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## Chapter 1

### Quantum++

Version 0.8.1 - development

Development branch, use it at your own risk!

Switch to the master branch for the latest stable version.

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. It uses the Eigen 3 linear algebra library and, if available, the OpenMP multi-processing library. For additional Eigen 3 documentation see http://eigen.tuxfamily.org/dox/. For a simple Eigen 3 quick AS CII reference see http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance.

If you are interesting in contributing, please contact me. To contribute, you need to have a decent knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and some working experience with Eigen 3.

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### **Building instructions**

#### Configuration:

- Compiler: g++ version 4.8 or later (for good C++11 support)
- Eigen 3 library located in \$HOME/eigen
- Quantum++ library located in \$HOME/qpp
- MATLAB compiler include header files: /Applications/MATLAB\_R2014b.app/extern/include
- MATLAB compiler shared library files: /Applications/MATLAB\_R2014b.app/bin/maci64

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#### Building without a build system

- Example file: \$HOME/qpp/examples/example.cpp
- Output executable: \$HOME/qpp/examples/example
- Must run the commands below from inside the directory \$HOME/qpp/examples

### Release version (without ${\tt MATLAB}$ support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Debug version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

#### Release version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2014b.app/extern/include \
    -L/Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

#### Debug version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2014b.app/extern/include \
    -L /Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

#### Building using cmake

The current version of the repository has a ./CMakeLists.txt configuration file for building examples using cmake. To build an example using cmake, I recommend an out-of-source build, i.e., from the root of the project (where ./include is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the relase version (default) executable qpp, from the source file ./examples/example.cpp, without MATLAB support (default), inside the directory ./build. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build  \begin{tabular}{ll} $\operatorname{rm}$ - & rf & $\star$ \\ $\operatorname{cmake}$ - DCMAKE_BUILD_TYPE=Debug - DWITH_MATLAB=ON .. \\ $\operatorname{make}$ \\ \end{tabular}
```

#### Or, to disable OpenMP support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the ./CMakeLists.txt file. See also ./CMakeLists.txt for additional options. Do not forget to remove everything from the ./build directory before a fresh build!

#### **Additional remarks**

- The C++ compiler must be C++11 compliant.
- If using Windows, I recommend compiling under cygwin via cmake and g++. See also http←://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2 for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file <cmath> using the provided patch./cmath\_cygwin.patch.
- If your compiler does not support OpenMP (as it is the case e.g with clang++), disable OpenMP in your build, as otherwise the linker may not find the gomp library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD\_L← IBRARY\_PATH is set to point to the MATLAB compiler library location, see the run\_OSX\_MATLAB script. Otherwise, you will get a runtime error like dyld: Library not loaded: @rpath/libmat.← dylib.

```
* I recommend running via a script, as otherwise setting the 
'DYLD_LIBRARY_PATH' globally may interfere with 
[macports] (https://www.macports.org/)' [cmake] (http://www.cmake.org/) 
installation (in case you use [cmake] (http://www.cmake.org/) from 
[macports] (https://www.macports.org/)). If you use a script, 
then the environment variable is local to the script and 
does not interfere with the rest of the system.

* Example of running script, run from inside the directory where 
the executable 'qpp' is located:

#!/bin/sh # Run Quantum++ under OS X with MATLAB support

export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:"/Applications/MATLAB_R2014b.app/bin/maci64"
./qpp
```

• If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See http://stackoverflow.← com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks for more details about this problem.

Quantum++

## Chapter 2

# Namespace Index

## 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

qpp	
Quantum++ main namespace	13
qpp::experimental	
Experimental/test functions/classes, do not use or modify	77
qpp::internal	
Internal utility functions, do not use/modify	77

6 Namespace Index

# **Chapter 3**

## **Hierarchical Index**

## 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

sta::exception
qpp::Exception
false_type
qpp::is_complex< T >
qpp::is_iterable < T, typename >
qpp::is_matrix_expression< Derived >
qpp::IDisplay
qpp::internal::IOManipEigen
$qpp::internal::IOManipPointer < PointerType > \dots $
qpp::internal::IOManipRange< InputIterator >
qpp::Timer
$qpp::internal::Singleton < T > \dots \dots$
$qpp::internal::Singleton < const \ Codes > \dots $
qpp::Codes
qpp::internal::Singleton < const Gates >
qpp::Gates
qpp::internal::Singleton < const Init >
qpp::Init
qpp::internal::Singleton < const States >
qpp::States
qpp::internal::Singleton < RandomDevices >
qpp::RandomDevices
true_type
qpp::is_complex < std::complex < T >>
qpp::is_iterable< T, my_void_t< decltype(std::declval< T >().begin()), decltype(std::declval< T
>().end()), typename T::value_type >>
qpp::is_matrix_expression< typename Eigen::MatrixBase< Derived >>

8 **Hierarchical Index** 

# **Chapter 4**

## **Class Index**

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

qpp::Codes	
Const Singleton class that defines quantum error correcting codes	81
qpp::Exception	
Generates custom exceptions, used when validating function parameters	83
qpp::Gates	
Const Singleton class that implements most commonly used gates	87
qpp::IDisplay	
Abstract class (interface) that mandates the definition of virtual std::ostream& display(std←	
::ostream& os) const	93
qpp::Init	
Const Singleton class that performs additional initializations/cleanups	95
qpp::internal::IOManipEigen	97
qpp::internal::IOManipPointer< PointerType >	98
qpp::internal::IOManipRange< InputIterator >	100
qpp::is_complex < T >	
Checks whether the type is a complex type	102
qpp::is_complex < std::complex < T > >	
Checks whether the type is a complex number type, specialization for complex types	103
qpp::is_iterable< T, typename >	
Checks whether T is compatible with an STL-like iterable container	104
qpp::is_iterable< T, my_void_t< decltype(std::declval< T >().begin()), decltype(std::declval< T >().↔	
end()), typename T::value_type >>	
Checks whether T is compatible with an STL-like iterable container, specialization for STL-like	
iterable containers	105
qpp::is_matrix_expression< Derived >	
Checks whether the type is an Eigen matrix expression	107
qpp::is_matrix_expression< typename Eigen::MatrixBase< Derived >>	
Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expres-	
sions	108
qpp::RandomDevices	
Singeleton class that manages the source of randomness in the library	109
qpp::internal::Singleton< T >	
Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously	
recurring template pattern)	111
qpp::States	
Const Singleton class that implements most commonly used states	112
qpp::Timer	
Chronometer	117

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# **Chapter 5**

# File Index

### 5.1 File List

Here is a	list of	all files	with brief	descriptions
-----------	---------	-----------	------------	--------------

constants.h
Constants
entanglement.h
Entanglement functions
entropies.h
Entropy functions
functions.h
Generic quantum computing functions
input_output.h
Input/output functions
instruments.h
Measurement functions
number_theory.h
Number theory functions
operations.h
Quantum operation functions
qpp.h
Quantum++ main header file, includes all other necessary headers
random.h
Randomness-related functions
traits.h
types.h
Type aliases
classes/codes.h
Quantum error correcting codes
classes/exception.h
Exceptions
classes/gates.h
Quantum gates
classes/idisplay.h
Display interface via the non-virtual interface (NVI)
classes/init.h
Initialization
classes/random_devices.h
Random devices
classes/states.h
Quantum states 125

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classes/timer.h	
Timing	26
experimental/test.h	
Experimental/test functions/classes	30
internal/util.h	
Internal utility functions	39
internal/classes/iomanip.h	
Input/output manipulators	37
internal/classes/singleton.h	
Singleton pattern via CRTP	38
MATLAB/matlab.h	
Input/output interfacing with MATLAB	40

## **Chapter 6**

## **Namespace Documentation**

### 6.1 qpp Namespace Reference

Quantum++ main namespace.

### **Namespaces**

· experimental

Experimental/test functions/classes, do not use or modify.

internal

Internal utility functions, do not use/modify.

### Classes

class Codes

const Singleton class that defines quantum error correcting codes

class Exception

Generates custom exceptions, used when validating function parameters.

• class Gates

const Singleton class that implements most commonly used gates

class IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

class Init

const Singleton class that performs additional initializations/cleanups

· struct is\_complex

Checks whether the type is a complex type.

- struct is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

• struct is\_iterable

Checks whether T is compatible with an STL-like iterable container.

struct is\_iterable< T, my\_void\_t< decltype(std::declval< T >().begin()), decltype(std::declval< T >().end()), typename T::value\_type > >

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

· struct is matrix expression

Checks whether the type is an Eigen matrix expression.

struct is matrix expression< typename Eigen::MatrixBase< Derived > >

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

· class RandomDevices

Singeleton class that manages the source of randomness in the library.

· class States

const Singleton class that implements most commonly used states

· class Timer

Chronometer.

### **Typedefs**

```
    template < class... >
        using my_void_t = void
        Alias template that implements the proposal for void_t.
```

• using idx = std::size\_t

Non-negative integer index.

• using bigint = long long int

Big integer.

• using ubigint = unsigned long long int

Non-negative big integer.

• using cplx = std::complex < double >

Complex number in double precision.

• using ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

using bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$ 

```
using dyn_mat = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >
```

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

```
using dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
```

Dynamic Eigen column vector over the field specified by Scalar.

template<typename Scalar >

```
using dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
```

Dynamic Eigen row vector over the field specified by Scalar.

#### **Functions**

```
    constexpr cplx operator"" i (unsigned long long int x) noexcept
```

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

constexpr cplx operator""\_i (long double x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

cplx omega (idx D)

D-th root of unity.

• template<typename Derived >

dyn\_col\_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
&dims)

Schmidt coefficients of the bi-partite pure state A.

```
• template<typename Derived >
  cmat schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Schmidt basis on Alice side.

    template<typename Derived >

  cmat schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Schmidt basis on Bob side.
• template<typename Derived >
  std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
  &dims)
     Schmidt probabilities of the bi-partite pure state A.

    template<typename Derived >

  double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Entanglement of the bi-partite pure state A.

    template<typename Derived >

  double gconcurrence (const Eigen::MatrixBase< Derived > &A)
      G-concurrence of the bi-partite pure state A.

    template<typename Derived >

  double negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Negativity of the bi-partite mixed state A.

    template<typename Derived >

  double lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Logarithmic negativity of the bi-partite mixed state A.
• template<typename Derived >
  double concurrence (const Eigen::MatrixBase< Derived > &A)
      Wootters concurrence of the bi-partite qubit mixed state A.
• template<typename Derived >
  double entropy (const Eigen::MatrixBase< Derived > &A)
      von-Neumann entropy of the density matrix A

    double entropy (const std::vector< double > &prob)

     Shannon entropy of the probability distribution prob.

    template<typename Derived >

  double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)
      Renyi- \alpha entropy of the density matrix A, for \alpha \geq 0.

    double renyi (const std::vector< double > &prob, double alpha)

     Renyi- \alpha entropy of the probability distribution prob, for \alpha \geq 0.

    template < typename Derived >

  double tsallis (const Eigen::MatrixBase< Derived > &A, double q)
      Tsallis- q entropy of the density matrix A, for q \ge 0.

    double tsallis (const std::vector< double > &prob, double q)

      Tsallis- q entropy of the probability distribution prob, for q \geq 0.
• template<typename Derived >
  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, const std::vector< idx > &dims)
      Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  double gmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector < idx > &subsysB, idx d=2)
      Quantum mutual information between 2 subsystems of a composite system.
template<typename Derived >
  dyn mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
```

Left singular vectors.

