Quantum++ v1.0.0-devel

Generated by Doxygen 1.8.10

Fri Oct 28 2016 18:49:03

Contents

1	Qua	ntum++			1			
2	Nam	espace	ace Index					
	2.1	Names	pace List		5			
3	Hier	archical	Index		7			
	3.1	Class I	Hierarchy		7			
4	Clas	s Index			9			
	4.1	Class I	_ist		9			
5	File	Index			11			
	5.1	File Lis	t		11			
6	Nam	nespace	Documer	ntation	13			
	6.1	qpp Na	ımespace	Reference	13			
		6.1.1	Detailed	Description	25			
		6.1.2	Typedef I	Documentation	25			
			6.1.2.1	bigint	25			
			6.1.2.2	bra	25			
			6.1.2.3	cmat	25			
			6.1.2.4	cplx	25			
			6.1.2.5	dmat	26			
			6.1.2.6	dyn_col_vect	26			
			6.1.2.7	dyn_mat	26			
			6.1.2.8	dyn_row_vect	26			
			6.1.2.9	$idx \ \ldots \ldots \ldots \ldots \ldots$	26			
			6.1.2.10	ket	26			
			6.1.2.11	to_void	26			
		6.1.3	Function	Documentation	26			
			6.1.3.1	absm(const Eigen::MatrixBase< Derived > &A)	26			
			6.1.3.2	abssq(InputIterator first, InputIterator last)	27			

iv CONTENTS

6.1.3.3	abssq(const Container &c, typename std::enable_if< is_iterable< Container > \cdot	
	::value >::type *=nullptr)	27
6.1.3.4	abssq(const Eigen::MatrixBase< Derived > &A)	27
6.1.3.5	adjoint(const Eigen::MatrixBase< Derived > &A)	27
6.1.3.6	anticomm(const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)	28
6.1.3.7	apply(const Eigen::MatrixBase< Derived1 $>$ &state, const Eigen::MatrixBase< Derived2 $>$ &A, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	28
6.1.3.8	apply(const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)	28
6.1.3.9	${\sf apply}({\sf const\ Eigen::MatrixBase} < {\sf Derived} > {\sf \&A}, {\sf const\ std::vector} < {\sf cmat} > {\sf \&Ks})$	29
6.1.3.10	apply(const Eigen::MatrixBase< Derived $>$ &A, const std::vector< cmat $>$ &Ks, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	29
6.1.3.11	apply(const Eigen::MatrixBase< Derived $>$ &A, const std::vector< cmat $>$ &Ks, const std::vector< idx $>$ &subsys, idx d=2)	29
6.1.3.12	applyCTRL(const Eigen::MatrixBase< Derived1 > &state, const Eigen::Matrix↔ Base< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const std::vector< idx > &dims)	30
6.1.3.13	applyCTRL(const Eigen::MatrixBase< Derived1 > &state, const Eigen::Matrix Base< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx d=2)	30
6.1.3.14	avg(const std::vector< double > &prob, const Container &X, typename std ← ::enable_if< is_iterable< Container >::value >::type ∗=nullptr)	31
6.1.3.15	bloch2rho(const std::vector< double > &r)	31
6.1.3.16	choi2kraus(const cmat &A)	31
6.1.3.17	choi2super(const cmat &A)	32
6.1.3.18	$\label{local_comm} \mbox{comm(const Eigen::MatrixBase} < \mbox{Derived1} > \&\mbox{A, const Eigen::MatrixBase} < \mbox{Derived2} > \&\mbox{B)} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	32
6.1.3.19	$complement(std::vector < T > subsys, idx \ N) \ \ \ldots \ \ \ldots \ \ \ldots \ \ \ldots$	32
6.1.3.20	$\label{eq:compperm} \mbox{compperm(const std::vector< idx > \&perm, const std::vector< idx > \σ)} .$	33
6.1.3.21	${\sf concurrence}({\sf const} \ {\sf Eigen::MatrixBase} {< } \ {\sf Derived} > \& {\sf A}) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	34
6.1.3.22	$conjugate (const\ Eigen:: Matrix Base < Derived > \&A) \ \ . \ \ . \ \ . \ \ . \ \ .$	34
6.1.3.23	$contfrac2x(const\ std::vector< int>\&cf,\ idx\ N) \qquad . \qquad .$	34
6.1.3.24	${\sf contfrac2x}({\sf const\ std::vector} < {\sf int} > \&{\sf cf})\ . \ . \ . \ . \ . \ . \ . \ . \ . \ .$	35
6.1.3.25	cor(const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)	35
6.1.3.26	cosm(const Eigen::MatrixBase< Derived > &A)	35
6.1.3.27	cov(const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)	35
6.1.3.28	cwise(const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const typename Derived::Scalar &))	36
6.1.3.29	det(const Eigen::MatrixBase< Derived > &A)	36
6.1.3.30	dirsum(const T &head)	36
6.1.3.31	dirsum(const T &head, const Args &tail)	37

CONTENTS

6.1.3.32	dirsum(const std::vector< Derived > &As)	37
6.1.3.33	dirsum(const std::initializer_list< Derived > &As)	37
6.1.3.34	dirsumpow(const Eigen::MatrixBase< Derived > &A, idx n)	38
6.1.3.35	$\label{eq:disp} \textit{disp}(\textit{const Eigen::MatrixBase} < \textit{Derived} > \&A, \textit{double chop=qpp::chop}) \ \ . \ \ . \ \ .$	38
6.1.3.36	disp(cplx z, double chop=qpp::chop)	38
6.1.3.37	disp(InputIterator first, InputIterator last, const std::string &separator, const std⇔ ::string &start=""["", const std::string &end=""]"")	38
6.1.3.38	disp(const Container &c, const std::string &separator, const std::string &start=""["", const std::string &end=""]"", typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)	39
6.1.3.39	disp(const PointerType *p, idx N, const std::string &separator, const std::string &start=""["", const std::string &end=""]"")	39
6.1.3.40	egcd(bigint a, bigint b)	39
6.1.3.41	eig(const Eigen::MatrixBase< Derived > &A)	40
6.1.3.42	entanglement(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	40
6.1.3.43	entanglement(const Eigen::MatrixBase< Derived > &A, idx d=2)	40
6.1.3.44	entropy(const Eigen::MatrixBase< Derived > &A)	41
6.1.3.45	entropy(const std::vector< double > &prob)	41
6.1.3.46	evals(const Eigen::MatrixBase< Derived > &A)	41
6.1.3.47	evects(const Eigen::MatrixBase< Derived > &A)	41
6.1.3.48	expm(const Eigen::MatrixBase< Derived > &A)	42
6.1.3.49	factors(bigint a)	42
6.1.3.50	$funm(const\ Eigen::MatrixBase < Derived > \&A,\ cplx(*f)(const\ cplx\ \&))\ .\ .\ .\ .\ .$	42
6.1.3.51	gcd(bigint a, bigint b)	42
6.1.3.52	gcd(const std::vector< bigint > &as)	43
6.1.3.53	gconcurrence(const Eigen::MatrixBase< Derived > &A)	43
6.1.3.54	grams(const std::vector< Derived > &As)	43
6.1.3.55	grams(const std::initializer_list< Derived > &As)	44
6.1.3.56	grams(const Eigen::MatrixBase< Derived > &A)	44
6.1.3.57	heig(const Eigen::MatrixBase< Derived > &A)	44
6.1.3.58	hevals(const Eigen::MatrixBase< Derived > &A)	45
6.1.3.59	hevects(const Eigen::MatrixBase< Derived > &A)	45
6.1.3.60	$inverse (const\ Eigen::MatrixBase < Derived > \&A) \\ \hspace*{0.5cm} \ldots \\ \hspace*{0.5cm} \ldots \\ \hspace*{0.5cm} \ldots$	45
6.1.3.61	$invperm(const\ std::vector < idx > \&perm)\ .\ .\ .\ .\ .\ .$	45
6.1.3.62	$\label{local-prop} $$ ip(const\ Eigen::MatrixBase < Derived > \φ,\ const\ Eigen::MatrixBase < Derived > \ψ,\ const\ std::vector < idx > \&dims) \ . \ . \ .$	46
6.1.3.63	ip(const Eigen::MatrixBase< Derived > φ, const Eigen::MatrixBase< Derived > ψ, const std::vector< idx > &subsys, idx d=2)	47
6.1.3.64	isprime(bigint p, idx k=80)	47
6.1.3.65	kraus2choi(const std::vector< cmat $>$ &Ks)	47
6.1.3.66	kraus2super(const std::vector< cmat > &Ks)	48

vi CONTENTS

6.1.3.67	kron(const T &head)	48
6.1.3.68	kron(const T &head, const Args &tail)	48
6.1.3.69	$kron(const\ std::vector < Derived > \&As) \qquad . \qquad .$	49
6.1.3.70	$kron(const\ std::initializer_list < Derived > \&As) \ \dots \ \dots \ \dots \ \dots$	49
6.1.3.71	$kronpow(const\ Eigen::MatrixBase < Derived > \&A,\ idx\ n) \\ . \\ .$	49
6.1.3.72	lcm(bigint a, bigint b)	50
6.1.3.73	${\sf lcm}({\sf const\ std}::{\sf vector}<{\sf bigint}> \&{\sf as}) $	50
6.1.3.74	load(const std::string &fname)	50
6.1.3.75	loadMATLABmatrix(const std::string &, const std::string &)	51
6.1.3.76	loadMATLABmatrix(const std::string &mat_file, const std::string &var_name)	51
6.1.3.77	loadMATLABmatrix(const std::string &mat_file, const std::string &var_name)	51
6.1.3.78	logdet(const Eigen::MatrixBase< Derived > &A)	52
6.1.3.79	${\sf logm}({\sf const}\ {\sf Eigen} {::} {\sf MatrixBase} {<}\ {\sf Derived} > \& {\sf A})\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .$	52
6.1.3.80	lognegativity(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	52
6.1.3.81	$lognegativity (const\ Eigen::MatrixBase < Derived > \&A,\ idx\ d=2)\ \dots \dots \dots$	52
6.1.3.82	marginalX(const dmat &probXY)	53
6.1.3.83	marginalY(const dmat &probXY)	53
6.1.3.84	measure(const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)	53
6.1.3.85	$\label{lem:measure} \mbox{measure(const Eigen::MatrixBase< Derived} > \&\mbox{A, const std::initializer_list} < \\ \mbox{cmat} > \&\mbox{Ks)} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	53
6.1.3.86	$measure (const\ Eigen:: Matrix Base < Derived > \&A,\ const\ cmat\ \&U) \ \ . \ \ . \ \ . \ \ .$	54
6.1.3.87	$\label{lem:measure} \mbox{measure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const std::vector} < \mbox{cmat} > \&\mbox{Ks, const std::vector} < \mbox{idx} > \&\mbox{subsys, const std::vector} < \mbox{idx} > \&\mbox{dims}) \ \ . \ . \ . \ . \ . \ . \ . \ . \ . $	54
6.1.3.88	$\label{lem:measure} \begin{tabular}{ll} measure(const Eigen::MatrixBase< Derived > \&A, const std::initializer_list< \\ cmat > \&Ks, const std::vector< idx > \&subsys, const std::vector< idx > \&dims) \\ \end{tabular}$	54
6.1.3.89	measure(const_Eigen::MatrixBase< Derived > &A, const_std::vector< cmat > &Ks, const_std::vector< idx > &subsys, idx d=2)	55
6.1.3.90	measure(const_Eigen::MatrixBase< Derived > &A, const_std::initializer_list< cmat > &Ks, const_std::vector< idx > &subsys, idx d=2)	55
6.1.3.91	$\label{eq:measure} \begin{tabular}{ll} measure(const Eigen::MatrixBase< Derived > &A, const cmat &V, const std $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$	56
6.1.3.92	$\label{lem:measure} \begin{tabular}{ll} measure(const Eigen::MatrixBase< Derived > &A, const cmat &V, const std \leftarrow ::vector< idx > &subsys, idx d=2)$	56
6.1.3.93	$\label{lem:measure_seq} \mbox{measure_seq(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, std::vector} < \mbox{idx} > \mbox{subsys, std::vector} < \mbox{idx} > \mbox{dims}) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	57
6.1.3.94	$\label{eq:measure_seq} \begin{split} \text{measure_seq(const Eigen::MatrixBase} < \text{Derived} > \&A, \ \text{std::vector} < \ \text{idx} > \text{subsys, idx d=2)} \\ & \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ \end{split}$	57
6.1.3.95	$\label{eq:mket} \mbox{mket(const std::vector< idx > \&mask, const std::vector< idx > \&dims)} \ \ . \ \ . \ \ . \ \ .$	58
6.1.3.96	$mket(const\ std::vector < idx > \&mask,\ idx\ d=2)\ \dots \dots \dots \dots$	58
6.1.3.97	modinv(bigint a, bigint p)	58
6.1.3.98	modmul(bigint a, bigint b, bigint p)	59

CONTENTS vii

6.1.3.99	modpow(bigint a, bigint n, bigint p)	59
6.1.3.100	mprj(const std::vector< idx > &mask, const std::vector< idx > &dims) \dots	59
6.1.3.101	mprj(const std::vector< idx > &mask, idx d=2)	60
6.1.3.102	$\label{eq:multiidx2n} \text{multiidx2n} \\ \text{(const std::vector< idx > \&midx, const std::vector< idx > \&dims)} \; . \; \; . \; \; .$	60
6.1.3.103	$n2 \\ \text{multiidx} \\ (\text{idx n, const std::vector} < \\ \text{idx} > \\ \\ \text{\&dims}) \\ \dots \\ \dots \\ \dots \\ \dots$	60
6.1.3.104	negativity(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	61
6.1.3.105	negativity(const Eigen::MatrixBase< Derived $>$ &A, idx d=2)	61
6.1.3.106	norm(const Eigen::MatrixBase< Derived > &A)	61
6.1.3.107	omega(idx D)	61
6.1.3.108	operator"""_i(unsigned long long int x) noexcept	62
6.1.3.109	operator"""_i(long double x) noexcept	62
6.1.3.110	$powm(const\ Eigen::MatrixBase < Derived > \&A,\ idx\ n)\ \ .\ \ .\ \ .\ \ .\ \ .$	62
6.1.3.111	$\label{eq:prj} \text{prj(const Eigen::MatrixBase} < \text{Derived} > \& A) \ \dots \dots \dots \dots \dots \dots \dots$	62
6.1.3.112	prod(const Eigen::MatrixBase< Derived > &A)	63
6.1.3.113	prod(InputIterator first, InputIterator last)	63
6.1.3.114	prod(const Container &c, typename std::enable_if< is_iterable< Container >← ::value >::type *=nullptr)	63
6.1.3.115	$\label{eq:ptrace} $	63
6.1.3.116	ptrace(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)	64
6.1.3.117	$ptrace1(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&dims)$	64
6.1.3.118	ptrace1(const Eigen::MatrixBase< Derived > &A, idx d=2)	65
6.1.3.119	$ptrace2(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&dims)$	65
6.1.3.120	ptrace2(const Eigen::MatrixBase< Derived > &A, idx d=2)	65
6.1.3.121	ptranspose(const Eigen::MatrixBase< Derived $>$ &A, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	66
6.1.3.122	ptranspose(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)	66
6.1.3.123	$\label{lem:matrixBase} $$\operatorname{Derived} > \&A, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{subsysA}, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{subsysB}, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{dims})$$	66
6.1.3.124	$\label{lem:matrixBase} $$\operatorname{Derived} > \&A, \ \operatorname{const} \ \operatorname{std}:: \ \operatorname{vector} < \ \operatorname{idx} > \& \operatorname{subsysA}, \ \operatorname{const} \ \operatorname{std}:: \ \operatorname{vector} < \ \operatorname{idx} > \& \operatorname{subsysB}, \ \operatorname{idx} \ \operatorname{d=2}) \ \ldots \ \ldots \ \ldots \ \ldots \ .$	67
6.1.3.125	rand(double a, double b)	67
6.1.3.126	rand(bigint a, bigint b)	67
6.1.3.127	rand(idx rows, idx cols, double a=0, double b=1)	68
6.1.3.128	rand(idx rows, idx cols, double a, double b)	68
6.1.3.129	rand(idx rows, idx cols, double a, double b)	68
6.1.3.130	randH(idx D=2)	68
6.1.3.131	randidx(idx a=std::numeric_limits< idx >::min(), idx b=std::numeric_limits< idx >::max())	69
6.1.3.132	randket(idx D=2)	69

viii CONTENTS

6.1.3.133	randkraus(idx N, idx D=2)	69
6.1.3.134	randn(idx rows, idx cols, double mean=0, double sigma=1)	69
6.1.3.135	randn(idx rows, idx cols, double mean, double sigma)	70
6.1.3.136	randn(idx rows, idx cols, double mean, double sigma)	70
6.1.3.137	randn(double mean=0, double sigma=1)	70
6.1.3.138	$randperm(idx\ N)\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$	71
6.1.3.139	randprime(bigint a, bigint b, idx N=1000)	71
6.1.3.140	$randprob(idx\;N) $	71
6.1.3.141	randrho(idx D=2)	71
6.1.3.142	randU(idx D=2)	72
6.1.3.143	randV(idx Din, idx Dout)	72
6.1.3.144	$renyi(const\ Eigen::MatrixBase < Derived > \&A,\ double\ alpha) \ \ . \ \ . \ \ . \ \ .$	72
6.1.3.145	$\label{eq:const_std} \textit{renyi}(\textit{const std::vector} < \textit{double} > \&\textit{prob}, \textit{double alpha}) \dots \dots \dots \dots$	72
6.1.3.146	${\sf reshape}({\sf const}\ {\sf Eigen::MatrixBase} {< }\ {\sf Derived} > {\sf \&A}, \ {\sf idx}\ {\sf rows}, \ {\sf idx}\ {\sf cols}) \ \ \ldots \ \ \ldots$	73
6.1.3.147	$\label{loch} \mbox{rho2bloch(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ \ . \ \ \ \ . \ \ \ . \ \ \ . \ \ . \ \ . \ \ . \ \ \ . \ \ \ \ . \ \ \ . \ \ \ \ \ . \ \ \ \ . \ \ . \ \ \ . \ \ \ \ \ \ \ . \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	73
6.1.3.148	$\label{local_pure} \mbox{rho2pure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) $	73
6.1.3.149	$save(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::string\ \&fname) . . .$	74
6.1.3.150	$save MATLAB matrix (const\ Eigen:: Matrix Base < Derived > \&,\ const\ std:: string\ \&,\ const\ std:: string\ \&)\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\$	74
6.1.3.151	saveMATLABmatrix(const_Eigen::MatrixBase< dmat > &A, const_std::string &mat_file, const_std::string &var_name, const_std::string &mode)	74
6.1.3.152	saveMATLABmatrix(const_Eigen::MatrixBase< cmat > &A, const_std::string &mat_file, const_std::string &var_name, const_std::string &mode)	74
6.1.3.153	schatten(const Eigen::MatrixBase< Derived > &A, double p)	75
6.1.3.154	schmidtA(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	75
6.1.3.155	schmidtA(const Eigen::MatrixBase< Derived > &A, idx d=2)	75
6.1.3.156	schmidtB(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	75
6.1.3.157	$schmidtB(const\ Eigen::MatrixBase < Derived > \&A,\ idx\ d=2)\ \dots \ \dots \ \dots$	76
6.1.3.158	schmidtcoeffs(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	76
6.1.3.159	schmidtcoeffs(const Eigen::MatrixBase< Derived > &A, idx d=2)	76
6.1.3.160	schmidtprobs(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	77
6.1.3.161	schmidtprobs(const Eigen::MatrixBase< Derived > &A, idx d=2)	77
6.1.3.162	sigma(const std::vector< double > &prob, const Container &X, typename std↔ ::enable_if< is_iterable< Container >::value >::type *=nullptr)	77
6.1.3.163	sinm(const Eigen::MatrixBase< Derived > &A)	78
6.1.3.164	spectralpowm(const Eigen::MatrixBase< Derived > &A, const cplx z)	78
6.1.3.165	sqrtm(const Eigen::MatrixBase< Derived > &A)	78
6.1.3.166	sum(const Eigen::MatrixBase< Derived > &A)	78

CONTENTS

	6.1.3.167	sum(InputIterator first, InputIterator last)	79
	6.1.3.168	$sum(const\ Container\ \&c,\ typename\ std::enable_if<\ is_iterable<\ Container\ > \hookrightarrow ::value\ >::type\ *=nullptr)\ .$	79
	6.1.3.169	super2choi(const cmat &A)	79
	6.1.3.170	svals(const Eigen::MatrixBase< Derived > &A)	79
	6.1.3.171	svd(const Eigen::MatrixBase< Derived > &A)	80
	6.1.3.172	svdU(const Eigen::MatrixBase< Derived > &A)	80
	6.1.3.173	svdV(const Eigen::MatrixBase< Derived > &A)	80
	6.1.3.174	$\label{lem:syspermute} syspermute (const \ Eigen::MatrixBase < Derived > \&A, \ const \ std::vector < idx > \&perm, \ const \ std::vector < idx > \&dims) \\ \\ \ldots \\ \ldots \\ \ldots$	80
	6.1.3.175	syspermute(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)	81
	6.1.3.176	$trace(const\ Eigen::MatrixBase < Derived > \&A)\ .\ .\ .\ .\ .\ .$	81
	6.1.3.177	$transpose(const\ Eigen::MatrixBase < Derived > \&A) \ \dots \dots \dots \dots \dots$	81
	6.1.3.178	$tsallis(const\ Eigen::MatrixBase < Derived > \&A,\ double\ q) \ \ . \ \ . \ \ . \ \ .$	81
	6.1.3.179	$tsallis(const\ std::vector < double > \&prob,\ double\ q) \ \ \ldots \ \ \ldots \ \ \ldots \ \ .$	82
	6.1.3.180	$uniform(idx\;N) \dots $	82
	6.1.3.181	$\label{eq:const_std::vector} $$ \ \ \ \ \ \ \ \ \ \ \ \ $	82
	6.1.3.182	x2contfrac(double x, idx N, idx cut=1e5)	82
6.1.4	Variable [Documentation	83
	6.1.4.1	chop	83
	6.1.4.2	ee	83
	6.1.4.3	eps	83
	6.1.4.4	infty	83
	6.1.4.5	maxn	83
	6.1.4.6	pi	83
qpp::ex	kperimenta	Namespace Reference	83
6.2.1	Detailed I	Description	83
qpp::in	ternal Nam	nespace Reference	84
6.3.1	Detailed I	Description	85
6.3.2	Function	Documentation	85
	6.3.2.1	check_cvector(const Eigen::MatrixBase< Derived > &A)	85
	6.3.2.2	$\label{eq:check_dims} \mbox{check_dims}(\mbox{const std::vector} < \mbox{idx} > \mbox{\&dims}) \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . $	85
	6.3.2.3	$\label{lem:check_dims_match_cvect} $	85
	6.3.2.4	$\label{lem:check_dims_match_mat} $$ \check_dims_match_mat(const\ std::vector<\ idx\ > \&dims,\ const\ Eigen::Matrix{\longleftrightarrow} $$ Base<\ Derived\ > \&A) \ .$	85
	6.3.2.5	check_dims_match_rvect(const std::vector< idx > &dims, const Eigen::Matrix← Base< Derived > &A)	85
	6.3.2.6	check_eq_dims(const std::vector< idx > &dims, idx dim) noexcept	85
	6.3.2.7	check_matching_sizes(const T1 &lhs, const T2 &rhs) noexcept	85
	qpp::ex 6.2.1 qpp::in 6.3.1	6.1.3.168 6.1.3.169 6.1.3.170 6.1.3.171 6.1.3.173 6.1.3.175 6.1.3.175 6.1.3.176 6.1.3.177 6.1.3.180 6.1.3.181 6.1.3.182 6.1.4 Variable I 6.1.4.1 6.1.4.2 6.1.4.3 6.1.4.4 6.1.4.5 6.1.4.6 qpp::experimenta 6.2.1 Detailed I qpp::internal Nam 6.3.1 Detailed I qpp::internal Nam 6.3.2 Function 6.3.2.1 6.3.2.2 6.3.2.3 6.3.2.4 6.3.2.5 6.3.2.6	6.1.3.168 sum(const Container &c, typename std::enable_lif< is_iterable< Container >== :value >::type ==nuliptr) 6.1.3.169 super2choi(const cmat &A) 6.1.3.170 svals(const Eigen::MatrixBase< Derived > &A) 6.1.3.171 svd(const Eigen::MatrixBase< Derived > &A) 6.1.3.173 svdV(const Eigen::MatrixBase< Derived > &A) 6.1.3.173 svdV(const Eigen::MatrixBase< Derived > &A) 6.1.3.173 svdV(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2) 6.1.3.175 syspermute(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2) 6.1.3.176 trace(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2) 6.1.3.178 tsallis(const Eigen::MatrixBase< Derived > &A, double q) 6.1.3.178 tsallis(const Eigen::MatrixBase< Derived > &A, double q) 6.1.3.180 uniform(idx N) 6.1.3.180 uniform(idx N) 6.1.3.181 var(const std::vector< double > &prob, const Container &X, typename std-:enable_if< is_iterable< Container >::value >::type *=nuliptr) 6.1.3.182 x2contfrac(double x, idx N, idx cut=1e5) 6.1.4.1 chop 6.1.4.2 ee 6.1.4.3 eps 6.1.4.4 infty. 6.1.4.5 maxn 6.1.4.6 pi qpp::experimental Namespace Reference 6.2.1 Detailed Description qpp::internal Namespace Reference 6.3.1 check_dims_match_cvect(const std::vector< idx > &dims, const Eigen::Matrix Base 6.3.2.3 check_dims_match_cvect(const std::vector< idx > &dims, const Eigen::Matrix Base 6.3.2.4 check_dims_match_mat(const std::vector< idx > &dims, const Eigen::Matrix Base 6.3.2.5 check_dims_match_mat(const std::vector< idx > &dims, const Eigen::Matrix Base 6.3.2.6 check_eq_dims(const std::vector< idx > &dims, const Eigen::Matrix Base 6.3.2.6 check_eq_dims(const std::vector< idx > &dims, const Eigen::Matrix Base

