qpp 0.1

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### **Chapter 1**

# quantum++ - A C++11 quantum computing library

Version

0.1

Author

Vlad Gheorghiu

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quantum++ - A C++	11 quantum	computing lib	rary

# **Chapter 2**

# Namespace Index

2.1	Namespace	List
-----	-----------	------

Her	is a list of all namespaces with brief descriptions:	paces with brief descriptions:	
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	pp::internal	66	

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## **Chapter 3**

### **Hierarchical Index**

### 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

qpp::DiscreteDistribution	69
qpp::DiscreteDistributionAbsSquare	70
exception	
qpp::Exception	71
qpp::NormalDistribution	80
qpp::Qudit	81
qpp::Singleton< T >	84
qpp::Gates	74
qpp::RandomDevices	
qpp::Singleton < const Gates >	84
qpp::Singleton < const States >	84
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## **Chapter 4**

### **Class Index**

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

u::DiscreteDistribution	69
::DiscreteDistributionAbsSquare	70
::Exception	71
o::Gates	74
::NormalDistribution	80
:::Qudit	81
::RandomDevices	83
::Singleton < T >	84
::States	85
o::Timer	88
::UniformIntDistribution	89
::UniformRealDistribution	89

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## **Chapter 5**

### File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

include/channels.h	91
include/constants.h	96
include/entanglement.h	97
include/entropies.h	98
include/functions.h	99
include/internal.h	02
include/io.h	03
	04
	05
	06
	07
include/classes/exception.h	92
include/classes/gates.h	92
include/classes/qudit.h	93
include/classes/randevs.h	93
include/classes/singleton.h	94
include/classes/stat.h	95
include/classes/states.h	95
include/classes/timer.h	96

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### **Chapter 6**

### **Namespace Documentation**

### 6.1 qpp Namespace Reference

#### **Namespaces**

· internal

#### Classes

- · class DiscreteDistribution
- · class DiscreteDistributionAbsSquare
- class Exception
- · class Gates
- · class NormalDistribution
- · class Qudit
- class RandomDevices
- class Singleton
- · class States
- class Timer
- class UniformIntDistribution
- · class UniformRealDistribution

#### **Typedefs**

```
    using cplx = std::complex < double >
        Complex number in double precision.
```

• using cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

using dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

using ket = Eigen::Matrix < cplx, Eigen::Dynamic, 1 >
 Complex (double precision) dynamic Eigen column matrix.

using bra = Eigen::Matrix < cplx, 1, Eigen::Dynamic >

Complex (double precision) dynamic Eigen row matrix.

```
    template < typename Scalar >
        using DynMat = Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic >
        Dynamic Eigen matrix over the field specified by Scalar.
```

#### **Functions**

Adjoint.

```
    cmat super (const std::vector < cmat > &Ks)

     Superoperator matrix representation.

    cmat choi (const std::vector < cmat > &Ks)

     Choi matrix representation.

    std::vector< cmat > choi2kraus (const cmat &A)

     Extracts orthogonal Kraus operators from Choi matrix.

    template<typename Derived >

  cmat channel (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)
     Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

    template<typename Derived >

  cmat channel (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
  std::size t > &subsys, const std::vector< std::size t > &dims)
     Applies the channel specified by the set of Kraus operators Ks to the part of the density matrix rho specified by
     subsys.

    constexpr std::complex< double > operator""_i (unsigned long long int x)

      User-defined literal for complex i = \sqrt{-1} (integer overload)

    constexpr std::complex< double > operator""_i (long double x)

      User-defined literal for complex i = \sqrt{-1} (real overload)

    std::complex< double > omega (std::size_t D)

     D-th root of unity.

    template<typename Derived >

  cmat schmidtcoeff (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size t > &dims)
• template<typename Derived >
  cmat schmidtU (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size_t > &dims)

    template<typename Derived >

  cmat schmidtV (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size t > &dims)

    template<typename Derived >

  cmat schmidtprob (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size t > &dims)
• template<typename Derived >
  double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size_t > &dims)

    template<typename Derived >

  double gconcurrence (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  double shannon (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  double renyi (const double alpha, const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  double renyi_inf (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  double tsallis (const double alpha, const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size t > &subsys,
  const std::vector< std::size_t > &dims)

    template<typename Derived >

  DynMat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.

    template<typename Derived >

  DynMat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  DynMat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)
```

```
• template<typename Derived >
  DynMat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)
      Trace.
• template<typename Derived >
  Derived::Scalar det (const Eigen::MatrixBase< Derived > &A)
     Determinant.

    template<typename Derived >

  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum.
• template<typename Derived >
  double norm (const Eigen::MatrixBase< Derived > &A)
      Trace norm.

    template<typename Derived >

  cmat evals (const Eigen::MatrixBase< Derived > &A)
     Eigenvalues.

    template<typename Derived >

  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  dmat hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.
• template<typename Derived >
  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.

    template<typename Derived >

  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)

    template<typename Derived >

  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.

    template<typename Derived >

  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.
• template<typename Derived >
  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
```

Matrix power.

• template<typename Derived >

DynMat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, std::size\_t n)

Matrix power.

• template<typename OutputScalar , typename Derived >

DynMat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(\*f)(const typename Derived::Scalar &))

Functor.

• template<typename T >

DynMat< typename T::Scalar > kron (const T &head)

Kronecker product (variadic overload)

• template<typename T , typename... Args>

DynMat< typename T::Scalar > kron (const T &head, const Args &...tail)

Kronecker product (variadic overload)

• template<typename Derived >

DynMat< typename Derived::Scalar > kron (const std::vector< Derived > &As)

Kronecker product (std::vector overload)

template<typename Derived >

DynMat< typename Derived::Scalar > kron (const std::initializer list< Derived > &As)

Kronecker product (std::initializer\_list overload)

• template<typename Derived >

DynMat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, std::size\_t n)

Kronecker power.

template<typename Derived >

DynMat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, std::size\_t rows, std::size\_t cols)

Reshape.

• template<typename Derived >

System permutation.

• template<typename Derived >

DynMat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std::vector<
std::size\_t > &dims)

Partial trace.

template<typename Derived >

DynMat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)

Partial trace.

template<typename Derived >

DynMat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &subsys, const std::vector< std::size\_t > &dims)

Partial trace.

• template<typename Derived >

Partial transpose.

• template<typename Derived1 , typename Derived2 >

Commutator.

• template<typename Derived1 , typename Derived2 >

DynMat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const Eigen ← ::MatrixBase< Derived2 > &B)

Anti-commutator.

• template<typename Derived >

DynMat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &V)

Projector.

• template<typename Derived >

DynMat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, std::size\_t pos, const std::vector< std::size\_t > &dims)

Expand out.

template<typename Derived >

DynMat< typename Derived::Scalar > grams (const std::vector< Derived > &Vs)

Gram-Schmidt orthogonalization (std::vector overload)

template<typename Derived >

DynMat< typename Derived::Scalar > grams (const std::initializer\_list< Derived > &Vs)

Gram-Schmidt orthogonalization (std::initializer list overload)

template<typename Derived >

DynMat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)

Gram-Schmidt orthogonalization (Eigen expression (matrix) overload)

std::vector< std::size\_t > n2multiidx (std::size\_t n, const std::vector< std::size\_t > &dims)

Non-negative integer index to multi-index.

• std::size\_t multiidx2n (const std::vector< std::size\_t > &midx, const std::vector< std::size\_t > &dims)

Multi-index to non-negative integer index.

ket mket (const std::vector< std::size t > &mask)

Multi-partite qubit ket.

ket mket (const std::vector< std::size t > &mask, const std::vector< std::size t > &dims)

Multi-partite qudit ket (different dimensions overload)

ket mket (const std::vector < std::size\_t > &mask, std::size\_t d)

Multi-partite qudit ket (same dimensions overload)

std::vector< std::size\_t > invperm (const std::vector< std::size\_t > &perm)

Inverse permutation.

std::vector< std::size\_t > compperm (const std::vector< std::size\_t > &perm, const std::vector< std::size\_t > &sigma)

Compose permutations.

template<typename T >

void disp (const T &x, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)

• template<typename T >

void displn (const T &x, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)

• template<typename T >

void disp (const T \*x, const std::size\_t n, const std::string &separator, const std::string &start="[", const std
::string &end="]", std::ostream &os=std::cout)

• template<typename T >

void displn (const T \*x, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)

• template<typename Derived >

void disp (const Eigen::MatrixBase< Derived > &A, double chop=chop, std::ostream &os=std::cout)

template<typename Derived >

void displn (const Eigen::MatrixBase< Derived > &A, double chop=chop, std::ostream &os=std::cout)

- void disp (const cplx c, double chop=chop, std::ostream &os=std::cout)
- void displn (const cplx c, double chop=chop, std::ostream &os=std::cout)
- template<typename Derived >

void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

• template<typename Derived >

DynMat< typename Derived::Scalar > load (const std::string &fname)

• template<typename Derived > Derived loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name) template<> dmat loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name) template<> cmat loadMATLABmatrix (const std::string &mat file, const std::string &var name)  $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{Derived} >$ void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat file, const std↔ ::string &var name, const std::string &mode) • template<> void saveMATLABmatrix (const Eigen::MatrixBase < dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode) template<> void saveMATLABmatrix (const Eigen::MatrixBase < cmat > &A, const std::string &mat file, const std::string &var name, const std::string &mode) template<typename Derived > Derived rand (std::size t rows, std::size t cols, double a=0, double b=1) template<> dmat rand (std::size\_t rows, std::size\_t cols, double a, double b) template<> cmat rand (std::size t rows, std::size t cols, double a, double b) double rand (double a=0, double b=1) • long long randint (long long a, long long b) template<typename Derived > Derived randn (std::size\_t rows, std::size\_t cols, double mean=0, double sigma=1) template<> dmat randn (std::size t rows, std::size t cols, double mean, double sigma) template<> cmat randn (std::size t rows, std::size t cols, double mean, double sigma) double randn (double mean=0, double sigma=1) cmat randU (std::size\_t D) cmat randV (std::size\_t Din, std::size\_t Dout) std::vector< cmat > randkraus (std::size t n, std::size t D) • cmat randH (std::size t D) ket randket (std::size\_t D) cmat randrho (std::size\_t D) std::vector< std::size\_t > randperm (std::size\_t n) constexpr double chop = 1e-10 ::chop. • constexpr double eps = 1e-12 Used to decide whether a number or expression in double precision is zero or not. constexpr std::size\_t maxn = 64 Maximum number of qubits.

#### **Variables**

Used in qpp::disp() and qpp::displn() for setting to zero numbers that have their absolute value smaller than qpp::ct←

constexpr double pi = 3.141592653589793238462643383279502884

constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

RandomDevices & rdevs = RandomDevices::get\_instance()

gpp::RandomDevices Singleton

const Gates & gt = Gates::get\_instance()

#### 6.1.1 Typedef Documentation

6.1.1.1 using qpp::bra = typedef Eigen::Matrix < cplx, 1, Eigen::Dynamic >

Complex (double precision) dynamic Eigen row matrix.

6.1.1.2 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.1.3 using qpp::cplx = typedef std::complex < double >

Complex number in double precision.