```
Complex conjugate.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)
     Adjoint.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar det (const Eigen::MatrixBase < Derived > &A)
     Determinant.

    template<typename Derived >

  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum of A.

    template<typename Derived >

  Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)
     Element-wise product of A.

    template<typename Derived >

  double norm (const Eigen::MatrixBase< Derived > &A)
     Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition.
template<typename Derived >
  dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)
     Eigenvalues.
• template<typename Derived >
  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.

    template<typename Derived >

  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn_col_vect< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.

    template<typename Derived >

  dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.
• template<typename Derived >
  cmat svdU (const Eigen::MatrixBase< Derived > &A)
```

```
• template<typename Derived >
  cmat svdV (const Eigen::MatrixBase< Derived > &A)
     Right singular vectors.

    template<typename Derived >

  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
• template<typename Derived >
  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.

    template<typename Derived >

  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template < typename Derived >

  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.
• template<typename Derived >
  cmat sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
template<typename Derived >
  cmat cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.
• template<typename Derived >
  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.
ullet template<typename Derived >
  double schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn_mat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const type-
  name Derived::Scalar &))
     Functor.

    template<typename T >

  dyn mat< typename T::Scalar > kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn mat< typename T::Scalar > kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > kron (const std::initializer_list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
```

```
• template<typename T >
  dyn_mat< typename T::Scalar > dirsum (const T &head)
     Direct sum.

    template<typename T, typename... Args>

  dyn mat< typename T::Scalar > dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsum (const std::vector< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsum (const std::initializer_list< Derived > &As)
      Direct sum.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
      Direct sum power.
\bullet \ \ {\it template}{<} {\it typename Derived}>
  dyn_mat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx
  cols)
     Reshape.
 template<typename Derived1 , typename Derived2 >
  dyn mat< typename Derived1::Scalar > comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::↔
  MatrixBase< Derived2 > &B)
      Commutator.
ullet template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > grams (const std::initializer list< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)
      Gram-Schmidt orthogonalization.

    std::vector< idx > n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx multiidx2n (const std::vector < idx > &midx, const std::vector < idx > &dims)

     Multi-index to non-negative integer index.

    ket mket (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Multi-partite qudit ket.

    ket mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
```

std::vector< double > abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of an STL-like range of complex numbers.

• template<typename Container >

std::vector< double > abssq (const Container &c, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Computes the absolute values squared of an STL-like container.

• template<typename Derived >

std::vector< double > abssq (const Eigen::MatrixBase< Derived > &A)

Computes the absolute values squared of an Eigen expression.

template<typename InputIterator >

std::iterator traits< InputIterator >::value type sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

template<typename Container >

Container::value\_type sum (const Container &c)

Element-wise sum of the elements of an STL-like container.

• template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

template<typename Container >

Container::value\_type prod (const Container &c)

Element-wise product of the elements of an STL-like container.

template<typename InputIterator >

std::conditional < is\_complex < typename std::iterator\_traits < InputIterator >::value\_type >::value, cplx, double >::type avg (InputIterator first, InputIterator last)

Average of the elements of an STL-like range.

• template<typename Container >

std::conditional< is\_complex< typename Container::value\_type >::value, cplx, double >::type avg (const Container &c)

Average of the elements of an STL-like container.

template<typename Derived >

std::conditional < is\_complex < typename Derived::Scalar >::value, cplx, double >::type avg (const Eigen ← ::MatrixBase < Derived > &A)

Average of the elements of an Eigen expression.

• template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

 $\bullet \ \ \text{template}{<} \text{typename T} >$ 

```
std::vector< T > complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

• template<typename Derived >

```
std::vector< double > rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

• template<typename Derived >

internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

Eigen expression ostream manipulator.

internal::IOManipEigen disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

• template<typename InputIterator >

internal::IOManipRange< InputIterator > disp (const InputIterator &first, const InputIterator &last, const std ::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >
 internal::IOManipRange< typename Container::const\_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >

internal::IOManipPointer< PointerType > disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

• template<typename Derived >

void save (const Eigen::MatrixBase < Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

• template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure\_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

• template<typename Derived >

Derived loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (gpp::dmat)

template

cmat loadMATLABmatrix (const std::string &mat file, const std::string &var name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >

void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std
::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

• template<>

void saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void saveMATLABmatrix (const Eigen::MatrixBase < cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

• std::vector< int > x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

double contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• ubigint gcd (ubigint m, ubigint n)

Greatest common divisor of two non-negative integers.

ubigint gcd (const std::vector< ubigint > &ns)

Greatest common divisor of a list of non-negative integers.

• ubigint lcm (ubigint m, ubigint n)

Least common multiple of two positive integers.

ubigint lcm (const std::vector< ubigint > &ns)

Least common multiple of a list of positive integers.

std::vector< idx > invperm (const std::vector< idx > &perm)

Inverse permutation.

• std::vector< idx > compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< ubigint > factors (ubigint n)

Prime factor decomposition.

• bool isprime (ubigint n)

Primality test.

ubigint modpow (ubigint a, ubigint n, ubigint p)

Integer power modulo p.

• template<typename Derived1 , typename Derived2 >

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx
d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >

dyn\_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived >

cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

• template<typename Derived >

cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

template<typename Derived >

cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat kraus2choi (const std::vector < cmat > &Ks)

Choi matrix.

std::vector< cmat > choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

template<typename Derived >

 $dyn_mat < typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase < Derived > &A, const std <math>\leftarrow ::vector < idx > &dims)$ 

Partial trace.

• template<typename Derived >

 $dyn_mat$ < typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std $\leftrightarrow$  ::vector< idx > &dims)

Partial trace.

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$ 

dyn\_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &subsys, const std::vector< idx > &dims)

Partial trace.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector
idx > &subsys, idx d=2)

Partial trace.

template<typename Derived >

Partial transpose.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial transpose.

template<typename Derived >

 $\frac{dyn_mat}{dyn_mat}$  typename Derived::Scalar >  $\frac{dyn_mat}{dyn_mat}$  (const Eigen::MatrixBase< Derived > &A, const  $\frac{dy}{dyn_mat}$ ) ::vector<  $\frac{dx}{dy}$  > &dims)

Subsystem permutation.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &perm, idx d=2)

Subsystem permutation.

double rand (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

• bigint rand (bigint a=std::numeric\_limits< bigint >::min(), bigint b=std::numeric\_limits< bigint >::max())

Generates a random big integer uniformly distributed in the interval [a, b].

• ubigint rand (ubigint a=std::numeric\_limits< ubigint >::min(), ubigint b=std::numeric\_limits< ubigint >↔ ::max())

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

idx randidx (idx a=std::numeric\_limits< idx >::min(), idx b=std::numeric\_limits< idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

• template<typename Derived >

Derived rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

template<typename Derived >

Derived randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

template<>

cmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat randU (idx D)

Generates a random unitary matrix.

cmat randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat randH (idx D)

Generates a random Hermitian matrix.

ket randket (idx D)

Generates a random normalized ket (pure state vector)

cmat randrho (idx D)

Generates a random density matrix.

std::vector< idx > randperm (idx n)

Generates a random uniformly distributed permutation.

## **Variables**

• constexpr double chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

• constexpr idx maxn = 64

Maximum number of allowed qu(d)its (subsystems)

• constexpr double pi = 3.141592653589793238462643383279502884

 $\pi$ 

• constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

constexpr double infty = std::numeric\_limits<double>::infinity()

Used to denote infinity in double precision.

## 6.1.1 Detailed Description

Quantum++ main namespace.

## 6.1.2 Typedef Documentation

6.1.2.1 using qpp::bigint = typedef long long int

Big integer.

6.1.2.2 using qpp::bra = typedef Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

6.1.2.3 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.2.4 using qpp::cplx = typedef std::complex < double >

Complex number in double precision.

6.1.2.5 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.2.6 template<typename Scalar > using qpp::dyn\_col\_vect = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by Scalar.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

6.1.2.7 template<typename Scalar > using qpp::dyn\_mat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic,
Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

6.1.2.8 template < typename Scalar > using qpp::dyn row vect = typedef Eigen::Matrix < Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

6.1.2.9 using qpp::idx = typedef std::size\_t

Non-negative integer index.

6.1.2.10 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

6.1.2.11 template < class... > using qpp::my\_void\_t = typedef void

Alias template that implements the proposal for void\_t.

See also

```
http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2014/n3911
```

6.1.2.12 using qpp::ubigint = typedef unsigned long long int

Non-negative big integer.

## **6.1.3 Function Documentation**

6.1.3.1 template < typename Derived > cmat qpp::absm ( const Eigen::MatrixBase < Derived > & A )

Matrix absolut value.

Α	Eigen expression
---	------------------

## Returns

Matrix absolut value of A

6.1.3.2 template<typename InputIterator > std::vector<double> qpp::abssq ( InputIterator first, InputIterator last )

Computes the absolute values squared of an STL-like range of complex numbers.

#### **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

#### Returns

Real vector consisting of the range absolut values squared

6.1.3.3 template<typename Container > std::vector<double> qpp::abssq ( const Container & c, typename std::enable\_if< is\_iterable< Container >::value >::type \* = nullptr )

Computes the absolute values squared of an STL-like container.

## **Parameters**

c STL-like container
----------------------

## Returns

Real vector consisting of the container's absolut values squared

6.1.3.4 template<typename Derived > std::vector<double> qpp::abssq ( const Eigen::MatrixBase< Derived > & A )

Computes the absolute values squared of an Eigen expression.

### **Parameters**

Α	Eigen expression

### Returns

Real vector consisting of the absolut values squared

6.1.3.5 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )

Adjoint.

A	Eigen expression
/ ·	Ligen expression

#### Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field as A

6.1.3.6 template<typename Derived1 , typename Derived2 > dyn\_mat<typename Derived1::Scalar> qpp::anticomm ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B )

Anti-commutator.

#### See also

## qpp::comm()

Anti-commutator  $\{A, B\} = AB + BA$ . Both A and B must be Eigen expressions over the same scalar field.

#### **Parameters**

Α	Eigen expression
В	Eigen expression

#### Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field as A

6.1.3.7 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > dpp::apply ( const Eigen::MatrixBase < Derived2 > & A, const std::vector < <math>idx > & subsys$ , const std::vector < idx > & dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

## **Parameters**

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

#### Returns

Gate A applied to the part subsys of state

6.1.3.8 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::apply ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & subsys, idx d = 2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

## Returns

Gate A applied to the part subsys of state

6.1.3.9 template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks )

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

## **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators

#### Returns

Output density matrix after the action of the channel

6.1.3.10 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

## **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
dims	Dimensions of the multi-partite system

# Returns

Output density matrix after the action of the channel

6.1.3.11 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho* 

## **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators

subsys	Subsystem indexes where the Kraus operators Ks are applied
d	Subsystem dimensions

#### Returns

Output density matrix after the action of the channel

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

## See also