CONTENTS

			6.3.2.8	check_nonzero_size(const T &x) noexcept	85
			6.3.2.9	$\label{eq:check_perm} \mbox{check_perm}(\mbox{const std::vector} < \mbox{idx} > \mbox{\&perm}) $	85
			6.3.2.10	$\label{lem:check_qubit_cvector} \mbox{check_qubit_cvector} (\mbox{const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) \ \mbox{noexcept} . . .$	85
			6.3.2.11	$\label{lem:check_qubit_matrix} \textbf{const Eigen::} \textbf{MatrixBase} < \textbf{Derived} > \textbf{\&A} \textbf{)} \ \textbf{noexcept} \ . \ . \ . \ .$	85
			6.3.2.12	$\label{lem:check_qubit_rvector} \mbox{check_qubit_rvector} (\mbox{const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) \ \mbox{noexcept} . . .$	85
			6.3.2.13	$\label{lem:check_qubit_vector} \mbox{check_qubit_vector} (\mbox{const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) \ \mbox{noexcept} \ \ . \ \ . \ \ .$	85
			6.3.2.14	check_rvector(const Eigen::MatrixBase< Derived > &A)	85
			6.3.2.15	check_square_mat(const Eigen::MatrixBase< Derived > &A)	85
			6.3.2.16	$\label{lem:check_subsys} $$ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	85
			6.3.2.17	$\label{local_check_vector} \mbox{check_vector(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) $	85
			6.3.2.18	dirsum2(const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)	86
			6.3.2.19	get_dim_subsys(idx sz, idx N)	86
			6.3.2.20	get_num_subsys(idx sz, idx d)	86
			6.3.2.21	$kron2 (const\ Eigen::MatrixBase < Derived1 > \&A,\ const\ Eigen::MatrixBase < Derived2 > \&B) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	86
			6.3.2.22	multiidx2n(const idx *const midx, idx numdims, const idx *const dims) noexcept	86
			6.3.2.23	n2multiidx(idx n, idx numdims, const idx $*$ const dims, idx $*$ result) noexcept	86
			6.3.2.24	$variadic_vector_emplace(std::vector < T > \&) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	86
			6.3.2.25	$variadic_vector_emplace(std::vector < T > \&v, First \&\&first, Args \&\&args) \ . \ . \ .$	86
7	Clas	s Docu	mentation		87
•	7.1			s Reference	87
		7.1.1		Description	88
		7.1.2		Enumeration Documentation	88
				Type	88
		7.1.3		tor & Destructor Documentation	88
			7.1.3.1	Codes()	88
			7.1.3.2	~Codes()=default	88
		7.1.4		Function Documentation	89
			7.1.4.1	codeword(Type type, idx i) const	89
		7.1.5		And Related Function Documentation	89
			7.1.5.1	internal::Singleton < const Codes >	89
	7.2	app::in	ternal::Dis	play_Impl_ Struct Reference	89
		7.2.1		Function Documentation	90
			7.2.1.1	display_impl_(const T &A, std::ostream &os, double chop=qpp::chop) const	90
	7.3	qpp::E		lass Reference	90
		7.3.1		Description	91
		7.3.2		Enumeration Documentation	91
		1.0.2	MCITIBEI	Enameration Documentation	•
		7.5.2	7.3.2.1	Type	91

CONTENTS xi

	7.3.3	Construc	ctor & Destructor Documentation	92
		7.3.3.1	Exception(const std::string &where, const Type &type)	92
		7.3.3.2	Exception(const std::string &where, const std::string &custom)	93
	7.3.4	Member	Function Documentation	93
		7.3.4.1	construct_exception_msg_()	93
		7.3.4.2	what() const noexcept override	93
	7.3.5	Member	Data Documentation	93
		7.3.5.1	custom	93
		7.3.5.2	msg	93
		7.3.5.3	type	93
		7.3.5.4	where	93
7.4	qpp::G	ates Class	s Reference	93
	7.4.1	Detailed	Description	96
	7.4.2	Construc	ctor & Destructor Documentation	96
		7.4.2.1	Gates()	96
		7.4.2.2	~Gates()=default	96
	7.4.3	Member	Function Documentation	96
		7.4.3.1	CTRL(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx N, idx d=2) const	96
		7.4.3.2	$\label{eq:const_expandout} \mbox{expandout(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, idx pos, const std::vector} < \mbox{idx} > \&\mbox{dims) const} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	96
		7.4.3.3	expandout(const Eigen::MatrixBase< Derived > &A, idx pos, const std ::initializer_list< idx > &dims) const	97
		7.4.3.4	expandout(const Eigen::MatrixBase< Derived > &A, idx pos, idx N, idx d=2) const	97
		7.4.3.5	Fd(idx D=2) const	98
		7.4.3.6	Id(idx D=2) const	98
		7.4.3.7	Rn(double theta, const std::vector< double > &n) const	98
		7.4.3.8	Xd(idx D=2) const	99
		7.4.3.9	Zd(idx D=2) const	99
	7.4.4	Friends A	And Related Function Documentation	99
		7.4.4.1	internal::Singleton < const Gates >	99
	7.4.5	Member	Data Documentation	99
		7.4.5.1	CNOT	99
		7.4.5.2	CNOTba	99
		7.4.5.3	CZ	99
		7.4.5.4	FRED	99
		7.4.5.5		100
		7.4.5.6		100
		7.4.5.7	S	100
		7.4.5.8		100

xii CONTENTS

		7.4.5.9	T	100
		7.4.5.10	TOF	100
		7.4.5.11	X	100
		7.4.5.12	Y	100
		7.4.5.13	Z	100
7.5	qpp::IC	Display Cla	ss Reference	100
	7.5.1	Detailed	Description	102
	7.5.2	Construc	tor & Destructor Documentation	102
		7.5.2.1	IDisplay()=default	102
		7.5.2.2	IDisplay(const IDisplay &)=default	102
		7.5.2.3	IDisplay(IDisplay &&)=default	102
		7.5.2.4	\sim IDisplay()=default	102
	7.5.3	Member	Function Documentation	102
		7.5.3.1	display(std::ostream &os) const =0	102
		7.5.3.2	operator=(const IDisplay &)=default	102
		7.5.3.3	operator=(IDisplay &&)=default	102
	7.5.4	Friends A	And Related Function Documentation	102
		7.5.4.1	operator<<	102
7.6	qpp::In	it Class Re	eference	103
	7.6.1	Detailed	Description	104
	7.6.2	Construc	tor & Destructor Documentation	104
		7.6.2.1	Init()	104
		7.6.2.2	~Init()	104
	7.6.3	Friends A	And Related Function Documentation	104
		7.6.3.1	internal::Singleton< const Init >	104
7.7	qpp::in	ternal::ION	ManipEigen Class Reference	104
	7.7.1	Construc	tor & Destructor Documentation	105
		7.7.1.1	IOManipEigen(const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)	105
		7.7.1.2	IOManipEigen(const cplx z, double chop=qpp::chop)	105
	7.7.2	Member	Function Documentation	105
		7.7.2.1	display(std::ostream &os) const override	105
	7.7.3	Member	Data Documentation	106
		7.7.3.1	A	106
		7.7.3.2	chop	106
7.8	qpp::in	ternal::ION	ManipPointer< PointerType > Class Template Reference	106
	7.8.1	Construc	tor & Destructor Documentation	107
		7.8.1.1	$IOManipPointer(const\ PointerType\ *p,\ idx\ N,\ const\ std::string\ \&separator,\ const\ std::string\ \&start=""""")\$	107
		7.8.1.2	IOManipPointer(const IOManipPointer &)=default	107
	7.8.2	Member	Function Documentation	107

CONTENTS xiii

		7.8.2.1	display(std::ostream &os) const override	107
		7.8.2.2	operator=(const IOManipPointer &)=default	107
	7.8.3	Member I	Data Documentation	107
		7.8.3.1	end	107
		7.8.3.2	$N_\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$	107
		7.8.3.3	p	107
		7.8.3.4	separator	108
		7.8.3.5	start	108
7.9	qpp::int	ternal::ION	ManipRange < InputIterator > Class Template Reference	108
	7.9.1	Construc	tor & Destructor Documentation	109
		7.9.1.1	IOManipRange(InputIterator first, InputIterator last, const std::string &separator, const std::string &start=""["", const std::string &end=""]"")	109
		7.9.1.2	IOManipRange(const IOManipRange &)=default	109
	7.9.2	Member I	Function Documentation	109
		7.9.2.1	display(std::ostream &os) const override	109
		7.9.2.2	operator=(const IOManipRange &)=default	109
	7.9.3	Member I	Data Documentation	109
		7.9.3.1	end	109
		7.9.3.2	first	109
		7.9.3.3	last	109
		7.9.3.4	separator	110
		7.9.3.5	start	110
7.10	qpp::is_	_complex<	T > Struct Template Reference	110
	7.10.1	Detailed I	Description	110
7.11	qpp::is_	_complex<	< std::complex $<$ T $>$ $>$ Struct Template Reference	111
	7.11.1	Detailed I	Description	111
7.12	qpp::is_	_iterable<	T, typename $>$ Struct Template Reference	112
	7.12.1	Detailed I	Description	112
7.13			T, to_void< decltype(std::declval< T >().begin()), decltype(std::declval< T ame T::value_type > > Struct Template Reference	113
	7.13.1	Detailed I	Description	114
7.14	qpp::is_	_matrix_ex	xpression < Derived > Struct Template Reference	114
	7.14.1	Detailed I	Description	115
7.15	qpp::Ra	andomDev	rices Class Reference	115
	7.15.1	Detailed I	Description	117
	7.15.2	Construc	tor & Destructor Documentation	117
		7.15.2.1	RandomDevices()	117
		7.15.2.2	\sim RandomDevices()=default	117
	7.15.3	Friends A	and Related Function Documentation	117
		7.15.3.1	internal::Singleton< RandomDevices >	117

XIV

	7.15.4	Member Data Documentation	17
		7.15.4.1 rd	17
		7.15.4.2 rng	17
7.16	qpp::int	ternal::Singleton< T > Class Template Reference	17
	7.16.1	Detailed Description	18
	7.16.2	Constructor & Destructor Documentation	18
		7.16.2.1 Singleton() noexcept=default	18
		7.16.2.2 Singleton(const Singleton &)=delete	18
		7.16.2.3 ~Singleton()=default	18
	7.16.3	Member Function Documentation	19
		$7.16.3.1 \text{get_instance() noexcept(std::is_nothrow_constructible} < T > ::value) \ . \ . \ . \ . \ . \ 1$	19
		$7.16.3.2 \text{get_thread_local_instance()} \ \ \text{noexcept(std::is_nothrow_constructible} < T > ::value) \ \ \textbf{1}$	19
		7.16.3.3 operator=(const Singleton &)=delete	19
7.17	qpp::St	tates Class Reference	19
	7.17.1	Detailed Description	21
	7.17.2	Constructor & Destructor Documentation	21
		7.17.2.1 States()	21
		7.17.2.2 ~States()=default	21
	7.17.3	Member Function Documentation	21
		7.17.3.1 jn(idx j, idx n, idx d=2) const	21
		7.17.3.2 mes(idx d=2) const	22
		7.17.3.3 minus(idx n) const	22
		7.17.3.4 one(idx n, idx d=2) const	22
		7.17.3.5 plus(idx n) const	22
		7.17.3.6 zero(idx n, idx d=2) const	23
	7.17.4	Friends And Related Function Documentation	23
		7.17.4.1 internal::Singleton < const States >	23
	7.17.5	Member Data Documentation	23
		7.17.5.1 b00	23
		7.17.5.2 b01	23
		7.17.5.3 b10	23
		7.17.5.4 b11	23
		7.17.5.5 GHZ	23
		7.17.5.6 pb00	23
		7.17.5.7 pb01	24
		7.17.5.8 pb10	24
		7.17.5.9 pb11	24
		7.17.5.10 pGHZ	24
		7.17.5.11 pW	24
		7.17.5.12 px0	24

CONTENTS xv

			7.17.5.13 px1	124
			7.17.5.14 py0	124
			7.17.5.15 py1	124
			7.17.5.16 pz0	124
			7.17.5.17 pz1	124
			7.17.5.18 W	124
			7.17.5.19 x0	125
			7.17.5.20 x1	125
			7.17.5.21 y0	125
			7.17.5.22 y1	125
			7.17.5.23 z0	125
			7.17.5.24 z1	125
	7.18	qpp::Tii	er< T, CLOCK_T > Class Template Reference	125
		7.18.1	Detailed Description	127
		7.18.2	Constructor & Destructor Documentation	128
			7.18.2.1 Timer() noexcept	128
			7.18.2.2 Timer(const Timer &)=default	128
			7.18.2.3 Timer(Timer &&)=default	128
			7.18.2.4 ~Timer()=default	128
		7.18.3	Member Function Documentation	128
			7.18.3.1 display(std::ostream &os) const override	128
			7.18.3.2 get_duration() const noexcept	128
			7.18.3.3 operator=(const Timer &)=default	129
			7.18.3.4 operator=(Timer &&)=default	129
			7.18.3.5 tic() noexcept	129
			7.18.3.6 tics() const noexcept	129
			7.18.3.7 toc() noexcept	129
		7.18.4	Member Data Documentation	129
			7.18.4.1 end	129
			7.18.4.2 start	129
•	Tile I	D = =		404
8		Docume	codes.h File Reference	131
	8.1			
	0.0	8.1.1	Detailed Description	
	8.2		exception.h File Reference	
	0.0	8.2.1	Detailed Description	
	8.3		gates.h File Reference	
	0.4	8.3.1	Detailed Description	
	8.4		idisplay.h File Reference	
		8.4.1	Detailed Description	133

xvi CONTENTS

8.5	classes/init.h File Reference	134
	8.5.1 Detailed Description	134
8.6	classes/random_devices.h File Reference	134
	8.6.1 Detailed Description	135
8.7	classes/states.h File Reference	135
	8.7.1 Detailed Description	135
8.8	classes/timer.h File Reference	136
	8.8.1 Detailed Description	136
8.9	constants.h File Reference	136
	8.9.1 Detailed Description	137
8.10	entanglement.h File Reference	137
	8.10.1 Detailed Description	139
8.11	entropies.h File Reference	139
	8.11.1 Detailed Description	140
8.12	experimental/experimental.h File Reference	140
	8.12.1 Detailed Description	140
8.13	functions.h File Reference	140
	8.13.1 Detailed Description	144
8.14	input_output.h File Reference	145
	8.14.1 Detailed Description	146
8.15	instruments.h File Reference	146
	8.15.1 Detailed Description	
8.16	internal/classes/iomanip.h File Reference	147
	8.16.1 Detailed Description	
8.17	internal/classes/singleton.h File Reference	148
	8.17.1 Detailed Description	149
8.18	internal/util.h File Reference	
	8.18.1 Detailed Description	150
8.19	macros.h File Reference	
	8.19.1 Detailed Description	151
	8.19.2 Macro Definition Documentation	151
	8.19.2.1 ERROR	
	8.19.2.2 ERRORLN	151
	8.19.2.3 PRINT	151
	8.19.2.4 PRINTLN	
8.20	MATLAB/matlab.h File Reference	
	8.20.1 Detailed Description	
8.21	number_theory.h File Reference	
	8.21.1 Detailed Description	
8.22	operations.h File Reference	154

	8.22.1 Detailed Description	156
8.23	qpp.h File Reference	156
	8.23.1 Detailed Description	158
	8.23.2 Macro Definition Documentation	158
	8.23.2.1 QPP_UNUSED	158
8.24	random.h File Reference	158
	8.24.1 Detailed Description	159
8.25	statistics.h File Reference	159
	8.25.1 Detailed Description	160
8.26	traits.h File Reference	161
	8.26.1 Detailed Description	161
8.27	types.h File Reference	162
	8.27.1 Detailed Description	163
Index		165
HIGGA		100

Chapter 1

Quantum++

Version 1.0.0-devel - development

Build status: Master ![Build Status] (https://api.travis-ci.org/vsoftco/qpp. ⇔ svg?branch=master) Devel![Build Status] (https://api.travis-ci.org/vsoftco/qpp. ⇔ svg?branch=v1.0.0-devel)

Quantum++ is a modern C++11 general purpose quantum computing library, composed solely of template header files. Quantum++ is written in standard C++11 and has very low external dependencies, using only the Eigen 3 linear algebra header-only template library and, if available, the OpenMP multi-processing library.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance. The library's simulation capabilities are only restricted by the amount of available physical memory. On a typical machine (Intel i5 8Gb RAM) Quantum++ can successfully simulate the evolution of 25 qubits in a pure state or of 12 qubits in a mixed state reasonably fast.

To report any bugs or ask for additional features/enhancements, please submit an issue with an appropriate label.

If you are interesting in contributing to this project, please contact me. To contribute, you need to have a solid knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and working experience with Eigen 3.

For additional Eigen 3 documentation see http://eigen.tuxfamily.org/dox/. For a simple Eigen 3 quick ASCII reference see http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt.

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You should have received a copy of the GNU General Public License along with Quantum++. If not, see http←://www.gnu.org/licenses/.

Building instructions

Configuration

- Compiler: g++ version 4.8.2 or later (for good C++11 support)
- Eigen 3 library located in \$HOME/eigen

2 Quantum++

Quantum++ library located in \$HOME/qpp

Optional

- MATLAB compiler include header files: /Applications/MATLAB_R2016a.app/extern/include
- MATLAB compiler shared library files: /Applications/MATLAB_R2016a.app/bin/maci64

Building using CMake (version 3.0.0 or later)

The current version of the repository has a ./CMakeLists.txt configuration file for building examples using CMake. To build an example using CMake, I recommend an out-of-source build, i.e., from the root of the project (where ./include is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The commands above build the relase version (default) executable qpp, from the source file ./examples/minimal.cpp, without MATLAB support (default), inside the directory ./build. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build
rm -rf *
cmake -DCMAKE_BUILD_TYPE=Debug -DWITH_MATLAB=ON ..
make
```

Or, to disable OpenMP support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the ./CMakeLists.txt file. See also ./CMakeLists.txt for additional options. Do not forget to clean the ./build directory before a fresh build!

Building without an automatic build system

- Example file: \$HOME/qpp/examples/minimal.cpp
- Output executable: \$HOME/qpp/examples/minimal
- You must run the commands below from inside the directory \$HOME/qpp/examples

Release version (without MATLAB support)

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

Debug version (without MATLAB support)

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

Release version (with MATLAB support)

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2016a.app/extern/include \
    -L/Applications/MATLAB_R2016a.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

Debug version (with MATLAB support)

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2016a.app/extern/include \
    -L /Applications/MATLAB_R2016a.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

Unit testing

Quantum++ was extensively tested via a suite of unit tests constructed with Google Test 1.8.0 (included with the project in ./unit_tests/lib/gtest-1.8.0). The source code of the unit tests is provided under ./unit_tests/tests. To build and run the unit tests, I strongly recommend to use CMake version 3.0.0 or later. Assuming you do use CMake, switch to the ./unit_tests directory, create a build directory inside it, then from the newly created ./unit_tests/build type

```
cmake ..
```

The commands above build ./unit_tests/build/tests/qpp_testing, which you then may run.

Note

The CMake configuration file ./unit_tests/CMakeLists.txt defines the same building options and default choices as the main ./CMakeLists.txt of Quantum++. Therefore you can use the same flags as the ones mentioned at the beginning of this document when customizing the build. You should modify ./unit_ctests/CMakeLists.txt accordingly in case your Eigen 3 library or MATLAB include/library files are in a different location than the one assumed in this document.

Additional remarks

- The C++ compiler must be fully standard-C++11 compliant.
- If using Windows, I recommend compiling under cygwin via CMake and g++. See also http-://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2 for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file <cmath> using the provided patch./cmath cygwin.patch.
- In case you use OS X and want to install clang++ version 3.7 or later, I highly recommend to install it via macports.
- If you use clang++ version 3.7 or later and want to use OpenMP (enabled by default), make sure to modify CLANG_LIBOMP and CLANG_LIBOMP_INCLUDE in CMakeLists.txt so they point to the correct location of the OpenMP library, as otherwise clang++ will not find <omp.h> and the libomp shared library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD_L← IBRARY_PATH is set to point to the MATLAB compiler library location, see the run_OSX_MATLAB script. Otherwise, you get a runtime error similar to

```
> dyld: Library not loaded: @rpath/libmat.dylib.
```

4 Quantum++

I recommend running via a script, as otherwise setting the DYLD_LIBRARY_PATH globally may interfere with macports' CMake installation (in case you use CMake from macports). If you use a script, then the environment variable is local to the script and does not interfere with the rest of the system.

- Example of script, assumed to be located in the root directory of Quantum++

```
#!/bin/sh
MATLAB=/Applications/MATLAB_R2016a.app
export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:$MATLAB/bin/maci64
./build/qpp
```

• If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See http://stackoverflow.← com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks for more details about this problem.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

qpp	
Quantum++ main namespace	13
qpp::experimental	
Experimental/test functions/classes, do not use or modify	83
qpp::internal	
Internal utility functions, do not use/modify	84

6 Namespace Index

Chapter 3

Hierarchical Index

3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:
qpp::internal::Display_Impl
qpp::internal::IOManipEigen
std::exception
qpp::Exception
false_type
qpp::is_complex< T >
qpp::is_iterable < T, typename >
qpp::IDisplay
qpp::internal::IOManipEigen
qpp::internal::IOManipPointer< PointerType >
qpp::internal::IOManipRange< InputIterator >
is_base_of qpp::is matrix expression< Derived >
qpp::internal::Singleton< T >
qpp::internal::Singleton < const Codes >
qpp::Codes
qpp::internal::Singleton < const Gates >
qpp::Gates
qpp::internal::Singleton < const Init >
qpp::Init
qpp::internal::Singleton < const States >
qpp::States
qpp::internal::Singleton < RandomDevices >
qpp::RandomDevices
true_type
qpp::is_complex< std::complex< T >>
$qpp::is_iterable < \hspace{0.1cm} T, \hspace{0.1cm} to_void < \hspace{0.1cm} decltype(std::declval < \hspace{0.1cm} T \hspace{0.1cm} > ().begin()), \hspace{0.1cm} decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::declty$
>().end()), typename T::value_type >>

8 **Hierarchical Index**

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

qpp::Codes	
Const Singleton class that defines quantum error correcting codes	87
qpp::internal::Display_Impl	89
qpp::Exception	
Generates custom exceptions, used when validating function parameters	90
qpp::Gates	
Const Singleton class that implements most commonly used gates	93
qpp::IDisplay	
Abstract class (interface) that mandates the definition of virtual std::ostream& display(std←	
::ostream& os) const	100
qpp::Init	
Const Singleton class that performs additional initializations/cleanups	103
qpp::internal::IOManipEigen	104
qpp::internal::IOManipPointer< PointerType >	106
qpp::internal::IOManipRange< InputIterator >	108
qpp::is_complex < T >	
Checks whether the type is a complex type	110
qpp::is_complex < std::complex < T > >	
Checks whether the type is a complex number type, specialization for complex types	111
qpp::is_iterable < T, typename >	
Checks whether T is compatible with an STL-like iterable container $\dots \dots \dots \dots$	112
$qpp::is_iterable < T, \ to_void < \ decltype(std::declval < T > ().begin()), \ decltype(std::declval < T > ().end()), \ decltype(std::decltype(std::declval < T > ().end()), \ decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::decltype(std::$	
typename T::value_type >>	
Checks whether T is compatible with an STL-like iterable container, specialization for STL-like	
iterable containers	113
qpp::is_matrix_expression< Derived >	
Checks whether the type is an Eigen matrix expression	114
qpp::RandomDevices	
Singeleton class that manages the source of randomness in the library	115
qpp::internal::Singleton< T >	
Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously	
recurring template pattern)	117
qpp::States	
Const Singleton class that implements most commonly used states	119
qpp::Timer< T, CLOCK_T >	
Chronometer	125

10 Class Index

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

constants.h	
Constants	. 136
entanglement.h	
Entanglement functions	. 137
entropies.h	
Entropy functions	. 139
functions.h	
Generic quantum computing functions	. 140
input_output.h	
Input/output functions	. 145
instruments.h	
Measurement functions	. 146
macros.h	
Preprocessor macros	. 151
number_theory.h	
Number theory functions	. 152
operations.h	
Quantum operation functions	. 154
qpp.h	
Quantum++ main header file, includes all other necessary headers	. 156
random.h	
Randomness-related functions	. 158
statistics.h	
Statistics functions	. 159
traits.h	
Type traits	. 161
types.h	
Type aliases	. 162
classes/codes.h	
Quantum error correcting codes	. 131
classes/exception.h	
Exceptions	. 131
classes/gates.h	
Quantum gates	. 132
classes/idisplay.h	
Display interface via the non-virtual interface (NVI)	. 133
classes/init.h	
Initialization	. 134

12 File Index

asses/random_devices.h	
Random devices	134
asses/states.h	
Quantum states	135
asses/timer.h	
Timing	136
perimental/experimental.h	
Experimental/test functions/classes	140
ternal/util.h	
Internal utility functions	149
ternal/classes/iomanip.h	
Input/output manipulators	147
ternal/classes/singleton.h	
Singleton pattern via CRTP	148
ATLAB/matlab.h	
Input/output interfacing with MATLAB	152

Chapter 6

Namespace Documentation

6.1 qpp Namespace Reference

Quantum++ main namespace.

Namespaces

experimental

Experimental/test functions/classes, do not use or modify.

internal

Internal utility functions, do not use/modify.

Classes

class Codes

const Singleton class that defines quantum error correcting codes

class Exception

Generates custom exceptions, used when validating function parameters.

• class Gates

const Singleton class that implements most commonly used gates

class IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

class Init

const Singleton class that performs additional initializations/cleanups

· struct is_complex

Checks whether the type is a complex type.

struct is_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

• struct is_iterable

Checks whether T is compatible with an STL-like iterable container.

struct is_iterable< T, to_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().end()), typename T::value_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

· struct is_matrix_expression

Checks whether the type is an Eigen matrix expression.

• class RandomDevices

Singeleton class that manages the source of randomness in the library.

· class States

const Singleton class that implements most commonly used states

· class Timer

Chronometer.

Typedefs

```
template<typename... > 
using to_void = void
```

Alias template that implements the proposal for void_t.

• using idx = std::size t

Non-negative integer index.

• using bigint = long long int

Big integer.

using cplx = std::complex < double >

Complex number in double precision.

using ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

using bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

using cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

using dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

template<typename Scalar >

```
using dyn_mat = Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic >
```

Dynamic Eigen matrix over the field specified by Scalar.

template<typename Scalar >

```
using dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
```

Dynamic Eigen column vector over the field specified by Scalar.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$

```
using dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
```

Dynamic Eigen row vector over the field specified by Scalar.

Functions

```
• constexpr cplx operator""_i (unsigned long long int x) noexcept
```

```
User-defined literal for complex i = \sqrt{-1} (integer overload)
```

constexpr cplx operator""_i (long double x) noexcept

User-defined literal for complex $i = \sqrt{-1}$ (real overload)

cplx omega (idx D)

D-th root of unity.

• template<typename Derived >

dyn_col_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
&dims)

Schmidt coefficients of the bi-partite pure state A.

• template<typename Derived >

```
dyn_col_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, idx d=2)
```

Schmidt coefficients of the bi-partite pure state A.

template<typename Derived >

```
{\color{blue}cmat schmidtA (const \ Eigen::MatrixBase< Derived > \&A, \ const \ std::vector < idx > \&dims)}
```

Schmidt basis on Alice side. • template<typename Derived > cmat schmidtA (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt basis on Alice side. template<typename Derived > cmat schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt basis on Bob side. • template<typename Derived > cmat schmidtB (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt basis on Bob side. template<typename Derived > std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt probabilities of the bi-partite pure state A. template<typename Derived > std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt probabilities of the bi-partite pure state A. template<typename Derived > double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Entanglement of the bi-partite pure state A. • template<typename Derived > double entanglement (const Eigen::MatrixBase< Derived > &A, idx d=2) Entanglement of the bi-partite pure state A. template<typename Derived > double gconcurrence (const Eigen::MatrixBase< Derived > &A) G-concurrence of the bi-partite pure state A. • template<typename Derived > double negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Negativity of the bi-partite mixed state A. template<typename Derived > double negativity (const Eigen::MatrixBase< Derived > &A, idx d=2) Negativity of the bi-partite mixed state A. template<typename Derived > double lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Logarithmic negativity of the bi-partite mixed state A. • template<typename Derived > double lognegativity (const Eigen::MatrixBase< Derived > &A, idx d=2) Logarithmic negativity of the bi-partite mixed state A. template<typename Derived > double concurrence (const Eigen::MatrixBase< Derived > &A) Wootters concurrence of the bi-partite qubit mixed state A. template<typename Derived > double entropy (const Eigen::MatrixBase< Derived > &A) von-Neumann entropy of the density matrix A double entropy (const std::vector< double > &prob) Shannon entropy of the probability distribution prob. • template<typename Derived > double renyi (const Eigen::MatrixBase< Derived > &A, double alpha) Renyi- α entropy of the density matrix A, for $\alpha \geq 0$. double renyi (const std::vector< double > &prob, double alpha)

template<typename Derived >

Renyi- α entropy of the probability distribution prob, for $\alpha \geq 0$.

double tsallis (const Eigen::MatrixBase< Derived > &A, double q)

```
Tsallis- q entropy of the density matrix A, for q \geq 0.

    double tsallis (const std::vector< double > &prob, double q)

      Tsallis- q entropy of the probability distribution prob, for q \geq 0.

    template<typename Derived >

  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, const std::vector< idx > &dims)
     Quantum mutual information between 2 subsystems of a composite system.
• template<typename Derived >
  double gmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, idx d=2)
     Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
     Transpose.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
     Complex conjugate.
template<typename Derived >
  dyn mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)
     Trace.
template<typename Derived >
  Derived::Scalar det (const Eigen::MatrixBase < Derived > &A)
     Determinant.
• template<typename Derived >
  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.
• template<typename Derived >
  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum of A.
• template<typename Derived >
  Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)
     Element-wise product of A.
• template<typename Derived >
  double norm (const Eigen::MatrixBase< Derived > &A)
     Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition.

    template<typename Derived >

  dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)
     Eigenvalues.
• template<typename Derived >
  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.
• template<typename Derived >
  std::pair< dyn_col_vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)
```

Full eigen decomposition of Hermitian expression.

