Quantum++ for the impatient

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Contents

1 Introduction

Quantum++ is a C++11 general purpose quantum computing library, composed solely of header files. It uses the Eigen 3 linear algebra library and, if available, the OpenMP multi-processing library. For additional Eigen 3 documentation see `http://eigen.tuxfamily.org/dox/`. For a simple Eigen 3 quick ASCII reference see `http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt`.

The simulator defines a large collection of (template) quantum computing related functions and few useful classes. The main data types are complex vectors and complex matrices, as I will describe below. Most functions operate on such vectors/matrices, and *always* return the result by value. Collection of objects are implemented via the standard library container `std::vector<>`, instantiated accordingly. Ease of use and performance were among the most important design factors of Quantum++.

2 Installation

To get started with Quantum++, first install the Eigen 3 library from `http://eigen.tuxfamily.org` into your home directory[1], as `$HOME/eigen`. You can change the name of the directory, but in the current document I will use `$HOME/eigen` as the location of the Eigen 3 library. Next, download the Quantum++ library from `http://vsoftco.github.io/qpp/` and unzip it into the home directory as `$HOME/qpp`. Finally, make sure that your compiler supports C++11 and preferably OpenMP. I recommend `g++`, version 4.8 or later. You are now ready to go!

We next build a simple minimal example to test that the installation was successful. Create a directory called `$HOME/qpp_examples`, and inside it create the file `minimal.cpp`, with the content listed in the listing below. A verbatim copy of the above program is also available at `$HOME/qpp/examples/minimal.cpp`.

```
1  // Minimal example
2  // Source: ./examples/minimal.cpp
3  #include <qpp.h>
4
5  int main()
6  {
7      std::cout << "Hello Quantum++!" << std::endl;
8  }
```

Next compile the file using a C++11 compliant compiler such as `g++` version 4.8 or later. From inside the directory `$HOME/qpp_examples`, type

```
g++ -std=c++11 -isystem $HOME/eigen -I $HOME/qpp/include minimal.cpp -o minimal
```

Your compile command may differ from the above, depending on the name of your C++ compiler and operating system. If everything went fine, then the above command builds the executable `minimal` in the directory `$HOME/qpp_examples`. To run it, type `./minimal` from inside the directory `$HOME/qpp_examples`. The output should look like

```
1  >>> Starting Quantum++...
2  >>> Sun Dec  7 16:50:06 2014
3
4  Hello Quantum++!
5
6  >>> Exiting Quantum++...
7  >>> Sun Dec  7 16:50:06 2014
```

In line 3 of the program we include the main header file of the library, `qpp.h`. This file includes all other necessary Quantum++ header files, as well as the following C++ standard library files

```
<algorithm>
<chrono>
<cmath>
<complex>
<cstdlib>
<cstring>
<ctime>
<exception>
```

[1] I implicitly assume from now on that you use a UNIX-based system, although everything should translate into Windows as well, with slight modifications

```
<fstream>
<functional>
<initializer_list>
<iomanip>
<iostream>
<iterator>
<limits>
<numeric>
<ostream>
<random>
<sstream>
<stdexcept>
<string>
<tuple>
<type_traits>
<utility>
<vector>
```

and Eigen 3 header files

```
<Eigen/Dense>
<Eigen/SVD>
```

Most of the time, you should be fine including only the header `qpp.h` in your main project, except when you want to use the MATLAB input/output interface support, in which case you have to explicitly include the header file `MATLAB/matlab.h`.

3 Data types, constants and global objects

All header files of Quantum++ are located inside the `include` directory. All functions, classes and global objects defined by the library belong to the `namespace qpp`. To avoid additional typing, I will omit the prefix `qpp::` in the rest of this document. I recommend to use

```
using namespace qpp;
```

in your main `.cpp` file.

3.1 Data types

The most important data types are defined via typedefs in the header file `types.h`. We list them in Table 1.

3.2 Constants

The important constants are defined in the header file `constants.h` and are listed in Table 2.

3.3 Singleton classes and their global instances

Some useful classes are defined as singletons and their instances are globally available, being initialized at runtime in the header file `qpp.h`, before `main()`. They are listed in Table 3.