6.1.1.4 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.1.5 template<typename Scalar > using qpp::DynMat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
auto mat = DynMat<float>(2,3); // type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
```

6.1.1.6 using qpp::ket = typedef Eigen::Matrix<cplx, Eigen::Dynamic, 1>

Complex (double precision) dynamic Eigen column matrix.

#### 6.1.2 Function Documentation

6.1.2.1 template<typename Derived > cmat qpp::absm ( const Eigen::MatrixBase< Derived > & A )

Matrix absolut value.

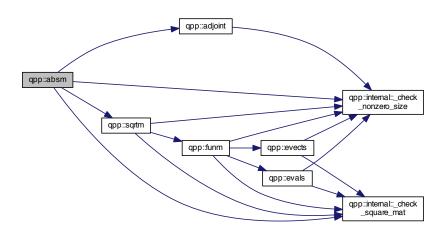
**Parameters** 

A | Eigen expression

#### Returns

Matrix absolut value of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.2 template<typename Derived > DynMat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )

#### Adjoint.

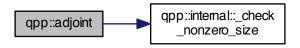
#### **Parameters**

A   Eigen expression
----------------------

#### Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.3 template<typename Derived1 , typename Derived2 > DynMat<typename Derived1::Scalar> qpp::anticomm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B)

#### Anti-commutator.

Anti-commutator  $\{A,B\} = AB + BA$ Both A and B must be Eigen expressions over the same scalar field

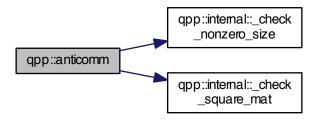
#### **Parameters**

A	Eigen expression
В	Eigen expression

#### Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.4 template<typename Derived > cmat qpp::channel ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks )

Applies the channel specified by the set of Kraus operators *Ks* to the density matrix *rho*.

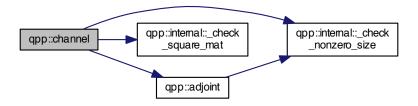
#### **Parameters**

rho	Eigen expression
Ks	std::vector of Eigen expressions representing the set of Kraus operators

#### Returns

Output density matrix, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.5 template<typename Derived > cmat qpp::channel ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks, const std::vector< std::size\_t > & subsys, const std::vector< std::size\_t > & dims )

Applies the channel specified by the set of Kraus operators *Ks* to the part of the density matrix *rho* specified by *subsys*.

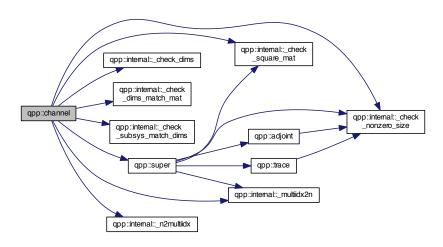
#### **Parameters**

rho	Eigen expression
Ks	std::vector of Eigen expressions representing the set of Kraus operators
subsys	Subsystems' indexes
dims	Dimensions of the multi-partite system

#### Returns

Output density matrix, as a dynamic matrix over the complex field

Here is the call graph for this function:



#### 6.1.2.6 cmat qpp::choi ( const std::vector < cmat > & Ks )

Choi matrix representation.

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $\mathit{Ks}$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|,\,|0\rangle\langle 1|$  etc.

#### Note

the superoperator matrix S and the Choi matrix C are related by  $S_{ab,mn} = C_{ma,nb}$ 

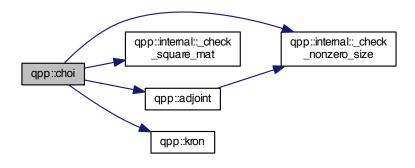
#### **Parameters**

Ks	std::vector of Eigen expressions representing the set of Kraus operators

#### Returns

Choi matrix representation, as a dynamic matrix over the complex field

Here is the call graph for this function:



#### 6.1.2.7 std::vector<cmat> qpp::choi2kraus ( const cmat & A )

Extracts orthogonal Kraus operators from Choi matrix.

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi representation *A* of the channel

#### Note

The Kraus operators satisfy  $Tr(K_i^\dagger K_j) = \delta_{ij}$  for all i 
eq j

#### Parameters

Α	Choi matrix
---	-------------

#### Returns

std::vector of dynamic matrices over the complex field representing the set of Kraus operators

Here is the call graph for this function:



 $\begin{array}{ll} \hbox{6.1.2.8} & \hbox{template$<$typename$ Derived1$ , typename Derived2$ > DynMat$<$typename Derived1::Scalar$> qpp::comm ( const Eigen::MatrixBase$< Derived2$ > & $B$ ) \\ \end{array}$ 

#### Commutator.

Commutator [A,B] = AB - BA

Both A and B must be Eigen expressions over the same scalar field

#### Parameters

Α	Eigen expression
В	Eigen expression

#### Returns

Commutator AB - BA, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



Compose permutations.

perm	Permutation
sigma	Permutation

## Returns

Composition of the permutations *perm* o *sigma* = perm(sigma)

Here is the call graph for this function:



6.1.2.10 template<typename Derived > DynMat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived > & A )

Complex conjugate.

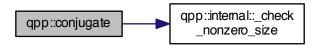
### **Parameters**

|--|

## Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



 $6.1.2.11 \quad template < typename \ Derived > cmat \ qpp::cosm \ ( \ const \ Eigen::MatrixBase < Derived > \& \ \textit{A} \ )$ 

Matrix cos.

Α	Eigen expression
---	------------------

### Returns

Matrix cosine of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.12 template < typename OutputScalar , typename Derived > DynMat < OutputScalar > qpp::cwise ( const Eigen::MatrixBase < Derived > & A, OutputScalar(\*)(const typename Derived::Scalar &) f)

## Functor.

## **Parameters**

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

## Returns

Component-wise f(A), as a dynamic matrix over the *OutputScalar* scalar field

Here is the call graph for this function:



6.1.2.13 template < typename Derived > Derived::Scalar qpp::det ( const Eigen::MatrixBase < Derived > & A )

Determinant.

Α	Eigen expression

#### Returns

Determinant of A, as a dynamic matrix over the same scalar field Returns  $\pm\infty$  when the determinant overflows/underflows

Here is the call graph for this function:



- 6.1.2.14 template<typename T > void qpp::disp ( const T & x, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] ", std::ostream & os = std::cout )
- 6.1.2.15 template < typename T > void qpp::disp ( const T \* x, const std::size\_t n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] ", std::ostream & os = std::cout )
- 6.1.2.16 template<typename Derived > void qpp::disp ( const Eigen::MatrixBase< Derived > & A, double chop = chop, std::ostream & os = std::cout )
- 6.1.2.17 void qpp::disp ( const cplx c, double chop = chop, std::ostream & os = std::cout )



6.1.2.18 template<typename T > void qpp::displn ( const T & x, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] ", std::ostream & os = std::cout )

Here is the call graph for this function:



6.1.2.19 template<typename T > void qpp::displn ( const T \* x, const std::size\_t n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] ", std::ostream & os = std::cout )

Here is the call graph for this function:



6.1.2.20 template < typename Derived > void qpp::displn ( const Eigen::MatrixBase < Derived > & A, double chop = chop, std::ostream & os = std::cout )



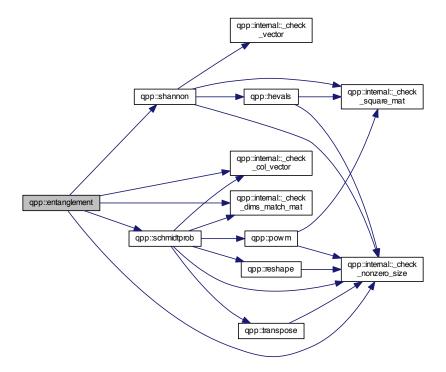
6.1.2.21 void qpp::displn ( const cplx c, double chop = chop, std::ostream & os = std::cout )

Here is the call graph for this function:



6.1.2.22 template < typename Derived > double qpp::entanglement ( const Eigen::MatrixBase < Derived > & A, const std::vector < std::size\_t > & dims )

Here is the call graph for this function:



6.1.2.23 template < typename Derived > cmat qpp::evals ( const Eigen::MatrixBase < Derived > & A )

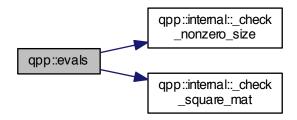
Eigenvalues.

Α	Eigen expression
---	------------------

## Returns

Eigenvalues of A, as a diagonal dynamic matrix over the complex field, with eigenvalues on the diagonal

Here is the call graph for this function:



6.1.2.24 template<typename Derived > cmat qpp::evects ( const Eigen::MatrixBase< Derived > & A )

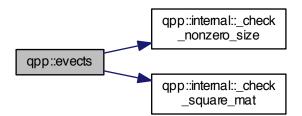
# Eigenvectors.

# **Parameters**

Α	Eigen expression

### Returns

Eigenvectors of A, as columns of a dynamic matrix over the complex field



## Expand out.

Expand out A as a matrix in a multi-partite system Faster than using <a href="mailto:qpp::kron(I, I, ..., I, A, I, ..., I">qpp::kron(I, I, ..., I, A, I, ..., I)</a>

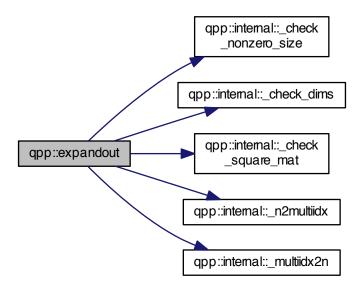
### **Parameters**

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

## Returns

Tensor product  $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$ , with A on position pos, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



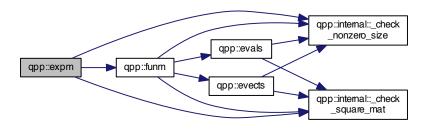
6.1.2.26 template<typename Derived > cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )

## Matrix exponential.

Α	Eigen expression

Matrix exponential of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.27 template < typename Derived > cmat qpp::funm ( const Eigen::MatrixBase < Derived > & A, cplx(\*)(const cplx &) f)

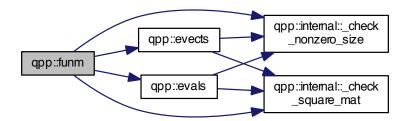
## Functional calculus f(A)

### **Parameters**

Α	Eigen expression
f	Pointer-to-function from complex to complex

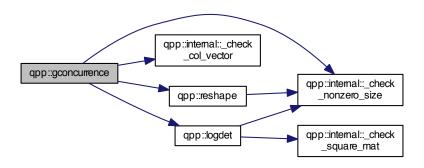
## Returns

f(A), as a dynamic matrix over the complex field



6.1.2.28 template < typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase < Derived > & A )

Here is the call graph for this function:



6.1.2.29 template<typename Derived > DynMat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )

Gram-Schmidt orthogonalization (std::vector overload)

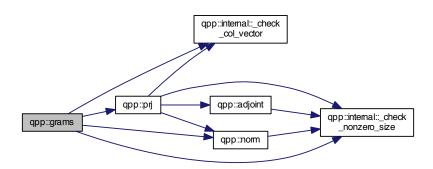
**Parameters** 

Vs	std::vector of Eigen expressions as column vectors
----	----------------------------------------------------

## Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.30 template<typename Derived > DynMat<typename Derived::Scalar> qpp::grams ( const std::initializer\_list< Derived > & Vs)

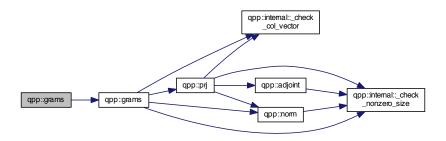
Gram-Schmidt orthogonalization (std::initializer list overload)

Vs	std::initializer list of Eigen expressions as column vectors

### Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.31 template < typename Derived > DynMat < typename Derived::Scalar > qpp::grams ( const Eigen::MatrixBase < Derived > & A )

Gram-Schmidt orthogonalization (Eigen expression (matrix) overload)

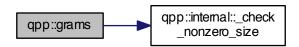
## **Parameters**

Α	Eigen expression, the input vectors are the columns of A

## Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.32 template<typename Derived > dmat qpp::hevals ( const Eigen::MatrixBase< Derived > & A )

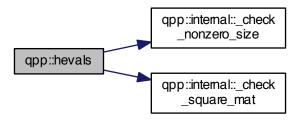
Hermitian eigenvalues.

Α	Eigen expression
---	------------------

## Returns

Eigenvalues of Hermitian A, as a diagonal dynamic matrix over the real field, with eigenvalues on the diagonal

Here is the call graph for this function:



6.1.2.33 template < typename Derived > cmat qpp::hevects ( const Eigen::MatrixBase < Derived > & A )

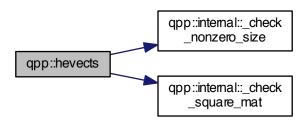
Hermitian eigenvectors.

# **Parameters**

Α	Eigen expression

### Returns

Eigenvectors of Hermitian A, as columns of a dynamic matrix over the complex field



6.1.2.34 template < typename Derived > DynMat < typename Derived::Scalar > qpp::inverse ( const Eigen::MatrixBase < Derived > & A )

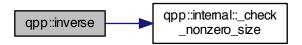
Inverse.

Α	Eigen expression

### Returns

Inverse of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.35 std::vector<std::size\_t> qpp::invperm ( const std::vector< std::size\_t> & perm )

Inverse permutation.

### **Parameters**

perm	Permutation

## Returns

Inverse of the permutation perm

Here is the call graph for this function:



6.1.2.36 template < typename T > DynMat < typename T::Scalar > qpp::kron ( const T & head )

Kronecker product (variadic overload)

Used to stop the recursion for the variadic template version of <a href="mailto:qpp::kron()">qpp::kron()</a>

head	Eigen expression

Its argument head

6.1.2.37 template<typename T , typename... Args> DynMat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )

Kronecker product (variadic overload)

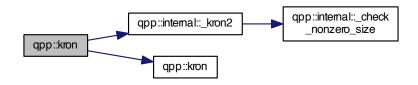
### **Parameters**

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

### Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.38 template < typename Derived > DynMat < typename Derived::Scalar > qpp::kron ( const std::vector < Derived > & As )

Kronecker product (std::vector overload)

	As	std::vector of Eigen expressions
--	----	----------------------------------

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.39 template < typename Derived > DynMat < typename Derived::Scalar > qpp::kron ( const std::initializer\_list < Derived > & As )

Kronecker product (std::initializer\_list overload)

#### **Parameters**

As	std::initializer list of Eigen expressions, such as {A1, A2,, Ak}
AS	stdinitializer_list of Ligeri expressions, such as {A1, A2, ,Ak}

## Returns

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



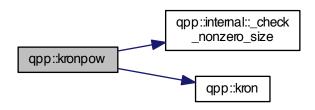
6.1.2.40 template<typename Derived > DynMat<typename Derived::Scalar> qpp::kronpow ( const Eigen::MatrixBase < Derived > & A, std::size\_t n)

Kronecker power.

Α	Eigen expression
n	Non-negative integer

Kronecker product of A with itself n times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field

Here is the call graph for this function:



- 6.1.2.41 template < typename Derived > DynMat < typename Derived::Scalar > qpp::load ( const std::string & fname )
- 6.1.2.42 template<typename Derived > Derived qpp::loadMATLABmatrix ( const std::string & mat\_file, const std::string & var\_name )
