

Quantum++  
v0.7.1

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# Contents

<b>1</b>	<b>Quantum++</b>	<b>1</b>
<b>2</b>	<b>Namespace Index</b>	<b>5</b>
2.1	Namespace List . . . . .	5
<b>3</b>	<b>Hierarchical Index</b>	<b>7</b>
3.1	Class Hierarchy . . . . .	7
<b>4</b>	<b>Class Index</b>	<b>9</b>
4.1	Class List . . . . .	9
<b>5</b>	<b>File Index</b>	<b>11</b>
5.1	File List . . . . .	11
<b>6</b>	<b>Namespace Documentation</b>	<b>13</b>
6.1	qpp Namespace Reference . . . . .	13
6.1.1	Detailed Description . . . . .	23
6.1.2	Typedef Documentation . . . . .	23
6.1.2.1	bra . . . . .	23
6.1.2.2	cmat . . . . .	23
6.1.2.3	cplx . . . . .	24
6.1.2.4	dmat . . . . .	24
6.1.2.5	dyn_col_vect . . . . .	24
6.1.2.6	dyn_mat . . . . .	24
6.1.2.7	dyn_row_vect . . . . .	24
6.1.2.8	idx . . . . .	24
6.1.2.9	ket . . . . .	24
6.1.3	Function Documentation . . . . .	24
6.1.3.1	absm . . . . .	24
6.1.3.2	abssq . . . . .	25
6.1.3.3	abssq . . . . .	25
6.1.3.4	adjoint . . . . .	25
6.1.3.5	anticomm . . . . .	25

6.1.3.6	apply	26
6.1.3.7	apply	26
6.1.3.8	apply	26
6.1.3.9	apply	27
6.1.3.10	apply	27
6.1.3.11	applyCTRL	27
6.1.3.12	applyCTRL	28
6.1.3.13	bloch2rho	28
6.1.3.14	choi2kraus	29
6.1.3.15	choi2super	29
6.1.3.16	comm	29
6.1.3.17	complement	29
6.1.3.18	compperm	30
6.1.3.19	concurrence	30
6.1.3.20	conjugate	30
6.1.3.21	contfrac2x	30
6.1.3.22	contfrac2x	31
6.1.3.23	cosm	31
6.1.3.24	cwise	31
6.1.3.25	det	31
6.1.3.26	dirsum	32
6.1.3.27	dirsum	32
6.1.3.28	dirsum	32
6.1.3.29	dirsum	33
6.1.3.30	dirsumpow	33
6.1.3.31	disp	33
6.1.3.32	disp	34
6.1.3.33	disp	34
6.1.3.34	disp	34
6.1.3.35	disp	35
6.1.3.36	eig	36
6.1.3.37	entanglement	36
6.1.3.38	entropy	36
6.1.3.39	entropy	37
6.1.3.40	evals	37
6.1.3.41	evects	37
6.1.3.42	expm	37
6.1.3.43	factors	38
6.1.3.44	funm	38
6.1.3.45	gcd	38

6.1.3.46	gcd	39
6.1.3.47	gconcurrency	39
6.1.3.48	grams	39
6.1.3.49	grams	39
6.1.3.50	grams	40
6.1.3.51	heig	40
6.1.3.52	hevals	40
6.1.3.53	hevects	41
6.1.3.54	inverse	41
6.1.3.55	invperm	41
6.1.3.56	isprime	41
6.1.3.57	kraus2choi	42
6.1.3.58	kraus2super	42
6.1.3.59	kron	42
6.1.3.60	kron	43
6.1.3.61	kron	43
6.1.3.62	kron	43
6.1.3.63	kronpow	44
6.1.3.64	lcm	44
6.1.3.65	lcm	44
6.1.3.66	load	44
6.1.3.67	loadMATLABmatrix	45
6.1.3.68	loadMATLABmatrix	45
6.1.3.69	loadMATLABmatrix	46
6.1.3.70	logdet	46
6.1.3.71	logm	46
6.1.3.72	lognegativity	46
6.1.3.73	measure	47
6.1.3.74	measure	47
6.1.3.75	measure	47
6.1.3.76	measure	48
6.1.3.77	measure	48
6.1.3.78	measure	49
6.1.3.79	measure	49
6.1.3.80	measure	50
6.1.3.81	measure	50
6.1.3.82	measure_seq	51
6.1.3.83	measure_seq	51
6.1.3.84	mket	51
6.1.3.85	mket	52

6.1.3.86	modpow	52
6.1.3.87	mprj	52
6.1.3.88	mprj	53
6.1.3.89	multiidx2n	53
6.1.3.90	n2multiidx	53
6.1.3.91	negativity	53
6.1.3.92	norm	54
6.1.3.93	omega	54
6.1.3.94	operator""_i	54
6.1.3.95	operator""_i	54
6.1.3.96	powm	54
6.1.3.97	prj	55
6.1.3.98	prod	55
6.1.3.99	prod	55
6.1.3.100	prod	55
6.1.3.101	ptrace	56
6.1.3.102	ptrace	56
6.1.3.103	ptrace1	56
6.1.3.104	ptrace2	58
6.1.3.105	ptranspose	58
6.1.3.106	ptranspose	58
6.1.3.107	qmutualinfo	59
6.1.3.108	qmutualinfo	59
6.1.3.109	rand	59
6.1.3.110	rand	60
6.1.3.111	rand	60
6.1.3.112	rand	60
6.1.3.113	randH	60
6.1.3.114	randidx	61
6.1.3.115	randket	61
6.1.3.116	randkraus	61
6.1.3.117	randn	61
6.1.3.118	randn	62
6.1.3.119	randn	62
6.1.3.120	randn	62
6.1.3.121	randperm	63
6.1.3.122	randrho	63
6.1.3.123	randU	63
6.1.3.124	randV	63
6.1.3.125	renyi	64

6.1.3.126	renyi	64
6.1.3.127	reshape	64
6.1.3.128	rho2bloch	65
6.1.3.129	rho2pure	65
6.1.3.130	save	65
6.1.3.131	saveMATLABmatrix	66
6.1.3.132	saveMATLABmatrix	66
6.1.3.133	saveMATLABmatrix	66
6.1.3.134	schatten	66
6.1.3.135	schmidtA	67
6.1.3.136	schmidtB	67
6.1.3.137	schmidtcoeffs	67
6.1.3.138	schmidtprobs	68
6.1.3.139	sinm	68
6.1.3.140	spectralpowm	68
6.1.3.141	sqrtn	69
6.1.3.142	sum	69
6.1.3.143	sum	69
6.1.3.144	sum	69
6.1.3.145	super2choi	70
6.1.3.146	svals	70
6.1.3.147	svd	70
6.1.3.148	svdU	70
6.1.3.149	svdV	71
6.1.3.150	syspermute	71
6.1.3.151	syspermute	71
6.1.3.152	trace	71
6.1.3.153	transpose	72
6.1.3.154	tsallis	72
6.1.3.155	tsallis	72
6.1.3.156	x2contfrac	73
6.1.4	Variable Documentation	73
6.1.4.1	chop	73
6.1.4.2	ee	73
6.1.4.3	eps	73
6.1.4.4	infty	73
6.1.4.5	maxn	73
6.1.4.6	pi	73
6.2	qpp::experimental Namespace Reference	73
6.2.1	Detailed Description	74

6.3	qpp::internal Namespace Reference	74
6.3.1	Detailed Description	75
6.3.2	Function Documentation	75
6.3.2.1	_check_cvector	75
6.3.2.2	_check_dims	75
6.3.2.3	_check_dims_match_cvect	75
6.3.2.4	_check_dims_match_mat	75
6.3.2.5	_check_dims_match_rvect	75
6.3.2.6	_check_eq_dims	75
6.3.2.7	_check_nonzero_size	75
6.3.2.8	_check_perm	75
6.3.2.9	_check_qubit_cvector	75
6.3.2.10	_check_qubit_matrix	75
6.3.2.11	_check_qubit_rvector	75
6.3.2.12	_check_qubit_vector	75
6.3.2.13	_check_rvector	75
6.3.2.14	_check_square_mat	75
6.3.2.15	_check_subsys_match_dims	75
6.3.2.16	_check_vector	75
6.3.2.17	_dirsum2	76
6.3.2.18	_kron2	76
6.3.2.19	_multiidx2n	76
6.3.2.20	_n2multiidx	76
6.3.2.21	variadic_vector_emplace	76
6.3.2.22	variadic_vector_emplace	76
<b>7</b>	<b>Class Documentation</b>	<b>77</b>
7.1	qpp::Codes Class Reference	77
7.1.1	Detailed Description	78
7.1.2	Member Enumeration Documentation	78
7.1.2.1	Type	78
7.1.3	Member Function Documentation	78
7.1.3.1	codeword	78
7.1.4	Friends And Related Function Documentation	79
7.1.4.1	internal::Singleton< const Codes >	79
7.2	qpp::Exception Class Reference	79
7.2.1	Detailed Description	80
7.2.2	Member Enumeration Documentation	80
7.2.2.1	Type	80
7.2.3	Constructor & Destructor Documentation	81



7.2.3.1	Exception	81
7.2.3.2	Exception	81
7.2.4	Member Function Documentation	82
7.2.4.1	what	82
7.3	qpp::Gates Class Reference	82
7.3.1	Detailed Description	84
7.3.2	Member Function Documentation	84
7.3.2.1	CTRL	84
7.3.2.2	expandout	85
7.3.2.3	Fd	85
7.3.2.4	Id	85
7.3.2.5	Rn	86
7.3.2.6	Xd	86
7.3.2.7	Zd	86
7.3.3	Friends And Related Function Documentation	87
7.3.3.1	internal::Singleton< const Gates >	87
7.3.4	Member Data Documentation	87
7.3.4.1	CNOT	87
7.3.4.2	CNOTba	87
7.3.4.3	CZ	87
7.3.4.4	FRED	87
7.3.4.5	H	87
7.3.4.6	Id2	87
7.3.4.7	S	87
7.3.4.8	SWAP	87
7.3.4.9	T	87
7.3.4.10	TOF	87
7.3.4.11	X	87
7.3.4.12	Y	88
7.3.4.13	Z	88
7.4	qpp::Init Class Reference	88
7.4.1	Detailed Description	89
7.4.2	Friends And Related Function Documentation	89
7.4.2.1	internal::Singleton< const Init >	89
7.5	qpp::internal::IOManipEigen Class Reference	89
7.5.1	Constructor & Destructor Documentation	89
7.5.1.1	IOManipEigen	89
7.5.1.2	IOManipEigen	89
7.5.2	Friends And Related Function Documentation	89
7.5.2.1	operator<<	89

7.6	<a href="#">qpp::internal::IOManipPointer&lt; PointerType &gt; Class Template Reference</a>	89
7.6.1	Constructor & Destructor Documentation	90
7.6.1.1	IOManipPointer	90
7.6.1.2	IOManipPointer	90
7.6.2	Member Function Documentation	90
7.6.2.1	operator=	90
7.6.3	Friends And Related Function Documentation	90
7.6.3.1	operator<<	90
7.7	<a href="#">qpp::internal::IOManipRange&lt; InputIterator &gt; Class Template Reference</a>	90
7.7.1	Constructor & Destructor Documentation	91
7.7.1.1	IOManipRange	91
7.7.2	Friends And Related Function Documentation	91
7.7.2.1	operator<<	91
7.8	<a href="#">qpp::RandomDevices Class Reference</a>	91
7.8.1	Detailed Description	92
7.8.2	Friends And Related Function Documentation	92
7.8.2.1	internal::Singleton< RandomDevices >	92
7.8.3	Member Data Documentation	92
7.8.3.1	_rng	92
7.9	<a href="#">qpp::internal::Singleton&lt; T &gt; Class Template Reference</a>	92
7.9.1	Detailed Description	93
7.9.2	Constructor & Destructor Documentation	93
7.9.2.1	Singleton	93
7.9.2.2	Singleton	93
7.9.2.3	~Singleton	93
7.9.3	Member Function Documentation	93
7.9.3.1	get_instance	93
7.9.3.2	get_thread_local_instance	94
7.9.3.3	operator=	94
7.10	<a href="#">qpp::States Class Reference</a>	94
7.10.1	Detailed Description	96
7.10.2	Friends And Related Function Documentation	96
7.10.2.1	internal::Singleton< const States >	96
7.10.3	Member Data Documentation	96
7.10.3.1	b00	96
7.10.3.2	b01	96
7.10.3.3	b10	96
7.10.3.4	b11	96
7.10.3.5	GHZ	96
7.10.3.6	pb00	96

7.10.3.7	pb01	96
7.10.3.8	pb10	96
7.10.3.9	pb11	96
7.10.3.10	pGHZ	97
7.10.3.11	pW	97
7.10.3.12	px0	97
7.10.3.13	px1	97
7.10.3.14	py0	97
7.10.3.15	py1	97
7.10.3.16	pz0	97
7.10.3.17	pz1	97
7.10.3.18	W	97
7.10.3.19	x0	97
7.10.3.20	x1	97
7.10.3.21	y0	97
7.10.3.22	y1	98
7.10.3.23	z0	98
7.10.3.24	z1	98
7.11	qpp::Timer Class Reference	98
7.11.1	Detailed Description	98
7.11.2	Constructor & Destructor Documentation	99
7.11.2.1	Timer	99
7.11.3	Member Function Documentation	99
7.11.3.1	seconds	99
7.11.3.2	tic	99
7.11.3.3	toc	99
7.11.4	Friends And Related Function Documentation	99
7.11.4.1	operator<<	99
7.11.5	Member Data Documentation	99
7.11.5.1	_end	99
7.11.5.2	_start	99
<b>8</b>	<b>File Documentation</b>	<b>101</b>
8.1	classes/codes.h File Reference	101
8.1.1	Detailed Description	101
8.2	classes/exception.h File Reference	101
8.2.1	Detailed Description	102
8.3	classes/gates.h File Reference	102
8.3.1	Detailed Description	103
8.4	classes/init.h File Reference	103

8.4.1 Detailed Description . . . . .	103
8.5 classes/random_devices.h File Reference . . . . .	104
8.5.1 Detailed Description . . . . .	104
8.6 classes/states.h File Reference . . . . .	104
8.6.1 Detailed Description . . . . .	105
8.7 classes/timer.h File Reference . . . . .	105
8.7.1 Detailed Description . . . . .	105
8.8 constants.h File Reference . . . . .	106
8.8.1 Detailed Description . . . . .	106
8.9 entanglement.h File Reference . . . . .	107
8.9.1 Detailed Description . . . . .	108
8.10 entropies.h File Reference . . . . .	108
8.10.1 Detailed Description . . . . .	109
8.11 experimental/test.h File Reference . . . . .	109
8.11.1 Detailed Description . . . . .	109
8.12 functions.h File Reference . . . . .	109
8.12.1 Detailed Description . . . . .	113
8.13 input_output.h File Reference . . . . .	113
8.13.1 Detailed Description . . . . .	114
8.14 instruments.h File Reference . . . . .	115
8.14.1 Detailed Description . . . . .	116
8.15 internal/classes/iomanip.h File Reference . . . . .	116
8.15.1 Detailed Description . . . . .	117
8.16 internal/classes/singleton.h File Reference . . . . .	117
8.16.1 Detailed Description . . . . .	117
8.17 internal/util.h File Reference . . . . .	118
8.17.1 Detailed Description . . . . .	119
8.18 MATLAB/matlab.h File Reference . . . . .	119
8.18.1 Detailed Description . . . . .	120
8.19 number_theory.h File Reference . . . . .	120
8.19.1 Detailed Description . . . . .	121
8.20 operations.h File Reference . . . . .	121
8.20.1 Detailed Description . . . . .	123
8.21 qpp.h File Reference . . . . .	123
8.21.1 Detailed Description . . . . .	125
8.22 random.h File Reference . . . . .	125
8.22.1 Detailed Description . . . . .	126
8.23 types.h File Reference . . . . .	126
8.23.1 Detailed Description . . . . .	127





# Chapter 1

## Quantum++

### Version 0.7.1 - development

Development branch, use it at your own risk!

Switch to the master branch for the latest stable version.

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. It uses the [Eigen 3](http://eigen.tuxfamily.org/dox/) linear algebra library and, if available, the [OpenMP](http://openmp.org/) multi-processing library. For additional [Eigen 3](http://eigen.tuxfamily.org/dox/) documentation see <http://eigen.tuxfamily.org/dox/>. For a simple [Eigen 3](http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt) quick ASCII reference see <http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt>.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance.

If you are interesting in contributing, please contact me. To contribute, you need to have a decent knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and some working experience with [Eigen 3](http://eigen.tuxfamily.org/dox/).

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### Building instructions

#### Configuration:

- Compiler: [g++](http://gcc.gnu.org/) version 4.8 or later (for good C++11 support)
- [Eigen 3](http://eigen.tuxfamily.org/dox/) library located in `$HOME/eigen`
- Quantum++ library located in `$HOME/qpp`
- [MATLAB](http://www.mathworks.com/) compiler include header files: `/Applications/MATLAB_R2014b.app/extern/include`
- [MATLAB](http://www.mathworks.com/) compiler shared library files: `/Applications/MATLAB_R2014b.app/bin/maci64`

### Building without a build system

- Example file: `$HOME/qpp/examples/example.cpp`
- Output executable: `$HOME/qpp/examples/example`
- Must run the commands below from inside the directory `$HOME/qpp/examples`

#### Release version (without **MATLAB** support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -O3 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Debug version (without **MATLAB** support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Release version (with **MATLAB** support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -O3 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2014b.app/extern/include \
    -L/Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

#### Debug version (with **MATLAB** support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2014b.app/extern/include \
    -L /Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

### Building using **cmake**

The current version of the repository has a `./CMakeLists.txt` configuration file for building examples using **cmake**. To build an example using **cmake**, I recommend an out-of-source build, i.e., from the root of the project (where `./include` is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the release version (default) executable `qpp`, from the source file `./examples/example.cpp`, without **MATLAB** support (default), inside the directory `./build`. To build a different configuration, e.g. debug version with **MATLAB** support, type from the root of the project

```
cd ./build
rm -rf *
cmake -DCMAKE_BUILD_TYPE=Debug -DWITH_MATLAB=ON ..
make
```

Or, to disable **OpenMP** support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```



To change the name of the example file, the location of the **Eigen 3** library or the location of **MATLAB** installation, edit the `./CMakeLists.txt` file. See also `./CMakeLists.txt` for additional options. Do not forget to remove everything from the `./build` directory before a fresh build!

#### Additional remarks

- The C++ compiler must be C++11 compliant.
- If using **Windows**, I recommend compiling under **cygwin** via **cmake** and **g++**. See also <http://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2> for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file `cmath` using the patch `./cmath_cygwin.patch`.
- If your compiler does not support **OpenMP** (as it is the case e.g with **clang++**), disable **OpenMP** in your build, as otherwise the linker may not find the **gomp** library.
- If you run the program on **OS X** with **MATLAB** support, make sure that the environment variable `DYLD_LIBRARY_PATH` is set to point to the **MATLAB** compiler library location, see the `run_OSX_MATLAB` script. Otherwise, you will get a runtime error like `dyld: Library not loaded: @rpath/libmat.dylib`.

```
* I recommend running via a script, as otherwise setting the
'DYLD_LIBRARY_PATH' globally may interfere with
[macports](https://www.macports.org/) [cmake](http://www.cmake.org/)
installation (in case you use [cmake](http://www.cmake.org/) from
[macports](https://www.macports.org/)). If you use a script,
then the environment variable is local to the script and
does not interfere with the rest of the system.
```

```
* Example of running script, run from inside the directory where
the executable 'qpp' is located:
```

```
#!/bin/sh # Run Quantum++ under OS X with MATLAB support
```

```
export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:"/Applications/MATLAB_R2014b.app/bin/maci64"
./qpp
```

- If you build a debug version with **g++** under **OS X** and use **gdb** to step inside template functions you may want to add `-fno-weak` compiler flag. See <http://stackoverflow.com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks> for more details about this problem.



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

<a href="#">qpp</a>	Quantum++ main namespace . . . . .	13
<a href="#">qpp::experimental</a>	Experimental/test functions/classes, do not use or modify . . . . .	73
<a href="#">qpp::internal</a>	Internal utility functions, do not use/modify . . . . .	74



## Chapter 3

# Hierarchical Index

### 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

std::exception	
qpp::Exception . . . . .	79
qpp::internal::IOManipEigen . . . . .	89
qpp::internal::IOManipPointer< PointerType > . . . . .	89
qpp::internal::IOManipRange< InputIterator > . . . . .	90
qpp::internal::Singleton< T > . . . . .	92
qpp::internal::Singleton< const Codes > . . . . .	92
qpp::Codes . . . . .	77
qpp::internal::Singleton< const Gates > . . . . .	92
qpp::Gates . . . . .	82
qpp::internal::Singleton< const Init > . . . . .	92
qpp::Init . . . . .	88
qpp::internal::Singleton< const States > . . . . .	92
qpp::States . . . . .	94
qpp::internal::Singleton< RandomDevices > . . . . .	92
qpp::RandomDevices . . . . .	91
qpp::Timer . . . . .	98



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">qpp::Codes</a>	Const Singleton class that defines quantum error correcting codes . . . . .	77
<a href="#">qpp::Exception</a>	Generates custom exceptions, used when validating function parameters . . . . .	79
<a href="#">qpp::Gates</a>	Const Singleton class that implements most commonly used gates . . . . .	82
<a href="#">qpp::Init</a>	Const Singleton class that performs additional initializations/cleanups . . . . .	88
<a href="#">qpp::internal::IOManipEigen</a>	. . . . .	89
<a href="#">qpp::internal::IOManipPointer&lt; PointerType &gt;</a>	. . . . .	89
<a href="#">qpp::internal::IOManipRange&lt; InputIterator &gt;</a>	. . . . .	90
<a href="#">qpp::RandomDevices</a>	Singleton class that manages the source of randomness in the library . . . . .	91
<a href="#">qpp::internal::Singleton&lt; T &gt;</a>	<a href="#">Singleton</a> policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern) . . . . .	92
<a href="#">qpp::States</a>	Const Singleton class that implements most commonly used states . . . . .	94
<a href="#">qpp::Timer</a>	Measures time . . . . .	98





## Chapter 5

# File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

<a href="#">constants.h</a>	
Constants . . . . .	106
<a href="#">entanglement.h</a>	
Entanglement functions . . . . .	107
<a href="#">entropies.h</a>	
Entropy functions . . . . .	108
<a href="#">functions.h</a>	
Generic quantum computing functions . . . . .	109
<a href="#">input_output.h</a>	
Input/output functions . . . . .	113
<a href="#">instruments.h</a>	
Measurement functions . . . . .	115
<a href="#">number_theory.h</a>	
Number theory functions . . . . .	120
<a href="#">operations.h</a>	
Quantum operation functions . . . . .	121
<a href="#">qpp.h</a>	
Quantum++ main header file, includes all other necessary headers . . . . .	123
<a href="#">random.h</a>	
Randomness-related functions . . . . .	125
<a href="#">types.h</a>	
Type aliases . . . . .	126
classes/ <a href="#">codes.h</a>	
Quantum error correcting codes . . . . .	101
classes/ <a href="#">exception.h</a>	
Exceptions . . . . .	101
classes/ <a href="#">gates.h</a>	
Quantum gates . . . . .	102
classes/ <a href="#">init.h</a>	
Initialization . . . . .	103
classes/ <a href="#">random_devices.h</a>	
Random devices . . . . .	104
classes/ <a href="#">states.h</a>	
Quantum states . . . . .	104
classes/ <a href="#">timer.h</a>	
Timing . . . . .	105
experimental/ <a href="#">test.h</a>	
Experimental/test functions/classes . . . . .	109

internal/ <a href="#">util.h</a>	
Internal utility functions . . . . .	118
internal/classes/ <a href="#">iomanip.h</a>	
Input/output manipulators . . . . .	116
internal/classes/ <a href="#">singleton.h</a>	
Singleton pattern via CRTP . . . . .	117
MATLAB/ <a href="#">matlab.h</a>	
Input/output interfacing with MATLAB . . . . .	119

## Chapter 6

# Namespace Documentation

### 6.1 qpp Namespace Reference

Quantum++ main namespace.