`cplx`	Complex number, alias for `std::complex<double>`
`idx`	Index (non-negative integer), alias for `std::size_t`
`cmat`	Complex dynamic matrices, alias for `Eigen::MatrixXcd`
`dmat`	Double dynamic matrices, alias for `Eigen::MatrixXd`
`ket`	Complex dynamic column vector, alias for `Eigen::VectorXcd`
`bra`	Complex dynamic row vector, alias for `Eigen::RowVectorXcd`
`dyn_mat<Scalar>`	Dynamic matrix template alias over the field `Scalar`, alias for `Eigen::Matrix<Scalar, Eigen::Dynamic, Eigen::Dynamic>`
`dyn_col_vect<Scalar>`	Dynamic column vector template alias over the field `Scalar`, alias for `Eigen::Matrix<Scalar, Eigen::Dynamic, 1>`
`dyn_row_vect<Scalar>`	Dynamic row vector template alias over the field `Scalar`, alias for `Eigen::Matrix<Scalar, 1, Eigen::Dynamic>`

Table 1: User-defined data types

`constexpr double pi = 3.1415...;`	π
`constexpr double ee = 2.7182...;`	e, base of natural logarithms
`constexpr idx infty = -1;`	Infinity
`constexpr idx maxn = 64;`	Maximum number of allowed qu(d)its (subsystems)
`constexpr double eps = 1e-12;`	Used in comparing floating point values to zero
`constexpr double chop = 1e-10;`	Used in display manipulators to set numbers to zero
`constexpr cplx operator""_i (unsigned long long int x)`	User-defined literal for the imaginary number $i := \sqrt{-1}$
`constexpr cplx operator""_i (unsigned long double int x)`	User-defined literal for the imaginary number $i := \sqrt{-1}$
`cplx omega(idx D)`	D-th root of unity $e^{2\pi i/D}$

Table 2: User-defined constants

`const Init& init = Init::get_instance();`	Library initialization
`const Codes& codes = Codes::get_instance();`	Quantum error correcting codes
`const Gates& gt = Gates::get_instance();`	Quantum gates
`const States& st = States::get_instance();`	Quantum states
`RandomDevices& rdevs = RandomDevices::get_instance()`	Random number generator engines

Table 3: Global singleton classes and instances

4 Simple examples

All examples of this section are copied verbatim from the directory `./examples` and compiled successfully. For convenience, the location of the source file is also displayed in the first line of each example, as a C++ comment.

The examples are simple and demonstrate the main feature of Quantum++. They cover only a small part of library functions, but enough to get the interested user started. For extensive about all library functions, including various overloads, the user should consult the complete reference located at `./doc/refman.pdf`. A more comprehensive (but also more complicated) example, that consists of a collection of quantum information processing routines, is located at `./examples/example.cpp`.

4.1 Gates and states

We introduce the main objects used by Quantum++: gates, states and basic operations. Consider the code in the listing below

```
// Gates and states
// Source: ./examples/gates_states.cpp
#include <qpp.h>
using namespace qpp;

int main()
{
    ket psi = st.z0; // |0> state
    cmat U = gt.X;
    ket result = U * psi;

    std::cout << "The result of applying the bit-flip gate X on |0> is:\n";
    std::cout << disp(result) << std::endl;

    psi = mket({1, 0}); // |10> state
    U = gt.CNOT; // Controlled-NOT
    result = U * psi;

    std::cout << "The result of applying the gate CNOT on |10> is:\n";
    std::cout << disp(result) << std::endl;

    U = randU(2);
    std::cout << "Generating a random one-qubit gate U:\n";
    std::cout << disp(U) << std::endl;

    result = applyCTRL(psi, U, {0}, {1});
    std::cout << "The result of applying the Controlled-U gate on |10> is:\n";
    std::cout << disp(result) << std::endl;
}
```

which may output

```
>>> Starting Quantum++...
>>> Sun Dec  7 16:50:06 2014

The result of applying the bit-flip gate X on |0> is:
     0
1.0000
The result of applying the gate CNOT on |10> is:
     0
     0
     0
1.0000
Generating a random one-qubit gate U:
0.4256 + 0.7524i  -0.4158 - 0.2826i
0.4977 + 0.0708i   0.8211 - 0.2703i
The result of applying the Controlled-U gate on |10> is:
```

```
16                0
17                0
18  0.4256 + 0.7524i
19  0.4977 + 0.0708i
20
21  >>> Exiting Quantum++...
22  >>> Sun Dec  7 16:50:06 2014
```

In line 4, we bring the namespace `qpp` into the global namespace.

