

# Quantum++ v0.1

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# Chapter 1

## Quantum++

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. It uses the [Eigen 3](http://eigen.tuxfamily.org/dox/) linear algebra library and, if available, the [OpenMP](#) multi-processing library. For additional [Eigen 3](http://eigen.tuxfamily.org/dox/) documentation see <http://eigen.tuxfamily.org/dox/>. For a simple [Eigen 3](http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt) quick ASCII reference see <http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt>.

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If you are interesting in contributing, please let me know. There is still work left to be done, and I can provide you with more details about what I have in mind. To contribute, you need to have a decent knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and some working experience with [Eigen 3](#).

The ultimate goal of this project is to build a universal quantum simulator, applicable to a vast majority of problems in quantum information/computation. The simulator should be fast but nevertheless user-friendly for anyone with a basic knowledge of C/C++.

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### Building instructions

#### Configuration:

- Compiler: `g++ >= 4.8` (for good C++11 support)
- [Eigen 3](#) library located in `$HOME/eigen`
- Quantum++ library located in `$HOME/qpp`
- MATLAB compiler include header files: `/Applications/MATLAB_R2014b.app/extern/include`
- MATLAB compiler shared library files: `/Applications/MATLAB_R2014b.app/bin/maci64`

#### Building without a build system

- Example file: `$HOME/qpp/examples/example.cpp`
- Output executable: `$HOME/qpp/examples/example`

- Must run the commands below from inside the directory `$HOME/qpp/examples`

#### Release version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -O3 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Debug version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Release version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -O3 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2014b.app/extern/include \
    -L/Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

#### Debug version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2014b.app/extern/include \
    -L /Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

#### Building using cmake

The current version of the repository has a `CMakeLists.txt` configuration file for building examples using `cmake` (`cmake` needs to be installed). To build an example using `cmake`, I recommend an out-of-source build, i.e., from the root of the project (where `./include` is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the release version (default) executable `qpp`, from the source file `./examples/example.cpp`, without MATLAB support (default), inside the directory `./build`. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build
rm -rf *
cmake -DCMAKE_BUILD_TYPE=Debug -DWITH_MATLAB=ON ..
make
```

Or, to disable **OpenMP** support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the `CMakeLists.txt` file. See also `CMakeLists.txt` for additional options. Do not forget to remove everything from the `./build` directory before a fresh build!



## Additional remarks

- The C++ compiler must be C++11 compliant.
- If your compiler does not support **OpenMP** (as it is the case e.g with clang++), disable **OpenMP** in your build, as otherwise the linker may not find the gomp library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD\_LIBRARY\_PATH is set to point to the MATLAB compiler library location, see the run\_OSX\_MATLAB script. Otherwise, you will get a runtime error like dyld: Library not loaded: @rpath/libmat.dylib.

\* I recommend running via a script, as otherwise setting the 'DYLD\_LIBRARY\_PATH' globally may interfere with Macports' 'cmake' installation (in case you use 'cmake' from 'macports'). If you use a script, then the environment variable is local to the script and does not interfere with the rest of the system.

\* Example of running script, run from inside the directory where the executable 'qpp' is located:

```
#!/bin/sh # Run Quantum++ under OS X with MATLAB support

export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:"/Applications/MATLAB_R2014b.app/bin/maci64"
./qpp
```

- If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See <http://stackoverflow.com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks> for more details about this problem.



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

<a href="#">qpp</a>	Quantum++ main namespace . . . . .	13
<a href="#">qpp::experimental</a>	Experimental/test functions/classes, do not use or modify . . . . .	68
<a href="#">qpp::internal</a>	Internal utility functions, do not use/modify . . . . .	68



## Chapter 3

# Hierarchical Index

### 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

std::exception	
qpp::Exception . . . . .	73
qpp::internal::IOManipEigen . . . . .	84
qpp::internal::IOManipPointer< PointerType > . . . . .	85
qpp::internal::IOManipRange< InputIterator > . . . . .	86
qpp::internal::Singleton< T > . . . . .	90
qpp::internal::Singleton< const Codes > . . . . .	90
qpp::Codes . . . . .	71
qpp::internal::Singleton< const Gates > . . . . .	90
qpp::Gates . . . . .	77
qpp::internal::Singleton< const Init > . . . . .	90
qpp::Init . . . . .	82
qpp::internal::Singleton< const States > . . . . .	90
qpp::States . . . . .	91
qpp::internal::Singleton< RandomDevices > . . . . .	90
qpp::RandomDevices . . . . .	88
qpp::Timer . . . . .	95



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">qpp::Codes</a>	Const Singleton class that defines quantum error correcting codes . . . . .	71
<a href="#">qpp::Exception</a>	Generates custom exceptions, used when validating function parameters . . . . .	73
<a href="#">qpp::Gates</a>	Const Singleton class that implements most commonly used gates . . . . .	77
<a href="#">qpp::Init</a>	Const Singleton class that performs additional initializations/cleanups . . . . .	82
<a href="#">qpp::internal::IOManipEigen</a>	. . . . .	84
<a href="#">qpp::internal::IOManipPointer&lt; PointerType &gt;</a>	. . . . .	85
<a href="#">qpp::internal::IOManipRange&lt; InputIterator &gt;</a>	. . . . .	86
<a href="#">qpp::RandomDevices</a>	Singleton class that manages the source of randomness in the library . . . . .	88
<a href="#">qpp::internal::Singleton&lt; T &gt;</a>	<a href="#">Singleton</a> policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern) . . . . .	90
<a href="#">qpp::States</a>	Const Singleton class that implements most commonly used states . . . . .	91
<a href="#">qpp::Timer</a>	Measures time . . . . .	95





## Chapter 5

# File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

<a href="#">constants.h</a>	
Constants . . . . .	104
<a href="#">entanglement.h</a>	
Entanglement functions . . . . .	105
<a href="#">entropies.h</a>	
Entropy functions . . . . .	106
<a href="#">functions.h</a>	
Generic quantum computing functions . . . . .	108
<a href="#">input_output.h</a>	
Input/output functions . . . . .	111
<a href="#">instruments.h</a>	
Measurement functions . . . . .	113
<a href="#">number_theory.h</a>	
Number theory functions . . . . .	118
<a href="#">operations.h</a>	
Quantum operation functions . . . . .	119
<a href="#">qpp.h</a>	
Quantum++ main header file, includes all other necessary headers . . . . .	121
<a href="#">random.h</a>	
Randomness-related functions . . . . .	122
<a href="#">types.h</a>	
Type aliases . . . . .	124
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classes/ <a href="#">exception.h</a>	
Exceptions . . . . .	99
classes/ <a href="#">gates.h</a>	
Quantum gates . . . . .	100
classes/ <a href="#">init.h</a>	
Initialization . . . . .	101
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Random devices . . . . .	102
classes/ <a href="#">states.h</a>	
Quantum states . . . . .	103
classes/ <a href="#">timer.h</a>	
Timing . . . . .	103
experimental/ <a href="#">test.h</a>	
Experimental/test functions/classes . . . . .	107

internal/ <a href="#">util.h</a>	
Internal utility functions . . . . .	116
internal/classes/ <a href="#">iomanip.h</a>	
Input/output manipulators . . . . .	114
internal/classes/ <a href="#">singleton.h</a>	
Singleton pattern via CRTP . . . . .	115
MATLAB/ <a href="#">matlab.h</a>	
Input/output interfacing with MATLAB . . . . .	117

## Chapter 6

# Namespace Documentation

### 6.1 qpp Namespace Reference

Quantum++ main namespace.

#### Namespaces

- [experimental](#)  
*Experimental/test functions/classes, do not use or modify.*
- [internal](#)  
*Internal utility functions, do not use/modify.*

#### Classes

- class [Codes](#)  
*const Singleton class that defines quantum error correcting codes*
- class [Exception](#)  
*Generates custom exceptions, used when validating function parameters.*
- class [Gates](#)  
*const Singleton class that implements most commonly used gates*
- class [Init](#)  
*const Singleton class that performs additional initializations/cleanups*
- class [RandomDevices](#)  
*Singleton class that manages the source of randomness in the library.*
- class [States](#)  
*const Singleton class that implements most commonly used states*
- class [Timer](#)  
*Measures time.*

#### Typedefs

- using [idx](#) = std::size\_t  
*Non-negative integer index.*
- using [cplx](#) = std::complex< double >  
*Complex number in double precision.*
- using [ket](#) = Eigen::VectorXcd

- *Complex (double precision) dynamic Eigen column vector.*
- using `bra` = Eigen::RowVectorXcd
- *Complex (double precision) dynamic Eigen row vector.*
- using `cmat` = Eigen::MatrixXcd
- *Complex (double precision) dynamic Eigen matrix.*
- using `dmat` = Eigen::MatrixXd
- *Real (double precision) dynamic Eigen matrix.*
- template<typename Scalar >  
using `dyn_mat` = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >  
*Dynamic Eigen matrix over the field specified by Scalar.*
- template<typename Scalar >  
using `dyn_col_vect` = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >  
*Dynamic Eigen column vector over the field specified by Scalar.*
- template<typename Scalar >  
using `dyn_row_vect` = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >  
*Dynamic Eigen row vector over the field specified by Scalar.*

## Functions

- constexpr `cplx operator""_i` (unsigned long long int x)  
*User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)*
- constexpr `cplx operator""_i` (long double x)  
*User-defined literal for complex  $i = \sqrt{-1}$  (real overload)*
- `cplx omega` (idx D)  
*D-th root of unity.*
- template<typename Derived >  
`dyn_col_vect`< double > `schmidtcoeffs` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt coefficients of the bi-partite pure state A.*
- template<typename Derived >  
`cmat schmidtA` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt basis on Alice's side.*
- template<typename Derived >  
`cmat schmidtB` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt basis on Bob's side.*
- template<typename Derived >  
std::vector< double > `schmidtprobs` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt probabilities of the bi-partite pure state A.*
- template<typename Derived >  
double `entanglement` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Entanglement of the bi-partite pure state A.*
- template<typename Derived >  
double `gconcurrence` (const Eigen::MatrixBase< Derived > &A)  
*G-concurrence of the bi-partite pure state A.*
- template<typename Derived >  
double `negativity` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Negativity of the bi-partite mixed state A.*
- template<typename Derived >  
double `lognegativity` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Logarithmic negativity of the bi-partite mixed state A.*

- `template<typename Derived >`  
`double concurrence (const Eigen::MatrixBase< Derived > &A)`  
*Wootters concurrence of the bi-partite qubit mixed state A.*
- `template<typename Derived >`  
`double entropy (const Eigen::MatrixBase< Derived > &A)`  
*von-Neumann entropy of the density matrix A*
- `double entropy (const std::vector< double > &prob)`  
*Shannon entropy of the probability distribution prob.*
- `template<typename Derived >`  
`double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)`  
*Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .*
- `double renyi (const std::vector< double > &prob, double alpha)`  
*Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .*
- `template<typename Derived >`  
`double tsallis (const Eigen::MatrixBase< Derived > &A, double q)`  
*Tsallis-  $q$  entropy of the density matrix A, for  $q \geq 0$ .*
- `double tsallis (const std::vector< double > &prob, double q)`  
*Tsallis-  $q$  entropy of the probability distribution prob, for  $q \geq 0$ .*
- `template<typename Derived >`  
`double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsA, const std::vector< idx > &subsB, const std::vector< idx > &dims)`  
*Quantum mutual information between 2 subsystems of a composite system.*
- `template<typename Derived >`  
`double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsA, const std::vector< idx > &subsB, idx d=2)`  
*Quantum mutual information between 2 subsystems of a composite system.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)`  
*Transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)`  
*Complex conjugate.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)`  
*Adjoint.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)`  
*Inverse.*
- `template<typename Derived >`  
`Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)`  
*Trace.*
- `template<typename Derived >`  
`Derived::Scalar det (const Eigen::MatrixBase< Derived > &A)`  
*Determinant.*
- `template<typename Derived >`  
`Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)`  
*Logarithm of the determinant.*
- `template<typename Derived >`  
`Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise sum of A.*
- `template<typename Derived >`  
`Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise product of A.*

- `template<typename Derived >`  
`double norm (const Eigen::MatrixBase< Derived > &A)`  
*Frobenius norm.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect< cplx >`  
`, cmat > eig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition.*
- `template<typename Derived >`  
`dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvalues.*
- `template<typename Derived >`  
`cmat evecs (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvectors.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect`  
`< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition of Hermitian expression.*
- `template<typename Derived >`  
`dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)`  
*Hermitian eigenvalues.*
- `template<typename Derived >`  
`cmat hevecs (const Eigen::MatrixBase< Derived > &A)`  
*Hermitian eigenvectors.*
- `template<typename Derived >`  
`std::tuple< cmat, dyn_col_vect`  
`< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)`  
*Full singular value decomposition.*
- `template<typename Derived >`  
`dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)`  
*Singular values.*
- `template<typename Derived >`  
`cmat svdU (const Eigen::MatrixBase< Derived > &A)`  
*Left singular vectors.*
- `template<typename Derived >`  
`cmat svdV (const Eigen::MatrixBase< Derived > &A)`  
*Right singular vectors.*
- `template<typename Derived >`  
`cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))`  
*Functional calculus  $f(A)$*
- `template<typename Derived >`  
`cmat sqrtm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix square root.*
- `template<typename Derived >`  
`cmat absm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix absolut value.*
- `template<typename Derived >`  
`cmat expm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix exponential.*
- `template<typename Derived >`  
`cmat logm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix logarithm.*
- `template<typename Derived >`  
`cmat sinm (const Eigen::MatrixBase< Derived > &A)`

- Matrix sin.*

  - `template<typename Derived >`  