- 6.1.2.43 template <> dmat qpp::loadMATLABmatrix ( const std::string & mat\_file, const std::string & var\_name )
- 6.1.2.44 template<> cmat qpp::loadMATLABmatrix ( const std::string & mat\_file, const std::string & var\_name )
- 6.1.2.45 template<typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase< Derived > & A )

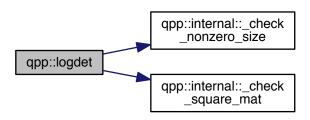
Logarithm of the determinant.

Especially useful when the determinant overflows/underflows

Α	Eigen expression

Logarithm of the determinant of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.46 template < typename Derived > cmat qpp::logm ( const Eigen::MatrixBase < Derived > & A )

Matrix logarithm.

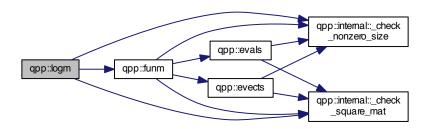
#### **Parameters**

Α	Eigen expression

### Returns

Matrix logarithm of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



# 6.1.2.47 ket qpp::mket ( const std::vector< std::size\_t > & mask )

Multi-partite qubit ket.

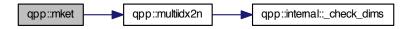
Constructs the multi-partite qubit ket  $|mask\rangle$ , where mask is a std::vector of 0's and 1's

mask	std::vector of 0's and 1's
------	----------------------------

### Returns

Multi-partite qubit state vector, as a dynamic column vector over the complex field

Here is the call graph for this function:



6.1.2.48 ket qpp::mket ( const std::vector < std::size\_t > & mask, const std::vector < std::size\_t > & dims )

Multi-partite qudit ket (different dimensions overload)

Constructs the multi-partite qudit ket  $|mask\rangle$ , where mask is a std::vector of non-negative integers Each element in mask has to be smaller than the corresponding element in dims

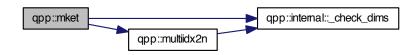
#### **Parameters**

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

## Returns

Multi-partite qudit state vector, as a dynamic column vector over the complex field

Here is the call graph for this function:



6.1.2.49 ket qpp::mket ( const std::vector< std::size\_t > & mask, std::size\_t d )

Multi-partite qudit ket (same dimensions overload)

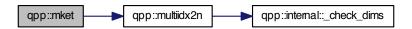
Constructs the multi-partite qudit ket  $|mask\rangle$  in a multi-partite system, all subsystem having equal dimension d mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d

mask	std::vector of non-negative integers
d	Subsystems' dimension

## Returns

Multi-partite qudit state vector, as a dynamic column vector over the complex field

Here is the call graph for this function:



6.1.2.50 std::size\_t qpp::multiidx2n ( const std::vector < std::size\_t > & midx, const std::vector < std::size\_t > & dims )

Multi-index to non-negative integer index.

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

#### **Parameters**

midx	Multi-index
dims	Dimensions of the multi-partite system

## Returns

Non-negative integer index

Here is the call graph for this function:



6.1.2.51 std::vector<std::size\_t> qpp::n2multiidx ( std::size\_t n, const std::vector< std::size\_t> & dims )

Non-negative integer index to multi-index.

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

n	Non-negative integer index
dims	Dimensions of the multi-partite system

## Returns

Multi-index of the same size as dims

Here is the call graph for this function:



6.1.2.52 template < typename Derived > double qpp::norm ( const Eigen::MatrixBase < Derived > & A )

Trace norm.

## **Parameters**

Α	Eigen expression

## Returns

Trace norm (Frobenius norm) of A, as a real number

Here is the call graph for this function:



6.1.2.53 std::complex<double> qpp::omega ( std::size\_t D )

D-th root of unity.

D	Non-negative integer

D-th root of unity  $\exp(2\pi i/D)$ 

6.1.2.54 constexpr std::complex<double> qpp::operator""\_i ( unsigned long long int x )

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

6.1.2.55 constexpr std::complex<double> qpp::operator""\_i ( long double x )

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.2.56 template<typename Derived > DynMat<typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase< Derived > & A, std::size\_t n )

Matrix power.

Explicitly multiplies the matrix A with itself n times

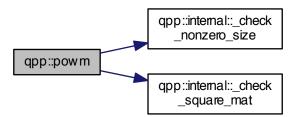
By convention  $A^0 = I$ 

**Parameters** 

Α	Eigen expression
n	Non-negative integer

## Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field



6.1.2.57 template<typename Derived > DynMat<typename Derived::Scalar> qpp::prj ( const Eigen::MatrixBase< Derived > & V )

# Projector.

Normalized projector onto state vector

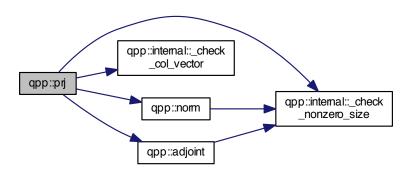
### **Parameters**

V	Eigen expression

### Returns

Projector onto the state vector V, or the matrix Zero if V has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.58 template<typename Derived > DynMat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase < Derived > & A, const std::vector< std::size\_t > & subsys, const std::vector< std::size\_t > & dims )

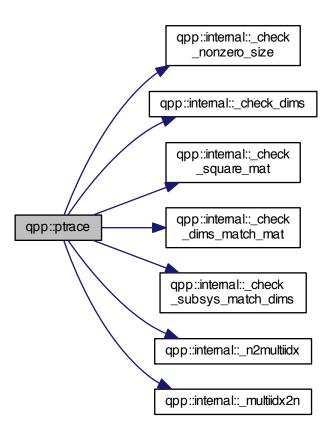
# Partial trace.

Partial trace of the multi-partite density matrix over a list of subsystems

Α	Eigen expression
subsys	Subsystems' indexes
dims	Dimensions of the multi-partite system

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.59 template<typename Derived > DynMat<typename Derived::Scalar> qpp::ptrace1 ( const Eigen::MatrixBase< Derived > & A, const std::vector< std::size\_t > & dims)

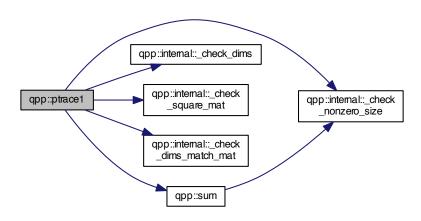
## Partial trace.

Partial trace of density matrix over the first subsystem in a bi-partite system

Α	Eigen expression
dims	Dimensions of bi-partite system (must be a std::vector with 2 elements)

Partial trace  $Tr_A(\cdot)$  over the first subsytem A in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field

Here is the call graph for this function:



# Partial trace.

## **Parameters**

Α	Eigen expression
dims	Dimensions of bi-partite system (must be a std::vector with 2 elements)

## Returns

Partial trace  $Tr_B(\cdot)$  over the second subsystem B in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field

Here is the call graph for this function:



# Partial transpose.

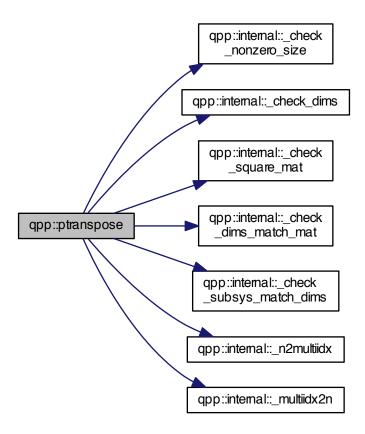
Partial transpose of the multi-partite density matrix over a list of subsystems

#### **Parameters**

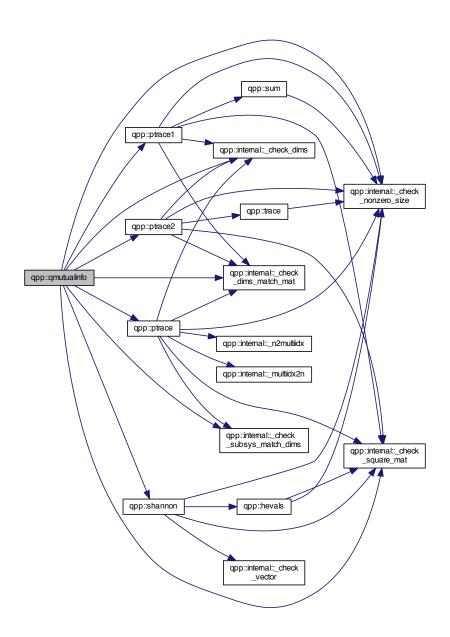
Α	Eigen expression
subsys	Subsystems' indexes
dims	Dimensions of the multi-partite system

## Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field



6.1.2.62 template < typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase < Derived > & A, const std::vector < std::size\_t > & subsys, const std::vector < std::size\_t > & dims )



- 6.1.2.63 template < typename Derived > Derived qpp::rand ( std::size\_t rows, std::size\_t cols, double a = 0, double b = 1 )
- 6.1.2.64 template <> dmat qpp::rand ( std::size\_t rows, std::size\_t cols, double a, double b )

6.1.2.65 template <> cmat qpp::rand ( std::size\_t rows, std::size\_t cols, double a, double b )

Here is the call graph for this function:



6.1.2.66 double qpp::rand ( double a = 0, double b = 1 )

Here is the call graph for this function:



6.1.2.67 cmat qpp::randH ( std::size\_t D )

Here is the call graph for this function:

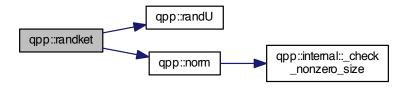


6.1.2.68 long long qpp::randint ( long long a, long long b )



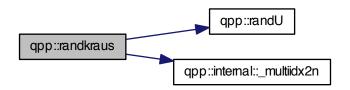
## 6.1.2.69 ket qpp::randket ( std::size\_t D )

Here is the call graph for this function:



# 6.1.2.70 std::vector<cmat> qpp::randkraus ( std::size\_t n, std::size\_t D )

Here is the call graph for this function:



- 6.1.2.71 template<typename Derived > Derived qpp::randn ( std::size\_t rows, std::size\_t cols, double mean = 0, double sigma = 1 )
- 6.1.2.72 template<> dmat qpp::randn ( std::size\_t rows, std::size\_t cols, double mean, double sigma )



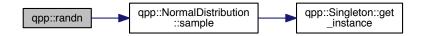
6.1.2.73 template<> cmat qpp::randn ( std::size\_t rows, std::size\_t cols, double mean, double sigma )

Here is the call graph for this function:



6.1.2.74 double qpp::randn ( double mean = 0, double sigma = 1 )

Here is the call graph for this function:

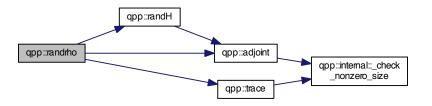


6.1.2.75 std::vector<std::size\_t> qpp::randperm ( std::size\_t n )



# 6.1.2.76 cmat qpp::randrho ( std::size\_t D )

Here is the call graph for this function:



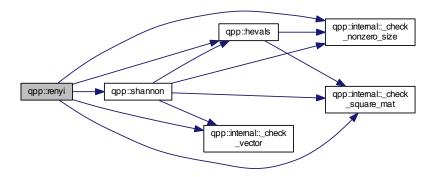
# 6.1.2.77 cmat qpp::randU ( std::size\_t D )

# 6.1.2.78 cmat qpp::randV ( std::size\_t Din, std::size\_t Dout )

Here is the call graph for this function:

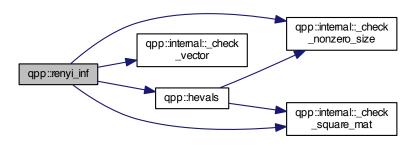


# 6.1.2.79 template<typename Derived > double qpp::renyi ( const double alpha, const Eigen::MatrixBase< Derived > & A )



6.1.2.80  $template < typename Derived > double qpp::renyi_inf ( const Eigen::MatrixBase < Derived <math>>$  & A )

Here is the call graph for this function:



6.1.2.81 template<typename Derived > DynMat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, std::size\_t rows, std::size\_t cols )

### Reshape.

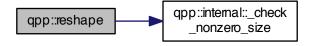
Uses column-major order when reshaping (same as MATLAB)

#### **Parameters**

Α	Eigen expression
rows	Number of rows of the reshaped matrix
cols	Number of columns of the reshaped matrix

## Returns

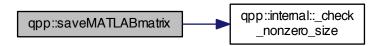
Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field



- 6.1.2.82 template < typename Derived > void qpp::save ( const Eigen::MatrixBase < Derived > & A, const std::string & fname )
- 6.1.2.83 template < typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < Derived > & A, const std::string & mat\_file, const std::string & mode )

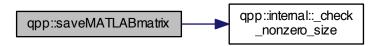
6.1.2.84 template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< dmat > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode )

Here is the call graph for this function:

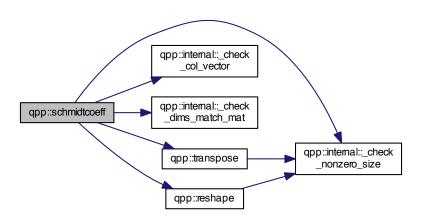


6.1.2.85 template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< cmat > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode )

Here is the call graph for this function:

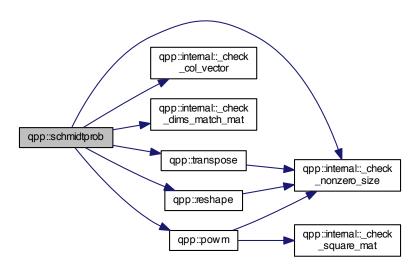


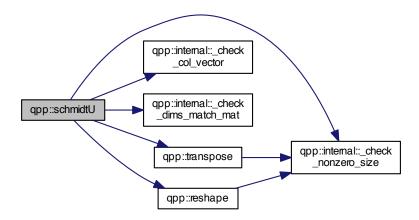
6.1.2.86 template < typename Derived > cmat qpp::schmidtcoeff ( const Eigen::MatrixBase < Derived > & A, const std::vector < std::size\_t > & dims )



6.1.2.87 template < typename Derived > cmat qpp::schmidtprob ( const Eigen::MatrixBase < Derived > & A, const std::vector < std::size\_t > & dims )

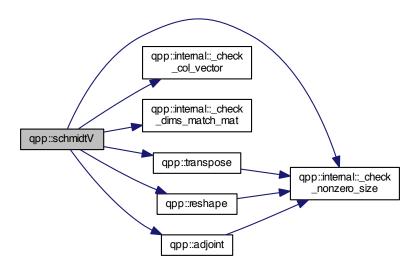
Here is the call graph for this function:





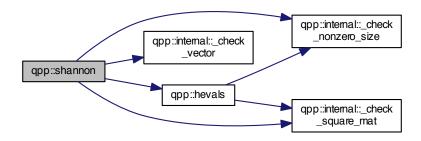
6.1.2.89 template<typename Derived > cmat qpp::schmidtV ( const Eigen::MatrixBase< Derived > & A, const std::vector< std::size\_t > & dims )

Here is the call graph for this function:



6.1.2.90 template<typename Derived > double qpp::shannon ( const Eigen::MatrixBase< Derived > & A )

Here is the call graph for this function:



6.1.2.91 template < typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase < Derived > & A )

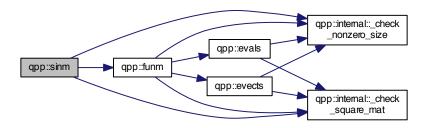
Matrix sin.

Α	Eigen expression
---	------------------

## Returns

Matrix sine of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.92 template<typename Derived > cmat qpp::spectralpowm ( const Eigen::MatrixBase< Derived > & A, const cplx z )

## Matrix power.

Uses the spectral decomposition of  $\emph{A}$  to compute the matrix power By convention  $\emph{A}^0 = \emph{I}$ 

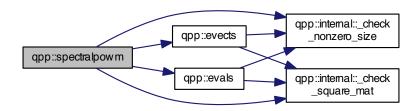
# **Parameters**

A	Eigen expression
Z	Complex number

## Returns

Matrix power  $A^z$ , as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.93 template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )

Matrix square root.

#### **Parameters**

Α	l Eigen expression
---	--------------------

#### Returns

Matrix square root of A, as a dynamic matrix over the complex field

Here is the call graph for this function:



6.1.2.94 template < typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase < Derived > & A )

Element-wise sum.

#### **Parameters**

A Eigen expression
--------------------

## Returns

Element-wise sum of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



# 6.1.2.95 cmat qpp::super ( const std::vector < cmat > & Ks )

Superoperator matrix representation.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

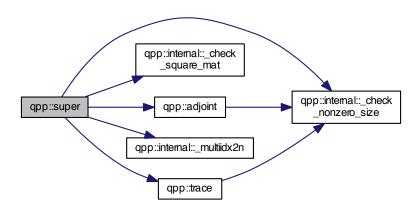
#### **Parameters**

Ks	std::vector of Eigen expressions representing the set of Kraus operators
----	--------------------------------------------------------------------------

#### Returns

Superoperator matrix representation, as a dynamic matrix over the complex field

Here is the call graph for this function:



System permutation.

Permutes the subsystems in a state vector or density matrix The qubit perm[i] is permuted to the location i

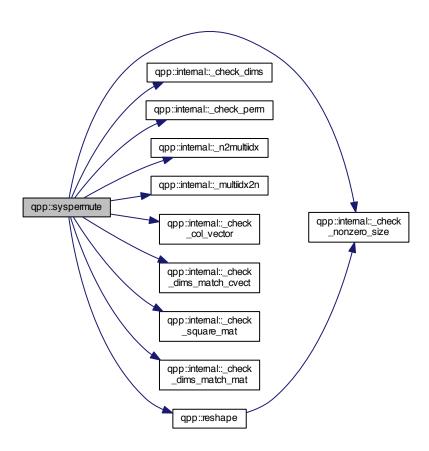
# Parameters

Α	Eigen expression
perm	Permutation
dims	Subsystems' dimensions

Returns

Permuted system, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.97 template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )

Trace.

**Parameters** 

Α	Eigen expression

#### Returns

Trace of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.98 template<typename Derived > DynMat<typename Derived::Scalar> qpp::transpose ( const Eigen::MatrixBase < Derived > &  $\bf A$  )

Transpose.

**Parameters** 

