus operators.

    cmat randH (idx D)

          Generates a random Hermitian matrix.

    ket randket (idx D)

          Generates a random normalized ket (pure state vector)

    cmat randrho (idx D)

          Generates a random density matrix.

    std::vector< idx > randperm (idx n)

          Generates a random uniformly distributed permutation.

    std::vector< double > uniform (idx N)

          Uniform probability distribution vector.

    std::vector< double > marginalX (const dmat &probXY)

          Marginal distribution.

    std::vector< double > marginalY (const dmat &probXY)

          Marginal distribution.
    • template<typename Container >
      double avg (const std::vector< double > &prob, const Container &X)
          Average.
    • template<typename Container >
      double cov (const dmat &probXY, const Container &X, const Container &Y)
          Covariance.
    • template<typename Container >
      double var (const std::vector< double > &prob, const Container &X)
          Variance.
    • template<typename Container >
      double sigma (const std::vector< double > &prob, const Container &X)
          Standard deviation.
    • template<typename Container >
      double cor (const dmat &probXY, const Container &X, const Container &Y)
          Correlation.
Variables
    • constexpr double chop = 1e-10
          Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

 constexpr double eps = 1e-12

          Used to decide whether a number or expression in double precision is zero or not.
    • constexpr idx maxn = 64
```

Maximum number of allowed qu(d)its (subsystems)

constexpr double pi = 3.141592653589793238462643383279502884

• constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

constexpr double infty = std::numeric\_limits<double>::infinity()

Used to denote infinity in double precision.

#### 6.1.1 **Detailed Description**

Quantum++ main namespace.

# 6.1.2 Typedef Documentation

6.1.2.1 using qpp::bigint = typedef long long int

Big integer.

6.1.2.2 using qpp::bra = typedef Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

6.1.2.3 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.2.4 using qpp::cplx = typedef std::complex < double >

Complex number in double precision.

6.1.2.5 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.2.6 template<typename Scalar > using qpp::dyn\_col\_vect = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by Scalar.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

6.1.2.7 template<typename Scalar > using qpp::dyn\_mat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic,
Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

6.1.2.8 template<typename Scalar > using qpp::dyn row vect = typedef Eigen::Matrix<Scalar, 1, Eigen::Dynamic>

Dynamic Eigen row vector over the field specified by Scalar.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

6.1.2.9 using qpp::idx = typedef std::size\_t

Non-negative integer index.

6.1.2.10 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

6.1.2.11 template < typename... > using qpp::to\_void = typedef void

Alias template that implements the proposal for void t.

See also

http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2014/n3911

6.1.2.12 using qpp::ubigint = typedef unsigned long long int

Non-negative big integer.

# 6.1.3 Function Documentation

6.1.3.1 template < typename Derived > cmat qpp::absm ( const Eigen::MatrixBase < Derived > & A )

Matrix absolut value.

**Parameters** 

Α	Eigen expression
---	------------------

#### Returns

Matrix absolut value of A

6.1.3.2 template<typename InputIterator > std::vector<double> qpp::abssq ( InputIterator first, InputIterator last )

Computes the absolute values squared of an STL-like range of complex numbers.

### **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

# Returns

Real vector consisting of the range absolut values squared

Computes the absolute values squared of an STL-like container.

# **Parameters**

С	STL-like container

# Returns

Real vector consisting of the container's absolut values squared

6.1 qpp Namespace Reference 27  $6.1.3.4 \quad template < typename \ Derived > std::vector < double > qpp::abssq \ ( \ const \ Eigen::MatrixBase < Derived > \& \ A \ )$ Computes the absolute values squared of an Eigen expression.

Α	Eigen expression
---	------------------

### Returns

Real vector consisting of the absolut values squared

6.1.3.5 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )

Adjoint.

**Parameters** 

Α	Eigen expression
	- '

#### Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field as A

6.1.3.6 template<typename Derived1 , typename Derived2 > dyn\_mat<typename Derived1::Scalar> qpp::anticomm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )

Anti-commutator.

See also

qpp::comm()

Anti-commutator  $\{A, B\} = AB + BA$ . Both A and B must be Eigen expressions over the same scalar field.

### **Parameters**

Α	Eigen expression
В	Eigen expression

# Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field as A

6.1.3.7 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > dpp::apply ( const Eigen::MatrixBase < Derived2 > & A, const std::vector < <math>idx > & subsys$ , const std::vector < idx > & dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

#### Returns

Gate A applied to the part subsys of state

6.1.3.8 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::apply ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & subsys, idx d = 2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

#### Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

# Returns

Gate A applied to the part subsys of state

6.1.3.9 template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks )

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

# **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators

# Returns

Output density matrix after the action of the channel

6.1.3.10 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
dims	Dimensions of the multi-partite system

### Returns

Output density matrix after the action of the channel

6.1.3.11 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho

#### **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
d	Subsystem dimensions

#### Returns

Output density matrix after the action of the channel

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

# See also

qpp::Gates::CTRL()

# Note

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

### **Parameters**

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

# Returns

CTRL-A gate applied to the part subsys of state

6.1.3.13 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > dpp::applyCTRL ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, idx d = 2 )$ 

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

### See also

qpp::Gates::CTRL()

#### Note

The dimension of the gate A must match the dimension of subsys

### **Parameters**

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

### Returns

CTRL-A gate applied to the part subsys of state

6.1.3.14 template < typename Container > double qpp::avg ( const std::vector < double > & prob, const Container & X )

# Average.

# **Parameters**

prob	Real probability vector representing the probability distribution of X

# Returns

Average of X

6.1.3.15 cmat qpp::bloch2rho ( const std::vector< double > & r ) [inline]

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

# See also

qpp::rho2bloch()

# **Parameters**

r	3-dimensional real vector

# Returns

Qubit density matrix

6.1.3.16 std::vector<cmat> qpp::choi2kraus ( const cmat & A ) [inline]

Orthogonal Kraus operators from Choi matrix.

See also

qpp::kraus2choi()

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix A

Note

The Kraus operators satisfy  $Tr(K_i^{\dagger}K_j) = \delta_{ij}$  for all  $i \neq j$ 

### **Parameters**

Α	Choi matrix

Returns

Set of orthogonal Kraus operators

6.1.3.17 cmat qpp::choi2super ( const cmat & A ) [inline]

Converts Choi matrix to superoperator matrix.

See also

qpp::super2choi()

#### **Parameters**

Α	Choi matrix
---	-------------

Returns

Superoperator matrix

6.1.3.18 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::comm ( const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B)

Commutator.

See also

qpp::anticomm()

Commutator [A, B] = AB - BA. Both A and B must be Eigen expressions over the same scalar field.

# **Parameters**

A	Eigen expression
В	Eigen expression

### Returns

Commutator AB - BA, as a dynamic matrix over the same scalar field as A

6.1.3.19 template < typename T > std::vector < T > qpp::complement ( std::vector < T > subsys, idx N )

Constructs the complement of a subsystem vector.

subsys	Subsystem vector
N	Total number of systems

### Returns

The complement of *subsys* with respect to the set  $\{0, 1, \dots, N-1\}$ 

6.1.3.20 std::vector<idx> qpp::compperm ( const std::vector< idx> & perm, const std::vector< idx> & sigma ) [inline]

Compose permutations.

#### **Parameters**

perm	Permutation
sigma	Permutation

#### Returns

Composition of the permutations  $perm \circ sigma = perm(sigma)$ 

6.1.3.21 template < typename Derived > double qpp::concurrence ( const Eigen::MatrixBase < Derived > & A )

Wootters concurrence of the bi-partite qubit mixed state A.

#### **Parameters**

ſ	Α	Eigen expression

# Returns

Wootters concurrence

6.1.3.22 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived > & A )

Complex conjugate.

**Parameters** 

Α	Eigen expression

# Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field as A

6.1.3.23 double qpp::contfrac2x ( const std::vector < int > & cf, idx n ) [inline]

Real representation of a simple continued fraction.

# See also

qpp::x2contfrac()

cf	Integer vector containing the simple continued fraction expansion
n	Number of terms considered in the continued fraction expansion. If <i>n</i> is greater than the size
	of <i>cf</i> ,then all terms in <i>cf</i> are considered.

# Returns

Real representation of the simple continued fraction

6.1.3.24 double qpp::contfrac2x ( const std::vector < int > & cf ) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

#### **Parameters**

cf	Integer vector containing the simple continued fraction expansion
----	---

### Returns

Real representation of the simple continued fraction

6.1.3.25 template<typename Container > double qpp::cor ( const dmat & probXY, const Container & X, const Container & Y )

# Correlation.

# **Parameters**

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)
X	Random variable values represented by an STL-like container
Y	Random variable values represented by an STL-like container

# Returns

Correlation of X and Y

6.1.3.26 template < typename Derived > cmat qpp::cosm ( const Eigen::MatrixBase < Derived > & A )

Matrix cos.

**Parameters** 

Α	Eigen expression

# Returns

Matrix cosine of A

6.1.3.27 template < typename Container > double qpp::cov ( const dmat & probXY, const Container & X, const Container & Y )

Covariance.

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)
X	Random variable values represented by an STL-like container
Y	Random variable values represented by an STL-like container

#### Returns

Covariance of X and Y

6.1.3.28 template<typename OutputScalar , typename Derived >  $dyn_mat$ <OutputScalar> qpp::cwise ( const Eigen::MatrixBase< Derived > & A, OutputScalar(\*)(const typename Derived::Scalar &) f )

#### Functor.

# **Parameters**

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

#### Returns

Component-wise f(A), as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.29 template<typename Derived > Derived::Scalar qpp::det ( const Eigen::MatrixBase< Derived > & A )

# Determinant.

# **Parameters**

Α	Eigen expression

### Returns

Determinant of A, as a scalar over the same scalar field as A. Returns  $\pm \infty$  when the determinant overflows/underflows.

 $6.1.3.30 \quad template < typename \ T > dyn\_mat < typename \ T :: Scalar > qpp:: dirsum \ ( \ const \ T \ \& \ head \ )$ 

Direct sum.

See also

qpp::dirsumpow()

Used to stop the recursion for the variadic template version of qpp::dirsum()

# **Parameters**

head	Eigen expression
------	------------------

# Returns

Its argument head

6.1.3.31 template < typename T, typename... Args > dyn\_mat < typename T::Scalar > qpp::dirsum ( const T & head, const Args &... tail )

Direct sum.

See also

qpp::dirsumpow()

#### **Parameters**

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

#### Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.32 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::dirsum ( const std::vector< Derived > & As )

Direct sum.

See also

qpp::dirsumpow()

#### **Parameters**

As	std::vector of Eigen expressions

### Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.33 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::dirsum ( const std::initializer\_list < Derived > & As )

Direct sum.

See also

qpp::dirsumpow()

### **Parameters**

As	std::initializer list of Eigen expressions, such as {A1, A2,, Ak}

#### Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.34 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::dirsumpow ( const Eigen::MatrixBase< Derived > & A, idx n )

Direct sum power.

See also

qpp::dirsum()

#### **Parameters**

Α	Eigen expression
n	Non-negative integer

# Returns

Direct sum of A with itself n times  $A^{\oplus n}$ , as a dynamic matrix over the same scalar field as A

6.1.3.35 template < typename Derived > internal::IOManipEigen qpp::disp ( const Eigen::MatrixBase < Derived > & A, double chop = qpp::chop )

Eigen expression ostream manipulator.

#### **Parameters**

Α	Eigen expression
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

# Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.36 internal::IOManipEigen qpp::disp(cplx z, double chop = qpp::chop) [inline]

Complex number ostream manipulator.

#### **Parameters**

Z	Complex number (or any other type implicitly cast-able to std::complex <double>)</double>
chop	Set to zero the elements smaller in absolute value than chop

#### Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.37 template < typename InputIterator > internal::IOManipRange < InputIterator > qpp::disp ( InputIterator first, InputIterator last, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

Range ostream manipulator.

**Parameters** 

first	Iterator to the first element of the range
last	Iterator to the last element of the range
separator	Separator
start	Left marking
end	Right marking

### Returns

Instance of qpp::internal::internal::IOManipRange

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

#### **Parameters**

С	Container
separator	Separator
start	Left marking
end	Right marking

#### Returns

Instance of qpp::internal::internal::IOManipRange

6.1.3.39 template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp ( const PointerType \* p, idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

C-style pointer ostream manipulator.

# **Parameters**

р	Pointer to the first element
n	Number of elements to be displayed
separator	Separator
start	Left marking
end	Right marking

# Returns

Instance of qpp::internal::internal::IOManipPointer

6.1.3.40 template < typename Derived > std::pair < dyn\_col\_vect < cplx>, cmat> qpp::eig ( const Eigen::MatrixBase < Derived > & A )

Full eigen decomposition.

#### See also

qpp::heig()

Α	Eigen expression
---	------------------

#### Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.41 template < typename Derived > double qpp::entanglement ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

#### See also

qpp::entropy()

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

# Returns

Entanglement, with the logarithm in base 2

6.1.3.42 template < typename Derived > double qpp::entropy ( const Eigen::MatrixBase < Derived > & A )

von-Neumann entropy of the density matrix A

**Parameters** 

Α	Eigen expression

# Returns

von-Neumann entropy, with the logarithm in base 2

6.1.3.43 double qpp::entropy ( const std::vector < double > & prob ) [inline]

Shannon entropy of the probability distribution prob.

**Parameters** 

prob	Real probability vector

# Returns

Shannon entropy, with the logarithm in base 2

6.1.3.44 template<typename Derived > dyn\_col\_vect<cplx> qpp::evals ( const Eigen::MatrixBase< Derived > & A )

Eigenvalues.

See also

qpp::hevals()

**Parameters** 

```
A | Eigen expression
```

Returns

Eigenvalues of A, as a complex dynamic column vector

6.1.3.45 template<typename Derived > cmat qpp::evects ( const Eigen::MatrixBase< Derived > & A )

Eigenvectors.

See also

qpp::hevects()

**Parameters** 

Α	Eigen expression

Returns

Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.46 template<typename Derived > cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )

Matrix exponential.

**Parameters** 

```
A Eigen expression
```

Returns

Matrix exponential of A

**6.1.3.47** std::vector<ubigint> qpp::factors( ubigint n ) [inline]

Prime factor decomposition.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

п	Integer strictly greater than 1
---	---------------------------------

# Returns

Integer vector containing the factors

6.1.3.48 template < typename Derived > cmat qpp::funm ( const Eigen::MatrixBase < Derived > & A, cplx(\*)(const cplx &) f

Functional calculus f(A)

### **Parameters**

Α	Eigen expression
f	Pointer-to-function from complex to complex

# Returns

f(A)

6.1.3.49 ubigint qpp::gcd ( ubigint m, ubigint n ) [inline]

Greatest common divisor of two non-negative integers.

# See also

qpp::lcm()

# **Parameters**

m	Non-negative integer
n	Non-negative integer

# Returns

Greatest common divisor of m and n

6.1.3.50 ubigint qpp::gcd ( const std::vector< ubigint > & ns ) [inline]

Greatest common divisor of a list of non-negative integers.

See also

qpp::lcm()

### **Parameters**

ns	List of non-negative integers

# Returns

Greatest common divisor of all numbers in ns

6.1.3.51 template < typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase < Derived > & A )

G-concurrence of the bi-partite pure state A.

Note

Both local dimensions must be equal

Uses qpp::logdet() to avoid overflows

See also

qpp::logdet()

#### **Parameters**

Α	Eigen expression
---	------------------

### Returns

G-concurrence

6.1.3.52 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )

Gram-Schmidt orthogonalization.

### **Parameters**

Vs	std::vector of Eigen expressions as column vectors

### Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.53 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::grams ( const std::initializer\_list < Derived > & Vs)

Gram-Schmidt orthogonalization.

### **Parameters**

Vs	std::initializer_list of Eigen expressions as column vectors

### Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.54 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const Eigen::MatrixBase< Derived > & A )

Gram-Schmidt orthogonalization.

A Eigen expression, the input vectors are the columns of A

Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field as A

```
6.1.3.55 template<typename Derived > std::pair<dyn_col_vect < double>, cmat> qpp::heig ( const Eigen::MatrixBase< Derived > & A )
```

Full eigen decomposition of Hermitian expression.

See also

qpp::eig()

**Parameters** 

```
A Eigen expression
```

Returns

Pair of: 1. Eigenvalues of A, as a real dynamic column vector, and 2. Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.56 template<typename Derived > dyn\_col\_vect<double> qpp::hevals ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvalues.

See also

qpp::evals()

Parameters