`cmat cosm` (const Eigen::MatrixBase< Derived > &A)
- Matrix cos.*

  - `template<typename Derived >`  
`cmat spectralpowm` (const Eigen::MatrixBase< Derived > &A, const `cplx` z)
- Matrix power.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > powm` (const Eigen::MatrixBase< Derived > &A, `idx` n)
- Matrix power.*

  - `template<typename Derived >`  
`double Schatten` (const Eigen::MatrixBase< Derived > &A, `idx` p)
- Schatten norm.*

  - `template<typename OutputScalar , typename Derived >`  
`dyn_mat< OutputScalar > cwise` (const Eigen::MatrixBase< Derived > &A, OutputScalar(\*f)(const type-name Derived::Scalar &))
- Functor.*

  - `template<typename T >`  
`dyn_mat< typename T::Scalar > kron` (const T &head)
- Kronecker product.*

  - `template<typename T , typename... Args>`  
`dyn_mat< typename T::Scalar > kron` (const T &head, const Args &...tail)
- Kronecker product.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > kron` (const std::vector< Derived > &As)
- Kronecker product.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > kron` (const std::initializer\_list< Derived > &As)
- Kronecker power.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > kronpow` (const Eigen::MatrixBase< Derived > &A, `idx` n)
- Reshape.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > reshape` (const Eigen::MatrixBase< Derived > &A, `idx` rows, `idx` cols)
- Commutator.*

  - `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > comm` (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)
- Anti-commutator.*

  - `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > anticomm` (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)
- Projector.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > prj` (const Eigen::MatrixBase< Derived > &V)
- Gram-Schmidt orthogonalization.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams` (const std::vector< Derived > &Vs)
- Gram-Schmidt orthogonalization.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams` (const std::initializer\_list< Derived > &Vs)

- Gram-Schmidt orthogonalization.*

  - `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)`  
*Gram-Schmidt orthogonalization.*
  - `std::vector< idx > n2multiidx (idx n, const std::vector< idx > &dims)`  
*Non-negative integer index to multi-index.*
  - `idx multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)`  
*Multi-index to non-negative integer index.*
  - `ket mket (const std::vector< idx > &mask, const std::vector< idx > &dims)`  
*Multi-partite qudit ket.*
  - `ket mket (const std::vector< idx > &mask, idx d=2)`  
*Multi-partite qudit ket.*
  - `cmat mprj (const std::vector< idx > &mask, const std::vector< idx > &dims)`  
*Projector onto multi-partite qudit ket.*
  - `cmat mprj (const std::vector< idx > &mask, idx d=2)`  
*Projector onto multi-partite qudit ket.*
  - `template<typename InputIterator >`  
`std::vector< double > abssq (InputIterator first, InputIterator last)`  
*Computes the absolut values squared of a range of complex numbers.*
  - `template<typename Derived >`  
`std::vector< double > abssq (const Eigen::MatrixBase< Derived > &V)`  
*Computes the absolut values squared of a column vector.*
  - `template<typename InputIterator >`  
`InputIterator::value_type sum (InputIterator first, InputIterator last)`  
*Element-wise sum of a range.*
  - `template<typename InputIterator >`  
`InputIterator::value_type prod (InputIterator first, InputIterator last)`  
*Element-wise product of a range.*
  - `template<typename Derived >`  
`dyn_col_vect< typename`  
`Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)`  
*Finds the pure state representation of a matrix proportional to a projector onto a pure state.*
  - `template<typename Derived >`  
`internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)`  
*Eigen expression ostream manipulator.*
  - `internal::IOManipEigen disp (cplx z, double chop=qpp::chop)`  
*Complex number ostream manipulator.*
  - `template<typename InputIterator >`  
`internal::IOManipRange`  
`< InputIterator > disp (const InputIterator &first, const InputIterator &last, const std::string &separator, const`  
`std::string &start="[", const std::string &end="]")`  
*Range ostream manipulator.*
  - `template<typename Container >`  
`internal::IOManipRange`  
`< typename`  
`Container::const_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[",`  
`const std::string &end="]")`  
*Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.*
  - `template<typename PointerType >`  
`internal::IOManipPointer`  
`< PointerType > disp (const PointerType *p, idx n, const std::string &separator, const std::string &start="[",`  
`const std::string &end="]")`  
*C-style pointer ostream manipulator.*



- `template<typename Derived >`  
`void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)`  
*Saves Eigen expression to a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > load (const std::string &fname)`  
*Loads Eigen matrix from a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U,`  
`const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U,`  
`const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector<`  
`::initializer_list< cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)`  
*Measures the state A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`Derived loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)`  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.*
- `template<>`  
`dmatrix loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)`

- Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*

  - `template<>`  
`cmat loadMATLABmatrix` (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- `template<typename Derived >`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.*
- `template<>`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< [dmat](#) > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< [cmat](#) > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- `std::vector< long long int > x2contfrac` (double x, [idx](#) n, [idx](#) cut=1e5)  
*Simple continued fraction expansion.*
- `double contfrac2x` (const std::vector< int > &cf, [idx](#) n)  
*Real representation of a simple continued fraction.*
- `double contfrac2x` (const std::vector< int > &cf)  
*Real representation of a simple continued fraction.*
- `idx gcd` ([idx](#) m, [idx](#) n)  
*Greatest common divisor of two non-negative integers.*
- `idx gcd` (const std::vector< [idx](#) > &ns)  
*Greatest common divisor of a list of non-negative integers.*
- `idx lcm` ([idx](#) m, [idx](#) n)  
*Least common multiple of two positive integers.*
- `idx lcm` (const std::vector< [idx](#) > &ns)  
*Least common multiple of a list of positive integers.*
- `std::vector< idx > invperm` (const std::vector< [idx](#) > &perm)  
*Inverse permutation.*
- `std::vector< idx > compperm` (const std::vector< [idx](#) > &perm, const std::vector< [idx](#) > &sigma)  
*Compose permutations.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > applyCTRL` (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< [idx](#) > &ctrl, const std::vector< [idx](#) > &subsys, const std::vector< [idx](#) > &dims)  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > applyCTRL` (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< [idx](#) > &ctrl, const std::vector< [idx](#) > &subsys, [idx](#) d=2)  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > apply` (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< [idx](#) > &subsys, const std::vector< [idx](#) > &dims)  
*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > apply` (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< [idx](#) > &subsys, [idx](#) d=2)

*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*

- `template<typename Derived >`  
`cmat apply` (const Eigen::MatrixBase< Derived > &rho, const std::vector< `cmat` > &Ks)

*Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.*

- `template<typename Derived >`  
`cmat apply` (const Eigen::MatrixBase< Derived > &rho, const std::vector< `cmat` > &Ks, const std::vector< `idx` > &subsys, const std::vector< `idx` > &dims)

*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*

- `template<typename Derived >`  
`cmat apply` (const Eigen::MatrixBase< Derived > &rho, const std::vector< `cmat` > &Ks, const std::vector< `idx` > &subsys, `idx` d=2)

*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*

- `cmat kraus2super` (const std::vector< `cmat` > &Ks)

*Superoperator matrix.*

- `cmat kraus2choi` (const std::vector< `cmat` > &Ks)

*Choi matrix.*

- `std::vector< cmat > choi2kraus` (const `cmat` &A)

*Orthogonal Kraus operators from Choi matrix.*

- `cmat choi2super` (const `cmat` &A)

*Converts Choi matrix to superoperator matrix.*

- `cmat super2choi` (const `cmat` &A)

*Converts superoperator matrix to Choi matrix.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace1` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &dims)

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace2` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &dims)

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &subsys, const std::vector< `idx` > &dims)

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &subsys, `idx` d=2)

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptranspose` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &subsys, const std::vector< `idx` > &dims)

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptranspose` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &subsys, `idx` d=2)

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > syspermute` (const Eigen::MatrixBase< Derived > &A, const std::vector< `idx` > &perm, const std::vector< `idx` > &dims)

*Subsystem permutation.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)`  
*Subsystem permutation.*
- `template<typename Derived >`  
`Derived rand (idx rows, idx cols, double a=0, double b=1)`  
*Generates a random matrix with entries uniformly distributed in the interval [a, b)*
- `template<>`  
`dmat rand (idx rows, idx cols, double a, double b)`  
*Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat rand (idx rows, idx cols, double a, double b)`  
*Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices ([qpp::cmat](#))*
- `double rand (double a=0, double b=1)`  
*Generates a random real number uniformly distributed in the interval [a, b)*
- `idx randidx (idx a=std::numeric_limits< idx >::min(), idx b=std::numeric_limits< idx >::max())`  
*Generates a random index (idx) uniformly distributed in the interval [a, b].*
- `template<typename Derived >`  
`Derived randn (idx rows, idx cols, double mean=0, double sigma=1)`  
*Generates a random matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$*
- `template<>`  
`dmat randn (idx rows, idx cols, double mean, double sigma)`  
*Generates a random real matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat randn (idx rows, idx cols, double mean, double sigma)`  
*Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices ([qpp::cmat](#))*
- `double randn (double mean=0, double sigma=1)`  
*Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$*
- `cmat randU (idx D)`  
*Generates a random unitary matrix.*
- `cmat randV (idx Din, idx Dout)`  
*Generates a random isometry matrix.*
- `std::vector< cmat > randkraus (idx N, idx D)`  
*Generates a set of random Kraus operators.*
- `cmat randH (idx D)`  
*Generates a random Hermitian matrix.*
- `ket randket (idx D)`  
*Generates a random normalized ket (pure state vector)*
- `cmat randrho (idx D)`  
*Generates a random density matrix.*
- `std::vector< idx > randperm (idx n)`  
*Generates a random uniformly distributed permutation.*

## Variables

- `constexpr double chop = 1e-10`  
*Used in [qpp::disp\(\)](#) for setting to zero numbers that have their absolute value smaller than [qpp::chop](#).*
- `constexpr double eps = 1e-12`

- Used to decide whether a number or expression in double precision is zero or not.*
- constexpr `idx maxn` = 64  
*Maximum number of allowed qu(d)its (subsystems)*
- constexpr double `pi` = 3.141592653589793238462643383279502884  
 $\pi$
- constexpr double `ee` = 2.718281828459045235360287471352662497  
*Base of natural logarithm,  $e$ .*
- constexpr double `infy` = std::numeric\_limits<double>::infinity()  
*Used to denote infinity in double precision.*
- const `Init` & `init` = `Init::get_instance()`  
`qpp::Init` const Singleton
- const `Codes` & `codes` = `Codes::get_instance()`  
`qpp::Codes` const Singleton
- const `Gates` & `gt` = `Gates::get_instance()`  
`qpp::Gates` const Singleton
- const `States` & `st` = `States::get_instance()`  
`qpp::States` const Singleton
- `RandomDevices` & `rdevs` = `RandomDevices::get_instance()`  
`qpp::RandomDevices` Singleton

### 6.1.1 Detailed Description

Quantum++ main namespace.

### 6.1.2 Typedef Documentation

#### 6.1.2.1 using `qpp::bra` = typedef `Eigen::RowVectorXcd`

Complex (double precision) dynamic Eigen row vector.

#### 6.1.2.2 using `qpp::cmat` = typedef `Eigen::MatrixXcd`

Complex (double precision) dynamic Eigen matrix.

#### 6.1.2.3 using `qpp::cplx` = typedef `std::complex<double>`

Complex number in double precision.

#### 6.1.2.4 using `qpp::dmat` = typedef `Eigen::MatrixXd`

Real (double precision) dynamic Eigen matrix.

#### 6.1.2.5 template<typename `Scalar`> using `qpp::dyn_col_vect` = typedef `Eigen::Matrix <Scalar, Eigen::Dynamic, 1>`

Dynamic Eigen column vector over the field specified by `Scalar`.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

**6.1.2.6** `template<typename Scalar > using qpp::dyn_mat = typedef Eigen::Matrix <Scalar, Eigen::Dynamic, Eigen::Dynamic>`

Dynamic Eigen matrix over the field specified by *Scalar*.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

**6.1.2.7** `template<typename Scalar > using qpp::dyn_row_vect = typedef Eigen::Matrix <Scalar, 1, Eigen::Dynamic>`

Dynamic Eigen row vector over the field specified by *Scalar*.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

**6.1.2.8** `using qpp::idx = typedef std::size_t`

Non-negative integer index.

**6.1.2.9** `using qpp::ket = typedef Eigen::VectorXcd`

Complex (double precision) dynamic Eigen column vector.

## 6.1.3 Function Documentation

**6.1.3.1** `template<typename Derived > cmat qpp::absm ( const Eigen::MatrixBase< Derived > & A )`

Matrix absolut value.

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

Matrix absolut value of *A*

**6.1.3.2** `template<typename InputIterator > std::vector<double> qpp::abssq ( InputIterator first, InputIterator last )`

Computes the absolut values squared of a range of complex numbers.

Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

Returns

Real vector consisting of the range's absolut values squared

**6.1.3.3** `template<typename Derived > std::vector<double> qpp::abssq ( const Eigen::MatrixBase< Derived > & V )`

Computes the absolut values squared of a column vector.

## Parameters

$V$	Eigen expression
-----	------------------

## Returns

Real vector consisting of the absolut values squared

6.1.3.4 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )`

Adjoint.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Adjoint (Hermitian conjugate) of  $A$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.5 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::anticomm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`

Anti-commutator.

Anti-commutator  $\{A, B\} = AB + BA$ . Both  $A$  and  $B$  must be Eigen expressions over the same scalar field.

## Parameters

$A$	Eigen expression
$B$	Eigen expression

## Returns

Anti-commutator  $AB + BA$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.6 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::apply ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Applies the gate  $A$  to the part *subsys* of the multi-partite state vector or density matrix *state*.

## Note

The dimension of the gate  $A$  must match the dimension of *subsys*

## Parameters

<i>state</i>	Eigen expression
$A$	Eigen expression
<i>subsys</i>	Subsystem indexes where the gate $A$ is applied

<i>dims</i>	Dimensions of the multi-partite system
-------------	--

**Returns**

Gate *A* applied to the part *subsys* of *state*