```
qpp::Gates::CTRL()
```

#### Note

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

#### **Parameters**

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

## Returns

CTRL-A gate applied to the part subsys of state

6.1.3.13 template<typename Derived1 , typename Derived2 > dyn\_mat<typename Derived1::Scalar> qpp::applyCTRL ( const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx d = 2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

### See also

```
qpp::Gates::CTRL()
```

### Note

The dimension of the gate A must match the dimension of subsys

## **Parameters**

state	Eigen expression

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

#### Returns

CTRL-A gate applied to the part subsys of state

```
6.1.3.14 template < typename InputIterator > std::conditional < is_complex < typename std::iterator_traits < Input ← Iterator >::value_type >::value, cplx, double >::type qpp::avg ( InputIterator first, InputIterator last )
```

Average of the elements of an STL-like range.

#### **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

#### Returns

Average of the range, as a scalar over the same scalar field as the range

6.1.3.15 template < typename Container > std::conditional < is\_complex < typename Container::value\_type >::value, cplx, double >::type qpp::avg ( const Container & c )

Average of the elements of an STL-like container.

#### **Parameters**

С	STL-like container

### Returns

Average of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.16 template<typename Derived > std::conditional< is\_complex< typename Derived::Scalar >::value, cplx, double>::type qpp::avg ( const Eigen::MatrixBase< Derived > & A )

Average of the elements of an Eigen expression.

### **Parameters**

Α	Eigen expression

## Returns

Average of the elements of the Eigen expression, as a scalar over the same scalar field as A

6.1.3.17 cmat qpp::bloch2rho ( const std::vector< double > & r ) [inline]

Computes the density matrix corresponding to the 3-dimensional real Bloch vector *r*.

#### See also

qpp::rho2bloch()

```
r 3-dimensional real vector
```

Returns

Qubit density matrix

6.1.3.18 std::vector < cmat > qpp::choi2kraus ( const cmat & A ) [inline]

Orthogonal Kraus operators from Choi matrix.

See also

qpp::kraus2choi()

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix A

Note

The Kraus operators satisfy  $Tr(K_i^\dagger K_j) = \delta_{ij}$  for all  $i \neq j$ 

#### **Parameters**

Α	Choi matrix

Returns

Set of orthogonal Kraus operators

6.1.3.19 cmat qpp::choi2super ( const cmat & A ) [inline]

Converts Choi matrix to superoperator matrix.

See also

qpp::super2choi()

**Parameters** 

```
A Choi matrix
```

Returns

Superoperator matrix

6.1.3.20 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > qpp::comm ( const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B )$ 

Commutator.

See also

qpp::anticomm()

Commutator [A, B] = AB - BA. Both A and B must be Eigen expressions over the same scalar field.

Α	Eigen expression
В	Eigen expression

## Returns

Commutator AB-BA, as a dynamic matrix over the same scalar field as  $\emph{A}$ 

6.1.3.21 template < typename T > std::vector < T > qpp::complement ( std::vector < T > subsys, idx N )

Constructs the complement of a subsystem vector.

#### **Parameters**

subsys	Subsystem vector
N	Total number of systems

## Returns

The complement of *subsys* with respect to the set  $\{0, 1, \dots, N-1\}$ 

6.1.3.22 std::vector<idx> qpp::compperm ( const std::vector< idx> & perm, const std::vector< idx> & sigma ) [inline]

Compose permutations.

#### **Parameters**

perm	Permutation
sigma	Permutation

## Returns

Composition of the permutations *perm* o *sigma* = perm(sigma)

6.1.3.23 template<typename Derived > double qpp::concurrence ( const Eigen::MatrixBase< Derived > & A )

Wootters concurrence of the bi-partite qubit mixed state A.

## **Parameters**

Α	Eigen expression

## Returns

Wootters concurrence

6.1.3.24 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived > & A )

Complex conjugate.

Α	Eigen expression
---	------------------

#### Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field as A

6.1.3.25 double qpp::contfrac2x ( const std::vector < int > & cf, idx n ) [inline]

Real representation of a simple continued fraction.

#### See also

qpp::x2contfrac()

## **Parameters**

cf	Integer vector containing the simple continued fraction expansion
n	Number of terms considered in the continued fraction expansion. If <i>n</i> is greater than the size
	of <i>cf</i> ,then all terms in <i>cf</i> are considered.

## Returns

Real representation of the simple continued fraction

6.1.3.26 double qpp::contfrac2x ( const std::vector < int > & cf ) [inline]

Real representation of a simple continued fraction.

## See also

qpp::x2contfrac()

## **Parameters**

cf	Integer vector containing the simple continued fraction expansion

## Returns

Real representation of the simple continued fraction

6.1.3.27 template < typename Derived > cmat qpp::cosm ( const Eigen::MatrixBase < Derived > & A )

Matrix cos.

## **Parameters**

Α	Eigen expression

## Returns

Matrix cosine of A

6.1.3.28 template<typename OutputScalar , typename Derived > dyn\_mat<OutputScalar> qpp::cwise ( const Eigen::MatrixBase< Derived > & A, OutputScalar(\*)(const typename Derived::Scalar &) f )

Functor.

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

## Returns

Component-wise f(A), as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.29 template<typename Derived > Derived::Scalar qpp::det ( const Eigen::MatrixBase< Derived > & A )

Determinant.

**Parameters** 

Α	Eigen expression

## Returns

Determinant of A, as a scalar over the same scalar field as A. Returns  $\pm \infty$  when the determinant overflows/underflows.

6.1.3.30 template < typename T > dyn\_mat < typename T::Scalar > qpp::dirsum ( const T & head )

Direct sum.

See also

qpp::dirsumpow()

Used to stop the recursion for the variadic template version of qpp::dirsum()

Parameters

head	Eigen expression

## Returns

Its argument head

6.1.3.31 template < typename T , typename... Args > dyn\_mat < typename T::Scalar > qpp::dirsum ( const T & head, const Args &... tail )

Direct sum.

See also

qpp::dirsumpow()

## **Parameters**

head	Eigen expression
------	------------------

tail Variadic Eigen expression (zero or more parameters)

## Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.32 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::dirsum ( const std::vector< Derived > & As )

Direct sum.

See also

qpp::dirsumpow()

### **Parameters**

As	std::vector of Eigen expressions

#### Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Direct sum.

See also

qpp::dirsumpow()

### **Parameters**

As std::initializer\_list of Eigen expressions, such as {A1, A2, ..., Ak}

### Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.34 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::dirsumpow ( const Eigen::MatrixBase < Derived > & A, idx n)

Direct sum power.

See also

qpp::dirsum()

Α	Eigen expression
n	Non-negative integer

#### Returns

Direct sum of A with itself n times  $A^{\oplus n}$ , as a dynamic matrix over the same scalar field as A

6.1.3.35 template < typename Derived > internal::IOManipEigen qpp::disp ( const Eigen::MatrixBase < Derived > & A, double chop = qpp::chop )

Eigen expression ostream manipulator.

#### **Parameters**

Г	Α	Eigen expression
	chop	Set to zero the elements smaller in absolute value than chop

## Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.36 internal::IOManipEigen qpp::disp(cplx z, double chop = qpp::chop) [inline]

Complex number ostream manipulator.

## **Parameters**

Z	Complex number (or any other type implicitly cast-able to std::complex <double>)</double>
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

### Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.37 template < typename InputIterator > internal::IOManipRange < InputIterator > qpp::disp ( const InputIterator & first, const InputIterator & last, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

Range ostream manipulator.

## **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range
separator	Separator
start	Left marking
end	Right marking

## Returns

Instance of qpp::internal::internal::IOManipRange

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

С	Container
separator	Separator
start	Left marking
end	Right marking

#### Returns

Instance of qpp::internal::internal::IOManipRange

6.1.3.39 template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp ( const PointerType \* p, idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

C-style pointer ostream manipulator.

#### **Parameters**

р	Pointer to the first element
n	Number of elements to be displayed
separator	Separator
start	Left marking
end	Right marking

## Returns

Instance of qpp::internal::internal::IOManipPointer

6.1.3.40 template<typename Derived > std::pair<dyn\_col\_vect<cplx>, cmat> qpp::eig ( const Eigen::MatrixBase< Derived > & A )

Full eigen decomposition.

See also

qpp::heig()

## Parameters

Α	Eigen expression

## Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.41 template < typename Derived > double qpp::entanglement ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

## See also

qpp::entropy()

Α	A Eigen expression	
dims	Dimensions of the bi-partite system	

## Returns

Entanglement, with the logarithm in base 2

6.1.3.42 template < typename Derived > double qpp::entropy ( const Eigen::MatrixBase < Derived > & A )

von-Neumann entropy of the density matrix A

**Parameters** 

Α	Eigen expression
---	------------------

### Returns

von-Neumann entropy, with the logarithm in base 2

6.1.3.43 double qpp::entropy ( const std::vector < double > & prob ) [inline]

Shannon entropy of the probability distribution prob.

**Parameters** 

prob	Real probability vector
	rear producting reasons

## Returns

Shannon entropy, with the logarithm in base 2

6.1.3.44 template<typename Derived > dyn\_col\_vect<cplx> qpp::evals ( const Eigen::MatrixBase< Derived > & A )

Eigenvalues.

See also

qpp::hevals()

Parameters

A	Eigen expression
---	------------------

## Returns

Eigenvalues of A, as a complex dynamic column vector

6.1.3.45 template < typename Derived > cmat qpp::evects ( const Eigen::MatrixBase < Derived > & A )

Eigenvectors.

See also

qpp::hevects()

```
A Eigen expression
```

Returns

Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.46 template<typename Derived > cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )

Matrix exponential.

**Parameters** 