```
• template<typename Derived >
  dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.
• template<typename Derived >
  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.
template<typename Derived >
  \verb|std::tuple| < \verb|cmat|, \verb|dyn_col_vect| < \verb|double| >, \verb|cmat| > \verb|svd| (const Eigen::MatrixBase| < Derived| > \&A) \\
     Full singular value decomposition.

    template<typename Derived >

  dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.

    template<typename Derived >

  cmat svdU (const Eigen::MatrixBase< Derived > &A)
     Left singular vectors.
• template<typename Derived >
  cmat svdV (const Eigen::MatrixBase< Derived > &A)
      Right singular vectors.
• template<typename Derived >
  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
template<typename Derived >
  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.
• template<typename Derived >
  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolute value.
• template<typename Derived >
  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.
• template<typename Derived >
  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat sinm (const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
  cmat cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.

    template < typename Derived >

  double schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn mat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)( const type-
  name Derived::Scalar &))
```

Functor.

```
• template<typename T >
  dyn_mat< typename T::Scalar > kron (const T &head)
     Kronecker product.

    template<typename T, typename... Args>

  dyn_mat< typename T::Scalar > kron (const T &head, const Args &...tail)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > kron (const std::initializer list< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn mat< typename T::Scalar > dirsum (const T &head)
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > dirsum (const std::vector< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsum (const std::initializer_list< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx
  cols)
     Reshape.

    template<typename Derived1 , typename Derived2 >

  dyn mat< typename Derived1::Scalar > comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::↔
  MatrixBase< Derived2 > &B)
     Commutator.

    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &A)
     Proiector.
template<typename Derived >
  dyn mat< typename Derived::Scalar > grams (const std::vector< Derived > &As)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const std::initializer_list< Derived > &As)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)
```

Gram-Schmidt orthogonalization.

• std::vector< idx > n2multiidx (idx n, const std::vector< idx > &dims)

Non-negative integer index to multi-index.

idx multiidx2n (const std::vector < idx > &midx, const std::vector < idx > &dims)

Multi-index to non-negative integer index.

ket mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

Multi-partite qudit ket.

ket mket (const std::vector < idx > &mask, idx d=2)

Multi-partite qudit ket.

cmat mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

Projector onto multi-partite qudit ket.

cmat mprj (const std::vector < idx > &mask, idx d=2)

Projector onto multi-partite qudit ket.

• template<typename InputIterator >

std::vector< double > abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of an STL-like range of complex numbers.

• template<typename Container >

std::vector< double > abssq (const Container &c, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Computes the absolute values squared of an STL-like container.

template<typename Derived >

std::vector< double > abssq (const Eigen::MatrixBase< Derived > &A)

Computes the absolute values squared of an Eigen expression.

template<typename InputIterator >

std::iterator_traits< InputIterator >::value_type sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

template<typename Container >

Container::value_type sum (const Container &c, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Element-wise sum of the elements of an STL-like container.

template<typename InputIterator >

std::iterator traits< InputIterator >::value type prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

• template<typename Container >

Container::value_type prod (const Container &c, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Element-wise product of the elements of an STL-like container.

• template<typename Derived >

```
dyn_col_vect< typename Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)
```

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

```
std::vector< T > complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

• template<typename Derived>

```
std::vector< double > rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

• template<typename Derived >

```
internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
```

Eigen expression ostream manipulator.

• internal::IOManipEigen disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

template<typename InputIterator >
 internal::IOManipRange
 InputIterator > disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

• template<typename Container >

internal::IOManipRange< typename Container::const_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]", typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >

internal::IOManipPointer< PointerType > disp (const PointerType *p, idx N, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

• template<typename Derived >

void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

template < typename Derived >

dyn mat< typename Derived::Scalar > load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

• template<typename Derived >

dyn_col_vect< typename Derived::Scalar > ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::←
MatrixBase< Derived > &psi, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Generalized inner product.

template<typename Derived >

dyn_col_vect< typename Derived::Scalar > ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::← MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >

```
std::tuple < idx, std::vector < cmat >> measure (const Eigen::MatrixBase < Derived > &A, const std::vector < cmat > &Ks, const std::vector < idx > &subsys, const std::vector < idx > &dims)\\
```

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

```
std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)
```

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple < idx, std::vector < double >, std::vector < cmat > > measure (const Eigen::MatrixBase < Derived > &A, const std::initializer_list < cmat > &Ks, const std::vector < idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template < typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template < typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, std::vector < idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

Derived loadMATLABmatrix (const std::string &, const std::string &)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

cmat loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

• template<typename Derived >

void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &, const std::string &, const std::string &, const std::string &)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

template<>

void saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

std::vector< int > x2contfrac (double x, idx N, idx cut=1e5)

Simple continued fraction expansion.

double contfrac2x (const std::vector< int > &cf, idx N)

Real representation of a simple continued fraction.

double contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

bigint gcd (bigint a, bigint b)

Greatest common divisor of two integers.

bigint gcd (const std::vector< bigint > &as)

Greatest common divisor of a list of integers.

bigint lcm (bigint a, bigint b)

Least common multiple of two integers.

bigint lcm (const std::vector< bigint > &as)

Least common multiple of a list of integers.

• std::vector< idx > invperm (const std::vector< idx > &perm)

Inverse permutation.

• std::vector< idx > compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< bigint > factors (bigint a)

Prime factor decomposition.

bigint modmul (bigint a, bigint b, bigint p)

Modular multiplication without overflow.

bigint modpow (bigint a, bigint n, bigint p)

Fast integer power modulo p based on the SQUARE-AND-MULTIPLY algorithm.

std::tuple < bigint, bigint, bigint > egcd (bigint a, bigint b)

Extended greatest common divisor of two integers.

bigint modinv (bigint a, bigint p)

Modular inverse of a mod p.

• bool isprime (bigint p, idx k=80)

Primality test based on the Miller-Rabin's algorithm.

bigint randprime (bigint a, bigint b, idx N=1000)

Generates a random big prime uniformly distributed in the interval [a, b].

template<typename Derived1 , typename Derived2 >

dyn_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const
std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx
d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)
```

Applies the channel specified by the set of Kraus operators Ks to the density matrix A.

template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx
> &subsys, const std::vector< idx > &dims)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix A.

• template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx
> &subsys, idx d=2)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix A.

cmat kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, idx d=2)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector < idx > &dims)

Partial trace.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, idx d=2)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &subsys, const std::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector
idx > &subsys, idx d=2)

Partial trace.

• template<typename Derived >

Partial transpose.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial transpose.

ullet template<typename Derived >

Subsystem permutation.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &perm, idx d=2)

Subsystem permutation.

• double rand (double a, double b)

Generates a random real number uniformly distributed in the interval [a, b)

• bigint rand (bigint a, bigint b)

Generates a random big integer uniformly distributed in the interval [a, b].

idx randidx (idx a=std::numeric_limits < idx >::min(), idx b=std::numeric_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$

Derived rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

• template<>

cmat rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

template<typename Derived >

Derived randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

template<>

cmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

double randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat randU (idx D=2)

Generates a random unitary matrix.

cmat randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > randkraus (idx N, idx D=2)

Generates a set of random Kraus operators.

cmat randH (idx D=2)

Generates a random Hermitian matrix.

• ket randket (idx D=2)

Generates a random normalized ket (pure state vector)

cmat randrho (idx D=2)

Generates a random density matrix.

std::vector< idx > randperm (idx N)

Generates a random uniformly distributed permutation.

std::vector< double > randprob (idx N)

Generates a random probability vector uniformly distributed over the probability simplex.

std::vector< double > uniform (idx N)

Uniform probability distribution vector.

std::vector< double > marginalX (const dmat &probXY)

Marginal distribution.

std::vector< double > marginalY (const dmat &probXY)

Marginal distribution.

 $\bullet \ \ \text{template}{<} \text{typename Container} >$

double avg (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Average.

 $\bullet \ \ \text{template}{<} \text{typename Container} >$

double cov (const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_← iterable< Container >::value >::type *=nullptr)

Covariance.

template<typename Container >
 double var (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_iterable
 Container >::value >::type *=nullptr)

Variance.

• template<typename Container > double sigma (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_← iterable< Container >::value >::type *=nullptr)

Standard deviation.

Correlation.

template<typename Container >
 double cor (const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_
 iterable< Container >::value >::type *=nullptr)

Variables

constexpr double chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx maxn = 64

Maximum number of allowed qubits/qudits (subsystems)

- constexpr double pi = 3.141592653589793238462643383279502884
- constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

constexpr double infty = std::numeric_limits<double>::infinity()

Used to denote infinity in double precision.

6.1.1 Detailed Description

Quantum++ main namespace.

6.1.2 Typedef Documentation

6.1.2.1 using qpp::bigint = typedef long long int

Big integer.

6.1.2.2 using qpp::bra = typedef Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

6.1.2.3 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.2.4 using qpp::cplx = typedef std::complex < double >

Complex number in double precision.

6.1.2.5 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.2.6 template<typename Scalar > using qpp::dyn_col_vect = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by Scalar.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

6.1.2.7 template<typename Scalar > using qpp::dyn_mat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic,
Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

6.1.2.8 template < typename Scalar > using qpp::dyn_row_vect = typedef Eigen::Matrix < Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

6.1.2.9 using qpp::idx = typedef std::size_t

Non-negative integer index.

6.1.2.10 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

6.1.2.11 template < typename... > using qpp::to_void = typedef void

Alias template that implements the proposal for void_t.

See also

```
http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2014/n3911
```

6.1.3 Function Documentation

6.1.3.1 template<typename Derived > cmat qpp::absm (const Eigen::MatrixBase< Derived > & A)

Matrix absolute value.

Α	Eigen expression
---	------------------

Returns

Matrix absolute value of A

6.1.3.2 template<typename InputIterator > std::vector<double> qpp::abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of an STL-like range of complex numbers.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Real vector consisting of the range absolute values squared

6.1.3.3 template<typename Container > std::vector<double> qpp::abssq (const Container & c, typename std::enable_if< is_iterable< Container >::value >::type * = nullptr)

Computes the absolute values squared of an STL-like container.

Parameters

c STL-like container

Returns

Real vector consisting of the container's absolute values squared

6.1.3.4 template<typename Derived > std::vector<double> qpp::abssq (const Eigen::MatrixBase< Derived > & A)

Computes the absolute values squared of an Eigen expression.

Parameters

Α	Eigen expression

Returns

Real vector consisting of the absolute values squared

6.1.3.5 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::adjoint (const Eigen::MatrixBase< Derived > & A)

Adjoint.

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A	Elderi expression
	9 1

Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field as A

6.1.3.6 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::anticomm (const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B)

Anti-commutator.

See also

qpp::comm()

Anti-commutator $\{A, B\} = AB + BA$. Both A and B must be Eigen expressions over the same scalar field.

Parameters

Α	Eigen expression
В	Eigen expression

Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field as A

6.1.3.7 template < typename Derived1 , typename Derived2 > $dyn_mat < typename Derived1::Scalar > dpp::apply (const Eigen::MatrixBase < Derived2 > & A, const std::vector < <math>idx > & subsys$, const std::vector < idx > & dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

Parameters

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

Returns

Gate A applied to the part subsys of state

6.1.3.8 template < typename Derived1 , typename Derived2 > dyn_mat < typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & subsys, idx d = 2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

Returns

Gate A applied to the part subsys of state

6.1.3.9 template<typename Derived > cmat qpp::apply (const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix A.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators

Returns

Output density matrix after the action of the channel

6.1.3.10 template<typename Derived > cmat qpp::apply (const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims)

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *A*.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
dims	Dimensions of the multi-partite system

Returns

Output density matrix after the action of the channel

6.1.3.11 template<typename Derived > cmat qpp::apply (const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, idx d = 2)

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *A*

Parameters

Α	Eigen expression
Ks	Set of Kraus operators

subsys	Subsystem indexes where the Kraus operators Ks are applied
d	Subsystem dimensions

Returns

Output density matrix after the action of the channel

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

See also

```
qpp::Gates::CTRL()
```

Note

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

Parameters

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

Returns

CTRL-A gate applied to the part subsys of state

6.1.3.13 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx d = 2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

See also

```
qpp::Gates::CTRL()
```

Note

The dimension of the gate A must match the dimension of subsys

Parameters

state	Eigen expression

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

Returns

CTRL-A gate applied to the part subsys of state

6.1.3.14 template<typename Container > double qpp::avg (const std::vector< double > & prob, const Container & X, typename std::enable_if< is_iterable< Container >::value >::type * = nullptr)

Average.

Parameters

prob	Real probability vector representing the probability distribution of X
X	Real random variable values represented by an STL-like container

Returns

Average of X

6.1.3.15 cmat qpp::bloch2rho (const std::vector< double > & r) [inline]

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

See also

qpp::rho2bloch()

Parameters

r	3-dimensional real vector

Returns

Qubit density matrix

6.1.3.16 std::vector<cmat>qpp::choi2kraus(const cmat & A) [inline]

Orthogonal Kraus operators from Choi matrix.

See also

qpp::kraus2choi()

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix A

Note

The Kraus operators satisfy $Tr(K_i^\dagger K_j) = \delta_{ij}$ for all $i \neq j$

Α	Choi matrix
---	-------------

Returns

Set of orthogonal Kraus operators

6.1.3.17 cmat qpp::choi2super (const cmat & A) [inline]

Converts Choi matrix to superoperator matrix.

See also

qpp::super2choi()

Parameters

A Choi matrix

Returns

Superoperator matrix

6.1.3.18 template < typename Derived1 , typename Derived2 > $dyn_mat < typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B)$

Commutator.

See also

qpp::anticomm()

Commutator [A, B] = AB - BA. Both A and B must be Eigen expressions over the same scalar field.

Parameters

Α	Eigen expression
В	Eigen expression

Returns

Commutator AB - BA, as a dynamic matrix over the same scalar field as A

6.1.3.19 template < typename T > std::vector < T > qpp::complement (std::vector < T > subsys, idx N)

Constructs the complement of a subsystem vector.

Parameters

subsys	Subsystem vector
N	Total number of systems

Returns

Complement of *subsys* with respect to the set $\{0, 1, ..., N-1\}$

6.1.3.20 std::vector < idx > qpp::compperm (const std::vector < idx > & perm, const std::vector < idx > & sigma) [inline]

Compose permutations.

perm	Permutation
sigma	Permutation

Returns

Composition of the permutations $perm \circ sigma = perm(sigma)$

6.1.3.21 template < typename Derived > double qpp::concurrence (const Eigen::MatrixBase < Derived > & A)

Wootters concurrence of the bi-partite qubit mixed state A.

Parameters

A Eigen expression	

Returns

Wootters concurrence

6.1.3.22 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::conjugate (const Eigen::MatrixBase< Derived > & A)

Complex conjugate.

Parameters

Α	Eigen expression

Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field as A

6.1.3.23 double qpp::contfrac2x (const std::vector < int > & cf, idx N) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

Parameters

cf	Integer vector containing the simple continued fraction expansion
N	Number of terms considered in the continued fraction expansion. If N is greater than the size
	of <i>cf</i> ,then all terms in <i>cf</i> are considered.

Returns

Real representation of the simple continued fraction

6.1.3.24 double qpp::contfrac2x (const std::vector < int > & cf) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

Parameters

cf	Integer vector containing the simple continued fraction expansion

Returns

Real representation of the simple continued fraction

6.1.3.25 template<typename Container > double qpp::cor (const dmat & probXY, const Container & X, const Container & Y, typename std::enable_if< is_iterable< Container >::value >::type * = nullptr)

Correlation.

Parameters

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)
X	Real random variable values represented by an STL-like container
Y	Real random variable values represented by an STL-like container

Returns

Correlation of X and Y

6.1.3.26 template < typename Derived > cmat qpp::cosm (const Eigen::MatrixBase < Derived > & A)

Matrix cos.

Parameters

Α	Eigen expression

Returns

Matrix cosine of A

6.1.3.27 template<typename Container > double qpp::cov (const dmat & probXY, const Container & X, const Container & Y, typename std::enable_if< is_iterable< Container >::value >::type * = nullptr)

Covariance.

Parameters

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

X	Real random variable values represented by an STL-like container
Y	Real random variable values represented by an STL-like container

Returns

Covariance of X and Y

6.1.3.28 template<typename OutputScalar , typename Derived > dyn_mat<OutputScalar> qpp::cwise (const Eigen::MatrixBase< Derived > & A, OutputScalar(*)(const typename Derived::Scalar &) f)

Functor.

Parameters

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

Returns

Component-wise f(A), as a dynamic matrix over the ${\it OutputScalar}$ scalar field

6.1.3.29 template<typename Derived > Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > & A)

Determinant.

Parameters

Α	Eigen expression
---	------------------

Returns

Determinant of A, as a scalar over the same scalar field as A. Returns $\pm \infty$ when the determinant overflows/underflows.

6.1.3.30 template<typename T > dyn_mat<typename T::Scalar> qpp::dirsum (const T & head)

Direct sum.

See also

qpp::dirsumpow()

Used to stop the recursion for the variadic template version of qpp::dirsum()

Parameters

head	Eigen expression

Returns

Its argument head

6.1.3.31 template<typename T , typename... Args> dyn_mat<typename T::Scalar> qpp::dirsum (const T & head, const Args &... tail)

Direct sum.

See also

qpp::dirsumpow()

Parameters

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.32 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::dirsum (const std::vector< Derived > & As)

Direct sum.

See also

qpp::dirsumpow()

Parameters

As	std::vector of Eigen expressions

Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.33 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::dirsum (const std::initializer_list < Derived > & As)

Direct sum.

See also

qpp::dirsumpow()

Parameters

As	std::initializer_list of Eigen expressions, such as {A1, A2, ,Ak}
----	-------------------------------------------------------------------

Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.34 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase < Derived > & A, idx n)

Direct sum power.

See also

qpp::dirsum()

Parameters

Α	Eigen expression
n	Non-negative integer

Returns

Direct sum of A with itself n times $A^{\oplus n}$, as a dynamic matrix over the same scalar field as A

6.1.3.35 template < typename Derived > internal::IOManipEigen qpp::disp (const Eigen::MatrixBase < Derived > & A, double chop = qpp::chop)

Eigen expression ostream manipulator.

Parameters

A	Eigen expression
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

Returns

Instance of qpp::internal::IOManipEigen

6.1.3.36 internal::IOManipEigen qpp::disp(cplx z, double chop = qpp::chop) [inline]

Complex number ostream manipulator.

Parameters

Z	Complex number (or any other type implicitly cast-able to std::complex <double>)</double>
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

Returns

Instance of qpp::internal::IOManipEigen

6.1.3.37 template < typename InputIterator > internal::IOManipRange < InputIterator > qpp::disp (InputIterator first, InputIterator last, const std::string & separator, const std::string & start = " [", const std::string & end = "] ")

Range ostream manipulator.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range
separator	Separator
start	Left marking
end	Right marking

Returns

Instance of qpp::internal::IOManipRange

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

Parameters

С	Container
separator	Separator
start	Left marking
end	Right marking

Returns

Instance of qpp::internal::IOManipRange

6.1.3.39 template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp (const PointerType * p, idx N, const std::string & separator, const std::string & start = " [", const std::string & end = "] ")

C-style pointer ostream manipulator.

Parameters

р	Pointer to the first element
N	Number of elements to be displayed
separator	Separator
start	Left marking
end	Right marking

Returns

Instance of qpp::internal::IOManipPointer

6.1.3.40 std::tuple < bigint, bigint > qpp::egcd (bigint a, bigint b) [inline]

Extended greatest common divisor of two integers.

See also

qpp::gcd()

а	Integer
b	Integer

Returns

Tuple of: 1. Integer m, 2. Integer n, and 3. Non-negative integer gcd(a,b) such that ma + nb = gcd(a,b)

Full eigen decomposition.

See also

qpp::heig()

Parameters

Α	Eigen expression

Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.42 template < typename Derived > double qpp::entanglement (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

See also

qpp::entropy()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Entanglement, with the logarithm in base 2

6.1.3.43 template < typename Derived > double qpp::entanglement (const Eigen::MatrixBase < Derived > & A, idx d = 2)

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

See also

qpp::entropy()

Α	Eigen expression
d	Subsystem dimensions

Returns

Entanglement, with the logarithm in base 2

6.1.3.44 template < typename Derived > double qpp::entropy (const Eigen::MatrixBase < Derived > & A)

von-Neumann entropy of the density matrix A

Parameters

Α	Eigen expression

Returns

von-Neumann entropy, with the logarithm in base 2

6.1.3.45 double qpp::entropy (const std::vector < double > & prob) [inline]

Shannon entropy of the probability distribution prob.

Parameters

prob	Real probability vector
	rear producting reasons

Returns

Shannon entropy, with the logarithm in base 2

6.1.3.46 template<typename Derived > dyn_col_vect<cplx> qpp::evals (const Eigen::MatrixBase< Derived > & A)

Eigenvalues.

See also

qpp::hevals()

Parameters

A	Eigen expression
---	------------------

Returns

Eigenvalues of A, as a complex dynamic column vector

6.1.3.47 template < typename Derived > cmat qpp::evects (const Eigen::MatrixBase < Derived > & A)

Eigenvectors.

See also

qpp::hevects()

```
A Eigen expression
```

Returns

Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.48 template<typename Derived > cmat qpp::expm (const Eigen::MatrixBase< Derived > & A)

Matrix exponential.

Parameters

```
A Eigen expression
```

Returns

Matrix exponential of A

6.1.3.49 std::vector
bigint> qpp::factors (bigint a) [inline]

Prime factor decomposition.

Note

Runs in $\mathcal{O}(\sqrt{n})$ time complexity

Parameters

а	Integer different from 0, 1 or -1

Returns

Integer vector containing the factors

6.1.3.50 template < typename Derived > cmat qpp::funm (const Eigen::MatrixBase < Derived > & A, cplx(*)(const cplx &) f

Functional calculus f(A)

Parameters

Α	Eigen expression
f	Pointer-to-function from complex to complex

Returns

f(A)

6.1.3.51 bigint qpp::gcd (bigint *a***, bigint** *b*) [inline]

Greatest common divisor of two integers.

See also

qpp::lcm()

а	Integer
b	Integer

Returns

Greatest common divisor of a and b

6.1.3.52 bigint qpp::gcd (const std::vector< bigint > & as) [inline]

Greatest common divisor of a list of integers.

See also

qpp::lcm()

Parameters

00	List of integers
as	LISTOFINITEGETS

Returns

Greatest common divisor of all numbers in as

6.1.3.53 template < typename Derived > double qpp::gconcurrence (const Eigen::MatrixBase < Derived > & A)

G-concurrence of the bi-partite pure state A.

Note

Both local dimensions must be equal

Uses qpp::logdet() to avoid overflows

See also

qpp::logdet()

Parameters

Α	Eigen expression
---	------------------

Returns

G-concurrence

6.1.3.54 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams (const std::vector< Derived > & As)

Gram-Schmidt orthogonalization.

As std::vector of Eigen expressions as column vectors

Returns

Gram-Schmidt vectors of As as columns of a dynamic matrix over the same scalar field as its arguments

Gram-Schmidt orthogonalization.

Parameters

As std::initializer_list of Eigen expressions as column vectors

Returns

Gram-Schmidt vectors of As as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.56 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams (const Eigen::MatrixBase< Derived > & A)

Gram-Schmidt orthogonalization.

Parameters

A | Eigen expression, the input vectors are the columns of A

Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field as A

 $6.1.3.57 \quad template < typename \ Derived > std::pair < dyn_col_vect < double >, cmat > qpp::heig (\ const \ Eigen::MatrixBase < Derived > \& \ A \)$

Full eigen decomposition of Hermitian expression.

See also

qpp::eig()

Parameters

A Eigen expression

Returns

Pair of: 1. Eigenvalues of A, as a real dynamic column vector, and 2. Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.58 template<typename Derived > dyn_col_vect<double> qpp::hevals (const Eigen::MatrixBase< Derived > & A)

Hermitian eigenvalues.

See also

qpp::evals()

Parameters

Returns

Eigenvalues of Hermitian A, as a real dynamic column vector

6.1.3.59 template<typename Derived > cmat qpp::hevects (const Eigen::MatrixBase< Derived > & A)

Hermitian eigenvectors.