In line 8 we use the `States` singleton `st` to declare `psi` as the zero eigenvector $|0\rangle$ of the Z Pauli operator. In line 9 we use the `Gates` singleton `gt` and assign to `U` the bit flip gate `gt.X`. In line 10 we compute the result of the operation $X|0\rangle$, and display the result $|1\rangle$ in lines 12 and 13. In line 13 we use the format manipulator `disp()`, which is especially useful when displaying complex matrices, as it displays the entries of the latter in the form $a + bi$, in contrast to the form (a, b) used by the C++ standard library. The manipulator also accepts additional parameters that allows e.g. setting to zero numbers smaller than some given value (useful to chop small values), and it is in addition overloaded for standard containers, iterators and C-style arrays.

In line 15 we reassign to `psi` the state $|10\rangle$ via the function `mket()`. We could have also used the Eigen 3 insertion operator

```
ket psi(4); // must specify the dimension before insertion of elements via <<
psi << 0, 0, 1, 0;
```

however the `mket()` function is more concise. In line 16 we declare a gate `U` as the Controlled-NOT with control as the first subsystem, and target as the last, using the global singleton `gt`. In line 17 we declare the ket `result` as the result of applying the Controlled-NOT gate to the state $|10\rangle$, i.e. $|11\rangle$. We then display the result of the computation in lines 19 and 20.

Next, in line 22 we generate a random unitary gate via the function `randU()`, then in line 26 apply the Controlled-U, with control as the first qubit and target as the second qubit, to the state `psi`. Finally, we display the result in lines 27 and 28.

4.2 Measurements

Let us now complicate things a bit and introduce measurements. Consider the example in the listing below

```
1   // Measurements
2   // Source: ./examples/measurements.cpp
3   #include <qpp.h>
4   using namespace qpp;
5
6   int main()
7   {
8       ket psi = mket({0, 0});
9       cmat U = gt.CNOT * kron(gt.H, gt.Id2);
10      ket result = U * psi; // we have the Bell state (|00>+|11>)/sqrt(2)
11
12      std::cout << "We just produced the Bell state:\n";
13      std::cout << disp(result) << std::endl;
14
15      // apply a bit flip on the second qubit
16      result = apply(result, gt.X, {1}); // we produced (|01>+|10>)/sqrt(2)
17      std::cout << "We produced the Bell state:\n";
18      std::cout << disp(result) << std::endl;
```

```
19
20      // measure the first qubit in the X basis
21      auto m = measure(result, gt.H, {0});
22      std::cout << "Measurement result: " << std::get<0>(m);
23      std::cout << std::endl << "Probabilities: ";
24      std::cout << disp(std::get<1>(m), ", ") << std::endl;
25      std::cout << "Resulting states: " << std::endl;
26      for (auto && elem : std::get<2>(m))
27          std::cout << disp(elem) << std::endl << std::endl;
28  }
```

which outputs

```
1   >>> Starting Quantum++...
2   >>> Sun Dec  7 16:50:06 2014
3
4   We just produced the Bell state:
5   0.7071
6        0
7        0
8   0.7071
9   We produced the Bell state:
10       0
11  0.7071
12  0.7071
13       0
14  Measurement result: 1
15  Probabilities: [0.5000, 0.5000]
16  Resulting states:
17  0.5000   0.5000
18  0.5000   0.5000
19
20   0.5000   -0.5000
21  -0.5000    0.5000
22
23
24  >>> Exiting Quantum++...
25  >>> Sun Dec  7 16:50:06 2014
```

In line 9, we use the function `kron()` to create the tensor product (Kronecker product) of the Hadamard gate on the first qubit and identity on the second qubit, then we left-multiply it by the Controlled-NOT gate. In line 10 we compute the result of the operation $CNOT_{ab}(H \otimes I)|00\rangle$, which is the Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$. We display it in lines 12 and 13.