```
6.1.3.7 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::apply ( const
Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > &
subsys, idx d = 2 )
```

Applies the gate *A* to the part *subsys* of the multi-partite state vector or density matrix *state*.

**Note**

The dimension of the gate *A* must match the dimension of *subsys*

**Parameters**

<i>state</i>	Eigen expression
<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>d</i>	Subsystem dimensions

**Returns**

Gate *A* applied to the part *subsys* of *state*

```
6.1.3.8 template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector<
cmat > & Ks )
```

Applies the channel specified by the set of Kraus operators *Ks* to the density matrix *rho*.

**Parameters**

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators

**Returns**

Output density matrix after the action of the channel

```
6.1.3.9 template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector<
cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )
```

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

**Parameters**

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators



<i>subsys</i>	Subsystem indexes where the Kraus operators <i>Ks</i> are applied
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Output density matrix after the action of the channel

6.1.3.10 `template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, idx d = 2 )`

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

**Parameters**

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators
<i>subsys</i>	Subsystem indexes where the Kraus operators <i>Ks</i> are applied
<i>d</i>	Subsystem dimensions

**Returns**

Output density matrix after the action of the channel

6.1.3.11 `template<typename Derived1, typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Applies the controlled-gate *A* to the part *subsys* of the multi-partite state vector or density matrix *state*.

**Note**

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

**Parameters**

<i>state</i>	Eigen expression
<i>A</i>	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

CTRL-A gate applied to the part *subsys* of *state*

6.1.3.12 `template<typename Derived1, typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx d = 2 )`

Applies the controlled-gate *A* to the part *subsys* of the multi-partite state vector or density matrix *state*.

**Note**

The dimension of the gate *A* must match the dimension of *subsys*

## Parameters

<i>state</i>	Eigen expression
<i>A</i>	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>d</i>	Subsystem dimensions

## Returns

CTRL-A gate applied to the part *subsys* of *state*

#### 6.1.3.13 `std::vector<cmat> qpp::choi2kraus ( const cmat & A )`

Orthogonal Kraus operators from Choi matrix.

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix *A*

## Note

The Kraus operators satisfy  $Tr(K_i^\dagger K_j) = \delta_{ij}$  for all  $i \neq j$

## Parameters

<i>A</i>	Choi matrix
----------	-------------

## Returns

Set of orthogonal Kraus operators

#### 6.1.3.14 `cmat qpp::choi2super ( const cmat & A )`

Converts Choi matrix to superoperator matrix.

## Parameters

<i>A</i>	Choi matrix
----------	-------------

## Returns

Superoperator matrix

#### 6.1.3.15 `template<typename Derived1, typename Derived2> dyn_mat<typename Derived1::Scalar> qpp::comm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`

Commutator.

Commutator  $[A, B] = AB - BA$ . Both *A* and *B* must be Eigen expressions over the same scalar field.

## Parameters

<i>A</i>	Eigen expression
<i>B</i>	Eigen expression

## Returns

Commutator  $AB - BA$ , as a dynamic matrix over the same scalar field as *A*

6.1.3.16 `std::vector<idx> qpp::compperm ( const std::vector< idx > & perm, const std::vector< idx > & sigma )`

Compose permutations.

## Parameters

<i>perm</i>	Permutation
<i>sigma</i>	Permutation

## Returns

Composition of the permutations  $perm \circ sigma = perm(sigma)$

6.1.3.17 `template<typename Derived> double qpp::concurrence ( const Eigen::MatrixBase< Derived> & A )`

Wootters concurrence of the bi-partite qubit mixed state *A*.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Wootters concurrence

6.1.3.18 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived> & A )`

Complex conjugate.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Complex conjugate of *A*, as a dynamic matrix over the same scalar field as *A*

6.1.3.19 `double qpp::contfrac2x ( const std::vector< int> & cf, idx n )`

Real representation of a simple continued fraction.

## Parameters

<i>cf</i>	Integer vector containing the simple continued fraction expansion
<i>n</i>	Number of terms considered in the continued fraction expansion. If <i>n</i> is greater than the size of <i>cf</i> , then all terms in <i>cf</i> are considered.

## Returns

Real representation of the simple continued fraction

6.1.3.20 `double qpp::contfrac2x ( const std::vector< int> & cf )`

Real representation of a simple continued fraction.

## Parameters

<i>cf</i>	Integer vector containing the simple continued fraction expansion
-----------	---

## Returns

Real representation of the simple continued fraction

6.1.3.21 `template<typename Derived> cmat qpp::cosm ( const Eigen::MatrixBase< Derived> & A )`

Matrix cos.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Matrix cosine of *A*

6.1.3.22 `template<typename OutputScalar, typename Derived> dyn_mat<OutputScalar> qpp::cwise ( const Eigen::MatrixBase< Derived> & A, OutputScalar*)(const typename Derived::Scalar &) f )`

Functor.

## Parameters

<i>A</i>	Eigen expression
<i>f</i>	Pointer-to-function from scalars of <i>A</i> to <i>OutputScalar</i>

## Returns

Component-wise  $f(A)$ , as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.23 `template<typename Derived> Derived::Scalar qpp::det ( const Eigen::MatrixBase< Derived> & A )`

Determinant.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Determinant of *A*, as a scalar in the same scalar field as *A*. Returns  $\pm\infty$  when the determinant overflows/underflows.

6.1.3.24 `template<typename Derived> internal::IOManipEigen qpp::disp ( const Eigen::MatrixBase< Derived> & A, double chop = qpp::chop )`

Eigen expression ostream manipulator.

## Parameters

<i>A</i>	Eigen expression
<i>chop</i>	Set to zero the elements smaller in absolute value than <i>chop</i>

## Returns

Instance of `qpp::internal::internal::LOManipEigen`

#### 6.1.3.25 `internal::LOManipEigen qpp::disp ( cplx z, double chop = qpp::chop )`

Complex number ostream manipulator.

## Parameters

<i>z</i>	Complex number (or any other type implicitly cast-able to <code>std::complex&lt;double&gt;</code> )
<i>chop</i>	Set to zero the elements smaller in absolute value than <i>chop</i>

## Returns

Instance of `qpp::internal::internal::LOManipEigen`

#### 6.1.3.26 `template<typename InputIterator > internal::LOManipRange<InputIterator> qpp::disp ( const InputIterator & first, const InputIterator & last, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )`

Range ostream manipulator.

## Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

## Returns

Instance of `qpp::internal::internal::LOManipRange`

#### 6.1.3.27 `template<typename Container > internal::LOManipRange<typename Container::const_iterator> qpp::disp ( const Container & c, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )`

Standard container ostream manipulator. The container must support `std::begin()`, `std::end()` and forward iteration.

## Parameters

<i>x</i>	Container
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

## Returns

Instance of `qpp::internal::internal::LOManipRange`

6.1.3.28 `template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp ( const PointerType * p,  
idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )`

C-style pointer ostream manipulator.

## Parameters

$x$	Pointer to the first element
$n$	Number of elements to be displayed
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

## Returns

Instance of `qpp::internal::internal::LOManipPointer`

**6.1.3.29** `template<typename Derived > std::pair<dyn_col_vect<cplx>, cmat> qpp::eig ( const Eigen::MatrixBase<Derived > & A )`

Full eigen decomposition.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Pair of: 1. Eigenvalues of  $A$ , as a complex dynamic column vector, and 2. Eigenvectors of  $A$ , as columns of a complex dynamic matrix

**6.1.3.30** `template<typename Derived > double qpp::entanglement ( const Eigen::MatrixBase<Derived > & A, const std::vector<idx > & dims )`

Entanglement of the bi-partite pure state  $A$ .

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

## See also

[qpp::entropy\(\)](#)

## Parameters

$A$	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

## Returns

Entanglement, with the logarithm in base 2

**6.1.3.31** `template<typename Derived > double qpp::entropy ( const Eigen::MatrixBase<Derived > & A )`

von-Neumann entropy of the density matrix  $A$

## Parameters

---



$A$	Eigen expression
-----	------------------

**Returns**

von-Neumann entropy, with the logarithm in base 2

**6.1.3.32** `double qpp::entropy ( const std::vector< double > & prob )`

Shannon entropy of the probability distribution *prob*.

**Parameters**

<i>prob</i>	Real probability vector
-------------	-------------------------

**Returns**

Shannon entropy, with the logarithm in base 2

**6.1.3.33** `template<typename Derived> dyn_col_vect<cplx> qpp::evals ( const Eigen::MatrixBase< Derived > & A )`

Eigenvalues.

**Parameters**

$A$	Eigen expression
-----	------------------

**Returns**

Eigenvalues of  $A$ , as a complex dynamic column vector

**6.1.3.34** `template<typename Derived> cmat qpp::evecs ( const Eigen::MatrixBase< Derived > & A )`

Eigenvectors.

**Parameters**

$A$	Eigen expression
-----	------------------

**Returns**

Eigenvectors of  $A$ , as columns of a complex dynamic matrix

**6.1.3.35** `template<typename Derived> cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )`

Matrix exponential.

**Parameters**

$A$	Eigen expression
-----	------------------

**Returns**

Matrix exponential of  $A$

**6.1.3.36** `template<typename Derived> cmat qpp::funm ( const Eigen::MatrixBase< Derived > & A, cplx(*) (const cplx &) f )`

Functional calculus  $f(A)$

## Parameters

$A$	Eigen expression
$f$	Pointer-to-function from complex to complex

## Returns

 $f(A)$ 
6.1.3.37 `idx qpp::gcd ( idx  $m$ , idx  $n$  )`

Greatest common divisor of two non-negative integers.

## Parameters

$m$	Non-negative integer
$n$	Non-negative integer

## Returns

Greatest common divisor of  $m$  and  $n$

6.1.3.38 `idx qpp::gcd ( const std::vector< idx > &  $ns$  )`

Greatest common divisor of a list of non-negative integers.

## Parameters

$ns$	List of non-negative integers
------	-------------------------------

## Returns

Greatest common divisor of all numbers in  $ns$

6.1.3.39 `template<typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase< Derived > &  $A$  )`

G-concurrence of the bi-partite pure state  $A$ .

## Note

Both local dimensions must be equal

Uses [qpp::logdet\(\)](#) to avoid overflows

## See also

[qpp::logdet\(\)](#)

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

G-concurrence

6.1.3.40 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )`

Gram-Schmidt orthogonalization.

## Parameters

<i>Vs</i>	std::vector of Eigen expressions as column vectors
-----------	--

## Returns

Gram-Schmidt vectors of *Vs* as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.41 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const std::initializer_list<Derived > & Vs )`

Gram-Schmidt orthogonalization.

## Parameters

<i>Vs</i>	std::initializer_list of Eigen expressions as column vectors
-----------	--

## Returns

Gram-Schmidt vectors of *Vs* as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.42 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const Eigen::MatrixBase<Derived > & A )`

Gram-Schmidt orthogonalization.

## Parameters

<i>A</i>	Eigen expression, the input vectors are the columns of <i>A</i>
----------	---

## Returns

Gram-Schmidt vectors of the columns of *A*, as columns of a dynamic matrix over the same scalar field as *A*

6.1.3.43 `template<typename Derived > std::pair<dyn_col_vect<double>, cmat> qpp::heig ( const Eigen::MatrixBase<Derived > & A )`

Full eigen decomposition of Hermitian expression.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Pair of: 1. Eigenvalues of *A*, as a real dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.44 `template<typename Derived > dyn_col_vect<double> qpp::hevals ( const Eigen::MatrixBase<Derived > & A )`

Hermitian eigenvalues.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Eigenvalues of Hermitian  $A$ , as a real dynamic column vector

6.1.3.45 `template<typename Derived> cmat qpp::hevects ( const Eigen::MatrixBase< Derived > & A )`

Hermitian eigenvectors.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Eigenvectors of Hermitian  $A$ , as columns of a complex matrix

6.1.3.46 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::inverse ( const Eigen::MatrixBase< Derived > & A )`

Inverse.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Inverse of  $A$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.47 `std::vector<idx> qpp::invperm ( const std::vector< idx > & perm )`

Inverse permutation.

## Parameters

$perm$	Permutation
--------	-------------

## Returns

Inverse of the permutation  $perm$

6.1.3.48 `cmat qpp::kraus2choi ( const std::vector< cmat > & Ks )`

Choi matrix.

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $K_s$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

## Note

The superoperator matrix  $S$  and the Choi matrix  $C$  are related by  $S_{ab,mn} = C_{ma,nb}$

## Parameters

<i>Ks</i>	Set of Kraus operators
-----------	------------------------

## Returns

Choi matrix

#### 6.1.3.49 `cmat qpp::kraus2super ( const std::vector< cmat > & Ks )`

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators *Ks* in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

## Parameters

<i>Ks</i>	Set of Kraus operators
-----------	------------------------

## Returns

Superoperator matrix

#### 6.1.3.50 `template<typename T> dyn_mat<typename T::Scalar> qpp::kron ( const T & head )`

Kronecker product.

Used to stop the recursion for the variadic template version of [qpp::kron\(\)](#)

## Parameters

<i>head</i>	Eigen expression
-------------	------------------

## Returns

Its argument *head*

#### 6.1.3.51 `template<typename T, typename... Args> dyn_mat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )`

Kronecker product.

## Parameters

<i>head</i>	Eigen expression
<i>tail</i>	Variadic Eigen expression (zero or more parameters)

## Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

#### 6.1.3.52 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::kron ( const std::vector< Derived > & As )`

Kronecker product.

## Parameters

<i>As</i>	std::vector of Eigen expressions
-----------	----------------------------------

## Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

**6.1.3.53** `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::kron ( const std::initializer_list<Derived> & As )`

Kronecker product.

## Parameters

<i>As</i>	std::initializer_list of Eigen expressions, such as { <i>A1</i> , <i>A2</i> , ... , <i>Ak</i> }
-----------	---

## Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

**6.1.3.54** `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::kronpow ( const Eigen::MatrixBase<Derived> & A, idx n )`

Kronecker power.

## Parameters

<i>A</i>	Eigen expression
<i>n</i>	Non-negative integer

## Returns

Kronecker product of *A* with itself *n* times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field as *A*

**6.1.3.55** `idx qpp::lcm ( idx m, idx n )`

Least common multiple of two positive integers.

## Parameters

<i>m</i>	Positive integer
<i>n</i>	Positive integer

## Returns

Least common multiple of *m* and *n*

**6.1.3.56** `idx qpp::lcm ( const std::vector< idx > & ns )`

Least common multiple of a list of positive integers.

## Parameters

<i>ns</i>	List of positive integers
-----------	---------------------------

## Returns

Least common multiple of all numbers in *ns*

**6.1.3.57** `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::load ( const std::string & fname )`

Loads Eigen matrix from a binary file (internal format) in double precision.

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

## See also

[qpp::loadMATLABmatrix\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>fname</i>	Output file name

**6.1.3.58** `template<typename Derived> Derived qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

This is the generic version that always throws [qpp::Exception::Type::UNDEFINED\\_TYPE](#). It is specialized only for [qpp::dmat](#) and [qpp::cmat](#) (the only matrix types that can be loaded)

**6.1.3.59** `template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`  
[inline]

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic double matrix from the
MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

## Note

If *var\_name* is a complex matrix, only the real part is loaded



## Parameters

<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be loaded

## Returns

Eigen double dynamic matrix ([qpp::dmat](#))

**6.1.3.60** `template<> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`  
`[inline]`

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic complex matrix from the
MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

## Parameters

<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be loaded

## Returns

Eigen complex dynamic matrix ([qpp::cmat](#))

**6.1.3.61** `template<typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase< Derived > & A )`

Logarithm of the determinant.

Useful when the determinant overflows/underflows

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Logarithm of the determinant of *A*, as a scalar in the same scalar field as *A*

**6.1.3.62** `template<typename Derived > cmat qpp::logm ( const Eigen::MatrixBase< Derived > & A )`

Matrix logarithm.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Matrix logarithm of *A*

**6.1.3.63** `template<typename Derived > double qpp::lognegativity ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Logarithmic negativity of the bi-partite mixed state *A*.

## Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

## Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.64 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

## Note

The dimension of all *Ks* must match the dimension of *subsys*.

## Parameters

<i>A</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Tuple consisting of 1. Result of the measurement, 2. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

6.1.3.65 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::initializer_list< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

## Note

The dimension of all *Ks* must match the dimension of *subsys*.

## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system
<i>Ks</i>	Set of Kraus operators

## Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.66** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const std::vector< cmat> & Ks, const std::vector< idx> & subsys, const idx d = 2 )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

#### Note

The dimension of all *Ks* must match the dimension of *subsys*.

#### Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions
<i>Ks</i>	Set of Kraus operators

#### Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.67** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const std::initializer_list< cmat> & Ks, const std::vector< idx> & subsys, const idx d = 2 )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

#### Note

The dimension of all *Ks* must match the dimension of *subsys*.

#### Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions
<i>Ks</i>	Set of Kraus operators

#### Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.68** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const cmat & U, const std::vector< idx> & subsys, const std::vector< idx> & dims )`

Measures the part *subsys* of the multi-partite state *A* in the orthonormal basis specified by the unitary matrix *U*.

#### Note

The dimension of *U* must match the dimension of *subsys*.

## Parameters

$A$	Eigen expression
$subsys$	Subsystem indexes that are measured
$dims$	Dimensions of the multi-partite system
$U$	Unitary matrix whose columns represent the measurement basis vectors

## Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.69** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const cmat & U, const std::vector< idx> & subsys, const idx d = 2 )`

Measures the part  $subsys$  of the multi-partite state  $A$  in the orthonormal basis specified by the unitary matrix  $U$ .

## Note

The dimension of  $U$  must match the dimension of  $subsys$ .

## Parameters

$A$	Eigen expression
$subsys$	Subsystem indexes that are measured
$d$	Subsystem dimensions
$U$	Unitary matrix whose columns represent the measurement basis vectors

## Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.70** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const std::vector< cmat> & Ks )`

Measures the state  $A$  using the set of Kraus operators  $Ks$ .

## Parameters

$A$	Eigen expression
$Ks$	Set of Kraus operators

## Returns

Tuple consisting of 1. Result of the measurement,

1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.71** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const std::initializer_list< cmat> & Ks )`

Measures the state  $A$  using the set of Kraus operators  $Ks$ .

## Parameters

<i>A</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators

## Returns

- Tuple consisting of
1. Result of the measurement,
  1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.72** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const cmat & U )`

Measures the state *A* in the orthonormal basis specified by the unitary matrix *U*.

## Parameters

<i>A</i>	Eigen expression
<i>U</i>	Unitary matrix whose columns represent the measurement basis vectors

## Returns

- Tuple consisting of
1. Result of the measurement,
  1. Vector of outcome probabilities and 3. Vector of post-measurement normalized states

**6.1.3.73** `ket qpp::mket ( const std::vector< idx> & mask, const std::vector< idx> & dims )`

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , where *mask* is a `std::vector` of non-negative integers. Each element in *mask* has to be smaller than the corresponding element in *dims*.

## Parameters

<i>mask</i>	<code>std::vector</code> of non-negative integers
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Multi-partite qudit state vector, as a complex dynamic column vector

**6.1.3.74** `ket qpp::mket ( const std::vector< idx> & mask, idx d = 2 )`

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , all subsystem having equal dimension *d*. *mask* is a `std::vector` of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

## Parameters

<i>mask</i>	<code>std::vector</code> of non-negative integers
<i>d</i>	Subsystem dimensions

## Returns

Multi-partite qudit state vector, as a complex dynamic column vector

#### 6.1.3.75 `cmat qpp::mprj ( const std::vector< idx > & mask, const std::vector< idx > & dims )`

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|\text{mask}\rangle$ , where *mask* is a `std::vector` of non-negative integers. Each element in *mask* has to be smaller than the corresponding element in *dims*.

##### Parameters

<i>mask</i>	<code>std::vector</code> of non-negative integers
<i>dims</i>	Dimensions of the multi-partite system

##### Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

#### 6.1.3.76 `cmat qpp::mprj ( const std::vector< idx > & mask, idx d = 2 )`

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|\text{mask}\rangle$ , all subsystem having equal dimension *d*. *mask* is a `std::vector` of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

##### Parameters

<i>mask</i>	<code>std::vector</code> of non-negative integers
<i>d</i>	Subsystem dimensions

##### Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

#### 6.1.3.77 `idx qpp::multiidx2n ( const std::vector< idx > & midx, const std::vector< idx > & dims )`

Multi-index to non-negative integer index.

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

##### Parameters

<i>midx</i>	Multi-index
<i>dims</i>	Dimensions of the multi-partite system

##### Returns

Non-negative integer index

#### 6.1.3.78 `std::vector<idx> qpp::n2multiidx ( idx n, const std::vector< idx > & dims )`

Non-negative integer index to multi-index.

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

##### Parameters

<i>n</i>	Non-negative integer index
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Multi-index of the same size as *dims*

**6.1.3.79** `template<typename Derived> double qpp::negativity ( const Eigen::MatrixBase< Derived> & A, const std::vector< idx> & dims )`

Negativity of the bi-partite mixed state *A*.

**Parameters**

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

**Returns**

Negativity

**6.1.3.80** `template<typename Derived> double qpp::norm ( const Eigen::MatrixBase< Derived> & A )`

Frobenius norm.

**Parameters**

<i>A</i>	Eigen expression
----------	------------------

**Returns**

Frobenius norm of *A*, as a real number

**6.1.3.81** `cplx qpp::omega ( idx D ) [inline]`

D-th root of unity.

**Parameters**

<i>D</i>	Non-negative integer
----------	----------------------

**Returns**

D-th root of unity  $\exp(2\pi i/D)$

**6.1.3.82** `constexpr cplx qpp::operator""_i ( unsigned long long int x )`

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

### 6.1.3.83 `constexpr cplx qpp::operator""_i ( long double x )`

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

### 6.1.3.84 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase<Derived> & A, idx n )`

Matrix power.

Explicitly multiplies the matrix  $A$  with itself  $n$  times. By convention  $A^0 = I$ .

Parameters

$A$	Eigen expression
$n$	Non-negative integer

Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field as  $A$

### 6.1.3.85 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::prj ( const Eigen::MatrixBase<Derived> & V )`

Projector.

Normalized projector onto state vector

Parameters

$V$	Eigen expression
-----	------------------

Returns

Projector onto the state vector  $V$ , or the matrix *Zero* if  $V$  has norm zero (i.e. smaller than `qpp::eps`), as a dynamic matrix over the same scalar field as  $A$

### 6.1.3.86 `template<typename Derived> Derived::Scalar qpp::prod ( const Eigen::MatrixBase<Derived> & A )`

Element-wise product of  $A$ .

Parameters

$A$	Eigen expression
-----	------------------

Returns

Element-wise product of  $A$ , as a scalar in the same scalar field as  $A$

### 6.1.3.87 `template<typename InputIterator> InputIterator::value_type qpp::prod ( InputIterator first, InputIterator last )`

Element-wise product of a range.



## Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

## Returns

Element-wise product of the range, as a scalar in the same scalar field as the range

6.1.3.88 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase<Derived> & A, const std::vector<idx> & subsys, const std::vector<idx> & dims )`

Partial trace.

Partial trace of the multi-partite density matrix over a list of subsystems

## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsystems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as *A*

6.1.3.89 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase<Derived> & A, const std::vector<idx> & subsys, idx d = 2 )`

Partial trace.

Partial trace of the multi-partite density matrix over a list of subsystems

## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>d</i>	Subsystem dimensions

## Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsystems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as *A*

6.1.3.90 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptrace1 ( const Eigen::MatrixBase<Derived> & A, const std::vector<idx> & dims )`

Partial trace.

Partial trace of density matrix over the first subsystem in a bi-partite system

## Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system (must be a <code>std::vector</code> with 2 elements)

**Returns**

Partial trace  $Tr_A(\cdot)$  over the first subsystem  $A$  in a bi-partite system  $A \otimes B$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.91 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptrace2 ( const Eigen::MatrixBase<Derived> & A, const std::vector< idx > & dims )`

Partial trace.

**Parameters**

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system (must be a <code>std::vector</code> with 2 elements)

**Returns**

Partial trace  $Tr_B(\cdot)$  over the second subsystem  $B$  in a bi-partite system  $A \otimes B$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.92 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase<Derived> & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Partial transpose.

Partial transpose of the multi-partite density matrix over a list of subsystems

**Parameters**

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsystems  $subsys$  in a multi-partite system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.93 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase<Derived> & A, const std::vector< idx > & subsys, idx d = 2 )`

Partial transpose.

Partial transpose of the multi-partite density matrix over a list of subsystems

**Parameters**

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>d</i>	Subsystem dimensions

**Returns**

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsystems  $subsys$  in a multi-partite system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.94 `template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, const std::vector< idx > & dims )`

Quantum mutual information between 2 subsystems of a composite system.

## Parameters

<i>A</i>	Eigen expression
<i>subsysA</i>	Indexes of the first subsystem
<i>subsysB</i>	Indexes of the second subsystem
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Mutual information between the 2 subsystems

6.1.3.95 `template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, idx d = 2 )`

Quantum mutual information between 2 subsystems of a composite system.

## Parameters

<i>A</i>	Eigen expression
<i>subsysA</i>	Indexes of the first subsystem
<i>subsysB</i>	Indexes of the second subsystem
<i>d</i>	Subsystem dimensions

## Returns

Mutual information between the 2 subsystems

6.1.3.96 `template<typename Derived > Derived qpp::rand ( idx rows, idx cols, double a = 0, double b = 1 )`

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws `qpp::Exception::Type::UNDEFINED_TYPE`. It is specialized only for `qpp::dmat` and `qpp::cmat`

6.1.3.97 `template<> dmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]`

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (`qpp::dmat`)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

## Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>a</i>	Beginning of the interval, belongs to it

<i>b</i>	End of the interval, does not belong to it
----------	--

**Returns**

Random real matrix

**6.1.3.98** `template<> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]`

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices ([qpp::cmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd,
// with entries (both real and imaginary) uniformly distributed in [-1,1)
auto mat = rand<cmat>(3, 3, -1, 1);
```