#### Namespaces

- [experimental](#)  
*Experimental/test functions/classes, do not use or modify.*
- [internal](#)  
*Internal utility functions, do not use/modify.*

#### Classes

- class [Codes](#)  
*const Singleton class that defines quantum error correcting codes*
- class [Exception](#)  
*Generates custom exceptions, used when validating function parameters.*
- class [Gates](#)  
*const Singleton class that implements most commonly used gates*
- class [Init](#)  
*const Singleton class that performs additional initializations/cleanups*
- class [RandomDevices](#)  
*Singleton class that manages the source of randomness in the library.*
- class [States](#)  
*const Singleton class that implements most commonly used states*
- class [Timer](#)  
*Measures time.*

#### Typedefs

- using [idx](#) = std::size\_t  
*Non-negative integer index.*
- using [cplx](#) = std::complex< double >  
*Complex number in double precision.*
- using [ket](#) = Eigen::VectorXcd

- *Complex (double precision) dynamic Eigen column vector.*
- using `bra` = Eigen::RowVectorXcd
- *Complex (double precision) dynamic Eigen row vector.*
- using `cmat` = Eigen::MatrixXcd
- *Complex (double precision) dynamic Eigen matrix.*
- using `dmat` = Eigen::MatrixXd
- *Real (double precision) dynamic Eigen matrix.*
- template<typename Scalar >  
using `dyn_mat` = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >  
*Dynamic Eigen matrix over the field specified by Scalar.*
- template<typename Scalar >  
using `dyn_col_vect` = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >  
*Dynamic Eigen column vector over the field specified by Scalar.*
- template<typename Scalar >  
using `dyn_row_vect` = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >  
*Dynamic Eigen row vector over the field specified by Scalar.*

## Functions

- constexpr `cplx operator""_i` (unsigned long long int x) noexcept  
*User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)*
- constexpr `cplx operator""_i` (long double x) noexcept  
*User-defined literal for complex  $i = \sqrt{-1}$  (real overload)*
- `cplx omega` (idx D)  
*D-th root of unity.*
- template<typename Derived >  
`dyn_col_vect`< double > `schmidtcoeffs` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt coefficients of the bi-partite pure state A.*
- template<typename Derived >  
`cmat schmidtA` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt basis on Alice side.*
- template<typename Derived >  
`cmat schmidtB` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt basis on Bob side.*
- template<typename Derived >  
std::vector< double > `schmidtprobs` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Schmidt probabilities of the bi-partite pure state A.*
- template<typename Derived >  
double `entanglement` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Entanglement of the bi-partite pure state A.*
- template<typename Derived >  
double `gconcurrence` (const Eigen::MatrixBase< Derived > &A)  
*G-concurrence of the bi-partite pure state A.*
- template<typename Derived >  
double `negativity` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Negativity of the bi-partite mixed state A.*
- template<typename Derived >  
double `lognegativity` (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)  
*Logarithmic negativity of the bi-partite mixed state A.*

- `template<typename Derived >`  
`double concurrence (const Eigen::MatrixBase< Derived > &A)`  
*Wootters concurrence of the bi-partite qubit mixed state A.*
- `template<typename Derived >`  
`double entropy (const Eigen::MatrixBase< Derived > &A)`  
*von-Neumann entropy of the density matrix A*
- `double entropy (const std::vector< double > &prob)`  
*Shannon entropy of the probability distribution prob.*
- `template<typename Derived >`  
`double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)`  
*Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .*
- `double renyi (const std::vector< double > &prob, double alpha)`  
*Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .*
- `template<typename Derived >`  
`double tsallis (const Eigen::MatrixBase< Derived > &A, double q)`  
*Tsallis-  $q$  entropy of the density matrix A, for  $q \geq 0$ .*
- `double tsallis (const std::vector< double > &prob, double q)`  
*Tsallis-  $q$  entropy of the probability distribution prob, for  $q \geq 0$ .*
- `template<typename Derived >`  
`double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsA, const std::vector< idx > &subsB, const std::vector< idx > &dims)`  
*Quantum mutual information between 2 subsystems of a composite system.*
- `template<typename Derived >`  
`double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsA, const std::vector< idx > &subsB, idx d=2)`  
*Quantum mutual information between 2 subsystems of a composite system.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)`  
*Transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)`  
*Complex conjugate.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)`  
*Adjoint.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)`  
*Inverse.*
- `template<typename Derived >`  
`Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)`  
*Trace.*
- `template<typename Derived >`  
`Derived::Scalar det (const Eigen::MatrixBase< Derived > &A)`  
*Determinant.*
- `template<typename Derived >`  
`Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)`  
*Logarithm of the determinant.*
- `template<typename Derived >`  
`Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise sum of A.*
- `template<typename Derived >`  
`Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise product of A.*

- `template<typename Derived >`  
`double norm (const Eigen::MatrixBase< Derived > &A)`  
*Frobenius norm.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition.*
- `template<typename Derived >`  
`dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvalues.*
- `template<typename Derived >`  
`cmat evecs (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvectors.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition of Hermitian expression.*
- `template<typename Derived >`  
`dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)`  
*Hermitian eigenvalues.*
- `template<typename Derived >`  
`cmat hevecs (const Eigen::MatrixBase< Derived > &A)`  
*Hermitian eigenvectors.*
- `template<typename Derived >`  
`std::tuple< cmat, dyn_col_vect< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)`  
*Full singular value decomposition.*
- `template<typename Derived >`  
`dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)`  
*Singular values.*
- `template<typename Derived >`  
`cmat svdU (const Eigen::MatrixBase< Derived > &A)`  
*Left singular vectors.*
- `template<typename Derived >`  
`cmat svdV (const Eigen::MatrixBase< Derived > &A)`  
*Right singular vectors.*
- `template<typename Derived >`  
`cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))`  
*Functional calculus  $f(A)$*
- `template<typename Derived >`  
`cmat sqrtm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix square root.*
- `template<typename Derived >`  
`cmat absm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix absolut value.*
- `template<typename Derived >`  
`cmat expm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix exponential.*
- `template<typename Derived >`  
`cmat logm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix logarithm.*
- `template<typename Derived >`  
`cmat sinm (const Eigen::MatrixBase< Derived > &A)`  
*Matrix sin.*
- `template<typename Derived >`  
`cmat cosm (const Eigen::MatrixBase< Derived > &A)`

*Matrix cos.*

- template<typename Derived >  
    `cmat spectralpowm` (const Eigen::MatrixBase< Derived > &A, const `cplx` z)

*Matrix power.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `powm` (const Eigen::MatrixBase< Derived > &A, `idx` n)

*Matrix power.*

- template<typename Derived >  
    double `schatten` (const Eigen::MatrixBase< Derived > &A, double p)

*Schatten matrix norm.*

- template<typename OutputScalar , typename Derived >  
    `dyn_mat`< OutputScalar > `cwise` (const Eigen::MatrixBase< Derived > &A, OutputScalar(\*)(const type-name Derived::Scalar &))

*Functor.*

- template<typename T >  
    `dyn_mat`< typename T::Scalar > `kron` (const T &head)

*Kronecker product.*

- template<typename T , typename... Args>  
    `dyn_mat`< typename T::Scalar > `kron` (const T &head, const Args &...tail)

*Kronecker product.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `kron` (const std::vector< Derived > &As)

*Kronecker product.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `kron` (const std::initializer\_list< Derived > &As)

*Kronecker product.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `kronpow` (const Eigen::MatrixBase< Derived > &A, `idx` n)

*Kronecker power.*

- template<typename T >  
    `dyn_mat`< typename T::Scalar > `dirsum` (const T &head)

*Direct sum.*

- template<typename T , typename... Args>  
    `dyn_mat`< typename T::Scalar > `dirsum` (const T &head, const Args &...tail)

*Direct sum.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `dirsum` (const std::vector< Derived > &As)

*Direct sum.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `dirsum` (const std::initializer\_list< Derived > &As)

*Direct sum.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `dirsumpow` (const Eigen::MatrixBase< Derived > &A, `idx` n)

*Direct sum power.*

- template<typename Derived >  
    `dyn_mat`< typename Derived::Scalar > `reshape` (const Eigen::MatrixBase< Derived > &A, `idx` rows, `idx` cols)

*Reshape.*

- template<typename Derived1 , typename Derived2 >  
    `dyn_mat`< typename Derived1::Scalar > `comm` (const Eigen::MatrixBase< Derived1 > &A, const Eigen::↔  
    MatrixBase< Derived2 > &B)

*Commutator.*

- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > anticomm` (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)  
*Anti-commutator.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > prj` (const Eigen::MatrixBase< Derived > &V)  
*Projector.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams` (const std::vector< Derived > &Vs)  
*Gram-Schmidt orthogonalization.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams` (const std::initializer\_list< Derived > &Vs)  
*Gram-Schmidt orthogonalization.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > grams` (const Eigen::MatrixBase< Derived > &A)  
*Gram-Schmidt orthogonalization.*
- `std::vector< idx > n2multiidx` (idx n, const std::vector< idx > &dims)  
*Non-negative integer index to multi-index.*
- `idx multiidx2n` (const std::vector< idx > &midx, const std::vector< idx > &dims)  
*Multi-index to non-negative integer index.*
- `ket mket` (const std::vector< idx > &mask, const std::vector< idx > &dims)  
*Multi-partite qudit ket.*
- `ket mket` (const std::vector< idx > &mask, idx d=2)  
*Multi-partite qudit ket.*
- `cmat mprj` (const std::vector< idx > &mask, const std::vector< idx > &dims)  
*Projector onto multi-partite qudit ket.*
- `cmat mprj` (const std::vector< idx > &mask, idx d=2)  
*Projector onto multi-partite qudit ket.*
- `template<typename InputIterator >`  
`std::vector< double > abssq` (InputIterator first, InputIterator last)  
*Computes the absolute values squared of a range of complex numbers.*
- `template<typename Derived >`  
`std::vector< double > abssq` (const Eigen::MatrixBase< Derived > &V)  
*Computes the absolute values squared of a column vector.*
- `template<typename InputIterator >`  
`std::iterator_traits< InputIterator >::value_type sum` (InputIterator first, InputIterator last)  
*Element-wise sum of an STL-like range.*
- `template<typename Container >`  
`Container::value_type sum` (const Container &c)  
*Element-wise sum of the elements of an STL-like container.*
- `template<typename InputIterator >`  
`std::iterator_traits< InputIterator >::value_type prod` (InputIterator first, InputIterator last)  
*Element-wise product of an STL-like range.*
- `template<typename Container >`  
`Container::value_type prod` (const Container &c)  
*Element-wise product of the elements of an STL-like container.*
- `template<typename Derived >`  
`dyn_col_vect< typename Derived::Scalar > rho2pure` (const Eigen::MatrixBase< Derived > &A)  
*Finds the pure state representation of a matrix proportional to a projector onto a pure state.*
- `template<typename T >`  
`std::vector< T > complement` (std::vector< T > subsys, idx N)  
*Constructs the complement of a subsystem vector.*



- `template<typename Derived >`  
`std::vector< double > rho2bloch (const Eigen::MatrixBase< Derived > &A)`  
*Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.*
- `cmat bloch2rho (const std::vector< double > &r)`  
*Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.*
- `template<typename Derived >`  
`internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)`  
*Eigen expression ostream manipulator.*
- `internal::IOManipEigen disp (cplx z, double chop=qpp::chop)`  
*Complex number ostream manipulator.*
- `template<typename InputIterator >`  
`internal::IOManipRange< InputIterator > disp (const InputIterator &first, const InputIterator &last, const std::string &separator, const std::string &start="[" , const std::string &end="]")`  
*Range ostream manipulator.*
- `template<typename Container >`  
`internal::IOManipRange< typename Container::const_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[" , const std::string &end="]")`  
*Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.*
- `template<typename PointerType >`  
`internal::IOManipPointer< PointerType > disp (const PointerType *p, idx n, const std::string &separator, const std::string &start="[" , const std::string &end="]")`  
*C-style pointer ostream manipulator.*
- `template<typename Derived >`  
`void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)`  
*Saves Eigen expression to a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > load (const std::string &fname)`  
*Loads Eigen matrix from a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)`  
*Measures the state A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure` (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure` (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)  
*Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > measure` (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const idx d=2)  
*Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< std::vector< idx >, double, cmat > measure_seq` (const Eigen::MatrixBase< Derived > &A, std::vector< idx > subsys, std::vector< idx > dims)  
*Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.*
- `template<typename Derived >`  
`std::tuple< std::vector< idx >, double, cmat > measure_seq` (const Eigen::MatrixBase< Derived > &A, std::vector< idx > subsys, idx d=2)  
*Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.*
- `template<typename Derived >`  
`Derived loadMATLABmatrix` (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.*
- `template<>`  
`dmat loadMATLABmatrix` (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat loadMATLABmatrix` (const std::string &mat\_file, const std::string &var\_name)  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- `template<typename Derived >`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.*
- `template<>`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`void saveMATLABmatrix` (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- `std::vector< int > x2confrac` (double x, idx n, idx cut=1e5)  
*Simple continued fraction expansion.*
- `double confrac2x` (const std::vector< int > &cf, idx n)  
*Real representation of a simple continued fraction.*
- `double confrac2x` (const std::vector< int > &cf)  
*Real representation of a simple continued fraction.*
- `unsigned long long int gcd` (unsigned long long int m, unsigned long long int n)  
*Greatest common divisor of two non-negative integers.*
- `unsigned long long int gcd` (const std::vector< unsigned long long int > &ns)  
*Greatest common divisor of a list of non-negative integers.*
- `unsigned long long int lcm` (unsigned long long int m, unsigned long long int n)

- Least common multiple of two positive integers.*

  - unsigned long long int **lcm** (const std::vector< unsigned long long int > &ns)
- Least common multiple of a list of positive integers.*

  - std::vector< **idx** > **invperm** (const std::vector< **idx** > &perm)
- Inverse permutation.*

  - std::vector< **idx** > **compperm** (const std::vector< **idx** > &perm, const std::vector< **idx** > &sigma)
- Compose permutations.*

  - std::vector< unsigned long long int > **factors** (unsigned long long int n)
- Prime factor decomposition.*

  - bool **isprime** (unsigned long long int n)
- Primality test.*

  - unsigned long long int **modpow** (unsigned long long int a, unsigned long long int n, unsigned long long int p)
- Integer power modulo p.*

  - template<typename Derived1 , typename Derived2 >  
**dyn\_mat**< typename Derived1::Scalar > **applyCTRL** (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< **idx** > &ctrl, const std::vector< **idx** > &subsys, const std::vector< **idx** > &dims)
- Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*

  - template<typename Derived1 , typename Derived2 >  
**dyn\_mat**< typename Derived1::Scalar > **applyCTRL** (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< **idx** > &ctrl, const std::vector< **idx** > &subsys, **idx** d=2)
- Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*

  - template<typename Derived1 , typename Derived2 >  
**dyn\_mat**< typename Derived1::Scalar > **apply** (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< **idx** > &subsys, const std::vector< **idx** > &dims)
- Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*

  - template<typename Derived1 , typename Derived2 >  
**dyn\_mat**< typename Derived1::Scalar > **apply** (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< **idx** > &subsys, **idx** d=2)
- Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*

  - template<typename Derived >  
**cmat apply** (const Eigen::MatrixBase< Derived > &rho, const std::vector< **cmat** > &Ks)
- Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.*

  - template<typename Derived >  
**cmat apply** (const Eigen::MatrixBase< Derived > &rho, const std::vector< **cmat** > &Ks, const std::vector< **idx** > &subsys, const std::vector< **idx** > &dims)
- Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*

  - template<typename Derived >  
**cmat apply** (const Eigen::MatrixBase< Derived > &rho, const std::vector< **cmat** > &Ks, const std::vector< **idx** > &subsys, **idx** d=2)
- Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*

  - **cmat kraus2super** (const std::vector< **cmat** > &Ks)
- Superoperator matrix.*

  - **cmat kraus2choi** (const std::vector< **cmat** > &Ks)
- Choi matrix.*

  - std::vector< **cmat** > **choi2kraus** (const **cmat** &A)
- Orthogonal Kraus operators from Choi matrix.*

  - **cmat choi2super** (const **cmat** &A)
- Converts Choi matrix to superoperator matrix.*

  - **cmat super2choi** (const **cmat** &A)

*Converts superoperator matrix to Choi matrix.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)`

*Partial trace.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)`

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, const std::vector< idx > &dims)`

*Subsystem permutation.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)`

*Subsystem permutation.*

- `double rand (double a=0, double b=1)`

*Generates a random real number uniformly distributed in the interval [a, b]*

- `idx randidx (idx a=std::numeric_limits< idx >::min(), idx b=std::numeric_limits< idx >::max())`

*Generates a random index (idx) uniformly distributed in the interval [a, b].*

- `template<typename Derived >`  
`Derived rand (idx rows, idx cols, double a=0, double b=1)`

*Generates a random matrix with entries uniformly distributed in the interval [a, b]*

- `template<>`  
`dmat rand (idx rows, idx cols, double a, double b)`

*Generates a random real matrix with entries uniformly distributed in the interval [a, b], specialization for double matrices ([qpp::dmat](#))*

- `template<>`  
`cmat rand (idx rows, idx cols, double a, double b)`

*Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b], specialization for complex matrices ([qpp::cmat](#))*

- `template<typename Derived >`  
`Derived randn (idx rows, idx cols, double mean=0, double sigma=1)`

*Generates a random matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$*

- `template<>`  
`dmat randn (idx rows, idx cols, double mean, double sigma)`

- Generates a random real matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for double matrices (`qpp::dmat`)*
- `template<>`  
`cmat randn` (`idx` rows, `idx` cols, double mean, double sigma)  
*Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices (`qpp::cmat`)*
- `double randn` (double mean=0, double sigma=1)  
*Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$*
- `cmat randU` (`idx` D)  
*Generates a random unitary matrix.*
- `cmat randV` (`idx` Din, `idx` Dout)  
*Generates a random isometry matrix.*
- `std::vector< cmat > randkraus` (`idx` N, `idx` D)  
*Generates a set of random Kraus operators.*
- `cmat randH` (`idx` D)  
*Generates a random Hermitian matrix.*
- `ket randket` (`idx` D)  
*Generates a random normalized ket (pure state vector)*
- `cmat randrho` (`idx` D)  
*Generates a random density matrix.*
- `std::vector< idx > randperm` (`idx` n)  
*Generates a random uniformly distributed permutation.*

## Variables

- `constexpr double chop` = 1e-10  
*Used in `qpp::disp()` for setting to zero numbers that have their absolute value smaller than `qpp::chop`.*
- `constexpr double eps` = 1e-12  
*Used to decide whether a number or expression in double precision is zero or not.*
- `constexpr idx maxn` = 64  
*Maximum number of allowed qu(d)its (subsystems)*
- `constexpr double pi` = 3.141592653589793238462643383279502884  
 $\pi$
- `constexpr double ee` = 2.718281828459045235360287471352662497  
*Base of natural logarithm,  $e$ .*
- `constexpr double infy` = `std::numeric_limits<double>::infinity()`  
*Used to denote infinity in double precision.*

### 6.1.1 Detailed Description

Quantum++ main namespace.

### 6.1.2 Typedef Documentation

#### 6.1.2.1 using `qpp::bra` = `typedef Eigen::RowVectorXcd`

Complex (double precision) dynamic Eigen row vector.

#### 6.1.2.2 using `qpp::cmat` = `typedef Eigen::MatrixXcd`

Complex (double precision) dynamic Eigen matrix.

#### 6.1.2.3 using qpp::cplx = typedef std::complex<double>

Complex number in double precision.

#### 6.1.2.4 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

#### 6.1.2.5 template<typename Scalar > using qpp::dyn\_col\_vect = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by *Scalar*.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

#### 6.1.2.6 template<typename Scalar > using qpp::dyn\_mat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by *Scalar*.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

#### 6.1.2.7 template<typename Scalar > using qpp::dyn\_row\_vect = typedef Eigen::Matrix<Scalar, 1, Eigen::Dynamic>

Dynamic Eigen row vector over the field specified by *Scalar*.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

#### 6.1.2.8 using qpp::idx = typedef std::size\_t

Non-negative integer index.

#### 6.1.2.9 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

### 6.1.3 Function Documentation

#### 6.1.3.1 template<typename Derived > cmat qpp::absm ( const Eigen::MatrixBase< Derived > & A )

Matrix absolut value.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Matrix absolut value of *A*

**6.1.3.2** `template<typename InputIterator > std::vector<double> qpp::abssq ( InputIterator first, InputIterator last )`

Computes the absolute values squared of a range of complex numbers.

## Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

## Returns

Real vector consisting of the range absolut values squared

**6.1.3.3** `template<typename Derived > std::vector<double> qpp::abssq ( const Eigen::MatrixBase< Derived > & V )`

Computes the absolute values squared of a column vector.

## Parameters

<i>V</i>	Eigen expression
----------	------------------

## Returns

Real vector consisting of the absolut values squared

**6.1.3.4** `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )`

Adjoint.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Adjoint (Hermitian conjugate) of *A*, as a dynamic matrix over the same scalar field as *A*

**6.1.3.5** `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::anticomm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`

Anti-commutator.

## See also

[qpp::comm\(\)](#)

Anti-commutator  $\{A, B\} = AB + BA$ . Both *A* and *B* must be Eigen expressions over the same scalar field.

## Parameters

$A$	Eigen expression
$B$	Eigen expression

## Returns

Anti-commutator  $AB + BA$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.6 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::apply ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Applies the gate  $A$  to the part  $subsys$  of the multi-partite state vector or density matrix  $state$ .

## Note

The dimension of the gate  $A$  must match the dimension of  $subsys$

## Parameters

$state$	Eigen expression
$A$	Eigen expression
$subsys$	Subsystem indexes where the gate $A$ is applied
$dims$	Dimensions of the multi-partite system

## Returns

Gate  $A$  applied to the part  $subsys$  of  $state$

6.1.3.7 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::apply ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & subsys, idx d = 2 )`

Applies the gate  $A$  to the part  $subsys$  of the multi-partite state vector or density matrix  $state$ .

## Note

The dimension of the gate  $A$  must match the dimension of  $subsys$

## Parameters

$state$	Eigen expression
$A$	Eigen expression
$subsys$	Subsystem indexes where the gate $A$ is applied
$d$	Subsystem dimensions

## Returns

Gate  $A$  applied to the part  $subsys$  of  $state$

6.1.3.8 `template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks )`

Applies the channel specified by the set of Kraus operators  $Ks$  to the density matrix  $rho$ .



## Parameters

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators

## Returns

Output density matrix after the action of the channel

6.1.3.9 `template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

## Parameters

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators
<i>subsys</i>	Subsystem indexes where the Kraus operators <i>Ks</i> are applied
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Output density matrix after the action of the channel

6.1.3.10 `template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, idx d = 2 )`

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

## Parameters

<i>rho</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators
<i>subsys</i>	Subsystem indexes where the Kraus operators <i>Ks</i> are applied
<i>d</i>	Subsystem dimensions

## Returns

Output density matrix after the action of the channel

6.1.3.11 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Applies the controlled-gate *A* to the part *subsys* of the multi-partite state vector or density matrix *state*.

## See also

[qpp::Gates::CTRL\(\)](#)

## Note

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

## Parameters

<i>state</i>	Eigen expression
<i>A</i>	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>dims</i>	Dimensions of the multi-partite system

## Returns

CTRL-A gate applied to the part *subsys* of *state*

```
6.1.3.12 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL (
    const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx >
    & ctrl, const std::vector< idx > & subsys, idx d = 2 )
```

Applies the controlled-gate *A* to the part *subsys* of the multi-partite state vector or density matrix *state*.

## See also

[qpp::Gates::CTRL\(\)](#)

## Note

The dimension of the gate *A* must match the dimension of *subsys*

## Parameters

<i>state</i>	Eigen expression
<i>A</i>	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>d</i>	Subsystem dimensions

## Returns

CTRL-A gate applied to the part *subsys* of *state*

```
6.1.3.13 cmat qpp::bloch2rho ( const std::vector< double > & r ) [inline]
```

Computes the density matrix corresponding to the 3-dimensional real Bloch vector *r*.

## See also

[qpp::rho2bloch\(\)](#)

## Parameters

<i>r</i>	3-dimensional real vector
----------	---------------------------

## Returns

Qubit density matrix

6.1.3.14 `std::vector<cmat> qpp::choi2kraus ( const cmat & A ) [inline]`

Orthogonal Kraus operators from Choi matrix.

See also

[qpp::kraus2choi\(\)](#)

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix  $A$

Note

The Kraus operators satisfy  $Tr(K_i^\dagger K_j) = \delta_{ij}$  for all  $i \neq j$

Parameters

$A$	Choi matrix
-----	-------------

Returns

Set of orthogonal Kraus operators

6.1.3.15 `cmat qpp::choi2super ( const cmat & A ) [inline]`

Converts Choi matrix to superoperator matrix.

See also

[qpp::super2choi\(\)](#)

Parameters

$A$	Choi matrix
-----	-------------

Returns

Superoperator matrix

6.1.3.16 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::comm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`

Commutator.

See also

[qpp::anticomm\(\)](#)

Commutator  $[A, B] = AB - BA$ . Both  $A$  and  $B$  must be Eigen expressions over the same scalar field.

Parameters

$A$	Eigen expression
$B$	Eigen expression

Returns

Commutator  $AB - BA$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.17 `template<typename T> std::vector<T> qpp::complement ( std::vector< T > subsys, idx N )`

Constructs the complement of a subsystem vector.

## Parameters

<i>subsys</i>	Subsystem vector
<i>N</i>	Total number of systems

## Returns

The complement of *subsys* with respect to the set  $\{0, 1, \dots, N - 1\}$

**6.1.3.18** `std::vector<idx> qpp::compperm ( const std::vector< idx > & perm, const std::vector< idx > & sigma )`  
`[inline]`

Compose permutations.

## Parameters

<i>perm</i>	Permutation
<i>sigma</i>	Permutation

## Returns

Composition of the permutations  $perm \circ sigma = perm(sigma)$

**6.1.3.19** `template<typename Derived> double qpp::concurrence ( const Eigen::MatrixBase< Derived > & A )`

Wootters concurrence of the bi-partite qubit mixed state *A*.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Wootters concurrence

**6.1.3.20** `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived > & A )`

Complex conjugate.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Complex conjugate of *A*, as a dynamic matrix over the same scalar field as *A*

**6.1.3.21** `double qpp::contfrac2x ( const std::vector< int > & cf, idx n )` `[inline]`

Real representation of a simple continued fraction.

## See also

[qpp::x2contfrac\(\)](#)

## Parameters

<i>cf</i>	Integer vector containing the simple continued fraction expansion
<i>n</i>	Number of terms considered in the continued fraction expansion. If <i>n</i> is greater than the size of <i>cf</i> , then all terms in <i>cf</i> are considered.

## Returns

Real representation of the simple continued fraction

6.1.3.22 `double qpp::contfrac2x ( const std::vector< int > & cf ) [inline]`

Real representation of a simple continued fraction.

## See also

[qpp::x2contfrac\(\)](#)

## Parameters

<i>cf</i>	Integer vector containing the simple continued fraction expansion
-----------	---

## Returns

Real representation of the simple continued fraction

6.1.3.23 `template<typename Derived> cmat qpp::cosm ( const Eigen::MatrixBase< Derived > & A )`

Matrix cos.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Matrix cosine of *A*

6.1.3.24 `template<typename OutputScalar, typename Derived> dyn_mat<OutputScalar> qpp::cwise ( const Eigen::MatrixBase< Derived > & A, OutputScalar*)(const typename Derived::Scalar &) f )`

Functor.

## Parameters

<i>A</i>	Eigen expression
<i>f</i>	Pointer-to-function from scalars of <i>A</i> to <i>OutputScalar</i>

## Returns

Component-wise  $f(A)$ , as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.25 `template<typename Derived> Derived::Scalar qpp::det ( const Eigen::MatrixBase< Derived > & A )`

Determinant.

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Determinant of *A*, as a scalar in the same scalar field as *A*. Returns  $\pm\infty$  when the determinant overflows/underflows.

6.1.3.26 `template<typename T> dyn_mat<typename T::Scalar> qpp::dirsum ( const T & head )`

Direct sum.

See also

[qpp::dirsumpow\(\)](#)

Used to stop the recursion for the variadic template version of [qpp::dirsum\(\)](#)

## Parameters

<i>head</i>	Eigen expression
-------------	------------------

## Returns

Its argument *head*

6.1.3.27 `template<typename T, typename... Args> dyn_mat<typename T::Scalar> qpp::dirsum ( const T & head, const Args &... tail )`

Direct sum.

See also

[qpp::dirsumpow\(\)](#)

## Parameters

<i>head</i>	Eigen expression
<i>tail</i>	Variadic Eigen expression (zero or more parameters)

## Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.28 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::dirsum ( const std::vector< Derived > & As )`

Direct sum.

See also

[qpp::dirsumpow\(\)](#)

## Parameters

<i>As</i>	std::vector of Eigen expressions
-----------	----------------------------------

## Returns

Direct sum of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.29 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::dirsum ( const std::initializer_list<Derived> & As )`

Direct sum.

See also

[qpp::dirsumpow\(\)](#)

## Parameters

<i>As</i>	std::initializer_list of Eigen expressions, such as { <i>A1</i> , <i>A2</i> , ... , <i>Ak</i> }
-----------	---

## Returns

Direct sum of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.30 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::dirsumpow ( const Eigen::MatrixBase<Derived> & A, idx n )`

Direct sum power.

See also

[qpp::dirsum\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>n</i>	Non-negative integer

## Returns

Direct sum of *A* with itself *n* times  $A^{\oplus n}$ , as a dynamic matrix over the same scalar field as *A*

6.1.3.31 `template<typename Derived> internal::IOManipEigen qpp::disp ( const Eigen::MatrixBase<Derived> & A, double chop = qpp::chop )`

Eigen expression ostream manipulator.

## Parameters

<i>A</i>	Eigen expression
<i>chop</i>	Set to zero the elements smaller in absolute value than <i>chop</i>

**Returns**

Instance of `qpp::internal::internal::IOManipEigen`

#### 6.1.3.32 `internal::IOManipEigen qpp::disp ( cplx z, double chop = qpp::chop ) [inline]`

Complex number ostream manipulator.

**Parameters**

<i>z</i>	Complex number (or any other type implicitly cast-able to <code>std::complex&lt;double&gt;</code> )
<i>chop</i>	Set to zero the elements smaller in absolute value than <i>chop</i>

**Returns**

Instance of `qpp::internal::internal::IOManipEigen`

#### 6.1.3.33 `template<typename InputIterator > internal::IOManipRange<InputIterator> qpp::disp ( const InputIterator & first, const InputIterator & last, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )`

Range ostream manipulator.

**Parameters**

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

**Returns**

Instance of `qpp::internal::internal::IOManipRange`

#### 6.1.3.34 `template<typename Container > internal::IOManipRange<typename Container::const_iterator> qpp::disp ( const Container & c, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )`

Standard container ostream manipulator. The container must support `std::begin()`, `std::end()` and forward iteration.

**Parameters**

<i>c</i>	Container
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

**Returns**

Instance of `qpp::internal::internal::IOManipRange`



```
6.1.3.35  template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp ( const PointerType * p,  
        idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )
```

C-style pointer ostream manipulator.

## Parameters

<i>p</i>	Pointer to the first element
<i>n</i>	Number of elements to be displayed
<i>separator</i>	Separator
<i>start</i>	Left marking
<i>end</i>	Right marking

## Returns

Instance of `qpp::internal::internal::LOManipPointer`

6.1.3.36 `template<typename Derived> std::pair<dyn_col_vect<cplx>, cmat> qpp::eig ( const Eigen::MatrixBase< Derived> & A )`

Full eigen decomposition.

## See also

[qpp::heig\(\)](#)

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.37 `template<typename Derived> double qpp::entanglement ( const Eigen::MatrixBase< Derived> & A, const std::vector< idx> & dims )`

Entanglement of the bi-partite pure state *A*.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

## See also

[qpp::entropy\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

## Returns

Entanglement, with the logarithm in base 2

6.1.3.38 `template<typename Derived> double qpp::entropy ( const Eigen::MatrixBase< Derived> & A )`

von-Neumann entropy of the density matrix *A*

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

von-Neumann entropy, with the logarithm in base 2

**6.1.3.39** `double qpp::entropy ( const std::vector< double > & prob ) [inline]`

Shannon entropy of the probability distribution *prob*.

## Parameters

<i>prob</i>	Real probability vector
-------------	-------------------------

## Returns

Shannon entropy, with the logarithm in base 2

**6.1.3.40** `template<typename Derived > dyn_col_vect<cplx> qpp::evals ( const Eigen::MatrixBase< Derived > & A )`

Eigenvalues.

## See also

[qpp::hevals\(\)](#)

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Eigenvalues of *A*, as a complex dynamic column vector

**6.1.3.41** `template<typename Derived > cmat qpp::evects ( const Eigen::MatrixBase< Derived > & A )`

Eigenvectors.

## See also

[qpp::hevects\(\)](#)

## Parameters

<i>A</i>	Eigen expression
----------	------------------

## Returns

Eigenvectors of *A*, as columns of a complex dynamic matrix

**6.1.3.42** `template<typename Derived > cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )`

Matrix exponential.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Matrix exponential of  $A$

6.1.3.43 `std::vector<unsigned long long int> qpp::factors ( unsigned long long int  $n$  )` `[inline]`

Prime factor decomposition.

## Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

## Parameters

$n$	Integer strictly greater than 1
-----	---------------------------------

## Returns

Integer vector containing the factors

6.1.3.44 `template<typename Derived > cmat qpp::funm ( const Eigen::MatrixBase< Derived > &  $A$ , cplx(*) (const cplx &)  $f$  )`

Functional calculus  $f(A)$

## Parameters

$A$	Eigen expression
$f$	Pointer-to-function from complex to complex

## Returns

$f(A)$

6.1.3.45 `unsigned long long int qpp::gcd ( unsigned long long int  $m$ , unsigned long long int  $n$  )` `[inline]`

Greatest common divisor of two non-negative integers.

## See also

[qpp::lcm\(\)](#)

## Parameters

$m$	Non-negative integer
$n$	Non-negative integer

## Returns

Greatest common divisor of  $m$  and  $n$

6.1.3.46 `unsigned long long int qpp::gcd ( const std::vector< unsigned long long int > & ns ) [inline]`

Greatest common divisor of a list of non-negative integers.

See also

[qpp::lcm\(\)](#)

Parameters

<i>ns</i>	List of non-negative integers
-----------	-------------------------------

Returns

Greatest common divisor of all numbers in *ns*

6.1.3.47 `template<typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase< Derived > & A )`

G-concurrence of the bi-partite pure state *A*.

Note

Both local dimensions must be equal

Uses [qpp::logdet\(\)](#) to avoid overflows

See also

[qpp::logdet\(\)](#)

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

G-concurrence

6.1.3.48 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )`

Gram-Schmidt orthogonalization.

Parameters

<i>Vs</i>	std::vector of Eigen expressions as column vectors
-----------	--

Returns

Gram-Schmidt vectors of *Vs* as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.49 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const std::initializer_list< Derived > & Vs )`

Gram-Schmidt orthogonalization.

## Parameters

$V$ s	std::initializer_list of Eigen expressions as column vectors
-------	--

## Returns

Gram-Schmidt vectors of  $V$ s as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.50 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams ( const Eigen::MatrixBase<Derived > & A )`

Gram-Schmidt orthogonalization.

## Parameters

$A$	Eigen expression, the input vectors are the columns of $A$
-----	--

## Returns

Gram-Schmidt vectors of the columns of  $A$ , as columns of a dynamic matrix over the same scalar field as  $A$

6.1.3.51 `template<typename Derived > std::pair<dyn_col_vect<double>, cmat> qpp::heig ( const Eigen::MatrixBase<Derived > & A )`

Full eigen decomposition of Hermitian expression.

## See also

[qpp::eig\(\)](#)

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Pair of: 1. Eigenvalues of  $A$ , as a real dynamic column vector, and 2. Eigenvectors of  $A$ , as columns of a complex dynamic matrix

6.1.3.52 `template<typename Derived > dyn_col_vect<double> qpp::hevals ( const Eigen::MatrixBase<Derived > & A )`

Hermitian eigenvalues.

## See also

[qpp::evals\(\)](#)

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Eigenvalues of Hermitian  $A$ , as a real dynamic column vector

6.1.3.53 `template<typename Derived > cmat qpp::hevects ( const Eigen::MatrixBase< Derived > & A )`

Hermitian eigenvectors.

See also

[qpp::evects\(\)](#)

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

Eigenvectors of Hermitian *A*, as columns of a complex matrix

6.1.3.54 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::inverse ( const Eigen::MatrixBase< Derived > & A )`

Inverse.

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

Inverse of *A*, as a dynamic matrix over the same scalar field as *A*

6.1.3.55 `std::vector<idx> qpp::invperm ( const std::vector< idx > & perm ) [inline]`

Inverse permutation.

Parameters

<i>perm</i>	Permutation
-------------	-------------

Returns

Inverse of the permutation *perm*

6.1.3.56 `bool qpp::isprime ( unsigned long long int n ) [inline]`

Primality test.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

Parameters

<i>n</i>	Integer strictly greater than 1
----------	---------------------------------

Returns

True if the number is prime, false otherwise

**6.1.3.57** `cmat qpp::kraus2choi ( const std::vector< cmat > & Ks ) [inline]`

Choi matrix.

See also

[qpp::choi2kraus\(\)](#)

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $Ks$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

**Note**

The superoperator matrix  $S$  and the Choi matrix  $C$  are related by  $S_{ab,mn} = C_{ma,nb}$

**Parameters**

$Ks$	Set of Kraus operators
------	------------------------

**Returns**

Choi matrix

**6.1.3.58** `cmat qpp::kraus2super ( const std::vector< cmat > & Ks ) [inline]`

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators  $Ks$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

**Parameters**

$Ks$	Set of Kraus operators
------	------------------------

**Returns**

Superoperator matrix

**6.1.3.59** `template<typename T> dyn_mat<typename T::Scalar> qpp::kron ( const T & head )`

Kronecker product.

See also

[qpp::kronpow\(\)](#)

Used to stop the recursion for the variadic template version of [qpp::kron\(\)](#)

**Parameters**

$head$	Eigen expression
--------	------------------

**Returns**

Its argument  $head$



6.1.3.60 `template<typename T, typename... Args> dyn_mat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )`

Kronecker product.

See also

[qpp::kronpow\(\)](#)

Parameters

<i>head</i>	Eigen expression
<i>tail</i>	Variadic Eigen expression (zero or more parameters)

Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.61 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::kron ( const std::vector< Derived > & As )`

Kronecker product.

See also

[qpp::kronpow\(\)](#)

Parameters

<i>As</i>	std::vector of Eigen expressions
-----------	----------------------------------

Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.62 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::kron ( const std::initializer_list< Derived > & As )`

Kronecker product.

See also

[qpp::kronpow\(\)](#)

Parameters

<i>As</i>	std::initializer_list of Eigen expressions, such as { <i>A1</i> , <i>A2</i> , ... , <i>Ak</i> }
-----------	---

Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.63 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::kronpow ( const Eigen::MatrixBase<Derived> & A, idx n )`

Kronecker power.

See also

[qpp::kron\(\)](#)

Parameters

$A$	Eigen expression
$n$	Non-negative integer

Returns

Kronecker product of  $A$  with itself  $n$  times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.64 `unsigned long long int qpp::lcm ( unsigned long long int m, unsigned long long int n ) [inline]`

Least common multiple of two positive integers.

See also

[qpp::gcd\(\)](#)

Parameters

$m$	Positive integer
$n$	Positive integer

Returns

Least common multiple of  $m$  and  $n$

6.1.3.65 `unsigned long long int qpp::lcm ( const std::vector< unsigned long long int> & ns ) [inline]`

Least common multiple of a list of positive integers.

See also

[qpp::gcd\(\)](#)

Parameters

$ns$	List of positive integers
------	---------------------------

Returns

Least common multiple of all numbers in  $ns$

6.1.3.66 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::load ( const std::string & fname )`

Loads Eigen matrix from a binary file (internal format) in double precision.

See also

[qpp::save\(\)](#)

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

Parameters

<i>A</i>	Eigen expression
<i>fname</i>	Output file name

**6.1.3.67** `template<typename Derived > Derived qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

See also

[qpp::saveMATLABmatrix\(\)](#)

This is the generic version that always throws [qpp::Exception::Type::UNDEFINED\\_TYPE](#). It is specialized only for [qpp::dmat](#) and [qpp::cmat](#) (the only matrix types that can be loaded)

**6.1.3.68** `template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`  
[inline]

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))

See also

[qpp::saveMATLABmatrix\(\)](#)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

Note

If *var\_name* is a complex matrix, only the real part is loaded

Parameters

<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen double dynamic matrix ([qpp::dmat](#))

**6.1.3.69** `template<> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )`  
`[inline]`

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))

See also

[qpp::saveMATLABmatrix\(\)](#)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

Parameters

<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen complex dynamic matrix ([qpp::cmat](#))

**6.1.3.70** `template<typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase< Derived > & A )`

Logarithm of the determinant.

Useful when the determinant overflows/underflows

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

Logarithm of the determinant of *A*, as a scalar in the same scalar field as *A*

**6.1.3.71** `template<typename Derived > cmat qpp::logm ( const Eigen::MatrixBase< Derived > & A )`

Matrix logarithm.

Parameters

<i>A</i>	Eigen expression
----------	------------------

Returns

Matrix logarithm of *A*

**6.1.3.72** `template<typename Derived > double qpp::lognegativity ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Logarithmic negativity of the bi-partite mixed state *A*.

## Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

## Returns

Logarithmic negativity, with the logarithm in base 2

**6.1.3.73** `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks )`

Measures the state *A* using the set of Kraus operators *Ks*.

## Parameters

<i>A</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

**6.1.3.74** `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::initializer_list< cmat > & Ks )`

Measures the state *A* using the set of Kraus operators *Ks*.

## Parameters

<i>A</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

**6.1.3.75** `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const cmat & U )`

Measures the state *A* in the orthonormal basis specified by the unitary matrix *U*.

## Parameters

<i>A</i>	Eigen expression
<i>U</i>	Unitary matrix whose columns represent the measurement basis vectors

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.76 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

See also

[qpp::measure\\_seq\(\)](#)

#### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### Parameters

<i>A</i>	Eigen expression
<i>Ks</i>	Set of Kraus operators
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.77 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::initializer_list< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

See also

[qpp::measure\\_seq\(\)](#)

#### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system
<i>Ks</i>	Set of Kraus operators

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.78 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat>> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const idx d = 2 )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

See also

[qpp::measure\\_seq\(\)](#)

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions
<i>Ks</i>	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.79 `template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat>> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const std::initializer_list< cmat > & Ks, const std::vector< idx > & subsys, const idx d = 2 )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* using the set of Kraus operators *Ks*.