In line 16 we use the function `apply()` to apply the gate X on the second qubit[2] of the previously produced Bell state. The function `apply()` takes as its third parameter a list of subsystems, and in our case `{1}` denotes the *second* subsystem, not the first. The function `apply()`, as well as many other functions that we will encounter, have a variety of useful overloads, see `doc/refman.pdf` for a detailed library reference. In lines 17 and 18 we display the newly created Bell state.

In line 21 we use the function `measure()` to perform a measurement of the first qubit (subsystem `{0}`) in the X basis. You may be confused by the apparition of `gt.H`, however this overload of the function

[2] Quantum++ uses the C/C++ numbering convention, with indexes starting from zero.

`measure()` takes as its second parameter the measurement basis, specified as the columns of a complex matrix. In our case, the eigenvectors of the X operator are just the columns of the Hadamard matrix. As mentioned before, as all other library functions, `measure()` returns by value, hence it does not modify its argument. The return of `measure` is a tuple consisting of the measurement result, the outcome probabilities, and the possible output states. Technically `measure()` returns a tuple of 3 elements

```
std::tuple<qpp::idx, std::vector<double>, std::vector<qpp::cmat>>
```

The first element represents the measurement result, the second the possible output probabilities and the third the output output states. Instead of using this long type definition, we use the new C++11 `auto` keyword to define the type of the result `m` of `measure()`. In lines 22–27 we use the standard `std::get<>()` function to retrieve each element of the tuple, then display the measurement result, the probabilities and the resulting output states.

4.3 Quantum operations

In the listing below we introduce quantum operations: quantum channels, as well as the partial trace and partial transpose operations.

```
1  // Quantum operations
2  // Source: ./examples/quantum_operations.cpp
3  #include <qpp.h>
4  using namespace qpp;
5
6  int main()
7  {
8      cmat rho = st.pb00; // projector onto the Bell state (|00>+|11>)/sqrt(2)
9      std::cout << "Initial state:\n";
10     std::cout << disp(rho) << std::endl;
11
12     // partial transpose of first subsystem
13     cmat rhoTA = ptranspose(rho, {0});
14     std::cout << "Eigenvalues of the partial transpose of Bell-0 state are:\n";
15     std::cout << disp(transpose(hevals(rhoTA))) << std::endl;
16
17     std::cout << "Measurement channel with 2 Kraus operators:\n";
18     std::vector<cmat> Ks {st.pz0, st.pz1}; // 2 Kraus operators
19     std::cout << disp(Ks[0]) << "\n    and \n" << disp(Ks[1]) << std::endl;
20
21     std::cout << "Superoperator matrix of the channel:\n";
22     std::cout << disp(super(Ks)) << std::endl;
23
24     std::cout << "Choi matrix of the channel:\n";
25     std::cout << disp(choi(Ks)) << std::endl;
26
27     // apply the channel onto the first subsystem
28     cmat rhoOut = apply(rho, Ks, {0});
29     std::cout << "After applying the measurement channel on the first qubit:\n";
30     std::cout << disp(rhoOut) << std::endl;
31
32     // take the partial trace over the second subsystem
33     cmat rhoA = ptrace(rhoOut, {1});
34     std::cout << "After partially tracing down the second subsystem:\n";
```

```
35      std::cout << disp(rhoA) << std::endl;
36
37      // compute the von-Neumann entropy
38      double ent = entropy(rhoA);
39      std::cout << "Entropy: " << ent << std::endl;
40  }
```