```
A Eigen expression
```

Returns

Matrix exponential of A

6.1.3.47 std::vector<ubigint> qpp::factors( ubigint n ) [inline]

Prime factor decomposition.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

### **Parameters**

n	Integer strictly greater than 1

Returns

Integer vector containing the factors

6.1.3.48 template < typename Derived > cmat qpp::funm ( const Eigen::MatrixBase < Derived > & A, cplx(\*)(const cplx &) f

Functional calculus f(A)

### **Parameters**

Α	Eigen expression
f	Pointer-to-function from complex to complex

Returns

f(A)

**6.1.3.49 ubigint qpp::gcd ( ubigint** *m***, ubigint** *n* **)** [inline]

Greatest common divisor of two non-negative integers.

See also

qpp::lcm()

m	Non-negative integer
n	Non-negative integer

## Returns

Greatest common divisor of m and n

6.1.3.50 ubigint qpp::gcd ( const std::vector< ubigint > & ns ) [inline]

Greatest common divisor of a list of non-negative integers.

See also

qpp::lcm()

#### **Parameters**

ns List of non-neg	ative integers
--------------------	----------------

## Returns

Greatest common divisor of all numbers in ns

6.1.3.51 template < typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase < Derived > & A )

G-concurrence of the bi-partite pure state A.

Note

Both local dimensions must be equal

Uses qpp::logdet() to avoid overflows

See also

qpp::logdet()

## **Parameters**

Α	Eigen expression
---	------------------

## Returns

G-concurrence

6.1.3.52 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )

Gram-Schmidt orthogonalization.

Vs std::vector of Eigen expressions as column vectors

#### Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

Gram-Schmidt orthogonalization.

#### **Parameters**

Vs std::initializer\_list of Eigen expressions as column vectors

## Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.54 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const Eigen::MatrixBase< Derived > & A )

Gram-Schmidt orthogonalization.

### **Parameters**

A Eigen expression, the input vectors are the columns of A

## Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field as A

Full eigen decomposition of Hermitian expression.

## See also

qpp::eig()

### **Parameters**

A | Eigen expression

## Returns

Pair of: 1. Eigenvalues of A, as a real dynamic column vector, and 2. Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.56 template<typename Derived > dyn\_col\_vect<double> qpp::hevals ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvalues.

See also

qpp::evals()

**Parameters** 

Returns

Eigenvalues of Hermitian A, as a real dynamic column vector

6.1.3.57 template<typename Derived > cmat qpp::hevects ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvectors.

See also

qpp::evects()

**Parameters** 

```
A Eigen expression
```

Returns

Eigenvectors of Hermitian A, as columns of a complex matrix

6.1.3.58 template < typename Derived >  $dyn_mat$  < typename Derived::Scalar >  $dyn_mat$  < typename Derived::Scalar >  $dyn_mat$  < Derived > & A )

Inverse.

**Parameters** 

```
A Eigen expression
```

Returns

Inverse of A, as a dynamic matrix over the same scalar field as A

6.1.3.59 std::vector<idx> qpp::invperm(const std::vector<idx> & perm) [inline]

Inverse permutation.

**Parameters** 

perm	Permutation

Returns

Inverse of the permutation perm

**6.1.3.60** bool qpp::isprime ( ubigint *n* ) [inline]

Primality test.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

#### **Parameters**

```
n Integer strictly greater than 1
```

#### Returns

True if the number is prime, false otherwise

6.1.3.61 cmat qpp::kraus2choi ( const std::vector < cmat > & Ks ) [inline]

Choi matrix.

#### See also

qpp::choi2kraus()

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $\mathit{Ks}$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|, |0\rangle\langle 1|$  etc.

Note

The superoperator matrix S and the Choi matrix C are related by  $S_{ab,mn}=C_{ma,nb}$ 

### **Parameters**

Ks
----

## Returns

Choi matrix

6.1.3.62 cmat qpp::kraus2super ( const std::vector < cmat > & Ks ) [inline]

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

**Parameters** 

Ks	Set of Kraus operators

## Returns

Superoperator matrix

6.1.3.63 template < typename T > dyn\_mat < typename T::Scalar > qpp::kron ( const T & head )

Kronecker product.

See also

qpp::kronpow()

Used to stop the recursion for the variadic template version of qpp::kron()

head	Eigen expression
------	------------------

## Returns

Its argument head

6.1.3.64 template<typename T , typename... Args> dyn\_mat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )

Kronecker product.

See also

qpp::kronpow()

#### **Parameters**

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

## Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.65 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::kron ( const std::vector< Derived > & As )

Kronecker product.

See also

qpp::kronpow()

## **Parameters**

As	std::vector of Eigen expressions

#### Returns

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.66 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::kron ( const std::initializer\_list< Derived > & As )

Kronecker product.

See also

qpp::kronpow()

As	std::initializer_list of Eigen expressions, such as {A1, A2, ,Ak}
----	---

## Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker power.

See also

qpp::kron()

#### **Parameters**

Α	Eigen expression
n	Non-negative integer

## Returns

Kronecker product of A with itself n times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field as A

6.1.3.68 ubigint qpp::lcm ( ubigint m, ubigint n ) [inline]

Least common multiple of two positive integers.

See also

qpp::gcd()

## **Parameters**

m	Positive integer
n	Positive integer

## Returns

Least common multiple of m and n

6.1.3.69 ubigint qpp::lcm ( const std::vector < ubigint > & ns ) [inline]

Least common multiple of a list of positive integers.

See also

qpp::gcd()

ns	List of positive integers
----	---------------------------

#### Returns

Least common multiple of all numbers in ns

6.1.3.70 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::load ( const std::string & fname )

Loads Eigen matrix from a binary file (internal format) in double precision.

See also

```
qpp::save()
```

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

## Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

#### **Parameters**

Α	Eigen expression
fname	Output file name

6.1.3.71 template<typename Derived > Derived qpp::loadMATLABmatrix ( const std::string & mat\_file, const std::string & var\_name )

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

See also

```
qpp::saveMATLABmatrix()
```

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be loaded)

```
6.1.3.72 template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

#### Note

If var\_name is a complex matrix, only the real part is loaded

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

## Returns

Eigen double dynamic matrix (qpp::dmat)

```
6.1.3.73 template<> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

qpp::saveMATLABmatrix()

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

#### **Parameters**

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

## Returns

Eigen complex dynamic matrix (qpp::cmat)

6.1.3.74 template < typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase < Derived > & A )

Logarithm of the determinant.

Useful when the determinant overflows/underflows

## **Parameters**

Δ	Figen expression
71	Ligen expression

## Returns

Logarithm of the determinant of A, as a scalar over the same scalar field as A

6.1.3.75 template < typename Derived > cmat qpp::logm ( const Eigen::MatrixBase < Derived > & A )

Matrix logarithm.

Α	Eigen expression
---	------------------

## Returns

Matrix logarithm of A

6.1.3.76 template < typename Derived > double qpp::lognegativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Logarithmic negativity of the bi-partite mixed state A.

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.77 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

## **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.78 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks )

Measures the state A using the set of Kraus operators Ks.

## **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.79 template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const cmat & U )

Measures the state A in the orthonormal basis specified by the unitary matrix *U*.

Α	Eigen expression
U	Unitary matrix whose columns represent the measurement basis vectors

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.80 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

#### See also

qpp::measure\_seq()

#### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.81 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

## See also

qpp::measure\_seq()

## Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system
Ks	Set of Kraus operators

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.82 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

#### See also

qpp::measure\_seq()

#### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
Ks	Set of Kraus operators

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.83 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, const idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

## See also

qpp::measure\_seq()

## Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
Ks	Set of Kraus operators

#### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.84 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & U, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis specified by the unitary matrix *U*.

#### See also

qpp::measure\_seq()

#### Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

## **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system
U	Unitary matrix whose columns represent the measurement basis vectors

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.85 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & U, const std::vector < idx > & subsys, const idx d = 2)

Measures the part *subsys* of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

#### See also

qpp::measure\_seq()

### Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
U	Unitary matrix whose columns represent the measurement basis vectors

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

```
6.1.3.86 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, std::vector < idx > dims)
```

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

#### See also

qpp::measure()

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

## Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

```
6.1.3.87 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, idx d = 2)
```

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

### See also

qpp::measure()

## **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

## Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.88 ket qpp::mket ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

#### Parameters 4 8 1

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

#### Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.89 ket qpp::mket ( const std::vector < idx > & mask, idx d=2 ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension *d. mask* is a std::vector of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

## **Parameters**

mask	std::vector of non-negative integers
d	Subsystem dimensions

#### **Returns**

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.90 ubigint qpp::modpow ( ubigint a, ubigint n, ubigint p ) [inline]

Integer power modulo p.

Computes  $a^n \mod p$ 

#### **Parameters**

а	Non-negative integer
n	Non-negative integer
р	Strictly positive integer

## Returns

 $a^n \bmod p$ 

6.1.3.91 cmat qpp::mprj ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

## Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.92 cmat qpp::mprj (const std::vector < idx > & mask, idx d = 2) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

#### **Parameters**

mask	std::vector of non-negative integers
d	Subsystem dimensions

## Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.93 idx qpp::multiidx2n( const std::vector < idx > & midx, const std::vector < idx > & dims ) [inline]

Multi-index to non-negative integer index.

## See also

qpp::n2multiidx()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

## **Parameters**

midx	Multi-index
dims	Dimensions of the multi-partite system

#### Returns

Non-negative integer index

6.1.3.94 std::vector<idx> qpp::n2multiidx ( idx n, const std::vector< idx > & dims ) [inline]

Non-negative integer index to multi-index.

## See also

qpp::multiidx2n()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

n	Non-negative integer index
dims	Dimensions of the multi-partite system

## Returns

Multi-index of the same size as dims

6.1.3.95 template < typename Derived > double qpp::negativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Negativity of the bi-partite mixed state A.

### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Negativity

6.1.3.96 template < typename Derived > double qpp::norm ( const Eigen::MatrixBase < Derived > & A )

Frobenius norm.

**Parameters** 

Α	Eigen expression

## Returns

Frobenius norm of A

6.1.3.97 cplx qpp::omega ( idx D ) [inline]

D-th root of unity.

**Parameters** 

D	Non-negative integer

## Returns

D-th root of unity  $\exp(2\pi i/D)$ 

**6.1.3.98** constexpr cplx qpp::operator""\_i ( unsigned long long int x ) [inline], [noexcept]

User-defined literal for complex  $i=\sqrt{-1}$  (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

**6.1.3.99 constexpr cplx qpp::operator""\_i( long double x )** [inline], [noexcept]

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.100 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase < Derived > & A, idx n)

Matrix power.

See also

qpp::spectralpowm()

Explicitly multiplies the matrix A with itself n times. By convention  $A^0 = I$ .

## **Parameters**

Α	Eigen expression
n	Non-negative integer

#### Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field as A

6.1.3.101 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::prj ( const Eigen::MatrixBase < Derived > & V )

Projector.

Normalized projector onto state vector

**Parameters** 

V	Eigen expression

## Returns

Projector onto the state vector V, or the matrix Zero if V has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field as A

6.1.3.102 template<typename Derived > Derived::Scalar qpp::prod ( const Eigen::MatrixBase< Derived > & A )

Element-wise product of A.

**Parameters** 

Α	Eigen expression

## Returns

Element-wise product of A, as a scalar over the same scalar field as A

6.1.3.103 template < typename InputIterator > std::iterator\_traits < InputIterator > ::value\_type qpp::prod ( InputIterator first, InputIterator last )

Element-wise product of an STL-like range.

first	Iterator to the first element of the range
last	Iterator to the last element of the range

#### Returns

Element-wise product of the range, as a scalar over the same scalar field as the range

6.1.3.104 template < typename Container > Container::value\_type qpp::prod ( const Container & c )

Element-wise product of the elements of an STL-like container.

#### **Parameters**

С	STL-like container

#### Returns

Element-wise product of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.105 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims)

Partial trace.

#### See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

#### Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.106 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2 )

Partial trace.

See also

qpp::ptrace1(), qpp::ptrace2()

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

#### Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.107 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptrace1 ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

#### Returns

Partial trace  $Tr_A(\cdot)$  over the first subsystem A in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.108 template < typename Derived >  $dyn_mat < typename Derived::Scalar > qpp::ptrace2 ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )$ 

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

#### Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

#### Returns

Partial trace  $Tr_B(\cdot)$  over the second subsystem B in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.109 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, const std::vector< idx > & dims )

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

#### Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.110 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2 )

#### Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

#### Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.111 template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, const std::vector< idx > & dims )

Quantum mutual information between 2 subsystems of a composite system.

#### **Parameters**

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
dims	Dimensions of the multi-partite system

#### Returns

Mutual information between the 2 subsystems

6.1.3.112 template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, idx d = 2 )

Quantum mutual information between 2 subsystems of a composite system.

#### **Parameters**

A Eigen expression
--------------------

subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
d	Subsystem dimensions

#### Returns

Mutual information between the 2 subsystems