See also

qpp::evects()

Parameters

```
A Eigen expression
```

Returns

Eigenvectors of Hermitian A, as columns of a complex matrix

6.1.3.60 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::inverse (const Eigen::MatrixBase< Derived > & A)

Inverse.

Parameters

```
A Eigen expression
```

Returns

Inverse of A, as a dynamic matrix over the same scalar field as A

6.1.3.61 std::vector<idx> qpp::invperm(const std::vector<idx> & perm) [inline]

Inverse permutation.

Parameters

perm	Permutation

Returns

Inverse of the permutation perm

6.1.3.62 template < typename Derived > dyn_col_vect < typename Derived::Scalar > dyn_i (const Eigen::MatrixBase < Derived > & dyn_i const Eigen::MatrixBase < Derived > & dyn_i const Std::vector < dyn_i const Std::v

Generalized inner product.

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined
dims	Dimensions of the multi-partite system

Returns

Inner product $\langle \phi_{subsys} | \psi \rangle$, as a scalar or column vector over the remaining Hilbert space

6.1.3.63 template<typename Derived > dyn_col_vect<typename Derived::Scalar> qpp::ip (const Eigen::MatrixBase< Derived > & phi, const Eigen::MatrixBase< Derived > & psi, const std::vector< idx > & subsys, idx d = 2)

Generalized inner product.

Parameters

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined
d	Subsystem dimensions

Returns

Inner product $\langle \phi_{subsys} | \psi \rangle$, as a scalar or column vector over the remaining Hilbert space

6.1.3.64 bool qpp::isprime (bigint p, idx k = 80) [inline]

Primality test based on the Miller-Rabin's algorithm.

Parameters

р	Integer different from 0, 1 or -1
k	Number of iterations. The probability of a false positive is 2^{-k} .

Returns

True if the number is (most-likely) prime, false otherwise

6.1.3.65 cmat qpp::kraus2choi (const std::vector < cmat > & Ks) [inline]

Choi matrix.

See also

qpp::choi2kraus()

Constructs the Choi matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis $\{|i\rangle\langle j|\}$ ordered in lexicographical order, i.e. $|0\rangle\langle 0|$, $|0\rangle\langle 1|$ etc.

Note

The superoperator matrix S and the Choi matrix C are related by $S_{ab,mn}=C_{ma,nb}$

Ks Set of Kraus operators

Returns

Choi matrix

6.1.3.66 cmat qpp::kraus2super (const std::vector < cmat > & Ks) [inline]

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis $\{|i\rangle\langle j|\}$ ordered in lexicographical order, i.e. $|0\rangle\langle 0|$, $|0\rangle\langle 1|$ etc.

Parameters

```
Ks Set of Kraus operators
```

Returns

Superoperator matrix

6.1.3.67 template<typename T > dyn_mat<typename T::Scalar> qpp::kron (const T & head)

Kronecker product.

See also

qpp::kronpow()

Used to stop the recursion for the variadic template version of app::kron()

Parameters

head	Eigen expression

Returns

Its argument head

6.1.3.68 template<typename T , typename... Args> dyn_mat<typename T::Scalar> qpp::kron (const T & head, const Args &... tail)

Kronecker product.

See also

qpp::kronpow()

Parameters

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.69 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::kron (const std::vector< Derived > & As)

Kronecker product.

See also

qpp::kronpow()

Parameters

As	std::vector of Eigen expressions
----	----------------------------------

Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker product.

See also

qpp::kronpow()

Parameters

As	std::initializer_list of Eigen expressions, such as {A1, A2, ,Ak}
----	-------------------------------------------------------------------

Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker power.

See also

qpp::kron()

Α	Eigen expression
n	Non-negative integer

Returns

Kronecker product of A with itself n times $A^{\otimes n}$, as a dynamic matrix over the same scalar field as A

```
6.1.3.72 bigint qpp::lcm (bigint a, bigint b) [inline]
```

Least common multiple of two integers.

See also

qpp::gcd()

Parameters

а	Integer
b	Integer

Returns

Least common multiple of a and b

```
6.1.3.73 bigint qpp::lcm ( const std::vector< bigint > & as ) [inline]
```

Least common multiple of a list of integers.

See also

qpp::gcd()

Parameters

as	List of integers

Returns

Least common multiple of all numbers in as

6.1.3.74 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::load (const std::string & fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

See also

```
qpp::save()
```

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

fname	Output file name
-------	------------------

6.1.3.75 template < typename Derived > Derived qpp::loadMATLABmatrix (const std::string & , const std::string &)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

See also

```
qpp::saveMATLABmatrix()
```

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be loaded)

```
6.1.3.76 template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

Note

If var_name is a complex matrix, only the real part is loaded

Parameters

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen double dynamic matrix (qpp::dmat)

```
6.1.3.77 template <> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen complex dynamic matrix (qpp::cmat)

6.1.3.78 template<typename Derived > Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > & A)

Logarithm of the determinant.

Useful when the determinant overflows/underflows

Parameters

Α	Eigen expression
---	------------------

Returns

Logarithm of the determinant of A, as a scalar over the same scalar field as A

6.1.3.79 template < typename Derived > cmat qpp::logm (const Eigen::MatrixBase < Derived > & A)

Matrix logarithm.

Parameters

Α	Eigen expression
---	------------------

Returns

Matrix logarithm of A

6.1.3.80 template<typename Derived > double qpp::lognegativity (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims)

Logarithmic negativity of the bi-partite mixed state A.

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.81 template < typename Derived > double qpp::lognegativity (const Eigen::MatrixBase < Derived > & A_1 , idx d = 2)

Logarithmic negativity of the bi-partite mixed state A.

Α	Eigen expression
d	Subsystem dimensions

Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.82 std::vector<double> qpp::marginalX (const dmat & probXY) [inline]

Marginal distribution.

Parameters

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

Returns

Real vector consisting of the marginal distribution of X

6.1.3.83 std::vector<double> qpp::marginalY (const dmat & probXY) [inline]

Marginal distribution.

Parameters

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

Returns

Real vector consisting of the marginal distribution of Y

6.1.3.84 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.85 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

Α	Eigen expression
Ks	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.86 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const cmat & U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

Parameters

Α	Eigen expression
U	Unitary matrix whose columns represent the measurement basis vectors

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.87 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.88 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.89 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

A	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.90 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.91 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis or rank-1 POVM specified by the matrix *V*.

See also

qpp::measure_seq()

Note

The dimension of V must match the dimension of subsys. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.92 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & subsys, idx d = 2)

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis or rank-1 POVM specified by the matrix *V*.

qpp::measure_seq()

Note

The dimension of *V* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

```
6.1.3.93 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, std::vector < idx > dims)
```

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

qpp::measure()

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

```
6.1.3.94 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, idx d = 2)
```

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

qpp::measure()

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.95 ket qpp::mket (const std::vector < idx > & mask, const std::vector < idx > & dims) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket $|\text{mask}\rangle$, where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

Parameters

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.96 ket qpp::mket (const std::vector < idx > & mask, idx d = 2) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket $|mask\rangle$, all subsystem having equal dimension *d. mask* is a std::vector of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

Parameters

mask	std::vector of non-negative integers
d	Subsystem dimensions

Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.97 bigint qpp::modinv (bigint a, bigint p) [inline]

Modular inverse of a mod p.

See also

qpp::egcd()

Note

a and p must be co-prime

а	Non-negative integer
р	Non-negative integer

Returns

Modular inverse $a^{-1} \mod p$

6.1.3.98 bigint qpp::modmul (bigint a, bigint b, bigint p) [inline]

Modular multiplication without overflow.

Computes $ab \bmod p$ without overflow

Parameters

а	Integer
b	Integer
р	Positive integer

Returns

 $ab \bmod p$ avoiding overflow

6.1.3.99 bigint qpp::modpow(bigint a, bigint n, bigint p) [inline]

Fast integer power modulo p based on the SQUARE-AND-MULTIPLY algorithm.

Note

Uses qpp::modmul() that avoids overflows

Computes $a^n \mod p$

Parameters

а	Non-negative integer
n	Non-negative integer
р	Strictly positive integer

Returns

 $a^n \bmod p$

6.1.3.100 cmat qpp::mprj (const std::vector < idx > & mask, const std::vector < idx > & dims) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket $|mask\rangle$, where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

Parameters

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.101 cmat qpp::mprj (const std::vector < idx > & mask, idx d = 2) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket $|mask\rangle$, all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

Parameters

mask	std::vector of non-negative integers
d	Subsystem dimensions

Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.102 idx = idx =

Multi-index to non-negative integer index.

See also

qpp::n2multiidx()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

Parameters

midx	Multi-index
dims	Dimensions of the multi-partite system

Returns

Non-negative integer index

6.1.3.103 std::vector<idx> qpp::n2multiidx (idx n, const std::vector< idx> & dims) [inline]

Non-negative integer index to multi-index.

See also

qpp::multiidx2n()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

n	Non-negative integer index
dims	Dimensions of the multi-partite system

Returns

Multi-index of the same size as dims

6.1.3.104 template<typename Derived > double qpp::negativity (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims)

Negativity of the bi-partite mixed state A.

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Negativity

6.1.3.105 template < typename Derived > double qpp::negativity (const Eigen::MatrixBase < Derived > & A, idx d = 2)

Negativity of the bi-partite mixed state A.

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Negativity

6.1.3.106 template<typename Derived > double qpp::norm (const Eigen::MatrixBase< Derived > & A)

Frobenius norm.

Parameters

Α	Eigen expression

Returns

Frobenius norm of A

6.1.3.107 cplx qpp::omega (idx D) [inline]

D-th root of unity.

D	Non-negative integer
---	----------------------

Returns

D-th root of unity $\exp(2\pi i/D)$

6.1.3.108 constexpr cplx qpp::operator""_i(unsigned long long int x) [inline], [noexcept]

User-defined literal for complex $i = \sqrt{-1}$ (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

6.1.3.109 constexpr cplx qpp::operator""_i (long double x) [inline], [noexcept]

User-defined literal for complex $i = \sqrt{-1}$ (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.110 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::powm (const Eigen::MatrixBase< Derived > & A, idx n)

Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.

See also

qpp::spectralpowm()

Explicitly multiplies the matrix ${\it A}$ with itself ${\it n}$ times. By convention ${\it A}^0={\it I}$.

Parameters

Α	Eigen expression
n	Non-negative integer

Returns

Matrix power A^n , as a dynamic matrix over the same scalar field as A

6.1.3.111 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::prj (const Eigen::MatrixBase< Derived > & A)

Projector.

Normalized projector onto state vector

Α	Eigen expression
---	------------------

Returns

Projector onto the state vector A, or the matrix Zero if A has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field as A

6.1.3.112 template < typename Derived > Derived::Scalar qpp::prod (const Eigen::MatrixBase < Derived > & A)

Element-wise product of A.

Parameters

Α	Eigen expression
---	------------------

Returns

Element-wise product of A, as a scalar over the same scalar field as A

6.1.3.113 template<typename InputIterator > std::iterator_traits<InputIterator>::value_type qpp::prod (InputIterator *first*, InputIterator *last*)

Element-wise product of an STL-like range.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Element-wise product of the range, as a scalar over the same scalar field as the range

Element-wise product of the elements of an STL-like container.

Parameters

С	STL-like container

Returns

Element-wise product of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.115 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Partial trace.

See also

qpp::ptrace1(), qpp::ptrace2()

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

	Α	Eigen expression
s	ubsys	Subsystem indexes
	dims	Dimensions of the multi-partite system

Returns

Partial trace $Tr_{subsys}(\cdot)$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.116 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, idx d = 2)

Partial trace.

See also

qpp::ptrace1(), qpp::ptrace2()

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

Returns

Partial trace $Tr_{subsys}(\cdot)$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.117 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptrace1 (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Partial trace $Tr_A(\cdot)$ over the first subsytem A in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.118 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace1 (const Eigen::MatrixBase< Derived > & A, idx d = 2)

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Partial trace $Tr_A(\cdot)$ over the first subsystem A in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.119 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptrace2 (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Partial trace $Tr_B(\cdot)$ over the second subsystem B in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.120 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace2 (const Eigen::MatrixBase< Derived > & A, idx d = 2)

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Partial trace $Tr_B(\cdot)$ over the second subsytem B in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.121 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptranspose (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims)

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

Returns

Partial transpose $(\cdot)^{T_{subsys}}$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.122 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptranspose (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2)

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

Returns

Partial transpose $(\cdot)^{T_{subsys}}$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.123 template<typename Derived > double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, const std::vector< idx > & dims)

Quantum mutual information between 2 subsystems of a composite system.

Parameters

Α	Eigen expression
subsysA	Indexes of the first subsystem

subsysB	Indexes of the second subsystem
dims	Dimensions of the multi-partite system

Returns

Mutual information between the 2 subsystems

6.1.3.124 template < typename Derived > double qpp::qmutualinfo (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsysA, const std::vector < idx > & subsysB, idx d = 2)

Quantum mutual information between 2 subsystems of a composite system.

Parameters

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
d	Subsystem dimensions

Returns

Mutual information between the 2 subsystems

6.1.3.125 double qpp::rand (double a, double b) [inline]

Generates a random real number uniformly distributed in the interval [a, b)

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random real number (double) uniformly distributed in the interval [a, b)

6.1.3.126 bigint qpp::rand (bigint a, bigint b) [inline]

Generates a random big integer uniformly distributed in the interval [a, b].

Note

To avoid ambiguity with double qpp::rand(double, double) cast at least one of the arguments to qpp::bigint

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

Returns

Random big integer uniformly distributed in the interval [a, b]

6.1.3.127 template < typename Derived > Derived qpp::rand (idx rows, idx cols, double a = 0, double b = 1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat

```
6.1.3.128 template <> dmat qpp::rand (idx rows, idx cols, double a, double b) [inline]
```

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random real matrix

```
6.1.3.129 template<> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) uniformly distributed in [-1,1) auto mat = rand<cmat>(3, 3, -1, 1);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random complex matrix

```
6.1.3.130 cmat qpp::randH ( idx D = 2 ) [inline]
```

Generates a random Hermitian matrix.

Dimension of the Hilbert space	

Returns

Random Hermitian matrix

Generates a random index (idx) uniformly distributed in the interval [a, b].

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

Returns

Random index (idx) uniformly distributed in the interval [a, b]

```
6.1.3.132 ket qpp::randket ( idx D = 2 ) [inline]
```

Generates a random normalized ket (pure state vector)

Parameters

D	Dimension of the Hilbert space

Returns

Random normalized ket

```
6.1.3.133 std::vector<cmat> qpp::randkraus(idx N, idx D = 2) [inline]
```

Generates a set of random Kraus operators.

Note

The set of Kraus operators satisfy the closure condition $\sum_i K_i^\dagger K_i = I$

Parameters

N	Number of Kraus operators
D	Dimension of the Hilbert space

Returns

Set of N Kraus operators satisfying the closure condition

6.1.3.134 template < typename Derived > Derived qpp::randn (idx rows, idx cols, double mean = 0, double sigma = 1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat

6.1.3.135 template<> dmat qpp::randn(idx rows, idx cols, double mean, double sigma) [inline]

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

Returns

Random real matrix

```
6.1.3.136 template <> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) normally distributed in N(0,2) auto mat = randn<cmat>(3, 3, 0, 2);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

Returns

Random complex matrix

```
6.1.3.137 double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]
```

Generates a random real number (double) normally distributed in N(mean, sigma)

Parameters

mean	Mean

sigma	Standard deviation

Returns

Random real number normally distributed in N(mean, sigma)

6.1.3.138 std::vector<idx> qpp::randperm(idx N) [inline]

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by std::shuffle), so that all permutations are equally probable Parameters

N Size of the permutation	
---------------------------	--

Returns

Random permutation of size N

6.1.3.139 bigint qpp::randprime (bigint a, bigint b, idx N = 1000) [inline]

Generates a random big prime uniformly distributed in the interval [a, b].

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it
N	Maximum number of candidates

Returns

Random big integer uniformly distributed in the interval [a, b]

6.1.3.140 std::vector<double> qpp::randprob(idx N) [inline]

Generates a random probability vector uniformly distributed over the probability simplex.

Parameters

N	Size of the probability vector

Returns

Random probability vector

6.1.3.141 cmat qpp::randrho(idx D = 2) [inline]

Generates a random density matrix.

Parameters

D	Dimension o	of the Hilbert space
---	-------------	----------------------

Returns

Random density matrix

6.1.3.142 cmat qpp::randU(idx *D* = 2) [inline]

Generates a random unitary matrix.

Parameters

D Dimension of the Hilbert space

Returns

Random unitary

6.1.3.143 cmat qpp::randV (idx Din, idx Dout) [inline]

Generates a random isometry matrix.

Parameters

Din	Size of the input Hilbert space
Dout	Size of the output Hilbert space

Returns

Random isometry matrix

6.1.3.144 template < typename Derived > double qpp::renyi (const Eigen::MatrixBase < Derived > & A, double alpha)

Renyi- α entropy of the density matrix A, for $\alpha \geq 0$.

Note

When $\alpha \to 1$ the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

Parameters

Α	Eigen expression
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

Returns

Renyi- α entropy, with the logarithm in base 2

6.1.3.145 double qpp::renyi (const std::vector < double > & prob, double alpha) [inline]

Renyi- α entropy of the probability distribution *prob*, for $\alpha \geq 0$.

Note

When $\alpha \to 1$ the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

prob	Real probability vector
alpha	Non-negative real number, use qpp::infty for $\alpha = \infty$

Returns

Renyi- α entropy, with the logarithm in base 2

6.1.3.146 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::reshape (const Eigen::MatrixBase< Derived > & A, idx rows, idx cols)

Reshape.

Uses column-major order when reshaping (same as MATLAB)

Parameters

Α	Eigen expression
rows	Number of rows of the reshaped matrix
cols	Number of columns of the reshaped matrix

Returns

Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field as A

6.1.3.147 template < typename Derived > std::vector < double > qpp::rho2bloch (const Eigen::MatrixBase < Derived > & A)

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

See also

qpp::bloch2rho()

Note

It is implicitly assumed that the density matrix is Hermitian

Parameters

Α	Eigen expression

Returns

3-dimensional Bloch vector

6.1.3.148 template<typename Derived > dyn_col_vect<typename Derived::Scalar> qpp::rho2pure (const Eigen::MatrixBase< Derived > & A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

Note

No purity check is done, the input state A must have rank one, otherwise the function returns the first non-zero eigenvector of A

Α	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. A is
	assumed to have rank one

Returns

The unique non-zero eigenvector of A, as a dynamic column vector over the same scalar field as A

6.1.3.149 template<typename Derived > void qpp::save (const Eigen::MatrixBase< Derived > & A, const std::string & fname)

Saves Eigen expression to a binary file (internal format) in double precision.

See also

qpp::load()

Parameters

Α	Eigen expression
fname	Output file name

 $6.1.3.150 \quad template < typename\ Derived > void\ qpp::saveMATLAB matrix\ (\ const\ Eigen::MatrixBase < Derived > \&\ ,\ const\ std::string\ \&\ ,\ const\ std::string\ \&\)$

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

See also

gpp::loadMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be saved)

6.1.3.151 template<> void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

qpp::loadMATLABmatrix()

Parameters

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.152 template<> void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > & A, const std::string & mat_file, const std::string & war_name, const std::string & mode) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

qpp::loadMATLABmatrix()

Parameters

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.153 template < typename Derived > double qpp::schatten (const Eigen::MatrixBase < Derived > & A, double p)

Schatten matrix norm.

Parameters

Α	Eigen expression
р	Real number, greater or equal to 1, use qpp::infty for $p=\infty$

Returns

Schatten-p matrix norm of A

6.1.3.154 template < typename Derived > cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt basis on Alice side.

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Unitary matrix ${\cal U}$ whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.155 template < typename Derived > cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > & A, idx d = 2)

Schmidt basis on Alice side.

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Unitary matrix ${\cal U}$ whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.156 template < typename Derived > cmat qpp::schmidtB (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt basis on Bob side.

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Unitary matrix ${\cal V}$ whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.157 template < typename Derived > cmat qpp::schmidtB (const Eigen::MatrixBase < Derived > & A, idx d = 2)

Schmidt basis on Bob side.

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Unitary matrix ${\cal V}$ whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.158 template<typename Derived > dyn_col_vect<double> qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims)

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

See also

qpp::schmidtprobs()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Schmidt coefficients of A, ordered in decreasing order, as a real dynamic column vector

6.1.3.159 template < typename Derived > dyn_col_vect < double > dyn_col_vect > dyn_col_vect < double >

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

See also

qpp::schmidtprobs()

Α	Eigen expression
d	Subsystem dimensions

Returns

Schmidt coefficients of A, ordered in decreasing order, as a real dynamic column vector

6.1.3.160 template < typename Derived > std::vector < double > qpp::schmidtprobs (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

See also

qpp::schmidtcoeffs()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Real vector consisting of the Schmidt probabilites of A, ordered in decreasing order

6.1.3.161 template < typename Derived > std::vector < double > qpp::schmidtprobs (const Eigen::MatrixBase < Derived > & A, idx d = 2)

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

See also

qpp::schmidtcoeffs()

Parameters

Α	Eigen expression
d	Subsystem dimensions

Returns

Real vector consisting of the Schmidt probabilites of A, ordered in decreasing order

6.1.3.162 template < typename Container > double qpp::sigma (const std::vector < double > & prob, const Container & X, typename std::enable_if < is iterable < Container >::value >::type * = nullptr)

Standard deviation.

prob	Real probability vector representing the probability distribution of X
X	Real random variable values represented by an STL-like container

Returns

Standard deviation of X

6.1.3.163 template < typename Derived > cmat qpp::sinm (const Eigen::MatrixBase < Derived > & A)

Matrix sin.

Parameters

Α	Eigen expression

Returns

Matrix sine of A

6.1.3.164 template < typename Derived > cmat qpp::spectralpowm (const Eigen::MatrixBase < Derived > & A, const cplx z

Matrix power.

See also

qpp::powm()

Uses the spectral decomposition of A to compute the matrix power. By convention $A^0=I$.

Parameters

Α	Eigen expression
Z	Complex number

Returns

Matrix power A^z

6.1.3.165 template < typename Derived > cmat qpp::sqrtm (const Eigen::MatrixBase < Derived > & A)

Matrix square root.

Parameters

Α	Eigen expression
---	------------------

Returns

Matrix square root of A

6.1.3.166 template < typename Derived > Derived::Scalar qpp::sum (const Eigen::MatrixBase < Derived > & A)

Element-wise sum of A.

Α	Eigen expression
---	------------------

Returns

Element-wise sum of A, as a scalar over the same scalar field as A

6.1.3.167 template < typename InputIterator > std::iterator_traits < InputIterator >::value_type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Element-wise sum of the range, as a scalar over the same scalar field as the range

6.1.3.168 template<typename Container > Container::value_type qpp::sum (const Container & c, typename std::enable_if<is_iterable< Container >::value >::type * = nullptr)

Element-wise sum of the elements of an STL-like container.

Parameters

c STL-like container	
----------------------	--

Returns

Element-wise sum of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.169 cmat qpp::super2choi(const cmat & A) [inline]

Converts superoperator matrix to Choi matrix.

See also

qpp::choi2super()

Parameters

Α	Superoperator matrix

Returns

Choi matrix

6.1.3.170 template < typename Derived > dyn_col_vect < double > qpp::svals (const Eigen::MatrixBase < Derived > & A)

Singular values.

Α	Eigen expression
---	------------------

Returns

Singular values of A, ordered in decreasing order, as a real dynamic column vector

6.1.3.171 template<typename Derived > std::tuple<cmat, dyn_col_vect<double>, cmat> qpp::svd (const Eigen::MatrixBase< Derived > & A)

Full singular value decomposition.

Parameters

```
A Eigen expression
```

Returns

Tuple of: 1. Left sigular vectors of *A*, as columns of a complex dynamic matrix, 2. Singular values of *A*, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of *A*, as columns of a complex dynamic matrix

6.1.3.172 template<typename Derived > cmat qpp::svdU (const Eigen::MatrixBase< Derived > & A)

Left singular vectors.

Parameters

Α	Eigen expression

Returns

Complex dynamic matrix, whose columns are the left singular vectors of A

6.1.3.173 template<typename Derived > cmat qpp::svdV (const Eigen::MatrixBase< Derived > & A)

Right singular vectors.

Parameters

```
A Eigen expression
```

Returns

Complex dynamic matrix, whose columns are the right singular vectors of A

6.1.3.174 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & perm

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

Α	Eigen expression
perm	Permutation
dims	Dimensions of the multi-partite system

Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.175 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::syspermute (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2)

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

Parameters

Α	Eigen expression
perm	Permutation
d	Subsystem dimensions

Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.176 template<typename Derived > Derived::Scalar qpp::trace (const Eigen::MatrixBase< Derived > & A)

Trace.

Parameters

4	Fine connection
A	Eigen expression

Returns

Trace of A, as a scalar over the same scalar field as A

6.1.3.177 template<typename Derived > dyn_mat <typename Derived::Scalar> dyn_mat <typename Derived > dyn_mat <typename Derived >

Transpose.

Parameters

Α	Eigen expression

Returns

Transpose of A, as a dynamic matrix over the same scalar field as A

6.1.3.178 template<typename Derived > double qpp::tsallis (const Eigen::MatrixBase< Derived > & A, double q)

Tsallis- q entropy of the density matrix A, for $q \ge 0$.

Note

When $q \to 1$ the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base e

Α	Eigen expression
q	Non-negative real number

Returns

Tsallis- q entropy

6.1.3.179 double qpp::tsallis (const std::vector < double > & prob, double q) [inline]

Tsallis- q entropy of the probability distribution *prob*, for $q \ge 0$.

Note

When $q \to 1$ the Tsallis entropy converges to the Shannon entropy, with the logarithm in base e

Parameters

prob	Real probability vector
q	Non-negative real number

Returns

Tsallis- q entropy

6.1.3.180 std::vector<double> qpp::uniform(idx N) [inline]

Uniform probability distribution vector.

Parameters

N	Size of the alphabet

Returns

Real vector consisting of a uniform distribution of size N

6.1.3.181 template < typename Container > double qpp::var (const std::vector < double > & prob, const Container & X, typename std::enable_if < is_iterable < Container >::value >::type * = nullptr)

Variance.

Parameters

prob	Real probability vector representing the probability distribution of X
X	Real random variable values represented by an STL-like container

Returns

Variance of X

6.1.3.182 std::vector<int> qpp::x2contfrac (double x, idx N, idx cut = 1e5) [inline]

Simple continued fraction expansion.

See also

qpp::contfrac2x()

X	Real number
N	Maximum number of terms in the expansion
cut	Stop the expansion when the next term is greater than <i>cut</i>

Returns

Integer vector containing the simple continued fraction expansion of x. If there are M less than N terms in the expansion, a shorter vector with M components is returned.