The output of this program is

```
1   >>> Starting Quantum++...
2   >>> Sun Dec  7 16:50:06 2014
3
4   Initial state:
5   0.5000   0   0   0.5000
6        0   0   0        0
7        0   0   0        0
8   0.5000   0   0   0.5000
9   Eigenvalues of the partial transpose of Bell-0 state are:
10  -0.5000   0.5000   0.5000   0.5000
11  Measurement channel with 2 Kraus operators:
12  1.0000   0
13       0   0
14      and
15  0        0
16  0   1.0000
17  Superoperator matrix of the channel:
18  1.0000   0   0        0
19       0   0   0        0
20       0   0   0        0
21       0   0   0   1.0000
22  Choi matrix of the channel:
23  1.0000   0   0        0
24       0   0   0        0
25       0   0   0        0
26       0   0   0   1.0000
27  After applying the measurement channel on the first qubit:
28  0.5000   0   0        0
29       0   0   0        0
30       0   0   0        0
31       0   0   0   0.5000
32  After partially tracing down the second subsystem:
33  0.5000        0
34       0   0.5000
35  Entropy: 1.0000
36
37  >>> Exiting Quantum++...
38  >>> Sun Dec  7 16:50:06 2014
```

The example should by now be self-explanatory.

In line 8 we define the input state `rho` as the projector onto the Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$, then display it in lines 9 and 10.

In lines 13–15 we partially transpose the first qubit, then display the eigenvalues of the resulting matrix `rhoTA`.

In lines 17–19 we define a quantum channel `Ks` consisting of two Kraus operators: $|0\rangle\langle 0|$ and $|1\rangle\langle 1|$, then display the latter. Note that Quantum++ uses the `std::vector<cmat>` container to store the Kraus operators and define a quantum channel.

In lines 21–25 we display the superoperator matrix as well as the Choi matrix of the channel `Ks`.

Next, in lines 28–30 we apply the channel `Ks` to the first qubit of the input state `rho`, then display the output state `rhoOut`.

In lines 33–35 we take the partial trace of the output state `rhoOut`, then display the resulting state `rhoA`.

Finally, in lines 38 and 39 we compute the von-Neumann entropy of the resulting state and display it.

4.4 Timing

To facilitate simple timing tasks, Quantum++ provides a `Timer` class that uses internally a `std::steady_clock`. The following program demonstrate its usage.

```
1  // Timing
2  // Source: ./examples/timing.cpp
3  #include <qpp.h>
4  using namespace qpp;
5
6  int main()
7  {
8      std::cout << std::setprecision(8); // increase the default output precision
9
10     // get the first codeword from Shor's [[9,1,3]] code
11     ket c0 = codes.codeword(Codes::Type::NINE_QUBIT_SHOR, 0);
12
13     Timer t; // declare and start a timer
14     std::vector<idx> perm = randperm(9); // declare a random permutation
15     ket c0perm = syspermute(c0, perm); // permute the system
16     t.toc(); // stops the timer
17     std::cout << "Permuting subsystems according to " << disp(perm, ", ");
18     std::cout << "\nIt took " << t << " seconds to permute the subsytems.\n";
19
20     t.tic(); // restart the timer
21     std::cout << "Inverse permutation: ";
22     std::cout << disp(invperm(perm), ", ") << std::endl;
23     ket c0invperm = syspermute(c0perm, invperm(perm)); // permute again
24     std::cout << "It took " << t.toc();
25     std::cout << " seconds to un-permute the subsystems.\n";
26
27     std::cout << "Norm difference: " << norm(c0invperm - c0) << std::endl;
28 }
```

A possible output of this program is

```
1  >>> Starting Quantum++...
2  >>> Sun Dec  7 16:50:06 2014
3
4  Permuting subsystems according to [8, 3, 4, 2, 6, 0, 5, 7, 1]
5  It took 0.00009200 seconds to permute the subsytems.
```

```
Inverse permutation: [5, 8, 3, 1, 2, 6, 4, 7, 0]
It took 0.00012900 seconds to un-permute the subsystems.
Norm difference: 0.00000000

>>> Exiting Quantum++...
>>> Sun Dec  7 16:50:06 2014
```

In line 8, we change the default output precision from 4 to 8 decimals after the delimiter.

In line 11, we use the `Codes` singleton `codes` to retrieve in `c0` the first codeword of the Shor's $[[9, 1, 3]]$ quantum error correcting code.