```
6.1.3.113 double qpp::rand ( double a = 0, double b = 1 ) [inline]
```

Generates a random real number uniformly distributed in the interval [a, b)

#### **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random real number (double) uniformly distributed in the interval [a, b)

Generates a random big integer uniformly distributed in the interval [a, b].

#### **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random big integer uniformly distributed in the interval [a, b]

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

#### **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random non-negative big integer uniformly distributed in the interval [a, b]

6.1.3.116 template < typename Derived > Derived qpp::rand ( idx rows, idx cols, double a = 0, double b = 1 )

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

6.1.3.117 template <> dmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXd, // with entries uniformly distributed in [-1,1) auto mat = rand<dmat>(3, 3, -1, 1);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random real matrix

```
6.1.3.118 template <> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) uniformly distributed in [-1,1) auto mat = rand<cmat>(3, 3, -1, 1);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

#### Returns

Random complex matrix

```
6.1.3.119 cmat qpp::randH(idx D) [inline]
```

Generates a random Hermitian matrix.

# **Parameters**

D	Dimension of the Hilbert space

#### Returns

Random Hermitian matrix

Generates a random index (idx) uniformly distributed in the interval [a, b].

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random index (idx) uniformly distributed in the interval [a, b]

**6.1.3.121** ket qpp::randket (idx D) [inline]

Generates a random normalized ket (pure state vector)

#### **Parameters**

D	Dimension of the Hilbert space

#### Returns

Random normalized ket

6.1.3.122 std::vector<cmat> qpp::randkraus(idx N, idx D) [inline]

Generates a set of random Kraus operators.

Note

The set of Kraus operators satisfy the closure condition  $\sum_i K_i^\dagger K_i = I$ 

#### **Parameters**

_		
	N	Number of Kraus operators
	D	Dimension of the Hilbert space

# Returns

Set of N Kraus operators satisfying the closure condition

6.1.3.123 template < typename Derived > Derived qpp::randn ( idx rows, idx cols, double mean = 0, double sigma = 1 )

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

6.1.3.124 template <> dmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

#### Returns

Random real matrix

```
6.1.3.125 template <> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

#### Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, // with entries (both real and imaginary) normally distributed in N(0,2) auto mat = randn<cmat>(3, 3, 0, 2);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

#### Returns

Random complex matrix

```
6.1.3.126 double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]
```

Generates a random real number (double) normally distributed in N(mean, sigma)

#### **Parameters**

mean	Mean
sigma	Standard deviation

#### Returns

Random real number normally distributed in N(mean, sigma)

```
6.1.3.127 std::vector<idx> qpp::randperm(idx n) [inline]
```

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by std::shuffle), so that all permutations are equally probable

n	Size of the permutation
---	-------------------------

#### Returns

Random permutation of size n

6.1.3.128 cmat qpp::randrho(idx D) [inline]

Generates a random density matrix.

**Parameters** 

```
D Dimension of the Hilbert space
```

# Returns

Random density matrix

6.1.3.129 cmat qpp::randU(idx D) [inline]

Generates a random unitary matrix.

**Parameters** 

D	Dimension of the Hilbert space
---	--------------------------------

#### Returns

Random unitary

6.1.3.130 cmat qpp::randV (idx Din, idx Dout) [inline]

Generates a random isometry matrix.

#### **Parameters**

Din	Size of the input Hilbert space
Dout	Size of the output Hilbert space

# Returns

Random isometry matrix

 $\textbf{6.1.3.131} \quad \textbf{template} < \textbf{typename Derived} > \textbf{double qpp::renyi ( const Eigen::MatrixBase} < \textbf{Derived} > \textbf{\& A, double alpha )}$ 

Renyi-  $\alpha$  entropy of the density matrix  ${\it A}$ , for  $\alpha \geq 0$ .

Note

When  $\alpha \to 1$  the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

Α	Eigen expression
alpha	Non-negative real number, use qpp::infty for $\alpha = \infty$

#### Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.132 double qpp::renyi ( const std::vector < double > & prob, double alpha ) [inline]

Renyi-  $\alpha$  entropy of the probability distribution *prob*, for  $\alpha \geq 0$ .

Note

When  $\alpha \to 1$  the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

#### **Parameters**

prob	Real probability vector
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

#### Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.133 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, idx rows, idx cols )

#### Reshape.

Uses column-major order when reshaping (same as MATLAB)

#### **Parameters**

Α	Eigen expression
rows	Number of rows of the reshaped matrix
cols	Number of columns of the reshaped matrix

### Returns

Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field as A

6.1.3.134 template<typename Derived > std::vector<double> qpp::rho2bloch ( const Eigen::MatrixBase< Derived > & A )

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

#### See also

qpp::bloch2rho()

#### Note

It is implicitly assumed that the density matrix is Hermitian

Α	Eigen expression
---	------------------

#### Returns

3-dimensional Bloch vector

6.1.3.135 template<typename Derived > dyn\_col\_vect<typename Derived::Scalar> qpp::rho2pure ( const Eigen::MatrixBase< Derived > & A )

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

#### Note

No purity check is done, the input state A must have rank one, otherwise the function returns the first non-zero eigenvector of A

#### **Parameters**

Α	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. A is
	assumed to have rank one

#### Returns

The unique non-zero eigenvector of A, as a dynamic column vector over the same scalar field as A

6.1.3.136 template < typename Derived > void qpp::save ( const Eigen::MatrixBase < Derived > & A, const std::string & fname )

Saves Eigen expression to a binary file (internal format) in double precision.

#### See also

qpp::load()

### Parameters

Α	Eigen expression
fname	Output file name

6.1.3.137 template<typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase< Derived > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode )

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

#### See also

qpp::loadMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be saved)

6.1.3.138 template <> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < dmat > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

#### See also

qpp::loadMATLABmatrix()

#### **Parameters**

	Α	Eigen expression over the complex field
	mat_file	MATALB .mat file
	var_name	Variable name in the .mat file representing the matrix to be saved
ĺ	mode	Saving mode (append, overwrite etc.), see MATLAB matOpen() documentation for details

6.1.3.139 template <> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < cmat > & A, const std::string & mat\_file, const std::string & var\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

#### See also

qpp::loadMATLABmatrix()

#### **Parameters**

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB matOpen() documentation for details

6.1.3.140 template < typename Derived > double qpp::schatten ( const Eigen::MatrixBase < Derived > & A, double p )

Schatten matrix norm.

#### **Parameters**

Α	Eigen expression
р	Real number, greater or equal to 1, use qpp::infty for $p=\infty$

#### Returns

Schatten-p matrix norm of A

6.1.3.141 template < typename Derived > cmat qpp::schmidtA ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt basis on Alice side.

**Parameters** 

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Unitary matrix U whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.142 template < typename Derived > cmat qpp::schmidtB ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt basis on Bob side.

#### **Parameters**

A	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Unitary matrix  ${\cal V}$  whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.143 template<typename Derived > dyn\_col\_vect<double> qpp::schmidtcoeffs ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

#### See also

qpp::schmidtprobs()

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

### Returns

Schmidt coefficients of A, as a real dynamic column vector

6.1.3.144 template < typename Derived > std::vector < double > qpp::schmidtprobs ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

#### See also

qpp::schmidtcoeffs()

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Real vector consisting of the Schmidt probabilites of A

6.1.3.145 template < typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase < Derived > & A )

Matrix sin.

**Parameters** 

Α	Eigen expression

#### Returns

Matrix sine of A

6.1.3.146 template < typename Derived > cmat qpp::spectralpowm (const Eigen::MatrixBase < Derived > & A, const cplx z

Matrix power.

See also

qpp::powm()

Uses the spectral decomposition of  $\emph{A}$  to compute the matrix power. By convention  $\emph{A}^0 = \emph{I}$ .

# **Parameters**

Α	Eigen expression
Z	Complex number

# Returns

Matrix power  $A^z$ 

6.1.3.147 template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )

Matrix square root.

**Parameters** 

Α	Eigen expression
---	------------------

# Returns

Matrix square root of A

6.1.3.148 template < typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase < Derived > & A )

Element-wise sum of A.

Α	Eigen expression
---	------------------

#### Returns

Element-wise sum of A, as a scalar over the same scalar field as A

6.1.3.149 template < typename InputIterator > std::iterator\_traits < InputIterator >::value\_type qpp::sum ( InputIterator first, InputIterator last )

Element-wise sum of an STL-like range.

#### **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

#### Returns

Element-wise sum of the range, as a scalar over the same scalar field as the range

6.1.3.150 template<typename Container > Container::value\_type qpp::sum ( const Container & c )

Element-wise sum of the elements of an STL-like container.

#### **Parameters**

С	STL-like container

#### Returns

Element-wise sum of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.151 cmat qpp::super2choi( const cmat & A ) [inline]

Converts superoperator matrix to Choi matrix.

# See also

qpp::choi2super()

# **Parameters**

Α	Superoperator matrix
---	----------------------

#### Returns

Choi matrix

 $6.1.3.152 \quad template < typename \ Derived > dyn\_col\_vect < double > qpp::svals \ ( \ const \ Eigen::MatrixBase < Derived > \& \ A \ )$ 

Singular values.

Α	Eigen expression
---	------------------

#### Returns

Singular values of A, ordered in decreasing order, as a real dynamic column vector

6.1.3.153 template<typename Derived > std::tuple<cmat, dyn\_col\_vect<double>, cmat> qpp::svd ( const Eigen::MatrixBase< Derived > & A )

Full singular value decomposition.

**Parameters** 