6.1.4 Variable Documentation

6.1.4.1 constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

6.1.4.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.4.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::eps) // x is zero</pre>
```

6.1.4.4 constexpr double qpp::infty = std::numeric_limits < double >::infinity()

Used to denote infinity in double precision.

6.1.4.5 constexpr idx qpp::maxn = 64

Maximum number of allowed qubits/qudits (subsystems)

Used internally to allocate arrays on the stack (for performance reasons):

6.1.4.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 π

6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

Classes

- struct Display_Impl_
- class IOManipEigen
- · class IOManipPointer
- class IOManipRange
- · class Singleton

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

Functions

- void n2multiidx (idx n, idx numdims, const idx *const dims, idx *result) noexcept
- idx multiidx2n (const idx *const midx, idx numdims, const idx *const dims) noexcept
- template<typename Derived >

bool check_square_mat (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$

bool check_vector (const Eigen::MatrixBase< Derived > &A)

ullet template<typename Derived >

bool check_rvector (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool check_cvector (const Eigen::MatrixBase< Derived > &A)

template<typename T >

bool check_nonzero_size (const T &x) noexcept

• template<typename T1 , typename T2 >

bool check_matching_sizes (const T1 &lhs, const T2 &rhs) noexcept

- bool check_dims (const std::vector< idx > &dims)
- $\bullet \ \ \mathsf{template}{<}\mathsf{typename} \ \mathsf{Derived}>$

 $bool\ check_dims_match_mat\ (const\ std::vector < idx > \&dims,\ const\ Eigen::MatrixBase < Derived > \&A)$

 $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{Derived} >$

bool check_dims_match_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool check_dims_match_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)

- bool check eq dims (const std::vector< idx > &dims, idx dim) noexcept
- bool check_subsys_match_dims (const std::vector< idx > &subsys, const std::vector< idx > &dims)
- template<typename Derived >

bool check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept

• template<typename Derived >

bool check_qubit_cvector (const Eigen::MatrixBase< Derived > &A) noexcept

• template<typename Derived >

bool check_qubit_rvector (const Eigen::MatrixBase< Derived > &A) noexcept

• template<typename Derived >

bool check_qubit_vector (const Eigen::MatrixBase< Derived > &A) noexcept

- bool check_perm (const std::vector< idx > &perm)
- template<typename Derived1 , typename Derived2 >

 $\frac{dyn_mat}{<} typename \ Derived1::Scalar > \frac{kron2}{(const \ Eigen::MatrixBase} < Derived1 > \&A, \ const \ Eigen:: \\ \\ MatrixBase < Derived2 > \&B)$

template < typename Derived1 , typename Derived2 > dyn_mat < typename Derived1::Scalar > dirsum2 (const Eigen::MatrixBase < Derived1 > &A, const Eigen ← ::MatrixBase < Derived2 > &B)

- template<typename T > void variadic_vector_emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
 void variadic_vector_emplace (std::vector< T > &v, First &&first, Args &&...args)
- idx get_num_subsys (idx sz, idx d)
- idx get dim subsys (idx sz, idx N)

6.3.1 Detailed Description

Internal utility functions, do not use/modify.

6.3.2 Function Documentation

- 6.3.2.1 template < typename Derived > bool qpp::internal::check_cvector (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.2 bool qpp::internal::check_dims (const std::vector < idx > & dims) [inline]
- 6.3.2.3 template<typename Derived > bool qpp::internal::check_dims_match_cvect (const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & A)
- 6.3.2.4 template<typename Derived > bool qpp::internal::check_dims_match_mat (const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & A)
- 6.3.2.5 template < typename Derived > bool qpp::internal::check_dims_match_rvect (const std::vector < idx > & dims, const Eigen::MatrixBase < Derived > & A)
- 6.3.2.6 bool qpp::internal::check_eq_dims (const std::vector < idx > & dims, idx dim) [inline], [noexcept]
- 6.3.2.7 template < typename T1 , typename T2 > bool qpp::internal::check_matching_sizes (const T1 & *Ihs*, const T2 & *rhs*) [noexcept]
- **6.3.2.8** template<typename T > bool qpp::internal::check_nonzero_size(const T & x) [noexcept]
- 6.3.2.9 bool qpp::internal::check_perm (const std::vector < idx > & perm) [inline]
- 6.3.2.10 template < typename Derived > bool qpp::internal::check_qubit_cvector (const Eigen::MatrixBase < Derived > & A) [noexcept]
- 6.3.2.11 template < typename Derived > bool qpp::internal::check_qubit_matrix (const Eigen::MatrixBase < Derived > & A) [noexcept]
- 6.3.2.12 template < typename Derived > bool qpp::internal::check_qubit_rvector (const Eigen::MatrixBase < Derived > & A) [noexcept]
- $\textbf{6.3.2.13} \quad \textbf{template} < \textbf{typename Derived} > \textbf{bool qpp::internal::check_qubit_vector(const Eigen::MatrixBase} < \textbf{Derived} > \textbf{\& A)} \\ [\texttt{noexcept}]$
- 6.3.2.14 template < typename Derived > bool qpp::internal::check_rvector (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.15 template < typename Derived > bool qpp::internal::check_square_mat (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.16 bool qpp::internal::check_subsys_match_dims (const std::vector < idx > & subsys, const std::vector < idx > & dims) [inline]
- 6.3.2.17 template < typename Derived > bool qpp::internal::check_vector (const Eigen::MatrixBase < Derived > & A)

- $6.3.2.18 \quad template < typename \ Derived 1 \ , typename \ Derived 2 > dyn_mat < typename \ Derived 1 :: Scalar > qpp::internal::dirsum 2 \\ (\ const \ Eigen::Matrix Base < Derived 1 > \& \ \textit{A}, \ const \ Eigen::Matrix Base < Derived 2 > \& \ \textit{B} \)$
- 6.3.2.19 idx qpp::internal::get_dim_subsys(idx sz, idx N) [inline]
- **6.3.2.20** idx qpp::internal::get_num_subsys (idx sz, idx d) [inline]
- **6.3.2.22** idx qpp::internal::multiidx2n (const idx *const *midx*, idx *numdims*, const idx *const *dims*) [inline], [noexcept]
- 6.3.2.23 void qpp::internal::n2multiidx (idx n, idx n numdims, const idx *const dims, idx * result) [inline], [noexcept]
- 6.3.2.24 template < typename T > void qpp::internal::variadic_vector_emplace (std::vector < T > &)
- 6.3.2.25 template < typename T , typename First , typename... Args > void qpp::internal::variadic_vector_emplace (std::vector < T > & ν , First && first, Args &&... args)

Chapter 7

Class Documentation

7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

#include <classes/codes.h>

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



88 Class Documentation

Public Types

enum Type { Type::FIVE_QUBIT = 1, Type::SEVEN_QUBIT_STEANE, Type::NINE_QUBIT_SHOR }
 Code types, add more codes here if needed.

Public Member Functions

ket codeword (Type type, idx i) const
 Returns the codeword of the specified code type.

Private Member Functions

• Codes ()

Default constructor.

Codes ()=default

Default destructor.

Friends

class internal::Singleton < const Codes >

Additional Inherited Members

7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

7.1.2 Member Enumeration Documentation

```
7.1.2.1 enum qpp::Codes::Type [strong]
```

Code types, add more codes here if needed.

See also

```
qpp::Codes::codeword()
```

Enumerator

```
FIVE_QUBIT [[5,1,3]] qubit code
SEVEN_QUBIT_STEANE [[7,1,3]] Steane qubit code
NINE_QUBIT_SHOR [[9,1,3]] Shor qubit code
```

7.1.3 Constructor & Destructor Documentation

```
7.1.3.1 qpp::Codes::Codes( ) [inline], [private]
Default constructor.
```

```
7.1.3.2 qpp::Codes::~Codes() [private], [default]
```

Default destructor.

7.1.4 Member Function Documentation

7.1.4.1 **ket** qpp::Codes::codeword (Type *type*, idx *i*) const [inline]

Returns the codeword of the specified code type.

See also

qpp::Codes::Type

Parameters

type	Code type
i	Codeword index

Returns

i-th codeword of the code type

7.1.5 Friends And Related Function Documentation

7.1.5.1 friend class internal::Singleton < const Codes > [friend]

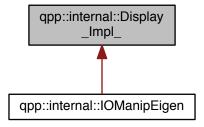
The documentation for this class was generated from the following file:

· classes/codes.h

7.2 qpp::internal::Display_Impl_ Struct Reference

#include <internal/util.h>

Inheritance diagram for qpp::internal::Display_Impl_:



Public Member Functions

template < typename T >
 std::ostream & display_impl_ (const T &A, std::ostream &os, double chop=qpp::chop) const

90 Class Documentation

7.2.1 Member Function Documentation

7.2.1.1 template<typename T > std::ostream & qpp::internal::Display_Impl_::display_impl_(const T & A, std::ostream & os, double chop = qpp::chop) const [inline]

The documentation for this struct was generated from the following file:

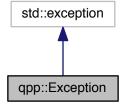
• internal/util.h

7.3 qpp::Exception Class Reference

Generates custom exceptions, used when validating function parameters.

#include <classes/exception.h>

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



Public Types

enum Type {

Type::UNKNOWN_EXCEPTION = 1, Type::ZERO_SIZE, Type::MATRIX_NOT_SQUARE, Type::MATRIX_← NOT_CVECTOR,

Type::MATRIX_NOT_RVECTOR, Type::MATRIX_NOT_VECTOR, Type::MATRIX_NOT_SQUARE_OR_C↔ VECTOR, Type::MATRIX_NOT_SQUARE_OR_RVECTOR,

Type::MATRIX_NOT_SQUARE_OR_VECTOR, Type::MATRIX_MISMATCH_SUBSYS, Type::DIMS_INVA← LID, Type::DIMS_NOT_EQUAL,

Type::DIMS_MISMATCH_MATRIX, Type::DIMS_MISMATCH_CVECTOR, Type::DIMS_MISMATCH_RVE←CTOR, Type::DIMS MISMATCH VECTOR,

Type::SUBSYS_MISMATCH_DIMS, Type::PERM_INVALID, Type::PERM_MISMATCH_DIMS, Type::NOT ← QUBIT_MATRIX,

Type::NOT_QUBIT_CVECTOR, Type::NOT_QUBIT_RVECTOR, Type::NOT_QUBIT_VECTOR, Type::NO← T QUBIT SUBSYS,

Type::NOT_BIPARTITE, Type::NO_CODEWORD, Type::OUT_OF_RANGE, Type::TYPE_MISMATCH, Type::SIZE MISMATCH, Type::UNDEFINED TYPE, Type::CUSTOM EXCEPTION }

Exception types, add more here if needed.

Public Member Functions

Exception (const std::string &where, const Type &type)
 Constructs an exception.

• Exception (const std::string &where, const std::string &custom)

Constructs an exception.

virtual const char * what () const noexcept override

Overrides std::exception::what()

Private Member Functions

void construct_exception_msg_ ()

Constructs the exception description from its type.

Private Attributes

- · std::string where_
- std::string msg
- Type type_
- · std::string custom_

7.3.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

7.3.2 Member Enumeration Documentation

7.3.2.1 enum qpp::Exception::Type [strong]

Exception types, add more here if needed.

See also

qpp::Exception::construct_exception_msg_()

Enumerator

UNKNOWN_EXCEPTION Unknown exception

ZERO_SIZE Zero sized object, e.g. empty Eigen::Matrix or std::vector with no elements

MATRIX_NOT_SQUARE Eigen::Matrix is not square

MATRIX_NOT_CVECTOR Eigen::Matrix is not a column vector

MATRIX_NOT_RVECTOR Eigen::Matrix is not a row vector

MATRIX_NOT_VECTOR Eigen::Matrix is not a row/column vector

MATRIX_NOT_SQUARE_OR_CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX_NOT_SQUARE_OR_RVECTOR Eigen::Matrix is not square nor a row vector

MATRIX_NOT_SQUARE_OR_VECTOR Eigen::Matrix is not square nor a row/column vector

MATRIX_MISMATCH_SUBSYS Matrix size mismatch subsystem sizes (e.g. in qpp::apply())

DIMS_INVALID std::vector<idx> of dimensions has zero size or contains zeros

DIMS_NOT_EQUAL Local/global dimensions are not equal

DIMS_MISMATCH_MATRIX Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

DIMS_MISMATCH_CVECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

DIMS_MISMATCH_RVECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

DIMS_MISMATCH_VECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

SUBSYS_MISMATCH_DIMS std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

PERM_INVALID std::vector<idx> does note represent a valid permutation

PERM_MISMATCH_DIMS Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

NOT_QUBIT_MATRIX Eigen::Matrix is not 2 x 2

NOT_QUBIT_CVECTOR Eigen::Matrix is not 2 x 1

NOT_QUBIT_RVECTOR Eigen::Matrix is not 1 x 2

NOT_QUBIT_VECTOR Eigen::Matrix is not 1 x 2 nor 2 x 1

NOT_QUBIT_SUBSYS Subsystems are not 2-dimensional

NOT_BIPARTITE std::vector<idx> of dimensions has size different from 2

NO_CODEWORD Codeword does not exist, thrown when calling qpp::Codes::codeword() with invalid index i

OUT_OF_RANGE Parameter out of range

TYPE_MISMATCH Scalar types do not match

SIZE_MISMATCH Sizes do not match

UNDEFINED_TYPE Templated specialization not defined for this type

CUSTOM_EXCEPTION Custom exception, the user must provide a custom message

7.3.3 Constructor & Destructor Documentation

7.3.3.1 qpp::Exception::Exception (const std::string & where, const Type & type) [inline]

Constructs an exception.

Parameters

where	Text representing where the exception occured
type	Exception type, defined in qpp::Exception::Type

7.3.3.2 qpp::Exception::Exception (const std::string & where, const std::string & custom) [inline]

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

Parameters

where	Text representing where the exception occured
custom	Exception description

7.3.4 Member Function Documentation

7.3.4.1 void qpp::Exception::construct_exception_msg_() [inline], [private]

Constructs the exception description from its type.

See also

qpp::Exception::Type

Must modify the code of this function if more exceptions are added

7.3.4.2 virtual const char* qpp::Exception::what () const [inline], [override], [virtual], [noexcept]

Overrides std::exception::what()

Returns

Exception description

7.3.5 Member Data Documentation

```
7.3.5.1 std::string qpp::Exception::custom_ [private]
```

7.3.5.2 std::string qpp::Exception::msg_ [private]

7.3.5.3 Type qpp::Exception::type [private]

7.3.5.4 std::string qpp::Exception::where [private]

The documentation for this class was generated from the following file:

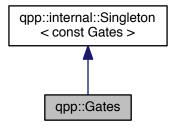
· classes/exception.h

7.4 qpp::Gates Class Reference

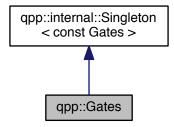
const Singleton class that implements most commonly used gates

#include <classes/gates.h>

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



Public Member Functions

- cmat Rn (double theta, const std::vector< double > &n) const
 Qubit rotation of theta about the 3-dimensional real (unit) vector n.
- cmat Zd (idx D=2) const

Generalized Z gate for qudits.

• cmat Fd (idx D=2) const

Fourier transform gate for qudits.

• cmat Xd (idx D=2) const

Generalized X gate for qudits.

template<typename Derived = Eigen::MatrixXcd>
 Derived Id (idx D=2) const

Identity gate.

• template<typename Derived >

 $\frac{dyn_mat}{dx} < typename\ Derived::Scalar > CTRL\ (const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&ctrl,\ const\ std::vector < idx > \&subsys,\ idx\ N,\ idx\ d=2)\ const$

Generates the multi-partite multiple-controlled-A gate in matrix form.

```
• template<typename Derived >
      dyn_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const
      std::vector< idx > &dims) const
          Expands out.
    template<typename Derived >
      dyn mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const
      std::initializer_list< idx > &dims) const
          Expands out.
    template<typename Derived >
      dyn_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, idx N,
      idx d=2) const
          Expands out.
Public Attributes

    cmat Id2 {cmat::Identity(2, 2)}

          Identity gate.

    cmat H {cmat::Zero(2, 2)}

          Hadamard gate.
    cmat X {cmat::Zero(2, 2)}
          Pauli Sigma-X gate.

    cmat Y {cmat::Zero(2, 2)}

          Pauli Sigma-Y gate.

    cmat Z {cmat::Zero(2, 2)}

          Pauli Sigma-Z gate.
    • cmat S {cmat::Zero(2, 2)}
          S gate.

    cmat T {cmat::Zero(2, 2)}

          T gate.
    cmat CNOT {cmat::Identity(4, 4)}
          Controlled-NOT control target gate.

    cmat CZ {cmat::Identity(4, 4)}

          Controlled-Phase gate.
    cmat CNOTba {cmat::Zero(4, 4)}
          Controlled-NOT target control gate.

    cmat SWAP {cmat::Identity(4, 4)}

          SWAP gate.
    cmat TOF {cmat::ldentity(8, 8)}
          Toffoli gate.
    • cmat FRED {cmat::ldentity(8, 8)}
```

Private Member Functions

Fredkin gate.

• Gates ()

Initializes the gates.

∼Gates ()=default

Default destructor.

Friends

class internal::Singleton < const Gates >

Additional Inherited Members

7.4.1 Detailed Description

const Singleton class that implements most commonly used gates

7.4.2 Constructor & Destructor Documentation

```
7.4.2.1 qpp::Gates::Gates( ) [inline],[private]
```

Initializes the gates.

```
7.4.2.2 qpp::Gates::~Gates() [private],[default]
```

Default destructor.

7.4.3 Member Function Documentation

7.4.3.1 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::Gates::CTRL (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, idx N, idx d = 2) const [inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

See also

```
qpp::applyCTRL()
```

Note

The dimension of the gate A must match the dimension of subsys

Parameters

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
N	Total number of subsystems
d	Subsystem dimensions

Returns

CTRL-A gate, as a matrix over the same scalar field as A

```
7.4.3.2 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::expandout ( const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]
```

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

Parameters

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

Returns

Tensor product $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$, with A on position pos, as a dynamic matrix over the same scalar field as A

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

Note

The std::initializer_list overload exists because otherwise, in the degenerate case when *dims* has only one element, the one element list is implicitly converted to the element's underlying type, i.e. qpp::idx, which has the net effect of picking the wrong (non-vector) qpp::expandout() overload

Parameters

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

Returns

Tensor product $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$, with A on position pos, as a dynamic matrix over the same scalar field as A

7.4.3.4 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::expandout (const Eigen::MatrixBase< Derived > & A, idx pos, idx N, idx d = 2) const [inline]

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

Parameters

Α	Eigen expression

pos	Position
N	Number of subsystems
d	Subsystem dimension

Returns

Tensor product $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$, with A on position pos, as a dynamic matrix over the same scalar field as A

7.4.3.5 cmat qpp::Gates::Fd (idx D = 2) const [inline]

Fourier transform gate for qudits.

Note

Defined as
$$F = \sum_{j,k=0}^{D-1} \exp(2\pi \mathrm{i} jk/D) |j\rangle\langle k|$$

Parameters

D Dimension of the Hilbert space	
----------------------------------	--

Returns

Fourier transform gate for qudits

7.4.3.6 template < typename Derived = Eigen::MatrixXcd > Derived qpp::Gates::Id (idx D = 2) const [inline] Identity gate.

Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

Parameters

D	Dimension of the Hilbert space
---	--------------------------------

Returns

Identity gate on a Hilbert space of dimension D

7.4.3.7 cmat qpp::Gates::Rn (double theta, const std::vector < double > & n) const [inline]

Qubit rotation of *theta* about the 3-dimensional real (unit) vector *n*.

Parameters

theta	Rotation angle
n	3-dimensional real (unit) vector

Returns

Rotation gate

7.4.3.8 cmat qpp::Gates::Xd (idx D = 2) const [inline]

Generalized X gate for qudits.

Note

Defined as
$$X=\sum_{j=0}^{D-1}|j\oplus 1\rangle\langle j|$$
, i.e. raising operator $X|j\rangle=|j\oplus 1\rangle$

Parameters

```
D Dimension of the Hilbert space
```

Returns

Generalized X gate for qudits

7.4.3.9 cmat qpp::Gates::Zd(idx D = 2) const [inline]

Generalized Z gate for qudits.

Note

Defined as
$$Z = \sum_{j=0}^{D-1} \exp(2\pi \mathrm{i} j/D) |j\rangle\langle j|$$

Parameters

D | Dimension of the Hilbert space

Returns

Generalized Z gate for qudits

- 7.4.4 Friends And Related Function Documentation
- 7.4.4.1 friend class internal::Singleton < const Gates > [friend]
- 7.4.5 Member Data Documentation
- 7.4.5.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

7.4.5.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

7.4.5.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

7.4.5.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}

Fredkin gate.

```
7.4.5.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}
Hadamard gate.
7.4.5.6 cmat qpp::Gates::ld2 {cmat::ldentity(2, 2)}
Identity gate.
7.4.5.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}
S gate.
7.4.5.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}
SWAP gate.
7.4.5.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}
T gate.
7.4.5.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}
Toffoli gate.
7.4.5.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}
Pauli Sigma-X gate.
7.4.5.12 cmat qpp::Gates::Y {cmat::Zero(2, 2)}
Pauli Sigma-Y gate.
7.4.5.13 cmat qpp::Gates::Z {cmat::Zero(2, 2)}
Pauli Sigma-Z gate.
The documentation for this class was generated from the following file:
    · classes/gates.h
```

7.5 qpp::IDisplay Class Reference

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const. #include <classes/idisplay.h>

Inheritance diagram for qpp::IDisplay:



Public Member Functions

• IDisplay ()=default

Default constructor.

• IDisplay (const IDisplay &)=default

Default copy constructor.

• IDisplay (IDisplay &&)=default

Default move constructor.

• IDisplay & operator= (const IDisplay &)=default

Default copy assignment operator.

• IDisplay & operator= (IDisplay &&)=default

Default move assignment operator.

virtual ∼IDisplay ()=default

Default virtual destructor.

Private Member Functions

• virtual std::ostream & display (std::ostream &os) const =0

Must be overridden by all derived classes.

Friends

std::ostream & operator<< (std::ostream &os, const IDisplay &rhs)

Overloads the extraction operator.

7.5.1 Detailed Description

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

This class defines friend inline std::ostream& operator<< (std::ostream& os, const qpp::IDisplay& rhs). The latter delegates the work to the pure private virtual function qpp::IDisplay::display() which has to be overridden by all derived classes.

7.5.2 Constructor & Destructor Documentation

```
7.5.2.1 qpp::IDisplay::IDisplay() [default]
```

Default constructor.

```
7.5.2.2 qpp::IDisplay::IDisplay ( const IDisplay & ) [default]
```

Default copy constructor.

```
7.5.2.3 qpp::IDisplay::IDisplay ( IDisplay && ) [default]
```

Default move constructor.

```
7.5.2.4 virtual qpp::|Display::~|Display( ) [virtual], [default]
```

Default virtual destructor.

7.5.3 Member Function Documentation

```
7.5.3.1 virtual std::ostream& qpp::IDisplay::display ( std::ostream & os ) const [private], [pure virtual]
```

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs).

Implemented in qpp::internal::IOManipEigen, qpp::Timer< T, CLOCK_T >, qpp::internal::IOManipPointer< PointerType >, and qpp::internal::IOManipRange< InputIterator >.

```
7.5.3.2 IDisplay& qpp::IDisplay::operator=( const IDisplay & ) [default]
```

Default copy assignment operator.

```
7.5.3.3 IDisplay& qpp::IDisplay::operator=(IDisplay&&) [default]
```

Default move assignment operator.

7.5.4 Friends And Related Function Documentation

```
7.5.4.1 std::ostream& operator<< ( std::ostream & os, const | Display & rhs ) [friend]
```

Overloads the extraction operator.

Delegates the work to the virtual function qpp::IDisplay::display()

The documentation for this class was generated from the following file:

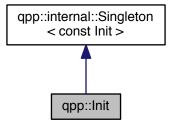
· classes/idisplay.h

7.6 qpp::Init Class Reference

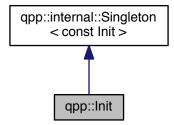
const Singleton class that performs additional initializations/cleanups

```
#include <classes/init.h>
```

Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



Private Member Functions

- Init ()
 - Additional initializations.
- ∼Init ()

Cleanups.

Friends

class internal::Singleton< const Init >

Additional Inherited Members

7.6.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

7.6.2 Constructor & Destructor Documentation

```
7.6.2.1 qpp::Init::Init( ) [inline],[private]
```

Additional initializations.