**Parameters**

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, does not belong to it

**Returns**

Random complex matrix

**6.1.3.99** `double qpp::rand ( double a = 0, double b = 1 )`

Generates a random real number uniformly distributed in the interval [a, b)

**Parameters**

<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, does not belong to it

**Returns**

Random real number (double) uniformly distributed in the interval [a, b)

**6.1.3.100** `cmat qpp::randH ( idx D )`

Generates a random Hermitian matrix.

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Random Hermitian matrix

**6.1.3.101** `idx qpp::randidx ( idx a = std::numeric_limits<idx>::min(), idx b = std::numeric_limits<idx>::max() )`

Generates a random index (idx) uniformly distributed in the interval [a, b].

## Parameters

<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, belongs to it

## Returns

Random index (*idx*) uniformly distributed in the interval [*a*, *b*]

6.1.3.102 `ket qpp::randket ( idx D )`

Generates a random normalized ket (pure state vector)

## Parameters

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

## Returns

Random normalized ket

6.1.3.103 `std::vector<cmat> qpp::randkraus ( idx N, idx D )`

Generates a set of random Kraus operators.

## Note

The set of Kraus operators satisfy the closure condition  $\sum_i K_i^\dagger K_i = I$

## Parameters

<i>N</i>	Number of Kraus operators
<i>D</i>	Dimension of the Hilbert space

## Returns

Set of *N* Kraus operators satisfying the closure condition

6.1.3.104 `template<typename Derived > Derived qpp::randn ( idx rows, idx cols, double mean = 0, double sigma = 1 )`

Generates a random matrix with entries normally distributed in N(*mean*, *sigma*)

If complex, then both real and imaginary parts are normally distributed in N(*mean*, *sigma*)

This is the generic version that always throws `qpp::Exception::Type::UNDEFINED_TYPE`. It is specialized only for `qpp::dmat` and `qpp::cmat`

6.1.3.105 `template<> dmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]`

Generates a random real matrix with entries normally distributed in N(*mean*, *sigma*), specialization for double matrices (`qpp::dmat`)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

## Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>mean</i>	Mean
<i>sigma</i>	Standard deviation

## Returns

Random real matrix

**6.1.3.106** `template<> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]`

Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices ([qpp::cmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd,
// with entries (both real and imaginary) normally distributed in N(0,2)
auto mat = randn<cmat>(3, 3, 0, 2);
```

## Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>mean</i>	Mean
<i>sigma</i>	Standard deviation

## Returns

Random complex matrix

**6.1.3.107** `double qpp::randn ( double mean = 0, double sigma = 1 )`

Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$

## Parameters

<i>mean</i>	Mean
<i>sigma</i>	Standard deviation

## Returns

Random real number normally distributed in  $N(\text{mean}, \text{sigma})$

**6.1.3.108** `std::vector<idx> qpp::randperm ( idx n )`

Generates a random uniformly distributed permutation.

Uses Knuth's shuffle method (as implemented by `std::shuffle`), so that all permutations are equally probable

## Parameters

$n$	Size of the permutation
-----	-------------------------

## Returns

Random permutation of size  $n$

6.1.3.109 `cmat qpp::randrho ( idx  $D$  )`

Generates a random density matrix.

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Random density matrix

6.1.3.110 `cmat qpp::randU ( idx  $D$  )`

Generates a random unitary matrix.

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Random unitary

6.1.3.111 `cmat qpp::randV ( idx  $D_{in}$ , idx  $D_{out}$  )`

Generates a random isometry matrix.

## Parameters

$D_{in}$	Size of the input Hilbert space
$D_{out}$	Size of the output Hilbert space

## Returns

Random isometry matrix

6.1.3.112 `template<typename Derived> double qpp::renyi ( const Eigen::MatrixBase< Derived > &  $A$ , double  $\alpha$  )`

Renyi-  $\alpha$  entropy of the density matrix  $A$ , for  $\alpha \geq 0$ .

## Note

When  $\alpha \rightarrow 1$  the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2



## Parameters

<i>A</i>	Eigen expression
<i>alpha</i>	Non-negative real number, use <a href="#">qpp::infy</a> for $\alpha = \infty$

## Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.113 `double qpp::renyi ( const std::vector< double > & prob, double alpha )`

Renyi-  $\alpha$  entropy of the probability distribution *prob*, for  $\alpha \geq 0$ .

## Note

When  $\alpha \rightarrow 1$  the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

## Parameters

<i>prob</i>	Real probability vector
<i>alpha</i>	Non-negative real number, use <a href="#">qpp::infy</a> for $\alpha = \infty$

## Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.114 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, idx rows, idx cols )`

Reshape.

Uses column-major order when reshaping (same as MATLAB)

## Parameters

<i>A</i>	Eigen expression
<i>rows</i>	Number of rows of the reshaped matrix
<i>cols</i>	Number of columns of the reshaped matrix

## Returns

Reshaped matrix with *rows* rows and *cols* columns, as a dynamic matrix over the same scalar field as *A*

6.1.3.115 `template<typename Derived > dyn_col_vect<typename Derived::Scalar> qpp::rho2pure ( const Eigen::MatrixBase< Derived > & A )`

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

## Note

No purity check is done, the input state *A* must have rank one, otherwise the function returns the first non-zero eigenvector of *A*

## Parameters

<i>A</i>	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. <i>A</i> is assumed to have rank one
----------	--

## Returns

The unique non-zero eigenvector of *A*, as a dynamic column vector over the same scalar field as *A*

6.1.3.116 `template<typename Derived > void qpp::save ( const Eigen::MatrixBase< Derived > & A, const std::string & fname )`

Saves Eigen expression to a binary file (internal format) in double precision.

## See also

[qpp::saveMATLABmatrix\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>fname</i>	Output file name

6.1.3.117 `template<typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< Derived > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode )`

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

This is the generic version that always throws [qpp::Exception::Type::UNDEFINED\\_TYPE](#). It is specialized only for [qpp::dmat](#) and [qpp::cmat](#) (the only matrix types that can be saved)

6.1.3.118 `template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< dmat > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode ) [inline]`

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))

## Parameters

<i>A</i>	Eigen expression over the complex field
<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be saved
<i>mode</i>	Saving mode (append, overwrite etc.), see MATLAB's <i>matOpen()</i> documentation for details

6.1.3.119 `template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< cmat > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode ) [inline]`

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))

## Parameters

<i>A</i>	Eigen expression over the complex field
<i>mat_file</i>	MATALB .mat file

<i>var_name</i>	Variable name in the .mat file representing the matrix to be saved
<i>mode</i>	Saving mode (append, overwrite etc.), see MATLAB's <i>matOpen()</i> documentation for details

6.1.3.120 `template<typename Derived > double qpp::schatten ( const Eigen::MatrixBase< Derived > & A, idx p )`

Schatten norm.

Parameters

<i>A</i>	Eigen expression
<i>p</i>	Integer, greater or equal to 1

Returns

Schatten-*p* norm of *A*, as a real number

6.1.3.121 `template<typename Derived > cmat qpp::schmidtA ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Schmidt basis on Alice's side.

Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

Returns

Unitary matrix *U* whose columns represent the Schmidt basis vectors on Alice's side.

6.1.3.122 `template<typename Derived > cmat qpp::schmidtB ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Schmidt basis on Bob's side.

Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

Returns

Unitary matrix *V* whose columns represent the Schmidt basis vectors on Bob's side.

6.1.3.123 `template<typename Derived > dyn_col_vect<double> qpp::schmidtcoeffs ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Schmidt coefficients of the bi-partite pure state *A*.

Note

The sum of the squares of the Schmidt coefficients equals 1

See also

[qpp::schmidtprobs\(\)](#)

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system

## Returns

Schmidt coefficients of  $A$ , as a real dynamic column vector

6.1.3.124 `template<typename Derived > std::vector<double> qpp::schmidtprobs ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Schmidt probabilities of the bi-partite pure state  $A$ .

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

## See also

[qpp::schmidtcoeffs\(\)](#)

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system

## Returns

Real vector consisting of the Schmidt probabilities of  $A$

6.1.3.125 `template<typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase< Derived > & A )`

Matrix sin.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Matrix sine of  $A$

6.1.3.126 `template<typename Derived > cmat qpp::spectralpowm ( const Eigen::MatrixBase< Derived > & A, const cplx z )`

Matrix power.

Uses the spectral decomposition of  $A$  to compute the matrix power. By convention  $A^0 = I$ .

## Parameters

$A$	Eigen expression
$z$	Complex number

## Returns

Matrix power  $A^z$

6.1.3.127 `template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )`

Matrix square root.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Matrix square root of  $A$

6.1.3.128 `template<typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase< Derived > & A )`

Element-wise sum of  $A$ .

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Element-wise sum of  $A$ , as a scalar in the same scalar field as  $A$

6.1.3.129 `template<typename InputIterator > InputIterator::value_type qpp::sum ( InputIterator first, InputIterator last )`

Element-wise sum of a range.

## Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

## Returns

Element-wise sum of the range, as a scalar in the same scalar field as the range

6.1.3.130 `cmat qpp::super2choi ( const cmat & A )`

Converts superoperator matrix to Choi matrix.

## Parameters

$A$	Superoperator matrix
-----	----------------------

## Returns

Choi matrix

6.1.3.131 `template<typename Derived > dyn_col_vect<double> qpp::svals ( const Eigen::MatrixBase< Derived > & A )`

Singular values.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Singular values of  $A$ , ordered in decreasing order, as a real dynamic column vector

6.1.3.132 `template<typename Derived > std::tuple<cmat, dyn_col_vect<double>, cmat> qpp::svd ( const Eigen::MatrixBase< Derived > & A )`

Full singular value decomposition.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Tuple of: 1. Left singular vectors of  $A$ , as columns of a complex dynamic matrix, 2. Singular values of  $A$ , ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of  $A$ , as columns of a complex dynamic matrix

6.1.3.133 `template<typename Derived > cmat qpp::svdU ( const Eigen::MatrixBase< Derived > & A )`

Left singular vectors.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Complex dynamic matrix, whose columns are the left singular vectors of  $A$

6.1.3.134 `template<typename Derived > cmat qpp::svdV ( const Eigen::MatrixBase< Derived > & A )`

Right singular vectors.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Complex dynamic matrix, whose columns are the right singular vectors of  $A$

6.1.3.135 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & dims )`

Subsystem permutation.

Permutes the subsystems in a state vector or density matrix. The qubit  $perm[i]$  is permuted to the location  $i$ .

## Parameters

$A$	Eigen expression
$perm$	Permutation
$dims$	Dimensions of the multi-partite system

## Returns

Permuted system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.136 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2 )`

Subsystem permutation.

Permutes the subsystems in a state vector or density matrix. The qubit  $perm[i]$  is permuted to the location  $i$ .

## Parameters

$A$	Eigen expression
$perm$	Permutation
$d$	Subsystem dimensions

## Returns

Permuted system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.137 `template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )`

Trace.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Trace of  $A$ , as a scalar in the same scalar field as  $A$

6.1.3.138 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::transpose ( const Eigen::MatrixBase< Derived > & A )`

Transpose.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Transpose of  $A$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.139 `template<typename Derived > double qpp::tsallis ( const Eigen::MatrixBase< Derived > & A, double q )`

Tsallis-  $q$  entropy of the density matrix  $A$ , for  $q \geq 0$ .

## Note

When  $q \rightarrow 1$  the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base  $e$

## Parameters

$A$	Eigen expression
$q$	Non-negative real number

## Returns

Tsallis-  $q$  entropy

6.1.3.140 `double qpp::tsallis ( const std::vector< double > & prob, double q )`

Tsallis-  $q$  entropy of the probability distribution  $prob$ , for  $q \geq 0$ .

## Note

When  $q \rightarrow 1$  the Tsallis entropy converges to the Shannon entropy, with the logarithm in base  $e$



## Parameters

<i>prob</i>	Real probability vector
<i>q</i>	Non-negative real number

## Returns

Tsallis-  $q$  entropy

6.1.3.141 `std::vector<long long int> qpp::x2contfrac ( double x, idx n, idx cut = 1e5 )`

Simple continued fraction expansion.

## Parameters

<i>x</i>	Real number
<i>n</i>	Number of terms in the expansion
<i>cut</i>	Stop the expansion when the next term is greater than <i>cut</i>

## Returns

Integer vector containing the simple continued fraction expansion of  $x$ . If there are  $m$  less than  $n$  terms in the expansion, a shorter vector with  $m$  components is returned.

## 6.1.4 Variable Documentation

6.1.4.1 `constexpr double qpp::chop = 1e-10`

Used in [qpp::disp\(\)](#) for setting to zero numbers that have their absolute value smaller than [qpp::chop](#).

6.1.4.2 `const Codes& qpp::codes = Codes::get_instance()`

[qpp::Codes](#) const Singleton

Initializes the codes, see the class [qpp::Codes](#)

6.1.4.3 `constexpr double qpp::ee = 2.718281828459045235360287471352662497`

Base of natural logarithm,  $e$ .

6.1.4.4 `constexpr double qpp::eps = 1e-12`

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if (std::abs(x) < qpp::eps) // x is zero
```

6.1.4.5 `const Gates& qpp::gt = Gates::get_instance()`

[qpp::Gates](#) const Singleton

Initializes the gates, see the class [qpp::Gates](#)

6.1.4.6 `constexpr double qpp::infy = std::numeric_limits<double>::infinity()`

Used to denote infinity in double precision.