```
A Eigen expression
```

Returns

Eigenvalues of Hermitian A, as a real dynamic column vector

6.1.3.57 template<typename Derived > cmat qpp::hevects ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvectors.

See also

qpp::evects()

Α	Eigen expression
---	------------------

### Returns

Eigenvectors of Hermitian A, as columns of a complex matrix

6.1.3.58 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::inverse ( const Eigen::MatrixBase < Derived > & A )

Inverse.

#### **Parameters**

A Eigen expression	
--------------------	--

### Returns

Inverse of A, as a dynamic matrix over the same scalar field as A

6.1.3.59 std::vector<idx> qpp::invperm(const std::vector<idx> & perm) [inline]

Inverse permutation.

#### **Parameters**

perm	Permutation
------	-------------

# Returns

Inverse of the permutation perm

6.1.3.60 template < typename Derived >  $dyn_col_vect$  < typename Derived::Scalar >  $dyn_i$  ( const Eigen::MatrixBase < Derived > &  $dyn_i$  const Eigen::MatrixBase < Derived > &  $dyn_i$  const Std::vector <  $dyn_i$  const Std::v

Generalized inner product.

# **Parameters**

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined
dims	Dimensions of the multi-partite system

# Returns

The inner product  $\langle \phi_{subsys} | \psi \rangle$ , as a scalar or column vector over the remaining Hilbert space

6.1.3.61 template<typename Derived > dyn\_col\_vect<typename Derived::Scalar> qpp::ip ( const Eigen::MatrixBase< Derived > & phi, const Eigen::MatrixBase< Derived > & psi, const std::vector< idx > & subsys, idx d = 2 )

Generalized inner product.

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined
d	Subsystem dimensions

#### Returns

The inner product  $\langle \phi_{subsys} | \psi \rangle$ , as a scalar or column vector over the remaining Hilbert space

**6.1.3.62** bool qpp::isprime ( ubigint *n* ) [inline]

Primality test.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

#### **Parameters**

n	Integer strictly greater than 1

# Returns

True if the number is prime, false otherwise

6.1.3.63 cmat qpp::kraus2choi ( const std::vector < cmat > & Ks ) [inline]

Choi matrix.

See also

qpp::choi2kraus()

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $\mathit{Ks}$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|, |0\rangle\langle 1|$  etc.

Note

The superoperator matrix S and the Choi matrix C are related by  $S_{ab,mn}=C_{ma,nb}$ 

# Parameters

Ks	Set of Kraus operators
	·

# Returns

Choi matrix

6.1.3.64 cmat qpp::kraus2super ( const std::vector < cmat > & Ks ) [inline]

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

Ks	Set of Kraus operators

### Returns

Superoperator matrix

6.1.3.65 template<typename T > dyn\_mat<typename T::Scalar> qpp::kron ( const T & head )

Kronecker product.

See also

qpp::kronpow()

Used to stop the recursion for the variadic template version of <a href="mailto:qpp::kron(">qpp::kron()</a>)

### **Parameters**

head	Eigen expression

### Returns

Its argument head

6.1.3.66 template<typename T , typename... Args> dyn\_mat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )

Kronecker product.

See also

qpp::kronpow()

### **Parameters**

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

### **Returns**

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.67 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::kron ( const std::vector< Derived > & As )

Kronecker product.

See also

qpp::kronpow()

As	std::vector of Eigen expressions
----	----------------------------------

### Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker product.

See also

qpp::kronpow()

#### **Parameters**

As	std::initializer list of Eigen expressions, such as {A1, A2, ,Ak}

### Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker power.

See also

qpp::kron()

# Parameters

Α	Eigen expression
n	Non-negative integer

# Returns

Kronecker product of A with itself n times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field as A

**6.1.3.70 ubigint qpp::lcm ( ubigint** *m***, ubigint** *n* **)** [inline]

Least common multiple of two positive integers.

See also

qpp::gcd()

m	Positive integer
n	Positive integer

### Returns

Least common multiple of m and n

6.1.3.71 ubigint qpp::lcm ( const std::vector < ubigint > & ns ) [inline]

Least common multiple of a list of positive integers.

### See also

qpp::gcd()

#### **Parameters**

ns	List of positive integers
----	---------------------------

#### Returns

Least common multiple of all numbers in ns

6.1.3.72 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::load ( const std::string & fname )

Loads Eigen matrix from a binary file (internal format) in double precision.

### See also

qpp::save()

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

# Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

### **Parameters**

Α	Eigen expression
fname	Output file name

6.1.3.73 template<typename Derived > Derived qpp::loadMATLABmatrix ( const std::string & mat\_file, const std::string & var\_name )

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

### See also

qpp::saveMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be loaded)

```
6.1.3.74 template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

Note

If var\_name is a complex matrix, only the real part is loaded

#### **Parameters**

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

#### Returns

Eigen double dynamic matrix (qpp::dmat)

```
6.1.3.75 template<> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

### **Parameters**

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

### Returns

Eigen complex dynamic matrix (qpp::cmat)

6.1.3.76 template < typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase < Derived > & A )

Logarithm of the determinant.

Useful when the determinant overflows/underflows

Α	Eigen expression
---	------------------

### Returns

Logarithm of the determinant of A, as a scalar over the same scalar field as A

6.1.3.77 template < typename Derived > cmat qpp::logm ( const Eigen::MatrixBase < Derived > & A )

Matrix logarithm.

**Parameters** 

Α	Eigen expression

# Returns

Matrix logarithm of A

6.1.3.78 template < typename Derived > double qpp::lognegativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Logarithmic negativity of the bi-partite mixed state A.

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

# Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.79 std::vector < double > qpp::marginalX ( const dmat & probXY ) [inline]

Marginal distribution.

# **Parameters**

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

# Returns

Real vector consisting of the marginal distribution of X

6.1.3.80 std::vector<double> qpp::marginalY ( const dmat & probXY ) [inline]

Marginal distribution.

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

#### Returns

Real vector consisting of the marginal distribution of Y

 $6.1.3.81 \quad template < typename \ Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( \ const \ Eigen::MatrixBase < Derived > & \textit{A}, \ const \ std::vector < cmat > & \textit{Ks} \ )$ 

Measures the state A using the set of Kraus operators Ks.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.82 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.83 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

# **Parameters**

Α	Eigen expression
U	Unitary matrix whose columns represent the measurement basis vectors

### Returns

6.1.3.84 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

#### See also

qpp::measure\_seq()

### Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.85 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

# See also

qpp::measure\_seq()

### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

### Returns

6.1.3.86 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

#### See also

qpp::measure\_seq()

### Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.87 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

# See also

qpp::measure\_seq()

### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

### Returns

6.1.3.88 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis or rank-1 POVM specified by the matrix *V*.

#### See also

qpp::measure\_seq()

#### Note

The dimension of V must match the dimension of subsys. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.89 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & S subsys, idx S = 2 )

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis or rank-1 POVM specified by the matrix *V*.

# See also

qpp::measure\_seq()

# Note

The dimension of *V* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

### **Parameters**

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

### Returns

6.1.3.90 template<typename Derived > std::tuple<std::vector<idx>, double, cmat> qpp::measure\_seq ( const Eigen::MatrixBase< Derived > & A, std::vector< idx > subsys, std::vector< idx > dims )

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

#### See also

qpp::measure()

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

#### Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.91 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure\_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, idx d = 2)

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

# See also

qpp::measure()

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

# Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.92 ket qpp::mket ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

### **Parameters**

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

#### Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.93 ket qpp::mket ( const std::vector < idx > & mask, idx d = 2 ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

#### **Parameters**

mask	std::vector of non-negative integers
d	Subsystem dimensions

#### Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.94 ubigint qpp::modpow ( ubigint a, ubigint n, ubigint p ) [inline]

Integer power modulo p.

Computes  $a^n \bmod p$ 

### **Parameters**

а	Non-negative integer
n	Non-negative integer
р	Strictly positive integer

# Returns

 $a^n \bmod p$ 

6.1.3.95 cmat qpp::mprj ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

### **Parameters**

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

# Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.96 cmat qpp::mprj ( const std::vector < idx > & mask, idx d = 2 ) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

mask	std::vector of non-negative integers
d	Subsystem dimensions

#### Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.97 idx qpp::multiidx2n( const std::vector < idx > & midx, const std::vector < idx > & dims ) [inline]

Multi-index to non-negative integer index.

#### See also

qpp::n2multiidx()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

#### **Parameters**

midx	Multi-index
dims	Dimensions of the multi-partite system

#### Returns

Non-negative integer index

6.1.3.98 std::vector<idx> qpp::n2multiidx ( idx n, const std::vector< idx > & dims ) [inline]

Non-negative integer index to multi-index.

# See also

qpp::multiidx2n()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

# **Parameters**

n	Non-negative integer index
dims	Dimensions of the multi-partite system

# Returns

Multi-index of the same size as dims

6.1.3.99 template < typename Derived > double qpp::negativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Negativity of the bi-partite mixed state A.

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Negativity

6.1.3.100 template < typename Derived > double qpp::norm ( const Eigen::MatrixBase < Derived > & A )

Frobenius norm.

**Parameters** 

Α	Eigen expression

Returns

Frobenius norm of A

```
6.1.3.101 cplx qpp::omega(idx D) [inline]
```

D-th root of unity.

**Parameters** 

D	Non-negative integer
---	----------------------

#### Returns

D-th root of unity  $\exp(2\pi i/D)$ 

```
6.1.3.102 constexpr cplx qpp::operator""_i( unsigned long long int x ) [inline], [noexcept]
```

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

# Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

**6.1.3.103** constexpr cplx qpp::operator""\_i ( long double x ) [inline], [noexcept]

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

#### Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.104 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase < Derived > & A, idx n )

Matrix power.

See also

qpp::spectralpowm()

Explicitly multiplies the matrix A with itself n times. By convention  $A^0 = I$ .

Α	Eigen expression
n	Non-negative integer

#### Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field as A

6.1.3.105 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::prj ( const Eigen::MatrixBase< Derived > & V )

#### Projector.

Normalized projector onto state vector

#### **Parameters**

V	Eigen expression

#### Returns

Projector onto the state vector V, or the matrix Zero if V has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field as A

6.1.3.106 template < typename Derived > Derived::Scalar qpp::prod ( const Eigen::MatrixBase < Derived > & A )

Element-wise product of A.

#### **Parameters**

Α	Eigen expression
---	------------------

# Returns

Element-wise product of A, as a scalar over the same scalar field as A

6.1.3.107 template<typename InputIterator > std::iterator\_traits<InputIterator>::value\_type qpp::prod ( InputIterator *first*, InputIterator *last* )

Element-wise product of an STL-like range.

# **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

## Returns

Element-wise product of the range, as a scalar over the same scalar field as the range

6.1.3.108 template < typename Container > Container::value\_type qpp::prod ( const Container & c )

Element-wise product of the elements of an STL-like container.

С	STL-like container
---	--------------------

#### Returns

Element-wise product of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.109 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Partial trace.

#### See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

#### Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.110 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2)

Partial trace.

See also

qpp::ptrace1(), qpp::ptrace2()

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

# **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

# Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.111 template < typename Derived >  $dyn_mat < typename Derived::Scalar > qpp::ptrace1 ( const Eigen::MatrixBase < Derived > & A, const std::vector < <math>idx > & dims$ )

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

#### Returns

Partial trace  $Tr_A(\cdot)$  over the first subsystem A in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.112 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptrace2 ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims)

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

#### Returns

Partial trace  $Tr_B(\cdot)$  over the second subsytem B in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.113 template < typename Derived >  $dyn_mat < typename Derived::Scalar > qpp::ptranspose ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims )$ 

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

# **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

#### Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.114 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2 )

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

#### Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.115 template < typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsysA, const std::vector < idx > & subsysB, const std::vector < idx > & dims )

Quantum mutual information between 2 subsystems of a composite system.

#### **Parameters**

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
dims	Dimensions of the multi-partite system

#### Returns

Mutual information between the 2 subsystems

6.1.3.116 template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, idx d = 2)

Quantum mutual information between 2 subsystems of a composite system.

# **Parameters**

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
d	Subsystem dimensions

#### Returns

Mutual information between the 2 subsystems

6.1.3.117 double qpp::rand ( double a = 0, double b = 1 ) [inline]

Generates a random real number uniformly distributed in the interval [a, b)

## **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random real number (double) uniformly distributed in the interval [a, b)

Generates a random big integer uniformly distributed in the interval [a, b].

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random big integer uniformly distributed in the interval [a, b]

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

#### **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random non-negative big integer uniformly distributed in the interval [a, b]

```
6.1.3.120 template < typename Derived > Derived qpp::rand ( idx rows, idx cols, double a = 0, double b = 1 )
```

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

```
6.1.3.121 template <> dmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]
```

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random real matrix

6.1.3.122 template <> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) uniformly distributed in [-1,1) auto mat = rand<cmat>(3, 3, -1, 1);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random complex matrix

```
6.1.3.123 cmat qpp::randH(idx D) [inline]
```

Generates a random Hermitian matrix.

# **Parameters**

D	Dimension of the Hilbert space
---	--------------------------------

# Returns

Random Hermitian matrix

Generates a random index (idx) uniformly distributed in the interval [a, b].

# **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

# Returns

Random index (idx) uniformly distributed in the interval [a, b]

```
6.1.3.125 ket qpp::randket(idx D) [inline]
```

Generates a random normalized ket (pure state vector)

D	Dimension of the Hilbert space
---	--------------------------------

#### Returns

Random normalized ket

```
6.1.3.126 std::vector<cmat> qpp::randkraus(idx N, idx D) [inline]
```

Generates a set of random Kraus operators.

Note

The set of Kraus operators satisfy the closure condition  $\sum_i K_i^\dagger K_i = I$ 

#### **Parameters**

N	Number of Kraus operators
D	Dimension of the Hilbert space

#### Returns

Set of N Kraus operators satisfying the closure condition

```
6.1.3.127 template < typename Derived > Derived qpp::randn ( idx rows, idx cols, double mean = 0, double sigma = 1 )
```

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

```
6.1.3.128 template <> dmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]
```

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXd, // with entries normally distributed in N(0,2) auto mat = randn<dmat>(3, 3, 0, 2);
```

### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

# Returns

Random real matrix

6.1.3.129 template <> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, // with entries (both real and imaginary) normally distributed in N(0,2) auto mat = randn<cmat>(3, 3, 0, 2);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

#### Returns

Random complex matrix

```
6.1.3.130 double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]
```

Generates a random real number (double) normally distributed in N(mean, sigma)

#### **Parameters**

mean	Mean
sigma	Standard deviation

#### Returns

Random real number normally distributed in N(mean, sigma)

```
6.1.3.131 std::vector<idx> qpp::randperm(idx n) [inline]
```

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by std::shuffle), so that all permutations are equally probable

# **Parameters**

n	Size of the permutation
---	-------------------------

#### Returns

Random permutation of size n

```
6.1.3.132 cmat qpp::randrho(idx D) [inline]
```

Generates a random density matrix.

D	Dimension of the Hilbert space

#### Returns

Random density matrix

6.1.3.133 cmat qpp::randU(idx D) [inline]

Generates a random unitary matrix.

#### **Parameters**

D   Dimension of the Hilbert space	
------------------------------------	--

#### Returns

Random unitary

6.1.3.134 cmat qpp::randV (idx Din, idx Dout) [inline]

Generates a random isometry matrix.

#### **Parameters**

Din	Size of the input Hilbert space
Dout	Size of the output Hilbert space

# Returns

Random isometry matrix

6.1.3.135 template < typename Derived > double qpp::renyi ( const Eigen::MatrixBase < Derived > & A, double alpha )

Renyi-  $\alpha$  entropy of the density matrix  ${\it A}$ , for  $\alpha \geq 0$ .

Note

When  $\alpha \to 1$  the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

#### **Parameters**

Α	Eigen expression
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

# Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.136 double qpp::renyi ( const std::vector < double > & prob, double alpha ) [inline]

Renyi-  $\alpha$  entropy of the probability distribution *prob*, for  $\alpha \geq 0$ .

Note

When lpha 
ightarrow 1 the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

prob	Real probability vector
alpha	Non-negative real number, use qpp::infty for $\alpha = \infty$

#### Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.137 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, idx rows, idx cols )

#### Reshape.

Uses column-major order when reshaping (same as MATLAB)

#### **Parameters**

Α	Eigen expression
rows	Number of rows of the reshaped matrix
cols	Number of columns of the reshaped matrix

#### Returns

Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field as A

6.1.3.138 template < typename Derived > std::vector < double > qpp::rho2bloch ( const Eigen::MatrixBase < Derived > & A )

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

#### See also

qpp::bloch2rho()

# Note

It is implicitly assumed that the density matrix is Hermitian

#### **Parameters**

A   Eigen expression
----------------------

## Returns

3-dimensional Bloch vector

6.1.3.139 template<typename Derived > dyn\_col\_vect<typename Derived::Scalar> qpp::rho2pure ( const Eigen::MatrixBase< Derived > & A )

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

# Note

No purity check is done, the input state A must have rank one, otherwise the function returns the first non-zero eigenvector of A

Α	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. A is
	assumed to have rank one

#### Returns

The unique non-zero eigenvector of A, as a dynamic column vector over the same scalar field as A

6.1.3.140 template<typename Derived > void qpp::save ( const Eigen::MatrixBase< Derived > & A, const std::string & fname )

Saves Eigen expression to a binary file (internal format) in double precision.

#### See also

qpp::load()

#### **Parameters**

Α	Eigen expression
fname	Output file name

6.1.3.141 template<typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< Derived > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode )

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

# See also

#### gpp::loadMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be saved)

6.1.3.142 template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< dmat > & A, const std::string & mat\_file, const std::string & war\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

#### See also

qpp::loadMATLABmatrix()

## **Parameters**

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.143 template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< cmat > & A, const std::string & mat\_file, const std::string & war\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

#### See also

# qpp::loadMATLABmatrix()

#### **Parameters**

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.144 template < typename Derived > double qpp::schatten ( const Eigen::MatrixBase < Derived > & A, double p )

Schatten matrix norm.

#### **Parameters**

Α	Eigen expression
р	Real number, greater or equal to 1, use qpp::infty for $p=\infty$

#### Returns

Schatten-p matrix norm of A

6.1.3.145 template<typename Derived > cmat qpp::schmidtA ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt basis on Alice side.

# **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Unitary matrix  ${\cal U}$  whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.146 template<typename Derived > cmat qpp::schmidtB ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt basis on Bob side.

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Unitary matrix V whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.147 template < typename Derived > dyn\_col\_vect < double > qpp::schmidtcoeffs ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

# See also

qpp::schmidtprobs()

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

# Returns

Schmidt coefficients of A, as a real dynamic column vector

6.1.3.148 template < typename Derived > std::vector < double > qpp::schmidtprobs ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

#### See also

qpp::schmidtcoeffs()

#### **Parameters**

A	Eigen expression
dims	Dimensions of the bi-partite system

# Returns

Real vector consisting of the Schmidt probabilites of A

6.1.3.149 template < typename Container > double qpp::sigma ( const std::vector < double > & prob, const Container & X )

Standard deviation.

# **Parameters**

	prob	Real probability vector representing the probability distribution of X
Ī	X	Random variable values represented by an STL-like container

#### Returns

Standard deviation of X

6.1.3.150 template < typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase < Derived > & A )

Matrix sin.

A Eigen expression

Returns

Matrix sine of A

6.1.3.151 template < typename Derived > cmat qpp::spectralpowm ( const Eigen::MatrixBase < Derived > & A, const cplx z )

Matrix power.

See also

qpp::powm()

Uses the spectral decomposition of A to compute the matrix power. By convention  $A^0=I$ .

#### **Parameters**

Α	Eigen expression
Z	Complex number

#### Returns

Matrix power  $A^z$ 

6.1.3.152 template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )

Matrix square root.