```
7.6.2.2 qpp::Init::~Init() [inline], [private]
```

Cleanups.

7.6.3 Friends And Related Function Documentation

7.6.3.1 friend class internal::Singleton < const lnit > [friend]

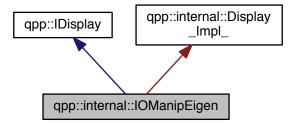
The documentation for this class was generated from the following file:

· classes/init.h

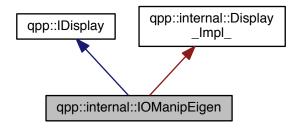
7.7 qpp::internal::IOManipEigen Class Reference

```
#include <internal/classes/iomanip.h>
```

Inheritance diagram for qpp::internal::IOManipEigen:



Collaboration diagram for qpp::internal::IOManipEigen:



Public Member Functions

- template < typename Derived >
 IOManipEigen (const Eigen::MatrixBase < Derived > &A, double chop=qpp::chop)
- IOManipEigen (const cplx z, double chop=qpp::chop)

Private Member Functions

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

Private Attributes

- cmat A
- double chop_

7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen (const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop) [inline], [explicit]
- 7.7.1.2 qpp::internal::IOManipEigen::IOManipEigen (const cplx z, double chop = qpp::chop) [inline], [explicit]

7.7.2 Member Function Documentation

7.7.2.1 std::ostream& qpp::internal::IOManipEigen::display(std::ostream & os) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

7.7.3 Member Data Documentation

7.7.3.1 cmat qpp::internal::IOManipEigen::A_ [private]

7.7.3.2 double qpp::internal::IOManipEigen::chop_ [private]

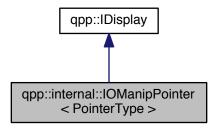
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

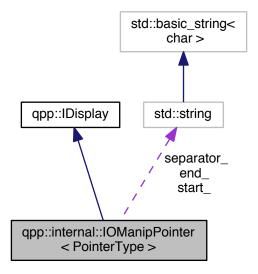
7.8 qpp::internal::IOManipPointer < PointerType > Class Template Reference

#include <internal/classes/iomanip.h>

 $Inheritance\ diagram\ for\ qpp::internal::IOManipPointer<\ PointerType>:$



Collaboration diagram for qpp::internal::IOManipPointer< PointerType >:



Public Member Functions

- IOManipPointer (const PointerType *p, idx N, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipPointer (const IOManipPointer &)=default
- IOManipPointer & operator= (const IOManipPointer &)=default

Private Member Functions

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

Private Attributes

- const PointerType * p_
- idx N
- std::string separator
- · std::string start_
- · std::string end_

7.8.1 Constructor & Destructor Documentation

- 7.8.1.1 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer (const PointerType * p, idx N, const std::string & separator, const std::string & start = " [", const std::string & end = "] ") [inline], [explicit]
- 7.8.1.2 template<typename PointerType> qpp::internal::IOManipPointer< PointerType>::IOManipPointer(const IOManipPointer<< PointerType> &) [default]
- 7.8.2 Member Function Documentation

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.8.2.2 template<typename PointerType> IOManipPointer& qpp::internal::IOManipPointer< PointerType
 >::operator=(const IOManipPointer< PointerType > &) [default]
- 7.8.3 Member Data Documentation
- 7.8.3.1 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::end_
 [private]
- **7.8.3.2** template<typename PointerType>idx qpp::internal::IOManipPointer< PointerType>::N_ [private]
- 7.8.3.3 template<typename PointerType> const PointerType* qpp::internal::IOManipPointer< PointerType >::p_ [private]

7.8.3.4 template < typename PointerType > std::string qpp::internal::IOManipPointer < PointerType >::separator_
[private]

7.8.3.5 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType >::start_ [private]

The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

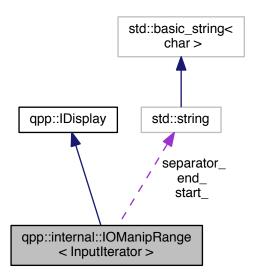
7.9 qpp::internal::IOManipRange < InputIterator > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipRange< InputIterator >:



Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



Public Member Functions

- IOManipRange (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipRange (const IOManipRange &)=default
- IOManipRange & operator= (const IOManipRange &)=default

Private Member Functions

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

Private Attributes

- InputIterator first
- InputIterator last
- std::string separator_
- std::string start
- · std::string end_

7.9.1 Constructor & Destructor Documentation

- 7.9.1.2 template<typename InputIterator> qpp::internal::IOManipRange< InputIterator>::IOManipRange (const IOManipRange < InputIterator > &) [default]
- 7.9.2 Member Function Documentation
- 7.9.2.1 template<typename InputIterator> std::ostream& qpp::internal::IOManipRange< InputIterator >::display (std::ostream & os) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.9.2.2 template<typename InputIterator> IOManipRange& qpp::internal::IOManipRange< InputIterator >::operator= (const IOManipRange< InputIterator > &) [default]
- 7.9.3 Member Data Documentation
- 7.9.3.1 template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::end_ [private]
- 7.9.3.2 template<typename InputIterator> InputIterator qpp::internal::IOManipRange< InputIterator>::first_
 [private]
- 7.9.3.3 template<typename InputIterator> InputIterator qpp::internal::IOManipRange< InputIterator>::last_ [private]

7.9.3.4 template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::separator_ [private]

7.9.3.5 template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::start_ [private]

The documentation for this class was generated from the following file:

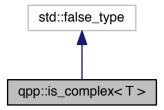
• internal/classes/iomanip.h

7.10 qpp::is_complex< T > Struct Template Reference

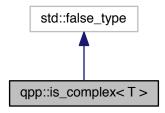
Checks whether the type is a complex type.

#include <traits.h>

Inheritance diagram for qpp::is_complex < T >:



Collaboration diagram for qpp::is_complex< T >:



7.10.1 Detailed Description

template < typename T > struct qpp::is_complex < T >

Checks whether the type is a complex type.

Provides the constant member value which is equal to true, if the type is a complex type, i.e. std::complex<T>

The documentation for this struct was generated from the following file:

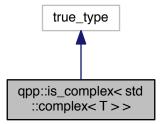
· traits.h

7.11 qpp::is_complex < std::complex < T > > Struct Template Reference

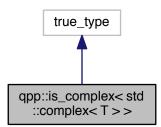
Checks whether the type is a complex number type, specialization for complex types.

#include <traits.h>

Inheritance diagram for qpp::is_complex< std::complex< T > >:



Collaboration diagram for qpp::is_complex < std::complex < T > >:



7.11.1 Detailed Description

 $template < typename \ T > struct \ qpp::is_complex < std::complex < T > >$

Checks whether the type is a complex number type, specialization for complex types.

The documentation for this struct was generated from the following file:

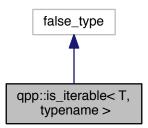
· traits.h

7.12 qpp::is_iterable < T, typename > Struct Template Reference

Checks whether *T* is compatible with an STL-like iterable container.

#include <traits.h>

Inheritance diagram for qpp::is_iterable < T, typename >:



Collaboration diagram for qpp::is_iterable < T, typename >:



7.12.1 Detailed Description

 $template < typename \ {\tt T}, typename \ {\tt = void} > {\tt struct \ qpp::is_iterable} < \ {\tt T}, typename >$

Checks whether *T* is compatible with an STL-like iterable container.

Provides the constant member *value* which is equal to *true*, if *T* is compatible with an iterable container, i.e. provides at least *begin()* and *end()* member functions. Otherwise, *value* is equal to *false*.

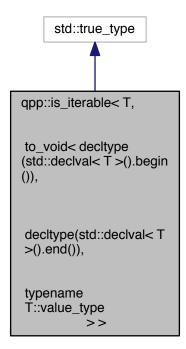
The documentation for this struct was generated from the following file:

• traits.h

7.13 qpp::is_iterable< T, to_void< decltype(std::declval< T >().begin()), decltype(std... ::declval< T >().end()), typename T::value_type > > Struct Template Reference

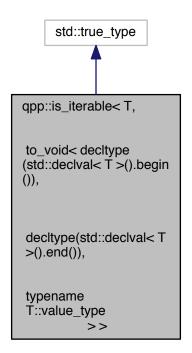
Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. #include <traits.h>

Inheritance diagram for qpp::is_iterable < T, to_void < decltype(std::declval < T >().begin()), decltype(std::declval < T >().end()), typename T::value_type > >:



 $Collaboration \ diagram \ for \ qpp::is_iterable < T, \ to_void < \ decltype(std::declval < T > ().begin()), \ decltype(std::declval < T > ().begin()),$

:declval< T >().end()), typename T::value_type > >:



7.13.1 Detailed Description

 $template < typename \ T > struct \ qpp::is_iterable < \ T, \ to_void < \ decltype(std::declval < \ T > ().begin()), \ decltype(std::declval < \ T > ().end()), \ typename \ T::value_type > >$

Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. The documentation for this struct was generated from the following file:

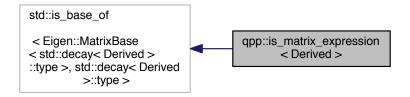
• traits.h

7.14 qpp::is_matrix_expression < Derived > Struct Template Reference

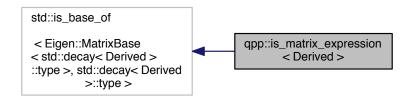
Checks whether the type is an Eigen matrix expression.

#include <traits.h>

Inheritance diagram for qpp::is_matrix_expression< Derived >:



Collaboration diagram for qpp::is_matrix_expression< Derived >:



7.14.1 Detailed Description

 $template < typename \ Derived > struct \ qpp::is_matrix_expression < \ Derived >$

Checks whether the type is an Eigen matrix expression.

Provides the constant member *value* which is equal to *true*, if the type is an Eigen matrix expression of type *Eigen ∷MatrixBase Derived >* . Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

· traits.h

7.15 qpp::RandomDevices Class Reference

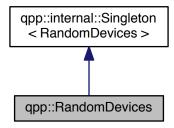
Singeleton class that manages the source of randomness in the library.

#include <classes/random_devices.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



Public Attributes

• std::mt19937 rng_

Mersenne twister random number generator.

Private Member Functions

• RandomDevices ()

Initializes and seeds the random number generators.

• \sim RandomDevices ()=default

Default destructor.

Private Attributes

• std::random_device rd_

used to seed std::mt19937 rng_

Friends

class internal::Singleton < RandomDevices >

Additional Inherited Members

7.15.1 Detailed Description

Singeleton class that manages the source of randomness in the library.

Consists of a wrapper around an std::mt19937 Mersenne twister random number generator engine and an std⇔ ::random_device engine. The latter is used to seed the Mersenne twister.

Warning

This class DOES NOT seed the standard C number generator used by Eigen::Matrix::Random(), since it is not thread safe. Do not use Eigen::Matrix::Random() or functions that depend on the C style random number engine, but use qpp::rand() instead!

7.15.2 Constructor & Destructor Documentation

```
7.15.2.1 qpp::RandomDevices::RandomDevices() [inline], [private]
```

Initializes and seeds the random number generators.

```
7.15.2.2 qpp::RandomDevices::~RandomDevices() [private], [default]
```

Default destructor.

7.15.3 Friends And Related Function Documentation

```
7.15.3.1 friend class internal::Singleton < RandomDevices > [friend]
```

7.15.4 Member Data Documentation

```
7.15.4.1 std::random_device qpp::RandomDevices::rd_ [private]
```

used to seed std::mt19937 rng_

7.15.4.2 std::mt19937 qpp::RandomDevices::rng_

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

• classes/random_devices.h

7.16 qpp::internal::Singleton< T > Class Template Reference

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

Static Public Member Functions

- static T & get_instance () noexcept(std::is_nothrow_constructible< T >::value)
- static T & get_thread_local_instance () noexcept(std::is_nothrow_constructible < T >::value)

Protected Member Functions

- Singleton () noexcept=default
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete
- virtual ∼Singleton ()=default

7.16.1 Detailed Description

template < typename T> class qpp::internal::Singleton < T>

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from qpp::internal::Singleton, make qpp::internal::Singleton a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function qpp::internal::Singleton::get_instance() (qpp::internal::Singleton::get_thread_local_← instance()), which returns a reference (thread_local_reference) to your newly created singleton (thread-safe in C++11).

Example:

See also

Code of qpp::Codes, qpp::Gates, qpp::Init, qpp::RandomDevices, qpp::States or qpp.h for real world examples of usage.

7.16.2 Constructor & Destructor Documentation

```
7.16.2.1 template < typename T > qpp::internal::Singleton < T >::Singleton ( ) [protected], [default], [noexcept]
```

```
7.16.2.2 template < typename T > qpp::internal::Singleton < T > ::Singleton ( const Singleton < T > & ) [protected], [delete]
```

```
7.16.2.3 template<typename T> virtual qpp::internal::Singleton< T>::\simSingleton( ) [protected], [virtual], [default]
```

7.16.3 Member Function Documentation

- 7.16.3.3 template<typename T> Singleton& qpp::internal::Singleton< T>::operator=(const Singleton< T>&) [protected], [delete]

The documentation for this class was generated from the following file:

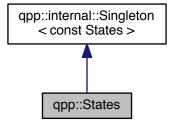
• internal/classes/singleton.h

7.17 qpp::States Class Reference

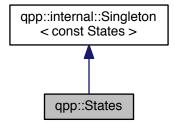
const Singleton class that implements most commonly used states

#include <classes/states.h>

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



Public Member Functions

• ket mes (idx d=2) const

Maximally entangled state of 2 qudits.

• ket zero (idx n, idx d=2) const

Zero state of n qudits.

ket one (idx n, idx d=2) const

One state of n qudits.

• ket jn (idx j, idx n, idx d=2) const

 $|j\rangle^{\otimes n}$ state of n qudits

• ket plus (idx n) const

Plus state of n qubits.

• ket minus (idx n) const

Minus state of n qubits.

Public Attributes

```
    ket x0 {ket::Zero(2)}
```

Pauli Sigma-X 0-eigenstate |+>

ket x1 {ket::Zero(2)}

Pauli Sigma-X 1-eigenstate |->

ket y0 {ket::Zero(2)}

Pauli Sigma-Y 0-eigenstate | y+>

ket y1 {ket::Zero(2)}

Pauli Sigma-Y 1-eigenstate |y->

ket z0 {ket::Zero(2)}

Pauli Sigma-Z 0-eigenstate | 0>

ket z1 {ket::Zero(2)}

Pauli Sigma-Z 1-eigenstate | 1>

cmat px0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.

cmat px1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.

cmat py0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 0-eigenstate $|y+\rangle < y+|$.

cmat py1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 1-eigenstate |y->< y-|.

cmat pz0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.

cmat pz1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.

ket b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

ket b01 {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

ket b10 {ket::Zero(4)}

Bell-10 state (following the convention in Nielsen and Chuang)

ket b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

cmat pb00 {cmat::Zero(4, 4)}

Projector onto the Bell-00 state.

```
    cmat pb01 {cmat::Zero(4, 4)}
```

Projector onto the Bell-01 state.

cmat pb10 {cmat::Zero(4, 4)}

Projector onto the Bell-10 state.

cmat pb11 {cmat::Zero(4, 4)}

Projector onto the Bell-11 state.

ket GHZ {ket::Zero(8)}

GHZ state.

ket W {ket::Zero(8)}

W state.

• cmat pGHZ {cmat::Zero(8, 8)}

Projector onto the GHZ state.

cmat pW {cmat::Zero(8, 8)}

Projector onto the W state.

Private Member Functions

- States ()
- ∼States ()=default

Default destructor.

Friends

class internal::Singleton < const States >

Additional Inherited Members

7.17.1 Detailed Description

const Singleton class that implements most commonly used states

7.17.2 Constructor & Destructor Documentation

```
7.17.2.1 qpp::States::States() [inline], [private]
Initialize the states
```

```
7.17.2.2 qpp::States:: \sim States( ) [private], [default]
```

Default destructor.

7.17.3 Member Function Documentation

```
7.17.3.1 ket qpp::States::jn ( idx j, idx n, idx d=2 ) const [inline] |j\rangle^{\otimes n} state of n qudits
```

Parameters

j	Non-negative integer
n	Non-negative integer
d	Subsystem dimensions

Returns

 $|j\rangle^{\otimes n}$ state of *n* qudits

7.17.3.2 ket qpp::States::mes (idx d = 2) const [inline]

Maximally entangled state of 2 qudits.

Parameters

d	Subsystem dimensions

Returns

Maximally entangled state $\frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |jj\rangle$ of 2 qudits

7.17.3.3 ket qpp::States::minus (idx n) const [inline]

Minus state of *n* qubits.

Parameters

n	Non-negative integer
- 11	Non-negative integer

Returns

Minus state $|-\rangle^{\otimes n}$ of n qubits

7.17.3.4 ket qpp::States::one (idx n, idx d = 2) const [inline]

One state of *n* qudits.

Parameters

n	Non-negative integer
d	Subsystem dimensions

Returns

One state $|1\rangle^{\otimes n}$ of n qudits

7.17.3.5 ket qpp::States::plus (idx n) const [inline]

Plus state of n qubits.

Parameters

_	New pagetive integral
[]	Non-negative integer

Returns

Plus state $|+\rangle^{\otimes n}$ of n qubits

7.17.3.6 ket qpp::States::zero (idx n, idx d = 2) const [inline]

Zero state of *n* qudits.

Parameters

n	Non-negative integer
d	Subsystem dimensions

Returns

Zero state $|0\rangle^{\otimes n}$ of n qudits

7.17.4 Friends And Related Function Documentation

7.17.4.1 friend class internal::Singleton < const States > [friend]

7.17.5 Member Data Documentation

7.17.5.1 ket qpp::States::b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

7.17.5.2 ket qpp::States::b01 {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

7.17.5.3 ket qpp::States::b10 {ket::Zero(4)}

Bell-10 state (following the convention in Nielsen and Chuang)

7.17.5.4 ket qpp::States::b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

7.17.5.5 ket qpp::States::GHZ {ket::Zero(8)}

GHZ state.

7.17.5.6 cmat qpp::States::pb00 {cmat::Zero(4, 4)}

Projector onto the Bell-00 state.

```
7.17.5.7 cmat qpp::States::pb01 {cmat::Zero(4, 4)}
Projector onto the Bell-01 state.
7.17.5.8 cmat qpp::States::pb10 {cmat::Zero(4, 4)}
Projector onto the Bell-10 state.
7.17.5.9 cmat qpp::States::pb11 {cmat::Zero(4, 4)}
Projector onto the Bell-11 state.
7.17.5.10 cmat qpp::States::pGHZ {cmat::Zero(8, 8)}
Projector onto the GHZ state.
7.17.5.11 cmat qpp::States::pW {cmat::Zero(8, 8)}
Projector onto the W state.
7.17.5.12 cmat qpp::States::px0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.
7.17.5.13 cmat qpp::States::px1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.
7.17.5.14 cmat qpp::States::py0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 0-eigenstate |y+>< y+|.
7.17.5.15 cmat qpp::States::py1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.
7.17.5.16 cmat qpp::States::pz0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.
7.17.5.17 cmat qpp::States::pz1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.
7.17.5.18 ket qpp::States::W {ket::Zero(8)}
W state.
```

7.17.5.19 ket qpp::States::x0 {ket::Zero(2)}

Pauli Sigma-X 0-eigenstate |+>

7.17.5.20 ket qpp::States::x1 {ket::Zero(2)}

Pauli Sigma-X 1-eigenstate |->

7.17.5.21 ket qpp::States::y0 {ket::Zero(2)}

Pauli Sigma-Y 0-eigenstate |y+>

7.17.5.22 ket qpp::States::y1 {ket::Zero(2)}

Pauli Sigma-Y 1-eigenstate |y->

7.17.5.23 ket qpp::States::z0 {ket::Zero(2)}

Pauli Sigma-Z 0-eigenstate |0>

7.17.5.24 ket qpp::States::z1 {ket::Zero(2)}

Pauli Sigma-Z 1-eigenstate |1>

The documentation for this class was generated from the following file:

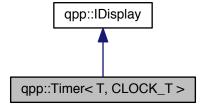
· classes/states.h

7.18 qpp::Timer < T, CLOCK_T > Class Template Reference

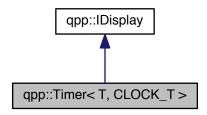
Chronometer.

#include <classes/timer.h>

Inheritance diagram for qpp::Timer< T, CLOCK_T >:



Collaboration diagram for qpp::Timer < T, CLOCK_T >:



Public Member Functions

• Timer () noexcept

Constructs an instance with the current time as the starting point.

· void tic () noexcept

Resets the chronometer.

• const Timer & toc () noexcept

Stops the chronometer.

· double tics () const noexcept

Time passed in the duration specified by T.

• template<typename U = T>

U get_duration () const noexcept

Duration specified by U.

• Timer (const Timer &)=default

Default copy constructor.

• Timer (Timer &&)=default

Default move constructor.

• Timer & operator= (const Timer &)=default

Default copy assignment operator.

• Timer & operator= (Timer &&)=default

Default move assignment operator.

virtual ∼Timer ()=default

Default virtual destructor.

Protected Attributes

CLOCK_T::time_point start_

• CLOCK_T::time_point end_

Private Member Functions

• std::ostream & display (std::ostream &os) const override

qpp::IDisplay::display() override

7.18.1 Detailed Description

 $template < typename\ T = std::chrono::duration < double >,\ typename\ CLOCK_T = std::chrono::steady_clock > class\ qpp::Timer < T,\ CLOCK_T >$

Chronometer.

128 Class Documentation

Template Parameters

T	Tics duration, default is std::chrono::duration <double, 1="">, i.e. seconds in double</double,>
	precision
CLOCK_T	Clock's type, default is std::chrono::steady_clock, not affected by wall clock
	changes during runtime

7.18.2 Constructor & Destructor Documentation

Constructs an instance with the current time as the starting point.

```
7.18.2.2 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> qpp::Timer< T, CLOCK_T > ::Timer( const Timer< T, CLOCK_T > & ) [default]
```

Default copy constructor.

Default move constructor.

```
7.18.2.4 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> virtual qpp::Timer < T, CLOCK_T > :: \sim Timer ( ) [virtual], [default]
```

Default virtual destructor.

7.18.3 Member Function Documentation

qpp::IDisplay::display() override

Parameters

OS	Output stream
03	Output stream

Returns

Writes to the output stream the number of tics (specified by T) that passed between the instantiation/reset and invocation of qpp::Timer::toc().

Implements qpp::IDisplay.

Duration specified by U.

Template Parameters

U	Duration, default is T, which defaults to std::chrono::duration <double, 1="">, i.e.</double,>
	seconds in double precision

Returns

Duration that passed between the instantiation/reset and invocation of qpp::Timer::toc()

Default copy assignment operator.

7.18.3.4 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> Timer& qpp::Timer< T, CLOCK_T > ::operator=(Timer< T, CLOCK_T > &&) [default]

Default move assignment operator.

7.18.3.5 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> void qpp::Timer< T, CLOCK_T >::tic() [inline], [noexcept]

Resets the chronometer.

Resets the starting/ending point to the current time

Time passed in the duration specified by T.

Returns

Number of tics (specified by T) that passed between the instantiation/reset and invocation of qpp::Timer::toc()

7.18.3.7 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> const Timer& qpp::Timer< T, CLOCK_T >::toc() [inline], [noexcept]

Stops the chronometer.

Set the current time as the ending point

Returns

Current instance

- 7.18.4 Member Data Documentation
- 7.18.4.1 template < typename T = std::chrono::duration < double >, typename CLOCK_T = std::chrono::steady_clock > CLOCK_T::time_point qpp::Timer < T, CLOCK_T >::end_ [protected]
- 7.18.4.2 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> CLOCK_T::time_point qpp::Timer< T, CLOCK_T >::start_ [protected]

The documentation for this class was generated from the following file:

130 Class Documentation

• classes/timer.h

Chapter 8

File Documentation

8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Codes const Singleton class that defines quantum error correcting codes

Namespaces

• qpp

Quantum++ main namespace.

8.1.1 Detailed Description

Quantum error correcting codes.

8.2 classes/exception.h File Reference

Exceptions.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Exception

Generates custom exceptions, used when validating function parameters.

Namespaces

• qpp

Quantum++ main namespace.

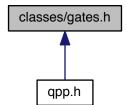
8.2.1 Detailed Description

Exceptions.

8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Gates

const Singleton class that implements most commonly used gates

Namespaces

qpp

Quantum++ main namespace.

8.3.1 Detailed Description

Quantum gates.

8.4 classes/idisplay.h File Reference

Display interface via the non-virtual interface (NVI)

This graph shows which files directly or indirectly include this file:



Classes

· class qpp::IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

Namespaces

• qpp

Quantum++ main namespace.

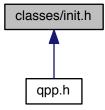
8.4.1 Detailed Description

Display interface via the non-virtual interface (NVI)

8.5 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Init

const Singleton class that performs additional initializations/cleanups

Namespaces

• qpp

Quantum++ main namespace.

8.5.1 Detailed Description

Initialization.

8.6 classes/random_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::RandomDevices

Singeleton class that manages the source of randomness in the library.

Namespaces

qpp

Quantum++ main namespace.

8.6.1 Detailed Description

Random devices.

8.7 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



Classes

class qpp::States

const Singleton class that implements most commonly used states

Namespaces

• qpp

Quantum++ main namespace.

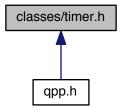
8.7.1 Detailed Description

Quantum states.

8.8 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



Classes

class qpp::Timer < T, CLOCK_T >
 Chronometer.

Namespaces

• qpp

Quantum++ main namespace.

8.8.1 Detailed Description

Timing.

8.9 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

• constexpr cplx qpp::operator""_i (unsigned long long int x) noexcept

User-defined literal for complex $i = \sqrt{-1}$ (integer overload)

• constexpr cplx qpp::operator""_i (long double x) noexcept

User-defined literal for complex $i = \sqrt{-1}$ (real overload)

• cplx qpp::omega (idx D)

D-th root of unity.

Variables

• constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx qpp::maxn = 64

Maximum number of allowed qubits/qudits (subsystems)

• constexpr double qpp::pi = 3.141592653589793238462643383279502884

 π

• constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

• constexpr double qpp::infty = std::numeric_limits<double>::infinity()

Used to denote infinity in double precision.

8.9.1 Detailed Description

Constants.

8.10 entanglement.h File Reference

Entanglement functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Negativity of the bi-partite mixed state A.

Logarithmic negativity of the bi-partite mixed state A.

double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, idx d=2)

• template<typename Derived >

template<typename Derived >

Functions

 template<typename Derived > dyn col vect< double > gpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt coefficients of the bi-partite pure state A. • template<typename Derived > dyn_col_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt coefficients of the bi-partite pure state A. template<typename Derived > cmat qpp::schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt basis on Alice side. template<typename Derived > cmat qpp::schmidtA (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt basis on Alice side. template<typename Derived > cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt basis on Bob side. template<typename Derived > cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt basis on Bob side. • template<typename Derived > std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt probabilities of the bi-partite pure state A. template<typename Derived > std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, idx d=2) Schmidt probabilities of the bi-partite pure state A. • template<typename Derived > double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Entanglement of the bi-partite pure state A. template<typename Derived > double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, idx d=2) Entanglement of the bi-partite pure state A. template<typename Derived > double qpp::gconcurrence (const Eigen::MatrixBase Derived > &A) G-concurrence of the bi-partite pure state A. • template<typename Derived > double qpp::negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Negativity of the bi-partite mixed state A. template<typename Derived > double qpp::negativity (const Eigen::MatrixBase Derived > &A, idx d=2)

double qpp::lognegativity (const Eigen::MatrixBase Derived > &A, const std::vector< idx > &dims)

Logarithmic negativity of the bi-partite mixed state A.

• template<typename Derived >

double qpp::concurrence (const Eigen::MatrixBase Derived > &A)

Wootters concurrence of the bi-partite qubit mixed state A.

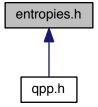
8.10.1 Detailed Description

Entanglement functions.

8.11 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

```
    template < typename Derived >
        double qpp::entropy (const Eigen::MatrixBase < Derived > &A)
```

von-Neumann entropy of the density matrix A

double qpp::entropy (const std::vector< double > &prob)

Shannon entropy of the probability distribution prob.

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$

double qpp::renyi (const Eigen::MatrixBase< Derived > &A, double alpha)

Renyi- α entropy of the density matrix A, for $\alpha \geq 0$.

double qpp::renyi (const std::vector< double > &prob, double alpha)

Renyi- α entropy of the probability distribution prob, for $\alpha \geq 0$.

• template<typename Derived >

double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)

Tsallis- q entropy of the density matrix A, for $q \geq 0$.

double qpp::tsallis (const std::vector< double > &prob, double q)

Tsallis- q entropy of the probability distribution prob, for $q \geq 0$.

template<typename Derived >
 double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)

Quantum mutual information between 2 subsystems of a composite system.

template<typename Derived >
 double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)

Quantum mutual information between 2 subsystems of a composite system.

8.11.1 Detailed Description

Entropy functions.

8.12 experimental/experimental.h File Reference

Experimental/test functions/classes.

Namespaces

• qpp

Quantum++ main namespace.

· qpp::experimental

Experimental/test functions/classes, do not use or modify.

8.12.1 Detailed Description

Experimental/test functions/classes.

8.13 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

```
    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > gpp::conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::trace">qpp::trace</a> (const Eigen::MatrixBase</a> Derived > &A)
      Trace.

    template<typename Derived >

  Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)
      Determinant.

    template<typename Derived >

  Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.
template<typename Derived >
  Derived::Scalar <a href="mailto:qpp::sum">qpp::sum</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise sum of A.
template<typename Derived >
  Derived::Scalar <a href="mailto:open:prod">open::prod</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise product of A.
• template<typename Derived >
  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase</a> Derived > &A)
      Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition.
• template<typename Derived >
  dyn col vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.

    template<typename Derived >

  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition of Hermitian expression.
• template<typename Derived >
  dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)
```

Hermitian eigenvalues.