```
A Eigen expression
```

#### Returns

Tuple of: 1. Left sigular vectors of A, as columns of a complex dynamic matrix, 2. Singular values of A, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of A, as columns of a complex dynamic matrix

6.1.3.154 template<typename Derived > cmat qpp::svdU ( const Eigen::MatrixBase< Derived > & A )

Left singular vectors.

**Parameters** 

Α	Eigen expression

#### Returns

Complex dynamic matrix, whose columns are the left singular vectors of A

6.1.3.155 template < typename Derived > cmat qpp::svdV ( const Eigen::MatrixBase < Derived > & A )

Right singular vectors.

**Parameters** 

```
A Eigen expression
```

## Returns

Complex dynamic matrix, whose columns are the right singular vectors of A

6.1.3.156 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & perm

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

Α	Eigen expression
perm	Permutation
dims	Dimensions of the multi-partite system

#### Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.157 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2 )

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

#### **Parameters**

Α	Eigen expression
perm	Permutation
d	Subsystem dimensions

#### Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.158 template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )

Trace.

**Parameters** 

4	Fine connection
A	Eigen expression

# Returns

Trace of A, as a scalar over the same scalar field as A

6.1.3.159 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar>  $dyn_mat$ <typename Derived >  $dyn_mat$ <typename Derived >

Transpose.

### **Parameters**

Α	Eigen expression

# Returns

Transpose of A, as a dynamic matrix over the same scalar field as A

6.1.3.160 template<typename Derived > double qpp::tsallis ( const Eigen::MatrixBase< Derived > & A, double q )

Tsallis- q entropy of the density matrix A, for  $q \ge 0$ .

Note

When  $q \to 1$  the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base e

Α	Eigen expression
q	Non-negative real number

#### Returns

Tsallis- q entropy

6.1.3.161 double qpp::tsallis ( const std::vector < double > & prob, double q ) [inline]

Tsallis- q entropy of the probability distribution *prob*, for  $q \ge 0$ .

#### Note

When  $q \to 1$  the Tsallis entropy converges to the Shannon entropy, with the logarithm in base e

#### **Parameters**

prob	Real probability vector
q	Non-negative real number

#### Returns

Tsallis- q entropy

6.1.3.162 std::vector<int> qpp::x2contfrac ( double x, idx n, idx cut = 1e5 ) [inline]

Simple continued fraction expansion.

#### See also

qpp::contfrac2x()

# **Parameters**

X	Real number
n	Number of terms in the expansion
cut	Stop the expansion when the next term is greater than <i>cut</i>

# Returns

Integer vector containing the simple continued fraction expansion of x. If there are m less than n terms in the expansion, a shorter vector with m components is returned.

#### 6.1.4 Variable Documentation

6.1.4.1 constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

6.1.4.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.4.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::eps) // x is zero</pre>
```

6.1.4.4 constexpr double qpp::infty = std::numeric\_limits < double > ::infinity()

Used to denote infinity in double precision.

6.1.4.5 constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

# 6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

# 6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

# 6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

### Classes

- class IOManipEigen
- · class IOManipPointer
- class IOManipRange
- class Singleton

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

### **Functions**

- void \_n2multiidx (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx \_multiidx2n (const idx \*midx, idx numdims, const idx \*dims) noexcept
- template < typename Derived >
   bool \_check\_square\_mat (const Eigen::MatrixBase < Derived > &A)

```
• template<typename Derived >
      bool <u>_check_vector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

      bool <u>_check_rvector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

      bool <u>check_cvector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename T >

      bool check nonzero size (const T &x) noexcept

    bool check dims (const std::vector < idx > &dims)

    template<typename Derived >

      bool _check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

      bool check dims match cvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

    template<typename Derived >

      bool check dims match rvect (const std::vector < idx > &dims, const Eigen::MatrixBase < Derived > &V)
    • bool check eq dims (const std::vector < idx > &dims, idx dim) noexcept

    bool _check_subsys_match_dims (const std::vector < idx > &subsys, const std::vector < idx > &dims)

    template<typename Derived >

      bool <u>_check_qubit_matrix</u> (const Eigen::MatrixBase< Derived > &A) noexcept
    • template<typename Derived >
      bool <u>_check_qubit_cvector</u> (const Eigen::MatrixBase< Derived > &V) noexcept

    template<typename Derived >

      bool check qubit rvector (const Eigen::MatrixBase< Derived > &V) noexcept

    template<typename Derived >

      bool <u>check_qubit_vector</u> (const Eigen::MatrixBase< Derived > &V) noexcept

    bool check perm (const std::vector < idx > &perm)

    template<typename Derived1 , typename Derived2 >

      dyn_mat< typename Derived1::Scalar > _kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen ↔
      ::MatrixBase< Derived2 > &B)

    template<typename Derived1 , typename Derived2 >

      dyn mat< typename Derived1::Scalar > _dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const
      Eigen::MatrixBase< Derived2 > &B)
    • template<typename T >
      void variadic_vector_emplace (std::vector< T > &)
    • template<typename T , typename First , typename... Args>
      void variadic_vector_emplace (std::vector< T > &v, First &&first, Args &&...args)
6.3.1 Detailed Description
```

Internal utility functions, do not use/modify.

#### 6.3.2 Function Documentation

- 6.3.2.1 template < typename Derived > bool qpp::internal::\_check\_cvector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.2 bool qpp::internal::\_check\_dims ( const std::vector < idx > & dims ) [inline]
- 6.3.2.3 template<typename Derived > bool qpp::internal:: check dims match cvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.3.2.4 template < typename Derived > bool qpp::internal::\_check\_dims\_match\_mat ( const std::vector < idx > & dims, const **Eigen::MatrixBase**< Derived > & A )
- 6.3.2.5 template < typename Derived > bool qpp::internal::\_check\_dims\_match\_rvect ( const std::vector < idx > & dims, const Eigen::MatrixBase< Derived > & V )

6.3.2.6 bool qpp::internal::\_check\_eq\_dims ( const std::vector < idx > & dims, idx dim ) [inline], [noexcept] 6.3.2.7 template < typename T > bool qpp::internal::\_check\_nonzero\_size( const T & x ) [noexcept] 6.3.2.8 bool qpp::internal::\_check\_perm ( const std::vector < idx > & perm ) [inline] 6.3.2.9 template<typename Derived > bool qpp::internal::\_check\_qubit\_cvector ( const Eigen::MatrixBase< Derived > & V ) [noexcept] 6.3.2.10 template < typename Derived > bool qpp::internal::\_check\_qubit\_matrix ( const Eigen::MatrixBase < Derived > & A ) [noexcept] template < typename Derived > bool qpp::internal::\_check\_qubit\_rvector ( const Eigen::MatrixBase < Derived > & V ) [noexcept] 6.3.2.12 template < typename Derived > bool qpp::internal::\_check\_qubit\_vector ( const Eigen::MatrixBase < Derived > & V ) [noexcept] 6.3.2.13 template < typename Derived > bool qpp::internal::\_check\_rvector ( const Eigen::MatrixBase < Derived > & A ) 6.3.2.14 template < typename Derived > bool qpp::internal::\_check\_square\_mat ( const Eigen::MatrixBase < Derived > & A ) 6.3.2.15 bool qpp::internal::\_check\_subsys\_match\_dims ( const std::vector < idx > & subsys, const std::vector < idx > & dims ) [inline] 6.3.2.16 template < typename Derived > bool qpp::internal::\_check\_vector ( const Eigen::MatrixBase < Derived > & A ) 6.3.2.17 template<typename Derived1 , typename Derived2 > dyn\_mat<typename Derived1::Scalar> qpp::internal::\_dirsum2 ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B ) 6.3.2.18 template < typename Derived1, typename Derived2 > dyn mat < typename Derived1::Scalar > qpp::internal:: kron2 ( const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B) 6.3.2.19 idx qpp::internal::\_multiidx2n ( const idx \* midx, idx numdims, const idx \* dims ) [inline], [noexcept] 6.3.2.20 void qpp::internal::\_n2multiidx ( idx n, idx numdims, const idx \* dims, idx \* result ) [inline], [noexcept] 6.3.2.21 template < typename T > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > & )

6.3.2.22 template < typename T , typename First , typename... Args > void qpp::internal::variadic\_vector\_emplace (

std::vector < T > & v, First && first, Args &&... args )

Names	pace	Docur	mentatior

# **Chapter 7**

# **Class Documentation**

# 7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

#include <classes/codes.h>

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



# **Public Types**

enum Type { Type::FIVE\_QUBIT = 1, Type::SEVEN\_QUBIT\_STEANE, Type::NINE\_QUBIT\_SHOR }
 Code types, add more codes here if needed.

#### **Public Member Functions**

ket codeword (Type type, idx i) const
 Returns the codeword of the specified code type.

#### **Private Member Functions**

• Codes ()

Default constructor.

Codes ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const Codes >

### **Additional Inherited Members**

# 7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

#### 7.1.2 Member Enumeration Documentation

```
7.1.2.1 enum qpp::Codes::Type [strong]
```

Code types, add more codes here if needed.

See also

```
qpp::Codes::codeword()
```

#### Enumerator

```
FIVE_QUBIT [[5,1,3]] qubit code
SEVEN_QUBIT_STEANE [[7,1,3]] Steane qubit code
NINE_QUBIT_SHOR [[9,1,3]] Shor qubit code
```

# 7.1.3 Constructor & Destructor Documentation

```
7.1.3.1 qpp::Codes::Codes( ) [inline],[private]
```

Default constructor.

```
7.1.3.2 qpp::Codes::~Codes() [private], [default]
```

Default destructor.