```
A Eigen expression
```

#### Returns

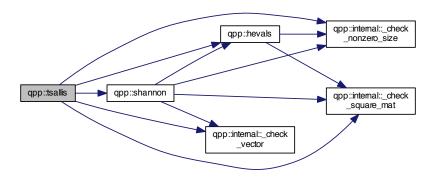
Transpose of A, as a dynamic matrix over the same scalar field

Here is the call graph for this function:



6.1.2.99 template < typename Derived > double qpp::tsallis ( const double alpha, const Eigen::MatrixBase < Derived > & A )

Here is the call graph for this function:



#### 6.1.3 Variable Documentation

#### 6.1.3.1 constexpr double qpp::chop = 1e-10

Used in *qpp::disp()* and *qpp::displn()* for setting to zero numbers that have their absolute value smaller than *qpp ⇔ ::ct::chop*.

6.1.3.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.3.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::ct::eps) // x is zero</pre>
```

6.1.3.4 const Gates& qpp::gt = Gates::get\_instance()

qpp::Gates const Singleton

Initializes the gates, see the class *qpp::Gates* 

6.1.3.5 constexpr std::size\_t qpp::maxn = 64

Maximum number of qubits.

Used internally to statically allocate arrays (for speed reasons)

6.1.3.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

#### 6.1.3.7 RandomDevices& qpp::rdevs = RandomDevices::get\_instance()

#### qpp::RandomDevices Singleton

Initializes the random devices, see the class *qpp::RandomDevices* 

6.1.3.8 const States& qpp::st = States::get\_instance()

qpp::States const Singleton

Initializes the states, see the class *qpp::States* 

# 6.2 qpp::internal Namespace Reference

#### **Functions**

- void n2multiidx (std::size t n, std::size t numdims, const std::size t \*dims, std::size t \*result)
- std::size t multiidx2n (const std::size t \*midx, std::size t numdims, const std::size t \*dims)
- template<typename Derived >

bool <u>\_check\_square\_mat</u> (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool <u>check\_vector</u> (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool <u>\_check\_row\_vector</u> (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool <u>\_check\_col\_vector</u> (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ template {<} typename \ T >$ 

bool <u>\_check\_nonzero\_size</u> (const T &x)

- bool <u>\_check\_dims</u> (const std::vector < std::size\_t > &dims)
- template<typename Derived >

 $\label{local_bool_check_dims_match_mat} \mbox{ (const std::vector} < \mbox{ std::size\_t} > \mbox{\&dims, const Eigen::MatrixBase} < \mbox{ Derived} > \mbox{\&A})$ 

template<typename Derived >

bool \_check\_dims\_match\_cvect (const std::vector< std::size\_t > &dims, const Eigen::MatrixBase< Derived > &V)

• template<typename Derived >

 $\label{local_check_dims_match_rvect} \mbox{ (const std::vector} < \mbox{ std::size\_t} > \& \mbox{ dims, const Eigen::MatrixBase} < \mbox{ Derived} > \& \mbox{ V)}$ 

- bool check eq dims (const std::vector < std::size t > &dims, std::size t dim)
- bool \_check\_subsys\_match\_dims (const std::vector< std::size\_t > &subsys, const std::vector< std::size\_t > &dims)
- bool \_check\_perm (const std::vector< std::size\_t > &perm)
- template<typename Derived1 , typename Derived2 >

DynMat< typename Derived1::Scalar > \_kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::

MatrixBase< Derived2 > &B)

template<typename T >

void variadic\_vector\_emplace (std::vector< T > &)

template<typename T, typename First, typename... Args>
 void variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)