**Parameters** 

Α	Eigen expression

Returns

Matrix square root of A

6.1.3.153 template < typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase < Derived > & A )

Element-wise sum of A.

**Parameters** 

```
A Eigen expression
```

Returns

Element-wise sum of A, as a scalar over the same scalar field as A

6.1.3.154 template < typename InputIterator > std::iterator\_traits < InputIterator >::value\_type qpp::sum ( InputIterator first, InputIterator last )

Element-wise sum of an STL-like range.

first	Iterator to the first element of the range
last	Iterator to the last element of the range

#### Returns

Element-wise sum of the range, as a scalar over the same scalar field as the range

6.1.3.155 template<typename Container > Container::value\_type qpp::sum ( const Container & c )

Element-wise sum of the elements of an STL-like container.

#### **Parameters**

С	STL-like container

# Returns

Element-wise sum of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.156 cmat qpp::super2choi(const cmat & A) [inline]

Converts superoperator matrix to Choi matrix.

#### See also

qpp::choi2super()

#### **Parameters**

Α	Superoperator matrix
---	----------------------

# Returns

Choi matrix

6.1.3.157 template<typename Derived > dyn\_col\_vect<double> qpp::svals ( const Eigen::MatrixBase< Derived > & A )

Singular values.

# **Parameters**

Α	Eigen expression

#### Returns

Singular values of A, ordered in decreasing order, as a real dynamic column vector

 $6.1.3.158 \quad template < typename \ Derived > std::tuple < cmat, \ dyn\_col\_vect < double >, \ cmat > qpp::svd \ ( \ const \ Eigen::MatrixBase < Derived > \& A \ )$ 

Full singular value decomposition.

Α	Eigen expression
---	------------------

#### Returns

Tuple of: 1. Left sigular vectors of *A*, as columns of a complex dynamic matrix, 2. Singular values of *A*, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of *A*, as columns of a complex dynamic matrix

6.1.3.159 template<typename Derived > cmat qpp::svdU ( const Eigen::MatrixBase< Derived > & A )

Left singular vectors.

#### **Parameters**

Α	Eigen expression

#### Returns

Complex dynamic matrix, whose columns are the left singular vectors of A

6.1.3.160 template < typename Derived > cmat qpp::svdV ( const Eigen::MatrixBase < Derived > & A )

Right singular vectors.

#### **Parameters**

Α	Eigen expression

# Returns

Complex dynamic matrix, whose columns are the right singular vectors of A

6.1.3.161 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & perm

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

#### **Parameters**

Α	Eigen expression
perm	Permutation
dims	Dimensions of the multi-partite system

#### Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.162 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2 )

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

Α	Eigen expression
perm	Permutation
d	Subsystem dimensions

#### Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.163 template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )

Trace.

#### **Parameters**

Λ	Figur averagion
I A	Eigen expression

#### Returns

Trace of A, as a scalar over the same scalar field as A

6.1.3.164 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::transpose ( const Eigen::MatrixBase< Derived > & A )

Transpose.

#### **Parameters**

A Eigen expression
--------------------

# Returns

Transpose of A, as a dynamic matrix over the same scalar field as A

6.1.3.165 template<typename Derived > double qpp::tsallis ( const Eigen::MatrixBase< Derived > & A, double q )

Tsallis- q entropy of the density matrix A, for  $q \ge 0$ .

Note

When  $q \to 1$  the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base e

# **Parameters**

Α	Eigen expression
q	Non-negative real number

#### Returns

Tsallis- q entropy

6.1.3.166 double qpp::tsallis ( const std::vector < double > & prob, double q ) [inline]

Tsallis- q entropy of the probability distribution *prob*, for  $q \ge 0$ .

Note

When  $q \to 1$  the Tsallis entropy converges to the Shannon entropy, with the logarithm in base e

prob	Real probability vector
q	Non-negative real number

#### Returns

Tsallis- q entropy

6.1.3.167 std::vector<double> qpp::uniform(idx N) [inline]

Uniform probability distribution vector.

#### **Parameters**

N Size of the alphabet	
------------------------	--

#### Returns

Real vector consisting of a uniform distribution of size N

6.1.3.168 template < typename Container > double qpp::var ( const std::vector < double > & prob, const Container & X )

Variance.

#### **Parameters**

prob	Real probability vector representing the probability distribution of X
X	Random variable values represented by an STL-like container

#### Returns

Variance of X

6.1.3.169 std::vector<int> qpp::x2contfrac ( double x, idx n, idx cut = 1e5 ) [inline]

Simple continued fraction expansion.

See also

qpp::contfrac2x()

# **Parameters**

Х	Real number
n	Number of terms in the expansion
cut	Stop the expansion when the next term is greater than <i>cut</i>

# Returns

Integer vector containing the simple continued fraction expansion of x. If there are m less than n terms in the expansion, a shorter vector with m components is returned.

# 6.1.4 Variable Documentation

6.1.4.1 constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

6.1.4.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.4.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::eps) // x is zero</pre>
```

6.1.4.4 constexpr double qpp::infty = std::numeric\_limits < double >::infinity()

Used to denote infinity in double precision.

6.1.4.5 constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

# 6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

# 6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

# 6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

#### **Classes**

- class IOManipEigen
- · class IOManipPointer
- class IOManipRange
- class Singleton

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

#### **Functions**

```
    void n2multiidx (idx n, idx numdims, const idx *dims, idx *result) noexcept

• idx _multiidx2n (const idx *midx, idx numdims, const idx *dims) noexcept

    template<typename Derived >

  bool check square mat (const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
  bool <u>_check_vector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

 bool <u>_check_rvector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  bool check cvector (const Eigen::MatrixBase< Derived > &A)
template<typename T >
  bool check nonzero size (const T &x) noexcept

    template<typename T1 , typename T2 >

  bool _check_matching_sizes (const T1 &lhs, const T2 &rhs) noexcept

    bool <u>_check_dims</u> (const std::vector< idx > &dims)

    template<typename Derived >

  bool _check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
 bool _check_dims_match_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

    template<typename Derived >

  bool _check_dims_match_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

    bool <u>_check_eq_dims</u> (const std::vector< idx > &dims, idx dim) noexcept

    bool <u>_check_subsys_match_dims</u> (const std::vector < idx > &subsys, const std::vector < idx > &dims)

    template<typename Derived >

  bool _check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept

    template<typename Derived >

  bool <u>_check_qubit_cvector</u> (const Eigen::MatrixBase< Derived > &V) noexcept

    template<typename Derived >

  bool _check_qubit_rvector (const Eigen::MatrixBase< Derived > &V) noexcept

    template<typename Derived >

  bool check qubit vector (const Eigen::MatrixBase< Derived > &V) noexcept

    bool check perm (const std::vector < idx > &perm)

    template<typename Derived1 , typename Derived2 >

  dyn mat< typename Derived1::Scalar > kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen ←
  ::MatrixBase< Derived2 > &B)
• template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > _dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
```

# 6.3.1 Detailed Description

template<typename T >

Internal utility functions, do not use/modify.

void variadic\_vector\_emplace (std::vector< T > &)
 template<typename T, typename First, typename... Args>

# 6.3.2 Function Documentation

- 6.3.2.1 template < typename Derived > bool qpp::internal::\_check\_cvector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.2 bool qpp::internal::\_check\_dims ( const std::vector < idx > & dims ) [inline]

void variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)

- 6.3.2.3 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_cvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.3.2.4 template < typename Derived > bool qpp::internal::\_check\_dims\_match\_mat ( const std::vector < idx > & dims, const Eigen::MatrixBase < Derived > & A)
- 6.3.2.5 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_rvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.3.2.6 bool qpp::internal::\_check\_eq\_dims ( const std::vector < idx > & dims, idx dim ) [inline], [noexcept]
- 6.3.2.7 template < typename T1 , typename T2 > bool qpp::internal::\_check\_matching\_sizes ( const T1 & *Ihs*, const T2 & *rhs* ) [noexcept]
- 6.3.2.8 template < typename T > bool qpp::internal:: check nonzero\_size ( const T & x ) [noexcept]
- 6.3.2.9 bool qpp::internal::\_check\_perm ( const std::vector < idx > & perm ) [inline]
- 6.3.2.11 template<typename Derived > bool qpp::internal::\_check\_qubit\_matrix ( const Eigen::MatrixBase< Derived > & A ) [noexcept]
- 6.3.2.12 template < typename Derived > bool qpp::internal::\_check\_qubit\_rvector ( const Eigen::MatrixBase < Derived > & V ) [noexcept]
- 6.3.2.13 template < typename Derived > bool qpp::internal::\_check\_qubit\_vector ( const Eigen::MatrixBase < Derived > & V ) [noexcept]
- 6.3.2.14 template < typename Derived > bool qpp::internal::\_check\_rvector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.15 template < typename Derived > bool qpp::internal::\_check\_square\_mat ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.16 bool qpp::internal::\_check\_subsys\_match\_dims ( const std::vector < idx > & subsys, const std::vector < idx > & dims ) [inline]
- 6.3.2.17 template < typename Derived > bool qpp::internal:: check\_vector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.18 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::internal::\_dirsum2 ( const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B )

- 6.3.2.21 void qpp::internal::\_n2multiidx ( idx n, idx numdims, const idx \* dims, idx \* result ) [inline], [noexcept]
- 6.3.2.22 template<typename T > void qpp::internal::variadic\_vector\_emplace(\_std::vector< T > &\_\_)
- 6.3.2.23 template < typename T , typename First , typename... Args > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > &  $\nu$ , First && first, Args &&... args )

# **Chapter 7**

# **Class Documentation**

# 7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

#include <classes/codes.h>

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



84 Class Documentation

# **Public Types**

enum Type { Type::FIVE\_QUBIT = 1, Type::SEVEN\_QUBIT\_STEANE, Type::NINE\_QUBIT\_SHOR }
 Code types, add more codes here if needed.

#### **Public Member Functions**

ket codeword (Type type, idx i) const
 Returns the codeword of the specified code type.

#### **Private Member Functions**

• Codes ()

Default constructor.

Codes ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const Codes >

# **Additional Inherited Members**

# 7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

# 7.1.2 Member Enumeration Documentation

```
7.1.2.1 enum qpp::Codes::Type [strong]
```

Code types, add more codes here if needed.

See also

```
qpp::Codes::codeword()
```

## Enumerator

```
FIVE_QUBIT [[5,1,3]] qubit code
SEVEN_QUBIT_STEANE [[7,1,3]] Steane qubit code
NINE_QUBIT_SHOR [[9,1,3]] Shor qubit code
```

# 7.1.3 Constructor & Destructor Documentation

```
7.1.3.1 qpp::Codes::Codes( ) [inline],[private]
```

Default constructor.

```
7.1.3.2 qpp::Codes::~Codes() [private], [default]
```

Default destructor.

# 7.1.4 Member Function Documentation

7.1.4.1 ket qpp::Codes::codeword ( Type type, idx i ) const [inline]

Returns the codeword of the specified code type.

See also

qpp::Codes::Type

#### **Parameters**

type	Code type
i	Codeword index

#### Returns

i-th codeword of the code type

#### 7.1.5 Friends And Related Function Documentation

**7.1.5.1 friend class internal::Singleton** < **const Codes** > [friend]

The documentation for this class was generated from the following file:

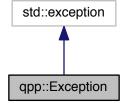
· classes/codes.h

# 7.2 qpp::Exception Class Reference

Generates custom exceptions, used when validating function parameters.

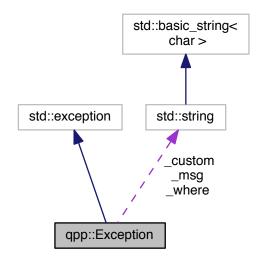
#include <classes/exception.h>

Inheritance diagram for qpp::Exception:



86 Class Documentation

Collaboration diagram for qpp::Exception:



# **Public Types**

enum Type {

Type::UNKNOWN\_EXCEPTION = 1, Type::ZERO\_SIZE, Type::MATRIX\_NOT\_SQUARE, Type::MATRIX\_← NOT\_CVECTOR,

Type::MATRIX\_NOT\_RVECTOR, Type::MATRIX\_NOT\_VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_C↔ VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_RVECTOR,

Type::MATRIX\_NOT\_SQUARE\_OR\_VECTOR, Type::MATRIX\_MISMATCH\_SUBSYS, Type::DIMS\_INVA← LID, Type::DIMS\_NOT\_EQUAL,

Type::DIMS\_MISMATCH\_MATRIX, Type::DIMS\_MISMATCH\_CVECTOR, Type::DIMS\_MISMATCH\_RVE CTOR, Type::DIMS\_MISMATCH\_VECTOR,

Type::SUBSYS\_MISMATCH\_DIMS, Type::PERM\_INVALID, Type::PERM\_MISMATCH\_DIMS, Type::NOT ← QUBIT\_MATRIX,

Type::NOT\_QUBIT\_CVECTOR, Type::NOT\_QUBIT\_RVECTOR, Type::NOT\_QUBIT\_VECTOR, Type::NO $\leftarrow$  T\_QUBIT\_SUBSYS,

Type::NOT\_BIPARTITE, Type::NO\_CODEWORD, Type::OUT\_OF\_RANGE, Type::TYPE\_MISMATCH, Type::SIZE\_MISMATCH, Type::UNDEFINED\_TYPE, Type::CUSTOM\_EXCEPTION }

Exception types, add more here if needed.

# **Public Member Functions**

• Exception (const std::string &where, const Type &type)

Constructs an exception.

• Exception (const std::string &where, const std::string &custom)

Constructs an exception.

virtual const char \* what () const noexceptoverride

Overrides std::exception::what()

#### **Private Member Functions**

void construct exception msg ()

Constructs the exception description from its type.

#### **Private Attributes**

- · std::string where
- std::string \_msg
- Type \_type
- std::string \_custom

# 7.2.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

#### 7.2.2 Member Enumeration Documentation

**7.2.2.1** enum qpp::Exception::Type [strong]

Exception types, add more here if needed.

See also

qpp::Exception:: construct exception msg()

#### Enumerator

UNKNOWN\_EXCEPTION Unknown exception

**ZERO\_SIZE** Zero sized object, e.g. empty Eigen::Matrix or std::vector<> with no elements

MATRIX\_NOT\_SQUARE Eigen::Matrix is not square

MATRIX\_NOT\_CVECTOR Eigen::Matrix is not a column vector

MATRIX\_NOT\_RVECTOR Eigen::Matrix is not a row vector

MATRIX\_NOT\_VECTOR Eigen::Matrix is not a row/column vector

MATRIX\_NOT\_SQUARE\_OR\_CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX\_NOT\_SQUARE\_OR\_RVECTOR Eigen::Matrix is not square nor a row vector

**MATRIX\_NOT\_SQUARE\_OR\_VECTOR** Eigen::Matrix is not square nor a row/column vector

MATRIX\_MISMATCH\_SUBSYS Matrix size mismatch subsystem sizes (e.g. in qpp::apply())

**DIMS\_INVALID** std::vector<idx> of dimensions has zero size or contains zeros

DIMS\_NOT\_EQUAL Local/global dimensions are not equal

**DIMS\_MISMATCH\_MATRIX** Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

**DIMS\_MISMATCH\_CVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

**DIMS\_MISMATCH\_RVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

**DIMS\_MISMATCH\_VECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

**SUBSYS\_MISMATCH\_DIMS** std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

88 Class Documentation

**PERM\_INVALID** std::vector<idx> does note represent a valid permutation

**PERM\_MISMATCH\_DIMS** Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

NOT\_QUBIT\_MATRIX Eigen::Matrix is not 2 x 2

NOT\_QUBIT\_CVECTOR Eigen::Matrix is not 2 x 1

NOT\_QUBIT\_RVECTOR Eigen::Matrix is not 1 x 2

NOT\_QUBIT\_VECTOR Eigen::Matrix is not 1 x 2 nor 2 x 1

NOT\_QUBIT\_SUBSYS Subsystems are not 2-dimensional

NOT\_BIPARTITE std::vector<idx> of dimensions has size different from 2

**NO\_CODEWORD** Codeword does not exist, thrown when calling qpp::Codes::codeword() with invalid index i

OUT\_OF\_RANGE Parameter out of range

TYPE\_MISMATCH Scalar types do not match

SIZE MISMATCH Sizes do not match

UNDEFINED\_TYPE Templated specialization not defined for this type

CUSTOM\_EXCEPTION Custom exception, user must provide a custom message

#### 7.2.3 Constructor & Destructor Documentation

7.2.3.1 qpp::Exception::Exception (const std::string & where, const Type & type ) [inline]

Constructs an exception.

#### **Parameters**

where	Text representing where the exception occured
type	Exception type, defined in qpp::Exception::Type

7.2.3.2 qpp::Exception::Exception ( const std::string & where, const std::string & custom ) [inline]

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

#### **Parameters**

where	Text representing where the exception occured
custom	Exception description

# 7.2.4 Member Function Documentation

7.2.4.1 void qpp::Exception::\_construct\_exception\_msg() [inline], [private]

Constructs the exception description from its type.

See also

qpp::Exception::Type

Must modify the code of this function if more exceptions are added

7.2.4.2 virtual const char\* qpp::Exception::what( ) const [inline], [override], [virtual], [noexcept]

Overrides std::exception::what()

#### Returns

**Exception** description

#### 7.2.5 Member Data Documentation