6.1.4.7 `const Init& qpp::init = Init::get_instance()`

[qpp::Init](#) const Singleton

Additional initializations/cleanups

6.1.4.8 `constexpr idx qpp::maxn = 64`

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.9 `constexpr double qpp::pi = 3.141592653589793238462643383279502884`

$\pi$

6.1.4.10 `RandomDevices& qpp::rdevs = RandomDevices::get_instance()`

[qpp::RandomDevices](#) Singleton

Initializes the random devices, see the class [qpp::RandomDevices](#)

6.1.4.11 `const States& qpp::st = States::get_instance()`

[qpp::States](#) const Singleton

Initializes the states, see the class [qpp::States](#)

## 6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

### 6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

## 6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

### Classes

- class [IOManipEigen](#)
- class [IOManipPointer](#)
- class [IOManipRange](#)
- class [Singleton](#)

*[Singleton](#) policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)*

## Functions

- void `_n2multiidx` (`idx` n, `idx` numdims, const `idx` \*dims, `idx` \*result)
- `idx` `_multiidx2n` (const `idx` \*midx, `idx` numdims, const `idx` \*dims)
- template<typename Derived >  
bool `_check_square_mat` (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool `_check_vector` (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool `_check_row_vector` (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool `_check_col_vector` (const Eigen::MatrixBase< Derived > &A)
- template<typename T >  
bool `_check_nonzero_size` (const T &x)
- bool `_check_dims` (const std::vector< `idx` > &dims)
- template<typename Derived >  
bool `_check_dims_match_mat` (const std::vector< `idx` > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool `_check_dims_match_cvect` (const std::vector< `idx` > &dims, const Eigen::MatrixBase< Derived > &V)
- template<typename Derived >  
bool `_check_dims_match_rvect` (const std::vector< `idx` > &dims, const Eigen::MatrixBase< Derived > &V)
- bool `_check_eq_dims` (const std::vector< `idx` > &dims, `idx` dim)
- bool `_check_subsys_match_dims` (const std::vector< `idx` > &subsys, const std::vector< `idx` > &dims)
- bool `_check_perm` (const std::vector< `idx` > &perm)
- template<typename Derived1 , typename Derived2 >  
`dyn_mat`< typename  
Derived1::Scalar > `_kron2` (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2  
> &B)
- template<typename T >  
void `variadic_vector_emplace` (std::vector< T > &)
- template<typename T , typename First , typename... Args>  
void `variadic_vector_emplace` (std::vector< T > &v, First &&first, Args &&...args)

### 6.3.1 Detailed Description

Internal utility functions, do not use/modify.

### 6.3.2 Function Documentation

6.3.2.1 template<typename Derived > bool qpp::internal::\_check\_col\_vector ( const Eigen::MatrixBase< Derived > & A )

6.3.2.2 bool qpp::internal::\_check\_dims ( const std::vector< `idx` > & dims )

6.3.2.3 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_cvect ( const std::vector< `idx` > & dims,  
const Eigen::MatrixBase< Derived > & V )

6.3.2.4 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_mat ( const std::vector< `idx` > & dims, const  
Eigen::MatrixBase< Derived > & A )

6.3.2.5 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_rvect ( const std::vector< `idx` > & dims,  
const Eigen::MatrixBase< Derived > & V )

6.3.2.6 bool qpp::internal::\_check\_eq\_dims ( const std::vector< `idx` > & dims, `idx` dim )

- 6.3.2.7 `template<typename T > bool qpp::internal::_check_nonzero_size ( const T & x )`
- 6.3.2.8 `bool qpp::internal::_check_perm ( const std::vector< idx > & perm )`
- 6.3.2.9 `template<typename Derived > bool qpp::internal::_check_row_vector ( const Eigen::MatrixBase< Derived > & A )`
- 6.3.2.10 `template<typename Derived > bool qpp::internal::_check_square_mat ( const Eigen::MatrixBase< Derived > & A )`
- 6.3.2.11 `bool qpp::internal::_check_subsys_match_dims ( const std::vector< idx > & subsys, const std::vector< idx > & dims )`
- 6.3.2.12 `template<typename Derived > bool qpp::internal::_check_vector ( const Eigen::MatrixBase< Derived > & A )`
- 6.3.2.13 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::internal::_kron2 ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`
- 6.3.2.14 `idx qpp::internal::_multiidx2n ( const idx * midx, idx numdims, const idx * dims )` [inline]
- 6.3.2.15 `void qpp::internal::_n2multiidx ( idx n, idx numdims, const idx * dims, idx * result )` [inline]
- 6.3.2.16 `template<typename T > void qpp::internal::variadic_vector_emplace ( std::vector< T > & )`
- 6.3.2.17 `template<typename T , typename First , typename... Args> void qpp::internal::variadic_vector_emplace ( std::vector< T > & v, First && first, Args &&... args )`

## Chapter 7

# Class Documentation

### 7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

```
#include <classes/codes.h>
```

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



## Public Types

- enum `Type` { `Type::FIVE_QUBIT` = 1, `Type::SEVEN_QUBIT_STEANE`, `Type::NINE_QUBIT_SHOR` }  
Code types, add more codes here if needed.

## Public Member Functions

- `ket codeword` (`Type` type, `idx` i) const  
Returns the codeword of the specified code.

## Private Member Functions

- `Codes` ()  
Default constructor.

## Friends

- class `internal::Singleton< const Codes >`

## Additional Inherited Members

### 7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

### 7.1.2 Member Enumeration Documentation

#### 7.1.2.1 enum `qpp::Codes::Type` [strong]

Code types, add more codes here if needed.

See also

`qpp::Codes::codeword()`

Enumerator

**`FIVE_QUBIT`** [[5,1,3]] qubit code  
**`SEVEN_QUBIT_STEANE`** [[7,1,3]] Steane qubit code  
**`NINE_QUBIT_SHOR`** [[9,1,3]] Shor qubit code

### 7.1.3 Constructor & Destructor Documentation

#### 7.1.3.1 `qpp::Codes::Codes ( )` [inline], [private]

Default constructor.

### 7.1.4 Member Function Documentation

#### 7.1.4.1 `ket qpp::Codes::codeword ( Type type, idx i ) const` [inline]

Returns the codeword of the specified code.

## Parameters

<i>type</i>	Code type, defined in the enum <code>qpp::Codes::Types</code>
<i>i</i>	Codeword index

## Returns

*i*-th codeword of the code *type*

## 7.1.5 Friends And Related Function Documentation

7.1.5.1 friend class `internal::Singleton< const Codes >` [friend]

The documentation for this class was generated from the following file:

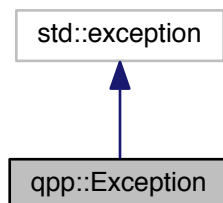
- [classes/codes.h](#)

## 7.2 qpp::Exception Class Reference

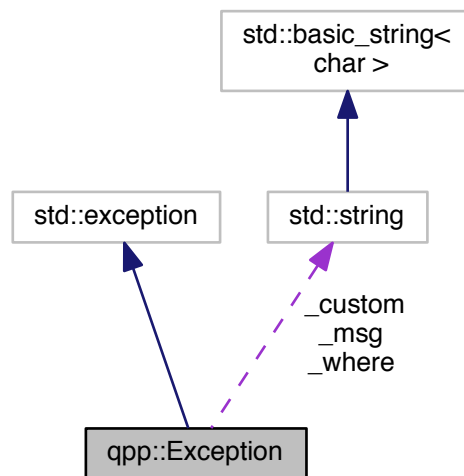
Generates custom exceptions, used when validating function parameters.

```
#include <classes/exception.h>
```

Inheritance diagram for `qpp::Exception`:



Collaboration diagram for `qpp::Exception`:



## Public Types

- enum `Type` {  
`Type::UNKNOWN_EXCEPTION = 1`, `Type::ZERO_SIZE`, `Type::MATRIX_NOT_SQUARE`, `Type::MATRIX_NOT_CVECTOR`,  
`Type::MATRIX_NOT_RVECTOR`, `Type::MATRIX_NOT_VECTOR`, `Type::MATRIX_NOT_SQUARE_OR_CVECTOR`, `Type::MATRIX_NOT_SQUARE_OR_RVECTOR`,  
`Type::MATRIX_NOT_SQUARE_OR_VECTOR`, `Type::MATRIX_MISMATCH_SUBSYS`, `Type::DIMS_INVALID`, `Type::DIMS_NOT_EQUAL`,  
`Type::DIMS_MISMATCH_MATRIX`, `Type::DIMS_MISMATCH_CVECTOR`, `Type::DIMS_MISMATCH_RVECTOR`, `Type::DIMS_MISMATCH_VECTOR`,  
`Type::SUBSYS_MISMATCH_DIMS`, `Type::PERM_INVALID`, `Type::PERM_MISMATCH_DIMS`, `Type::NOT_QUBIT_GATE`,  
`Type::NOT_QUBIT_SUBSYS`, `Type::NOT_BIPARTITE`, `Type::NO_CODEWORD`, `Type::OUT_OF_RANGE`,  
`Type::TYPE_MISMATCH`, `Type::UNDEFINED_TYPE`, `Type::CUSTOM_EXCEPTION` }  
*Exception types, add more here if needed.*

## Public Member Functions

- `Exception` (const `std::string` &where, const `Type` &type)  
*Constructs an exception.*
- `Exception` (const `std::string` &where, const `std::string` &custom)  
*Constructs an exception.*
- virtual const char \* `what` () const noexcept override  
*Overrides `std::exception::what()`*

## Private Member Functions

- void `_construct_exception_msg` ()  
*Constructs the exception's description from its type.*



## Private Attributes

- `std::string _where`
- `std::string _msg`
- `Type _type`
- `std::string _custom`

### 7.2.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

### 7.2.2 Member Enumeration Documentation

#### 7.2.2.1 `enum qpp::Exception::Type` [strong]

[Exception](#) types, add more here if needed.

See also

[qpp::Exception::\\_construct\\_exception\\_msg\(\)](#)

#### Enumerator

- UNKNOWN\_EXCEPTION** Unknown exception
- ZERO\_SIZE** Zero sized object, e.g. empty `Eigen::Matrix` or `std::vector<>` with no elements
- MATRIX\_NOT\_SQUARE** `Eigen::Matrix` is not square
- MATRIX\_NOT\_CVECTOR** `Eigen::Matrix` is not a column vector
- MATRIX\_NOT\_RVECTOR** `Eigen::Matrix` is not a row vector
- MATRIX\_NOT\_VECTOR** `Eigen::Matrix` is not a row/column vector
- MATRIX\_NOT\_SQUARE\_OR\_CVECTOR** `Eigen::Matrix` is not square nor a column vector
- MATRIX\_NOT\_SQUARE\_OR\_RVECTOR** `Eigen::Matrix` is not square nor a row vector
- MATRIX\_NOT\_SQUARE\_OR\_VECTOR** `Eigen::Matrix` is not square nor a row/column vector
- MATRIX\_MISMATCH\_SUBSYS** Matrix size mismatch subsystem sizes (e.g. in [qpp::apply\(\)](#))
- DIMS\_INVALID** `std::vector<idx>` of dimensions has zero size or contains zeros
- DIMS\_NOT\_EQUAL** Local/global dimensions are not equal
- DIMS\_MISMATCH\_MATRIX** Product of the elements of `std::vector<idx>` of dimensions is not equal to the number of rows of `Eigen::Matrix` (assumed to be a square matrix)
- DIMS\_MISMATCH\_CVECTOR** Product of the elements of `std::vector<idx>` of dimensions is not equal to the number of elements of `Eigen::Matrix` (assumed to be a column vector)
- DIMS\_MISMATCH\_RVECTOR** Product of the elements of `std::vector<idx>` of dimensions is not equal to the number of elements of `Eigen::Matrix` (assumed to be a row vector)
- DIMS\_MISMATCH\_VECTOR** Product of the elements of `std::vector<idx>` of dimensions is not equal to the number of elements of `Eigen::Matrix` (assumed to be a row/column vector)
- SUBSYS\_MISMATCH\_DIMS** `std::vector<idx>` of subsystem labels has duplicates, or has entries that are larger than the size of the `std::vector<idx>` of dimensions
- PERM\_INVALID** `std::vector<idx>` does not represent a valid permutation
- PERM\_MISMATCH\_DIMS** Size of the `std::vector<idx>` representing the permutation is different from the size of the `std::vector<idx>` of dimensions
- NOT\_QUBIT\_GATE** `Eigen::Matrix` is not 2 x 2

**NOT\_QUBIT\_SUBSYS** Subsystems are not 2-dimensional

**NOT\_BIPARTITE** `std::vector<idx>` of dimensions has size different from 2

**NO\_CODEWORD** Codeword does not exist, thrown when calling `qpp::Codes::codeword()` with invalid index *i*

**OUT\_OF\_RANGE** Parameter out of range

**TYPE\_MISMATCH** Scalar types do not match

**UNDEFINED\_TYPE** Templated specialization not defined for this type

**CUSTOM\_EXCEPTION** Custom exception, user must provide a custom message

## 7.2.3 Constructor & Destructor Documentation

7.2.3.1 `qpp::Exception::Exception ( const std::string & where, const Type & type )` `[inline]`

Constructs an exception.

Parameters

<i>where</i>	Text representing where the exception occurred
<i>type</i>	<a href="#">Exception</a> 's type, see the strong enumeration <a href="#">qpp::Exception::Type</a>

7.2.3.2 `qpp::Exception::Exception ( const std::string & where, const std::string & custom )` `[inline]`

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

Parameters

<i>where</i>	Text representing where the exception occurred
<i>custom</i>	<a href="#">Exception</a> 's description

## 7.2.4 Member Function Documentation

7.2.4.1 `void qpp::Exception::_construct_exception_msg ( )` `[inline]`, `[private]`

Constructs the exception's description from its type.

See also

[qpp::Exception::Type](#)

Must modify the code of this function if more exceptions are added

7.2.4.2 `virtual const char* qpp::Exception::what ( ) const` `[inline]`, `[override]`, `[virtual]`, `[noexcept]`

Overrides `std::exception::what()`

Returns

[Exception](#)'s description

### 7.2.5 Member Data Documentation

7.2.5.1 `std::string qpp::Exception::_custom` [private]

7.2.5.2 `std::string qpp::Exception::_msg` [private]

7.2.5.3 `Type qpp::Exception::_type` [private]

7.2.5.4 `std::string qpp::Exception::_where` [private]

The documentation for this class was generated from the following file:

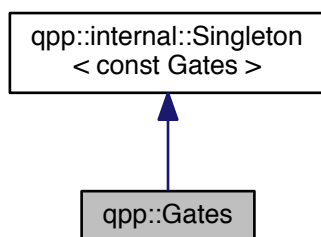
- [classes/exception.h](#)

## 7.3 qpp::Gates Class Reference

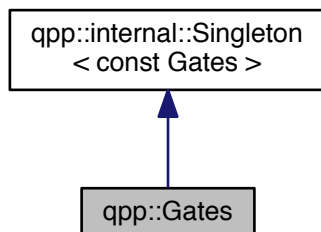
const Singleton class that implements most commonly used gates

```
#include <classes/gates.h>
```

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



## Public Member Functions

- **cmat Rn** (double theta, std::vector< double > n) const  
*Rotation of theta about the 3-dimensional real unit vector n.*
- **cmat Zd** (idx D) const  
*Generalized Z gate for qudits.*
- **cmat Fd** (idx D) const  
*Fourier transform gate for qudits.*
- **cmat Xd** (idx D) const  
*Generalized X gate for qudits.*
- template<typename Derived = Eigen::MatrixXcd>  
Derived **Id** (idx D) const  
*Identity gate.*
- template<typename Derived >  
**dyn\_mat**< typename Derived::Scalar > **CTRL** (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx n, idx d=2) const  
*Generates the multi-partite multiple-controlled-A gate in matrix form.*
- template<typename Derived >  
**dyn\_mat**< typename Derived::Scalar > **expandout** (const Eigen::MatrixBase< Derived > &A, idx pos, const std::vector< idx > &dims) const  
*Expands out.*

## Public Attributes

- **cmat Id2** {cmat::Identity(2, 2)}  
*Identity gate.*
- **cmat H** {cmat::Zero(2, 2)}  
*Hadamard gate.*
- **cmat X** {cmat::Zero(2, 2)}  
*Pauli Sigma-X gate.*
- **cmat Y** {cmat::Zero(2, 2)}  
*Pauli Sigma-Y gate.*
- **cmat Z** {cmat::Zero(2, 2)}  
*Pauli Sigma-Z gate.*
- **cmat S** {cmat::Zero(2, 2)}  
*S gate.*
- **cmat T** {cmat::Zero(2, 2)}  
*T gate.*
- **cmat CNOT** {cmat::Identity(4, 4)}  
*Controlled-NOT control target gate.*
- **cmat CZ** {cmat::Identity(4, 4)}  
*Controlled-Phase gate.*
- **cmat CNOTba** {cmat::Zero(4, 4)}  
*Controlled-NOT target control gate.*
- **cmat SWAP** {cmat::Identity(4, 4)}  
*SWAP gate.*
- **cmat TOF** {cmat::Identity(8, 8)}  
*Toffoli gate.*
- **cmat FRED** {cmat::Identity(8, 8)}  
*Fredkin gate.*

## Private Member Functions

- [Gates](#) ()  
*Initializes the gates.*

## Friends

- class [internal::Singleton](#)< const [Gates](#) >

## Additional Inherited Members

### 7.3.1 Detailed Description

const Singleton class that implements most commonly used gates

### 7.3.2 Constructor & Destructor Documentation

#### 7.3.2.1 qpp::Gates::Gates ( ) [inline], [private]

Initializes the gates.

### 7.3.3 Member Function Documentation

#### 7.3.3.1 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::CTRL ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx n, idx d = 2 ) const [inline]

Generates the multi-partite multiple-controlled- $A$  gate in matrix form.