# 6.2.1 Detailed Description

Internal functions, do not modify or use directly

- 6.2.2 Function Documentation
- 6.2.2.1 template < typename Derived > bool qpp::internal::\_check\_col\_vector ( const Eigen::MatrixBase < Derived > & A )
- 6.2.2.2 bool qpp::internal::\_check\_dims ( const std::vector < std::size\_t > & dims )
- 6.2.2.3 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_cvect ( const std::vector< std::size\_t > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.2.2.4 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_mat ( const std::vector< std::size\_t > & dims, const Eigen::MatrixBase< Derived > & A )
- 6.2.2.5 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_rvect ( const std::vector< std::size\_t > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.2.2.6 bool qpp::internal::\_check\_eq\_dims ( const std::vector< std::size\_t > & dims, std::size\_t dim )
- 6.2.2.7 template<typename T > bool qpp::internal::\_check\_nonzero\_size ( const T & x )
- 6.2.2.8 bool qpp::internal::\_check\_perm ( const std::vector< std::size\_t > & perm )
- 6.2.2.9 template < typename Derived > bool qpp::internal::\_check\_row\_vector ( const Eigen::MatrixBase < Derived > & A )
- 6.2.2.10 template < typename Derived > bool qpp::internal::\_check\_square\_mat ( const Eigen::MatrixBase < Derived > & A )
- 6.2.2.11 bool qpp::internal::\_check\_subsys\_match\_dims ( const std::vector< std::size\_t > & subsys, const std::vector< std::size\_t > & dims )
- $6.2.2.12 \quad template < typename \ Derived > bool \ qpp::internal::\_check\_vector \ ( \ const \ Eigen::MatrixBase < Derived > \& \ A \ )$
- $\begin{array}{ll} \textbf{6.2.2.13} & \textbf{template} < \textbf{typename Derived1} \ , \ \textbf{typename Derived2} > \textbf{DynMat} < \textbf{typename Derived1::Scalar} > \textbf{qpp::internal::\_kron2} \ ( \\ \textbf{const Eigen::MatrixBase} < \textbf{Derived1} > \& \textit{A}, \ \textbf{const Eigen::MatrixBase} < \textbf{Derived2} > \& \textit{B} \ ) \end{array}$

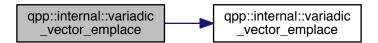
Here is the call graph for this function:



- 6.2.2.14 std::size\_t app::internal::\_multiidx2n ( const std::size\_t \* midx, std::size\_t numdims, const std::size\_t \* dims )
- 6.2.2.15 void qpp::internal::\_n2multiidx ( std::size\_t n, std::size\_t numdims, const std::size\_t \* dims, std::size\_t \* result )
- 6.2.2.16 template < typename T > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > & )

6.2.2.17 template < typename T , typename First , typename... Args > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > & v, First && first, Args &&... args )

Here is the call graph for this function:



# **Chapter 7**

# **Class Documentation**

# 7.1 qpp::DiscreteDistribution Class Reference

```
#include <stat.h>
```

#### **Public Member Functions**

- template<typename InputIterator >
   DiscreteDistribution (InputIterator first, InputIterator last)
- Discrete Distribution (std::initializer\_list< double > weights)
- Discrete Distribution (std::vector< double > weights)
- std::size\_t sample ()
- std::vector< double > probabilities () const

#### **Protected Attributes**

```
std::discrete_distributionstd::size_t > _d
```

#### 7.1.1 Constructor & Destructor Documentation

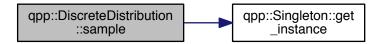
- 7.1.1.1 template < typename InputIterator > qpp::DiscreteDistribution::DiscreteDistribution ( InputIterator first, InputIterator last ) [inline]
- 7.1.1.2 qpp::DiscreteDistribution::DiscreteDistribution ( std::initializer\_list < double > weights ) [inline]
- 7.1.1.3 qpp::DiscreteDistribution::DiscreteDistribution ( std::vector< double > weights ) [inline]

#### 7.1.2 Member Function Documentation

```
7.1.2.1 std::vector<double> qpp::DiscreteDistribution::probabilities ( ) const [inline]
```

7.1.2.2 std::size\_t qpp::DiscreteDistribution::sample() [inline]

Here is the call graph for this function:



# 7.1.3 Member Data Documentation

7.1.3.1 std::discrete\_distribution<std::size\_t> qpp::DiscreteDistribution::\_d [protected]

The documentation for this class was generated from the following file:

· include/classes/stat.h

# 7.2 qpp::DiscreteDistributionAbsSquare Class Reference

#include <stat.h>

#### **Public Member Functions**

- template<typename InputIterator >
   DiscreteDistributionAbsSquare (InputIterator first, InputIterator last)
- DiscreteDistributionAbsSquare (std::initializer\_list< cplx > amplitudes)
- DiscreteDistributionAbsSquare (std::vector< cplx > amplitudes)
- template<typename Derived >
   DiscreteDistributionAbsSquare (const Eigen::MatrixBase< Derived > &V)
- std::size\_t sample ()
- std::vector< double > probabilities () const

#### **Protected Member Functions**

template<typename InputIterator >
 std::vector< double > cplx2weights (InputIterator first, InputIterator last) const

#### **Protected Attributes**

std::discrete\_distributionstd::size\_t > \_d

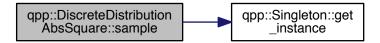
#### 7.2.1 Constructor & Destructor Documentation

- 7.2.1.1 template < typename InputIterator > qpp::DiscreteDistributionAbsSquare::DiscreteDistributionAbsSquare ( InputIterator *lirst*, InputIterator *last* ) <code>[inline]</code>
- 7.2.1.2 qpp::DiscreteDistributionAbsSquare::DiscreteDistributionAbsSquare ( std::initializer\_list< cplx > amplitudes ) [inline]
- 7.2.1.3 qpp::DiscreteDistributionAbsSquare::DiscreteDistributionAbsSquare ( std::vector < cplx > amplitudes ) [inline]
- 7.2.1.4 template<typename Derived > qpp::DiscreteDistributionAbsSquare::DiscreteDistributionAbsSquare ( const Eigen::MatrixBase< Derived > & V ) [inline]

#### 7.2.2 Member Function Documentation

- 7.2.2.1 template<typename InputIterator > std::vector<double> qpp::DiscreteDistributionAbsSquare::cplx2weights ( InputIterator first, InputIterator last ) const [inline], [protected]
- **7.2.2.2** std::vector<double> qpp::DiscreteDistributionAbsSquare::probabilities ( ) const [inline]
- **7.2.2.3** std::size\_t qpp::DiscreteDistributionAbsSquare::sample() [inline]

Here is the call graph for this function:



#### 7.2.3 Member Data Documentation

**7.2.3.1** std::discrete\_distribution<std::size\_t> qpp::DiscreteDistributionAbsSquare::\_d [protected]

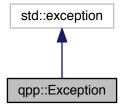
The documentation for this class was generated from the following file:

• include/classes/stat.h

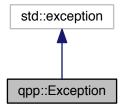
# 7.3 qpp::Exception Class Reference

#include <exception.h>

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



#### **Public Types**

• enum Type {

Type::UNKNOWN\_EXCEPTION = 1, Type::ZERO\_SIZE, Type::MATRIX\_NOT\_SQUARE, Type::MATRIX\_← NOT\_CVECTOR,

Type::MATRIX\_NOT\_RVECTOR, Type::MATRIX\_NOT\_VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_C↔ VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_RVECTOR,

Type::MATRIX\_NOT\_SQUARE\_OR\_VECTOR, Type::DIMS\_INVALID, Type::DIMS\_NOT\_EQUAL, Type::D↔ IMS\_MISMATCH\_MATRIX,

 $\label{type::DIMS_MISMATCH_CVECTOR} Type::DIMS\_MISMATCH\_RVECTOR, Type::DIMS\_MISMATCH\_VE \leftarrow CTOR, Type::SUBSYS\_MISMATCH\_DIMS,$ 

Type::PERM\_INVALID, Type::NOT\_QUBIT\_GATE, Type::NOT\_QUBIT\_SUBSYS, Type::NOT\_BIPARTITE, Type::OUT\_OF\_RANGE, Type::TYPE\_MISMATCH, Type::UNDEFINED\_TYPE, Type::CUSTOM\_EXCEPT → ION }

#### **Public Member Functions**

- Exception (const std::string &where, const Type &type)
- Exception (const std::string &where, const std::string &custom)
- virtual const char \* what () const noexceptoverride

#### **Private Member Functions**

• std::string \_construct\_exception\_msg ()

#### **Private Attributes**

- · std::string where
- std::string \_msg
- Type type
- std::string custom

#### 7.3.1 Member Enumeration Documentation

7.3.1.1 enum qpp::Exception::Type [strong]

#### Enumerator

UNKNOWN\_EXCEPTION Unknown exception

ZERO\_SIZE Zero sized object, e.g. empty Eigen::Matrix or std::vector with no elements

MATRIX\_NOT\_SQUARE Eigen::Matrix is not square

MATRIX\_NOT\_CVECTOR Eigen::Matrix is not a column vector

MATRIX\_NOT\_RVECTOR Eigen::Matrix is not a row vector

MATRIX\_NOT\_VECTOR Eigen::Matrix is not a row/column vector

MATRIX NOT SQUARE OR CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX\_NOT\_SQUARE\_OR\_RVECTOR Eigen::Matrix is not square nor a row vector

MATRIX\_NOT\_SQUARE\_OR\_VECTOR Eigen::Matrix is not square nor a row/column vector

DIMS\_INVALID std::vector<std::size\_t> representing the dimensions has zero size or contains zeros

*DIMS\_NOT\_EQUAL* std::vector<std::size\_t> representing the dimensions contains non-equal elements

**DIMS\_MISMATCH\_MATRIX** Product of the dimenisons' std::vector<std::size\_t> is not equal to the number of rows of Eigen::Matrix (assumed to be square)

**DIMS\_MISMATCH\_CVECTOR** Product of the dimenisons' std::vector<std::size\_t> is not equal to the number of cols of Eigen::Matrix (assumed to be a column vector)

**DIMS\_MISMATCH\_RVECTOR** Product of the dimenisons' std::vector<std::size\_t> is not equal to the number of cols of Eigen::Matrix (assumed to be a row vector)

**DIMS\_MISMATCH\_VECTOR** Product of the dimenisons' std::vector<std::size\_t> is not equal to the number of cols of Eigen::Matrix (assumed to be a row/column vector)

**SUBSYS\_MISMATCH\_DIMS** std::vector<std::size\_t> representing the subsystems' labels has duplicatates, or has entries that are larger than the size of the std::vector<std::size\_t> representing the dimensions

PERM\_INVALID Invalid std::vector<std::size\_t> permutation

NOT\_QUBIT\_GATE Eigen::Matrix is not 2 x 2

NOT\_QUBIT\_SUBSYS Subsystems are not 2-dimensional

NOT\_BIPARTITE std::vector<std::size t> representing the dimensions has size different from 2

OUT\_OF\_RANGE Parameter out of range

**TYPE\_MISMATCH** Types do not match (i.e. Matrix<double> vs Matrix<cplx>)

UNDEFINED\_TYPE Templated function not defined for this type

CUSTOM\_EXCEPTION Custom exception, user must provide a custom message

#### 7.3.2 Constructor & Destructor Documentation

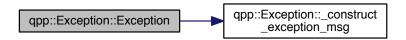
7.3.2.1 qpp::Exception::Exception ( const std::string & where, const Type & type ) [inline]

Here is the call graph for this function:



7.3.2.2 qpp::Exception::Exception ( const std::string & where, const std::string & custom ) [inline]

Here is the call graph for this function:



#### 7.3.3 Member Function Documentation

- 7.3.3.1 std::string qpp::Exception::\_construct\_exception\_msg( ) [inline], [private]
- 7.3.3.2 virtual const char\* qpp::Exception::what ( ) const [inline], [override], [virtual], [noexcept]
- 7.3.4 Member Data Documentation
- **7.3.4.1 std::string qpp::Exception::\_custom** [private]
- **7.3.4.2 std::string qpp::Exception::\_msg** [private]
- 7.3.4.3 Type qpp::Exception::\_type [private]
- **7.3.4.4 std::string qpp::Exception::\_where** [private]