```
7.2.5.1 std::string qpp::Exception::_custom [private]
```

**7.2.5.2 std::string qpp::Exception::\_msg** [private]

7.2.5.3 Type qpp::Exception::\_type [private]

**7.2.5.4 std::string qpp::Exception::\_where** [private]

The documentation for this class was generated from the following file:

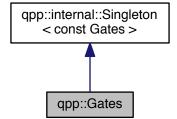
· classes/exception.h

# 7.3 qpp::Gates Class Reference

const Singleton class that implements most commonly used gates

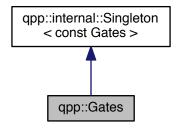
#include <classes/gates.h>

Inheritance diagram for qpp::Gates:



90 Class Documentation

Collaboration diagram for qpp::Gates:



#### **Public Member Functions**

• cmat Rn (double theta, const std::vector< double > &n) const

Rotation of theta about the 3-dimensional real unit vector n.

• cmat Zd (idx D) const

Generalized Z gate for qudits.

• cmat Fd (idx D) const

Fourier transform gate for qudits.

• cmat Xd (idx D) const

Generalized X gate for qudits.

• template<typename Derived = Eigen::MatrixXcd>

Derived Id (idx D) const

Identity gate.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > CTRL (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &ctrl, const std::vector< idx > &subsys, idx n, idx d=2) const

Generates the multi-partite multiple-controlled-A gate in matrix form.

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$ 

dyn\_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const
std::vector< idx > &dims) const

Expands out.

# **Public Attributes**

• cmat Id2 {cmat::Identity(2, 2)}

Identity gate.

cmat H {cmat::Zero(2, 2)}

Hadamard gate.

cmat X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

cmat Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

cmat Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

cmat S {cmat::Zero(2, 2)}

```
S gate.

    cmat T {cmat::Zero(2, 2)}

     T gate.
• cmat CNOT {cmat::Identity(4, 4)}
     Controlled-NOT control target gate.
cmat CZ {cmat::Identity(4, 4)}
     Controlled-Phase gate.
cmat CNOTba {cmat::Zero(4, 4)}
     Controlled-NOT target control gate.
```

cmat SWAP {cmat::Identity(4, 4)}

SWAP gate.

 cmat TOF {cmat::Identity(8, 8)} Toffoli gate.

• cmat FRED {cmat::Identity(8, 8)} Fredkin gate.

# **Private Member Functions**

• Gates ()

Initializes the gates.

∼Gates ()=default

Default destructor.

# **Friends**

class internal::Singleton < const Gates >

# **Additional Inherited Members**

#### 7.3.1 **Detailed Description**

Default destructor.

const Singleton class that implements most commonly used gates

# 7.3.2 Constructor & Destructor Documentation

```
7.3.2.1 qpp::Gates::Gates() [inline], [private]
Initializes the gates.
7.3.2.2 qpp::Gates::~Gates() [private], [default]
```

# 7.3.3 Member Function Documentation

 $7.3.3.1 \quad template < typename \ Derived > dyn\_mat < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived > dyn\_mat < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived > dyn\_mat < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Scalar > qpp:: Gates:: CTRL \ ( \ const \ Eigen:: Matrix Base < typename \ Derived :: Matrix Base < typename \ Derived :: Matrix Base < typename \ Derived :: Matrix Base < typename \ Deriv$ Derived > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, idx n, idx d = 2 ) const [inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

92 Class Documentation

#### See also

qpp::applyCTRL()

Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
n	Total number of subsystems
d	Subsystem dimensions

#### Returns

CTRL-A gate, as a matrix over the same scalar field as A

7.3.3.2 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::expandout ( const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

# **Parameters**

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

# Returns

Tensor product  $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$ , with A on position pos, as a dynamic matrix over the same scalar field as A

7.3.3.3 cmat qpp::Gates::Fd(idx D)const [inline]

Fourier transform gate for qudits.

Note

Defined as  $F = \sum_{jk} \exp(2\pi i j k/D) |j\rangle\langle k|$ 

**Parameters** 

D Dimension of the Hilbert space

Returns

Fourier transform gate for qudits

7.3.3.4 template < typename Derived = Eigen::MatrixXcd > Derived qpp::Gates::Id ( idx D ) const [inline]

Identity gate.

Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

#### **Parameters**

D	Dimension of the Hilbert space			

Returns

Identity gate

7.3.3.5 cmat qpp::Gates::Rn ( double theta, const std::vector < double > & n ) const [inline]

Rotation of theta about the 3-dimensional real unit vector n.

#### **Parameters**

theta	Rotation angle
n	3-dimensional real unit vector

#### Returns

Rotation gate

7.3.3.6 cmat qpp::Gates::Xd (idx D) const [inline]

Generalized X gate for qudits.

Note

Defined as 
$$X = \sum_j |j \oplus 1 \rangle \langle j|$$

#### **Parameters**

D	Dimension of the Hilbert space

Returns

Generalized X gate for qudits

7.3.3.7 cmat qpp::Gates::Zd(idx D) const [inline]

Generalized Z gate for qudits.

Note

Defined as 
$$Z = \sum_{j} \exp(2\pi i j/D) |j\rangle\langle j|$$

**Parameters** 

D Dimension of the Hilbert space

Returns

Generalized Z gate for qudits

7.3.4 Friends And Related Function Documentation

**7.3.4.1** friend class internal::Singleton < const Gates > [friend]

7.3.5 Member Data Documentation

7.3.5.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

7.3.5.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

7.3.5.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

7.3.5.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}

Fredkin gate.

7.3.5.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}

Hadamard gate.

7.3.5.6 cmat qpp::Gates::ld2 {cmat::Identity(2, 2)}

Identity gate.

7.3.5.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}

S gate.

7.3.5.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}

SWAP gate.

7.3.5.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}

T gate.

7.3.5.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}

Toffoli gate.

7.3.5.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

7.3.5.12 cmat qpp::Gates::Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

7.3.5.13 cmat qpp::Gates::Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

The documentation for this class was generated from the following file:

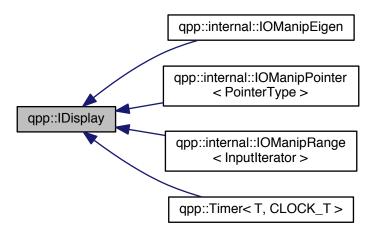
· classes/gates.h

## 7.4 qpp::IDisplay Class Reference

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

#include <classes/idisplay.h>

Inheritance diagram for qpp::IDisplay:



#### **Public Member Functions**

- IDisplay ()=default

  Default constructor.
- IDisplay (const IDisplay &)=default

Default copy constructor.

• IDisplay (IDisplay &&)=default

Default move constructor.

• IDisplay & operator= (const IDisplay &)=default

Default copy assignment operator.

• IDisplay & operator= (IDisplay &&)=default

Default move assignment operator.

virtual ∼IDisplay ()=default

Default virtual destructor.

#### **Private Member Functions**

virtual std::ostream & display (std::ostream &os) const =0
 Must be overridden by all derived classes.

#### **Friends**

std::ostream & operator<< (std::ostream &os, const IDisplay &rhs)</li>
 Overloads the extraction operator.

#### 7.4.1 Detailed Description

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

This class defines friend inline std::ostream& operator<< (std::ostream& os, const qpp::IDisplay& rhs). The latter delegates the work to the pure private virtual function qpp::IDisplay::display() which has to be overridden by all derived classes.

#### 7.4.2 Constructor & Destructor Documentation

```
7.4.2.1 qpp::IDisplay::IDisplay( ) [default]
Default constructor.
7.4.2.2 qpp::IDisplay::IDisplay( const IDisplay & ) [default]
Default copy constructor.
7.4.2.3 qpp::IDisplay::IDisplay( IDisplay && ) [default]
```

7.4.2.4 virtual qpp::IDisplay::~IDisplay( ) [virtual], [default]

Default virtual destructor.

Default move constructor.

#### 7.4.3 Member Function Documentation

7.4.3.1 virtual std::ostream& qpp::IDisplay::display( std::ostream & os ) const [private], [pure virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs).

Implemented in qpp::internal::IOManipEigen, qpp::Timer< T, CLOCK\_T >, qpp::internal::IOManipPointer< PointerType >, and qpp::internal::IOManipRange< InputIterator >.

7.4.3.2 IDisplay& qpp::IDisplay::operator=( const IDisplay & ) [default]

Default copy assignment operator.

7.4.3.3 IDisplay& qpp::IDisplay::operator=(IDisplay && ) [default]

Default move assignment operator.

#### 7.4.4 Friends And Related Function Documentation

7.4.4.1 std::ostream& operator<< ( std::ostream & os, const IDisplay & rhs ) [friend]

Overloads the extraction operator.

Delegates the work to the virtual function <a href="mailto:qpp::IDisplay::display">qpp::IDisplay::display()</a>

The documentation for this class was generated from the following file:

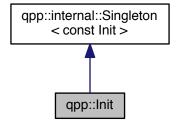
· classes/idisplay.h

#### 7.5 qpp::Init Class Reference

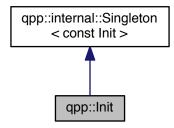
const Singleton class that performs additional initializations/cleanups

#include <classes/init.h>

Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



#### **Private Member Functions**

• Init ()

Additional initializations.

• ∼Init ()

Cleanups.

#### **Friends**

class internal::Singleton < const Init >

#### **Additional Inherited Members**

#### 7.5.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

#### 7.5.2 Constructor & Destructor Documentation

```
7.5.2.1 qpp::Init::Init( ) [inline],[private]
```

Additional initializations.

```
7.5.2.2 qpp::Init::\simInit( ) [inline],[private]
```

Cleanups.

#### 7.5.3 Friends And Related Function Documentation

**7.5.3.1** friend class internal::Singleton < const lnit > [friend]

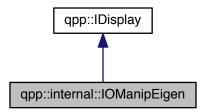
The documentation for this class was generated from the following file:

· classes/init.h

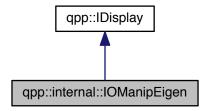
## 7.6 qpp::internal::IOManipEigen Class Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipEigen:



Collaboration diagram for qpp::internal::IOManipEigen:



#### **Public Member Functions**

- template<typename Derived >
   IOManipEigen (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
- IOManipEigen (const cplx z, double chop=qpp::chop)

## **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- cmat \_A
- double \_chop

#### 7.6.1 Constructor & Destructor Documentation

- 7.6.1.1 template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen ( const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop ) [inline], [explicit]
- 7.6.1.2 qpp::internal::IOManipEigen::IOManipEigen ( const cplx z, double chop = qpp::chop ) [inline], [explicit]

#### 7.6.2 Member Function Documentation

7.6.2.1 std::ostream& qpp::internal::IOManipEigen::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

#### 7.6.3 Member Data Documentation

- 7.6.3.1 cmat qpp::internal::IOManipEigen::\_A [private]
- **7.6.3.2** double qpp::internal::IOManipEigen::\_chop [private]

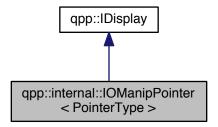
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

#### 7.7 qpp::internal::IOManipPointer < PointerType > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipPointer< PointerType >:



Collaboration diagram for qpp::internal::IOManipPointer< PointerType >:



#### **Public Member Functions**

- IOManipPointer (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipPointer (const IOManipPointer &)=default
- IOManipPointer & operator= (const IOManipPointer &)=default

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- const PointerType \* \_p
- idx \_n
- std::string \_separator
- std::string \_start
- std::string \_end

#### 7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer ( const PointerType \* p, idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " ) [inline], [explicit]
- 7.7.1.2 template<typename PointerType> qpp::internal::IOManipPointer< PointerType>::IOManipPointer( const IOManipPointer<< PointerType> & ) [default]

#### 7.7.2 Member Function Documentation

7.7.2.1 template<typename PointerType> std::ostream& qpp::internal::IOManipPointer< PointerType >::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.7.2.2 template<typename PointerType> IOManipPointer& qpp::internal::IOManipPointer< PointerType
  >::operator=( const IOManipPointer< PointerType > & ) [default]
- 7.7.3 Member Data Documentation
- 7.7.3.1 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_end [private]
- 7.7.3.2 template<typename PointerType>idx qpp::internal::IOManipPointer< PointerType>::\_n [private]
- 7.7.3.3 template<typename PointerType> const PointerType\* qpp::internal::IOManipPointer< PointerType >::\_p [private]
- 7.7.3.4 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_separator [private]
- 7.7.3.5 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType >::\_start [private]

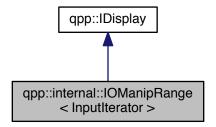
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

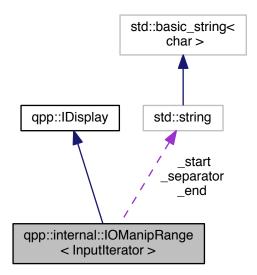
## 7.8 qpp::internal::IOManipRange < InputIterator > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipRange< InputIterator >:



Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



#### **Public Member Functions**

- IOManipRange (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipRange (const IOManipRange &)=default
- IOManipRange & operator= (const IOManipRange &)=default

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- · InputIterator first
- · InputIterator \_last
- · std::string \_separator
- std::string \_start
- std::string \_end

#### 7.8.1 Constructor & Destructor Documentation

- 7.8.1.2 template<typename InputIterator> qpp::internal::IOManipRange< InputIterator>::IOManipRange ( const IOManipRange < InputIterator > & ) [default]

#### 7.8.2 Member Function Documentation

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

7.8.2.2 template<typename InputIterator> IOManipRange& qpp::internal::IOManipRange< InputIterator >::operator=( const IOManipRange< InputIterator > & ) [default]

#### 7.8.3 Member Data Documentation

- **7.8.3.1** template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_end [private]
- **7.8.3.2** template<typename InputIterator > InputIterator qpp::internal::IOManipRange< InputIterator >::\_first [private]
- **7.8.3.3** template<typename InputIterator > InputIterator qpp::internal::IOManipRange< InputIterator >::\_last [private]
- **7.8.3.4** template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_separator [private]
- **7.8.3.5** template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_start [private]

The documentation for this class was generated from the following file:

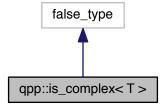
• internal/classes/iomanip.h

## 7.9 qpp::is\_complex< T > Struct Template Reference

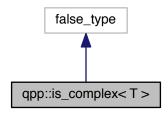
Checks whether the type is a complex type.

#include <traits.h>

Inheritance diagram for qpp::is complex< T >:



Collaboration diagram for qpp::is\_complex< T >:



#### 7.9.1 Detailed Description

template<typename T>struct qpp::is\_complex< T>

Checks whether the type is a complex type.

Provides the member constant value which is equal to true, if the type is a complex type (i.e. std::complex < T >)

The documentation for this struct was generated from the following file:

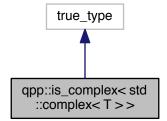
· traits.h

## 7.10 qpp::is\_complex< std::complex< T > > Struct Template Reference

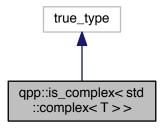
Checks whether the type is a complex number type, specialization for complex types.

#include <traits.h>

Inheritance diagram for qpp::is\_complex< std::complex< T > >:



Collaboration diagram for qpp::is\_complex< std::complex< T > >:



#### 7.10.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_complex < std::complex < T > >$ 

Checks whether the type is a complex number type, specialization for complex types.

The documentation for this struct was generated from the following file:

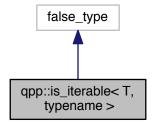
· traits.h

## 7.11 qpp::is\_iterable < T, typename > Struct Template Reference

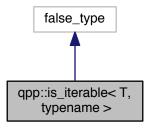
Checks whether *T* is compatible with an STL-like iterable container.

#include <traits.h>

Inheritance diagram for qpp::is\_iterable < T, typename >:



Collaboration diagram for qpp::is\_iterable < T, typename >:



#### 7.11.1 Detailed Description

 $template < typename \ {\tt T}, \ typename \ {\tt = void} > {\tt struct \ qpp::is\_iterable} < \ {\tt T}, \ typename >$ 

Checks whether T is compatible with an STL-like iterable container.

Provides the member constant *value* which is equal to *true*, if *T* is compatible with an iterable container, i.e. provides at least *begin()* and *end()* member functions. Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

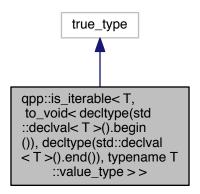
· traits.h

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

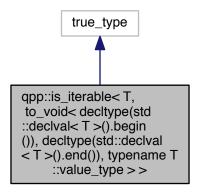
#include <traits.h>

 $Inheritance\ diagram\ for\ qpp:: is\_iterable < T,\ to\_void < \ decltype(std::declval < T > ().begin()),\ decltype(std::decltyp$ 

T >().end()), typename T::value\_type > >:



Collaboration diagram for qpp::is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().begin()), typename T::value\_type > >:



#### 7.12.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_iterable < \ T, \ to\_void < \ decltype(std::declval < \ T > ().begin()), \ decltype(std::declval < \ T > ().end()), \ typename \ T::value\_type > >$ 

Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. The documentation for this struct was generated from the following file:

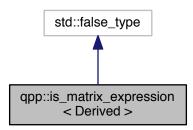
• traits.h

## 7.13 qpp::is\_matrix\_expression < Derived > Struct Template Reference

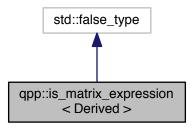
Checks whether the type is an Eigen matrix expression.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< Derived >:



Collaboration diagram for qpp::is\_matrix\_expression< Derived >:



#### 7.13.1 Detailed Description

 $template {<} typename \ Derived {>} struct \ qpp::is\_matrix\_expression {<} \ Derived {>}$ 

Checks whether the type is an Eigen matrix expression.

Provides the member constant *value* which is equal to *true*, if the type is an Eigen matrix expression of type *Eigen ∷MatrixBase Oerived >*. Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

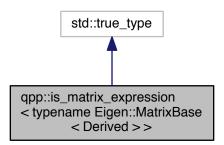
• traits.h

# 7.14 qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >> Struct Template Reference

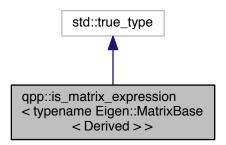
Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



Collaboration diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



#### 7.14.1 Detailed Description

 $template < typename\ Derived > struct\ qpp::is\_matrix\_expression < \ typename\ Eigen::MatrixBase < Derived > >$ 

 ${\it Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.}$ 

The documentation for this struct was generated from the following file:

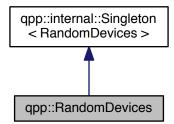
· traits.h

## 7.15 qpp::RandomDevices Class Reference

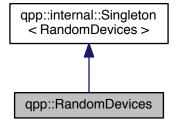
Singeleton class that manages the source of randomness in the library.

#include <classes/random\_devices.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



#### **Public Attributes**

std::mt19937 \_rng

Mersenne twister random number generator.

#### **Private Member Functions**

• RandomDevices ()

Initializes and seeds the random number generators.

∼RandomDevices ()=default

Default destructor.

#### **Private Attributes**

```
std::random_device _rd
used to seed std::mt19937 _rng
```

#### **Friends**

class internal::Singleton < RandomDevices >

#### **Additional Inherited Members**

#### 7.15.1 Detailed Description

Singeleton class that manages the source of randomness in the library.

Consists of a wrapper around an std::mt19937 Mersenne twister random number generator engine and an std⇔ ::random\_device engine. The latter is used to seed the Mersenne twister.

#### Warning

This class DOES NOT seed the standard C number generator used by Eigen::Matrix::Random(), since it is not thread safe. Do not use Eigen::Matrix::Random() or functions that depend on the C style random number engine, but use <a href="mailto:qpp::rand()">qpp::rand()</a> instead!

#### 7.15.2 Constructor & Destructor Documentation

```
7.15.2.1 qpp::RandomDevices::RandomDevices() [inline], [private]
```

Initializes and seeds the random number generators.