See also

[qpp::measure\\_seq\(\)](#)

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions
<i>Ks</i>	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

**6.1.3.80** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const cmat & U, const std::vector< idx> & subsys, const std::vector< idx> & dims )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis specified by the unitary matrix *U*.

See also

[qpp::measure\\_seq\(\)](#)

#### Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system
<i>U</i>	Unitary matrix whose columns represent the measurement basis vectors

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

**6.1.3.81** `template<typename Derived> std::tuple<idx, std::vector<double>, std::vector<cmat>> qpp::measure ( const Eigen::MatrixBase< Derived> & A, const cmat & U, const std::vector< idx> & subsys, const idx d = 2 )`

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis specified by the unitary matrix *U*.

See also

[qpp::measure\\_seq\(\)](#)

#### Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions
<i>U</i>	Unitary matrix whose columns represent the measurement basis vectors

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states



**6.1.3.82** `template<typename Derived > std::tuple<std::vector<idx>, double, cmat> qpp::measure_seq ( const Eigen::MatrixBase< Derived > & A, std::vector< idx > subsys, std::vector< idx > dims )`

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

[qpp::measure\(\)](#)

Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>dims</i>	Dimensions of the multi-partite system

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

**6.1.3.83** `template<typename Derived > std::tuple<std::vector<idx>, double, cmat> qpp::measure_seq ( const Eigen::MatrixBase< Derived > & A, std::vector< idx > subsys, idx d = 2 )`

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

[qpp::measure\(\)](#)

Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes that are measured
<i>d</i>	Subsystem dimensions

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

**6.1.3.84** `ket qpp::mket ( const std::vector< idx > & mask, const std::vector< idx > & dims ) [inline]`

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|mask\rangle$ , where *mask* is a `std::vector` of non-negative integers. Each element in *mask* has to be smaller than the corresponding element in *dims*.

Parameters

<i>mask</i>	std::vector of non-negative integers
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Multi-partite qudit state vector, as a complex dynamic column vector

**6.1.3.85** `ket qpp::mket ( const std::vector< idx > & mask, idx d = 2 ) [inline]`

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , all subsystem having equal dimension  $d$ .  $mask$  is a std::vector of non-negative integers, and each element in  $mask$  has to be strictly smaller than  $d$ .

**Parameters**

<i>mask</i>	std::vector of non-negative integers
<i>d</i>	Subsystem dimensions

**Returns**

Multi-partite qudit state vector, as a complex dynamic column vector

**6.1.3.86** `unsigned long long int qpp::modpow ( unsigned long long int a, unsigned long long int n, unsigned long long int p ) [inline]`

Integer power modulo p.

Computes  $a^n \bmod p$

**Parameters**

<i>a</i>	Non-negative integer
<i>n</i>	Non-negative integer
<i>p</i>	Strictly positive integer

**Returns**

$a^n \bmod p$

**6.1.3.87** `cmat qpp::mprj ( const std::vector< idx > & mask, const std::vector< idx > & dims ) [inline]`

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|\text{mask}\rangle$ , where  $mask$  is a std::vector of non-negative integers. Each element in  $mask$  has to be smaller than the corresponding element in  $dims$ .

**Parameters**

<i>mask</i>	std::vector of non-negative integers
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

**6.1.3.88** `cmat qpp::mprj ( const std::vector< idx > & mask, idx d = 2 ) [inline]`

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|\text{mask}\rangle$ , all subsystem having equal dimension  $d$ .  $\text{mask}$  is a `std::vector` of non-negative integers, and each element in  $\text{mask}$  has to be strictly smaller than  $d$ .

Parameters

<i>mask</i>	<code>std::vector</code> of non-negative integers
<i>d</i>	Subsystem dimensions

Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

**6.1.3.89** `idx qpp::multiidx2n ( const std::vector< idx > & midx, const std::vector< idx > & dims ) [inline]`

Multi-index to non-negative integer index.

See also

[qpp::n2multiidx\(\)](#)

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

Parameters

<i>midx</i>	Multi-index
<i>dims</i>	Dimensions of the multi-partite system

Returns

Non-negative integer index

**6.1.3.90** `std::vector<idx> qpp::n2multiidx ( idx n, const std::vector< idx > & dims ) [inline]`

Non-negative integer index to multi-index.

See also

[qpp::multiidx2n\(\)](#)

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

Parameters

<i>n</i>	Non-negative integer index
<i>dims</i>	Dimensions of the multi-partite system

Returns

Multi-index of the same size as *dims*

**6.1.3.91** `template<typename Derived > double qpp::negativity ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Negativity of the bi-partite mixed state  $A$ .

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system

## Returns

Negativity

6.1.3.92 `template<typename Derived> double qpp::norm ( const Eigen::MatrixBase< Derived > & A )`

Frobenius norm.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Frobenius norm of  $A$

6.1.3.93 `cplx qpp::omega ( idx D ) [inline]`

D-th root of unity.

## Parameters

$D$	Non-negative integer
-----	----------------------

## Returns

D-th root of unity  $\exp(2\pi i/D)$

6.1.3.94 `constexpr cplx qpp::operator""_i ( unsigned long long int x ) [inline],[noexcept]`

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

6.1.3.95 `constexpr cplx qpp::operator""_i ( long double x ) [inline],[noexcept]`

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.96 `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase< Derived > & A, idx n )`

Matrix power.

See also

[qpp::spectralpowm\(\)](#)

Explicitly multiplies the matrix  $A$  with itself  $n$  times. By convention  $A^0 = I$ .

## Parameters

$A$	Eigen expression
$n$	Non-negative integer

## Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field as  $A$

**6.1.3.97** `template<typename Derived> dyn_mat<typename Derived::Scalar> qpp::prj ( const Eigen::MatrixBase< Derived> & V )`

Projector.

Normalized projector onto state vector

## Parameters

$V$	Eigen expression
-----	------------------

## Returns

Projector onto the state vector  $V$ , or the matrix *Zero* if  $V$  has norm zero (i.e. smaller than `qpp::eps`), as a dynamic matrix over the same scalar field as  $A$

**6.1.3.98** `template<typename Derived> Derived::Scalar qpp::prod ( const Eigen::MatrixBase< Derived> & A )`

Element-wise product of  $A$ .

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Element-wise product of  $A$ , as a scalar in the same scalar field as  $A$

**6.1.3.99** `template<typename InputIterator> std::iterator_traits<InputIterator>::value_type qpp::prod ( InputIterator first, InputIterator last )`

Element-wise product of an STL-like range.

## Parameters

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

## Returns

Element-wise product of the range, as a scalar in the same scalar field as the range

**6.1.3.100** `template<typename Container> Container::value_type qpp::prod ( const Container & c )`

Element-wise product of the elements of an STL-like container.

## Parameters

<i>c</i>	STL-like container
----------	--------------------

## Returns

Element-wise product of the elements of the container, as a scalar in the same scalar field as the container

6.1.3.101 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase<Derived > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Partial trace.

See also

[qpp::ptrace1\(\)](#), [qpp::ptrace2\(\)](#)

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsystems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as *A*

6.1.3.102 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase<Derived > & A, const std::vector< idx > & subsys, idx d = 2 )`

Partial trace.

See also

[qpp::ptrace1\(\)](#), [qpp::ptrace2\(\)](#)

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>d</i>	Subsystem dimensions

## Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsystems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as *A*

6.1.3.103 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace1 ( const Eigen::MatrixBase<Derived > & A, const std::vector< idx > & dims )`

Partial trace.

See also

[qpp::ptrace2\(\)](#)

Partial trace over the first subsystem of bi-partite state vector or density matrix

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system (must be a <code>std::vector</code> with 2 elements)

## Returns

Partial trace  $Tr_A(\cdot)$  over the first subsystem  $A$  in a bi-partite system  $A \otimes B$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.104 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace2 ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Partial trace.

See also

[qpp::ptrace1\(\)](#)

Partial trace over the second subsystem of bi-partite state vector or density matrix

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system (must be a <code>std::vector</code> with 2 elements)

## Returns

Partial trace  $Tr_B(\cdot)$  over the second subsystem  $B$  in a bi-partite system  $A \otimes B$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.105 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrtranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )`

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

## Parameters

$A$	Eigen expression
$subsys$	Subsystem indexes
$dims$	Dimensions of the multi-partite system

## Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsystems  $subsys$  in a multi-partite system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.106 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrtranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2 )`

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems



## Parameters

<i>A</i>	Eigen expression
<i>subsys</i>	Subsystem indexes
<i>d</i>	Subsystem dimensions

## Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsystems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as *A*

**6.1.3.107** `template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, const std::vector< idx > & dims )`

Quantum mutual information between 2 subsystems of a composite system.

## Parameters

<i>A</i>	Eigen expression
<i>subsysA</i>	Indexes of the first subsystem
<i>subsysB</i>	Indexes of the second subsystem
<i>dims</i>	Dimensions of the multi-partite system

## Returns

Mutual information between the 2 subsystems

**6.1.3.108** `template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, idx d = 2 )`

Quantum mutual information between 2 subsystems of a composite system.

## Parameters

<i>A</i>	Eigen expression
<i>subsysA</i>	Indexes of the first subsystem
<i>subsysB</i>	Indexes of the second subsystem
<i>d</i>	Subsystem dimensions

## Returns

Mutual information between the 2 subsystems

**6.1.3.109** `double qpp::rand ( double a = 0, double b = 1 ) [inline]`

Generates a random real number uniformly distributed in the interval [a, b)

## Parameters

<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, does not belong to it

## Returns

Random real number (double) uniformly distributed in the interval [a, b)

**6.1.3.110** `template<typename Derived > Derived qpp::rand ( idx rows, idx cols, double a = 0, double b = 1 )`

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws `qpp::Exception::Type::UNDEFINED_TYPE`. It is specialized only for `qpp::dmat` and `qpp::cmat`

**6.1.3.111** `template<> dmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]`

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (`qpp::dmat`)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

#### Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, does not belong to it

#### Returns

Random real matrix

**6.1.3.112** `template<> cmatrix qpp::rand ( idx rows, idx cols, double a, double b ) [inline]`

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (`qpp::cmat`)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd,
// with entries (both real and imaginary) uniformly distributed in [-1,1)
auto mat = rand<cmat>(3, 3, -1, 1);
```

#### Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, does not belong to it

#### Returns

Random complex matrix

**6.1.3.113** `cmatrix qpp::randH ( idx D ) [inline]`

Generates a random Hermitian matrix.

## Parameters

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

## Returns

Random Hermitian matrix

**6.1.3.114** `idx qpp::randidx ( idx a = std::numeric_limits<idx>::min(), idx b = std::numeric_limits<idx>::max() ) [inline]`

Generates a random index (idx) uniformly distributed in the interval [a, b].

## Parameters

<i>a</i>	Beginning of the interval, belongs to it
<i>b</i>	End of the interval, belongs to it

## Returns

Random index (idx) uniformly distributed in the interval [a, b]

**6.1.3.115** `ket qpp::randket ( idx D ) [inline]`

Generates a random normalized ket (pure state vector)

## Parameters

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

## Returns

Random normalized ket

**6.1.3.116** `std::vector<cmat> qpp::randkraus ( idx N, idx D ) [inline]`

Generates a set of random Kraus operators.

## Note

The set of Kraus operators satisfy the closure condition  $\sum_i K_i^\dagger K_i = I$

## Parameters

<i>N</i>	Number of Kraus operators
<i>D</i>	Dimension of the Hilbert space

## Returns

Set of *N* Kraus operators satisfying the closure condition

**6.1.3.117** `template<typename Derived > Derived qpp::randn ( idx rows, idx cols, double mean = 0, double sigma = 1 )`

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws `qpp::Exception::Type::UNDEFINED_TYPE`. It is specialized only for `qpp::dmat` and `qpp::cmat`

### 6.1.3.118 `template<> dmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]`

Generates a random real matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for double matrices ([qpp::dmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

#### Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>mean</i>	Mean
<i>sigma</i>	Standard deviation

#### Returns

Random real matrix

### 6.1.3.119 `template<> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]`

Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices ([qpp::cmat](#))

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd,
// with entries (both real and imaginary) normally distributed in N(0,2)
auto mat = randn<cmat>(3, 3, 0, 2);
```

#### Parameters

<i>rows</i>	Number of rows of the random generated matrix
<i>cols</i>	Number of columns of the random generated matrix
<i>mean</i>	Mean
<i>sigma</i>	Standard deviation

#### Returns

Random complex matrix

### 6.1.3.120 `double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]`

Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$

#### Parameters

<i>mean</i>	Mean
-------------	------

<i>sigma</i>	Standard deviation
--------------	--------------------

**Returns**

Random real number normally distributed in  $N(\text{mean}, \text{sigma})$

**6.1.3.121** `std::vector<idx> qpp::randperm ( idx n ) [inline]`

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by `std::shuffle`), so that all permutations are equally probable

**Parameters**

<i>n</i>	Size of the permutation
----------	-------------------------

**Returns**

Random permutation of size *n*

**6.1.3.122** `cmat qpp::randrho ( idx D ) [inline]`

Generates a random density matrix.

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Random density matrix

**6.1.3.123** `cmat qpp::randU ( idx D ) [inline]`

Generates a random unitary matrix.

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Random unitary

**6.1.3.124** `cmat qpp::randV ( idx Din, idx Dout ) [inline]`

Generates a random isometry matrix.

**Parameters**

<i>Din</i>	Size of the input Hilbert space
------------	---------------------------------

<i>Dout</i>	Size of the output Hilbert space
-------------	----------------------------------

**Returns**

Random isometry matrix

**6.1.3.125** `template<typename Derived > double qpp::renyi ( const Eigen::MatrixBase< Derived > & A, double alpha )`

Renyi-  $\alpha$  entropy of the density matrix  $A$ , for  $\alpha \geq 0$ .

**Note**

When  $\alpha \rightarrow 1$  the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

**Parameters**

<i>A</i>	Eigen expression
<i>alpha</i>	Non-negative real number, use <a href="#">qpp::infty</a> for $\alpha = \infty$

**Returns**

Renyi-  $\alpha$  entropy, with the logarithm in base 2

**6.1.3.126** `double qpp::renyi ( const std::vector< double > & prob, double alpha ) [inline]`

Renyi-  $\alpha$  entropy of the probability distribution  $prob$ , for  $\alpha \geq 0$ .

**Note**

When  $\alpha \rightarrow 1$  the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

**Parameters**

<i>prob</i>	Real probability vector
<i>alpha</i>	Non-negative real number, use <a href="#">qpp::infty</a> for $\alpha = \infty$

**Returns**

Renyi-  $\alpha$  entropy, with the logarithm in base 2

**6.1.3.127** `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, idx rows, idx cols )`

Reshape.

Uses column-major order when reshaping (same as MATLAB)

**Parameters**

<i>A</i>	Eigen expression
<i>rows</i>	Number of rows of the reshaped matrix

<i>cols</i>	Number of columns of the reshaped matrix
-------------	--

**Returns**

Reshaped matrix with *rows* rows and *cols* columns, as a dynamic matrix over the same scalar field as *A*

6.1.3.128 `template<typename Derived > std::vector<double> qpp::rho2bloch ( const Eigen::MatrixBase< Derived > & A )`

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix *A*.

**See also**

[qpp::bloch2rho\(\)](#)

**Note**

It is implicitly assumed that the density matrix is Hermitian

**Parameters**

<i>A</i>	Eigen expression
----------	------------------

**Returns**

3-dimensional Bloch vector

6.1.3.129 `template<typename Derived > dyn_col_vect<typename Derived::Scalar> qpp::rho2pure ( const Eigen::MatrixBase< Derived > & A )`

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

**Note**

No purity check is done, the input state *A* must have rank one, otherwise the function returns the first non-zero eigenvector of *A*

**Parameters**

<i>A</i>	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. <i>A</i> is assumed to have rank one
----------	--

**Returns**

The unique non-zero eigenvector of *A*, as a dynamic column vector over the same scalar field as *A*

6.1.3.130 `template<typename Derived > void qpp::save ( const Eigen::MatrixBase< Derived > & A, const std::string & fname )`

Saves Eigen expression to a binary file (internal format) in double precision.

**See also**

[qpp::load\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>fname</i>	Output file name

6.1.3.131 `template<typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< Derived > &A, const std::string & mat_file, const std::string & var_name, const std::string & mode )`

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

See also

[qpp::loadMATLABmatrix\(\)](#)

This is the generic version that always throws [qpp::Exception::Type::UNDEFINED\\_TYPE](#). It is specialized only for [qpp::dmat](#) and [qpp::cmat](#) (the only matrix types that can be saved)

6.1.3.132 `template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< dmat > &A, const std::string & mat_file, const std::string & var_name, const std::string & mode ) [inline]`

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))

See also

[qpp::loadMATLABmatrix\(\)](#)

## Parameters

<i>A</i>	Eigen expression over the complex field
<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be saved
<i>mode</i>	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.133 `template<> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< cmat > &A, const std::string & mat_file, const std::string & var_name, const std::string & mode ) [inline]`

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))

See also

[qpp::loadMATLABmatrix\(\)](#)

## Parameters

<i>A</i>	Eigen expression over the complex field
<i>mat_file</i>	MATALB .mat file
<i>var_name</i>	Variable name in the .mat file representing the matrix to be saved
<i>mode</i>	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.134 `template<typename Derived > double qpp::schatten ( const Eigen::MatrixBase< Derived > &A, double p )`

Schatten matrix norm.



## Parameters

$A$	Eigen expression
$p$	Real number, greater or equal to 1, use <a href="#">qpp::infty</a> for $p = \infty$

## Returns

Schatten- $p$  matrix norm of  $A$

6.1.3.135 `template<typename Derived> cmat qpp::schmidtA ( const Eigen::MatrixBase< Derived> & A, const std::vector< idx> & dims )`

Schmidt basis on Alice side.

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system

## Returns

Unitary matrix  $U$  whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.136 `template<typename Derived> cmat qpp::schmidtB ( const Eigen::MatrixBase< Derived> & A, const std::vector< idx> & dims )`

Schmidt basis on Bob side.

## Parameters

$A$	Eigen expression
$dims$	Dimensions of the bi-partite system

## Returns

Unitary matrix  $V$  whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.137 `template<typename Derived> dyn_col_vect<double> qpp::schmidtcoeffs ( const Eigen::MatrixBase< Derived> & A, const std::vector< idx> & dims )`

Schmidt coefficients of the bi-partite pure state  $A$ .

## Note

The sum of the squares of the Schmidt coefficients equals 1

## See also

[qpp::schmidtprobs\(\)](#)

## Parameters

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

**Returns**

Schmidt coefficients of  $A$ , as a real dynamic column vector

6.1.3.138 `template<typename Derived > std::vector<double> qpp::schmidtprobs ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )`

Schmidt probabilities of the bi-partite pure state  $A$ .

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

**See also**

[qpp::schmidtcoeffs\(\)](#)

**Parameters**

<i>A</i>	Eigen expression
<i>dims</i>	Dimensions of the bi-partite system

**Returns**

Real vector consisting of the Schmidt probabilities of  $A$

6.1.3.139 `template<typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase< Derived > & A )`

Matrix sin.

**Parameters**

<i>A</i>	Eigen expression
----------	------------------

**Returns**

Matrix sine of  $A$

6.1.3.140 `template<typename Derived > cmat qpp::spectralpowm ( const Eigen::MatrixBase< Derived > & A, const cplx z )`

Matrix power.

**See also**

[qpp::powm\(\)](#)

Uses the spectral decomposition of  $A$  to compute the matrix power. By convention  $A^0 = I$ .

**Parameters**

$A$	Eigen expression
$z$	Complex number

**Returns**

Matrix power  $A^z$

6.1.3.141 `template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )`

Matrix square root.

**Parameters**

$A$	Eigen expression
-----	------------------

**Returns**

Matrix square root of  $A$

6.1.3.142 `template<typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase< Derived > & A )`

Element-wise sum of  $A$ .

**Parameters**

$A$	Eigen expression
-----	------------------

**Returns**

Element-wise sum of  $A$ , as a scalar in the same scalar field as  $A$

6.1.3.143 `template<typename InputIterator > std::iterator_traits<InputIterator>::value_type qpp::sum ( InputIterator first, InputIterator last )`

Element-wise sum of an STL-like range.

**Parameters**

<i>first</i>	Iterator to the first element of the range
<i>last</i>	Iterator to the last element of the range

**Returns**

Element-wise sum of the range, as a scalar in the same scalar field as the range

6.1.3.144 `template<typename Container > Container::value_type qpp::sum ( const Container & c )`

Element-wise sum of the elements of an STL-like container.

**Parameters**

c	STL-like container
---	--------------------

**Returns**

Element-wise sum of the elements of the container, as a scalar in the same scalar field as the container

**6.1.3.145** `cmat qpp::super2choi ( const cmat & A ) [inline]`

Converts superoperator matrix to Choi matrix.

**See also**

[qpp::choi2super\(\)](#)

**Parameters**

A	Superoperator matrix
---	----------------------

**Returns**

Choi matrix

**6.1.3.146** `template<typename Derived > dyn_col_vect<double> qpp::svals ( const Eigen::MatrixBase< Derived > & A )`

Singular values.

**Parameters**

A	Eigen expression
---	------------------

**Returns**

Singular values of A, ordered in decreasing order, as a real dynamic column vector

**6.1.3.147** `template<typename Derived > std::tuple<cmat, dyn_col_vect<double>, cmat> qpp::svd ( const Eigen::MatrixBase< Derived > & A )`

Full singular value decomposition.

**Parameters**

A	Eigen expression
---	------------------

**Returns**

Tuple of: 1. Left singular vectors of A, as columns of a complex dynamic matrix, 2. Singular values of A, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of A, as columns of a complex dynamic matrix

**6.1.3.148** `template<typename Derived > cmat qpp::svdU ( const Eigen::MatrixBase< Derived > & A )`

Left singular vectors.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Complex dynamic matrix, whose columns are the left singular vectors of  $A$

6.1.3.149 `template<typename Derived > cmat qpp::svdV ( const Eigen::MatrixBase< Derived > & A )`

Right singular vectors.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Complex dynamic matrix, whose columns are the right singular vectors of  $A$

6.1.3.150 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & dims )`

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit  $perm[i]$  is permuted to the location  $i$ .