In line 13 we declare an instance `timer` of the class `Timer`. In line 14 we declare a random permutation `perm` via the function `randperm()`. In line 15 we permute the codeword according to the permutation `perm` using the function `syspermute()` and store the result . In line 16 we stop the timer. In line 17 we display the permutation, using an overloaded form of the `disp()` manipulator for C++ standard library containers. The latter takes a `std::string` as its second parameter to specify the delimiter between the elements of the container. In line 18 we display the elapsed time using the `ostream operator<<()` operator overload for `Timer` objects.

Next, in line 20 we reset the timer, then display the inverse permutation of `perm` in lines 21 and 22. In line 23 we permute the already permuted state `c0perm` according to the inverse permutation of `perm`, and store the result in `c0invperm`. In lines 24 and 25 we display the elapsed time. Note that in line 24 we used directly `t.toc()` in the stream insertion operator, since, for convenience, the member function `Timer::toc()` returns a `const Timer&`.

Finally, in line 27, we verify that by permuting and permuting again using the inverse permutation we recover the initial codeword, i.e. the norm difference has to be zero.

4.5 Input/output

We now introduce the input/output functions of Quantum++, as well as the input/output interfacing with MATLAB. The program below saves a matrix in both Quantum++ internal format as well as in MATLAB format, then loads it back and tests that the norm difference between the saved/loaded matrix is zero.

```
1  // Input/output
2  // Source: ./examples/input_output.cpp
3  #include <qpp.h>
4  #include <MATLAB/matlab.h> // must be explicitly included
5  using namespace qpp;
6
7  int main()
8  {
9      // Quantum++ native input/output
10     cmat rho = randrho(256); // an 8 qubit density operator
11     save(rho, "rho.dat"); // save it
12     cmat loaded_rho = load<cmat>("rho.dat"); // load it back
13     // display the difference in norm, should be 0
14     std::cout << "Norm difference load/save: ";
15     std::cout << norm(loaded_rho - rho) << std::endl;
16
17     // interfacing with MATLAB
18     saveMATLABmatrix(rho, "rho.mat", "rho", "w");
19     loaded_rho = loadMATLABmatrix<cmat>("rho.mat", "rho");
20     // display the difference in norm, should be 0
21     std::cout << "Norm difference MATLAB load/save: ";
```

```
    std::cout << norm(loaded_rho - rho) << std::endl;
}
```

The output of this program is

```
>>> Starting Quantum++...
>>> Sun Dec  7 16:50:06 2014

Norm difference load/save: 0.0000
Norm difference MATLAB load/save: 0.0000

>>> Exiting Quantum++...
>>> Sun Dec  7 16:50:06 2014
```

Note that in order to use the MATLAB input/output interface support, you need to explicitly include the header file `MATLAB/matlab.h`, and you also need to have MATLAB or MATLAB compiler installed, otherwise the program fails to compile. See the file `README.md` from the root directory of Quantum++ for more details about how to compile with MATLAB support.

4.6 Exceptions

Most Quantum++ functions throw exceptions in the case of unrecoverable errors, such as out-of-range input parameters, input/output errors etc. The exceptions are handled via the class `Exception`, derived from `std::exception`. The exception types are hard-coded inside the strongly-typed enumeration (enum class) `Exception::Type`. If you want to add more exceptions, augment the enumeration `Exception::Type` and also modify accordingly the member function `Exception::_construct_exception_msg()`, which constructs the exception message displayed via the overridden virtual function `Exception::what()`. Below is an illustrative example on exception handling.

```
// Exceptions
// Source: ./examples/exceptions.cpp
#include <qpp.h>
using namespace qpp;

int main()
{
    cmat rho = randrho(16); // 4 qubits (subsystems)
    try
    {
        double mInfo = qmutualinfo(rho, {0}, {4}); // throws qpp::Exception
        std::cout << "Mutual information between first and last subsystem: ";
        std::cout << mInfo << std::endl;
    }
    catch (const std::exception& e)
    {
        std::cout << "Exception caught: " << e.what() << std::endl;
    }
}
```