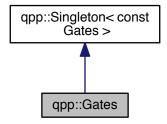
The documentation for this class was generated from the following file:

• include/classes/exception.h

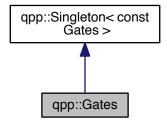
# 7.4 qpp::Gates Class Reference

#include <gates.h>

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



#### **Public Member Functions**

- cmat Rn (double theta, std::vector< double > n) const
- cmat Zd (std::size t D) const
- cmat Fd (std::size t D) const
- cmat Xd (std::size\_t D) const
- template<typename Derived = Eigen::MatrixXcd>
   Derived Id (std::Size\_t D) const
- template<typename Derived1 , typename Derived2 >
   DynMat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< std::size\_t > &ctrl, const std::vector< std::size\_t > &subsys, std::size t n, std::size t d=2) const
- template<typename Derived >
   DynMat< typename Derived::Scalar > CTRL (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &ctrl, const std::vector< std::size\_t > &subsys, std::size\_t n, std::size\_t d=2) const

# **Public Attributes**

```
cmat Id2 { cmat::Identity(2, 2) }
cmat H { cmat::Zero(2, 2) }
cmat X { cmat::Zero(2, 2) }
cmat Y { cmat::Zero(2, 2) }
cmat Z { cmat::Zero(2, 2) }
cmat S { cmat::Zero(2, 2) }
cmat T { cmat::Zero(2, 2) }
cmat CNOTab { cmat::Identity(4, 4) }
cmat CX { cmat::Identity(4, 4) }
cmat CNOTba { cmat::Zero(4, 4) }
cmat SWAP { cmat::Identity(4, 4) }
cmat TOF { cmat::Identity(8, 8) }
cmat FRED { cmat::Identity(8, 8) }
```

# **Private Member Functions**

• Gates ()

#### **Friends**

class Singleton < const Gates >

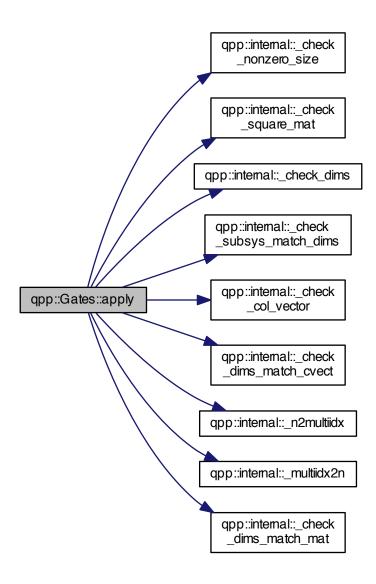
# **Additional Inherited Members**

## 7.4.1 Constructor & Destructor Documentation

```
7.4.1.1 qpp::Gates::Gates() [inline], [private]
```

#### 7.4.2 Member Function Documentation

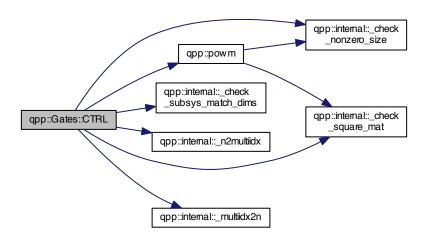
Here is the call graph for this function:



7.4.2.2 template<typename Derived1 , typename Derived2 > DynMat<typename Derived1::Scalar> qpp::Gates::applyCTRL ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< std::size\_t > & ctrl, const std::vector< std::size\_t n, std::size\_t n, std::size\_t d = 2 ) const [inline]

7.4.2.3 template<typename Derived > DynMat<typename Derived::Scalar> qpp::Gates::CTRL ( const Eigen::MatrixBase< Derived > & A, const std::vector< std::size\_t > & ctrl, const std::vector< std::size\_t > & subsys, std::size\_t n, std::size\_t d = 2 ) const [inline]

Here is the call graph for this function:



7.4.2.4 cmat qpp::Gates::Fd ( std::size\_t D ) const [inline]

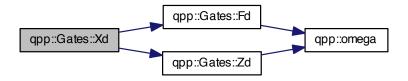
Here is the call graph for this function:



- 7.4.2.5 template<typename Derived = Eigen::MatrixXcd> Derived qpp::Gates::Id ( std::size\_t D ) const [inline]
- 7.4.2.6 cmat qpp::Gates::Rn ( double theta, std::vector < double > n ) const [inline]

7.4.2.7 cmat qpp::Gates::Xd ( std::size\_t D ) const [inline]

Here is the call graph for this function:



7.4.2.8 cmat qpp::Gates::Zd ( std::size\_t D ) const [inline]

Here is the call graph for this function:



- 7.4.3 Friends And Related Function Documentation
- **7.4.3.1** friend class Singleton < const Gates > [friend]
- 7.4.4 Member Data Documentation
- 7.4.4.1 cmat qpp::Gates::CNOTab { cmat::Identity(4, 4) }
- 7.4.4.2 cmat qpp::Gates::CNOTba { cmat::Zero(4, 4) }
- 7.4.4.3 cmat qpp::Gates::CZ { cmat::Identity(4, 4) }
- 7.4.4.4 cmat qpp::Gates::FRED { cmat::Identity(8, 8) }
- 7.4.4.5 cmat qpp::Gates::H { cmat::Zero(2, 2) }
- 7.4.4.6 cmat qpp::Gates::ld2 { cmat::ldentity(2, 2) }
- 7.4.4.7 cmat qpp::Gates::S { cmat::Zero(2, 2) }
- 7.4.4.8 cmat qpp::Gates::SWAP { cmat::Identity(4, 4) }
- 7.4.4.9 cmat qpp::Gates::T { cmat::Zero(2, 2) }

```
    7.4.4.10 cmat qpp::Gates::TOF { cmat::Identity(8, 8) }
    7.4.4.11 cmat qpp::Gates::X { cmat::Zero(2, 2) }
    7.4.4.12 cmat qpp::Gates::Y { cmat::Zero(2, 2) }
    7.4.4.13 cmat qpp::Gates::Z { cmat::Zero(2, 2) }
```

The documentation for this class was generated from the following file:

• include/classes/gates.h

# 7.5 qpp::NormalDistribution Class Reference

```
#include <stat.h>
```

#### **Public Member Functions**

- NormalDistribution (double mean=0, double sigma=1)
- double sample ()

#### **Protected Attributes**

• std::normal\_distribution\_d

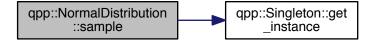
#### 7.5.1 Constructor & Destructor Documentation

7.5.1.1 qpp::NormalDistribution::NormalDistribution ( double mean = 0, double sigma = 1 ) [inline]

# 7.5.2 Member Function Documentation

**7.5.2.1** double qpp::NormalDistribution::sample() [inline]

Here is the call graph for this function:



## 7.5.3 Member Data Documentation

**7.5.3.1 std::normal\_distribution qpp::NormalDistribution::\_d** [protected]

The documentation for this class was generated from the following file:

• include/classes/stat.h

# 7.6 qpp::Qudit Class Reference

```
#include <qudit.h>
```

#### **Public Member Functions**

- Qudit (const cmat &rho=States::get\_instance().pz0)
- std::size\_t measure (const cmat &U, bool destructive=false)
- std::size\_t measure (bool destructive=false)
- cmat getRho () const
- std::size\_t getD () const

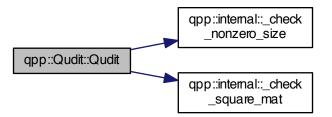
# **Private Attributes**

- cmat \_rho
- std::size\_t \_D

#### 7.6.1 Constructor & Destructor Documentation

```
7.6.1.1 qpp::Qudit::Qudit ( const cmat & rho = States::get_instance () .pz0 ) [inline]
```

Here is the call graph for this function:

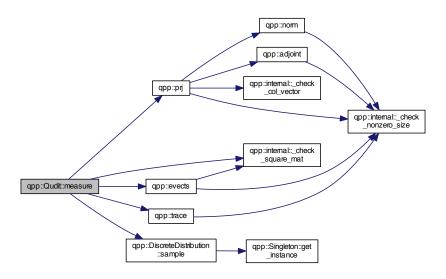


#### 7.6.2 Member Function Documentation

- 7.6.2.1 std::size\_t qpp::Qudit::getD ( ) const [inline]
- 7.6.2.2 cmat qpp::Qudit::getRho() const [inline]

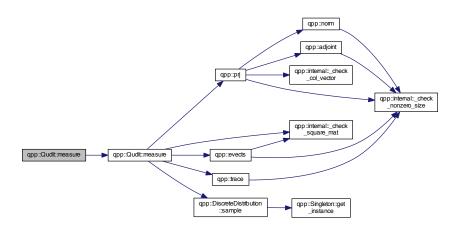
7.6.2.3 std::size\_t qpp::Qudit::measure ( const cmat & U, bool destructive = false ) [inline]

Here is the call graph for this function:



7.6.2.4 std::size\_t qpp::Qudit::measure ( bool destructive = false ) [inline]

Here is the call graph for this function:



## 7.6.3 Member Data Documentation

7.6.3.1 std::size\_t qpp::Qudit::\_D [private]

**7.6.3.2 cmat qpp::Qudit::\_rho** [private]

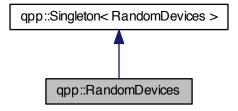
The documentation for this class was generated from the following file:

• include/classes/qudit.h

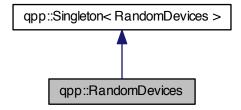
# 7.7 qpp::RandomDevices Class Reference

#include <randevs.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



# **Public Attributes**

• std::mt19937 \_rng

# **Private Member Functions**

• RandomDevices ()

# **Private Attributes**

• std::random\_device \_rd

# **Friends**

class Singleton < RandomDevices >

#### **Additional Inherited Members**

#### 7.7.1 Constructor & Destructor Documentation

7.7.1.1 qpp::RandomDevices::RandomDevices() [inline], [private]

#### 7.7.2 Friends And Related Function Documentation

**7.7.2.1** friend class Singleton < Random Devices > [friend]

#### 7.7.3 Member Data Documentation

**7.7.3.1** std::random\_device qpp::RandomDevices::\_rd [private]

7.7.3.2 std::mt19937 qpp::RandomDevices::\_rng

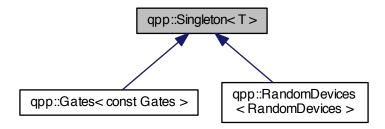
The documentation for this class was generated from the following file:

• include/classes/randevs.h

# 7.8 qpp::Singleton < T > Class Template Reference

#include <singleton.h>

Inheritance diagram for qpp::Singleton < T >:



#### **Static Public Member Functions**

• static T & get\_instance ()

#### **Protected Member Functions**

- Singleton ()=default
- virtual ∼Singleton ()
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete

#### 7.8.1 Constructor & Destructor Documentation

- 7.8.1.1 template < typename T > qpp::Singleton < T >::Singleton ( ) [protected], [default]
- 7.8.1.2 template<typename T> virtual qpp::Singleton < T>:: $\sim$ Singleton ( ) [inline], [protected], [virtual]

#### 7.8.2 Member Function Documentation

- 7.8.2.1 template<typename T> static T& qpp::Singleton < T>::get\_instance( ) [inline], [static]
- 7.8.2.2 template<typename T> Singleton& qpp::Singleton< T>::operator= ( const Singleton< T>& ) [protected], [delete]

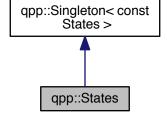
The documentation for this class was generated from the following file:

• include/classes/singleton.h

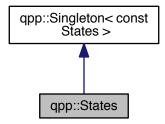
# 7.9 qpp::States Class Reference

#include <states.h>

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



#### **Public Attributes**

```
    ket x0 { ket::Zero(2) }

ket x1 { ket::Zero(2) }
ket y0 { ket::Zero(2) }

    ket y1 { ket::Zero(2) }

    ket z0 { ket::Zero(2) }

ket z1 { ket::Zero(2) }

    cmat px0 { cmat::Zero(2, 2) }

cmat px1 { cmat::Zero(2, 2) }
cmat py0 { cmat::Zero(2, 2) }
cmat py1 { cmat::Zero(2, 2) }
• cmat pz0 { cmat::Zero(2, 2) }

    cmat pz1 { cmat::Zero(2, 2) }

ket b00 { ket::Zero(4) }
ket b01 { ket::Zero(4) }
ket b10 { ket::Zero(4) }
ket b11 { ket::Zero(4) }

    cmat pb00 { cmat::Zero(4, 4) }

cmat pb01 { cmat::Zero(4, 4) }

    cmat pb10 { cmat::Zero(4, 4) }

cmat pb11 { cmat::Zero(4, 4) }

    ket GHZ { ket::Zero(8) }

    ket W { ket::Zero(8) }

    cmat pGHZ { cmat::Zero(8, 8) }

    cmat pW { cmat::Zero(8, 8) }
```

#### **Private Member Functions**

• States ()

### **Friends**

class Singleton < const States >

#### **Additional Inherited Members**

```
Constructor & Destructor Documentation
7.9.1.1
        qpp::States::States() [inline],[private]
7.9.2
        Friends And Related Function Documentation
7.9.2.1 friend class Singleton < const States > [friend]
7.9.3
        Member Data Documentation
7.9.3.1
        ket qpp::States::b00 { ket::Zero(4) }
7.9.3.2
        ket qpp::States::b01 { ket::Zero(4) }
        ket qpp::States::b10 { ket::Zero(4) }
7.9.3.3
        ket qpp::States::b11 { ket::Zero(4) }
        ket qpp::States::GHZ { ket::Zero(8) }
        cmat qpp::States::pb00 { cmat::Zero(4, 4) }
7.9.3.7
        cmat qpp::States::pb01 { cmat::Zero(4, 4) }
7.9.3.8 cmat qpp::States::pb10 { cmat::Zero(4, 4) }
7.9.3.9
        cmat qpp::States::pb11 { cmat::Zero(4, 4) }
7.9.3.10 cmat qpp::States::pGHZ { cmat::Zero(8, 8) }
7.9.3.11 cmat qpp::States::pW { cmat::Zero(8, 8) }
7.9.3.12 cmat qpp::States::px0 { cmat::Zero(2, 2) }
7.9.3.13 cmat qpp::States::px1 { cmat::Zero(2, 2) }
7.9.3.14 cmat qpp::States::py0 { cmat::Zero(2, 2) }
7.9.3.15 cmat qpp::States::py1 { cmat::Zero(2, 2) }
7.9.3.16 cmat qpp::States::pz0 { cmat::Zero(2, 2) }
7.9.3.17 cmat qpp::States::pz1 { cmat::Zero(2, 2) }
7.9.3.18 ket qpp::States::W { ket::Zero(8) }
7.9.3.19 ket qpp::States::x0 { ket::Zero(2) }
7.9.3.20 ket qpp::States::x1 { ket::Zero(2) }
7.9.3.21 ket qpp::States::y0 { ket::Zero(2) }
7.9.3.22 ket qpp::States::y1 { ket::Zero(2) }
```

```
7.9.3.23 ket qpp::States::z0 { ket::Zero(2) }7.9.3.24 ket qpp::States::z1 { ket::Zero(2) }
```

The documentation for this class was generated from the following file:

• include/classes/states.h

# 7.10 qpp::Timer Class Reference

```
#include <timer.h>
```

#### **Public Member Functions**

- Timer ()
- void tic ()
- void toc ()
- double seconds () const

#### **Protected Attributes**

- std::chrono::steady\_clock::time\_point \_start
- · std::chrono::steady\_clock::time\_point\_end

#### **Friends**

std::ostream & operator<< (std::ostream &os, const Timer &rhs)</li>

#### 7.10.1 Constructor & Destructor Documentation

```
7.10.1.1 qpp::Timer::Timer( ) [inline]
```

#### 7.10.2 Member Function Documentation

```
7.10.2.1 double qpp::Timer::seconds ( ) const [inline]
```

```
7.10.2.2 void qpp::Timer::tic() [inline]
```

- 7.10.2.3 void qpp::Timer::toc( ) [inline]
- 7.10.3 Friends And Related Function Documentation
- 7.10.3.1 std::ostream& operator << ( std::ostream & os, const Timer & rhs ) [friend]

#### 7.10.4 Member Data Documentation

```
7.10.4.1 std::chrono::steady_clock::time_point qpp::Timer::_end [protected]
```

**7.10.4.2** std::chrono::steady\_clock::time\_point qpp::Timer::\_start [protected]

The documentation for this class was generated from the following file:

• include/classes/timer.h

# 7.11 qpp::UniformIntDistribution Class Reference

```
#include <stat.h>
```

#### **Public Member Functions**

- UniformIntDistribution (int a=0, int b=1)
- int sample ()

#### **Protected Attributes**

· std::uniform\_int\_distribution\_d

#### 7.11.1 Constructor & Destructor Documentation

7.11.1.1 qpp::UniformIntDistribution::UniformIntDistribution (int a = 0, int b = 1) [inline]

#### 7.11.2 Member Function Documentation

7.11.2.1 int qpp::UniformIntDistribution::sample() [inline]

Here is the call graph for this function:



#### 7.11.3 Member Data Documentation

**7.11.3.1** std::uniform\_int\_distribution qpp::UniformIntDistribution::\_d [protected]

The documentation for this class was generated from the following file:

• include/classes/stat.h

# 7.12 qpp::UniformRealDistribution Class Reference

```
#include <stat.h>
```

## **Public Member Functions**

- UniformRealDistribution (double a=0, double b=1)
- double sample ()

# **Protected Attributes**

· std::uniform\_real\_distribution\_d

#### 7.12.1 Constructor & Destructor Documentation

7.12.1.1 qpp::UniformRealDistribution::UniformRealDistribution ( double a = 0, double b = 1 ) [inline]

# 7.12.2 Member Function Documentation

**7.12.2.1** double qpp::UniformRealDistribution::sample( ) [inline]

Here is the call graph for this function:



#### 7.12.3 Member Data Documentation

**7.12.3.1 std::uniform\_real\_distribution qpp::UniformRealDistribution::\_d** [protected]

The documentation for this class was generated from the following file:

• include/classes/stat.h

# **Chapter 8**

# **File Documentation**

# 8.1 include/channels.h File Reference

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

#### **Functions**

- cmat qpp::super (const std::vector< cmat > &Ks)
  - Superoperator matrix representation.
- cmat qpp::choi (const std::vector< cmat > &Ks)

Choi matrix representation.

- std::vector< cmat > qpp::choi2kraus (const cmat &A)
  - Extracts orthogonal Kraus operators from Choi matrix.
- $\bullet \ \ \text{template}{<} \text{typename Derived} >$ 
  - cmat qpp::channel (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

 $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{Derived} >$ 

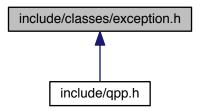
cmat qpp::channel (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std $\leftrightarrow$  ::vector< std::size\_t > &subsys, const std::vector< std::size\_t > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part of the density matrix rho specified by subsys.

92 File Documentation

# 8.2 include/classes/exception.h File Reference

This graph shows which files directly or indirectly include this file:



#### Classes

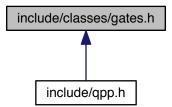
· class qpp::Exception

# **Namespaces**

• qpp

# 8.3 include/classes/gates.h File Reference

This graph shows which files directly or indirectly include this file:



# Classes

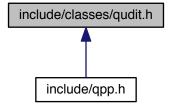
class qpp::Gates

# **Namespaces**

qpp

# 8.4 include/classes/qudit.h File Reference

This graph shows which files directly or indirectly include this file:



#### Classes

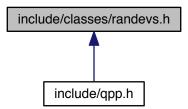
· class qpp::Qudit

# **Namespaces**

• qpp

# 8.5 include/classes/randevs.h File Reference

This graph shows which files directly or indirectly include this file:



# Classes

• class qpp::RandomDevices

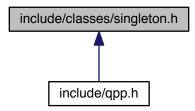
# **Namespaces**

qpp

94 File Documentation

# 8.6 include/classes/singleton.h File Reference

This graph shows which files directly or indirectly include this file:



#### **Classes**

class qpp::Singleton< T >

# **Namespaces**

qpp

## Macros

- #define CLASS\_SINGLETON(Foo)
- #define CLASS\_CONST\_SINGLETON(Foo)

#### 8.6.1 Macro Definition Documentation

# 8.6.1.1 #define CLASS\_CONST\_SINGLETON( Foo )

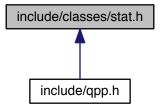
# Value:

#### 8.6.1.2 #define CLASS\_SINGLETON( Foo )

#### Value:

# 8.7 include/classes/stat.h File Reference

This graph shows which files directly or indirectly include this file:



# **Classes**

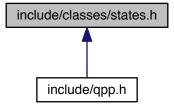
- class qpp::NormalDistribution
- class qpp::UniformRealDistribution
- class qpp::UniformIntDistribution
- class qpp::DiscreteDistribution
- class qpp::DiscreteDistributionAbsSquare

# **Namespaces**

• qpp

# 8.8 include/classes/states.h File Reference

This graph shows which files directly or indirectly include this file:



#### Classes

class qpp::States

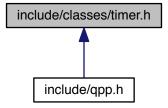
96 File Documentation

# **Namespaces**

• qpp

# 8.9 include/classes/timer.h File Reference

This graph shows which files directly or indirectly include this file:



#### Classes

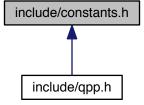
class qpp::Timer

# **Namespaces**

qpp

# 8.10 include/constants.h File Reference

This graph shows which files directly or indirectly include this file:



# **Namespaces**

qpp

## **Functions**

- constexpr std::complex< double > qpp::operator""\_i (unsigned long long int x)
  - User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)
- constexpr std::complex< double > qpp::operator"" i (long double x)
  - User-defined literal for complex  $i = \sqrt{-1}$  (real overload)
- std::complex < double > qpp::omega (std::size\_t D)
  - D-th root of unity.

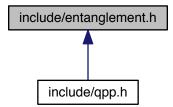
#### **Variables**

- constexpr double qpp::chop = 1e-10
  - Used in qpp::disp() and qpp::displn() for setting to zero numbers that have their absolute value smaller than qpp::ct← ::chop.
- constexpr double qpp::eps = 1e-12
  - Used to decide whether a number or expression in double precision is zero or not.
- constexpr std::size\_t qpp::maxn = 64
  - Maximum number of qubits.
- constexpr double qpp::pi = 3.141592653589793238462643383279502884
  - π
- constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

# 8.11 include/entanglement.h File Reference

This graph shows which files directly or indirectly include this file:



# **Namespaces**

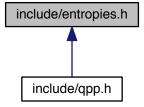
qpp

- template<typename Derived >
   cmat qpp::schmidtcoeff (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)
- template<typename Derived >
   cmat qpp::schmidtU (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)

- template<typename Derived >
   cmat qpp::schmidtV (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)
- template<typename Derived >
   cmat qpp::schmidtprob (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)
- template<typename Derived >
   double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &dims)
- template<typename Derived >
   double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)

# 8.12 include/entropies.h File Reference

This graph shows which files directly or indirectly include this file:



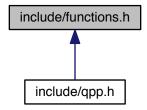
#### **Namespaces**

• qpp

- template<typename Derived > double qpp::shannon (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   double qpp::renyi (const double alpha, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   double qpp::renyi\_inf (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   double qpp::tsallis (const double alpha, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &subsys,
   const std::vector< std::size\_t > &dims)

## 8.13 include/functions.h File Reference

This graph shows which files directly or indirectly include this file:



# **Namespaces**

qpp

## **Functions**

```
    template<typename Derived >

  DynMat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.

    template<typename Derived >

  DynMat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  DynMat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
  DynMat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)
      Inverse.
template<typename Derived >
  Derived::Scalar qpp::trace (const Eigen::MatrixBase< Derived > &A)
      Trace.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::det">qpp::det</a> (const Eigen::MatrixBase</a> Derived > &A)
      Determinant.
template<typename Derived >
  Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > &A)
      Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar qpp::sum (const Eigen::MatrixBase< Derived > &A)
      Element-wise sum.
• template<typename Derived >
  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase</a> Derived > &A)
      Trace norm.

    template<typename Derived >
```

cmat qpp::evals (const Eigen::MatrixBase< Derived > &A)

```
Eigenvalues.
• template<typename Derived >
  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
      Eigenvectors.
• template<typename Derived >
  dmat qpp::hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.
• template<typename Derived >
  cmat <a href="mailto:qpp::hevects">qpp::hevects</a> (const Eigen::MatrixBase</a> Derived > &A)
     Hermitian eigenvectors.

    template<typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
      Functional calculus f(A)

    template<typename Derived >

  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
      Matrix square root.
• template<typename Derived >
  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
      Matrix absolut value.

    template<typename Derived >

  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat <a href="mailto:qpp::logm">qpp::logm</a> (const Eigen::MatrixBase</a> Derived > &A)
      Matrix logarithm.
• template<typename Derived >
  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.

    template<typename Derived >

  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
      Matrix cos.

    template<typename Derived >

  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
      Matrix power.
• template<typename Derived >
  DynMat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, std::size_t n)
      Matrix power.
• template<typename OutputScalar , typename Derived >
  DynMat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const type-
  name Derived::Scalar &))
     Functor.
template<typename T >
  DynMat< typename T::Scalar > qpp::kron (const T &head)
      Kronecker product (variadic overload)

    template<typename T, typename... Args>

  DynMat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
      Kronecker product (variadic overload)

    template<typename Derived >

  DynMat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
      Kronecker product (std::vector overload)

    template<typename Derived >

  DynMat< typename Derived::Scalar > qpp::kron (const std::initializer list< Derived > &As)
      Kronecker product (std::initializer list overload)
```

template<typename Derived >

DynMat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, std::size\_t n)

Kronecker power.

template<typename Derived >

DynMat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, std::size\_t rows, std::size\_t cols)

Reshape.

template<typename Derived >

DynMat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size t > &perm, const std::vector< std::size t > &dims)

System permutation.

• template<typename Derived >

 $\label{lem:def:def:DynMat} DynMat < typename \ Derived::Scalar > qpp::ptrace1 \ (const \ Eigen::MatrixBase < Derived > &A, \ const \ std \\ ::vector < std::size\_t > &dims)$ 

Partial trace.

• template<typename Derived >

 $\label{lem:def:DynMat} \mbox{DynMat} < \mbox{typename Derived::Scalar} > \mbox{qpp::ptrace2} \mbox{ (const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const std} \\ \mbox{::vector} < \mbox{std::size\_t} > \&\mbox{dims})$ 

Partial trace.

template<typename Derived >

DynMat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std⇔ ::vector< std::size t > &subsys, const std::vector< std::size t > &dims)

Partial trace.

template<typename Derived >

DynMat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< std::size\_t > &subsys, const std::vector< std::size\_t > &dims)

Partial transpose.

• template<typename Derived1 , typename Derived2 >

 $\label{lem:def:def:DynMat} \mbox{ Derived1::Scalar } > \mbox{qpp::comm (const Eigen::MatrixBase} < \mbox{ Derived1 } > \mbox{\&A, const Eigen::MatrixBase} < \mbox{ Derived2 } > \mbox{\&B)}$ 

Commutator.

template<typename Derived1 , typename Derived2 >

DynMat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)

Anti-commutator.

 $\bullet \ \ {\sf template}{<} {\sf typename \ Derived} >$ 

DynMat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)

Projector.

• template<typename Derived >

DynMat< typename Derived::Scalar > qpp::expandout (const Eigen::MatrixBase< Derived > &A, std::size ← \_t pos, const std::vector< std::size\_t > &dims)

Expand out.

• template<typename Derived >

DynMat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)

Gram-Schmidt orthogonalization (std::vector overload)

• template<typename Derived >

DynMat< typename Derived::Scalar > qpp::grams (const std::initializer list< Derived > &Vs)

Gram-Schmidt orthogonalization (std::initializer\_list overload)

• template<typename Derived >

DynMat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)

Gram-Schmidt orthogonalization (Eigen expression (matrix) overload)

std::vector< std::size t > qpp::n2multiidx (std::size t n, const std::vector< std::size t > &dims)

Non-negative integer index to multi-index.

std::size\_t qpp::multiidx2n (const std::vector < std::size\_t > &midx, const std::vector < std::size\_t > &dims)
 Multi-index to non-negative integer index.

ket qpp::mket (const std::vector< std::size t > &mask)

Multi-partite qubit ket.

ket qpp::mket (const std::vector < std::size\_t > &mask, const std::vector < std::size\_t > &dims)

Multi-partite qudit ket (different dimensions overload)

ket qpp::mket (const std::vector< std::size\_t > &mask, std::size\_t d)

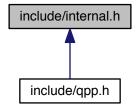
Multi-partite qudit ket (same dimensions overload)

- std::vector< std::size\_t > qpp::invperm (const std::vector< std::size\_t > &perm)
   Inverse permutation.
- std::vector< std::size\_t > app::compperm (const std::vector< std::size\_t > aperm, const std::vector< std
   ::size\_t > aperm, const std::vector< std
   ::size\_t

Compose permutations.

#### 8.14 include/internal.h File Reference

This graph shows which files directly or indirectly include this file:



# **Namespaces**

- qpp::internal
- qpp

## **Functions**

- void qpp::internal::\_n2multiidx (std::size\_t n, std::size\_t numdims, const std::size\_t \*dims, std::size\_t \*result)
- std::size t qpp::internal:: multiidx2n (const std::size t \*midx, std::size t numdims, const std::size t \*dims)
- template<typename Derived >

bool qpp::internal::\_check\_square\_mat (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ \text{template}{<} \text{typename Derived} >$ 

bool qpp::internal::\_check\_vector (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool qpp::internal::\_check\_row\_vector (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ \mathsf{template}{<} \mathsf{typename} \ \mathsf{Derived} >$ 

bool qpp::internal::\_check\_col\_vector (const Eigen::MatrixBase< Derived > &A)

• template<typename T >

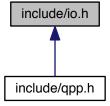
bool qpp::internal::\_check\_nonzero\_size (const T &x)

bool qpp::internal::\_check\_dims (const std::vector< std::size\_t > &dims)

- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_mat (const std::vector< std::size\_t > &dims, const Eigen::Matrix
   Base< Derived > &A)
- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_cvect (const std::vector< std::size\_t > &dims, const Eigen::Matrix
   Base< Derived > &V)
- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_rvect (const std::vector< std::size\_t > &dims, const Eigen::Matrix
   Base< Derived > &V)
- bool qpp::internal::\_check\_eq\_dims (const std::vector< std::size\_t > &dims, std::size\_t dim)
- bool qpp::internal::\_check\_subsys\_match\_dims (const std::vector< std::size\_t > &subsys, const std
   ::vector< std::size\_t > &dims)
- bool qpp::internal::\_check\_perm (const std::vector< std::size\_t > &perm)
- template<typename Derived1, typename Derived2 >
   DynMat< typename Derived1::Scalar > qpp::internal::\_kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >
   void qpp::internal::variadic\_vector\_emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
   void qpp::internal::variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)

# 8.15 include/io.h File Reference

This graph shows which files directly or indirectly include this file:



# **Namespaces**

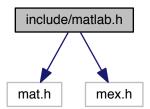
qpp

- template<typename T >
   void qpp::disp (const T &x, const std::string &separator, const std::string &start="[", const std::string &end="]",
   std::ostream &os=std::cout)
- template<typename T >
   void qpp::displn (const T &x, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)
- template<typename T >
   void qpp::disp (const T \*x, const std::size\_t n, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)

- template<typename T >
   void qpp::displn (const T \*x, const std::size\_t n, const std::string &separator, const std::string &start="[", const std::string &end="]", std::ostream &os=std::cout)
- template<typename Derived > void qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=chop, std::ostream &os=std::cout)
- template<typename Derived >
   void qpp::displn (const Eigen::MatrixBase< Derived > &A, double chop=chop, std::ostream &os=std::cout)
- void qpp::disp (const cplx c, double chop=chop, std::ostream &os=std::cout)
- void qpp::displn (const cplx c, double chop=chop, std::ostream &os=std::cout)
- template<typename Derived >
   void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)
- template<typename Derived >
   DynMat< typename Derived::Scalar > qpp::load (const std::string &fname)

#### 8.16 include/matlab.h File Reference

```
#include "mat.h"
#include "mex.h"
Include dependency graph for matlab.h:
```



#### **Namespaces**

• qpp

- template < typename Derived >
   Derived qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)
- template<>
   dmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)
- template<>
   cmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)
- template<typename Derived >
   void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)
- template<>
   void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std
   ::string &var\_name, const std::string &mode)

template<>
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std
 ::string &var\_name, const std::string &mode)

# 8.17 include/qpp.h File Reference

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <exception>
#include <fstream>
#include <functional>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <numeric>
#include <ostream>
#include <random>
#include <stdexcept>
#include <string>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "constants.h"
#include "types.h"
#include "classes/exception.h"
#include "classes/singleton.h"
#include "classes/states.h"
#include "classes/randevs.h"
#include "internal.h"
#include "functions.h"
#include "classes/gates.h"
#include "classes/stat.h"
#include "entropies.h"
#include "entanglement.h"
#include "channels.h"
#include "io.h"
#include "random.h"
#include "classes/qudit.h"
#include "classes/timer.h"
Include dependency graph for qpp.h:
```



# **Namespaces**

• qpp

#### **Variables**

```
    RandomDevices & qpp::rdevs = RandomDevices::get_instance()
    qpp::RandomDevices Singleton
```

const Gates & qpp::gt = Gates::get\_instance()

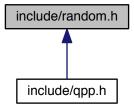
qpp::Gates const Singleton

• const States & qpp::st = States::get\_instance()

qpp::States const Singleton

# 8.18 include/random.h File Reference

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

# **Functions**

```
    template<typename Derived >
        Derived qpp::rand (std::size_t rows, std::size_t cols, double a=0, double b=1)
```

template<>

dmat qpp::rand (std::size\_t rows, std::size\_t cols, double a, double b)

template<>

cmat qpp::rand (std::size\_t rows, std::size\_t cols, double a, double b)

- double qpp::rand (double a=0, double b=1)
- long long qpp::randint (long long a, long long b)
- $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{Derived} >$

Derived <a href="mailto:qpp::randn">qpp::randn</a> (std::size\_t rows, std::size\_t cols, double mean=0, double sigma=1)

template<>

dmat qpp::randn (std::size\_t rows, std::size\_t cols, double mean, double sigma)

• template/>

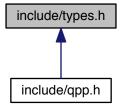
cmat qpp::randn (std::size\_t rows, std::size\_t cols, double mean, double sigma)

- double <a href="mailto:qpp::randn">qpp::randn</a> (double mean=0, double sigma=1)
- cmat qpp::randU (std::size\_t D)
- cmat qpp::randV (std::size\_t Din, std::size\_t Dout)
- std::vector < cmat > qpp::randkraus (std::size\_t n, std::size\_t D)
- cmat qpp::randH (std::size\_t D)

- ket qpp::randket (std::size\_t D)
- cmat qpp::randrho (std::size\_t D)
- std::vector< std::size\_t > qpp::randperm (std::size\_t n)

# 8.19 include/types.h File Reference

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

# **Typedefs**

- using qpp::cplx = std::complex < double >
  - Complex number in double precision.
- using qpp::cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using qpp::dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

- using qpp::ket = Eigen::Matrix< cplx, Eigen::Dynamic, 1 >
  - Complex (double precision) dynamic Eigen column matrix.
- using qpp::bra = Eigen::Matrix < cplx, 1, Eigen::Dynamic >

Complex (double precision) dynamic Eigen row matrix.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$ 

 $using \ qpp::DynMat = Eigen::Matrix < Scalar, \ Eigen::Dynamic, \ Eigen::Dynamic > \\$ 

Dynamic Eigen matrix over the field specified by Scalar.

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