```
• template<typename Derived >
  cmat qpp::hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.
• template<typename Derived >
  std::tuple< cmat, dyn_col_vect< double >, cmat > qpp::svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.
• template<typename Derived >
  dyn_col_vect< double > qpp::svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.

    template<typename Derived >

  cmat qpp::svdU (const Eigen::MatrixBase< Derived > &A)
     Left singular vectors.

    template<typename Derived >

  cmat qpp::svdV (const Eigen::MatrixBase< Derived > &A)
     Right singular vectors.

    template < typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
• template<typename Derived >
  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolute value.

    template<typename Derived >

  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat qpp::logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.
• template<typename Derived >
  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.
template<typename Derived >
  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.
• template<typename Derived >
  double qpp::schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn mat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)( const
  typename Derived::Scalar &))
     Functor.

    template<typename T >

  dyn_mat< typename T::Scalar > qpp::kron (const T &head)
     Kronecker product.
```

```
• template<typename T, typename... Args>
  dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)
     Kronecker product.
\bullet \ \ {\it template}{<} {\it typename Derived}>
  dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn mat< typename T::Scalar > qpp::dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows,
  idx cols)
     Reshape.

    template<typename Derived1 , typename Derived2 >

  dyn mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Commutator.
• template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &A)
     Projector.

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &As)
     Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &As)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.
• std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)
     Non-negative integer index to multi-index.
```

idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

Multi-index to non-negative integer index.

ket qpp::mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

Multi-partite qudit ket.

ket qpp::mket (const std::vector< idx > &mask, idx d=2)

Multi-partite qudit ket.

cmat qpp::mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

Projector onto multi-partite qudit ket.

cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)

Projector onto multi-partite qudit ket.

template<typename InputIterator >

std::vector< double > qpp::abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of an STL-like range of complex numbers.

• template<typename Container >

```
std::vector< double > qpp::abssq (const Container &c, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)
```

Computes the absolute values squared of an STL-like container.

template<typename Derived >

```
std::vector< double > qpp::abssq (const Eigen::MatrixBase< Derived > &A)
```

Computes the absolute values squared of an Eigen expression.

• template<typename InputIterator >

```
std::iterator traits < InputIterator >::value type qpp::sum (InputIterator first, InputIterator last)
```

Element-wise sum of an STL-like range.

• template<typename Container >

```
Container::value_type qpp::sum (const Container &c, typename std::enable_if< is_iterable< Container >--- ::value >::type *=nullptr)
```

Element-wise sum of the elements of an STL-like container.

• template<typename InputIterator >

```
std::iterator traits< InputIterator >::value type qpp::prod (InputIterator first, InputIterator last)
```

Element-wise product of an STL-like range.

template<typename Container >

```
Container::value_type qpp::prod (const Container &c, typename std::enable_if< is_iterable< Container >\leftarrow::value >::type *=nullptr)
```

Element-wise product of the elements of an STL-like container.

• template<typename Derived >

```
dyn_col_vect< typename Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)
```

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

```
std::vector< T > qpp::complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$

```
std::vector< double > qpp::rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat qpp::bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

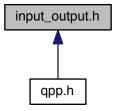
8.13.1 Detailed Description

Generic quantum computing functions.

8.14 input_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

template<typename Derived >
 internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

Eigen expression ostream manipulator.

template<typename InputIterator >
 internal::IOManipRange< InputIterator > qpp::disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container>
 internal::IOManipRange< typename Container::const_iterator > qpp::disp (const Container &c, const std
 ::string &separator, const std::string &start="[", const std::string &end="]", typename std::enable_if< is_←
 iterable< Container >::value >::type *=nullptr)

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >
 internal::IOManipPointer< PointerType > qpp::disp (const PointerType *p, idx N, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

ullet template<typename Derived >

void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

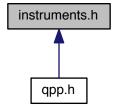
8.14.1 Detailed Description

Input/output functions.

8.15 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

template<typename Derived >
 dyn_col_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, const std::vector< idx > &dims)

• template<typename Derived >

dyn_col_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

Generalized inner product.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

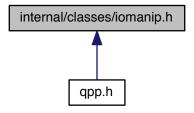
8.15.1 Detailed Description

Measurement functions.

8.16 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



Classes

- class qpp::internal::IOManipRange< InputIterator >
- class qpp::internal::IOManipPointer< PointerType >
- class qpp::internal::IOManipEigen

Namespaces

• qpp

Quantum++ main namespace.

• qpp::internal

Internal utility functions, do not use/modify.

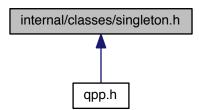
8.16.1 Detailed Description

Input/output manipulators.

8.17 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



Classes

class qpp::internal::Singleton< T >

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

Namespaces

• qpp

Quantum++ main namespace.

• qpp::internal

Internal utility functions, do not use/modify.

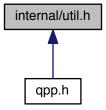
8.17.1 Detailed Description

Singleton pattern via CRTP.

8.18 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



Classes

• struct qpp::internal::Display_Impl_

Namespaces

• qpp

Quantum++ main namespace.

· qpp::internal

Internal utility functions, do not use/modify.

Functions

- void qpp::internal::n2multiidx (idx n, idx numdims, const idx *const dims, idx *result) noexcept
- idx qpp::internal::multiidx2n (const idx *const midx, idx numdims, const idx *const dims) noexcept
- template<typename Derived >
 bool qpp::internal::check_square_mat (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::check_vector (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::check_rvector (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::check_cvector (const Eigen::MatrixBase< Derived > &A)
- template < typename T >
 bool qpp::internal::check_nonzero_size (const T &x) noexcept
- template<typename T1, typename T2 >
 bool qpp::internal::check_matching_sizes (const T1 &lhs, const T2 &rhs) noexcept
- bool qpp::internal::check_dims (const std::vector< idx > &dims)
- template<typename Derived >
 bool qpp::internal::check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::check_dims_match_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::check_dims_match_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- bool qpp::internal::check_eq_dims (const std::vector< idx > &dims, idx dim) noexcept
- bool qpp::internal::check_subsys_match_dims (const std::vector< idx > &subsys, const std::vector< idx > &dims)
- template<typename Derived >
 bool qpp::internal::check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >
 bool qpp::internal::check_qubit_cvector (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >
 bool qpp::internal::check_qubit_rvector (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >
 bool qpp::internal::check_qubit_vector (const Eigen::MatrixBase< Derived > &A) noexcept
- bool qpp::internal::check_perm (const std::vector< idx > &perm)
- template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::internal::kron2 (const Eigen::MatrixBase< Derived1 > &A,
 const Eigen::MatrixBase< Derived2 > &B)
- template<typename Derived1 , typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::internal::dirsum2 (const Eigen::MatrixBase< Derived1 > &A,
 const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >
 void qpp::internal::variadic_vector_emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
 void qpp::internal::variadic_vector_emplace (std::vector< T > &v, First &&first, Args &&...args)
- idx qpp::internal::get_num_subsys (idx sz, idx d)
- idx qpp::internal::get_dim_subsys (idx sz, idx N)

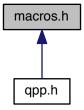
8.18.1 Detailed Description

Internal utility functions.

8.19 macros.h File Reference

Preprocessor macros.

This graph shows which files directly or indirectly include this file:



Macros

- #define PRINT(x) std::cout << (x)
- #define PRINTLN(x) std::cout << (x) << std::endl
- #define ERROR(x) std::cerr << (x)
- #define ERRORLN(x) std::cerr << (x) << std::endl

8.19.1 Detailed Description

Preprocessor macros.

8.19.2 Macro Definition Documentation

8.19.2.1 #define ERROR(
$$x$$
) std::cerr $<<$ (x)

Prints an error message to std::cerr

8.19.2.2 #define ERRORLN(
$$x$$
) std::cerr $<<$ (x) $<<$ std::endl

Prints an error message to std::cerr and adds a new line

8.19.2.3 #define PRINT(
$$x$$
) std::cout $<<$ (x)

Prints a message

8.19.2.4 #define PRINTLN(
$$x$$
) std::cout $<<$ (x) $<<$ std::endl

Prints a message and adds a new line

8.20 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

Namespaces

• qpp

Quantum++ main namespace.

Functions

template < typename Derived >
 Derived qpp::loadMATLABmatrix (const std::string &, const std::string &)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>
 dmat qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

cmat qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &, const std::string &, const std::string &)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

• template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat_file, const std ::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

• template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat_file, const std ::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

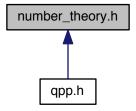
8.20.1 Detailed Description

Input/output interfacing with MATLAB.

8.21 number_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

• std::vector< int > qpp::x2contfrac (double x, idx N, idx cut=1e5)

Simple continued fraction expansion.

double qpp::contfrac2x (const std::vector< int > &cf, idx N)

Real representation of a simple continued fraction.

double qpp::contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• bigint qpp::gcd (bigint a, bigint b)

Greatest common divisor of two integers.

bigint qpp::gcd (const std::vector< bigint > &as)

Greatest common divisor of a list of integers.

bigint qpp::lcm (bigint a, bigint b)

Least common multiple of two integers.

bigint qpp::lcm (const std::vector< bigint > &as)

Least common multiple of a list of integers.

• std::vector< idx > qpp::invperm (const std::vector< idx > &perm)

Inverse permutation.

• std::vector< idx > qpp::compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< bigint > qpp::factors (bigint a)

Prime factor decomposition.

bigint qpp::modmul (bigint a, bigint b, bigint p)

Modular multiplication without overflow.

• bigint qpp::modpow (bigint a, bigint n, bigint p)

Fast integer power modulo p based on the SQUARE-AND-MULTIPLY algorithm.

std::tuple < bigint, bigint, bigint > qpp::egcd (bigint a, bigint b)

Extended greatest common divisor of two integers.

• bigint gpp::modinv (bigint a, bigint p)

Modular inverse of a mod p.

• bool qpp::isprime (bigint p, idx k=80)

Primality test based on the Miller-Rabin's algorithm.

• bigint qpp::randprime (bigint a, bigint b, idx N=1000)

Generates a random big prime uniformly distributed in the interval [a, b].

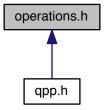
8.21.1 Detailed Description

Number theory functions.

8.22 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



Namespaces

dbb

Quantum++ main namespace.

Functions

template<typename Derived1 , typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 const std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 idx d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const
 Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix A.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix A.

• template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix A.

cmat qpp::kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat qpp::kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > qpp::choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat qpp::choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat qpp::super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

template<typename Derived >

Partial trace

• template<typename Derived >

 $\label{local_dyn_mat} \textit{dyn_mat} < \textit{typename Derived::} Scalar > \textit{qpp::ptrace1} \; (\textit{const Eigen::} MatrixBase < Derived > \&A, \; idx \; d=2)$

template < typename Derived >

 $dyn_mat < typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase < Derived > &A, const std <math>\leftrightarrow$::vector < idx > &dims)

Partial trace.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase< Derived > &A, idx d=2)

Partial trace.

• template<typename Derived >

Partial trace.

template<typename Derived >

 $\label{lem:dyn_mat} $$ dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std $$::vector< idx > &subsys, idx d=2) $$$

Partial trace.

 $\bullet \ \ {\sf template}{<} {\sf typename \ Derived} >$

dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Partial transpose.

template<typename Derived >

 $\label{lem:dyn_mat} $$ dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2) $$$

Partial transpose.
 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, const std::vector< idx > &dims)</typename>
Subsystem permutation.
 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)</typename>
Subsystem permutation.
8.22.1 Detailed Description
Quantum operation functions.
8.23 qpp.h File Reference
Quantum++ main header file, includes all other necessary headers.

156

File Documentation

```
#include <algorithm>
#include <cassert>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <memory>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "macros.h"
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "traits.h"
#include "classes/idisplay.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "statistics.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

Namespaces

qpp

Quantum++ main namespace.

Macros

#define QPP UNUSED

8.23.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

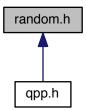
8.23.2 Macro Definition Documentation

8.23.2.1 #define QPP_UNUSED_

8.24 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

• double qpp::rand (double a, double b)

Generates a random real number uniformly distributed in the interval [a, b)

bigint qpp::rand (bigint a, bigint b)

Generates a random big integer uniformly distributed in the interval [a, b].

idx qpp::randidx (idx a=std::numeric_limits < idx >::min(), idx b=std::numeric_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

ullet template<typename Derived >

Derived qpp::rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

template<typename Derived >

Derived qpp::randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

• template<>

dmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

• template<>

cmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

double qpp::randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat qpp::randU (idx D=2)

Generates a random unitary matrix.

cmat qpp::randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > qpp::randkraus (idx N, idx D=2)

Generates a set of random Kraus operators.

cmat qpp::randH (idx D=2)

Generates a random Hermitian matrix.

ket qpp::randket (idx D=2)

Generates a random normalized ket (pure state vector)

• cmat qpp::randrho (idx D=2)

Generates a random density matrix.

std::vector< idx > qpp::randperm (idx N)

Generates a random uniformly distributed permutation.

std::vector< double > qpp::randprob (idx N)

Generates a random probability vector uniformly distributed over the probability simplex.

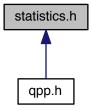
8.24.1 Detailed Description

Randomness-related functions.

8.25 statistics.h File Reference

Statistics functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

std::vector< double > qpp::uniform (idx N)

Uniform probability distribution vector.

std::vector< double > qpp::marginalX (const dmat &probXY)

Marginal distribution.

std::vector< double > qpp::marginalY (const dmat &probXY)

Marginal distribution.

• template<typename Container >

double qpp::avg (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_← iterable< Container >::value >::type *=nullptr)

Average.

• template<typename Container >

double qpp::cov (const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)

Covariance.

• template<typename Container >

double qpp::var (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_← iterable< Container >::value >::type *=nullptr)

Variance.

• template<typename Container >

double qpp::sigma (const std::vector< double > &prob, const Container &X, typename std::enable_if< is_← iterable< Container >::value >::type *=nullptr)

Standard deviation.

• template<typename Container >

 $\label{local-container} \begin{tabular}{ll} double & qpp::cor (const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if < is_iterable < Container >::value >::type *=nullptr) \end{tabular}$

Correlation.

8.25.1 Detailed Description

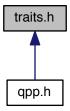
Statistics functions.

8.26 traits.h File Reference 161

8.26 traits.h File Reference

Type traits.

This graph shows which files directly or indirectly include this file:



Classes

struct qpp::is_iterable < T, typename >

Checks whether T is compatible with an STL-like iterable container.

struct qpp::is_iterable< T, to_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().← end()), typename T::value_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

struct qpp::is_matrix_expression< Derived >

Checks whether the type is an Eigen matrix expression.

struct qpp::is_complex< T >

Checks whether the type is a complex type.

struct qpp::is_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

Namespaces

• qpp

Quantum++ main namespace.

Typedefs

template<typename... > using qpp::to_void = void

Alias template that implements the proposal for void_t.

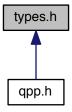
8.26.1 Detailed Description

Type traits.

8.27 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Typedefs

• using qpp::idx = std::size t

Non-negative integer index.

• using qpp::bigint = long long int

Big integer.

• using qpp::cplx = std::complex< double >

Complex number in double precision.

using qpp::ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

• using qpp::bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using qpp::cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using qpp::dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$

 $using \ \ qpp:: dyn_mat = Eigen:: Matrix < Scalar, \ Eigen:: Dynamic, \ Eigen:: Dynamic > \\$

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

using qpp::dyn_col_vect = Eigen::Matrix < Scalar, Eigen::Dynamic, 1 >

Dynamic Eigen column vector over the field specified by Scalar.

• template<typename Scalar >

using qpp::dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

8.27.1 Detailed Description

Type aliases.

Index

\sim Codes	qpp::Gates, 99
qpp::Codes, 88	CNOTba
\sim Gates	qpp::Gates, 99
qpp::Gates, 96	CTRL
\sim IDisplay	qpp::Gates, 96
qpp::IDisplay, 102	CUSTOM_EXCEPTION
∼Init	qpp::Exception, 92
qpp::Init, 104	CZ
~RandomDevices	qpp::Gates, 99
qpp::RandomDevices, 117	check_cvector
~Singleton	qpp::internal, 85
qpp::internal::Singleton, 118	check_dims
~States	qpp::internal, 85
qpp::States, 121	check_dims_match_cvect
~Timer	qpp::internal, 85
qpp::Timer, 128	check_dims_match_mat
۸	qpp::internal, 85
A_ qpp::internal::IOManipEigen, 106	check_dims_match_rvect
absm	qpp::internal, 85
qpp, 26	check_eq_dims
abssq	qpp::internal, 85
qpp, 27	check_matching_sizes
adjoint	qpp::internal, 85
qpp, 27	check_nonzero_size
anticomm	qpp::internal, 85
qpp, 28	check_perm
apply	qpp::internal, 85
qpp, 28, 29	check_qubit_cvector
applyCTRL	qpp::internal, 85
qpp, 30	check_qubit_matrix
avg	qpp::internal, 85
qpp, 31	check_qubit_rvector
-11-1-7 -	qpp::internal, 85
b00	check_qubit_vector
qpp::States, 123	qpp::internal, 85
b01	check_rvector
qpp::States, 123	qpp::internal, 85
b10	check_square_mat
qpp::States, 123	qpp::internal, 85
b11	check_subsys_match_dims
qpp::States, 123	qpp::internal, 85
bigint	check_vector
qpp, 25	qpp::internal, 85
bloch2rho	choi2kraus
qpp, 31	qpp, 31
bra	choi2super
qpp, 25	qpp, 32
	chop
CNOT	qpp, 83

chop_	qpp, 36, 37
qpp::internal::IOManipEigen, 106	dirsum2
classes/codes.h, 131	qpp::internal, 85
classes/exception.h, 131	dirsumpow
classes/gates.h, 132	qpp, 37
classes/idisplay.h, 133	disp
classes/init.h, 134	qpp, 38, 39
classes/random_devices.h, 134	display
classes/states.h, 135	qpp::IDisplay, 102
classes/timer.h, 136	qpp::Timer, 128
cmat	qpp::internal::IOManipEigen, 105
qpp, 25	qpp::internal::IOManipPointer, 107
Codes	qpp::internal::IOManipRange, 109
qpp::Codes, 88	display_impl_
codeword	qpp::internal::Display_Impl_, 90
qpp::Codes, 89	dmat
comm	qpp, 25
qpp, 32	dyn_col_vect
complement	qpp, 26
qpp, 32	dyn_mat
compperm	qpp, 26
qpp, 32	dyn_row_vect
concurrence	qpp, <u>26</u>
qpp, 34	
conjugate	ERROR
qpp, 34	macros.h, 151
constants.h, 136	ERRORLN
construct_exception_msg_	macros.h, 151
qpp::Exception, 93	ee
contfrac2x	qpp, <mark>83</mark>
qpp, 34	egcd
cor	qpp, 39
qpp, 35	eig
cosm	qpp, 40
qpp, 35	end_
COV	qpp::Timer, 129
qpp, 35	gpp::internal::IOManipPointer, 107
cplx	qpp::internal::IOManipRange, 109
qpp, 25	entanglement
custom	qpp, 40
qpp::Exception, 93	entanglement.h, 137
cwise	entropies.h, 139
	3.11. 3p. 33.11, 100
ann 36	entropy
qpp, 36	entropy
	qpp, 41
DIMS_INVALID	qpp, 41 eps
DIMS_INVALID qpp::Exception, 92	qpp, 41 eps qpp, 83
DIMS_INVALID	qpp, 41 eps qpp, 83 evals
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout qpp::Gates, 96, 97
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout qpp::Gates, 96, 97
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout qpp::Gates, 96, 97 experimental/experimental.h, 140
DIMS_INVALID	qpp, 41 eps
DIMS_INVALID	qpp, 41 eps qpp, 83 evals qpp, 41 evects qpp, 41 Exception qpp::Exception, 92, 93 expandout qpp::Gates, 96, 97 experimental/experimental.h, 140 expm

qpp::Codes, 88	Init
FRED	qpp::Init, 104
qpp::Gates, 99	input_output.h, 145
factors	instruments.h, 146
qpp, 42	internal/classes/iomanip.h, 147
Fd	internal/classes/singleton.h, 148
qpp::Gates, 98	internal/util.h, 149
first	internal::Singleton< const Codes >
qpp::internal::IOManipRange, 109	qpp::Codes, 89
functions.h, 140	internal::Singleton< const Gates >
funm	gpp::Gates, 99
qpp, 42	internal::Singleton< const Init >
4 F F	qpp::Init, 104
GHZ	internal::Singleton< const States >
qpp::States, 123	qpp::States, 123
Gates	internal::Singleton< RandomDevices >
qpp::Gates, 96	qpp::RandomDevices, 117
gcd	
qpp, 42, 43	inverse
gconcurrence	qpp, 45
qpp, 43	invperm
	qpp, 45
get_dim_subsys	ip
qpp::internal, 86	qpp, 45, 47
get_duration	isprime
qpp::Timer, 128	qpp, 47
get_instance	
qpp::internal::Singleton, 119	jn
get_num_subsys	qpp::States, 121
qpp::internal, 86	
get_thread_local_instance	ket
qpp::internal::Singleton, 119	qpp, 26
grams	kraus2choi
qpp, 43, 44	qpp, 47
	kraus2super
Н	qpp, 48
qpp::Gates, 99	kron
heig	qpp, 48, 49
qpp, 44	kron2
hevals	qpp::internal, 86
qpp, 44	kronpow
hevects	qpp, 49
qpp, 45	
	last_
IDisplay	qpp::internal::IOManipRange, 109
qpp::IDisplay, 102	lcm
IOManipEigen	qpp, 50
qpp::internal::IOManipEigen, 105	load
IOManipPointer	qpp, 50
qpp::internal::IOManipPointer, 107	loadMATLABmatrix
IOManipRange	qpp, 51
qpp::internal::IOManipRange, 109	logdet
ld	qpp, 52
qpp::Gates, 98	logm
ld2	qpp, 52
qpp::Gates, 100	lognegativity
idx	qpp, 52
qpp, 26	ALL,
infty	MATLAB/matlab.h, 152
qpp, 83	MATRIX MISMATCH SUBSYS
н т /	

qpp::Exception, 92	NOT_BIPARTITE
MATRIX_NOT_CVECTOR	qpp::Exception, 92
qpp::Exception, 92	NOT_QUBIT_CVECTOR
MATRIX_NOT_RVECTOR	qpp::Exception, 92
qpp::Exception, 92	NOT_QUBIT_MATRIX
MATRIX_NOT_SQUARE	qpp::Exception, 92
qpp::Exception, 92	NOT QUBIT RVECTOR
MATRIX_NOT_SQUARE_OR_CVECTOR	qpp::Exception, 92
qpp::Exception, 92	NOT_QUBIT_SUBSYS
MATRIX_NOT_SQUARE_OR_RVECTOR	qpp::Exception, 92
qpp::Exception, 92	NOT_QUBIT_VECTOR
MATRIX_NOT_SQUARE_OR_VECTOR	qpp::Exception, 92
qpp::Exception, 92	negativity
MATRIX NOT VECTOR	qpp, 61
qpp::Exception, 92	norm
macros.h, 151	qpp, 61
ERROR, 151	number theory.h, 152
ERRORLN, 151	nambor_treery.n, roz
PRINT, 151	OUT OF RANGE
PRINTLN, 151	qpp::Exception, 92
	omega
marginalX	qpp, 61
qpp, 53	one
marginalY	
qpp, 53	qpp::States, 122 operations.h, 154
maxn	•
qpp, 83	operator<<
measure	qpp::IDisplay, 102
qpp, 53–56	operator=
measure_seq	qpp::IDisplay, 102
qpp, 5 7	qpp::Timer, 129
mes	qpp::internal::IOManipPointer, 107
qpp::States, 122	qpp::internal::IOManipRange, 109
minus	qpp::internal::Singleton, 119
0: : 400	operator"" i
qpp::States, 122	• –
qpp::States, 122 mket	qpp, 6 2
	qpp, 62
mket	qpp, 62 p_
mket qpp, 58	qpp, 62 p_ qpp::internal::IOManipPointer, 107
mket qpp, 58 modinv	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID
mket qpp, 58 modinv qpp, 58	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92
mket qpp, 58 modinv qpp, 58 modmul	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92 PERM_MISMATCH_DIMS
mket qpp, 58 modinv qpp, 58 modmul qpp, 59	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92 PERM_MISMATCH_DIMS qpp::Exception, 92
mket qpp, 58 modinv qpp, 58 modmul qpp, 59 modpow qpp, 59	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92 PERM_MISMATCH_DIMS
mket qpp, 58 modinv qpp, 58 modmul qpp, 59 modpow qpp, 59 mprj	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92 PERM_MISMATCH_DIMS qpp::Exception, 92
mket	qpp, 62 p_
mket qpp, 58 modinv qpp, 58 modmul qpp, 59 modpow qpp, 59 mprj qpp, 59, 60 msg_	qpp, 62 p_
mket	qpp, 62 p_ qpp::internal::IOManipPointer, 107 PERM_INVALID qpp::Exception, 92 PERM_MISMATCH_DIMS qpp::Exception, 92 pGHZ qpp::States, 124 PRINT
mket qpp, 58 modinv qpp, 58 modmul qpp, 59 modpow qpp, 59 mprj qpp, 59, 60 msg_ qpp::Exception, 93 multiidx2n	qpp, 62 p_
mket	qpp, 62 p_
mket qpp, 58 modinv qpp, 58 modmul qpp, 59 modpow qpp, 59 mprj qpp, 59, 60 msg_ qpp::Exception, 93 multiidx2n	qpp, 62 p_
mket	p_
mket	p_
mket	qpp, 62 p_
mket	p_