The output of this program is

```
>>> Starting Quantum++...
>>> Sun Dec  7 16:50:06 2014

Exception caught: IN qpp::qmutualinfo(): Subsystems mismatch dimensions!

>>> Exiting Quantum++...
>>> Sun Dec  7 16:50:06 2014
```

In line 8 we declare a random density matrix on four qubits (dimension 16). In line 11, we compute the mutual information between the first and the 5-th subsystems. Line 11 throws an `Exception` of type `Exception::Type::SUBSYS_MISMATCH_DIMS`, as there are only four systems. We next catch the exception in line 15 via the `std::exception` base class. We could have also used directly the class `Exception`, however using the base class allows the catching of other exceptions, not just of the type `Exception`. Finally in line 17 we display the corresponding exception message.

5 Brief description of Quantum++ file structure

A brief description of the Quantum++ file structure is presented below. The directories and their brief descriptions are emphasized using **bold fonts**. The main header file qpp.h is emphasized in red fonts.

```
./
├── doc/ ..... Documentation
│   ├── html/ ..... HTML documentation
│   │   └── index.html ..... Main HTML documentation file
│   ├── quick.pdf ..... Quick reference guide
│   └── refman.pdf ..... Complete reference
├── examples/ ..... Usage examples
│   └── ex*.cpp ..... Example source files
└── include/ ..... Header files
    ├── MATLAB/ ..... MATLAB support
    │   └── matlab.h ..... Input/output interfacing with MATLAB
    ├── classes/ ..... Class definitions
    │   ├── codes.h ..... Quantum error correcting codes
    │   ├── exception.h ..... Exceptions
    │   ├── gates.h ..... Quantum gates
    │   ├── init.h ..... Initialization
    │   ├── random_devices.h ..... Random devices
    │   ├── states.h ..... Quantum states
    │   └── timer.h ..... Timing
    ├── experimental/ ..... Experimental/test functions/classes, do not use or modify
    │   └── test.h ..... Experimental/test functions/classes
    ├── internal/ ..... Internal implementation details, do not use/modify
    │   ├── classes/ ..... Internal class definitions
    │   │   ├── iomanip.h ..... Input/output manipulators
    │   │   └── singleton.h ..... Singleton pattern via CRTP
    │   └── util.h ..... Internal utility functions
    ├── constants.h ..... Constants
    ├── entanglement.h ..... Entanglement functions
    ├── entropies.h ..... Entropy functions
    ├── functions.h ..... Generic quantum computing functions
    ├── input_output.h ..... Input/output functions
    └── instruments.h ..... Measurement functions
```

```
    ├── number_theory.h ..... Number theory functions
    ├── operations.h ..... Quantum operation functions
    ├── qpp.h ..... Quantum++ main header file, includes all other necessary headers
    ├── random.h ..... Randomness-related functions
    └── types.h ..... Type aliases
├── CMakeLists.txt ..... cmake configuration file, builds ./examples/example.cpp
├── COPYING ..... GNU General Public License version 3
├── README.md ..... Building instructions
├── RELEASE.md ..... Release notes
├── VERSION ..... Version number
└── run_OSX_MATLAB ..... Script for running with MATLAB support under OS X
```

6 Advanced topics

6.1 Aliasing

Aliasing occurs whenever the same Eigen 3 matrix/vector appears on both sides of the assignment operator, and happens because of Eigen 3's lazy evaluation system. Examples that exhibit aliasing:

```
mat = 2 * mat;
```

or

```
mat = mat.transpose();
```

Aliasing *does not* occur in statements like

```
mat = f(mat);
```

where `f()` returns by value. Aliasing produces in general unexpected results, and should be avoided at all costs.