```
7.15.2.2 qpp::RandomDevices::~RandomDevices() [private], [default]
```

Default destructor.

#### 7.15.3 Friends And Related Function Documentation

```
7.15.3.1 friend class internal::Singleton < RandomDevices > [friend]
```

#### 7.15.4 Member Data Documentation

```
7.15.4.1 std::random_device qpp::RandomDevices::_rd [private]
```

used to seed std::mt19937 rng

7.15.4.2 std::mt19937 qpp::RandomDevices::\_rng

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

classes/random\_devices.h

### 7.16 qpp::internal::Singleton < T > Class Template Reference

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

#### **Static Public Member Functions**

- static T & get\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)
- static thread\_local T & get\_thread\_local\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)

#### **Protected Member Functions**

- Singleton () noexcept=default
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete
- virtual ∼Singleton ()=default

#### 7.16.1 Detailed Description

template<typename T>class qpp::internal::Singleton< T>

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from qpp::internal::Singleton, make qpp::internal::Singleton a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function qpp::internal::Singleton::get\_instance() (qpp::internal::Singleton::get\_thread\_local\_cinstance()), which returns a reference (thread\_local reference) to your newly created singleton (thread-safe in C++11).

#### Example:

#### See also

Code of qpp::Codes, qpp::Gates, qpp::Init, qpp::RandomDevices, qpp::States or qpp.h for real world examples of usage.

#### 7.16.2 Constructor & Destructor Documentation

```
7.16.2.1 template<typename T> qpp::internal::Singleton< T>::Singleton( ) [protected], [default], [noexcept]
```

7.16.2.3 template virtual qpp::internal::Singleton< T>::
$$\sim$$
Singleton( ) [protected], [virtual], [default]

#### 7.16.3 Member Function Documentation

The documentation for this class was generated from the following file:

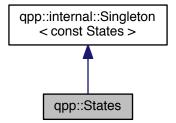
• internal/classes/singleton.h

#### 7.17 qpp::States Class Reference

const Singleton class that implements most commonly used states

```
#include <classes/states.h>
```

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



#### **Public Attributes**

```
    ket x0 {ket::Zero(2)}
```

Pauli Sigma-X 0-eigenstate |+>

ket x1 {ket::Zero(2)}

Pauli Sigma-X 1-eigenstate |->

ket y0 {ket::Zero(2)}

Pauli Sigma-Y 0-eigenstate | y+>

ket y1 {ket::Zero(2)}

Pauli Sigma-Y 1-eigenstate | y->

ket z0 {ket::Zero(2)}

Pauli Sigma-Z 0-eigenstate |0>

ket z1 {ket::Zero(2)}

Pauli Sigma-Z 1-eigenstate | 1>

cmat px0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.

• cmat px1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.

cmat py0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle < y+|$ .

cmat py1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 1-eigenstate |y->< y-|.

• cmat pz0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.

cmat pz1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 1-eigenstate | 1><1|.

ket b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

ket b01 {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

ket b10 {ket::Zero(4)}

Bell-10 state (following the convention in Nielsen and Chuang)

ket b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

cmat pb00 {cmat::Zero(4, 4)}

Projector onto the Bell-00 state.

cmat pb01 {cmat::Zero(4, 4)}

Projector onto the Bell-01 state.

cmat pb10 {cmat::Zero(4, 4)}

Projector onto the Bell-10 state.

cmat pb11 {cmat::Zero(4, 4)}

Projector onto the Bell-11 state.

ket GHZ {ket::Zero(8)}

GHZ state.

ket W {ket::Zero(8)}

W state.

cmat pGHZ {cmat::Zero(8, 8)}

Projector onto the GHZ state.

cmat pW {cmat::Zero(8, 8)}

Projector onto the W state.

#### **Private Member Functions**

- States ()
- $\sim$ States ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const States >

#### **Additional Inherited Members**

#### 7.17.1 Detailed Description

const Singleton class that implements most commonly used states

#### 7.17.2 Constructor & Destructor Documentation

```
7.17.2.1 qpp::States::States( ) [inline],[private]
```

Initialize the states

```
7.17.2.2 qpp::States::~States() [private], [default]
```

Default destructor.

#### 7.17.3 Friends And Related Function Documentation

**7.17.3.1** friend class internal::Singleton < const States > [friend]

#### 7.17.4 Member Data Documentation

7.17.4.1 ket qpp::States::b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

```
7.17.4.2 ket qpp::States::b01 {ket::Zero(4)}
Bell-01 state (following the convention in Nielsen and Chuang)
7.17.4.3 ket qpp::States::b10 {ket::Zero(4)}
Bell-10 state (following the convention in Nielsen and Chuang)
7.17.4.4 ket qpp::States::b11 {ket::Zero(4)}
Bell-11 state (following the convention in Nielsen and Chuang)
7.17.4.5 ket qpp::States::GHZ {ket::Zero(8)}
GHZ state.
7.17.4.6 cmat qpp::States::pb00 {cmat::Zero(4, 4)}
Projector onto the Bell-00 state.
7.17.4.7 cmat qpp::States::pb01 {cmat::Zero(4, 4)}
Projector onto the Bell-01 state.
7.17.4.8 cmat qpp::States::pb10 {cmat::Zero(4, 4)}
Projector onto the Bell-10 state.
7.17.4.9 cmat qpp::States::pb11 {cmat::Zero(4, 4)}
Projector onto the Bell-11 state.
7.17.4.10 cmat qpp::States::pGHZ {cmat::Zero(8, 8)}
Projector onto the GHZ state.
7.17.4.11 cmat qpp::States::pW {cmat::Zero(8, 8)}
Projector onto the W state.
7.17.4.12 cmat qpp::States::px0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.
7.17.4.13 cmat qpp::States::px1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.
```

```
7.17.4.14 cmat qpp::States::py0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 0-eigenstate |y+><y+|.
7.17.4.15 cmat qpp::States::py1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.
7.17.4.16 cmat qpp::States::pz0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.
7.17.4.17 cmat qpp::States::pz1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.
7.17.4.18 ket qpp::States::W {ket::Zero(8)}
W state.
7.17.4.19 ket qpp::States::x0 {ket::Zero(2)}
Pauli Sigma-X 0-eigenstate |+>
7.17.4.20 ket qpp::States::x1 {ket::Zero(2)}
Pauli Sigma-X 1-eigenstate |->
7.17.4.21 ket qpp::States::y0 {ket::Zero(2)}
Pauli Sigma-Y 0-eigenstate |y+>
7.17.4.22 ket qpp::States::y1 {ket::Zero(2)}
Pauli Sigma-Y 1-eigenstate |y->
7.17.4.23 ket qpp::States::z0 {ket::Zero(2)}
Pauli Sigma-Z 0-eigenstate |0>
7.17.4.24 ket qpp::States::z1 {ket::Zero(2)}
Pauli Sigma-Z 1-eigenstate |1>
The documentation for this class was generated from the following file:
```

classes/states.h

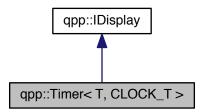
Generated on Wed Oct 14 2015 08:59:16 for Quantum++ by Doxygen

## 7.18 qpp::Timer < T, CLOCK\_T > Class Template Reference

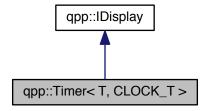
#### Chronometer.

#include <classes/timer.h>

Inheritance diagram for qpp::Timer< T, CLOCK\_T >:



Collaboration diagram for qpp::Timer < T, CLOCK\_T >:



#### **Public Member Functions**

· Timer () noexcept

Constructs an instance with the current time as the starting point.

• void tic () noexcept

Resets the chronometer.

· const Timer & toc () noexcept

Stops the chronometer.

• double tics () const noexcept

Time passed in the duration specified by T.

• template<typename U = T>

U get\_duration ()

Duration specified by U.

• Timer (const Timer &)=default

Default copy constructor.

Timer (Timer &&)=default

Default move constructor.

• Timer & operator= (const Timer &)=default

Default copy assignment operator.

• Timer & operator= (Timer &&)=default

Default move assignment operator.

virtual ~Timer ()=default

Default virtual destructor.

#### **Protected Attributes**

- CLOCK\_T::time\_point \_start
- CLOCK\_T::time\_point \_end

#### **Private Member Functions**

#### 7.18.1 Detailed Description

 $\label{template} $$ \textbf{T} = \textbf{std}:: \textbf{chrono}:: \textbf{duration} < \textbf{double}>, \ \textbf{typename CLOCK\_T} = \textbf{std}:: \textbf{chrono}:: \textbf{steady\_clock}> \textbf{class qpp}:: \textbf{Timer} < \textbf{T, CLOCK T}> $$ \textbf{T} = \textbf{Std}:: \textbf{chrono}:: \textbf{steady\_clock}> \textbf{class qpp}:: \textbf{Timer} < \textbf{T, CLOCK T}> $$ \textbf{T} = \textbf{Std}:: \textbf{chrono}:: \textbf{steady\_clock}> \textbf{class qpp}:: \textbf{Timer} < \textbf{T, CLOCK T}> $$ \textbf{T} = \textbf{Std}:: \textbf{chrono}:: \textbf{Steady\_clock}> \textbf{class qpp}:: \textbf{Timer} < \textbf{T, CLOCK T}> $$ \textbf{T} = \textbf{Std}:: \textbf{Clock} = \textbf{T, CLOCK} = \textbf{$ 

Chronometer.

**Template Parameters** 

T	Tics duration, default is std::chrono::duration <double, 1=""> i.e. seconds in double</double,>
	precision
CLOCK_T	Clock's type, default is std::chrono::steady_clock, not affected by wall clock
	changes during runtime

#### 7.18.2 Constructor & Destructor Documentation

Constructs an instance with the current time as the starting point.

```
7.18.2.2 template < typename T = std::chrono::duration < double >, typename CLOCK_T = std::chrono::steady_clock > qpp::Timer < T, CLOCK_T > ::Timer ( const Timer < T, CLOCK_T > & ) [default]
```

Default copy constructor.

```
7.18.2.3 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> qpp::Timer< T, CLOCK_T > ::Timer( Timer< T, CLOCK_T > && ) [default]
```

Default move constructor.

7.18.2.4 template<typename T = std::chrono::duration<double>, typename CLOCK\_T = std::chrono::steady\_clock> virtual qpp::Timer< T, CLOCK\_T>::~Timer( ) [virtual], [default]

Default virtual destructor.

#### 7.18.3 Member Function Documentation

qpp::IDisplay::display() override

**Parameters** 

```
os Output stream
```

#### Returns

Writes to the output stream the number of tics (specified by T) that passed between the instantiation/reset and invocation of <a href="mailto:qpp::Timer::toc()">qpp::Timer::toc()</a>.

Implements qpp::IDisplay.

```
7.18.3.2 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> template<typename U = T> U qpp::Timer< T, CLOCK_T >::get_duration( ) [inline]
```

Duration specified by U.

**Template Parameters** 

U	Duration, default is T, which defaults to std::chrono::duration <double, 1="">, i.e.</double,>
	seconds in double precision

#### Returns

Duration that passed between the instantiation/reset and invocation of qpp::Timer::toc()

Default copy assignment operator.

7.18.3.4 template<typename T = std::chrono::duration<double>, typename CLOCK\_T = std::chrono::steady\_clock> Timer& qpp::Timer< T, CLOCK\_T > ::operator=( Timer< T, CLOCK\_T > && ) [default]

Default move assignment operator.

7.18.3.5 template < typename T = std::chrono::duration < double >, typename CLOCK\_T = std::chrono::steady\_clock > void qpp::Timer < T, CLOCK\_T >::tic() [inline], [noexcept]

Resets the chronometer.

Resets the starting/ending point to the current time

Time passed in the duration specified by T.

#### Returns

Number of tics (specified by T) that passed between the instantiation/reset and invocation of qpp::Timer::toc()

Stops the chronometer.

Set the current time as the ending point

Returns

Current instance

#### 7.18.4 Member Data Documentation

- 7.18.4.1 template<typename T = std::chrono::duration<double>, typename CLOCK\_T = std::chrono::steady\_clock> CLOCK\_T::time\_point qpp::Timer< T, CLOCK\_T >::\_end [protected]
- 7.18.4.2 template < typename T = std::chrono::duration < double >, typename CLOCK\_T = std::chrono::steady\_clock > CLOCK\_T::time\_point qpp::Timer < T, CLOCK\_T >::\_start [protected]

The documentation for this class was generated from the following file:

· classes/timer.h

## **Chapter 8**

## **File Documentation**

## 8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Codes

const Singleton class that defines quantum error correcting codes

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### 8.1.1 Detailed Description

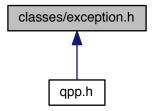
Quantum error correcting codes.

## 8.2 classes/exception.h File Reference

Exceptions.

124 File Documentation

This graph shows which files directly or indirectly include this file:



#### **Classes**

• class qpp::Exception

Generates custom exceptions, used when validating function parameters.

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### 8.2.1 Detailed Description

Exceptions.

## 8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Gates

const Singleton class that implements most commonly used gates

#### **Namespaces**

qpp

Quantum++ main namespace.

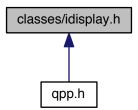
#### 8.3.1 Detailed Description

Quantum gates.

## 8.4 classes/idisplay.h File Reference

Display interface via the non-virtual interface (NVI)

This graph shows which files directly or indirectly include this file:



#### Classes

· class qpp::IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### 8.4.1 Detailed Description

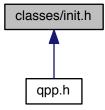
Display interface via the non-virtual interface (NVI)

126 File Documentation

## 8.5 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Init

const Singleton class that performs additional initializations/cleanups

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### 8.5.1 Detailed Description

Initialization.

## 8.6 classes/random\_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



#### **Classes**

• class qpp::RandomDevices

Singeleton class that manages the source of randomness in the library.

#### **Namespaces**

qpp

Quantum++ main namespace.

#### 8.6.1 Detailed Description

Random devices.

#### 8.7 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



#### **Classes**

• class qpp::States

const Singleton class that implements most commonly used states

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### 8.7.1 Detailed Description

Quantum states.

128 File Documentation

## 8.8 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



#### Classes

class qpp::Timer < T, CLOCK\_T >
 Chronometer.

#### **Namespaces**

qpp

Quantum++ main namespace.

#### 8.8.1 Detailed Description

Timing.

## 8.9 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

qpp

Quantum++ main namespace.

#### **Functions**

• constexpr cplx qpp::operator""\_i (unsigned long long int x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

• constexpr cplx qpp::operator""\_i (long double x) noexcept

User-defined literal for complex  $i=\sqrt{-1}$  (real overload)

• cplx qpp::omega (idx D)

D-th root of unity.

#### **Variables**

• constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

• constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

• constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

• constexpr double qpp::infty = std::numeric\_limits<double>::infinity()

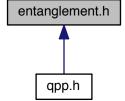
Used to denote infinity in double precision.

## 8.9.1 Detailed Description

Constants.

## 8.10 entanglement.h File Reference

Entanglement functions.



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 dyn\_col\_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector<
 idx > &dims)

Schmidt coefficients of the bi-partite pure state A.

template < typename Derived >
 cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)

template<typename Derived >
 cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Schmidt basis on Bob side.

Schmidt basis on Alice side.

template<typename Derived >
 std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx
 > &dims)

Schmidt probabilities of the bi-partite pure state A.

template<typename Derived >
 double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
 Entanglement of the bi-partite pure state A.

template<typename Derived >
 double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)

G-concurrence of the bi-partite pure state A.

template < typename Derived >
 double qpp::negativity (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)
 Negativity of the bi-partite mixed state A.

template<typename Derived >
 double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

template<typename Derived >
 double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)

Wootters concurrence of the bi-partite qubit mixed state A.

Logarithmic negativity of the bi-partite mixed state A.

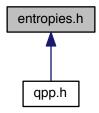
## 8.10.1 Detailed Description

Entanglement functions.

## 8.11 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 double qpp::entropy (const Eigen::MatrixBase< Derived > &A)

von-Neumann entropy of the density matrix A

double qpp::entropy (const std::vector< double > &prob)

Shannon entropy of the probability distribution prob.

• template<typename Derived >

double <a href="mailto:qpp::renyi">qpp::renyi</a> (const Eigen::MatrixBase< Derived > &A, double alpha)

Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .

double qpp::renyi (const std::vector< double > &prob, double alpha)

Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .

• template<typename Derived >

double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)

Tsallis- q entropy of the density matrix A, for  $q \geq 0$ .

double qpp::tsallis (const std::vector< double > &prob, double q)

Tsallis- q entropy of the probability distribution prob, for  $q \geq 0$ .

• template<typename Derived >

double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)

Quantum mutual information between 2 subsystems of a composite system.

ullet template<typename Derived >

double <a href="mailto:qpp::qmutualinfo">qpp::qmutualinfo</a> (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)

Quantum mutual information between 2 subsystems of a composite system.

#### 8.11.1 Detailed Description

Entropy functions.

## 8.12 experimental/experimental.h File Reference

Experimental/test functions/classes.

#### **Namespaces**

• qpp

Quantum++ main namespace.

• qpp::experimental

Experimental/test functions/classes, do not use or modify.

## 8.12.1 Detailed Description

Experimental/test functions/classes.

## 8.13 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)
   Transpose.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)
   Complex conjugate.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)
   Adjoint.