## Parameters

$A$	Eigen expression
$perm$	Permutation
$dims$	Dimensions of the multi-partite system

## Returns

Permuted system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.151 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2 )`

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit  $perm[i]$  is permuted to the location  $i$ .

## Parameters

$A$	Eigen expression
$perm$	Permutation
$d$	Subsystem dimensions

## Returns

Permuted system, as a dynamic matrix over the same scalar field as  $A$

6.1.3.152 `template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )`

Trace.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Trace of  $A$ , as a scalar in the same scalar field as  $A$

6.1.3.153 `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::transpose ( const Eigen::MatrixBase<Derived > & A )`

Transpose.

## Parameters

$A$	Eigen expression
-----	------------------

## Returns

Transpose of  $A$ , as a dynamic matrix over the same scalar field as  $A$

6.1.3.154 `template<typename Derived > double qpp::tsallis ( const Eigen::MatrixBase<Derived > & A, double  $q$  )`

Tsallis-  $q$  entropy of the density matrix  $A$ , for  $q \geq 0$ .

## Note

When  $q \rightarrow 1$  the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base  $e$

## Parameters

$A$	Eigen expression
$q$	Non-negative real number

## Returns

Tsallis-  $q$  entropy

6.1.3.155 `double qpp::tsallis ( const std::vector< double > & prob, double  $q$  ) [inline]`

Tsallis-  $q$  entropy of the probability distribution  $prob$ , for  $q \geq 0$ .

## Note

When  $q \rightarrow 1$  the Tsallis entropy converges to the Shannon entropy, with the logarithm in base  $e$

## Parameters

$prob$	Real probability vector
$q$	Non-negative real number

## Returns

Tsallis-  $q$  entropy

6.1.3.156 `std::vector<int> qpp::x2contfrac ( double x, idx n, idx cut = 1e5 ) [inline]`

Simple continued fraction expansion.

See also

[qpp::contfrac2x\(\)](#)

Parameters

<i>x</i>	Real number
<i>n</i>	Number of terms in the expansion
<i>cut</i>	Stop the expansion when the next term is greater than <i>cut</i>

Returns

Integer vector containing the simple continued fraction expansion of *x*. If there are *m* less than *n* terms in the expansion, a shorter vector with *m* components is returned.

## 6.1.4 Variable Documentation

6.1.4.1 `constexpr double qpp::chop = 1e-10`

Used in [qpp::disp\(\)](#) for setting to zero numbers that have their absolute value smaller than [qpp::chop](#).

6.1.4.2 `constexpr double qpp::ee = 2.718281828459045235360287471352662497`

Base of natural logarithm, *e*.

6.1.4.3 `constexpr double qpp::eps = 1e-12`

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if (std::abs(x) < qpp::eps) // x is zero
```

6.1.4.4 `constexpr double qpp::infy = std::numeric_limits<double>::infinity()`

Used to denote infinity in double precision.

6.1.4.5 `constexpr idx qpp::maxn = 64`

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.6 `constexpr double qpp::pi = 3.141592653589793238462643383279502884`

$\pi$

## 6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

### 6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

## 6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

### Classes

- class [IOManipEigen](#)
- class [IOManipPointer](#)
- class [IOManipRange](#)
- class [Singleton](#)

*[Singleton](#) policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)*

### Functions

- void [\\_n2multiidx](#) (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx [\\_multiidx2n](#) (const idx \*midx, idx numdims, const idx \*dims) noexcept
- template<typename Derived >  
bool [\\_check\\_square\\_mat](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [\\_check\\_vector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [\\_check\\_rvector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [\\_check\\_cvector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename T >  
bool [\\_check\\_nonzero\\_size](#) (const T &x) noexcept
- bool [\\_check\\_dims](#) (const std::vector< idx > &dims)
- template<typename Derived >  
bool [\\_check\\_dims\\_match\\_mat](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [\\_check\\_dims\\_match\\_cvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- template<typename Derived >  
bool [\\_check\\_dims\\_match\\_rvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- bool [\\_check\\_eq\\_dims](#) (const std::vector< idx > &dims, idx dim) noexcept
- bool [\\_check\\_subsys\\_match\\_dims](#) (const std::vector< idx > &subsys, const std::vector< idx > &dims)
- template<typename Derived >  
bool [\\_check\\_qubit\\_matrix](#) (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >  
bool [\\_check\\_qubit\\_cvector](#) (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >  
bool [\\_check\\_qubit\\_rvector](#) (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >  
bool [\\_check\\_qubit\\_vector](#) (const Eigen::MatrixBase< Derived > &V) noexcept
- bool [\\_check\\_perm](#) (const std::vector< idx > &perm)
- template<typename Derived1 , typename Derived2 >  
[dyn\\_mat](#)< typename Derived1::Scalar > [\\_kron2](#) (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)



- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > _dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)`
- `template<typename T >`  
`void variadic_vector_emplace (std::vector< T > &)`
- `template<typename T , typename First , typename... Args>`  
`void variadic_vector_emplace (std::vector< T > &v, First &&first, Args &&...args)`

### 6.3.1 Detailed Description

Internal utility functions, do not use/modify.

### 6.3.2 Function Documentation

6.3.2.1 `template<typename Derived > bool qpp::internal::_check_cvector ( const Eigen::MatrixBase< Derived > & A )`

6.3.2.2 `bool qpp::internal::_check_dims ( const std::vector< idx > & dims ) [inline]`

6.3.2.3 `template<typename Derived > bool qpp::internal::_check_dims_match_cvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )`

6.3.2.4 `template<typename Derived > bool qpp::internal::_check_dims_match_mat ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & A )`

6.3.2.5 `template<typename Derived > bool qpp::internal::_check_dims_match_rvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )`

6.3.2.6 `bool qpp::internal::_check_eq_dims ( const std::vector< idx > & dims, idx dim ) [inline], [noexcept]`

6.3.2.7 `template<typename T > bool qpp::internal::_check_nonzero_size ( const T & x ) [noexcept]`

6.3.2.8 `bool qpp::internal::_check_perm ( const std::vector< idx > & perm ) [inline]`

6.3.2.9 `template<typename Derived > bool qpp::internal::_check_qubit_cvector ( const Eigen::MatrixBase< Derived > & V ) [noexcept]`

6.3.2.10 `template<typename Derived > bool qpp::internal::_check_qubit_matrix ( const Eigen::MatrixBase< Derived > & A ) [noexcept]`

6.3.2.11 `template<typename Derived > bool qpp::internal::_check_qubit_rvector ( const Eigen::MatrixBase< Derived > & V ) [noexcept]`

6.3.2.12 `template<typename Derived > bool qpp::internal::_check_qubit_vector ( const Eigen::MatrixBase< Derived > & V ) [noexcept]`

6.3.2.13 `template<typename Derived > bool qpp::internal::_check_rvector ( const Eigen::MatrixBase< Derived > & A )`

6.3.2.14 `template<typename Derived > bool qpp::internal::_check_square_mat ( const Eigen::MatrixBase< Derived > & A )`

6.3.2.15 `bool qpp::internal::_check_subsys_match_dims ( const std::vector< idx > & subsys, const std::vector< idx > & dims ) [inline]`

6.3.2.16 `template<typename Derived > bool qpp::internal::_check_vector ( const Eigen::MatrixBase< Derived > & A )`

- 6.3.2.17 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar>  
qpp::internal::_dirsum2 ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`
- 6.3.2.18 `template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::internal::_kron2 (   
const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )`
- 6.3.2.19 `idx qpp::internal::_multiidx2n ( const idx * midx, idx numdims, const idx * dims ) [inline],  
[noexcept]`
- 6.3.2.20 `void qpp::internal::_n2multiidx ( idx n, idx numdims, const idx * dims, idx * result ) [inline],  
[noexcept]`
- 6.3.2.21 `template<typename T > void qpp::internal::variadic_vector_emplace ( std::vector< T > & )`
- 6.3.2.22 `template<typename T , typename First , typename... Args> void qpp::internal::variadic_vector_emplace (   
std::vector< T > & v, First && first, Args &&... args )`

## Chapter 7

# Class Documentation

### 7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

```
#include <classes/codes.h>
```

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



## Public Types

- enum [Type](#) { [Type::FIVE\\_QUBIT](#) = 1, [Type::SEVEN\\_QUBIT\\_STEANE](#), [Type::NINE\\_QUBIT\\_SHOR](#) }  
*Code types, add more codes here if needed.*

## Public Member Functions

- [ket codeword](#) ([Type](#) type, [idx](#) i) const  
*Returns the codeword of the specified code type.*

## Friends

- class [internal::Singleton< const Codes >](#)

## Additional Inherited Members

### 7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

### 7.1.2 Member Enumeration Documentation

#### 7.1.2.1 enum [qpp::Codes::Type](#) [[strong](#)]

Code types, add more codes here if needed.

See also

[qpp::Codes::codeword\(\)](#)

Enumerator

***FIVE\_QUBIT*** [\[\[5,1,3\]\]](#) qubit code  
***SEVEN\_QUBIT\_STEANE*** [\[\[7,1,3\]\]](#) Steane qubit code  
***NINE\_QUBIT\_SHOR*** [\[\[9,1,3\]\]](#) Shor qubit code

### 7.1.3 Member Function Documentation

#### 7.1.3.1 [ket qpp::Codes::codeword](#) ( [Type](#) type, [idx](#) i ) const [[inline](#)]

Returns the codeword of the specified code type.

See also

[qpp::Codes::Type](#)

Parameters

<i>type</i>	Code type
-------------	-----------

<i>i</i>	Codeword index
----------	----------------

### Returns

*i*-th codeword of the code *type*

## 7.1.4 Friends And Related Function Documentation

### 7.1.4.1 friend class internal::Singleton< const Codes > [friend]

The documentation for this class was generated from the following file:

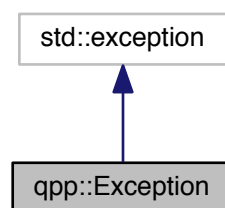
- [classes/codes.h](#)

## 7.2 qpp::Exception Class Reference

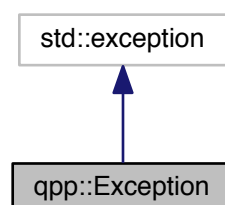
Generates custom exceptions, used when validating function parameters.

```
#include <classes/exception.h>
```

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



## Public Types

- enum `Type` {  
`Type::UNKNOWN_EXCEPTION = 1`, `Type::ZERO_SIZE`, `Type::MATRIX_NOT_SQUARE`, `Type::MATRIX_↵  
NOT_CVECTOR`,  
`Type::MATRIX_NOT_RVECTOR`, `Type::MATRIX_NOT_VECTOR`, `Type::MATRIX_NOT_SQUARE_OR_C↵  
VECTOR`, `Type::MATRIX_NOT_SQUARE_OR_RVECTOR`,  
`Type::MATRIX_NOT_SQUARE_OR_VECTOR`, `Type::MATRIX_MISMATCH_SUBSYS`, `Type::DIMS_INVA↵  
LID`, `Type::DIMS_NOT_EQUAL`,  
`Type::DIMS_MISMATCH_MATRIX`, `Type::DIMS_MISMATCH_CVECTOR`, `Type::DIMS_MISMATCH_RVE↵  
CTOR`, `Type::DIMS_MISMATCH_VECTOR`,  
`Type::SUBSYS_MISMATCH_DIMS`, `Type::PERM_INVALID`, `Type::PERM_MISMATCH_DIMS`, `Type::NOT↵  
_QUBIT_MATRIX`,  
`Type::NOT_QUBIT_CVECTOR`, `Type::NOT_QUBIT_RVECTOR`, `Type::NOT_QUBIT_VECTOR`, `Type::NO↵  
T_QUBIT_SUBSYS`,  
`Type::NOT_BIPARTITE`, `Type::NO_CODEWORD`, `Type::OUT_OF_RANGE`, `Type::TYPE_MISMATCH`,  
`Type::UNDEFINED_TYPE`, `Type::CUSTOM_EXCEPTION` }

*Exception types, add more here if needed.*

## Public Member Functions

- `Exception` (const std::string &where, const `Type` &type)  
*Constructs an exception.*
- `Exception` (const std::string &where, const std::string &custom)  
*Constructs an exception.*
- virtual const char \* `what` () const noexcept override  
*Overrides std::exception::what()*

### 7.2.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

### 7.2.2 Member Enumeration Documentation

#### 7.2.2.1 enum `qpp::Exception::Type` [strong]

`Exception` types, add more here if needed.

See also

`qpp::Exception::_construct_exception_msg()`

Enumerator

**UNKNOWN\_EXCEPTION** Unknown exception  
**ZERO\_SIZE** Zero sized object, e.g. empty `Eigen::Matrix` or `std::vector<>` with no elements  
**MATRIX\_NOT\_SQUARE** `Eigen::Matrix` is not square  
**MATRIX\_NOT\_CVECTOR** `Eigen::Matrix` is not a column vector  
**MATRIX\_NOT\_RVECTOR** `Eigen::Matrix` is not a row vector  
**MATRIX\_NOT\_VECTOR** `Eigen::Matrix` is not a row/column vector  
**MATRIX\_NOT\_SQUARE\_OR\_CVECTOR** `Eigen::Matrix` is not square nor a column vector  
**MATRIX\_NOT\_SQUARE\_OR\_RVECTOR** `Eigen::Matrix` is not square nor a row vector

**MATRIX\_NOT\_SQUARE\_OR\_VECTOR** Eigen::Matrix is not square nor a row/column vector

**MATRIX\_MISMATCH\_SUBSYS** Matrix size mismatch subsystem sizes (e.g. in [qpp::apply\(\)](#))

**DIMS\_INVALID** std::vector<idx> of dimensions has zero size or contains zeros

**DIMS\_NOT\_EQUAL** Local/global dimensions are not equal

**DIMS\_MISMATCH\_MATRIX** Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

**DIMS\_MISMATCH\_CVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

**DIMS\_MISMATCH\_RVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

**DIMS\_MISMATCH\_VECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

**SUBSYS\_MISMATCH\_DIMS** std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

**PERM\_INVALID** std::vector<idx> does not represent a valid permutation

**PERM\_MISMATCH\_DIMS** Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

**NOT\_QUBIT\_MATRIX** Eigen::Matrix is not 2 x 2

**NOT\_QUBIT\_CVECTOR** Eigen::Matrix is not 2 x 1

**NOT\_QUBIT\_RVECTOR** Eigen::Matrix is not 1 x 2

**NOT\_QUBIT\_VECTOR** Eigen::Matrix is not 1 x 2 nor 2 x 1

**NOT\_QUBIT\_SUBSYS** Subsystems are not 2-dimensional

**NOT\_BIPARTITE** std::vector<idx> of dimensions has size different from 2

**NO\_CODEWORD** Codeword does not exist, thrown when calling [qpp::Codes::codeword\(\)](#) with invalid index *i*

**OUT\_OF\_RANGE** Parameter out of range

**TYPE\_MISMATCH** Scalar types do not match

**UNDEFINED\_TYPE** Templated specialization not defined for this type

**CUSTOM\_EXCEPTION** Custom exception, user must provide a custom message

### 7.2.3 Constructor & Destructor Documentation

7.2.3.1 `qpp::Exception::Exception ( const std::string & where, const Type & type )` `[inline]`

Constructs an exception.

Parameters

<i>where</i>	Text representing where the exception occurred
<i>type</i>	<a href="#">Exception</a> type, defined in <a href="#">qpp::Exception::Type</a>

7.2.3.2 `qpp::Exception::Exception ( const std::string & where, const std::string & custom )` `[inline]`

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

## Parameters

<i>where</i>	Text representing where the exception occurred
<i>custom</i>	<a href="#">Exception</a> description

## 7.2.4 Member Function Documentation

**7.2.4.1** `virtual const char* qpp::Exception::what ( ) const` `[inline], [override], [virtual], [noexcept]`

Overrides `std::exception::what()`

## Returns

[Exception](#) description

The documentation for this class was generated from the following file:

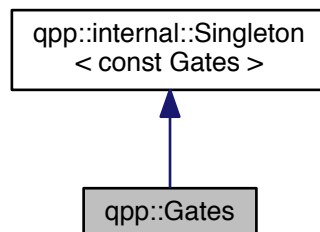
- [classes/exception.h](#)

## 7.3 qpp::Gates Class Reference

const Singleton class that implements most commonly used gates

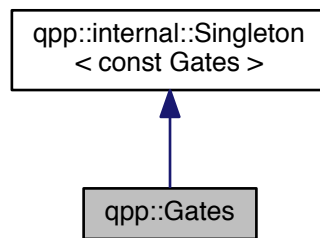
```
#include <classes/gates.h>
```

Inheritance diagram for `qpp::Gates`:





Collaboration diagram for qpp::Gates:



## Public Member Functions

- **cmat Rn** (double theta, const std::vector< double > &n) const  
*Rotation of theta about the 3-dimensional real unit vector n.*
- **cmat Zd** (idx D) const  
*Generalized Z gate for qudits.*
- **cmat Fd** (idx D) const  
*Fourier transform gate for qudits.*
- **cmat Xd** (idx D) const  
*Generalized X gate for qudits.*
- template<typename Derived = Eigen::MatrixXcd>  
Derived **Id** (idx D) const  
*Identity gate.*
- template<typename Derived >  
**dyn\_mat**< typename Derived::Scalar > **CTRL** (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx n, idx d=2) const  
*Generates the multi-partite multiple-controlled-A gate in matrix form.*
- template<typename Derived >  
**dyn\_mat**< typename Derived::Scalar > **expandout** (const Eigen::MatrixBase< Derived > &A, idx pos, const std::vector< idx > &dims) const  
*Expands out.*

## Public Attributes

- **cmat Id2** {cmat::Identity(2, 2)}  
*Identity gate.*
- **cmat H** {cmat::Zero(2, 2)}  
*Hadamard gate.*
- **cmat X** {cmat::Zero(2, 2)}  
*Pauli Sigma-X gate.*
- **cmat Y** {cmat::Zero(2, 2)}  
*Pauli Sigma-Y gate.*
- **cmat Z** {cmat::Zero(2, 2)}  
*Pauli Sigma-Z gate.*
- **cmat S** {cmat::Zero(2, 2)}

- S gate.*
- [cmat T](#) {cmat::Zero(2, 2)}
- T gate.*
- [cmat CNOT](#) {cmat::Identity(4, 4)}
- Controlled-NOT control target gate.*
- [cmat CZ](#) {cmat::Identity(4, 4)}
- Controlled-Phase gate.*
- [cmat CNOTba](#) {cmat::Zero(4, 4)}
- Controlled-NOT target control gate.*
- [cmat SWAP](#) {cmat::Identity(4, 4)}
- SWAP gate.*
- [cmat TOF](#) {cmat::Identity(8, 8)}
- Toffoli gate.*
- [cmat FRED](#) {cmat::Identity(8, 8)}
- Fredkin gate.*

## Friends

- class [internal::Singleton< const Gates >](#)

## Additional Inherited Members

### 7.3.1 Detailed Description

const Singleton class that implements most commonly used gates

### 7.3.2 Member Function Documentation

**7.3.2.1** `template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::CTRL ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx n, idx d = 2 ) const`  
[inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

See also

[qpp::applyCTRL\(\)](#)

Note

The dimension of the gate *A* must match the dimension of *subsys*

Parameters

<i>A</i>	Eigen expression
<i>ctrl</i>	Control subsystem indexes
<i>subsys</i>	Subsystem indexes where the gate <i>A</i> is applied
<i>n</i>	Total number of subsystems

<i>d</i>	Subsystem dimensions
----------	----------------------

**Returns**

CTRL-A gate, as a matrix over the same scalar field as *A*

```
7.3.2.2 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::expandout ( const
Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]
```

Expands out.

**See also**

[qpp::kron\(\)](#)

Expands out *A* as a matrix in a multi-partite system. Faster than using [qpp::kron](#)(*I*, *I*, ..., *I*, *A*, *I*, ..., *I*).

**Parameters**

<i>A</i>	Eigen expression
<i>pos</i>	Position
<i>dims</i>	Dimensions of the multi-partite system

**Returns**

Tensor product  $I \otimes \dots \otimes I \otimes A \otimes I \otimes \dots \otimes I$ , with *A* on position *pos*, as a dynamic matrix over the same scalar field as *A*

```
7.3.2.3 cmat qpp::Gates::Fd ( idx D ) const [inline]
```

Fourier transform gate for qudits.

**Note**

Defined as  $F = \sum_{jk} \exp(2\pi i j k / D) |j\rangle \langle k|$

**Parameters**

<i>D</i>	Dimension of the Hilbert space
----------	--------------------------------

**Returns**

Fourier transform gate for qudits

```
7.3.2.4 template<typename Derived = Eigen::MatrixXcd> Derived qpp::Gates::Id ( idx D ) const [inline]
```

Identity gate.

**Note**

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Identity gate

**7.3.2.5** `cmat qpp::Gates::Rn ( double theta, const std::vector< double > & n ) const` `[inline]`

Rotation of *theta* about the 3-dimensional real unit vector *n*.

## Parameters

<i>theta</i>	Rotation angle
<i>n</i>	3-dimensional real unit vector

## Returns

Rotation gate

**7.3.2.6** `cmat qpp::Gates::Xd ( idx D ) const` `[inline]`

Generalized X gate for qudits.

## Note

Defined as  $X = \sum_j |j \oplus 1\rangle \langle j|$

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Generalized X gate for qudits

**7.3.2.7** `cmat qpp::Gates::Zd ( idx D ) const` `[inline]`

Generalized Z gate for qudits.

## Note

Defined as  $Z = \sum_j \exp(2\pi i j / D) |j\rangle \langle j|$

## Parameters

$D$	Dimension of the Hilbert space
-----	--------------------------------

## Returns

Generalized Z gate for qudits

### 7.3.3 Friends And Related Function Documentation

7.3.3.1 friend class internal::Singleton< const Gates > [friend]

### 7.3.4 Member Data Documentation

7.3.4.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

7.3.4.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

7.3.4.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

7.3.4.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}

Fredkin gate.

7.3.4.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}

Hadamard gate.

7.3.4.6 cmat qpp::Gates::Id2 {cmat::Identity(2, 2)}

Identity gate.

7.3.4.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}

S gate.

7.3.4.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}

SWAP gate.

7.3.4.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}

T gate.

7.3.4.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}

Toffoli gate.

7.3.4.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

#### 7.3.4.12 `cmat qpp::Gates::Y {cmat::Zero(2, 2)}`

Pauli Sigma-Y gate.

#### 7.3.4.13 `cmat qpp::Gates::Z {cmat::Zero(2, 2)}`

Pauli Sigma-Z gate.

The documentation for this class was generated from the following file:

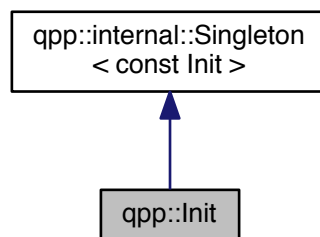
- [classes/gates.h](#)

## 7.4 `qpp::Init` Class Reference

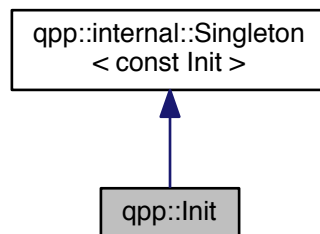
const Singleton class that performs additional initializations/cleanups