Whereas the first line produces aliasing, it is not dangerous, since the assignment is done in a one-to-one manner, i.e. each element (i, j) on the left hand side of the assignment operator is solely a function of the the *same* (i, j) element on the right hand side, i.e. $mat(i, j) = f(mat(i, j)), \forall i, j$. The problem appears whenever coefficients are being combined and overlap, such as in the second example, where $mat(i, j) = mat(j, i), \forall i, j$. To avoid aliasing, use the member function `eval()` to transform the right hand side object into a temporary, such as

```
mat = 2 * mat.eval();
```

In general, aliasing can not be detected at compile time, but can be detected at runtime whenever the compile flag `EIGEN_NO_DEBUG` is not set. Quantum++ does not set this flag in debug mode. I highly recommend to first compile your program in debug mode to detect aliasing run-time assertions, as well as other possible issues that may have escaped you, such as assigning to a matrix another matrix of different dimension etc.

For more details about aliasing, see the official Eigen 3 documentation at `http://eigen.tuxfamily.org/dox/group__TopicAliasing.html`.

6.2 Optimizations

Whenever testing your application, I recommend compiling in debug mode, as Eigen 3 run-time assertions can provide extremely helpful feedback on potential issues. Whenever the code is production-ready, you should *always* compile with optimization flags turned on, such as `-O3` (for `g++`) and `-DEIGEN_NO_DEBUG` flags set. If available, you should turn on the OpenMP multi-processing flag (`-fopenmp` for `g++`), as it enables multi-core/multi-processing.

Since most Quantum++ functions return by value, in assigments of the form

```
mat = f(another_mat);
```

there is an additional copy assignment operator when assigning the temporary returned by `f()` back to `mat`. As far as I know, this extra copy operation is not elided. Unfortunately, Eigen 3 does not yet support move semantics, which would have got rid of this additional assignment via the corresponding move assignment operator. If in the future Eigen 3 will support move semantics, the additional assignment operator will be "free", and you won't have to modify any existing code to enable the optimization; the Eigen 3 move assignment operator should take care of it for you.

Note that in a line of the form

```
cmat mat = f(another_mat);
```

most compilers perform return value optimization (RVO), i.e. the temporary on the right hand side is constructed directly inside the object `mat`, the copy constructor being elided.

6.3 Extending Quantum++

Most Quantum++ operate on Eigen 3 matrices/vectors, and return either a matrix or a scalar. In principle, you may be tempted to write a new function such as

```
cmat f(const cmat& A){...}
```

The problem with the approach above is that Eigen 3 uses *expression templates* as the type of each expression, i.e. different expressions have in general different types, see `http://eigen.tuxfamily.org/dox/TopicFunctionTakingEigenTypes.html` for more details. The correct way to write a generic function that is guaranteed to work with any matrix expression is to make your function template and declare the input parameter as `Eigen::MatrixBase<Derived>`, where `Derived` is the template parameter. For example, the Quantum++ `transpose()` function is defined as

```
1  template<typename Derived>
2  dyn_mat<typename Derived::Scalar>
3  transpose(const Eigen::MatrixBase<Derived>& A)
4  {
5      const dyn_mat<typename Derived::Scalar>& rA = A;
6
7      // check zero-size
8      if (!internal::_check_nonzero_size(rA))
9          throw Exception("qpp::transpose()", Exception::Type::ZERO_SIZE);
10
11     return rA.transpose();
12 }
```

It takes an Eigen 3 matrix expression, line 2, and returns a dynamic matrix over the scalar field of the expression, line 2. In line 5 we implicitly convert the input expression `A` to a dynamic matrix `rA` over the same scalar field as the expression, via binding to a `const` reference, therefore paying no copying cost. We then use `rA` instead of the original expression `A` in the rest of the function. Note that most of the time it is OK to use the original expression, however there are some cases where you may get a compile time error if the expression is not explicitly casted to a matrix. For consistency, I use this reference binding trick in the code of all Quantum++ functions.

As you may have already seen, Quantum++ consists mainly of a collection of functions and few classes. There is no complicated class hierarchy, and you can regard the Quantum++ API as a medium-level API. You may extend it to incorporate graphical input, e.g. use a graphical library such as Qt, or build a more sophisticated library on top of it. I recommend to read the source code and make yourself familiar with the library before deciding to extend it. You should also check the complete reference manual `.doc/refman.pdf` for an extensive documentation of all functions and classes.

I hope you find Quantum++ useful and wish you happy usage!