#### Note

The dimension of the gate  $A$  must match the dimension of *subsys*

#### Parameters

$A$	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate $A$ is applied
$n$	Total number of subsystems
$d$	Subsystem dimensions

#### Returns

CTRL- $A$  gate, as a matrix over the same scalar field as  $A$

#### 7.3.3.2 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::expandout ( const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]

Expands out.

Expands out  $A$  as a matrix in a multi-partite system. Faster than using [qpp::kron](#)( $I, I, \dots, I, A, I, \dots, I$ ).

## Parameters

$A$	Eigen expression
$pos$	Position
$dims$	Dimensions of the multi-partite system

## Returns

Tensor product  $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$ , with  $A$  on position  $pos$ , as a dynamic matrix over the same scalar field as  $A$

7.3.3.3 `cmat qpp::Gates::Fd ( idx  $D$  ) const` `[inline]`

Fourier transform gate for qudits.

## Note

Defined as  $F = \sum_{jk} \exp(2\pi i jk/D) |j\rangle\langle k|$

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Fourier transform gate for qudits

7.3.3.4 `template<typename Derived = Eigen::MatrixXcd> Derived qpp::Gates::Id ( idx  $D$  ) const` `[inline]`

Identity gate.

## Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Identity gate

7.3.3.5 `cmat qpp::Gates::Rn ( double  $\theta$ , std::vector< double >  $n$  ) const` `[inline]`

Rotation of  $\theta$  about the 3-dimensional real unit vector  $n$ .

## Parameters

$\theta$	Rotation angle
$n$	3-dimensional real unit vector

## Returns

Rotation gate

**7.3.3.6** `cmat qpp::Gates::Xd ( idx D ) const` `[inline]`

Generalized X gate for qudits.

**Note**

Defined as  $X = \sum_j |j \oplus 1\rangle \langle j|$

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Generalized X gate for qudits

**7.3.3.7** `cmat qpp::Gates::Zd ( idx D ) const` `[inline]`

Generalized Z gate for qudits.

**Note**

Defined as  $Z = \sum_j \exp(2\pi i j / D) |j\rangle \langle j|$

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Generalized Z gate for qudits

**7.3.4 Friends And Related Function Documentation****7.3.4.1** `friend class internal::Singleton< const Gates >` `[friend]`**7.3.5 Member Data Documentation****7.3.5.1** `cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}`

Controlled-NOT control target gate.

**7.3.5.2** `cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}`

Controlled-NOT target control gate.

**7.3.5.3** `cmat qpp::Gates::CZ {cmat::Identity(4, 4)}`

Controlled-Phase gate.

**7.3.5.4** `cmat qpp::Gates::FRED {cmat::Identity(8, 8)}`

Fredkin gate.

7.3.5.5 `cmat qpp::Gates::H {cmat::Zero(2, 2)}`

Hadamard gate.

7.3.5.6 `cmat qpp::Gates::Id2 {cmat::Identity(2, 2)}`

Identity gate.

7.3.5.7 `cmat qpp::Gates::S {cmat::Zero(2, 2)}`

S gate.

7.3.5.8 `cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}`

SWAP gate.

7.3.5.9 `cmat qpp::Gates::T {cmat::Zero(2, 2)}`

T gate.

7.3.5.10 `cmat qpp::Gates::TOF {cmat::Identity(8, 8)}`

Toffoli gate.

7.3.5.11 `cmat qpp::Gates::X {cmat::Zero(2, 2)}`

Pauli Sigma-X gate.

7.3.5.12 `cmat qpp::Gates::Y {cmat::Zero(2, 2)}`

Pauli Sigma-Y gate.

7.3.5.13 `cmat qpp::Gates::Z {cmat::Zero(2, 2)}`

Pauli Sigma-Z gate.

The documentation for this class was generated from the following file:

- [classes/gates.h](#)

## 7.4 qpp::Init Class Reference

const Singleton class that performs additional initializations/cleanups

```
#include <classes/init.h>
```



Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



### Public Member Functions

- [Init \(\)](#)  
*Additional initializations.*

### Private Member Functions

- [~Init \(\)](#)  
*Cleanups.*

### Friends

- class [internal::Singleton< const Init >](#)

### Additional Inherited Members

#### 7.4.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

## 7.4.2 Constructor & Destructor Documentation

### 7.4.2.1 `qpp::Init::Init ( )` `[inline]`

Additional initializations.

### 7.4.2.2 `qpp::Init::~~Init ( )` `[inline], [private]`

Cleanups.

## 7.4.3 Friends And Related Function Documentation

### 7.4.3.1 `friend class internal::Singleton<const Init>` `[friend]`

The documentation for this class was generated from the following file:

- [classes/init.h](#)

## 7.5 `qpp::internal::IOManipEigen` Class Reference

```
#include <internal/classes/iomanip.h>
```

### Public Member Functions

- `template<typename Derived>`  
`IOManipEigen (const Eigen::MatrixBase< Derived> &A, double chop=qpp::chop)`
- `IOManipEigen (const cplx z, double chop=qpp::chop)`

### Private Attributes

- `cmat_A`
- `double _chop`

### Friends

- `template<typename charT, typename traits>`  
`std::basic_ostream< charT,`  
`traits> &operator<< (std::basic_ostream< charT, traits> &os, const IOManipEigen &rhs)`

## 7.5.1 Constructor & Destructor Documentation

### 7.5.1.1 `template<typename Derived> qpp::internal::IOManipEigen::IOManipEigen ( const Eigen::MatrixBase< Derived> &A, double chop = qpp::chop )` `[inline], [explicit]`

### 7.5.1.2 `qpp::internal::IOManipEigen::IOManipEigen ( const cplx z, double chop = qpp::chop )` `[inline], [explicit]`

## 7.5.2 Friends And Related Function Documentation

7.5.2.1 `template<typename charT , typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const IOManipEigen & rhs ) [friend]`

### 7.5.3 Member Data Documentation

7.5.3.1 `cmat qpp::internal::IOManipEigen::_A [private]`

7.5.3.2 `double qpp::internal::IOManipEigen::_chop [private]`

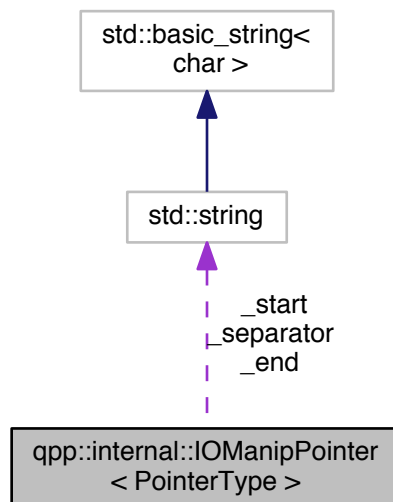
The documentation for this class was generated from the following file:

- [internal/classes/iomanip.h](#)

## 7.6 qpp::internal::IOManipPointer< PointerType > Class Template Reference

`#include <internal/classes/iomanip.h>`

Collaboration diagram for `qpp::internal::IOManipPointer< PointerType >`:



### Public Member Functions

- `IOManipPointer` (`const PointerType *p`, `const idx n`, `const std::string &separator`, `const std::string &start="["`, `const std::string &end="]"`)
- `IOManipPointer` (`const IOManipPointer &`)=default
- `IOManipPointer & operator=` (`const IOManipPointer &`)=default

### Private Attributes

- `const PointerType * _p`
- `idx _n`

- `std::string _separator`
- `std::string _start`
- `std::string _end`

## Friends

- `template<typename charT, typename traits >  
std::basic_ostream< charT,  
traits > & operator<< (std::basic_ostream< charT, traits > &os, const IManipPointer &rhs)`

## 7.6.1 Constructor & Destructor Documentation

- 7.6.1.1 `template<typename PointerType> qpp::internal::IManipPointer< PointerType >::IManipPointer ( const PointerType * p, const idx n, const std::string & separator, const std::string & start = " [", const std::string & end = "]" ) [inline],[explicit]`
- 7.6.1.2 `template<typename PointerType> qpp::internal::IManipPointer< PointerType >::IManipPointer ( const IManipPointer< PointerType > & ) [default]`

## 7.6.2 Member Function Documentation

- 7.6.2.1 `template<typename PointerType> IManipPointer& qpp::internal::IManipPointer< PointerType >::operator= ( const IManipPointer< PointerType > & ) [default]`

## 7.6.3 Friends And Related Function Documentation

- 7.6.3.1 `template<typename PointerType> template<typename charT, typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const IManipPointer< PointerType > & rhs ) [friend]`

## 7.6.4 Member Data Documentation

- 7.6.4.1 `template<typename PointerType> std::string qpp::internal::IManipPointer< PointerType >::_end [private]`
- 7.6.4.2 `template<typename PointerType> idx qpp::internal::IManipPointer< PointerType >::_n [private]`
- 7.6.4.3 `template<typename PointerType> const PointerType* qpp::internal::IManipPointer< PointerType >::_p [private]`
- 7.6.4.4 `template<typename PointerType> std::string qpp::internal::IManipPointer< PointerType >::_separator [private]`
- 7.6.4.5 `template<typename PointerType> std::string qpp::internal::IManipPointer< PointerType >::_start [private]`

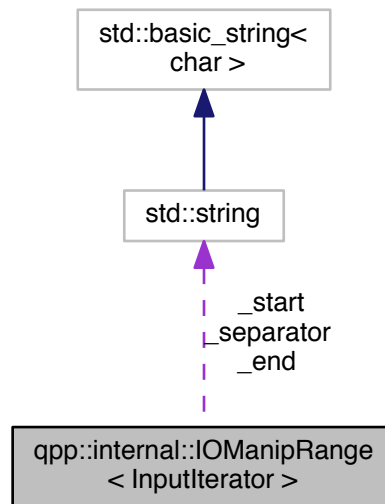
The documentation for this class was generated from the following file:

- `internal/classes/iomanip.h`

## 7.7 qpp::internal::IManipRange< InputIterator > Class Template Reference

```
#include <internal/classes/iomanip.h>
```

Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



## Public Member Functions

- [IOManipRange](#) (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[, const std::string &end="]")

## Private Attributes

- InputIterator [\\_first](#)
- InputIterator [\\_last](#)
- std::string [\\_separator](#)
- std::string [\\_start](#)
- std::string [\\_end](#)

## Friends

- template<typename charT, typename traits >  
std::basic\_ostream< charT,  
traits > & [operator<<](#) (std::basic\_ostream< charT, traits > &os, const [IOManipRange](#) &rhs)

## 7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template<typename InputIterator > qpp::internal::IOManipRange< InputIterator >::IOManipRange ( InputIterator *first*, InputIterator *last*, const std::string & *separator*, const std::string & *start* = "[", const std::string & *end* = "]" ) [inline],[explicit]

## 7.7.2 Friends And Related Function Documentation

7.7.2.1 `template<typename InputIterator > template<typename charT , typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const IManipRange< InputIterator > & rhs )`  
`[friend]`

### 7.7.3 Member Data Documentation

7.7.3.1 `template<typename InputIterator > std::string qpp::internal::IManipRange< InputIterator >::_end`  
`[private]`

7.7.3.2 `template<typename InputIterator > InputIterator qpp::internal::IManipRange< InputIterator >::_first`  
`[private]`

7.7.3.3 `template<typename InputIterator > InputIterator qpp::internal::IManipRange< InputIterator >::_last`  
`[private]`

7.7.3.4 `template<typename InputIterator > std::string qpp::internal::IManipRange< InputIterator >::_separator`  
`[private]`

7.7.3.5 `template<typename InputIterator > std::string qpp::internal::IManipRange< InputIterator >::_start`  
`[private]`

The documentation for this class was generated from the following file:

- [internal/classes/iomanip.h](#)

## 7.8 qpp::RandomDevices Class Reference

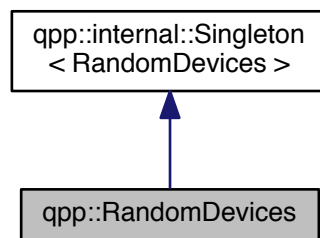
Singleton class that manages the source of randomness in the library.

```
#include <classes/random_devices.h>
```

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



### Public Attributes

- `std::mt19937 _rng`  
*Mersenne twister random number generator.*

### Private Member Functions

- `RandomDevices ()`  
*Initializes and seeds the random number generators.*

### Private Attributes

- `std::random_device _rd`  
*used to seed std::mt19937 \_rng*

### Friends

- class `internal::Singleton < RandomDevices >`

### Additional Inherited Members

#### 7.8.1 Detailed Description

Singleton class that manages the source of randomness in the library.

It consists of a wrapper around an `std::mt19937` Mersenne twister random number generator engine and an `std::random_device` engine. The latter is used to seed the Mersenne twister. The class also seeds the standard `std::srand` C number generator, as it is used by Eigen.

#### 7.8.2 Constructor & Destructor Documentation

##### 7.8.2.1 `qpp::RandomDevices::RandomDevices ( )` `[inline]`, `[private]`

Initializes and seeds the random number generators.

### 7.8.3 Friends And Related Function Documentation

7.8.3.1 friend class `internal::Singleton< RandomDevices >` [`friend`]

### 7.8.4 Member Data Documentation

7.8.4.1 `std::random_device qpp::RandomDevices::_rd` [`private`]

used to seed `std::mt19937 _rng`

7.8.4.2 `std::mt19937 qpp::RandomDevices::_rng`

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

- [classes/random\\_devices.h](#)

## 7.9 `qpp::internal::Singleton< T >` Class Template Reference

`Singleton` policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

### Static Public Member Functions

- static `T & get_instance ()`

### Protected Member Functions

- `Singleton ()`
- virtual `~Singleton ()`
- `Singleton (const Singleton &)=delete`
- `Singleton & operator= (const Singleton &)=delete`

### 7.9.1 Detailed Description

```
template<typename T>class qpp::internal::Singleton< T >
```

`Singleton` policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from `qpp::internal::Singleton`, make `qpp::internal::Singleton` a friend of your class, then declare the constructor of your class as private. To get an instance, use the static member function `qpp::internal::Singleton::get_instance()`, which returns a reference to your newly created singleton (thread-safe in C++11).

Example:

```
class MySingleton: public qpp::internal::Singleton<MySingleton>
{
    friend class qpp::internal::Singleton<MySingleton>;
public:
    // Declare all public members here
private:
    MySingleton()
```



```

    {
        // Implement the constructor here
    }
};

MySingleton& mySingleton = MySingleton::get_instance(); // Get an instance

```

See also

Code of [qpp::Codes](#), [qpp::Gates](#), [qpp::RandomDevices](#), [qpp::States](#) or [qpp.h](#) for real world examples of usage.