```
#include <classes/init.h>
```

Inheritance diagram for `qpp::Init`:



Collaboration diagram for `qpp::Init`:



### Friends

- class [internal::Singleton< const Init >](#)

## Additional Inherited Members

### 7.4.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

### 7.4.2 Friends And Related Function Documentation

#### 7.4.2.1 friend class internal::Singleton< const Init > [friend]

The documentation for this class was generated from the following file:

- [classes/init.h](#)

## 7.5 qpp::internal::IOManipEigen Class Reference

```
#include <internal/classes/iomanip.h>
```

### Public Member Functions

- `template<typename Derived >`  
[IOManipEigen](#) (const Eigen::MatrixBase< Derived > &A, double [chop](#)=[qpp::chop](#))
- [IOManipEigen](#) (const [cplx](#) z, double [chop](#)=[qpp::chop](#))

### Friends

- `template<typename charT , typename traits >`  
`std::basic_ostream< charT, traits > & operator<< (std::basic_ostream< charT, traits > &os, const IO↵ManipEigen &rhs)`

### 7.5.1 Constructor & Destructor Documentation

7.5.1.1 `template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen ( const Eigen::MatrixBase< Derived > &A, double chop = qpp::chop ) [inline],[explicit]`

7.5.1.2 `qpp::internal::IOManipEigen::IOManipEigen ( const cplx z, double chop = qpp::chop ) [inline],[explicit]`

### 7.5.2 Friends And Related Function Documentation

7.5.2.1 `template<typename charT , typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const IOManipEigen & rhs ) [friend]`

The documentation for this class was generated from the following file:

- [internal/classes/iomanip.h](#)

## 7.6 qpp::internal::IOManipPointer< PointerType > Class Template Reference

```
#include <internal/classes/iomanip.h>
```

## Public Member Functions

- [IOManipPointer](#) (const PointerType \*p, const [idx](#) n, const std::string &separator, const std::string &start="[,", const std::string &end="]")
- [IOManipPointer](#) (const [IOManipPointer](#) &)=default
- [IOManipPointer](#) & [operator=](#) (const [IOManipPointer](#) &)=default

## Friends

- template<typename charT , typename traits >  
std::basic\_ostream< charT, traits > & [operator<<](#) (std::basic\_ostream< charT, traits > &os, const [IOManipPointer](#) &rhs)

## 7.6.1 Constructor & Destructor Documentation

- 7.6.1.1 template<typename PointerType> [qpp::internal::IOManipPointer](#)< PointerType >::IOManipPointer ( const PointerType \* p, const [idx](#) n, const std::string & separator, const std::string & start = " [,", const std::string & end = "]" ) [inline],[explicit]
- 7.6.1.2 template<typename PointerType> [qpp::internal::IOManipPointer](#)< PointerType >::IOManipPointer ( const [IOManipPointer](#)< PointerType > & ) [default]

## 7.6.2 Member Function Documentation

- 7.6.2.1 template<typename PointerType> [IOManipPointer](#)& [qpp::internal::IOManipPointer](#)< PointerType >::operator= ( const [IOManipPointer](#)< PointerType > & ) [default]

## 7.6.3 Friends And Related Function Documentation

- 7.6.3.1 template<typename PointerType> template<typename charT , typename traits > std::basic\_ostream<charT, traits>& [operator<<](#) ( std::basic\_ostream< charT, traits > & os, const [IOManipPointer](#)< PointerType > & rhs ) [friend]

The documentation for this class was generated from the following file:

- internal/classes/[iomanip.h](#)

## 7.7 [qpp::internal::IOManipRange](#)< InputIterator > Class Template Reference

```
#include <internal/classes/iomanip.h>
```

## Public Member Functions

- [IOManipRange](#) (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[,", const std::string &end="]")

## Friends

- template<typename charT , typename traits >  
std::basic\_ostream< charT, traits > & [operator<<](#) (std::basic\_ostream< charT, traits > &os, const [IOManipRange](#) &rhs)



### 7.7.1 Constructor & Destructor Documentation

7.7.1.1 `template<typename InputIterator > qpp::internal::IOManipRange< InputIterator >::IOManipRange ( InputIterator first, InputIterator last, const std::string & separator, const std::string & start = " [", const std::string & end = "]" ) [inline],[explicit]`

### 7.7.2 Friends And Related Function Documentation

7.7.2.1 `template<typename InputIterator > template<typename charT , typename traits > std::basic_ostream<charT, traits>& operator<< ( std::basic_ostream< charT, traits > & os, const IOManipRange< InputIterator > & rhs ) [friend]`

The documentation for this class was generated from the following file:

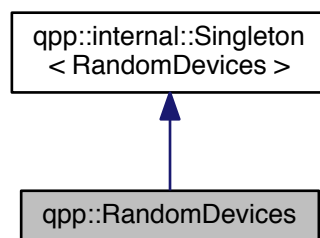
- [internal/classes/iomanip.h](#)

## 7.8 qpp::RandomDevices Class Reference

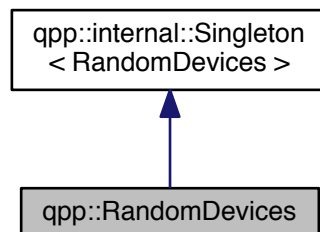
Singleton class that manages the source of randomness in the library.

```
#include <classes/random_devices.h>
```

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



## Public Attributes

- `std::mt19937 _rng`  
*Mersenne twister random number generator.*

## Friends

- class `internal::Singleton< RandomDevices >`

## Additional Inherited Members

### 7.8.1 Detailed Description

Singleton class that manages the source of randomness in the library.

Consists of a wrapper around an `std::mt19937` Mersenne twister random number generator engine and an `std::random_device` engine. The latter is used to seed the Mersenne twister.

#### Warning

This class DOES NOT seed the standard C number generator used by `Eigen::Matrix::Random()`, since it is not thread safe. Do not use `Eigen::Matrix::Random()` or functions that depend on the C style random number engine, but use `qpp::rand()` instead!

### 7.8.2 Friends And Related Function Documentation

7.8.2.1 friend class `internal::Singleton< RandomDevices >` `[friend]`

### 7.8.3 Member Data Documentation

7.8.3.1 `std::mt19937 qpp::RandomDevices::_rng`

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

- `classes/random_devices.h`

## 7.9 `qpp::internal::Singleton< T >` Class Template Reference

`Singleton` policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

### Static Public Member Functions

- static `T & get_instance ()` `noexcept(std::is_nothrow_constructible< T >::value)`
- static `thread_local T & get_thread_local_instance ()` `noexcept(std::is_nothrow_constructible< T >::value)`

## Protected Member Functions

- [Singleton](#) () noexcept=default
- [Singleton](#) (const [Singleton](#) &)=delete
- [Singleton](#) & [operator=](#) (const [Singleton](#) &)=delete
- virtual [~Singleton](#) ()=default

### 7.9.1 Detailed Description

template<typename T>class qpp::internal::Singleton< T >

[Singleton](#) policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from [qpp::internal::Singleton](#), make [qpp::internal::Singleton](#) a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function [qpp::internal::Singleton::get\\_instance\(\)](#) ([qpp::internal::Singleton::get\\_thread\\_local\\_instance\(\)](#)), which returns a reference (thread\_local reference) to your newly created singleton (thread-safe in C++11).

Example:

```
class MySingleton: public qpp::internal::Singleton<MySingleton>
{
    friend class qpp::internal::Singleton<MySingleton>;
public:
    // Declare all public members here
private:
    MySingleton()
    {
        // Implement the constructor here
    }
    ~MySingleton()
    {
        // Implement the destructor here
    }
};

MySingleton& mySingleton = MySingleton::get_instance(); // Get an instance
thread_local MySingleton& tls = MySingleton::get_thread_local_instance();
// Get a thread_local instance
```

See also

Code of [qpp::Codes](#), [qpp::Gates](#), [qpp::Init](#), [qpp::RandomDevices](#), [qpp::States](#) or [qpp.h](#) for real world examples of usage.

### 7.9.2 Constructor & Destructor Documentation

7.9.2.1 template<typename T> [qpp::internal::Singleton< T >::Singleton](#) ( ) [protected], [default], [noexcept]

7.9.2.2 template<typename T> [qpp::internal::Singleton< T >::Singleton](#) ( const [Singleton< T >](#) & ) [protected], [delete]

7.9.2.3 template<typename T> virtual [qpp::internal::Singleton< T >::~~Singleton](#) ( ) [protected], [virtual], [default]

### 7.9.3 Member Function Documentation

7.9.3.1 template<typename T> static T& [qpp::internal::Singleton< T >::get\\_instance](#) ( ) [inline], [static], [noexcept]

7.9.3.2 `template<typename T> static thread_local T& qpp::internal::Singleton< T >::get_thread_local_instance ( )`  
`[inline], [static], [noexcept]`

7.9.3.3 `template<typename T> Singleton& qpp::internal::Singleton< T >::operator= ( const Singleton< T > & )`  
`[protected], [delete]`

The documentation for this class was generated from the following file:

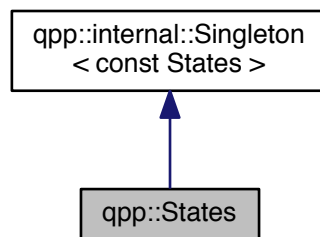
- [internal/classes/singleton.h](#)

## 7.10 qpp::States Class Reference

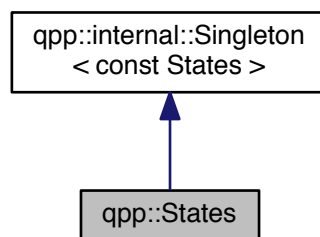
const Singleton class that implements most commonly used states

```
#include <classes/states.h>
```

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



### Public Attributes

- `ket x0 {ket::Zero(2)}`  
*Pauli Sigma-X 0-eigenstate  $|+\rangle$*

- [ket x1](#) {ket::Zero(2)}  
*Pauli Sigma-X 1-eigenstate  $|-\rangle$*
- [ket y0](#) {ket::Zero(2)}  
*Pauli Sigma-Y 0-eigenstate  $|y+\rangle$*
- [ket y1](#) {ket::Zero(2)}  
*Pauli Sigma-Y 1-eigenstate  $|y-\rangle$*
- [ket z0](#) {ket::Zero(2)}  
*Pauli Sigma-Z 0-eigenstate  $|0\rangle$*
- [ket z1](#) {ket::Zero(2)}  
*Pauli Sigma-Z 1-eigenstate  $|1\rangle$*
- [cmat px0](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-X 0-eigenstate  $|+\rangle\langle+|$ .*
- [cmat px1](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-X 1-eigenstate  $|-\rangle\langle-|$ .*
- [cmat py0](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle\langle y+|$ .*
- [cmat py1](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Y 1-eigenstate  $|y-\rangle\langle y-|$ .*
- [cmat pz0](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Z 0-eigenstate  $|0\rangle\langle 0|$ .*
- [cmat pz1](#) {cmat::Zero(2, 2)}  
*Projector onto the Pauli Sigma-Z 1-eigenstate  $|1\rangle\langle 1|$ .*
- [ket b00](#) {ket::Zero(4)}  
*Bell-00 state (following the convention in Nielsen and Chuang)*
- [ket b01](#) {ket::Zero(4)}  
*Bell-01 state (following the convention in Nielsen and Chuang)*
- [ket b10](#) {ket::Zero(4)}  
*Bell-10 state (following the convention in Nielsen and Chuang)*
- [ket b11](#) {ket::Zero(4)}  
*Bell-11 state (following the convention in Nielsen and Chuang)*
- [cmat pb00](#) {cmat::Zero(4, 4)}  
*Projector onto the Bell-00 state.*
- [cmat pb01](#) {cmat::Zero(4, 4)}  
*Projector onto the Bell-01 state.*
- [cmat pb10](#) {cmat::Zero(4, 4)}  
*Projector onto the Bell-10 state.*
- [cmat pb11](#) {cmat::Zero(4, 4)}  
*Projector onto the Bell-11 state.*
- [ket GHZ](#) {ket::Zero(8)}  
*GHZ state.*
- [ket W](#) {ket::Zero(8)}  
*W state.*
- [cmat pGHZ](#) {cmat::Zero(8, 8)}  
*Projector onto the GHZ state.*
- [cmat pW](#) {cmat::Zero(8, 8)}  
*Projector onto the W state.*

## Friends

- class [internal::Singleton](#)< [const States](#) >

## Additional Inherited Members

### 7.10.1 Detailed Description

const Singleton class that implements most commonly used states

### 7.10.2 Friends And Related Function Documentation

7.10.2.1 `friend class internal::Singleton< const States > [friend]`

### 7.10.3 Member Data Documentation

7.10.3.1 `ket qpp::States::b00 {ket::Zero(4)}`

Bell-00 state (following the convention in Nielsen and Chuang)

7.10.3.2 `ket qpp::States::b01 {ket::Zero(4)}`

Bell-01 state (following the convention in Nielsen and Chuang)

7.10.3.3 `ket qpp::States::b10 {ket::Zero(4)}`

Bell-10 state (following the convention in Nielsen and Chuang)

7.10.3.4 `ket qpp::States::b11 {ket::Zero(4)}`

Bell-11 state (following the convention in Nielsen and Chuang)

7.10.3.5 `ket qpp::States::GHZ {ket::Zero(8)}`

GHZ state.

7.10.3.6 `cmat qpp::States::pb00 {cmat::Zero(4, 4)}`

Projector onto the Bell-00 state.

7.10.3.7 `cmat qpp::States::pb01 {cmat::Zero(4, 4)}`

Projector onto the Bell-01 state.

7.10.3.8 `cmat qpp::States::pb10 {cmat::Zero(4, 4)}`

Projector onto the Bell-10 state.

7.10.3.9 `cmat qpp::States::pb11 {cmat::Zero(4, 4)}`

Projector onto the Bell-11 state.

7.10.3.10 `cmat qpp::States::pGHZ {cmat::Zero(8, 8)}`

Projector onto the GHZ state.

7.10.3.11 `cmat qpp::States::pW {cmat::Zero(8, 8)}`

Projector onto the W state.

7.10.3.12 `cmat qpp::States::px0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-X 0-eigenstate  $|+\rangle\langle+|$ .

7.10.3.13 `cmat qpp::States::px1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-X 1-eigenstate  $|-\rangle\langle-|$ .

7.10.3.14 `cmat qpp::States::py0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle\langle y+|$ .

7.10.3.15 `cmat qpp::States::py1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Y 1-eigenstate  $|y-\rangle\langle y-|$ .

7.10.3.16 `cmat qpp::States::pz0 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Z 0-eigenstate  $|0\rangle\langle 0|$ .

7.10.3.17 `cmat qpp::States::pz1 {cmat::Zero(2, 2)}`

Projector onto the Pauli Sigma-Z 1-eigenstate  $|1\rangle\langle 1|$ .

7.10.3.18 `ket qpp::States::W {ket::Zero(8)}`

W state.

7.10.3.19 `ket qpp::States::x0 {ket::Zero(2)}`

Pauli Sigma-X 0-eigenstate  $|+\rangle$

7.10.3.20 `ket qpp::States::x1 {ket::Zero(2)}`

Pauli Sigma-X 1-eigenstate  $|-\rangle$

7.10.3.21 `ket qpp::States::y0 {ket::Zero(2)}`

Pauli Sigma-Y 0-eigenstate  $|y+\rangle$

### 7.10.3.22 `ket qpp::States::y1 {ket::Zero(2)}`

Pauli Sigma-Y 1-eigenstate  $|y\rangle$

### 7.10.3.23 `ket qpp::States::z0 {ket::Zero(2)}`

Pauli Sigma-Z 0-eigenstate  $|0\rangle$

### 7.10.3.24 `ket qpp::States::z1 {ket::Zero(2)}`

Pauli Sigma-Z 1-eigenstate  $|1\rangle$

The documentation for this class was generated from the following file:

- [classes/states.h](#)

## 7.11 `qpp::Timer` Class Reference

Measures time.