```
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar qpp::trace (const Eigen::MatrixBase < Derived > &A)
      Trace
• template<typename Derived >
  Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)
      Determinant.
\bullet \ \ {\it template}{<} {\it typename Derived} >
  Derived::Scalar <a href="mailto:qpp::logdet">qpp::logdet</a> (const Eigen::MatrixBase</a> Derived > &A)
      Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::sum">qpp::sum</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise sum of A.
• template<typename Derived >
  Derived::Scalar <a href="mailto:open:prod">open:prod</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise product of A.

    template<typename Derived >

  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase< Derived > &A)
      Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition.
\bullet \ \ {\it template}{<} {\it typename Derived} >
  dyn_col_vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.

    template<typename Derived >

  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
      Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvalues.

    template<typename Derived >

  cmat qpp::hevects (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn col vect< double >, cmat > qpp::svd (const Eigen::MatrixBase< Derived > &A)
      Full singular value decomposition.
• template<typename Derived >
  dyn_col_vect< double > qpp::svals (const Eigen::MatrixBase< Derived > &A)
      Singular values.

    template<typename Derived >

  cmat qpp::svdU (const Eigen::MatrixBase< Derived > &A)
      Left singular vectors.

    template<typename Derived >

  cmat qpp::svdV (const Eigen::MatrixBase< Derived > &A)
      Right singular vectors.

    template<typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
```

```
Functional calculus f(A)
• template<typename Derived >
  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.
template<typename Derived >
  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat qpp::logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.
• template<typename Derived >
  double qpp::schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn_mat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const
  typename Derived::Scalar &))
     Functor.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)
     Direct sum.
```

```
• template<typename Derived >
  dyn mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows,
  idx cols)
     Reshape.
- template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Commutator.

    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.

    std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

     Multi-index to non-negative integer index.

    ket qpp::mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

     Multi-partite qudit ket.

    ket qpp::mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat qpp::mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
  std::vector< double > qpp::abssq (InputIterator first, InputIterator last)
     Computes the absolute values squared of an STL-like range of complex numbers.

    template<typename Container >

  std::vector< double > qpp::abssq (const Container &c, typename std::enable_if< is_iterable< Container
  >::value >::type *=nullptr)
```

Computes the absolute values squared of an STL-like container.

template < typename Derived >
 std::vector < double > qpp::abssq (const Eigen::MatrixBase < Derived > &A)

Computes the absolute values squared of an Eigen expression.

• template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

• template<typename Container >

Container::value\_type qpp::sum (const Container &c)

Element-wise sum of the elements of an STL-like container.

template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type qpp::prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

• template<typename Container >

Container::value\_type <a href="mailto:qpp::prod">qpp::prod</a> (const Container &c)

Element-wise product of the elements of an STL-like container.

• template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

```
std::vector< T > qpp::complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

• template<typename Derived >

```
std::vector< double > qpp::rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat qpp::bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

#### 8.13.1 Detailed Description

Generic quantum computing functions.

## 8.14 input\_output.h File Reference

Input/output functions.



#### **Namespaces**

qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

Eigen expression ostream manipulator.

internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

template<typename InputIterator >
 internal::IOManipRange< InputIterator > qpp::disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >
 internal::IOManipRange< typename Container::const\_iterator > qpp::disp (const Container &c, const std
 ::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >
 internal::IOManipPointer< PointerType > qpp::disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

template<typename Derived >
 void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

#### 8.14.1 Detailed Description

Input/output functions.

#### 8.15 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 dyn\_col\_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, const std::vector< idx > &dims)
 Generalized inner product.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$ 

dyn\_col\_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$ 

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

#### 8.15.1 Detailed Description

Measurement functions.

## 8.16 internal/classes/iomanip.h File Reference

Input/output manipulators.



#### Classes

- $\bullet \ \, {\it class qpp::internal::IOManipRange} < {\it InputIterator} > \\$
- class qpp::internal::IOManipPointer< PointerType >
- class qpp::internal::IOManipEigen

#### **Namespaces**

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

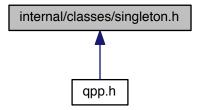
## 8.16.1 Detailed Description

Input/output manipulators.

## 8.17 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



## Classes

- class qpp::internal::Singleton < T >

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

#### **Namespaces**

qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

#### 8.17.1 Detailed Description

Singleton pattern via CRTP.

### 8.18 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

#### **Functions**

- void qpp::internal::\_n2multiidx (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx qpp::internal::\_multiidx2n (const idx \*midx, idx numdims, const idx \*dims) noexcept

bool qpp::internal::\_check\_square\_mat (const Eigen::MatrixBase< Derived > &A)

template<typename Derived >

bool qpp::internal::\_check\_vector (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$ 

bool qpp::internal:: check rvector (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool qpp::internal::\_check\_cvector (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ template\!<\!typename\ T>$ 

bool qpp::internal::\_check\_nonzero\_size (const T &x) noexcept

• template<typename T1 , typename T2 >

bool qpp::internal::\_check\_matching\_sizes (const T1 &lhs, const T2 &rhs) noexcept

- bool qpp::internal:: check dims (const std::vector < idx > &dims)
- template<typename Derived >

 $bool\ qpp::internal::\_check\_dims\_match\_mat\ (const\ std::vector < idx > \&dims,\ const\ Eigen::MatrixBase < Derived > \&A)$ 

template<typename Derived >
 bool qpp::internal::\_check\_dims\_match\_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase
 Derived > &V)

- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase
   Derived > &V)
- bool qpp::internal::\_check\_eq\_dims (const std::vector< idx > &dims, idx dim) noexcept
- bool qpp::internal::\_check\_subsys\_match\_dims (const std::vector< idx > &subsys, const std::vector< idx > &dims)
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_matrix (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_cvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_rvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_vector (const Eigen::MatrixBase< Derived > &V) noexcept
- bool qpp::internal::\_check\_perm (const std::vector< idx > &perm)
- template<typename Derived1, typename Derived2 >
   dyn\_mat< typename Derived1::Scalar > qpp::internal::\_kron2 (const Eigen::MatrixBase< Derived1 > &A,
   const Eigen::MatrixBase< Derived2 > &B)
- template<typename Derived1, typename Derived2 >
   dyn\_mat< typename Derived1::Scalar > qpp::internal::\_dirsum2 (const Eigen::MatrixBase< Derived1 > &A,
   const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >
   void qpp::internal::variadic vector emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
   void qpp::internal::variadic vector emplace (std::vector< T > &v, First &&first, Args &&...args)

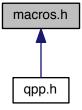
#### 8.18.1 Detailed Description

Internal utility functions.

## 8.19 macros.h File Reference

Preprocessor macros.

This graph shows which files directly or indirectly include this file:



#### **Macros**

- #define PRINT(x)
- #define PRINTLN(x)

```
• #define ERROR(x)
```

• #define ERRORLN(x)

#### 8.19.1 Detailed Description

Preprocessor macros.

#### 8.19.2 Macro Definition Documentation

```
8.19.2.1 #define ERROR( x )
```

Prints an error message to std::cerr

```
8.19.2.2 #define ERRORLN( x )
```

Prints an error message to std::cerr and adds a new line

```
8.19.2.3 #define PRINT( x )
```

Prints a message

```
8.19.2.4 #define PRINTLN( x )
```

Prints a message and adds a new line

#### 8.20 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

#### **Namespaces**

• qpp

Quantum++ main namespace.

## **Functions**

template < typename Derived >
 Derived qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)
 Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

cmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std ::string &var name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std ::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

## 8.20.1 Detailed Description

Input/output interfacing with MATLAB.

## 8.21 number\_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

- std::vector < int > qpp::x2contfrac (double x, idx n, idx cut=1e5)
- Simple continued fraction expansion.
- double qpp::contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double qpp::contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• ubigint qpp::gcd (ubigint m, ubigint n)

Greatest common divisor of two non-negative integers.

ubigint qpp::gcd (const std::vector< ubigint > &ns)

Greatest common divisor of a list of non-negative integers.

• ubigint qpp::lcm (ubigint m, ubigint n)

Least common multiple of two positive integers.

ubigint qpp::lcm (const std::vector< ubigint > &ns)

Least common multiple of a list of positive integers.

• std::vector< idx > qpp::invperm (const std::vector< idx > &perm)

Inverse permutation.

 $\bullet \; \mathsf{std} :: \mathsf{vector} < \mathsf{idx} > \mathsf{qpp} :: \mathsf{compperm} \; (\mathsf{const} \; \mathsf{std} :: \mathsf{vector} < \mathsf{idx} > \& \mathsf{perm}, \; \mathsf{const} \; \mathsf{std} :: \mathsf{vector} < \mathsf{idx} > \& \mathsf{sigma}) \\$ 

Compose permutations.

• std::vector< ubigint > qpp::factors (ubigint n)

Prime factor decomposition.

• bool <a href="mailto:app::isprime">app::isprime</a> (ubigint n)

Primality test.

• ubigint <a href="mailto:qpp::modpow">qpp::modpow</a> (ubigint a, ubigint n, ubigint p)

Integer power modulo p.

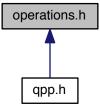
#### 8.21.1 Detailed Description

Number theory functions.

## 8.22 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived1 , typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 const std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

ullet template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ← ::vector< idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat qpp::kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat qpp::kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > qpp::choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

• cmat qpp::choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat qpp::super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

template<typename Derived >

Partial trace.

• template<typename Derived >

Partial trace.

template<typename Derived >

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std
 ::vector< idx > &subsys, idx d=2)

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Partial transpose.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)

Partial transpose.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, const std::vector< idx > &dims)

Subsystem permutation.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)

Subsystem permutation.

## 8.22.1 Detailed Description

Quantum operation functions.

# 8.23 qpp.h File Reference

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "traits.h"
#include "classes/idisplay.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "statistics.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
#include "macros.h"
```

## **Namespaces**

qpp

Quantum++ main namespace.

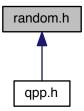
#### 8.23.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

#### 8.24 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

• double qpp::rand (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

- bigint qpp::rand (bigint a=std::numeric\_limits< bigint >::min(), bigint b=std::numeric\_limits< bigint >::max())

  Generates a random big integer uniformly distributed in the interval [a, b].
- ubigint qpp::rand (ubigint a=std::numeric\_limits< ubigint >::min(), ubigint b=std::numeric\_limits< ubigint >↔ ::max())

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

idx qpp::randidx (idx a=std::numeric\_limits < idx >::min(), idx b=std::numeric\_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

• template<typename Derived >

Derived qpp::rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

• template<typename Derived >

Derived qpp::randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

• template<>

cmat gpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double <a href="mailto:qpp::randn">qpp::randn</a> (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat qpp::randU (idx D)

Generates a random unitary matrix.

• cmat qpp::randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > qpp::randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat qpp::randH (idx D)

Generates a random Hermitian matrix.

ket qpp::randket (idx D)

Generates a random normalized ket (pure state vector)

cmat qpp::randrho (idx D)

Generates a random density matrix.

std::vector< idx > qpp::randperm (idx n)

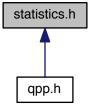
Generates a random uniformly distributed permutation.

#### 8.24.1 Detailed Description

Randomness-related functions.

#### 8.25 statistics.h File Reference

Statistics functions.



8.26 traits.h File Reference 151

#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

```
• std::vector < double > qpp::uniform (idx N)
```

Uniform probability distribution vector.

std::vector< double > qpp::marginalX (const dmat &probXY)

Marginal distribution.

std::vector< double > qpp::marginalY (const dmat &probXY)

Marginal distribution.

 $\bullet \ \ \text{template}{<} \text{typename Container} >$ 

double <a href="mailto:qpp::avg">qpp::avg</a> (const std::vector< double > &prob, const Container &X)

Average

• template<typename Container >

double qpp::cov (const dmat &probXY, const Container &X, const Container &Y)

Covariance.

• template<typename Container >

double qpp::var (const std::vector< double > &prob, const Container &X)

Variance.

• template<typename Container >

double qpp::sigma (const std::vector< double > &prob, const Container &X)

Standard deviation.

• template<typename Container >

double <a href="mailto:qpp::cor">qpp::cor</a> (const dmat &probXY, const Container &X, const Container &Y)

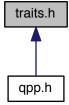
Correlation.

#### 8.25.1 Detailed Description

Statistics functions.

## 8.26 traits.h File Reference

Type traits.



#### Classes

struct qpp::is\_iterable < T, typename >

Checks whether T is compatible with an STL-like iterable container.

struct qpp::is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().← end()), typename T::value\_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

struct qpp::is\_matrix\_expression< Derived >

Checks whether the type is an Eigen matrix expression.

struct qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

struct qpp::is\_complex< T >

Checks whether the type is a complex type.

struct qpp::is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

#### **Namespaces**

• qpp

Quantum++ main namespace.

## **Typedefs**

template<typename... > using qpp::to\_void = void

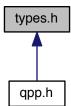
Alias template that implements the proposal for void\_t.

#### 8.26.1 Detailed Description

Type traits.

## 8.27 types.h File Reference

Type aliases.



#### **Namespaces**

• qpp

Quantum++ main namespace.

## **Typedefs**

```
• using qpp::idx = std::size_t
```

Non-negative integer index.

• using qpp::bigint = long long int

Big integer.

• using qpp::ubigint = unsigned long long int

Non-negative big integer.

• using qpp::cplx = std::complex< double >

Complex number in double precision.

using qpp::ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

using qpp::bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using qpp::cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using qpp::dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

• template<typename Scalar >

```
using <a href="mailto:qpp::dyn_mat">qpp::dyn_mat</a> = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >
```

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

```
using qpp::dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
```

Dynamic Eigen column vector over the field specified by Scalar.

• template<typename Scalar >

```
using qpp::dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
```

Dynamic Eigen row vector over the field specified by Scalar.

## 8.27.1 Detailed Description

Type aliases.

# Index

_A	qpp::internal::IOManipRange, 104
qpp::internal::IOManipEigen, 100	_kron2
_check_cvector	qpp::internal, 82
qpp::internal, 81	_last
_check_dims	qpp::internal::IOManipRange, 104
qpp::internal, 81	_msg
_check_dims_match_cvect	qpp::Exception, 89
qpp::internal, 81	_multiidx2n
_check_dims_match_mat	qpp::internal, 82
qpp::internal, 82	_n
_check_dims_match_rvect	qpp::internal::IOManipPointer, 102
qpp::internal, 82	_n2multiidx
_check_eq_dims	qpp::internal, 82
qpp::internal, 82	_p
_check_matching_sizes	qpp::internal::IOManipPointer, 102
qpp::internal, 82	_rd
_check_nonzero_size	qpp::RandomDevices, 112
qpp::internal, 82	_rng
_check_perm	qpp::RandomDevices, 112
qpp::internal, 82	_separator
_check_qubit_cvector	qpp::internal::IOManipPointer, 102
qpp::internal, 82	qpp::internal::IOManipRange, 104
_check_qubit_matrix	_start
qpp::internal, 82	qpp::Timer, 122
_check_qubit_rvector	qpp::internal::IOManipPointer, 102
qpp::internal, 82	qpp::internal::IOManipRange, 104
_check_qubit_vector	_type
qpp::internal, 82	qpp::Exception, 89
_check_rvector	_where
qpp::internal, 82	qpp::Exception, 89
_check_square_mat	~Codes
qpp::internal, 82	qpp::Codes, 84
_check_subsys_match_dims	~Gates
qpp::internal, 82	qpp::Gates, 91
_check_vector	~IDisplay
qpp::internal, 82	qpp::IDisplay, 96 ∼Init
_chop	
qpp::internal::IOManipEigen, 100	qpp::Init, 98 ∼RandomDevices
_construct_exception_msg	qpp::RandomDevices, 112
qpp::Exception, 88	~Singleton
custom	qpp::internal::Singleton, 114
qpp::Exception, 89	~States
dirsum2	qpp::States, 116
qpp::internal, 82	~Timer
_end	qpp::Timer, 120
qpp::Timer, 122	App. Timor, TEO
qpp::internal::IOManipPointer, 102	absm
qpp::internal::IOManipRange, 104	qpp, 26
first	abssq

qpp, 26	complement
adjoint	qpp, 32
qpp, 28	compperm
anticomm	qpp, 33
qpp, 28	concurrence
apply	qpp, 33
qpp, 28–30	conjugate
applyCTRL	qpp, 33
qpp, 30	constants.h, 128
avg	contfrac2x
qpp, 31	qpp, 33, 34
41FF ; • .	cor
b00	qpp, 34
qpp::States, 116	cosm
b01	qpp, 34
qpp::States, 116	COV
b10	qpp, 34
qpp::States, 117	cplx
b11	
qpp::States, 117	qpp, 25 cwise
bigint	
qpp, 25	qpp, 35
bloch2rho	DIMS INVALID
	qpp::Exception, 87
qpp, 31	DIMS MISMATCH CVECTOR
bra	
qpp, 25	qpp::Exception, 87
CNOT	DIMS_MISMATCH_MATRIX
	qpp::Exception, 87
qpp::Gates, 94 CNOTba	DIMS_MISMATCH_RVECTOR
	qpp::Exception, 87
qpp::Gates, 94	DIMS_MISMATCH_VECTOR
CTRL	qpp::Exception, 87
qpp::Gates, 91	DIMS_NOT_EQUAL
CUSTOM_EXCEPTION	qpp::Exception, 87
qpp::Exception, 88	det
CZ	qpp, 35
qpp::Gates, 94	dirsum
choi2kraus	qpp, 35, 36
qpp, 31	dirsumpow
choi2super	qpp, 36
qpp, 32	disp
chop	qpp, 37, 38
qpp, 79	display
classes/codes.h, 123	qpp::IDisplay, 97
classes/exception.h, 123	qpp::Timer, 121
classes/gates.h, 124	qpp::internal::IOManipEigen, 100
classes/idisplay.h, 125	qpp::internal::IOManipPointer, 102
classes/init.h, 126	qpp::internal::IOManipRange, 104
classes/random_devices.h, 126	dmat
classes/states.h, 127	qpp, 25
classes/timer.h, 128	dyn_col_vect
cmat	qpp, 25
qpp, 25	dyn_mat
Codes	
qpp::Codes, 84	qpp, 25
codeword	dyn_row_vect
qpp::Codes, 85	qpp, 25
	ERROR
comm	
qpp, 32	macros.h, 143