### 7.1.4 Member Function Documentation

7.1.4.1 ket qpp::Codes::codeword ( Type type, idx i ) const [inline]

Returns the codeword of the specified code type.

See also

qpp::Codes::Type

#### **Parameters**

type	Code type
i	Codeword index

#### Returns

i-th codeword of the code type

#### 7.1.5 Friends And Related Function Documentation

**7.1.5.1 friend class internal::Singleton** < **const Codes** > [friend]

The documentation for this class was generated from the following file:

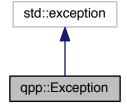
· classes/codes.h

# 7.2 qpp::Exception Class Reference

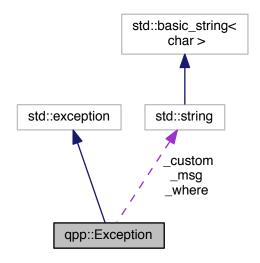
Generates custom exceptions, used when validating function parameters.

#include <classes/exception.h>

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



# **Public Types**

enum Type {

Type::UNKNOWN\_EXCEPTION = 1, Type::ZERO\_SIZE, Type::MATRIX\_NOT\_SQUARE, Type::MATRIX\_← NOT\_CVECTOR,

Type::MATRIX\_NOT\_RVECTOR, Type::MATRIX\_NOT\_VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_C↔ VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_RVECTOR,

Type::MATRIX\_NOT\_SQUARE\_OR\_VECTOR, Type::MATRIX\_MISMATCH\_SUBSYS, Type::DIMS\_INVA← LID, Type::DIMS\_NOT\_EQUAL,

Type::DIMS\_MISMATCH\_MATRIX, Type::DIMS\_MISMATCH\_CVECTOR, Type::DIMS\_MISMATCH\_RVE← CTOR, Type::DIMS\_MISMATCH\_VECTOR,

Type::SUBSYS\_MISMATCH\_DIMS, Type::PERM\_INVALID, Type::PERM\_MISMATCH\_DIMS, Type::NOT ← QUBIT\_MATRIX,

Type::NOT\_QUBIT\_CVECTOR, Type::NOT\_QUBIT\_RVECTOR, Type::NOT\_QUBIT\_VECTOR, Type::NO← T\_QUBIT\_SUBSYS,

Type::NOT\_BIPARTITE, Type::NO\_CODEWORD, Type::OUT\_OF\_RANGE, Type::TYPE\_MISMATCH, Type::UNDEFINED\_TYPE, Type::CUSTOM\_EXCEPTION }

Exception types, add more here if needed.

# **Public Member Functions**

• Exception (const std::string &where, const Type &type)

Constructs an exception.

• Exception (const std::string &where, const std::string &custom)

Constructs an exception.

virtual const char \* what () const noexceptoverride

Overrides std::exception::what()

#### **Private Member Functions**

• void construct exception msg ()

Constructs the exception description from its type.

#### **Private Attributes**

- · std::string \_where
- std::string \_msg
- Type \_type
- std::string \_custom

#### 7.2.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

#### 7.2.2 Member Enumeration Documentation

**7.2.2.1 enum qpp::Exception::Type** [strong]

Exception types, add more here if needed.

See also

qpp::Exception:: construct exception msg()

#### Enumerator

UNKNOWN\_EXCEPTION Unknown exception

**ZERO\_SIZE** Zero sized object, e.g. empty Eigen::Matrix or std::vector<> with no elements

MATRIX\_NOT\_SQUARE Eigen::Matrix is not square

MATRIX\_NOT\_CVECTOR Eigen::Matrix is not a column vector

MATRIX\_NOT\_RVECTOR Eigen::Matrix is not a row vector

MATRIX\_NOT\_VECTOR Eigen::Matrix is not a row/column vector

MATRIX\_NOT\_SQUARE\_OR\_CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX\_NOT\_SQUARE\_OR\_RVECTOR Eigen::Matrix is not square nor a row vector

MATRIX\_NOT\_SQUARE\_OR\_VECTOR Eigen::Matrix is not square nor a row/column vector

MATRIX\_MISMATCH\_SUBSYS Matrix size mismatch subsystem sizes (e.g. in qpp::apply())

**DIMS\_INVALID** std::vector<idx> of dimensions has zero size or contains zeros

DIMS\_NOT\_EQUAL Local/global dimensions are not equal

**DIMS\_MISMATCH\_MATRIX** Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

**DIMS\_MISMATCH\_CVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

**DIMS\_MISMATCH\_RVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

**DIMS\_MISMATCH\_VECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

**SUBSYS\_MISMATCH\_DIMS** std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

**PERM\_INVALID** std::vector<idx> does note represent a valid permutation

**PERM\_MISMATCH\_DIMS** Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

NOT\_QUBIT\_MATRIX Eigen::Matrix is not 2 x 2

NOT\_QUBIT\_CVECTOR Eigen::Matrix is not 2 x 1

NOT\_QUBIT\_RVECTOR Eigen::Matrix is not 1 x 2

NOT\_QUBIT\_VECTOR Eigen::Matrix is not 1 x 2 nor 2 x 1

NOT\_QUBIT\_SUBSYS Subsystems are not 2-dimensional

NOT\_BIPARTITE std::vector<idx> of dimensions has size different from 2

**NO\_CODEWORD** Codeword does not exist, thrown when calling qpp::Codes::codeword() with invalid index i

OUT\_OF\_RANGE Parameter out of range

TYPE\_MISMATCH Scalar types do not match

UNDEFINED\_TYPE Templated specialization not defined for this type

CUSTOM\_EXCEPTION Custom exception, user must provide a custom message

#### 7.2.3 Constructor & Destructor Documentation

7.2.3.1 qpp::Exception::Exception ( const std::string & where, const Type & type ) [inline]

Constructs an exception.

#### **Parameters**

where	Text representing where the exception occured
type	Exception type, defined in qpp::Exception::Type

7.2.3.2 qpp::Exception::Exception ( const std::string & where, const std::string & custom ) [inline]

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

### Parameters

where	Text representing where the exception occured
custom	Exception description

# 7.2.4 Member Function Documentation

7.2.4.1 void qpp::Exception::\_construct\_exception\_msg() [inline], [private]

Constructs the exception description from its type.

#### See also

qpp::Exception::Type

Must modify the code of this function if more exceptions are added

7.2.4.2 virtual const char\* qpp::Exception::what ( ) const [inline], [override], [virtual], [noexcept]

Overrides std::exception::what()

#### Returns

**Exception** description

#### 7.2.5 Member Data Documentation

```
7.2.5.1 std::string qpp::Exception::_custom [private]
```

**7.2.5.2 std::string qpp::Exception::\_msg** [private]

7.2.5.3 Type qpp::Exception::\_type [private]

**7.2.5.4 std::string qpp::Exception::\_where** [private]

The documentation for this class was generated from the following file:

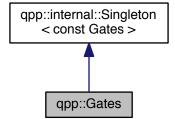
· classes/exception.h

# 7.3 qpp::Gates Class Reference

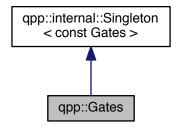
const Singleton class that implements most commonly used gates

#include <classes/gates.h>

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



#### **Public Member Functions**

• cmat Rn (double theta, const std::vector< double > &n) const

Rotation of theta about the 3-dimensional real unit vector n.

• cmat Zd (idx D) const

Generalized Z gate for gudits.

• cmat Fd (idx D) const

Fourier transform gate for qudits.

• cmat Xd (idx D) const

Generalized X gate for qudits.

• template<typename Derived = Eigen::MatrixXcd>

Derived Id (idx D) const

Identity gate.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > CTRL (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &ctrl, const std::vector< idx > &subsys, idx n, idx d=2) const

Generates the multi-partite multiple-controlled-A gate in matrix form.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const
std::vector< idx > &dims) const

Expands out.

# **Public Attributes**

• cmat Id2 {cmat::Identity(2, 2)}

Identity gate.

cmat H {cmat::Zero(2, 2)}

Hadamard gate.

cmat X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

cmat Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

cmat Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

cmat S {cmat::Zero(2, 2)}

```
S gate.

cmat T {cmat::Zero(2, 2)}
T gate.

cmat CNOT {cmat::Identity(4, 4)}
Controlled-NOT control target gate.

cmat CZ {cmat::Identity(4, 4)}
Controlled-Phase gate.

cmat CNOTba {cmat::Zero(4, 4)}
Controlled-NOT target control gate.

cmat SWAP {cmat::Identity(4, 4)}
SWAP gate.

cmat TOF {cmat::Identity(8, 8)}
```

• cmat FRED {cmat::Identity(8, 8)}

# **Private Member Functions**

Toffoli gate.

Fredkin gate.

• Gates ()

Initializes the gates.

∼Gates ()=default

Default destructor.

### **Friends**

class internal::Singleton < const Gates >

### **Additional Inherited Members**

#### 7.3.1 Detailed Description

Default destructor.

const Singleton class that implements most commonly used gates

#### 7.3.2 Constructor & Destructor Documentation

```
7.3.2.1 qpp::Gates::Gates( ) [inline],[private]
Initializes the gates.
7.3.2.2 qpp::Gates::~Gates( ) [private],[default]
```

### 7.3.3 Member Function Documentation

7.3.3.1 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::CTRL ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx n, idx d = 2 ) const [inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

#### See also

qpp::applyCTRL()

### Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
n	Total number of subsystems
d	Subsystem dimensions

#### Returns

CTRL-A gate, as a matrix over the same scalar field as A

7.3.3.2 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::expandout ( const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

#### **Parameters**

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

# Returns

Tensor product  $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$ , with A on position pos, as a dynamic matrix over the same scalar field as A

7.3.3.3 cmat qpp::Gates::Fd(idx D)const [inline]

Fourier transform gate for qudits.

Note

Defined as  $F = \sum_{jk} \exp(2\pi i j k/D) |j\rangle\langle k|$ 

**Parameters** 

D Dimension of the Hilbert space

Returns

Fourier transform gate for qudits

7.3.3.4 template < typename Derived = Eigen::MatrixXcd > Derived qpp::Gates::Id ( idx D ) const [inline]

Identity gate.

Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

#### **Parameters**

D	Dimension of the Hilbert space

Returns

Identity gate

7.3.3.5 cmat qpp::Gates::Rn ( double theta, const std::vector< double > & n ) const [inline]

Rotation of theta about the 3-dimensional real unit vector n.

#### **Parameters**

theta	Rotation angle
n	3-dimensional real unit vector

#### Returns

Rotation gate

7.3.3.6 cmat qpp::Gates::Xd (idx D) const [inline]

Generalized X gate for qudits.