```
#include <classes/timer.h>
```

### Public Member Functions

- [Timer](#) () noexcept  
*Constructs an instance with the current time as the starting point.*
- void [tic](#) () noexcept  
*Resets the chronometer.*
- const [Timer](#) & [toc](#) () noexcept  
*Stops the chronometer.*
- double [seconds](#) () const noexcept  
*Time passed in seconds.*

### Protected Attributes

- `std::chrono::steady_clock::time_point` [\\_start](#)
- `std::chrono::steady_clock::time_point` [\\_end](#)

### Friends

- `template<typename charT, typename traits>`  
`std::basic_ostream< charT, traits > & operator<< (std::basic_ostream< charT, traits > &os, const Timer &rhs)`  
*Overload for std::ostream operators.*

### 7.11.1 Detailed Description

Measures time.

Uses a `std::chrono::steady_clock`. It is not affected by wall clock changes during runtime.



## 7.11.2 Constructor & Destructor Documentation

### 7.11.2.1 qpp::Timer::Timer ( ) [inline], [noexcept]

Constructs an instance with the current time as the starting point.

## 7.11.3 Member Function Documentation

### 7.11.3.1 double qpp::Timer::seconds ( ) const [inline], [noexcept]

Time passed in seconds.

#### Returns

Number of seconds that passed between the instantiation/reset and invocation of [qpp::Timer::toc\(\)](#)

### 7.11.3.2 void qpp::Timer::tic ( ) [inline], [noexcept]

Resets the chronometer.

Resets the starting/ending point to the current time

### 7.11.3.3 const Timer& qpp::Timer::toc ( ) [inline], [noexcept]

Stops the chronometer.

Set the current time as the ending point

#### Returns

Current instance

## 7.11.4 Friends And Related Function Documentation

### 7.11.4.1 template<typename charT , typename traits > std::basic\_ostream<charT, traits>& operator<< ( std::basic\_ostream< charT, traits > & os, const Timer & rhs ) [friend]

Overload for std::ostream operators.

#### Parameters

<i>os</i>	Output stream
<i>rhs</i>	<a href="#">Timer</a> instance

#### Returns

Writes to the output stream the number of seconds that passed between the instantiation/reset and invocation of [qpp::Timer::toc\(\)](#).

## 7.11.5 Member Data Documentation

### 7.11.5.1 std::chrono::steady\_clock::time\_point qpp::Timer::\_end [protected]

### 7.11.5.2 std::chrono::steady\_clock::time\_point qpp::Timer::\_start [protected]

The documentation for this class was generated from the following file:

- [classes/timer.h](#)

## Chapter 8

# File Documentation

### 8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



#### Classes

- class [qpp::Codes](#)  
*const Singleton class that defines quantum error correcting codes*

#### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

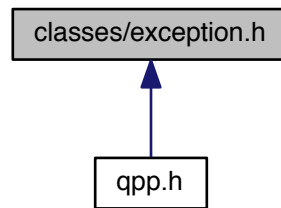
#### 8.1.1 Detailed Description

Quantum error correcting codes.

### 8.2 classes/exception.h File Reference

Exceptions.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Exception](#)  
*Generates custom exceptions, used when validating function parameters.*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### 8.2.1 Detailed Description

Exceptions.

## 8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Gates](#)  
*const Singleton class that implements most commonly used gates*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

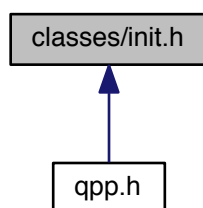
### 8.3.1 Detailed Description

Quantum gates.

## 8.4 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Init](#)  
*const Singleton class that performs additional initializations/cleanups*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### 8.4.1 Detailed Description

Initialization.

## 8.5 classes/random\_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



### Classes

- class [qpp::RandomDevices](#)  
*Singleton class that manages the source of randomness in the library.*

### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

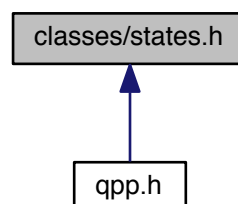
### 8.5.1 Detailed Description

Random devices.

## 8.6 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::States](#)  
*const Singleton class that implements most commonly used states*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

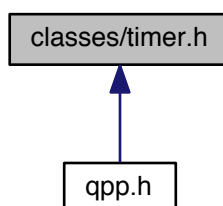
### 8.6.1 Detailed Description

Quantum states.

## 8.7 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::Timer](#)  
*Measures time.*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### 8.7.1 Detailed Description

Timing.

## 8.8 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- constexpr cplx [qpp::operator""\\_i](#) (unsigned long long int x) noexcept  
*User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)*
- constexpr cplx [qpp::operator""\\_i](#) (long double x) noexcept  
*User-defined literal for complex  $i = \sqrt{-1}$  (real overload)*
- cplx [qpp::omega](#) (idx D)  
*D-th root of unity.*

### Variables

- constexpr double [qpp::chop](#) = 1e-10  
*Used in [qpp::disp\(\)](#) for setting to zero numbers that have their absolute value smaller than [qpp::chop](#).*
- constexpr double [qpp::eps](#) = 1e-12  
*Used to decide whether a number or expression in double precision is zero or not.*
- constexpr idx [qpp::maxn](#) = 64  
*Maximum number of allowed qu(d)its (subsystems)*
- constexpr double [qpp::pi](#) = 3.141592653589793238462643383279502884  
 $\pi$
- constexpr double [qpp::ee](#) = 2.718281828459045235360287471352662497  
*Base of natural logarithm, e.*
- constexpr double [qpp::infy](#) = std::numeric\_limits<double>::infinity()  
*Used to denote infinity in double precision.*

### 8.8.1 Detailed Description

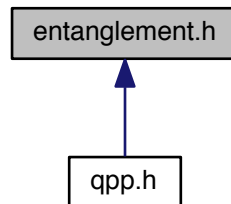
Constants.



## 8.9 entanglement.h File Reference

Entanglement functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >  
dyn_col_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Schmidt coefficients of the bi-partite pure state A.*
- `template<typename Derived >  
cmat qpp::schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Schmidt basis on Alice side.*
- `template<typename Derived >  
cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Schmidt basis on Bob side.*
- `template<typename Derived >  
std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Schmidt probabilities of the bi-partite pure state A.*
- `template<typename Derived >  
double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Entanglement of the bi-partite pure state A.*
- `template<typename Derived >  
double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)`  
*G-concurrence of the bi-partite pure state A.*
- `template<typename Derived >  
double qpp::negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Negativity of the bi-partite mixed state A.*
- `template<typename Derived >  
double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Logarithmic negativity of the bi-partite mixed state A.*

- `template<typename Derived >`  
`double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)`  
*Wootters concurrence of the bi-partite qubit mixed state A.*

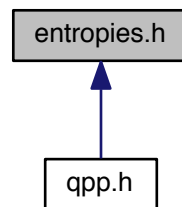
### 8.9.1 Detailed Description

Entanglement functions.

## 8.10 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- `qpp`  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >`  
`double qpp::entropy (const Eigen::MatrixBase< Derived > &A)`  
*von-Neumann entropy of the density matrix A*
- `double qpp::entropy (const std::vector< double > &prob)`  
*Shannon entropy of the probability distribution prob.*
- `template<typename Derived >`  
`double qpp::renyi (const Eigen::MatrixBase< Derived > &A, double alpha)`  
*Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .*
- `double qpp::renyi (const std::vector< double > &prob, double alpha)`  
*Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .*
- `template<typename Derived >`  
`double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)`  
*Tsallis- q entropy of the density matrix A, for  $q \geq 0$ .*
- `double qpp::tsallis (const std::vector< double > &prob, double q)`  
*Tsallis- q entropy of the probability distribution prob, for  $q \geq 0$ .*

- `template<typename Derived >`  
`double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)`

*Quantum mutual information between 2 subsystems of a composite system.*

- `template<typename Derived >`  
`double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)`

*Quantum mutual information between 2 subsystems of a composite system.*

### 8.10.1 Detailed Description

Entropy functions.

## 8.11 experimental/test.h File Reference

Experimental/test functions/classes.

### Namespaces

- `qpp`  
*Quantum++ main namespace.*
- `qpp::experimental`  
*Experimental/test functions/classes, do not use or modify.*

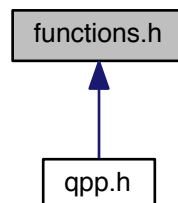
### 8.11.1 Detailed Description

Experimental/test functions/classes.

## 8.12 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



## Namespaces

- [qpp](#)

*Quantum++ main namespace.*

## Functions

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)`  
*Transpose.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)`  
*Complex conjugate.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)`  
*Adjoint.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)`  
*Inverse.*
- `template<typename Derived >`  
`Derived::Scalar qpp::trace (const Eigen::MatrixBase< Derived > &A)`  
*Trace.*
- `template<typename Derived >`  
`Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)`  
*Determinant.*
- `template<typename Derived >`  
`Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > &A)`  
*Logarithm of the determinant.*
- `template<typename Derived >`  
`Derived::Scalar qpp::sum (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise sum of A.*
- `template<typename Derived >`  
`Derived::Scalar qpp::prod (const Eigen::MatrixBase< Derived > &A)`  
*Element-wise product of A.*
- `template<typename Derived >`  
`double qpp::norm (const Eigen::MatrixBase< Derived > &A)`  
*Frobenius norm.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition.*
- `template<typename Derived >`  
`dyn_col_vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvalues.*
- `template<typename Derived >`  
`cmat qpp::evecs (const Eigen::MatrixBase< Derived > &A)`  
*Eigenvectors.*
- `template<typename Derived >`  
`std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)`  
*Full eigen decomposition of Hermitian expression.*
- `template<typename Derived >`  
`dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)`  
*Hermitian eigenvalues.*

- template<typename Derived >  
`cmat qpp::hevects` (const Eigen::MatrixBase< Derived > &A)  
*Hermitian eigenvectors.*
- template<typename Derived >  
`std::tuple< cmat, dyn_col_vect< double >, cmat > qpp::svd` (const Eigen::MatrixBase< Derived > &A)  
*Full singular value decomposition.*
- template<typename Derived >  
`dyn_col_vect< double > qpp::svals` (const Eigen::MatrixBase< Derived > &A)  
*Singular values.*
- template<typename Derived >  
`cmat qpp::svdU` (const Eigen::MatrixBase< Derived > &A)  
*Left singular vectors.*
- template<typename Derived >  
`cmat qpp::svdV` (const Eigen::MatrixBase< Derived > &A)  
*Right singular vectors.*
- template<typename Derived >  
`cmat qpp::funm` (const Eigen::MatrixBase< Derived > &A, cplx(\*f)(const cplx &))  
*Functional calculus  $f(A)$*
- template<typename Derived >  
`cmat qpp::sqrtm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix square root.*
- template<typename Derived >  
`cmat qpp::absm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix absolut value.*
- template<typename Derived >  
`cmat qpp::expm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix exponential.*
- template<typename Derived >  
`cmat qpp::logm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix logarithm.*
- template<typename Derived >  
`cmat qpp::sinm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix sin.*
- template<typename Derived >  
`cmat qpp::cosm` (const Eigen::MatrixBase< Derived > &A)  
*Matrix cos.*
- template<typename Derived >  
`cmat qpp::spectralpowm` (const Eigen::MatrixBase< Derived > &A, const cplx z)  
*Matrix power.*
- template<typename Derived >  
`dyn_mat< typename Derived::Scalar > qpp::powm` (const Eigen::MatrixBase< Derived > &A, idx n)  
*Matrix power.*
- template<typename Derived >  
`double qpp::schatten` (const Eigen::MatrixBase< Derived > &A, double p)  
*Schatten matrix norm.*
- template<typename OutputScalar, typename Derived >  
`dyn_mat< OutputScalar > qpp::cwise` (const Eigen::MatrixBase< Derived > &A, OutputScalar(\*f)(const  
typename Derived::Scalar &))  
*Functor.*
- template<typename T >  
`dyn_mat< typename T::Scalar > qpp::kron` (const T &head)  
*Kronecker product.*

- `template<typename T , typename... Args>`  
`dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)`  
*Kronecker product.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)`  
*Kronecker product.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)`  
*Kronecker product.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)`  
*Kronecker power.*
- `template<typename T >`  
`dyn_mat< typename T::Scalar > qpp::dirsum (const T &head)`  
*Direct sum.*
- `template<typename T , typename... Args>`  
`dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)`  
*Direct sum.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)`  
*Direct sum.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)`  
*Direct sum.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)`  
*Direct sum power.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx cols)`  
*Reshape.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)`  
*Commutator.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)`  
*Anti-commutator.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)`  
*Projector.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)`  
*Gram-Schmidt orthogonalization.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &Vs)`  
*Gram-Schmidt orthogonalization.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)`  
*Gram-Schmidt orthogonalization.*
- `std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)`  
*Non-negative integer index to multi-index.*

- `idx qpp::multiidx2n` (const std::vector< idx > &midx, const std::vector< idx > &dims)  
*Multi-index to non-negative integer index.*
- `ket qpp::mket` (const std::vector< idx > &mask, const std::vector< idx > &dims)  
*Multi-partite qudit ket.*
- `ket qpp::mket` (const std::vector< idx > &mask, idx d=2)  
*Multi-partite qudit ket.*
- `cmat qpp::mprj` (const std::vector< idx > &mask, const std::vector< idx > &dims)  
*Projector onto multi-partite qudit ket.*
- `cmat qpp::mprj` (const std::vector< idx > &mask, idx d=2)  
*Projector onto multi-partite qudit ket.*
- `template<typename InputIterator >`  
`std::vector< double > qpp::abssq` (InputIterator first, InputIterator last)  
*Computes the absolute values squared of a range of complex numbers.*
- `template<typename Derived >`  
`std::vector< double > qpp::abssq` (const Eigen::MatrixBase< Derived > &V)  
*Computes the absolute values squared of a column vector.*
- `template<typename InputIterator >`  
`std::iterator_traits< InputIterator >::value_type qpp::sum` (InputIterator first, InputIterator last)  
*Element-wise sum of an STL-like range.*
- `template<typename Container >`  
`Container::value_type qpp::sum` (const Container &c)  
*Element-wise sum of the elements of an STL-like container.*
- `template<typename InputIterator >`  
`std::iterator_traits< InputIterator >::value_type qpp::prod` (InputIterator first, InputIterator last)  
*Element-wise product of an STL-like range.*
- `template<typename Container >`  
`Container::value_type qpp::prod` (const Container &c)  
*Element-wise product of the elements of an STL-like container.*
- `template<typename Derived >`  
`dyn_col_vect< typename Derived::Scalar > qpp::rho2pure` (const Eigen::MatrixBase< Derived > &A)  
*Finds the pure state representation of a matrix proportional to a projector onto a pure state.*
- `template<typename T >`  
`std::vector< T > qpp::complement` (std::vector< T > subsys, idx N)  
*Constructs the complement of a subsystem vector.*
- `template<typename Derived >`  
`std::vector< double > qpp::rho2bloch` (const Eigen::MatrixBase< Derived > &A)  
*Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.*
- `cmat qpp::bloch2rho` (const std::vector< double > &r)  
*Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.*

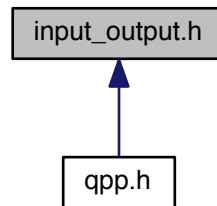
### 8.12.1 Detailed Description

Generic quantum computing functions.

## 8.13 input\_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

## Functions

- `template<typename Derived >`  
`internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)`  
*Eigen expression ostream manipulator.*
- `internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)`  
*Complex number ostream manipulator.*
- `template<typename InputIterator >`  
`internal::IOManipRange< InputIterator > qpp::disp (const InputIterator &first, const InputIterator &last, const std::string &separator, const std::string &start="[", const std::string &end="]")`  
*Range ostream manipulator.*
- `template<typename Container >`  
`internal::IOManipRange< typename Container::const_iterator > qpp::disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]")`  
*Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.*
- `template<typename PointerType >`  
`internal::IOManipPointer< PointerType > qpp::disp (const PointerType *p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")`  
*C-style pointer ostream manipulator.*
- `template<typename Derived >`  
`void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)`  
*Saves Eigen expression to a binary file (internal format) in double precision.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::load (const std::string &fname)`  
*Loads Eigen matrix from a binary file (internal format) in double precision.*

### 8.13.1 Detailed Description

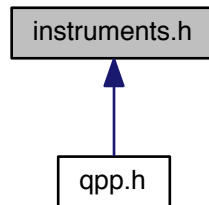
Input/output functions.



## 8.14 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks)`  
*Measures the state A using the set of Kraus operators Ks.*
- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)`  
*Measures the state A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >  
std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)`  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*

- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure` (`const Eigen::MatrixBase< Derived > &A`, `const std::initializer_list< cmat > &Ks`, `const std::vector< idx > &subsys`, `const idx d=2`)  
*Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure` (`const Eigen::MatrixBase< Derived > &A`, `const cmat &U`, `const std::vector< idx > &subsys`, `const std::vector< idx > &dims`)  
*Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure` (`const Eigen::MatrixBase< Derived > &A`, `const cmat &U`, `const std::vector< idx > &subsys`, `const idx d=2`)  
*Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.*
- `template<typename Derived >`  
`std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq` (`const Eigen::MatrixBase< Derived > &A`, `std::vector< idx > subsys`, `std::vector< idx > dims`)  
*Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.*
- `template<typename Derived >`  
`std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq` (`const Eigen::MatrixBase< Derived > &A`, `std::vector< idx > subsys`, `idx d=2`)  
*Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.*

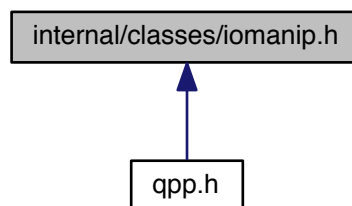
### 8.14.1 Detailed Description

Measurement functions.

## 8.15 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



### Classes

- class `qpp::internal::IOManipRange< InputIterator >`
- class `qpp::internal::IOManipPointer< PointerType >`
- class `qpp::internal::IOManipEigen`

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

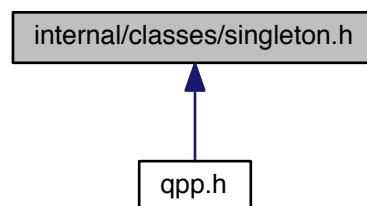
### 8.15.1 Detailed Description

Input/output manipulators.

## 8.16 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



## Classes

- class [qpp::internal::Singleton< T >](#)  
*[Singleton](#) policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)*

## Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

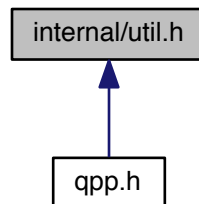
### 8.16.1 Detailed Description

Singleton pattern via CRTP.

## 8.17 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*
- [qpp::internal](#)  
*Internal utility functions, do not use/modify.*

### Functions

- void [qpp::internal::\\_n2multiidx](#) (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx [qpp::internal::\\_multiidx2n](#) (const idx \*midx, idx numdims, const idx \*dims) noexcept
- template<typename Derived >  
bool [qpp::internal::\\_check\\_square\\_mat](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_vector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_rvector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_cvector](#) (const Eigen::MatrixBase< Derived > &A)
- template<typename T >  
bool [qpp::internal::\\_check\\_nonzero\\_size](#) (const T &x) noexcept
- bool [qpp::internal::\\_check\\_dims](#) (const std::vector< idx > &dims)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_mat](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_cvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- template<typename Derived >  
bool [qpp::internal::\\_check\\_dims\\_match\\_rvect](#) (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)
- bool [qpp::internal::\\_check\\_eq\\_dims](#) (const std::vector< idx > &dims, idx dim) noexcept
- bool [qpp::internal::\\_check\\_subsys\\_match\\_dims](#) (const std::vector< idx > &subsys, const std::vector< idx > &dims)

- `template<typename Derived >`  
`bool qpp::internal::\_check\_qubit\_matrix (const Eigen::MatrixBase< Derived > &A) noexcept`
- `template<typename Derived >`  
`bool qpp::internal::\_check\_qubit\_cvector (const Eigen::MatrixBase< Derived > &V) noexcept`
- `template<typename Derived >`  
`bool qpp::internal::\_check\_qubit\_rvector (const Eigen::MatrixBase< Derived > &V) noexcept`
- `template<typename Derived >`  
`bool qpp::internal::\_check\_qubit\_vector (const Eigen::MatrixBase< Derived > &V) noexcept`
- `bool qpp::internal::\_check\_perm (const std::vector< idx > &perm)`
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::internal::\_kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)`
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::internal::\_dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)`
- `template<typename T >`  
`void qpp::internal::variadic\_vector\_emplace (std::vector< T > &)`
- `template<typename T , typename First , typename... Args>`  
`void qpp::internal::variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)`

### 8.17.1 Detailed Description

Internal utility functions.