## 7.9.2 Constructor & Destructor Documentation

**7.9.2.1** `template<typename T> qpp::internal::Singleton< T >::Singleton ( )` `[inline]`, `[protected]`

**7.9.2.2** `template<typename T> virtual qpp::internal::Singleton< T >::~~Singleton ( )` `[inline]`, `[protected]`, `[virtual]`

**7.9.2.3** `template<typename T> qpp::internal::Singleton< T >::Singleton ( const Singleton< T > & )` `[protected]`, `[delete]`

## 7.9.3 Member Function Documentation

**7.9.3.1** `template<typename T> static T& qpp::internal::Singleton< T >::get_instance ( )` `[inline]`, `[static]`

**7.9.3.2** `template<typename T> Singleton& qpp::internal::Singleton< T >::operator= ( const Singleton< T > & )` `[protected]`, `[delete]`

The documentation for this class was generated from the following file:

- [internal/classes/singleton.h](#)

## 7.10 qpp::States Class Reference

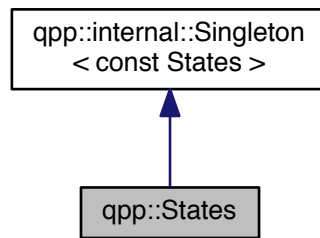
const Singleton class that implements most commonly used states

```
#include <classes/states.h>
```

Inheritance diagram for qpp::States:



Collaboration diagram for `qpp::States`:



## Public Attributes

- `ket x0` {ket::Zero(2)}  
*Pauli Sigma-X 0-eigenstate  $|+\rangle$*
- `ket x1` {ket::Zero(2)}  
*Pauli Sigma-X 1-eigenstate  $|-\rangle$*
- `ket y0` {ket::Zero(2)}  
*Pauli Sigma-Y 0-eigenstate  $|y+\rangle$*
- `ket y1` {ket::Zero(2)}  
*Pauli Sigma-Y 1-eigenstate  $|y-\rangle$*
- `ket z0` {ket::Zero(2)}  
*Pauli Sigma-Z 0-eigenstate  $|0\rangle$*
- `ket z1` {ket::Zero(2)}  
*Pauli Sigma-Z 1-eigenstate  $|1\rangle$*
- `cmat px0` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-X 0-eigenstate  $|+\rangle\langle+|$ .*
- `cmat px1` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-X 1-eigenstate  $|-\rangle\langle-|$ .*
- `cmat py0` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle\langle y+|$ .*
- `cmat py1` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Y 1-eigenstate  $|y-\rangle\langle y-|$ .*
- `cmat pz0` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Z 0-eigenstate  $|0\rangle\langle 0|$ .*
- `cmat pz1` {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Z 1-eigenstate  $|1\rangle\langle 1|$ .*
- `ket b00` {ket::Zero(4)}  
*Bell-00 state (following the convention in Nielsen and Chuang)*
- `ket b01` {ket::Zero(4)}  
*Bell-01 state (following the convention in Nielsen and Chuang)*
- `ket b10` {ket::Zero(4)}  
*Bell-10 state (following the convention in Nielsen and Chuang)*
- `ket b11` {ket::Zero(4)}  
*Bell-11 state (following the convention in Nielsen and Chuang)*
- `cmat pb00` {cmat::Zero(4, 4)}

- Projector onto the Bell-00 state.*
- [cmat pb01](#) {cmat::Zero(4, 4)}
- Projector onto the Bell-01 state.*
- [cmat pb10](#) {cmat::Zero(4, 4)}
- Projector onto the Bell-10 state.*
- [cmat pb11](#) {cmat::Zero(4, 4)}
- Projector onto the Bell-11 state.*
- [ket GHZ](#) {ket::Zero(8)}
- GHZ state.*
- [ket W](#) {ket::Zero(8)}
- W state.*
- [cmat pGHZ](#) {cmat::Zero(8, 8)}
- Projector onto the GHZ state.*
- [cmat pW](#) {cmat::Zero(8, 8)}
- Projector onto the W state.*

## Private Member Functions

- [States](#) ()

## Friends

- class [internal::Singleton< const States >](#)

## Additional Inherited Members

### 7.10.1 Detailed Description

const Singleton class that implements most commonly used states

### 7.10.2 Constructor & Destructor Documentation

#### 7.10.2.1 [qpp::States::States \( \)](#) `[inline], [private]`

Initialize the states

### 7.10.3 Friends And Related Function Documentation

#### 7.10.3.1 `friend class internal::Singleton< const States >` `[friend]`

### 7.10.4 Member Data Documentation

#### 7.10.4.1 [ket qpp::States::b00](#) {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

#### 7.10.4.2 [ket qpp::States::b01](#) {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

7.10.4.3 `ket qpp::States::b10 {ket::Zero(4)}`

Bell-10 state (following the convention in Nielsen and Chuang)

7.10.4.4 `ket qpp::States::b11 {ket::Zero(4)}`

Bell-11 state (following the convention in Nielsen and Chuang)

7.10.4.5 `ket qpp::States::GHZ {ket::Zero(8)}`

GHZ state.

7.10.4.6 `cmat qpp::States::pb00 {cmat::Zero(4, 4)}`

Projector onto the Bell-00 state.

7.10.4.7 `cmat qpp::States::pb01 {cmat::Zero(4, 4)}`

Projector onto the Bell-01 state.

7.10.4.8 `cmat qpp::States::pb10 {cmat::Zero(4, 4)}`

Projector onto the Bell-10 state.

7.10.4.9 `cmat qpp::States::pb11 {cmat::Zero(4, 4)}`

Projector onto the Bell-11 state.

7.10.4.10 `cmat qpp::States::pGHZ {cmat::Zero(8, 8)}`

Projector onto the GHZ state.

7.10.4.11 `cmat qpp::States::pW {cmat::Zero(8, 8)}`

Projector onto the W state.

7.10.4.12 `cmat qpp::States::px0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-X 0-eigenstate  $|+\rangle\langle+|$ .

7.10.4.13 `cmat qpp::States::px1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-X 1-eigenstate  $|-\rangle\langle-|$ .

7.10.4.14 `cmat qpp::States::py0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle\langle y+|$ .

7.10.4.15 `cmat qpp::States::py1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Y 1-eigenstate  $|y-\rangle\langle y-|$ .

7.10.4.16 `cmat qpp::States::pz0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Z 0-eigenstate  $|0\rangle\langle 0|$ .

7.10.4.17 `cmat qpp::States::pz1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Z 1-eigenstate  $|1\rangle\langle 1|$ .

7.10.4.18 `ket qpp::States::W {ket::Zero(8)}`

W state.

7.10.4.19 `ket qpp::States::x0 {ket::Zero(2)}`

Pauli Sigma-X 0-eigenstate  $|+\rangle$

7.10.4.20 `ket qpp::States::x1 {ket::Zero(2)}`

Pauli Sigma-X 1-eigenstate  $|-\rangle$

7.10.4.21 `ket qpp::States::y0 {ket::Zero(2)}`

Pauli Sigma-Y 0-eigenstate  $|y+\rangle$

7.10.4.22 `ket qpp::States::y1 {ket::Zero(2)}`

Pauli Sigma-Y 1-eigenstate  $|y-\rangle$

7.10.4.23 `ket qpp::States::z0 {ket::Zero(2)}`

Pauli Sigma-Z 0-eigenstate  $|0\rangle$

7.10.4.24 `ket qpp::States::z1 {ket::Zero(2)}`

Pauli Sigma-Z 1-eigenstate  $|1\rangle$

The documentation for this class was generated from the following file:

- [classes/states.h](#)

## 7.11 qpp::Timer Class Reference

Measures time.

```
#include <classes/timer.h>
```

## Public Member Functions

- [Timer](#) ()  
*Constructs an instance with the current time as the starting point.*
- void [tic](#) ()  
*Resets the chronometer.*
- const [Timer](#) & [toc](#) ()  
*Stops the chronometer.*
- double [seconds](#) () const  
*Time passed in seconds.*

## Protected Attributes

- std::chrono::steady\_clock::time\_point [\\_start](#)
- std::chrono::steady\_clock::time\_point [\\_end](#)

## Friends

- template<typename charT , typename traits >  
std::basic\_ostream< charT,  
traits > & [operator<<](#) (std::basic\_ostream< charT, traits > &os, const [Timer](#) &rhs)  
*Overload for std::ostream operators.*

### 7.11.1 Detailed Description

Measures time.

Uses a std::chrono::steady\_clock. It is not affected by wall clock changes during runtime.

### 7.11.2 Constructor & Destructor Documentation

#### 7.11.2.1 `qpp::Timer::Timer ( )` `[inline]`

Constructs an instance with the current time as the starting point.

### 7.11.3 Member Function Documentation

#### 7.11.3.1 `double qpp::Timer::seconds ( )` `const` `[inline]`

Time passed in seconds.

#### Returns

Number of seconds that passed between the instantiation/reset and invocation of `qpp::Timer::toc()`

#### 7.11.3.2 `void qpp::Timer::tic ( )` `[inline]`

Resets the chronometer.

Resets the starting/ending point to the current time

7.11.3.3 `const Timer& qpp::Timer::toc ( ) [inline]`

Stops the chronometer.

Set the current time as the ending point

**Returns**

Current instance

## 7.11.4 Friends And Related Function Documentation

7.11.4.1 `template<typename charT , typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const Timer & rhs ) [friend]`

Overload for std::ostream operators.

**Parameters**

<i>os</i>	Output stream
<i>rhs</i>	<a href="#">Timer</a> instance

**Returns**

Writes to the output stream the number of seconds that passed between the instantiation/reset and invocation of [qpp::Timer::toc\(\)](#).

## 7.11.5 Member Data Documentation

7.11.5.1 `std::chrono::steady_clock::time_point qpp::Timer::_end [protected]`7.11.5.2 `std::chrono::steady_clock::time_point qpp::Timer::_start [protected]`

The documentation for this class was generated from the following file:

- [classes/timer.h](#)





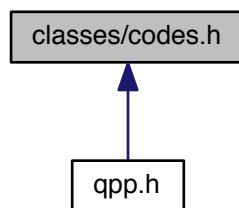
## Chapter 8

# File Documentation

### 8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



#### Classes

- class [qpp::Codes](#)  
*const Singleton class that defines quantum error correcting codes*

#### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

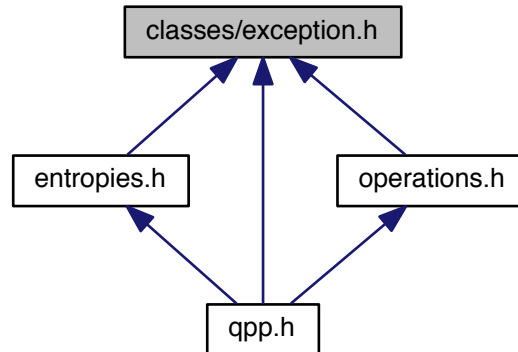
#### 8.1.1 Detailed Description

Quantum error correcting codes.

### 8.2 classes/exception.h File Reference

Exceptions.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Exception](#)

*Generates custom exceptions, used when validating function parameters.*

## Namespaces

- [qpp](#)

*Quantum++ main namespace.*

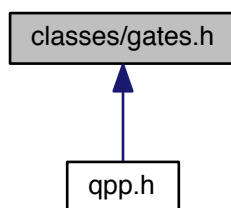
### 8.2.1 Detailed Description

Exceptions.

## 8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Gates](#)  
*const Singleton class that implements most commonly used gates*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

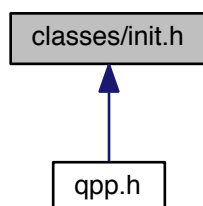
### 8.3.1 Detailed Description

Quantum gates.

## 8.4 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Init](#)  
*const Singleton class that performs additional initializations/cleanups*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

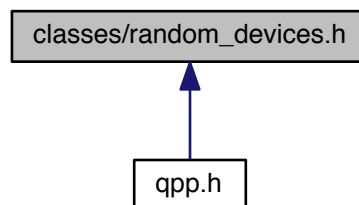
### 8.4.1 Detailed Description

Initialization.

## 8.5 classes/random\_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::RandomDevices](#)  
*Singleton class that manages the source of randomness in the library.*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

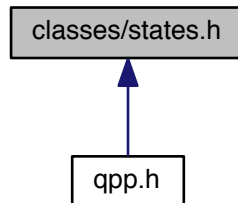
### 8.5.1 Detailed Description

Random devices.

## 8.6 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



### Classes

- class [qpp::States](#)  
*const Singleton class that implements most commonly used states*

### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

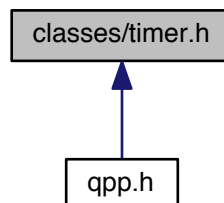
#### 8.6.1 Detailed Description

Quantum states.

## 8.7 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Timer](#)  
*Measures time.*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

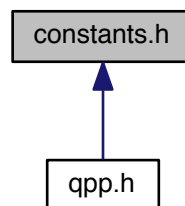
### 8.7.1 Detailed Description

Timing.

## 8.8 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

## Functions

- constexpr cplx [qpp::operator""\\_i](#) (unsigned long long int x)  
*User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)*
- constexpr cplx [qpp::operator""\\_i](#) (long double x)  
*User-defined literal for complex  $i = \sqrt{-1}$  (real overload)*
- cplx [qpp::omega](#) (idx D)  
*D-th root of unity.*

## Variables

- constexpr double `qpp::chop` = 1e-10  
*Used in `qpp::disp()` for setting to zero numbers that have their absolute value smaller than `qpp::chop`.*
- constexpr double `qpp::eps` = 1e-12  
*Used to decide whether a number or expression in double precision is zero or not.*
- constexpr idx `qpp::maxn` = 64  
*Maximum number of allowed qu(d)its (subsystems)*
- constexpr double `qpp::pi` = 3.141592653589793238462643383279502884  
 $\pi$
- constexpr double `qpp::ee` = 2.718281828459045235360287471352662497  
*Base of natural logarithm,  $e$ .*
- constexpr double `qpp::infy` = std::numeric\_limits<double>::infinity()  
*Used to denote infinity in double precision.*

### 8.8.1 Detailed Description

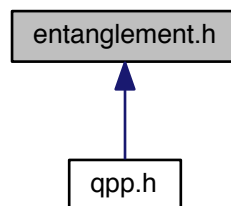
Constants.

## 8.9 entanglement.h File Reference

Entanglement functions.

```
#include <types.h>
#include <vector>
```

This graph shows which files directly or indirectly include this file:



## Namespaces

- `qpp`  
*Quantum++ main namespace.*

## Functions

- template<typename Derived >  
dyn\_col\_vect< double > `qpp::schmidtcoeffs` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

*Schmidt coefficients of the bi-partite pure state A.*

- `template<typename Derived >`  
`cmat qpp::schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Schmidt basis on Alice's side.*

- `template<typename Derived >`  
`cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Schmidt basis on Bob's side.*

- `template<typename Derived >`  
`std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Schmidt probabilities of the bi-partite pure state A.*

- `template<typename Derived >`  
`double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Entanglement of the bi-partite pure state A.*

- `template<typename Derived >`  
`double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)`

*G-concurrence of the bi-partite pure state A.*

- `template<typename Derived >`  
`double qpp::negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Negativity of the bi-partite mixed state A.*

- `template<typename Derived >`  
`double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Logarithmic negativity of the bi-partite mixed state A.*

- `template<typename Derived >`  
`double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)`

*Wootters concurrence of the bi-partite qubit mixed state A.*

### 8.9.1 Detailed Description

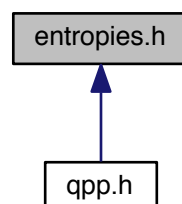
Entanglement functions.

## 8.10 entropies.h File Reference

Entropy functions.

```
#include <classes/exception.h>
```

This graph shows which files directly or indirectly include this file:





## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

## Functions

- `template<typename Derived >`  
`double qpp::entropy (const Eigen::MatrixBase< Derived > &A)`  
*von-Neumann entropy of the density matrix A*
- `double qpp::entropy (const std::vector< double > &prob)`  
*Shannon entropy of the probability distribution prob.*
- `template<typename Derived >`  
`double qpp::renyi (const Eigen::MatrixBase< Derived > &A, double alpha)`  
*Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .*
- `double qpp::renyi (const std::vector< double > &prob, double alpha)`  
*Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .*
- `template<typename Derived >`  
`double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)`  
*Tsallis-  $q$  entropy of the density matrix A, for  $q \geq 0$ .*
- `double qpp::tsallis (const std::vector< double > &prob, double q)`  
*Tsallis-  $q$  entropy of the probability distribution prob, for  $q \geq 0$ .*
- `template<typename Derived >`  
`double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)`  
*Quantum mutual information between 2 subsystems of a composite system.*
- `template<typename Derived >`  
`double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)`  
*Quantum mutual information between 2 subsystems of a composite system.*

### 8.10.1 Detailed Description

Entropy functions.

## 8.11 experimental/test.h File Reference

Experimental/test functions/classes.

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::experimental](#)  
*Experimental/test functions/classes, do not use or modify.*

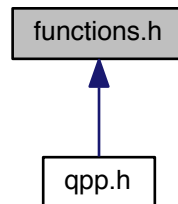
### 8.11.1 Detailed Description

Experimental/test functions/classes.