ERRORLN	hevals
macros.h, 143	qpp, 43
ee	hevects
qpp, 79	qpp, 43
eig	
qpp, 38	IDisplay
entanglement	qpp::IDisplay, 96
qpp, 39	IOManipEigen
entanglement.h, 129	qpp::internal::IOManipEigen, 100
entropies.h, 130	IOManipPointer
entropy	qpp::internal::IOManipPointer, 101
qpp, 39	IOManipRange
eps	qpp::internal::IOManipRange, 103
qpp, 80	ld
evals	qpp::Gates, 93
qpp, 39	ld2
evects	qpp::Gates, 94
qpp, 40	idx
Exception	qpp, 25
qpp::Exception, 88	infty
expandout	qpp, 80
qpp::Gates, 92	Init
experimental/experimental.h, 132	qpp::Init, 98
expm	input_output.h, 136
qpp, 40	instruments.h, 137
	internal/classes/iomanip.h, 139
FIVE_QUBIT	internal/classes/singleton.h, 140
qpp::Codes, 84	internal/util.h, 141
FRED	internal::Singleton< const Codes >
qpp::Gates, 94	qpp::Codes, 85
factors	internal::Singleton< const Gates >
qpp, 40	qpp::Gates, 94
Fd	internal::Singleton< const Init >
qpp::Gates, 92	qpp::Init, 98
functions.h, 132	internal::Singleton< const States >
funm	qpp::States, 116
qpp, 41	internal::Singleton < RandomDevices >
0117	qpp::RandomDevices, 112
GHZ	inverse
qpp::States, 117	qpp, 44
Gates	invperm
qpp::Gates, 91	qpp, 44
gcd	ip
qpp, 41	qpp, 44
gconcurrence	isprime
qpp, 41	qpp, 45
get_duration	kot
qpp::Timer, 121	ket
get_instance qpp::internal::Singleton, 114	qpp, 25 kraus2choi
get_thread_local_instance	qpp, 45
qpp::internal::Singleton, 114	kraus2super
grams	qpp, 45
qpp, 42	kron
Н	qpp, 46, 47
qpp::Gates, 94	kronpow
heig	qpp, <del>4</del> 7
qpp, 43	lcm
4PP, TO	IVIII

qpp, 47, 48	NOT_BIPARTITE
load	qpp::Exception, 88
qpp, 48	NOT_QUBIT_CVECTOR
IoadMATLABmatrix	qpp::Exception, 88
qpp, 48, 49	NOT_QUBIT_MATRIX
logdet	qpp::Exception, 88
qpp, 49	NOT_QUBIT_RVECTOR
logm	qpp::Exception, 88
qpp, 50	NOT_QUBIT_SUBSYS
lognegativity	qpp::Exception, 88
qpp, 50	NOT QUBIT VECTOR
	qpp::Exception, 88
MATLAB/matlab.h, 143	negativity
MATRIX_MISMATCH_SUBSYS	qpp, 58
qpp::Exception, 87	norm
MATRIX_NOT_CVECTOR	gpp, 59
qpp::Exception, 87	number theory.h, 144
MATRIX NOT RVECTOR	,,
qpp::Exception, 87	OUT OF RANGE
MATRIX NOT SQUARE	qpp::Exception, 88
qpp::Exception, 87	omega
MATRIX NOT SQUARE OR CVECTOR	qpp, 59
qpp::Exception, 87	operations.h, 145
MATRIX NOT SQUARE OR RVECTOR	operator<<
qpp::Exception, 87	qpp::IDisplay, 97
MATRIX NOT SQUARE OR VECTOR	operator=
qpp::Exception, 87	qpp::IDisplay, 97
MATRIX_NOT_VECTOR	qpp::Timer, 121
qpp::Exception, 87	qpp::internal::IOManipPointer, 102
macros.h, 142	qpp::internal::IOManipRange, 104
ERROR, 143	qpp::internal::Singleton, 114
ERRORLN, 143	operator""_i
PRINT, 143	qpp, 59
PRINTLN, 143	DEDM INVALID
marginalX	PERM_INVALID
qpp, 50	qpp::Exception, 87
	PERM MISMATCH DIMS
marginalY	
marginalY qpp, 50	qpp::Exception, 88
qpp, 50 maxn	qpp::Exception, 88 pGHZ
qpp, 50	qpp::Exception, 88 pGHZ qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT
qpp, 50 maxn qpp, 80	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143
qpp, 50 maxn qpp, 80 measure	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN
qpp, 50 maxn qpp, 80 measure qpp, 51–54	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143
qpp, 50 maxn qpp, 80 measure qpp, 51–54 measure_seq	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW
qpp, 50 maxn qpp, 80 measure qpp, 51–54 measure_seq qpp, 54, 55	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117
qpp, 50 maxn qpp, 80 measure qpp, 51–54 measure_seq qpp, 54, 55 mket	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117 pb11 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117 pb11 qpp::States, 117
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117 pb11 qpp::States, 117 pb11 qpp::States, 117
qpp, 50 maxn qpp, 80 measure qpp, 51–54 measure_seq qpp, 54, 55 mket qpp, 55, 56 modpow qpp, 56 mprj qpp, 56 multiidx2n qpp, 58 NINE_QUBIT_SHOR qpp::Codes, 84	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117 pb11 qpp::States, 117 pb11 qpp::States, 117 pi qpp, 80 powm
qpp, 50 maxn	qpp::Exception, 88 pGHZ qpp::States, 117 PRINT macros.h, 143 PRINTLN macros.h, 143 pW qpp::States, 117 pb00 qpp::States, 117 pb01 qpp::States, 117 pb10 qpp::States, 117 pb11 qpp::States, 117 pb11 qpp::States, 117

qpp, 60	dyn_row_vect, 25
prod	ee, 79
qpp, 60	eig, 38
ptrace	entanglement, 39
qpp, 61	entropy, 39
ptrace1	eps, 80
qpp, 61	evals, 39
ptrace2	evects, 40
qpp, 63	expm, 40
ptranspose	factors, 40
qpp, 63	
рх0	funm, 41
qpp::States, 117	gcd, 41
	gconcurrence, 41
px1	grams, 42
qpp::States, 117	heig, 43
py0	hevals, 43
qpp::States, 117	hevects, 43
py1	idx, 25
qpp::States, 118	infty, 80
pz0	inverse, 44
qpp::States, 118	invperm, 44
pz1	ip, 44
qpp::States, 118	isprime, 45
	ket, 25
qmutualinfo	kraus2choi, 45
qpp, 64	
qpp, 13	kraus2super, 45
absm, 26	kron, 46, 47
abssq, 26	kronpow, 47
adjoint, 28	lcm, 47, 48
anticomm, 28	load, 48
apply, 28–30	loadMATLABmatrix, 48, 49
applyCTRL, 30	logdet, 49
avg, 31	logm, 50
bigint, 25	lognegativity, 50
bloch2rho, 31	marginalX, 50
	marginalY, 50
bra, 25	maxn, 80
choi2kraus, 31	measure, 51–54
choi2super, 32	measure_seq, 54, 55
chop, 79	mket, 55, 56
cmat, 25	
comm, 32	modpow, 56
complement, 32	mprj, 56
compperm, 33	multiidx2n, 58
concurrence, 33	n2multiidx, 58
conjugate, 33	negativity, 58
contfrac2x, 33, 34	norm, 59
cor, 34	omega, 59
cosm, 34	operator""_i, 59
cov, 34	pi, 80
cplx, 25	powm, 59
cwise, 35	prj, 60
det, 35	prod, 60
der, 35 dirsum, 35, 36	ptrace, 61
	ptrace1, 61
dirsumpow, 36	•
disp, 37, 38	ptrace2, 63
dmat, 25	ptranspose, 63
dyn_col_vect, 25	qmutualinfo, 64
dyn_mat, 25	rand, 64, 66