qpp, 83	cwise, 36
plus	det, 36
qpp::States, 122	dirsum, 36, 37
powm	dirsumpow, 37
qpp, 62	disp, 38, 39
prj	dmat, 25
qpp, 62	dyn_col_vect, 26
prod	dyn_mat, 26
qpp, 63	dyn_row_vect, 26
ptrace	ee, <mark>83</mark>
qpp, 63, 64	egcd, 39
ptrace1	eig, 40
qpp, 64	entanglement, 40
ptrace2	entropy, 41
qpp, 65	eps, <mark>83</mark>
ptranspose	evals, 41
qpp, 66	evects, 41
px0	expm, 42
qpp::States, 124	factors, 42
px1	funm, 42
qpp::States, 124	gcd, 42, 43
py0	gconcurrence, 43
qpp::States, 124	grams, 43, 44
py1	heig, 44
qpp::States, 124	hevals, 44
pz0	hevects, 45
qpp::States, 124	idx, 26
pz1	infty, 83
qpp::States, 124	inverse, 45
	invperm, 45
QPP_UNUSED_	ip, 45, 47
qpp.h, 158	isprime, 47
qmutualinfo	ket, 26
qpp, 66, 67	kraus2choi, 47
qpp, 13	•
absm, 26	kraus2super, 48 kron, 48, 49
abssq, 27	
adjoint, 27	kronpow, 49
anticomm, 28	lcm, 50 load, 50
apply, 28, 29	*
applyCTRL, 30	loadMATLABmatrix, 51
avg, 31	logdet, 52
bigint, 25	logm, 52
bloch2rho, 31	lognegativity, 52
bra, 25	marginalX, 53
choi2kraus, 31	marginalY, 53
choi2super, 32	maxn, 83
chop, 83	measure, 53–56
cmat, 25	measure_seq, 57
comm, 32	mket, 58
complement, 32	modinv, 58
compperm, 32	modmul, 59
concurrence, 34	modpow, 59
conjugate, 34	mprj, 59, 60
contfrac2x, 34	multiidx2n, 60
cor, 35	n2multiidx, 60
cosm, 35	negativity, 61
cov, 35	norm, 61
cplx, 25	omega, 61

operator""_i, 62	internal::Singleton< const Codes >, 89
pi, 83	NINE_QUBIT_SHOR, 88
powm, 62	SEVEN_QUBIT_STEANE, 88
prj, 62	Type, 88
prod, 63	qpp::Exception, 90
ptrace, 63, 64	CUSTOM_EXCEPTION, 92
ptrace1, 64	construct_exception_msg_, 93
ptrace2, 65	custom_, 93
ptranspose, 66	DIMS_INVALID, 92
qmutualinfo, 66, 67	DIMS_MISMATCH_CVECTOR, 92
rand, 67, 68	DIMS_MISMATCH_MATRIX, 92
randH, 68	DIMS_MISMATCH_RVECTOR, 92
randU, 72	DIMS_MISMATCH_VECTOR, 92
randV, 72	DIMS_NOT_EQUAL, 92
randidx, 69	Exception, 92, 93
randket, 69	MATRIX_MISMATCH_SUBSYS, 92
randkraus, 69	MATRIX_NOT_CVECTOR, 92
randn, 69, 70	MATRIX_NOT_RVECTOR, 92
randperm, 71	MATRIX_NOT_SQUARE, 92
randprime, 71	MATRIX_NOT_SQUARE_OR_CVECTOR, 92
randprob, 71	MATRIX_NOT_SQUARE_OR_RVECTOR, 92
randrho, 71	MATRIX_NOT_SQUARE_OR_VECTOR, 92
renyi, 72	MATRIX_NOT_VECTOR, 92
reshape, 73	msg_, 93
rho2bloch, 73	NO_CODEWORD, 92
rho2pure, 73	NOT_BIPARTITE, 92
save, 74	NOT_QUBIT_CVECTOR, 92
saveMATLABmatrix, 74	NOT_QUBIT_MATRIX, 92
schatten, 75	NOT_QUBIT_RVECTOR, 92
schmidtA, 75	NOT_QUBIT_SUBSYS, 92
schmidtB, 75, 76	NOT_QUBIT_VECTOR, 92
schmidtcoeffs, 76	OUT_OF_RANGE, 92
schmidtprobs, 77	PERM_INVALID, 92
sigma, 77	PERM_MISMATCH_DIMS, 92
sinm, 78	SIZE_MISMATCH, 92
spectralpowm, 78	SUBSYS_MISMATCH_DIMS, 92
sqrtm, 78	TYPE_MISMATCH, 92
sum, 78, 79	Type, 91
super2choi, 79	type_, 93
svals, 79	UNDEFINED_TYPE, 92
svd, 80	UNKNOWN_EXCEPTION, 92
svdU, 80	what, 93
svdV, 80	where_, 93
syspermute, 80, 81	ZERO_SIZE, 92
to_void, 26	qpp::Gates, 93
trace, 81	\sim Gates, 96
transpose, 81	CNOT, 99
tsallis, 81, 82	CNOTba, 99
uniform, 82	CTRL, 96
var, <mark>82</mark>	CZ, 99
x2contfrac, 82	expandout, 96, 97
qpp.h, 156	FRED, 99
QPP_UNUSED_, 158	Fd, 98
qpp::Codes, 87	Gates, 96
∼Codes, 88	H, 99
Codes, 88	ld, 98
codeword, 89	ld2, 100
FIVE_QUBIT, 88	internal::Singleton< const Gates >, 99

S, 100 SWAP, 100 T, 100 TOF, 100 Xd, 98 Y, 100 Zd, 98 Qpp::IDisplay, 100 Zd, 99 Qpp::IDisplay, 102 display, 102 Operator<, 102 Operator<, 102 Operator=, 102 Qpp::Init, 104 Init, 104 Init, 104 Init, 104 Init, 104 Init, 104 Init, 104 Sear AndomDevices, 117 RandomDevices, 117 RandomDevices, 117 rd _ 117 rg _ , 117 qpp::States, 119 States, 121 bob, 123 b10, 123 b10, 123 b11, 123 cnes, 122 gnes, 123 gpointerial, 124 gpointerial, 124 gpointerial, 124 gpointerial, 125 gpointerial, 126 gpointerial, 127 gpointerial, 128 gpointerial, 128 gpointerial, 128 gpointerial, 128 gpointerial, 129 gpt duration, 128 operator=, 129 gpt duration, 128 operator=, 129 gpt duration, 128 operator=, 129 gpt itics, 129 tics, 129 gpt itics, 129 gpt:Timer < T, CLOCK T >, 125 qpp::Experimental, 83 qpp::Internal, 83	_	
SWAP, 100 T, 100 T, 100 X, 100	Rn, 98	z1, 125
T, 100 TOF, 100 X, 100 X, 100 X, 98 Y, 100 Z, 102 display, 102 display, 102 display, 102 operator<, 102 operator<, 102 operator, 102 qpp::Init, 103 ~Init, 104 internal::Singleton< const Init >, 104 qpp::RandomDevices, 115 ∴RandomDevices, 117 rd_niternal::Singleton< RandomDevices >, 117 RandomDevices, 117 rd_nitranal::Singleton< RandomDevices >, 117 qpp::States, 119 ~States, 121 bo0, 123 b10, 123 b10, 123 b10, 123 b11, 123 GHZ, 123 internal::Singleton< const States >, 123 jn, 121 mes, 122 minus, 122 one, 122 pGHZ, 124 pW, 124 pb00, 123 pb01, 123 pb01, 123 pb01, 124 pb01, 124 pb11, 124 plus, 122 px, 124 pv, 125 pv, 124 pv, 125 pv, 124 pv, 125 pv, 124 pv, 124 pv, 125 pv, 125 pv,		•
TOF, 100		
X, 100 Xd, 98 Y, 100 Z, 100 Zd, 99 qpp:://display, 102 clipsplay, 102 display, 102 operator<, 102 operator<, 102 operator, 102 qpp:://display, 103		
Xd, 98 Y, 100 Z, 100 Zd, 99 qpp::IDisplay, 100		
V, 100 Z,		
Z, 100		- -
Zd, 99 qpp:://lisplay, 100 -/ Display, 102 display, 102 libisplay, 102 operator <<, 102 operator <<, 102 operator, 102 qpp:://lin, 103 -/ Init, 104 Init, 104 Init, 104 Init, 104 Init, 107 RandomDevices, 115 -/ RandomDevices, 117 RandomDevices, 117 RandomDevices, 117 rd, 117 qpp:://listes, 119 -/ States, 121 boo, 123 bot, 123 bot, 123 bot, 123 bot, 123 internal::Singleton < const States >, 123 jn, 121 mes, 122 minus, 122 one, 122 pGHZ, 124 pb00, 123 pb01, 123 pb01, 123 pb01, 124 pb01, 124 pb11, 124 pb11, 124 pb11, 124 pb11, 124 pb1, 122 px, 124 pv, 124 pv		•
qpp::IDisplay, 100		
~IDisplay, 102 display, 102 display, 102 Display, 102 operator <<, 104 Init, 104 Init, 104 Init, 104 Internal::Singleton < const lnit >, 104 opp::RandomDevices, 115		
display, 102		•
IDisplay, 102	· ·	•
operator<<, 102 operator=, 103 opera	• •	
operator=, 102 qpp::init_103	• •	
qpp::Init, 103 check_evector, 85 ~Init, 104 check_dims_match_cvect, 85 Init int, 104 check_dims_match_cvect, 85 internal::Singleton < const Init >, 104 check_dims_match_mat, 85 qpp::RandomDevices, 115 check_dims_match_rvect, 85 ~RandomDevices, 117 check_de_dims_s5 internal::Singleton < RandomDevices >, 117 check_matching_sizes, 85 RandomDevices, 117 check_nonzero_size, 85 rd, 117 check_qubit_vector, 85 rd, 117 check_qubit_vector, 85 rd, 117 check_qubit_vector, 85 rd, 123 check_qubit_vector, 85 b00, 123 check_qubit_vector, 85 b11, 123 check_vector, 85 b11, 123 check_subsys_match_dims, 85 gd_T, 121 get_dim_subsys, 86 get_num_subsys, 86 get_num_subsys, 86 get_num_subsys, 86 get_num_subsys, 86 get_num_subsys, 86 ronultidx, 86 none, 122 numtidix, 86 pBDO, 123 pb10, 124 numtidix, 86 pb11, 124 numtidix, 86 pb11, 124	•	
~Init, 104 Init, 104 Internal::Singleton < const Init >, 104 qpp::RandomDevices, 115 ~RandomDevices, 117 Internal::Singleton < RandomDevices >, 117 RandomDevices, 117 RandomDevices, 117 RandomDevices, 117 RandomDevices, 117 RandomDevices, 117 RandomDevices, 117 Check_eq_dims, 85 Check_matching_sizes, 85 Check_matching_sizes, 85 Check_monzero_size, 85 Check_nonzero_size, 85 Check_qubit_evector, 85 Check_qubit_vector, 85 Check_qubit_vector, 85 Check_qubit_vector, 85 Check_qubit_vector, 85 Check_vector, 85 Check_square_mat, 85 Check_square_mat, 85 Check_square_mat, 85 Check_vector, 85 Check_square_mat, 85 Check_vector, 85 Check_square_mat, 85 Check_vector, 85 Check_vector, 85 Check_vector, 85 Check_square_mat, 85 Check_qubit_vector, 85 Check_qubit_vec	·	
Init, 104 internal::Singleton < const Init >, 104 qpp::RandomDevices, 115 ~RandomDevices, 117 internal::Singleton < RandomDevices >, 117 RandomDevices, 117 rd_, 117 rd_, 117 qpp::Rates, 119 ~States, 121 b00, 123 b01, 123 b11, 123 GHZ, 123 internal::Singleton < const States >, 123 jn, 121 mes, 122 minus, 122 one, 122 pGHZ, 124 pb00, 123 pb00, 123 pb01, 123 pb01, 124 pb00, 123 pb01, 124 pb00, 123 pb01, 124 pb11, 124 pb01, 124 pb11, 124 pb11, 124 pb11, 124 pb0, 124 pp0, 124 pv1, 124		
internal::Singleton < const Init >, 104 qpp::RandomDevices, 115		
qpp::RandomDevices, 115 check_dlims_match_rvect, 85 ~RandomDevices, 117 check_eq_dlims, 85 internal::Singleton < RandomDevices >, 117 check_matching_sizes, 85 rd_, 117 check_perm, 85 rng_, 117 check_qubit_cvector, 85 qpp::States, 119 check_qubit_cvector, 85 ~States, 121 check_qubit_vector, 85 b00, 123 check_qubit_vector, 85 b01, 123 check_square_mat, 85 b10, 123 check_square_mat, 85 b11, 123 check_vector, 85 b11, 123 check_square_mat, 85 check_square_mat, 85 check_vector, 85 internal::Singleton < const States >, 123 dirsum2, 85 jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2rmultiidx, 86 pW, 124 variadic_vector_emplace, 86 qpp::internal::Display_impl_, 90 display_impl_, 90 pb10, 124 pp::internal::OManipEigen, 104 px1, 124 pp::internal::OManipPointer	•	
~RandomDevices, 117 internal::Singleton < RandomDevices >, 117 RandomDevices, 117 rd_, 117 rg_, 117 qpp::States, 119		
internal::Singleton < RandomDevices >, 117 RandomDevices, 117 rd_, 117 rg_, 117 check_perm, 85 check_perm, 85 check_qubit_cvector, 85 check_qubit_trector, 85 check_qubit_trector, 85 check_qubit_vector, 85 check_qubit_vector, 85 check_qubit_vector, 85 check_qubit_vector, 85 check_qubit_vector, 85 check_square_mat, 85 check_subsys_match_dims, 85 check_subsys_match_dims, 85 check_vector, 85 dirsum2, 85 internal::Singleton < const States >, 123 internal::Singleton < const States >, 123 jn, 121 mes, 122 minus, 122 one, 122 pGHZ, 124 pW, 124 pb00, 123 pb01, 123 pb01, 123 pb10, 124 pb11, 124 pb11, 124 pv1, 124 pv2, 124 pv3, 124 pv4, 124 pv6, 124 pv7,		
RandomDevices, 117 rd_, 117 rd_, 117 check_nonzero_size, 85 rd_, 117 rng_, 117 check_qubit_cvector, 85 check_qubit_trector, 85 check_qubit_rvector, 85 b00, 123 check_qubit_vector, 85 b01, 123 check_qubit_vector, 85 check_vector, 85 check_square_mat, 85 check_subsys_match_dims, 85 check_vector, 85 dirsum2, 85 internal::Singleton< const States >, 123 jn, 121 mes, 122 mes, 122 minus, 122 one, 122 pGHZ, 124 pW, 124 pb00, 123 pb10, 124 pb11, 124 pb11, 124 pv1, 124 pv3, 124 pv1, 124 pv2, 124 pv2, 124 pv3, 125 pv3, 124 pv4, 124 pv6, 124 pv6, 124 pv6, 124 pv7, 124 pv1, 124 pv2, 125 pv3 pv3 pv4 pv1, 124 pv1, 124 pv2, 125 pv3 pv3 pv4 pv1, 124 pv1, 124 pv1, 124 pv1, 124 pv1, 124 pv2 pv2 pv3 pv3 pv4 pv1 pv1 pv2 pv2 pv3 pv3 pv4 pv2 pv3 pv3 pv4 pv3 pv6 pv6 pv7 pv7 pv7 pv8 pv7 pv8		_ ·_
rd_, 117 rng_, 117 rng_, 117 check_perm, 85 check_qubit_cvector, 85 check_qubit_rvector, 85 check_qubi		_
rng_, 117 qpp::States, 119		
qpp::States, 119 check_qubit_matrix, 85 ~States, 121 check_qubit_rvector, 85 b00, 123 check_qubit_vector, 85 b01, 123 check_rvector, 85 b10, 123 check_square_mat, 85 b11, 123 check_square_mat, 85 cHZ, 123 check_vector, 85 internal::Singleton< const States >, 123 dirsum2, 85 jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2multiidx, 86 pW, 124 variadic_vector_emplace, 86 pb00, 123 qpp::internal::Display_impl_, 89 display_impl_, 90 qpp::internal::IOManipEigen, 104 pb11, 124 A, 106 px0, 124 chop_, 106 px1, 124 lOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer qy1, 124 display, 107		
~States, 121 b00, 123 b01, 123 check_qubit_rvector, 85 b01, 123 check_rvector, 85 check_rvector, 85 check_square_mat, 85 check_subsys_match_dims, 85 check_vector, 85 check_subsys_match_dims, 85 check_vector, 85 check_subsys_match_dims, 85 check_subsys_match_dims, 85 check_square_mat, 85 check_rvector, 85 check_rvector, 85 check_square_mat, 85 check_rvector, 85 check_square_mat, 85 check_rvector, 85 check_square_mat, 85 check_rvector, 85 check_square_mat, 85 check_square_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_stare_		_ · _
b00, 123 b01, 123 b01, 123 check_qubit_vector, 85 check_rvector, 85 check_square_mat, 85 check_subsys_match_dims, 85 check_vector, 85 check_subsys_match_dims, 85 check_vector, 85 check_vector, 85 check_vector, 85 check_vector, 85 check_vector, 85 check_vector, 85 dirsum2, 85 get_dim_subsys, 86 get_num_subsys, 86 minus, 122 get_dim_subsys, 86 multiidx2n, 86 ne, 122 pGHZ, 124 pW, 124 pW, 124 pb00, 123 pb10, 124 pb11, 124 pb11, 124 pb11, 124 pb11, 124 pp2, 124 py0, 124 py1, 124 display, 105 pp:internal::IOManipPointer display, 107		—· —
b01, 123 b10, 123 b10, 123 check_rvector, 85 b11, 123 check_square_mat, 85 b11, 123 check_subsys_match_dims, 85 check_subsys_match_dims, 85 check_vector, 85 internal::Singleton < const States >, 123 internal::Singleton < const States >, 123 jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 multiidx2n, 86 n2multiidx2n, 86 n2multiidx, 86 pW, 124 pW, 124 pb00, 123 pb10, 124 pb11, 124 pb11, 124 pb11, 124 plus, 122 px0, 124 py1, 124 check_vector, 85 check_subsys_match_dims, 85 check_subsys_selling subsys_selling get_dim_subsys, 86 mes, 122 get_dim_subsys, 86 mes,		_ · _
b10, 123 b11, 123 check_square_mat, 85 b11, 123 check_subsys_match_dims, 85 GHZ, 123 internal::Singleton < const States >, 123 jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 pGHZ, 124 pW, 124 pb1, 123 pb10, 124 plus, 122 px0, 124 py1, 124 check_square_mat, 85 check_square_mat, 85 check_subsys_match_dims, 85 check_vector, 85 dirsum2, 85 get_dim_subsys, 86 kron2, 86 pet_num_subsys, 86 kron2, 86 n2multiidx2n, 86 n2multiidx2n, 86 n2multiidx, 86 variadic_vector_emplace, 86 pp0::internal::Display_Impl_, 89 display_impl_, 90 qpp::internal::IOManipEigen, 104 ph11, 124 ph11, 124 ph11, 124 py0, 124 py0, 124 py0, 124 py0, 124 qpp::internal::IOManipPointer display, 107		
b11, 123		
GHZ, 123 check_vector, 85 internal::Singleton < const States > , 123 dirsum2, 85 jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2multiidx, 86 pW, 124 variadic_vector_emplace, 86 pb00, 123 qpp::internal::Display_Impl_, 89 pb10, 124 qpp::internal::IOManipEigen, 104 pb11, 124 A, 106 pus, 122 chop, 106 px0, 124 display, 105 py0, 124 qpp::internal::IOManipPointer py0, 124 qpp::internal::IOManipPointer display, 107	•	_ · _
internal::Singleton < const States >, 123 jn, 121 mes, 122 minus, 122 one, 122 one, 124 pW, 124 pb00, 123 pb01, 123 pb10, 124 pb11, 124 pvx, 124 px, 124 px, 124 px, 124 py, 124 py, 124 dirsum2, 85 get_dim_subsys, 86 get_num_subsys, 86 multiidx2n, 86 n2multiidx, 86 variadic_vector_emplace, 86 qpp::internal::Display_lmpl_, 89 display_impl_, 90 qpp::internal::IOManipEigen, 104 A_, 106 chop_, 106 chop_, 106 display, 105 px0, 124 px1, 124 py1, 124 qpp::internal::IOManipPointer display, 107		_ •
jn, 121 get_dim_subsys, 86 mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2multiidx, 86 pb00, 123 qpp::internal::Display_lmpl_, 89 pb01, 123 display_impl_, 90 pb10, 124 qpp::internal::IOManipEigen, 104 pb11, 124 A_, 106 plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107		
mes, 122 get_num_subsys, 86 minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2multiidx, 86 pb00, 123 qpp::internal::Display_Impl_, 89 pb01, 123 display_impl_, 90 pb10, 124 qpp::internal::IOManipEigen, 104 pb11, 124 A_, 106 plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107		
minus, 122 kron2, 86 one, 122 multiidx2n, 86 pGHZ, 124 n2multiidx, 86 pW, 124 variadic_vector_emplace, 86 pb00, 123 qpp::internal::Display_lmpl_, 89 pb01, 123 display_impl_, 90 pb10, 124 qpp::internal::IOManipEigen, 104 pb11, 124 A_, 106 plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 qpp::internal::IOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer qy1, 124 display, 107	• •	
one, 122 pGHZ, 124 pW, 124 pw, 124 pb00, 123 pb01, 123 pb10, 124 pb11, 124 plus, 122 px0, 124 py0, 124 py1, 124 pwinterial::IOManipEigen, 104 pilidix2n, 86 n2multiidx2n, 86 n2multiidx, 86 puntation, 86 n2multiidx, 86 puntation, 86 n2multiidx, 86 puntation, 86 n2multiidx, 86 puntation, 86 n2multiidx, 86 n2multiidx, 86 pp::internal::IOManipEigen, 109 ndisplay, 105 py0, 124 py1, 124 display, 107		
pGHZ, 124 pW, 124 variadic_vector_emplace, 86 pb00, 123 qpp::internal::Display_lmpl_, 89 pb01, 123 display_impl_, 90 pb10, 124 pb11, 124 A_, 106 plus, 122 chop_, 106 px0, 124 px1, 124 px1, 124 py0, 124 py1, 124 qpp::internal::IOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer display, 107	•	
pW, 124		
pb00, 123 pb01, 123 pb10, 124 pb11, 124 plus, 122 px0, 124 px1, 124 py1, 124 py2, 105 py2, 124 py1, 124 py1, 124 py1, 124 py1, 124 py1, 124 py2, 107 py2, 106 py3, 105 py3, 107	•	
pb01, 123 display_impl_, 90 pb10, 124 qpp::internal::IOManipEigen, 104 pb11, 124 A_, 106 plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 IOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	·
pb10, 124	•	
pb11, 124 plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 py0, 124 py0, 124 py1, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	
plus, 122 chop_, 106 px0, 124 display, 105 px1, 124 lOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	
px0, 124 display, 105 px1, 124 IOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	
px1, 124 IOManipEigen, 105 py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	• —
py0, 124 qpp::internal::IOManipPointer py1, 124 display, 107	•	
py1, 124 display, 107	•	
· · ·	• •	
N71) 124 And 1117	pz0, 124	end_, 107
pz0, 124 end_, 107 pz1, 124 IOManipPointer, 107	•	
States, 121 N_, 107	•	•
W, 124 operator=, 107		
x0, 124		•
x0, 124 p_, 107 x1, 125 separator_, 107		• —
y0, 125 separator_, 107 y1, 125		• —
y1, 125 start_, 106 y1, 125 qpp::internal::IOManipPointer< PointerType >, 106	•	
z0, 125 qpp::internal::IOManipRange	-	
20, 120 чррынанноманрнануе	20, 120	apptomaiomainprainge

display 400	ula a Our surra
display, 109	rho2pure
end_, 109 first , 109	qpp, 73 Rn
IOManipRange, 109	qpp::Gates, 98
last_, 109	rng
operator=, 109	qpp::RandomDevices, 117
separator_, 109	qpprandombevices, 117
start_, 110	S
gpp::internal::IOManipRange< InputIterator >, 108	qpp::Gates, 100
qpp::internal::Singleton	SEVEN_QUBIT_STEANE
~Singleton, 118	qpp::Codes, 88
get instance, 119	SIZE_MISMATCH
get_thread_local_instance, 119	qpp::Exception, 92
operator=, 119	SUBSYS_MISMATCH_DIMS
Singleton, 118	qpp::Exception, 92
qpp::internal::Singleton< T >, 117	SWAP
qpp::is_complex< std::complex< T >>, 111	qpp::Gates, 100
qpp::is_complex< T >, 110	save
qpp::is_iterable< T, to_void< decltype(std::declval< T	qpp, 74
>().begin()), decltype(std::declval< T >().	saveMATLABmatrix
end()), typename T::value type > >, 113	qpp, 74
qpp::is_iterable< T, typename >, 112	schatten
qpp::is_matrix_expression< Derived >, 114	qpp, 75
4	schmidtA
rand	qpp, 75
qpp, 67, 68	schmidtB
randH	qpp, 75, 76
qpp, 68	schmidtcoeffs
randU	qpp, 76
qpp, 72	schmidtprobs
randV	qpp, 77
qpp, 72	separator_
randidx	qpp::internal::IOManipPointer, 107
qpp, 69	qpp::internal::IOManipRange, 109
randket	sigma
qpp, 69	qpp, 77
randkraus	Singleton
qpp, 69	qpp::internal::Singleton, 118
randn	sinm
qpp, 69, 70	qpp, 78
random.h, 158	spectralpowm
RandomDevices	qpp, 78
qpp::RandomDevices, 117	sqrtm
randperm	qpp, 78
qpp, 71	start_
randprime	qpp::Timer, 129
qpp, 71	qpp::internal::IOManipPointer, 108
randprob	qpp::internal::IOManipRange, 110
qpp, 71	States
randrho	qpp::States, 121
qpp, 71	statistics.h, 159
rd_	sum
qpp::RandomDevices, 117	qpp, 78, 79
renyi	super2choi
qpp, 72	qpp, 79
reshape	svals
qpp, 73	qpp, 79
rho2bloch	svd
qpp, 73	qpp, 80

svdU		qpp::States, 124
qpp, 80 svdV	x1	qpp::States, 125
qpp, 80	x2c	ontfrac
syspermute		qpp, <mark>82</mark>
qpp, 80, 81	Xd	qpp::Gates, 98
Т		qppdates, 30
qpp::Gates, 100	Υ	
TOF qpp::Gates, 100	y0	qpp::Gates, 100
TYPE MISMATCH	, ,	qpp::States, 125
qpp::Exception, 92	y1	
tic Time 100		qpp::States, 125
qpp::Timer, 129 tics	Z	
qpp::Timer, 129		qpp::Gates, 100
Timer	z0	annu Otataa 105
qpp::Timer, 128	z1	qpp::States, 125
to_void qpp, 26		qpp::States, 125
toc	ZEF	RO_SIZE
qpp::Timer, 129	7.	qpp::Exception, 92
trace	Zd	qpp::Gates, 99
qpp, 81	zero	
traits.h, 161 transpose		qpp::States, 123
qpp, 81		
tsallis		
qpp, 81, 82		
Type qpp::Codes, 88		
qpp::Exception, 91		
type_		
qpp::Exception, 93		
types.h, 162		
UNDEFINED_TYPE		
qpp::Exception, 92		
UNKNOWN_EXCEPTION		
qpp::Exception, 92 uniform		
qpp, 82		
var		
qpp, 82 variadic vector emplace		
qpp::internal, 86		
W		
qpp::States, 124 what		
qpp::Exception, 93		
where_		
qpp::Exception, 93		
X		
qpp::Gates, 100		
x0		