## 8.18 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `template<typename Derived >`  
`Derived qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)`  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.*
- `template<>`  
`dmat qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)`  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
`cmat qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)`  
*Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*
- `template<typename Derived >`  
`void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)`  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.*

- `template<>`  
 void [qpp::saveMATLABmatrix](#) (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices ([qpp::dmat](#))*
- `template<>`  
 void [qpp::saveMATLABmatrix](#) (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)  
*Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices ([qpp::cmat](#))*

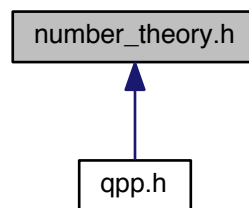
### 8.18.1 Detailed Description

Input/output interfacing with MATLAB.

## 8.19 number\_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- `std::vector< int >` [qpp::x2contfrac](#) (double x, idx n, idx cut=1e5)  
*Simple continued fraction expansion.*
- `double` [qpp::contfrac2x](#) (const std::vector< int > &cf, idx n)  
*Real representation of a simple continued fraction.*
- `double` [qpp::contfrac2x](#) (const std::vector< int > &cf)  
*Real representation of a simple continued fraction.*
- `unsigned long long int` [qpp::gcd](#) (unsigned long long int m, unsigned long long int n)  
*Greatest common divisor of two non-negative integers.*
- `unsigned long long int` [qpp::gcd](#) (const std::vector< unsigned long long int > &ns)  
*Greatest common divisor of a list of non-negative integers.*
- `unsigned long long int` [qpp::lcm](#) (unsigned long long int m, unsigned long long int n)

- Least common multiple of two positive integers.*
  - unsigned long long int [qpp::lcm](#) (const std::vector< unsigned long long int > &ns)
- Least common multiple of a list of positive integers.*
  - std::vector< idx > [qpp::invperm](#) (const std::vector< idx > &perm)
- Inverse permutation.*
  - std::vector< idx > [qpp::compperm](#) (const std::vector< idx > &perm, const std::vector< idx > &sigma)
- Compose permutations.*
  - std::vector< unsigned long long int > [qpp::factors](#) (unsigned long long int n)
- Prime factor decomposition.*
  - bool [qpp::isprime](#) (unsigned long long int n)
- Primality test.*
  - unsigned long long int [qpp::modpow](#) (unsigned long long int a, unsigned long long int n, unsigned long long int p)
- Integer power modulo p.*

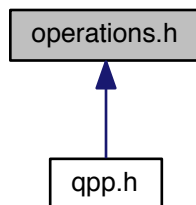
### 8.19.1 Detailed Description

Number theory functions.

## 8.20 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- template<typename Derived1 , typename Derived2 >  
dyn\_mat< typename Derived1::Scalar > [qpp::applyCTRL](#) (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const std::vector< idx > &dims)  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*

- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx d=2)`  
*Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived1 , typename Derived2 >`  
`dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)`  
*Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.*
- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)`  
*Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.*
- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*
- `template<typename Derived >`  
`cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)`  
*Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.*
- `cmat qpp::kraus2super (const std::vector< cmat > &Ks)`  
*Superoperator matrix.*
- `cmat qpp::kraus2choi (const std::vector< cmat > &Ks)`  
*Choi matrix.*
- `std::vector< cmat > qpp::choi2kraus (const cmat &A)`  
*Orthogonal Kraus operators from Choi matrix.*
- `cmat qpp::choi2super (const cmat &A)`  
*Converts Choi matrix to superoperator matrix.*
- `cmat qpp::super2choi (const cmat &A)`  
*Converts superoperator matrix to Choi matrix.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace1 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)`  
*Partial trace.*
- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)`  
*Partial trace.*



- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const`  
`std::vector< idx > &subsys, const std::vector< idx > &dims)`

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const`  
`std::vector< idx > &subsys, idx d=2)`

*Partial transpose.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const`  
`std::vector< idx > &perm, const std::vector< idx > &dims)`

*Subsystem permutation.*

- `template<typename Derived >`  
`dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const`  
`std::vector< idx > &perm, idx d=2)`

*Subsystem permutation.*

### 8.20.1 Detailed Description

Quantum operation functions.

## 8.21 qpp.h File Reference

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

## Namespaces

- [qpp](#)

*Quantum++ main namespace.*

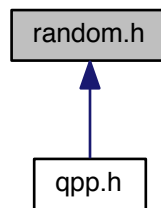
### 8.21.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

## 8.22 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



### Namespaces

- [qpp](#)  
*Quantum++ main namespace.*

### Functions

- double [qpp::rand](#) (double a=0, double b=1)  
*Generates a random real number uniformly distributed in the interval [a, b]*
- idx [qpp::randidx](#) (idx a=std::numeric\_limits< idx >::min(), idx b=std::numeric\_limits< idx >::max())  
*Generates a random index (idx) uniformly distributed in the interval [a, b].*
- template<typename Derived >  
Derived [qpp::rand](#) (idx rows, idx cols, double a=0, double b=1)  
*Generates a random matrix with entries uniformly distributed in the interval [a, b]*
- template<>  
dmat [qpp::rand](#) (idx rows, idx cols, double a, double b)  
*Generates a random real matrix with entries uniformly distributed in the interval [a, b], specialization for double matrices ([qpp::dmat](#))*
- template<>  
cmat [qpp::rand](#) (idx rows, idx cols, double a, double b)  
*Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b], specialization for complex matrices ([qpp::cmat](#))*
- template<typename Derived >  
Derived [qpp::randn](#) (idx rows, idx cols, double mean=0, double sigma=1)  
*Generates a random matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$*
- template<>  
dmat [qpp::randn](#) (idx rows, idx cols, double mean, double sigma)  
*Generates a random real matrix with entries normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for double matrices ([qpp::dmat](#))*

- `template<>`  
`cmat qpp::randn` (idx rows, idx cols, double mean, double sigma)  
*Generates a random complex matrix with entries (both real and imaginary) normally distributed in  $N(\text{mean}, \text{sigma})$ , specialization for complex matrices (`qpp::cmat`)*
- `double qpp::randn` (double mean=0, double sigma=1)  
*Generates a random real number (double) normally distributed in  $N(\text{mean}, \text{sigma})$*
- `cmat qpp::randU` (idx D)  
*Generates a random unitary matrix.*
- `cmat qpp::randV` (idx Din, idx Dout)  
*Generates a random isometry matrix.*
- `std::vector< cmat > qpp::randkraus` (idx N, idx D)  
*Generates a set of random Kraus operators.*
- `cmat qpp::randH` (idx D)  
*Generates a random Hermitian matrix.*
- `ket qpp::randket` (idx D)  
*Generates a random normalized ket (pure state vector)*
- `cmat qpp::randrho` (idx D)  
*Generates a random density matrix.*
- `std::vector< idx > qpp::randperm` (idx n)  
*Generates a random uniformly distributed permutation.*

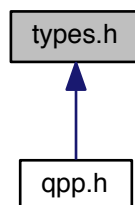
### 8.22.1 Detailed Description

Randomness-related functions.

## 8.23 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



## Namespaces

- `qpp`  
*Quantum++ main namespace.*

## Typedefs

- using [qpp::idx](#) = std::size\_t  
*Non-negative integer index.*
- using [qpp::cplx](#) = std::complex< double >  
*Complex number in double precision.*
- using [qpp::ket](#) = Eigen::VectorXcd  
*Complex (double precision) dynamic Eigen column vector.*
- using [qpp::bra](#) = Eigen::RowVectorXcd  
*Complex (double precision) dynamic Eigen row vector.*
- using [qpp::cmat](#) = Eigen::MatrixXcd  
*Complex (double precision) dynamic Eigen matrix.*
- using [qpp::dmat](#) = Eigen::MatrixXd  
*Real (double precision) dynamic Eigen matrix.*
- template<typename Scalar >  
using [qpp::dyn\\_mat](#) = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >  
*Dynamic Eigen matrix over the field specified by Scalar.*
- template<typename Scalar >  
using [qpp::dyn\\_col\\_vect](#) = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >  
*Dynamic Eigen column vector over the field specified by Scalar.*
- template<typename Scalar >  
using [qpp::dyn\\_row\\_vect](#) = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >  
*Dynamic Eigen row vector over the field specified by Scalar.*

### 8.23.1 Detailed Description

Type aliases.



# Index

- `_check_cvector`
  - `qpp::internal`, 75
- `_check_dims`
  - `qpp::internal`, 75
- `_check_dims_match_cvect`
  - `qpp::internal`, 75
- `_check_dims_match_mat`
  - `qpp::internal`, 75
- `_check_dims_match_rvect`
  - `qpp::internal`, 75
- `_check_eq_dims`
  - `qpp::internal`, 75
- `_check_nonzero_size`
  - `qpp::internal`, 75
- `_check_perm`
  - `qpp::internal`, 75
- `_check_qubit_cvector`
  - `qpp::internal`, 75
- `_check_qubit_matrix`
  - `qpp::internal`, 75
- `_check_qubit_rvector`
  - `qpp::internal`, 75
- `_check_qubit_vector`
  - `qpp::internal`, 75
- `_check_rvector`
  - `qpp::internal`, 75
- `_check_square_mat`
  - `qpp::internal`, 75
- `_check_subsys_match_dims`
  - `qpp::internal`, 75
- `_check_vector`
  - `qpp::internal`, 75
- `_dirsum2`
  - `qpp::internal`, 75
- `_end`
  - `qpp::Timer`, 99
- `_kron2`
  - `qpp::internal`, 76
- `_multiidx2n`
  - `qpp::internal`, 76
- `_n2multiidx`
  - `qpp::internal`, 76
- `_rng`
  - `qpp::RandomDevices`, 92
- `_start`
  - `qpp::Timer`, 99
- `~Singleton`
  - `qpp::internal::Singleton`, 93
- `absm`
  - `qpp`, 24
- `abssq`
  - `qpp`, 25
- `adjoint`
  - `qpp`, 25
- `anticomm`
  - `qpp`, 25
- `apply`
  - `qpp`, 26, 27
- `applyCTRL`
  - `qpp`, 27, 28
- `b00`
  - `qpp::States`, 96
- `b01`
  - `qpp::States`, 96
- `b10`
  - `qpp::States`, 96
- `b11`
  - `qpp::States`, 96
- `bloch2rho`
  - `qpp`, 28
- `bra`
  - `qpp`, 23
- `CNOT`
  - `qpp::Gates`, 87
- `CNOTba`
  - `qpp::Gates`, 87
- `CTRL`
  - `qpp::Gates`, 84
- `CUSTOM_EXCEPTION`
  - `qpp::Exception`, 81
- `CZ`
  - `qpp::Gates`, 87
- `choi2kraus`
  - `qpp`, 28
- `choi2super`
  - `qpp`, 29
- `chop`
  - `qpp`, 73
- `classes/codes.h`, 101
- `classes/exception.h`, 101
- `classes/gates.h`, 102
- `classes/init.h`, 103
- `classes/random_devices.h`, 104
- `classes/states.h`, 104
- `classes/timer.h`, 105
- `cmat`
  - `qpp`, 23

- codeword
  - qpp::Codes, 78
- comm
  - qpp, 29
- complement
  - qpp, 29
- compperm
  - qpp, 30
- concurrency
  - qpp, 30
- conjugate
  - qpp, 30
- constants.h, 106
- contrac2x
  - qpp, 30, 31
- cosm
  - qpp, 31
- cplx
  - qpp, 23
- cwise
  - qpp, 31
- DIMS\_INVALID
  - qpp::Exception, 81
- DIMS\_MISMATCH\_CVECTOR
  - qpp::Exception, 81
- DIMS\_MISMATCH\_MATRIX
  - qpp::Exception, 81
- DIMS\_MISMATCH\_RVECTOR
  - qpp::Exception, 81
- DIMS\_MISMATCH\_VECTOR
  - qpp::Exception, 81
- DIMS\_NOT\_EQUAL
  - qpp::Exception, 81
- det
  - qpp, 31
- dirsum
  - qpp, 32, 33
- dirsumpow
  - qpp, 33
- disp
  - qpp, 33, 34
- dmat
  - qpp, 24
- dyn\_col\_vect
  - qpp, 24
- dyn\_mat
  - qpp, 24
- dyn\_row\_vect
  - qpp, 24
- ee
  - qpp, 73
- eig
  - qpp, 36
- entanglement
  - qpp, 36
- entanglement.h, 107
- entropies.h, 108
- entropy
  - qpp, 36, 37
- eps
  - qpp, 73
- evals
  - qpp, 37
- evects
  - qpp, 37
- Exception
  - qpp::Exception, 81
- expandout
  - qpp::Gates, 85
- experimental/test.h, 109
- expm
  - qpp, 37
- FIVE\_QUBIT
  - qpp::Codes, 78
- FRED
  - qpp::Gates, 87
- factors
  - qpp, 38
- Fd
  - qpp::Gates, 85
- functions.h, 109
- funm
  - qpp, 38
- GHZ
  - qpp::States, 96
- gcd
  - qpp, 38
- gconcurrency
  - qpp, 39
- get\_instance
  - qpp::internal::Singleton, 93
- get\_thread\_local\_instance
  - qpp::internal::Singleton, 93
- grams
  - qpp, 39, 40
- H
  - qpp::Gates, 87
- heig
  - qpp, 40
- hevals
  - qpp, 40
- hevects
  - qpp, 40
- IOManipEigen
  - qpp::internal::IOManipEigen, 89
- IOManipPointer
  - qpp::internal::IOManipPointer, 90
- IOManipRange
  - qpp::internal::IOManipRange, 91
- Id
  - qpp::Gates, 85
- Id2



- qpp::Gates, 87
- idx
  - qpp, 24
- infty
  - qpp, 73
- input\_output.h, 113
- instruments.h, 115
- internal/classes/iomanip.h, 116
- internal/classes/singleton.h, 117
- internal/util.h, 118
- internal::Singleton< const Codes >
  - qpp::Codes, 79
- internal::Singleton< const Gates >
  - qpp::Gates, 87
- internal::Singleton< const Init >
  - qpp::Init, 89
- internal::Singleton< const States >
  - qpp::States, 96
- internal::Singleton< RandomDevices >
  - qpp::RandomDevices, 92
- inverse
  - qpp, 41
- invperm
  - qpp, 41
- isprime
  - qpp, 41
- ket
  - qpp, 24
- kraus2choi
  - qpp, 41
- kraus2super
  - qpp, 42
- kron
  - qpp, 42, 43
- kronpow
  - qpp, 43
- lcm
  - qpp, 44
- load
  - qpp, 44
- loadMATLABmatrix
  - qpp, 45
- logdet
  - qpp, 46
- logm
  - qpp, 46
- lognegativity
  - qpp, 46
- MATLAB/matlab.h, 119
- MATRIX\_MISMATCH\_SUBSYS
  - qpp::Exception, 81
- MATRIX\_NOT\_CVECTOR
  - qpp::Exception, 80
- MATRIX\_NOT\_RVECTOR
  - qpp::Exception, 80
- MATRIX\_NOT\_SQUARE
  - qpp::Exception, 80
- MATRIX\_NOT\_SQUARE\_OR\_CVECTOR
  - qpp::Exception, 80
- MATRIX\_NOT\_SQUARE\_OR\_RVECTOR
  - qpp::Exception, 80
- MATRIX\_NOT\_SQUARE\_OR\_VECTOR
  - qpp::Exception, 80
- MATRIX\_NOT\_VECTOR
  - qpp::Exception, 80
- maxn
  - qpp, 73
- measure
  - qpp, 47–50
- measure\_seq
  - qpp, 50, 51
- mket
  - qpp, 51, 52
- modpow
  - qpp, 52
- mprj
  - qpp, 52
- multiidx2n
  - qpp, 53
- n2multiidx
  - qpp, 53
- NINE\_QUBIT\_SHOR
  - qpp::Codes, 78
- NO\_CODEWORD
  - qpp::Exception, 81
- NOT\_BIPARTITE
  - qpp::Exception, 81
- NOT\_QUBIT\_CVECTOR
  - qpp::Exception, 81
- NOT\_QUBIT\_MATRIX
  - qpp::Exception, 81
- NOT\_QUBIT\_RVECTOR
  - qpp::Exception, 81
- NOT\_QUBIT\_SUBSYS
  - qpp::Exception, 81
- NOT\_QUBIT\_VECTOR
  - qpp::Exception, 81
- negativity
  - qpp, 53
- norm
  - qpp, 54
- number\_theory.h, 120
- OUT\_OF\_RANGE
  - qpp::Exception, 81
- omega
  - qpp, 54
- operations.h, 121
- operator<<
  - qpp::Timer, 99
  - qpp::internal::IOManipEigen, 89
  - qpp::internal::IOManipPointer, 90
  - qpp::internal::IOManipRange, 91
- operator=

- qpp::internal::IOManipPointer, 90
  - qpp::internal::Singleton, 94
- operator""\_i
  - qpp, 54
- PERM\_INVALID
  - qpp::Exception, 81
- PERM\_MISMATCH\_DIMS
  - qpp::Exception, 81
- pGHZ
  - qpp::States, 96
- pW
  - qpp::States, 97
- pb00
  - qpp::States, 96
- pb01
  - qpp::States, 96
- pb10
  - qpp::States, 96
- pb11
  - qpp::States, 96
- pi
  - qpp, 73
- powm
  - qpp, 54
- prj
  - qpp, 55
- prod
  - qpp, 55
- ptrace
  - qpp, 56
- ptrace1
  - qpp, 56
- ptrace2
  - qpp, 58
- ptranspose
  - qpp, 58
- px0
  - qpp::States, 97
- px1
  - qpp::States, 97
- py0
  - qpp::States, 97
- py1
  - qpp::States, 97
- pz0
  - qpp::States, 97
- pz1
  - qpp::States, 97
- qmutualinfo
  - qpp, 59
- qpp, 13
  - absm, 24
  - abssq, 25
  - adjoint, 25
  - anticomm, 25
  - apply, 26, 27
  - applyCTRL, 27, 28
  - bloch2rho, 28
  - bra, 23
  - choi2kraus, 28
  - choi2super, 29
  - chop, 73
  - cmat, 23
  - comm, 29
  - complement, 29
  - compperm, 30
  - concurrence, 30
  - conjugate, 30
  - contfrac2x, 30, 31
  - cosm, 31
  - cplx, 23
  - cwise, 31
  - det, 31
  - dirsum, 32, 33
  - dirsumpow, 33
  - disp, 33, 34
  - dmat, 24
  - dyn\_col\_vect, 24
  - dyn\_mat, 24
  - dyn\_row\_vect, 24
  - ee, 73
  - eig, 36
  - entanglement, 36
  - entropy, 36, 37
  - eps, 73
  - evals, 37
  - evects, 37
  - expm, 37
  - factors, 38
  - funm, 38
  - gcd, 38
  - gconcurrence, 39
  - grams, 39, 40
  - heig, 40
  - hevals, 40
  - hevects, 40
  - idx, 24
  - infty, 73
  - inverse, 41
  - invperm, 41
  - isprime, 41
  - ket, 24
  - kraus2choi, 41
  - kraus2super, 42
  - kron, 42, 43
  - kronpow, 43
  - lcm, 44
  - load, 44
  - loadMATLABmatrix, 45
  - logdet, 46
  - logm, 46
  - lognegativity, 46
  - maxn, 73
  - measure, 47–50
  - measure\_seq, 50, 51

- mket, [51](#), [52](#)
- modpow, [52](#)
- mprj, [52](#)
- multiidx2n, [53](#)
- n2multiidx, [53](#)
- negativity, [53](#)
- norm, [54](#)
- omega, [54](#)
- operator""\_i, [54](#)
- pi, [73](#)
- powm, [54](#)
- prj, [55](#)
- prod, [55](#)
- ptrace, [56](#)
- ptrace1, [56](#)
- ptrace2, [58](#)
- ptranspose, [58](#)
- qmutualinfo, [59](#)
- rand, [59](#), [60](#)
- randH, [60](#)
- randU, [63](#)
- randV, [63](#)
- randidx, [61](#)
- randket, [61](#)
- randkraus, [61](#)
- randn, [61](#), [62](#)
- randperm, [63](#)
- randrho, [63](#)
- renyi, [64](#)
- reshape, [64](#)
- rho2bloch, [65](#)
- rho2pure, [65](#)
- save, [65](#)
- saveMATLABmatrix, [66](#)
- schatten, [66](#)
- schmidtA, [67](#)
- schmidtB, [67](#)
- schmidtcoeffs, [67](#)
- schmidtprobs, [68](#)
- sinm, [68](#)
- spectralpowm, [68](#)
- sqrtn, [69](#)
- sum, [69](#)
- super2choi, [70](#)
- svals, [70](#)
- svd, [70](#)
- svdU, [70](#)
- svdV, [71](#)
- syspermute, [71](#)
- trace, [71](#)
- transpose, [72](#)
- tsallis, [72](#)
- x2contfrac, [72](#)
- qpp.h, [123](#)
- qpp::Codes, [77](#)
  - codeword, [78](#)
  - FIVE\_QUBIT, [78](#)
  - internal::Singleton< const Codes >, [79](#)
  - NINE\_QUBIT\_SHOR, [78](#)
  - SEVEN\_QUBIT\_STEANE, [78](#)
  - Type, [78](#)
- qpp::Exception, [79](#)
  - CUSTOM\_EXCEPTION, [81](#)
  - DIMS\_INVALID, [81](#)
  - DIMS\_MISMATCH\_CVECTOR, [81](#)
  - DIMS\_MISMATCH\_MATRIX, [81](#)
  - DIMS\_MISMATCH\_RVECTOR, [81](#)
  - DIMS\_MISMATCH\_VECTOR, [81](#)
  - DIMS\_NOT\_EQUAL, [81](#)
  - Exception, [81](#)
  - MATRIX\_MISMATCH\_SUBSYS, [81](#)
  - MATRIX\_NOT\_CVECTOR, [80](#)
  - MATRIX\_NOT\_RVECTOR, [80](#)
  - MATRIX\_NOT\_SQUARE, [80](#)
  - MATRIX\_NOT\_SQUARE\_OR\_CVECTOR, [80](#)
  - MATRIX\_NOT\_SQUARE\_OR\_RVECTOR, [80](#)
  - MATRIX\_NOT\_SQUARE\_OR\_VECTOR, [80](#)
  - MATRIX\_NOT\_VECTOR, [80](#)
  - NO\_CODEWORD, [81](#)
  - NOT\_BIPARTITE, [81](#)
  - NOT\_QUBIT\_CVECTOR, [81](#)
  - NOT\_QUBIT\_MATRIX, [81](#)
  - NOT\_QUBIT\_RVECTOR, [81](#)
  - NOT\_QUBIT\_SUBSYS, [81](#)
  - NOT\_QUBIT\_VECTOR, [81](#)
  - OUT\_OF\_RANGE, [81](#)
  - PERM\_INVALID, [81](#)
  - PERM\_MISMATCH\_DIMS, [81](#)
  - SUBSYS\_MISMATCH\_DIMS, [81](#)
  - TYPE\_MISMATCH, [81](#)
  - Type, [80](#)
  - UNDEFINED\_TYPE, [81](#)
  - UNKNOWN\_EXCEPTION, [80](#)
  - what, [82](#)
  - ZERO\_SIZE, [80](#)
- qpp::Gates, [82](#)
  - CNOT, [87](#)
  - CNOTba, [87](#)
  - CTRL, [84](#)
  - CZ, [87](#)
  - expandout, [85](#)
  - FRED, [87](#)
  - Fd, [85](#)
  - H, [87](#)
  - Id, [85](#)
  - Id2, [87](#)
  - internal::Singleton< const Gates >, [87](#)
  - Rn, [86](#)
  - S, [87](#)
  - SWAP, [87](#)
  - T, [87](#)
  - TOF, [87](#)
  - X, [87](#)
  - Xd, [86](#)
  - Y, [87](#)
  - Z, [88](#)

- Zd, 86
- qpp::Init, 88
  - internal::Singleton< const Init >, 89
- qpp::RandomDevices, 91
  - \_rng, 92
  - internal::Singleton< RandomDevices >, 92
- qpp::States, 94
  - b00, 96
  - b01, 96
  - b10, 96
  - b11, 96
  - GHZ, 96
  - internal::Singleton< const States >, 96
  - pGHZ, 96
  - pW, 97
  - pb00, 96
  - pb01, 96
  - pb10, 96
  - pb11, 96
  - px0, 97
  - px1, 97
  - py0, 97
  - py1, 97
  - pz0, 97
  - pz1, 97
  - W, 97
  - x0, 97
  - x1, 97
  - y0, 97
  - y1, 97
  - z0, 98
  - z1, 98
- qpp::Timer, 98
  - \_end, 99
  - \_start, 99
  - operator<<, 99
  - seconds, 99
  - tic, 99
  - Timer, 99
  - toc, 99
- qpp::experimental, 73
- qpp::internal, 74
  - \_check\_cvector, 75
  - \_check\_dims, 75
  - \_check\_dims\_match\_cvect, 75
  - \_check\_dims\_match\_mat, 75
  - \_check\_dims\_match\_rvect, 75
  - \_check\_eq\_dims, 75
  - \_check\_nonzero\_size, 75
  - \_check\_perm, 75
  - \_check\_qubit\_cvector, 75
  - \_check\_qubit\_matrix, 75
  - \_check\_qubit\_rvector, 75
  - \_check\_qubit\_vector, 75
  - \_check\_rvector, 75
  - \_check\_square\_mat, 75
  - \_check\_subsys\_match\_dims, 75
  - \_check\_vector, 75
  - \_dirsum2, 75
  - \_kron2, 76
  - \_multiidx2n, 76
  - \_n2multiidx, 76
  - variadic\_vector\_emplace, 76
- qpp::internal::IOManipEigen, 89
  - IOManipEigen, 89
  - operator<<, 89
- qpp::internal::IOManipPointer
  - IOManipPointer, 90
  - operator<<, 90
  - operator=, 90
- qpp::internal::IOManipPointer< PointerType >, 89
- qpp::internal::IOManipRange
  - IOManipRange, 91
  - operator<<, 91
- qpp::internal::IOManipRange< InputIterator >, 90
- qpp::internal::Singleton
  - ~Singleton, 93
  - get\_instance, 93
  - get\_thread\_local\_instance, 93
  - operator=, 94
  - Singleton, 93
- qpp::internal::Singleton< T >, 92
- rand
  - qpp, 59, 60
- randH
  - qpp, 60
- randU
  - qpp, 63
- randV
  - qpp, 63
- randidx
  - qpp, 61
- randket
  - qpp, 61
- randkraus
  - qpp, 61
- randn
  - qpp, 61, 62
- random.h, 125
- randperm
  - qpp, 63
- randrho
  - qpp, 63
- renyi
  - qpp, 64
- reshape
  - qpp, 64
- rho2bloch
  - qpp, 65
- rho2pure
  - qpp, 65
- Rn
  - qpp::Gates, 86
- S
  - qpp::Gates, 87

SEVEN\_QUBIT\_STEANE  
     qpp::Codes, 78  
 SUBSYS\_MISMATCH\_DIMS  
     qpp::Exception, 81  
 SWAP  
     qpp::Gates, 87  
 save  
     qpp, 65  
 saveMATLABmatrix  
     qpp, 66  
 schatten  
     qpp, 66  
 schmidtA  
     qpp, 67  
 schmidtB  
     qpp, 67  
 schmidtcoeffs  
     qpp, 67  
 schmidtprobs  
     qpp, 68  
 seconds  
     qpp::Timer, 99  
 Singleton  
     qpp::internal::Singleton, 93  
 sinm  
     qpp, 68  
 spectralpowm  
     qpp, 68  
 sqrtm  
     qpp, 69  
 sum  
     qpp, 69  
 super2choi  
     qpp, 70  
 svals  
     qpp, 70  
 svd  
     qpp, 70  
 svdU  
     qpp, 70  
 svdV  
     qpp, 71  
 syspermute  
     qpp, 71  
 T  
     qpp::Gates, 87  
 TOF  
     qpp::Gates, 87  
 TYPE\_MISMATCH  
     qpp::Exception, 81  
 tic  
     qpp::Timer, 99  
 Timer  
     qpp::Timer, 99  
 toc  
     qpp::Timer, 99  
 trace  
     qpp, 71  
 transpose  
     qpp, 72  
 tsallis  
     qpp, 72  
 Type  
     qpp::Codes, 78  
     qpp::Exception, 80  
 types.h, 126  
 UNDEFINED\_TYPE  
     qpp::Exception, 81  
 UNKNOWN\_EXCEPTION  
     qpp::Exception, 80  
 variadic\_vector\_emplace  
     qpp::internal, 76  
 W  
     qpp::States, 97  
 what  
     qpp::Exception, 82  
 X  
     qpp::Gates, 87  
 x0  
     qpp::States, 97  
 x1  
     qpp::States, 97  
 x2contfrac  
     qpp, 72  
 Xd  
     qpp::Gates, 86  
 Y  
     qpp::Gates, 87  
 y0  
     qpp::States, 97  
 y1  
     qpp::States, 97  
 Z  
     qpp::Gates, 88  
 z0  
     qpp::States, 98  
 z1  
     qpp::States, 98  
 ZERO\_SIZE  
     qpp::Exception, 80  
 Zd  
     qpp::Gates, 86