randU, 70 randV,		
randV. 70 randidx, 67 randkraus, 68 randidx, 67 randkraus, 68 randr, 68, 69 randremm, 69 renyi, 70 reshape, 71 rho2pure, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 73 schmidtle, 73 schmidtle, 73 schmidtle, 73 schmidtleoeffs, 73 schmidtleoeffs, 73 schmidtleoeffs, 73 schmidtleoeffs, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sum, 75, 76 super2choi, 76 svals, 76 sval, 76 svalv, 77 svdV, 77 svalv, 79 spermule, 77 to_void, 26 trace, 78 transpose, 78 transpo	randH, 67	DIMS_MISMATCH_MATRIX, 87
randket, 67 randket, 67 randket, 67 randket, 67 randket, 68 randh, 68, 69 randpern, 69 randpern, 69 randpern, 69 randpern, 69 randpern, 70 reshape, 71 rho2pure, 71 save, 72 saveMATLABmatrix, 72 sochmidta, 73 schmidta, 73 schmi		<del>-</del>
randkraus, 68 randraus, 68 randr, 68, 69 randrhos, 69 renyi, 70 reshape, 71 rho2buch, 71 rho2pure, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 73 schmidtle, 74 spectralpown, 75 sqrtm,		
randkraus, 68 randn, 68, 69 randpern, 69 renyi, 70 reshape, 71 rhozbure, 71 save, 72 saveMATIX, NOT, SOUARE, 67 rhozbure, 71 save, 72 saveMATLABmatrix, 72 sohatten, 73 schmidtle, 74 sigma, 74 spectralpown, 75 sqrim, 75 subsystantial, 76 svals, 78 svals, 76 sval		
randn, 68, 69 randperm, 69 randrho, 69 randrho, 69 renyi, 70 reshape, 71 rhozbloch, 71 rhozbloch, 71 rhozbure, 71 save, 72 saveMATLABmatrix, 72 schatter, 73 schmidtb, 73 schmidtb, 73 schmidtbors, 74 signa, 74 signa, 74 signa, 74 signa, 74 signa, 75 squr, 76 squr, 76 squr, 76 squr, 76 squr, 77 squr, 79 squr,		•
randperm, 69 randrho, 69 randrho, 69 randrho, 69 randrho, 69 randrho, 69 renyi, 70 reshape, 71 rhoebloch, 71 rhoebloch, 71 rhoebloch, 71 rhoebloch, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 schatten, 73 schmidta, 7		
randrho, 69 renyi, 70 renhape, 71 reshape, 71 rhozbloch, 71 rhozbloch, 71 rhozbloch, 71 rhozbloch, 71 save, 72 saveMATLABmatrix, 72 schatten, 73 schmidta, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 svalv, 77 syspermute, 77 to void, 26 trace, 78 transpose, 79 tra		
renyi, 70 reshape, 71 rhozbolcch, 71 rhozbure, 71 rhozbure, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 schatten, 73 schmidtA, 73 schmidtA, 73 schmidtoeffs, 73 schmidtprobs, 74 sigm, 74 sigm, 74 sigm, 74 sigm, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 76 sval, 76 sval, 76 sval, 76 sval, 76 sval, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 traling, 79 var, 79 var, 79 var, 79 var, 79 qpph, 147 qpp, 147 qpp, 147 qpp, 147 qpp, 147 qpp, 148 Qpp:Exception, 85 construct, exception, 85 construct, ex	randperm, 69	
reshape, 71 rho2bloch, 71 rho2bloch, 71 rho2pure, 71 save, 72 save, 72 save, 72 save, 72 save, 73 schmidta, 73 schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sub, 76 sub, 76 svals, 78 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 78 svals, 76 svals, 76 svals, 78 svals, 76 svals, 76 svals, 78 svals, 76 svals, 78 sv	randrho, 69	MATRIX_NOT_SQUARE, 87
rho2bioch, 71 rho2pure, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 schmidtA, 73 schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtcoeffs, 73 schmidtprobs, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 svals, 76 svals, 76 svals, 76 svdl, 77 svdV, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 transpose, 78 transpose, 78 transpose, 79 qpp.h, 147 qpp::Codes, 84 Codes, 85 NINE_OUBIT_SHOR, 84 SEVEN_OUBIT_SHOR, 84 SEVEN_OUBIT_SHOR, 84 SEVEN_OUBIT_STEANE, 84 Type, 89msg, 89msg, 89msg, 89msg, 89msg, 89msg, 89msg, 89msg, 89where, 89 CUSTOM_EXCEPTION, 88 CISTOM ATRIX. NOT_VECTOR, 87 NO_CODEWORD, 88 NOT_QIBIT_VECTOR, 88 NOT_QUBIT_STEANE, 84 Type, 89msg, 89where, 89 CUSTOM_EXCEPTION, 88 CISTOM_STEATURE, 89 CUSTOM_EXCEPTION, 88 CISTOM_STEATURE, 96 CISTOM_STEATURE, 97 Codes, 97 Codes, 98 Custom, 89 Custom, 80 Custo	renyi, 70	MATRIX_NOT_SQUARE_OR_CVECTOR, 87
rho2pure, 71 save, 72 saveMATLABmatrix, 72 saveMATLABmatrix, 72 schmidtA, 73 schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtB, 73 schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 svalv, 77 svdV, 77 svdV, 77 svdV, 77 svdV, 77 svdV, 77 svalv, 79 var, 79 var, 79 var, 79 var, 79 var, 79 var, 79 qpp.h, 147 qpp:Codes, 83 codeword, 85 FIVE_OUBIT, 84 internal::Singleton < const Codes >, 85 NINE_OUBIT, Steep in, 85 construct_exception_msg, 88 custom, 89 _msg, 89 _msg, 89 _where, 89 CUSTOM_EXCEPTION, 88 NOT_QUBIT_VECTOR, 88 NOT_QUBIT_Subsys, 88 NOT_QUBIT_su	reshape, 71	MATRIX_NOT_SQUARE_OR_RVECTOR, 87
save, 72 saveMATLABmatrix, 72 schatten, 73 schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtB, 73 schmidtoeffs, 73 schmidtprobs, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sval, 76 super2choi, 76 sval, 76 sval, 77 syspermute, 79 sy	rho2bloch, 71	MATRIX_NOT_SQUARE_OR_VECTOR, 87
saveMATLABmatrix, 72 schatten, 73 schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtB, 73 schmidtD, 73 schmidtD, 73 schmidtD, 73 schmidtD, 73 schmidtD, 74 schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 svals, 76 sval, 77 svotV, 77 svpspermute, 77 svotV, 77 svpspermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 83	rho2pure, 71	MATRIX_NOT_VECTOR, 87
schatten, 73 schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtcoeffs, 83 schmidteoeffs, 83 schmidtcoeffs, 83 schmidteoeffs,	save, 72	NO_CODEWORD, 88
schmidtA, 73 schmidtB, 73 schmidtB, 73 schmidtB, 73 schmidtCoeffs, 73 schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 svals, 76 svdl, 77 svdl, 77 svdl, 77 svdl, 77 svdl, 77 svdl, 77 svspermute, 77 svspermute, 77 subigint, 26 uniform, 79 var, 79 xazcontfrac, 79 qpp.h, 147 qpp::Codes, 83 construct_exception_msg, 88 _ custorn, 89 _ msg, 89 _ msg, 89 _ msg, 89 _ where, 89 custorn, Exception, 87 schmidt North Colors splaye, 96 culstown, 85 culstown, 89 culstown, 85 culstown, 89	saveMATLABmatrix, 72	NOT_BIPARTITE, 88
schmidtB, 73 schmidtcoeffs, 73 schmidtcoeffs, 73 schmidtcoeffs, 73 schmidtcoeffs, 73 schmidtcoeffs, 74 schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 spectralpowm, 75 sqrtm, 75 sum, 75, 76 super2choi, 76 svals, 76 svals, 76 sval, 77 svdV, 77 svdV, 77 svdV, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 transpose, 78 transpose, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp:Codes, 83	schatten, 73	NOT_QUBIT_CVECTOR, 88
schmidtcoeffs, 73 schmidtprobs, 74 schmidtprobs, 74 sigma, 74 sigma, 74 sinm, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 75 super2chol, 76 svals, 76 svals, 76 svals, 76 sval, 77 sval, 78 spermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 var, 79 svan, 79 svan, 79 svan, 79 svalcontfrac, 79 stpp., 147 spp::Codes, 83 Codes, 84 Codes, 84 Codes, 84 Codes, 84 Codes, 84 Codes, 84 codeword, 85 FIVE_QUBIT_s41 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 84 Type, 84 construct_exception_msg, 88custom, 89msg, 89msg, 89msg, 89msg, 89myer, 89 CUSTOM_EXCEPTION, 88 DIMS_INVALID, 87  NOT_QUBIT_SIDIANS, 88 SIZBISSYS, 88 NOT_QUBIT_SIDIANS, 88 SIZBIS VACIONT, 88 SIZBIS VACIONT, 88 SIZBIS VALID, 87  NOT_QUBIT_SIDIANS, 88 SIZDIS Splay, 96	schmidtA, 73	NOT_QUBIT_MATRIX, 88
schmidtprobs, 74 sigma, 74 sigma, 74 sigma, 74 sigma, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sqrtm, 76 super2choi, 76 svals, 76 svals, 76 svals, 76 svals, 76 svalv, 77 svdV, 77 svdV, 77 svdV, 77 svdV, 77 svalv, 26 trace, 78 transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 var, 79 var, 79 var, 79 sycontfrac, 79 qpp.:Codes, 83 ~Codes, 84 codeword, 85 FIVE_QUBIT_SHOR, 84 SEVEN_QUBIT_SHOR, 84 SEVEN_QUBIT_SHOR, 84 spulse, 89 Lype, 96 Lyst MSMATCH_DIMS, 88 SUEZE, MISMATCH_DIMS, 88 SUESY_SIMISMATCH_DIMS, 8	schmidtB, 73	NOT_QUBIT_RVECTOR, 88
sigma, 74 simm, 74 simm, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 super2choi, 76 svals, 76 svals, 76 svd, 76 svd, 77 svdV, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 tablish, 78 ubigint, 26 uniform, 79 var, 79 var, 79 var, 79 var, 79 var, 79 sycontfrac, 79 qpp.it, 147 qpp::Codes, 83	schmidtcoeffs, 73	NOT_QUBIT_SUBSYS, 88
sim, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 sum, 75, 76 super2choi, 76 svd, 76 svd, 76 svd, 76 svd, 77 svd, 77 svd, 77 svd, 77 svd, 77 svgpermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 tuligint, 26 uniform, 79 var, 79 var, 79 var, 79 var, 79 spectodes, 83 ~Codes, 84 Codes, 84 Codes, 84 codeword, 85 FIVE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 89 Lype, 89 L	schmidtprobs, 74	
sim, 74 spectralpowm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 sum, 75, 76 super2choi, 76 svd, 76 svd, 76 svd, 76 svd, 77 svd, 77 svd, 77 svd, 77 svd, 77 svgpermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 tuligint, 26 uniform, 79 var, 79 var, 79 var, 79 var, 79 spectodes, 83 ~Codes, 84 Codes, 84 Codes, 84 codeword, 85 FIVE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 89 Lype, 89 L	sigma, 74	OUT OF RANGE, 88
spectralpowm, 75 sqrtm, 75 sqrtm, 75 sum, 75, 76 super2choi, 76 super2choi, 76 svals, 76 svalv, 77 svalv, 77 svalv, 77 svalv, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 transpose, 78 transpose, 78 transpose, 78 tuniform, 79 var, 79 var, 79 var, 79 syzcontfrac, 79 dpp.h, 147 dpp::Codes, 83	_	
sqrtm, 75 sum, 75, 76 super2choi, 76 super2choi, 76 super2choi, 76 super2choi, 76 super2choi, 76 super2choi, 76 svals, 76 svals, 76 svd, 76 svd, 77 svdV, 77 svdV, 77 superachoid, 78 superachoid, 78 superachoid, 78 superachoid, 78 superachoid, 78 superachoid, 89 superach		PERM MISMATCH DIMS, 88
sum, 75, 76         SUBSYS_MISMATCH_DIMS, 87           super2choi, 76         TYPE_MISMATCH, 88           svals, 76         UNDEFINED_TYPE, 88           svdU, 77         UNDEFINED_TYPE, 88           svdV, 77         UNDEFINED_TYPE, 88           svdV, 77         What, 88           syspermute, 77         ZERO_SIZE, 87           to_void, 26         qpp::Gates, 89           trace, 78         CNOT, 94           tsallis, 78         CNOTba, 94           ubigint, 26         CTRL, 91           uniform, 79         CZ, 94           var, 79         expandout, 92           x2contfrac, 79         FRED, 94           qpp::Codes, 83         Gates, 91           Codes, 84         Id, 92           qpp::Codes, 84         Id, 93           codeword, 85         Id, 93           rodeword, 85         Id, 93           Internal::Singleton < const Gates >, 94           internal::Singleton < const Gates >, 94           Rn, 93         S, 94           SEVEN_QUBIT_STEANE, 84         SWAP, 94           Type, 84         T, 94           qpp::Exception, 85         TOF, 94           _construct_exception_msg, 88         X, 95           _custom, 89	·	
super2choi, 76       TYPE_MISMATCH, 88         svals, 76       Type, 87         svd, 76       UNDEFINED_TYPE, 88         svdU, 77       UNKNOWN_EXCEPTION, 87         svdV, 77       what, 88         syspermute, 77       ZERO_SIZE, 87         to_void, 26       qpp::Gates, 89         trace, 78       CNOT, 94         tsallis, 78       CNOTba, 94         ubigint, 26       CTRL, 91         uniform, 79       CZ, 94         var, 79       expandout, 92         x2contfrac, 79       FRED, 94         qpp.h, 147       Fd, 92         qpp:Codes, 83       Gates, 91         ~Codes, 84       Id, 93         codeword, 85       Id2, 94         FIVE_QUBIT_84       internal::Singleton< const Gates >, 94         internal::Singleton< const Gates >, 94       internal::Singleton< const Gates >, 94         NINE_QUBIT_SHOR, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _ustom, 89       Y, 95         _type, 89       Z, 95         _type, 89       Z, 95         _type, 80       Z, 93         <	·	
svals, 76 svd, 76 svd, 76 svd, 77 svdV, 77 svdV, 77 svdV, 77 svdV, 77 syspermute, 77 to_void, 26 trace, 78 transpose, 78 tubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 84 Codes, 84 Codes, 84 Codes, 84 Codeword, 85 FIVE_QUBIT_84 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 84 qpp::Exception, 85custom_ 89msg, 89msg, 89msg, 89msg, 89mype, 80USTOM_EXCEPTION, 88 UNDEFINED_TYPE, 88 UNDKENED_TYPE, 88 UNDKENED_TYP		
svd, 76 svdU, 77 svdV, 77 svdV, 77 svgpermute, 77 to_void, 26 trace, 78 transpose, 78 tallis, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp:Codes, 83 ~Codes, 84 Codes, 84 Codes, 84 Codes, 84 codeword, 85 FIVE_QUBIT_S4 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 Type, 84 qpp:Exception, 85 _construct_exception_msg, 88 _usdV, 78 custom, 89 _msg, 89 _msg, 89 _where, 89 CUSTOM_EXCEPTION, 88 DIMS_INVALID, 87  IERO_SIZE, 87 unknown, 88 CREPS_OSIZE, 87 CORS, 89 CVERO_SIZE, 87 CONT, 94 CNOT, 94 C	•	
svdU, 77       UNKNOWN_EXCEPTION, 87         svdV, 77       what, 88         syspermute, 77       ZERO_SIZE, 87         to_void, 26       qpp::Gates, 89         trace, 78       ~Gates, 91         transpose, 78       CNOT, 94         tsallis, 78       CNOTba, 94         ubigint, 26       CTRL, 91         uniform, 79       expandout, 92         vacontfrac, 79       FRED, 94         qpp.h, 147       Fd, 92         qpp::Codes, 83       Gates, 91         ~Codes, 84       Id, 93         codeword, 85       Id, 93         riternal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       S, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::Display, 96		· ·
svdV, 77 syspermute, 77 syspermute, 77 to_void, 26 trace, 78 trace, 78 trace, 78 cNOT, 94 tsallis, 78 ubigint, 26 uniform, 79 var, 79 var, 79 vaccontfrac, 79 qpp::Codes, 83  ~Codes, 84 Codes, 84 Codes, 84 Codes, 84 codeword, 85 FIVE_QUBIT, 84 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 SEVEN_QUBIT_SHOR, 84 qpp::Exception, 85 _construct_exception_msg, 88 _use, 89 _msg, 89 _msg, 89 _msg, 89 _where, 89 CUSTOM_EXCEPTION, 88 DIMS_INVALID, 87  qpp::Display, 96  COORD, 88  CERO_SIZE, 87 QERO_SIZE, 87 QERO_		
syspermute, 77       ZERO_SIZE, 87         to_void, 26       qpp::Gates, 89         trace, 78       ~Gates, 91         transpose, 78       CNOT, 94         tsallis, 78       CNOTba, 94         ubigint, 26       CTRL, 91         uniform, 79       CZ, 94         var, 79       expandout, 92         x2contfrac, 79       FRED, 94         qpp.h, 147       Fd, 92         qpp::Codes, 83       Gates, 91         ~Codes, 84       Id, 93         codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_STEANE, 84       S, 94         SEVEN_QUBIT_STEANE, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Zd, 93         _USTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		
to_void, 26 trace, 78 trace, 78 transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 83 ~Codes, 84 Codeword, 85 FIVE_QUBIT, 84 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 84 qpp::Exception, 85 _construct_exception_msg, 88 _msg, 89 _msg, 89 _msg, 89 _uyer, 89 _USTOM_EXCEPTION, 88 DIMS_INVALID, 87 CNOT ba, 94 Internal::Singleton < const Gates >, 94 Internal::Singleton < const Gates , 91 Int		
trace, 78 transpose, 78 transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 83	• •	
transpose, 78 tsallis, 78 ubigint, 26 uniform, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 83		
tsallis, 78 ubigint, 26 ubigint, 26 uniform, 79 var, 79 var, 79 x2contfrac, 79 qpp.h, 147 qpp::Codes, 83		
ubigint, 26 uniform, 79 var, 79 var, 79 x2contfrac, 79  qpp.h, 147 qpp::Codes, 83	•	
uniform, 79       CZ, 94         var, 79       expandout, 92         x2contfrac, 79       FRED, 94         qpp.h, 147       Fd, 92         qpp::Codes, 83       Gates, 91         ~Codes, 84       H, 94         Codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		
var, 79       expandout, 92         x2contfrac, 79       FRED, 94         qpp.h, 147       Fd, 92         qpp::Codes, 83       Gates, 91         ~Codes, 84       H, 94         Codes, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton< const Gates >, 94         internal::Singleton< const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96	. T	
x2contfrac, 79 qpp.h, 147		
qpp.h, 147       Fd, 92         qpp::Codes, 83       Gates, 91         ~Codes, 84       H, 94         Codes, 84       Id, 93         codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Gates >, 94       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		•
qpp::Codes, 83       Gates, 91         ~Codes, 84       H, 94         Codes, 84       Id, 93         codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		
Codes, 84 Codes, 84 Codes, 84 Codeword, 85 FIVE_QUBIT, 84 internal::Singleton < const Codes >, 85 NINE_QUBIT_SHOR, 84 SEVEN_QUBIT_STEANE, 84 Type, 84 Type, 84 Type, 85construct_exception_msg, 88custom, 89msg, 89type, 89where, 89 CUSTOM_EXCEPTION, 88 DIMS_INVALID, 87 Id2, 93 Id2, 94 Id2, 94 Id2, 94 Id2, 94 Internal::Singleton < const Gates >, 94 Internal::Singleton < const Gates >, 94 Id2, 94 Internal::Singleton < const Gates >, 94 Id2, 94 Id2		
Codes, 84       Id, 93         codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		,
codeword, 85       Id2, 94         FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		
FIVE_QUBIT, 84       internal::Singleton < const Gates >, 94         internal::Singleton < const Codes >, 85       Rn, 93         NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ~IDisplay, 96		
internal::Singleton < const Codes > , 85  NINE_QUBIT_SHOR, 84  SEVEN_QUBIT_STEANE, 84  Type, 84  qpp::Exception, 85  _construct_exception_msg, 88  _custom, 89  _msg, 89  _type, 89  _where, 89  CUSTOM_EXCEPTION, 88  NINE_QUBIT_SHOR, 84  S, 94  SWAP, 94  T, 94  TOF, 94  X, 95  Xd, 93  Y, 95  Z, 95  Zd, 93  Qpp::IDisplay, 95  □IDisplay, 96		
NINE_QUBIT_SHOR, 84       S, 94         SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ∼IDisplay, 96	_ · · ·	_
SEVEN_QUBIT_STEANE, 84       SWAP, 94         Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ∼IDisplay, 96		
Type, 84       T, 94         qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ∼IDisplay, 96		
qpp::Exception, 85       TOF, 94         _construct_exception_msg, 88       X, 95         _custom, 89       Xd, 93         _msg, 89       Y, 95         _type, 89       Z, 95         _where, 89       Zd, 93         CUSTOM_EXCEPTION, 88       qpp::IDisplay, 95         DIMS_INVALID, 87       ∼IDisplay, 96		
construct_exception_msg, 88		
_custom, 89		
_msg, 89 Y, 95 _type, 89 Z, 95 _where, 89 Zd, 93 CUSTOM_EXCEPTION, 88 qpp::IDisplay, 95 DIMS_INVALID, 87 ∼IDisplay, 96	· <del>-</del>	
_type, 89		
_where, 89 Zd, 93 CUSTOM_EXCEPTION, 88 qpp::IDisplay, 95 DIMS_INVALID, 87 ~IDisplay, 96		
CUSTOM_EXCEPTION, 88 qpp::IDisplay, 95 DIMS_INVALID, 87 ~IDisplay, 96	_ · ·	
DIMS_INVALID, 87 ~IDisplay, 96		
DIIVIS_IVIISIVIATUR_UVEUTUR, o/ UISPIAY, 9/		· ·
		αιορια <b>y</b> , στ

IDisplay, 96	_check_dims_match_mat, 82
operator<<, 97	_check_dims_match_rvect, 82
operator=, 97	_check_eq_dims, 82
qpp::Init, 97	_check_matching_sizes, 82
∼Init, 98	_check_nonzero_size, 82
Init, 98	_check_perm, 82
internal::Singleton< const Init >, 98	_check_qubit_cvector, 82
qpp::RandomDevices, 111	_check_qubit_matrix, 82
_rd, 112	_check_qubit_rvector, 82
_rng, 112	_check_qubit_vector, 82
~RandomDevices, 112	check rvector, 82
internal::Singleton < RandomDevices >, 112	_check_square_mat, 82
RandomDevices, 112	_check_subsys_match_dims, 82
qpp::States, 114	_check_vector, 82
∼States, 116	_dirsum2, 82
b00, 116	_kron2, 82
b01, 116	_multiidx2n, 82
b10, 117	_n2multiidx, 82
b11, 117	variadic_vector_emplace, 82
GHZ, 117	qpp::internal::IOManipEigen, 99
internal::Singleton< const States >, 116	_А, 100
pGHZ, 117	_A, 100 _chop, 100
•	
pW, 117	display, 100
pb00, 117	IOManipEigen, 100
pb01, 117	qpp::internal::IOManipPointer
pb10, 117	_end, 102
pb11, 117	_n, 102
px0, 117	_p, 102
px1, 117	_separator, 102
py0, 117	_start, 102
py1, 118	display, 102
pz0, 118	IOManipPointer, 101
pz1, 118	operator=, 102
States, 116	qpp::internal::IOManipPointer< PointerType >, 100
W, 118	qpp::internal::IOManipRange
x0, 118	_end, 104
x1, 118	_first, 104
y0, 118	_last, 104
y1, 118	_separator, 104
z0, 118	start, 104
z1, 118	display, 104
qpp::Timer	IOManipRange, 103
_end, 122	operator=, 104
start, 122	qpp::internal::IOManipRange< InputIterator >, 102
$\sim$ Timer, 120	qpp::internal::Singleton
display, 121	~Singleton, 114
get_duration, 121	get_instance, 114
operator=, 121	get_thread_local_instance, 114
tic, 121	operator=, 114
tics, 121	Singleton, 114
Timer, 120	qpp::internal::Singleton < T >, 113
toc, 122	qpp::is_complex < std::complex < T > >, 105
qpp::Timer< T, CLOCK_T >, 119	app::is_complex< T >, 104
qpp::experimental, 80	qpp::is_iterable< T, to_void< decltype(std::declval< T
qpp::internal, 80	>().begin()), decltype(std::declval< T >().←
_check_cvector, 81	end()), typename T::value_type > >, 107
_check_dims, 81	qpp::is_iterable < T, typename >, 106
_check_dims_match_cvect, 81	qpp::is_matrix_expression< Derived >, 109

qpp::is_matrix_expression< typename Eigen::Matrix↔	schmidtprobs
Base< Derived > >, 110	qpp, 74
Base Control > 7, 110	sigma
rand	qpp, 74
qpp, 64, 66	Singleton
randH	qpp::internal::Singleton, 114
qpp, 67	sinm
randU	qpp, 74
qpp, 70	spectralpowm
randV	qpp, <b>75</b>
qpp, 70	sqrtm
randidx	qpp, 75
qpp, 67	States
randket	qpp::States, 116
qpp, 67	statistics.h, 150
randkraus	sum
qpp, 68 randn	qpp, 75, 76
qpp, 68, 69	super2choi
random.h, 149	qpp, 76
RandomDevices	svals
qpp::RandomDevices, 112	qpp, 76
randperm	svd
qpp, 69	qpp, 76 svdU
randrho	
qpp, 69	qpp, 77 svdV
renyi	
qpp, 70	qpp, 77 syspermute
reshape	qpp, 77
qpp, 71	чрр, 77
rho2bloch	Т
qpp, 71	qpp::Gates, 94
rho2pure	TOF
qpp, 71	qpp::Gates, 94
Rn	TYPE_MISMATCH
qpp::Gates, 93	qpp::Exception, 88
	tic
S	qpp::Timer, 121
qpp::Gates, 94	tics
SEVEN_QUBIT_STEANE	qpp::Timer, 121
qpp::Codes, 84	Timer
SIZE_MISMATCH	qpp::Timer, 120
qpp::Exception, 88	to_void
SUBSYS_MISMATCH_DIMS	qpp, <mark>26</mark>
qpp::Exception, 87	toc
SWAP	qpp::Timer, 122
qpp::Gates, 94	trace
save	qpp, 78
qpp, 72	traits.h, 151
saveMATLABmatrix	transpose
qpp, 72 schatten	qpp, 78
	tsallis
qpp, 73 schmidtA	qpp, 78
qpp, 73	Type
schmidtB	qpp::Codes, 84
qpp, 73	qpp::Exception, 87
schmidtcoeffs	types.h, 152
qpp, 73	UNDEFINED TYPE
APP, 10	5.15E1 INED_1 11 E

```
qpp::Exception, 88
UNKNOWN_EXCEPTION
    qpp::Exception, 87
ubigint
    qpp, 26
uniform
    qpp, 79
var
    qpp, 79
variadic_vector_emplace
    qpp::internal, 82
W
    qpp::States, 118
what
    qpp::Exception, 88
Χ
    qpp::Gates, 95
х0
    qpp::States, 118
x1
    qpp::States, 118
x2contfrac
    qpp, 79
Xd
    qpp::Gates, 93
Υ
    qpp::Gates, 95
y0
    qpp::States, 118
у1
    qpp::States, 118
Ζ
    qpp::Gates, 95
z0
    qpp::States, 118
z1
    qpp::States, 118
ZERO_SIZE
    qpp::Exception, 87
Zd
    qpp::Gates, 93
```