## 8.12 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)`  
*Transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)`  
*Complex conjugate.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)`  
*Adjoint.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)`  
*Inverse.*
- `template<typename Derived >`  
`Derived::Scalar qpp::trace (const Eigen::MatrixBase< Derived > &A)`  
*Trace.*
- `template<typename Derived >`  
`Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)`  
*Determinant.*
- `template<typename Derived >`  
`Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > &A)`  
*Logarithm of the determinant.*
- `template<typename Derived >`  
`Derived::Scalar qpp::sum (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise sum of A.*
- `template<typename Derived >`  
`Derived::Scalar qpp::prod (const Eigen::MatrixBase< Derived > &A)`

*Element-wise product of A.*

- template<typename Derived >  
double [qpp::norm](#) (const Eigen::MatrixBase< Derived > &A)

*Frobenius norm.*

- template<typename Derived >  
std::pair< dyn\_col\_vect< cplx >  
, cmat > [qpp::eig](#) (const Eigen::MatrixBase< Derived > &A)

*Full eigen decomposition.*

- template<typename Derived >  
dyn\_col\_vect< cplx > [qpp::evals](#) (const Eigen::MatrixBase< Derived > &A)

*Eigenvalues.*

- template<typename Derived >  
cmat [qpp::evecs](#) (const Eigen::MatrixBase< Derived > &A)

*Eigenvectors.*

- template<typename Derived >  
std::pair< dyn\_col\_vect  
< double >, cmat > [qpp::heig](#) (const Eigen::MatrixBase< Derived > &A)

*Full eigen decomposition of Hermitian expression.*

- template<typename Derived >  
dyn\_col\_vect< double > [qpp::hevals](#) (const Eigen::MatrixBase< Derived > &A)

*Hermitian eigenvalues.*

- template<typename Derived >  
cmat [qpp::hevecs](#) (const Eigen::MatrixBase< Derived > &A)

*Hermitian eigenvectors.*

- template<typename Derived >  
std::tuple< cmat, dyn\_col\_vect  
< double >, cmat > [qpp::svd](#) (const Eigen::MatrixBase< Derived > &A)

*Full singular value decomposition.*

- template<typename Derived >  
dyn\_col\_vect< double > [qpp::svals](#) (const Eigen::MatrixBase< Derived > &A)

*Singular values.*

- template<typename Derived >  
cmat [qpp::svdU](#) (const Eigen::MatrixBase< Derived > &A)

*Left singular vectors.*

- template<typename Derived >  
cmat [qpp::svdV](#) (const Eigen::MatrixBase< Derived > &A)

*Right singular vectors.*

- template<typename Derived >  
cmat [qpp::funm](#) (const Eigen::MatrixBase< Derived > &A, cplx(\*f)(const cplx &))

*Functional calculus  $f(A)$*

- template<typename Derived >  
cmat [qpp::sqrtm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix square root.*

- template<typename Derived >  
cmat [qpp::absm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix absolut value.*

- template<typename Derived >  
cmat [qpp::expm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix exponential.*

- template<typename Derived >  
cmat [qpp::logm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix logarithm.*

- template<typename Derived >  
cmat [qpp::sinm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix sin.*

- template<typename Derived >  
cmat [qpp::cosm](#) (const Eigen::MatrixBase< Derived > &A)

*Matrix cos.*

- template<typename Derived >  
cmat [qpp::spectralpowm](#) (const Eigen::MatrixBase< Derived > &A, const cplx z)

*Matrix power.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::powm](#) (const Eigen::MatrixBase< Derived > &A, idx n)

*Matrix power.*

- template<typename Derived >  
double [qpp::schatten](#) (const Eigen::MatrixBase< Derived > &A, idx p)

*Schatten norm.*

- template<typename OutputScalar , typename Derived >  
dyn\_mat< OutputScalar > [qpp::cwise](#) (const Eigen::MatrixBase< Derived > &A, OutputScalar(\*)(f))(const  
typename Derived::Scalar &))

*Functor.*

- template<typename T >  
dyn\_mat< typename T::Scalar > [qpp::kron](#) (const T &head)

*Kronecker product.*

- template<typename T , typename... Args>  
dyn\_mat< typename T::Scalar > [qpp::kron](#) (const T &head, const Args &...tail)

*Kronecker product.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::kron](#) (const std::vector< Derived > &As)

*Kronecker product.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::kron](#) (const std::initializer\_list< Derived > &As)

*Kronecker product.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::kronpow](#) (const Eigen::MatrixBase< Derived > &A, idx n)

*Kronecker power.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::reshape](#) (const Eigen::MatrixBase< Derived > &A, idx rows,  
idx cols)

*Reshape.*

- template<typename Derived1 , typename Derived2 >  
dyn\_mat< typename  
Derived1::Scalar > [qpp::comm](#) (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< De-  
rived2 > &B)

*Commutator.*

- template<typename Derived1 , typename Derived2 >  
dyn\_mat< typename  
Derived1::Scalar > [qpp::anticomm](#) (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase<  
Derived2 > &B)

*Anti-commutator.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::prj](#) (const Eigen::MatrixBase< Derived > &V)

*Projector.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::grams](#) (const std::vector< Derived > &Vs)

*Gram-Schmidt orthogonalization.*

- template<typename Derived >  
dyn\_mat< typename Derived::Scalar > [qpp::grams](#) (const std::initializer\_list< Derived > &Vs)

*Gram-Schmidt orthogonalization.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)`

*Gram-Schmidt orthogonalization.*

- `std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)`

*Non-negative integer index to multi-index.*

- `idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)`

*Multi-index to non-negative integer index.*

- `ket qpp::mket (const std::vector< idx > &mask, const std::vector< idx > &dims)`

*Multi-partite qudit ket.*

- `ket qpp::mket (const std::vector< idx > &mask, idx d=2)`

*Multi-partite qudit ket.*

- `cmat qpp::mprj (const std::vector< idx > &mask, const std::vector< idx > &dims)`

*Projector onto multi-partite qudit ket.*

- `cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)`

*Projector onto multi-partite qudit ket.*

- `template<typename InputIterator >`  
`std::vector< double > qpp::abssq (InputIterator first, InputIterator last)`

*Computes the absolut values squared of a range of complex numbers.*

- `template<typename Derived >`  
`std::vector< double > qpp::abssq (const Eigen::MatrixBase< Derived > &V)`

*Computes the absolut values squared of a column vector.*

- `template<typename InputIterator >`  
`InputIterator::value_type qpp::sum (InputIterator first, InputIterator last)`

*Element-wise sum of a range.*

- `template<typename InputIterator >`  
`InputIterator::value_type qpp::prod (InputIterator first, InputIterator last)`

*Element-wise product of a range.*

- `template<typename Derived >`  
`dyn_col_vect< typename`  
`Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)`

*Finds the pure state representation of a matrix proportional to a projector onto a pure state.*

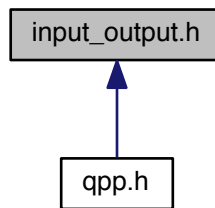
### 8.12.1 Detailed Description

Generic quantum computing functions.

## 8.13 input\_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

## Functions

- `template<typename Derived >`  
`internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)`  
*Eigen expression ostream manipulator.*
- `internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)`  
*Complex number ostream manipulator.*
- `template<typename InputIterator >`  
`internal::IOManipRange`  
`< InputIterator > qpp::disp (const InputIterator &first, const InputIterator &last, const std::string &separator,`  
`const std::string &start="["`, `const std::string &end=""]")`  
*Range ostream manipulator.*
- `template<typename Container >`  
`internal::IOManipRange`  
`< typename`  
`Container::const_iterator > qpp::disp (const Container &c, const std::string &separator, const std::string`  
`&start="["`, `const std::string &end=""]")`  
*Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.*
- `template<typename PointerType >`  
`internal::IOManipPointer`  
`< PointerType > qpp::disp (const PointerType *p, idx n, const std::string &separator, const std::string`  
`&start="["`, `const std::string &end=""]")`  
*C-style pointer ostream manipulator.*
- `template<typename Derived >`  
`void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)`  
*Saves Eigen expression to a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::load (const std::string &fname)`  
*Loads Eigen matrix from a binary file (internal format) in double precision.*

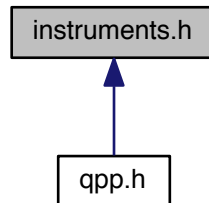
### 8.13.1 Detailed Description

Input/output functions.

## 8.14 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state A in the orthonormal basis specified by the unitary matrix U.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat`  
`&U, const std::vector< idx > &subsys, const idx d=2)`

*Measures the part subsys of the multi-partite state A in the orthonormal basis specified by the unitary matrix U.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::`  
`::vector< cmat > &Ks)`

*Measures the state A using the set of Kraus operators Ks.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::`  
`::initializer_list< cmat > &Ks)`

*Measures the state A using the set of Kraus operators Ks.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector`  
`< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat`  
`&U)`

*Measures the state A in the orthonormal basis specified by the unitary matrix U.*

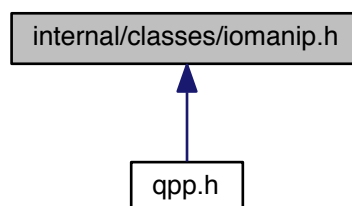
### 8.14.1 Detailed Description

Measurement functions.

## 8.15 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



### Classes

- class [qpp::internal::IOManipRange< InputIterator >](#)
- class [qpp::internal::IOManipPointer< PointerType >](#)
- class [qpp::internal::IOManipEigen](#)



## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

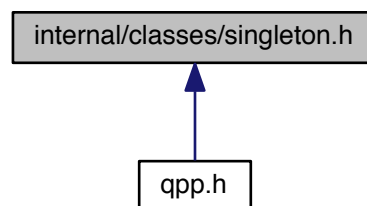
### 8.15.1 Detailed Description

Input/output manipulators.

## 8.16 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::internal::Singleton< T >](#)  
*[Singleton](#) policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

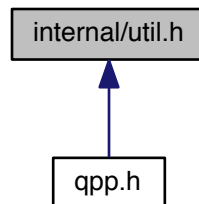
### 8.16.1 Detailed Description

Singleton pattern via CRTP.

## 8.17 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

### Functions

- void [qpp::internal::\\_n2multiidx](#) (idx n, idx numdims, const idx \*dims, idx \*result)
- idx [qpp::internal::\\_multiidx2n](#) (const idx \*midx, idx numdims, const idx \*dims)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_square\\_mat](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_vector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_row\\_vector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_col\\_vector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename T >  
bool [qpp::internal::\\_check\\_nonzero\\_size](#) (const T &x)
- bool [qpp::internal::\\_check\\_dims](#) (const std::vector< idx > &dims)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_mat](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_cvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_rvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- bool [qpp::internal::\\_check\\_eq\\_dims](#) (const std::vector< idx > &dims, idx dim)
- bool [qpp::internal::\\_check\\_subsys\\_match\\_dims](#) (const std::vector< idx > &subsys, const std::vector< idx > &dims)

- bool [qpp::internal::\\_check\\_perm](#) (const std::vector< idx > &perm)
- template<typename Derived1 , typename Derived2 >  
dyn\_mat< typename  
Derived1::Scalar > [qpp::internal::\\_kron2](#) (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >  
void [qpp::internal::variadic\\_vector\\_emplace](#) (std::vector< T > &)
- template<typename T , typename First , typename... Args>  
void [qpp::internal::variadic\\_vector\\_emplace](#) (std::vector< T > &v, First &&first, Args &&...args)

### 8.17.1 Detailed Description

Internal utility functions.

## 8.18 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- template<typename Derived >  
Derived [qpp::loadMATLABmatrix](#) (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.*
- template<>  
dmat [qpp::loadMATLABmatrix](#) (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- template<>  
cmat [qpp::loadMATLABmatrix](#) (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- template<typename Derived >  
void [qpp::saveMATLABmatrix](#) (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.*
- template<>  
void [qpp::saveMATLABmatrix](#) (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- template<>  
void [qpp::saveMATLABmatrix](#) (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*

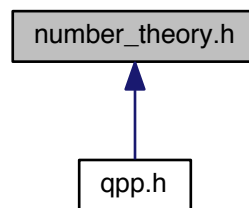
### 8.18.1 Detailed Description

Input/output interfacing with MATLAB.

## 8.19 number\_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `std::vector< long long int > qpp::x2contfrac (double x, idx n, idx cut=1e5)`  
*Simple continued fraction expansion.*
- `double qpp::contfrac2x (const std::vector< int > &cf, idx n)`  
*Real representation of a simple continued fraction.*
- `double qpp::contfrac2x (const std::vector< int > &cf)`  
*Real representation of a simple continued fraction.*
- `idx qpp::gcd (idx m, idx n)`  
*Greatest common divisor of two non-negative integers.*
- `idx qpp::gcd (const std::vector< idx > &ns)`  
*Greatest common divisor of a list of non-negative integers.*
- `idx qpp::lcm (idx m, idx n)`  
*Least common multiple of two positive integers.*
- `idx qpp::lcm (const std::vector< idx > &ns)`  
*Least common multiple of a list of positive integers.*
- `std::vector< idx > qpp::invperm (const std::vector< idx > &perm)`  
*Inverse permutation.*
- `std::vector< idx > qpp::compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)`  
*Compose permutations.*

### 8.19.1 Detailed Description

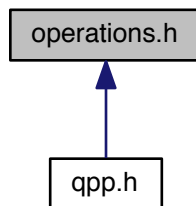
Number theory functions.

## 8.20 operations.h File Reference

Quantum operation functions.

```
#include <classes/exception.h>
```

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)

*Quantum++ main namespace.*

### Functions

- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx d=2)`  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename`  
`Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)`  
*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*

- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)`  
*Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.*
- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*
- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)`  
*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*
- `cmat qpp::kraus2super (const std::vector< cmat > &Ks)`  
*Superoperator matrix.*
- `cmat qpp::kraus2choi (const std::vector< cmat > &Ks)`  
*Choi matrix.*
- `std::vector< cmat > qpp::choi2kraus (const cmat &A)`  
*Orthogonal Kraus operators from Choi matrix.*
- `cmat qpp::choi2super (const cmat &A)`  
*Converts Choi matrix to superoperator matrix.*
- `cmat qpp::super2choi (const cmat &A)`  
*Converts superoperator matrix to Choi matrix.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace1 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Partial transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)`  
*Partial transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, const std::vector< idx > &dims)`  
*Subsystem permutation.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)`  
*Subsystem permutation.*

### 8.20.1 Detailed Description

Quantum operation functions.

## 8.21 qpp.h File Reference

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "types.h"
#include "constants.h"
#include "classes/exception.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

## Namespaces

- [qpp](#)

*Quantum++ main namespace.*

## Variables

- const Init & [qpp::init](#) = Init::get\_instance()  
*[qpp::Init](#) const Singleton*
- const Codes & [qpp::codes](#) = Codes::get\_instance()  
*[qpp::Codes](#) const Singleton*
- const Gates & [qpp::gt](#) = Gates::get\_instance()  
*[qpp::Gates](#) const Singleton*
- const States & [qpp::st](#) = States::get\_instance()  
*[qpp::States](#) const Singleton*
- RandomDevices & [qpp::rdevs](#) = RandomDevices::get\_instance()  
*[qpp::RandomDevices](#) Singleton*

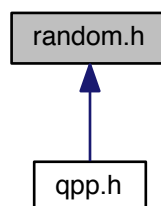
### 8.21.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

## 8.22 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



## Namespaces

- [qpp](#)

*Quantum++ main namespace.*



## Functions

- `template<typename Derived >`  
`Derived qpp::rand` (idx rows, idx cols, double a=0, double b=1)  
*Generates a random matrix with entries uniformly distributed in the interval [a, b]*
- `template<>`  
`dmat qpp::rand` (idx rows, idx cols, double a, double b)  
*Generates a random real matrix with entries uniformly distributed in the interval [a, b], specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat qpp::rand` (idx rows, idx cols, double a, double b)  
*Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b], specialization for complex matrices ([qpp::cmat](#))*
- `double qpp::rand` (double a=0, double b=1)  
*Generates a random real number uniformly distributed in the interval [a, b]*
- `idx qpp::randidx` (idx a=std::numeric\_limits< idx >::min(), idx b=std::numeric\_limits< idx >::max())  
*Generates a random index (idx) uniformly distributed in the interval [a, b].*
- `template<typename Derived >`  
`Derived qpp::randn` (idx rows, idx cols, double mean=0, double sigma=1)  
*Generates a random matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$*
- `template<>`  
`dmat qpp::randn` (idx rows, idx cols, double mean, double sigma)  
*Generates a random real matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat qpp::randn` (idx rows, idx cols, double mean, double sigma)  
*Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices ([qpp::cmat](#))*
- `double qpp::randn` (double mean=0, double sigma=1)  
*Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$*
- `cmat qpp::randU` (idx D)  
*Generates a random unitary matrix.*
- `cmat qpp::randV` (idx Din, idx Dout)  
*Generates a random isometry matrix.*
- `std::vector< cmat > qpp::randkraus` (idx N, idx D)  
*Generates a set of random Kraus operators.*
- `cmat qpp::randH` (idx D)  
*Generates a random Hermitian matrix.*
- `ket qpp::randket` (idx D)  
*Generates a random normalized ket (pure state vector)*
- `cmat qpp::randrho` (idx D)  
*Generates a random density matrix.*
- `std::vector< idx > qpp::randperm` (idx n)  
*Generates a random uniformly distributed permutation.*

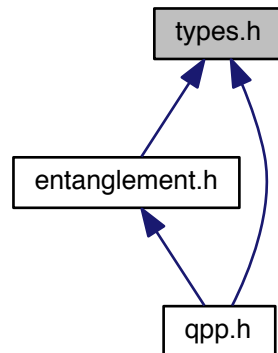
### 8.22.1 Detailed Description

Randomness-related functions.

## 8.23 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Typedefs

- using [qpp::idx](#) = std::size\_t  
*Non-negative integer index.*
- using [qpp::cplx](#) = std::complex< double >  
*Complex number in double precision.*
- using [qpp::ket](#) = Eigen::VectorXcd  
*Complex (double precision) dynamic Eigen column vector.*
- using [qpp::bra](#) = Eigen::RowVectorXcd  
*Complex (double precision) dynamic Eigen row vector.*
- using [qpp::cmat](#) = Eigen::MatrixXcd  
*Complex (double precision) dynamic Eigen matrix.*
- using [qpp::dmat](#) = Eigen::MatrixXd  
*Real (double precision) dynamic Eigen matrix.*
- template<typename Scalar >  
using [qpp::dyn\\_mat](#) = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >  
*Dynamic Eigen matrix over the field specified by Scalar.*
- template<typename Scalar >  
using [qpp::dyn\\_col\\_vect](#) = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >  
*Dynamic Eigen column vector over the field specified by Scalar.*
- template<typename Scalar >  
using [qpp::dyn\\_row\\_vect](#) = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >  
*Dynamic Eigen row vector over the field specified by Scalar.*

### 8.23.1 Detailed Description

Type aliases.

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