Note

Defined as 
$$X = \sum_j |j \oplus 1 \rangle \langle j|$$

#### **Parameters**

D	Dimension of the Hilbert space

Returns

Generalized X gate for qudits

7.3.3.7 cmat qpp::Gates::Zd(idx D) const [inline]

Generalized Z gate for qudits.

Note

Defined as  $Z = \sum_{j} \exp(2\pi i j/D) |j\rangle\langle j|$ 

**Parameters** 

D Dimension of the Hilbert space

Returns

Generalized Z gate for qudits

7.3.4 Friends And Related Function Documentation

**7.3.4.1** friend class internal::Singleton < const Gates > [friend]

7.3.5 Member Data Documentation

7.3.5.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

7.3.5.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

7.3.5.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

7.3.5.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}

Fredkin gate.

7.3.5.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}

Hadamard gate.

7.3.5.6 cmat qpp::Gates::ld2 {cmat::Identity(2, 2)}

Identity gate.

7.3.5.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}

S gate.

7.3.5.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}

SWAP gate.

7.3.5.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}

T gate.

7.3.5.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}

Toffoli gate.

7.3.5.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

7.3.5.12 cmat qpp::Gates::Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

7.3.5.13 cmat qpp::Gates::Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

The documentation for this class was generated from the following file:

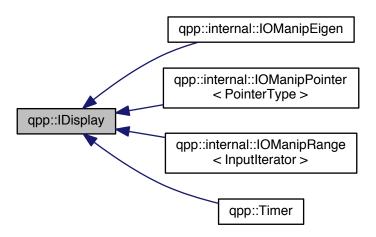
· classes/gates.h

# 7.4 qpp::IDisplay Class Reference

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

#include <classes/idisplay.h>

Inheritance diagram for qpp::IDisplay:



# **Public Member Functions**

- IDisplay ()=default

  Default constructor.
- IDisplay (const IDisplay &)=default

Default copy constructor.

• IDisplay (IDisplay &&)=default

Default move constructor.

• IDisplay & operator= (const IDisplay &)=default

Default copy assignment operator.

• IDisplay & operator= (IDisplay &&)=default

Default move assignment operator.

virtual ∼IDisplay ()=default

Default virtual destructor.

#### **Private Member Functions**

virtual std::ostream & display (std::ostream &os) const =0
 Must be overridden by all derived classes.

#### **Friends**

std::ostream & operator<< (std::ostream &os, const IDisplay &rhs)</li>
 Overloads the extraction operator.

### 7.4.1 Detailed Description

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

This class defines friend inline std::ostream& operator<< (std::ostream& os, const qpp::IDisplay& rhs). The latter delegates the work to the pure private virtual function qpp::IDisplay::display() which has to be overridden by all derived classes.

#### 7.4.2 Constructor & Destructor Documentation

```
7.4.2.1 qpp::IDisplay::IDisplay( ) [default]
Default constructor.
7.4.2.2 qpp::IDisplay::IDisplay( const IDisplay & ) [default]
Default copy constructor.
7.4.2.3 qpp::IDisplay::IDisplay( IDisplay && ) [default]
```

7.4.2.4 virtual qpp::IDisplay::~IDisplay( ) [virtual], [default]

Default virtual destructor.

Default move constructor.

#### 7.4.3 Member Function Documentation

7.4.3.1 virtual std::ostream& qpp::IDisplay::display( std::ostream & os ) const [private], [pure virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs).

Implemented in qpp::internal::IOManipEigen, qpp::Timer, qpp::internal::IOManipPointer< PointerType >, and qpp::internal::IOManipRange< InputIterator >.

7.4.3.2 IDisplay& qpp::IDisplay::operator=( const IDisplay & ) [default]

Default copy assignment operator.

7.4.3.3 IDisplay& qpp::IDisplay::operator=(IDisplay && ) [default]

Default move assignment operator.

#### 7.4.4 Friends And Related Function Documentation

7.4.4.1 std::ostream& operator<< ( std::ostream & os, const IDisplay & rhs ) [friend]

Overloads the extraction operator.

Delegates the work to the virtual function qpp::IDisplay::display()

The documentation for this class was generated from the following file:

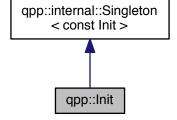
· classes/idisplay.h

# 7.5 qpp::Init Class Reference

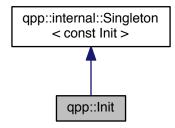
const Singleton class that performs additional initializations/cleanups

#include <classes/init.h>

Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



#### **Private Member Functions**

• Init ()

Additional initializations.

• ∼Init ()

Cleanups.

#### **Friends**

class internal::Singleton < const Init >

# **Additional Inherited Members**

# 7.5.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

#### 7.5.2 Constructor & Destructor Documentation

```
7.5.2.1 qpp::Init::Init( ) [inline],[private]
```

Additional initializations.

```
7.5.2.2 qpp::Init::\simInit( ) [inline],[private]
```

Cleanups.

### 7.5.3 Friends And Related Function Documentation

**7.5.3.1** friend class internal::Singleton < const lnit > [friend]

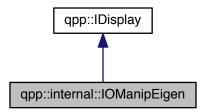
The documentation for this class was generated from the following file:

· classes/init.h

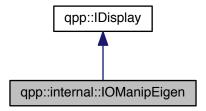
# 7.6 qpp::internal::IOManipEigen Class Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipEigen:



Collaboration diagram for qpp::internal::IOManipEigen:



#### **Public Member Functions**

- template<typename Derived >
   IOManipEigen (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
- IOManipEigen (const cplx z, double chop=qpp::chop)

# **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- cmat \_A
- double <u>\_chop</u>

#### 7.6.1 Constructor & Destructor Documentation

- 7.6.1.1 template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen ( const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop ) [inline], [explicit]
- 7.6.1.2 qpp::internal::IOManipEigen::IOManipEigen ( const cplx z, double chop = qpp::chop ) [inline], [explicit]

#### 7.6.2 Member Function Documentation

7.6.2.1 std::ostream& qpp::internal::IOManipEigen::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

#### 7.6.3 Member Data Documentation

- 7.6.3.1 cmat qpp::internal::IOManipEigen::\_A [private]
- **7.6.3.2** double qpp::internal::IOManipEigen::\_chop [private]

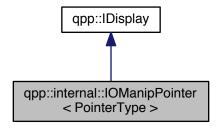
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

# 7.7 qpp::internal::IOManipPointer < PointerType > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipPointer< PointerType >:



Collaboration diagram for qpp::internal::IOManipPointer< PointerType >:



#### **Public Member Functions**

- IOManipPointer (const PointerType \*p, const idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipPointer (const IOManipPointer &)=default
- IOManipPointer & operator= (const IOManipPointer &)=default

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- const PointerType \* \_p
- idx \_n
- std::string \_separator
- std::string \_start
- std::string \_end

# 7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer ( const PointerType \* p, const idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " ) [inline], [explicit]
- 7.7.1.2 template<typename PointerType> qpp::internal::IOManipPointer< PointerType>::IOManipPointer( const IOManipPointer<< PointerType> & ) [default]

#### 7.7.2 Member Function Documentation

7.7.2.1 template<typename PointerType> std::ostream& qpp::internal::IOManipPointer< PointerType >::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.7.2.2 template<typename PointerType> IOManipPointer& qpp::internal::IOManipPointer< PointerType
  >::operator=( const IOManipPointer< PointerType > & ) [default]
- 7.7.3 Member Data Documentation
- 7.7.3.1 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_end [private]
- 7.7.3.2 template<typename PointerType>idx qpp::internal::IOManipPointer< PointerType>::\_n [private]
- 7.7.3.3 template<typename PointerType> const PointerType\* qpp::internal::IOManipPointer< PointerType >::\_p [private]
- 7.7.3.4 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_separator [private]
- 7.7.3.5 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType >::\_start [private]

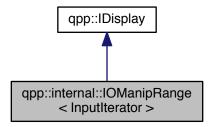
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

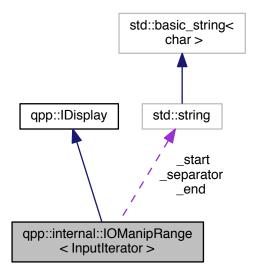
# 7.8 qpp::internal::IOManipRange < InputIterator > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipRange< InputIterator >:



Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



#### **Public Member Functions**

• IOManipRange (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- · InputIterator \_first
- · InputIterator \_last
- std::string \_separator
- · std::string \_start
- std::string \_end

# 7.8.1 Constructor & Destructor Documentation

#### 7.8.2 Member Function Documentation

7.8.2.1 template<typename InputIterator> std::ostream& qpp::internal::IOManipRange< InputIterator >::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

#### 7.8.3 Member Data Documentation

- **7.8.3.1** template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_end [private]
- **7.8.3.2** template<typename InputIterator > InputIterator qpp::internal::IOManipRange< InputIterator >::\_first [private]
- 7.8.3.3 template<typename InputIterator> InputIterator qpp::internal::IOManipRange< InputIterator>::\_last [private]
- **7.8.3.4 template**<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_separator [private]
- **7.8.3.5 template**<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_start [private]

The documentation for this class was generated from the following file:

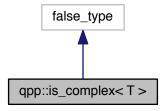
• internal/classes/iomanip.h

# 7.9 qpp::is\_complex< T > Struct Template Reference

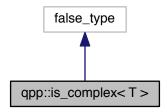
Checks whether the type is a complex type.

#include <traits.h>

Inheritance diagram for qpp::is\_complex< T >:



Collaboration diagram for qpp::is\_complex< T >:



# 7.9.1 Detailed Description

template<typename T>struct qpp::is\_complex< T>

Checks whether the type is a complex type.

Provides the member constant value which is equal to true, if the type is a complex type (i.e. std::complex < T >)

The documentation for this struct was generated from the following file:

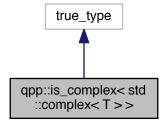
· traits.h

# 7.10 qpp::is\_complex< std::complex< T > > Struct Template Reference

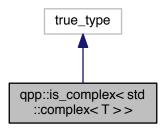
Checks whether the type is a complex number type, specialization for complex types.

#include <traits.h>

Inheritance diagram for qpp::is\_complex< std::complex< T > >:



Collaboration diagram for qpp::is\_complex < std::complex < T > >:



### 7.10.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_complex < std::complex < T > >$ 

Checks whether the type is a complex number type, specialization for complex types.

The documentation for this struct was generated from the following file:

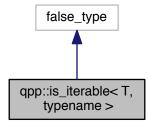
· traits.h

# 7.11 qpp::is\_iterable < T, typename > Struct Template Reference

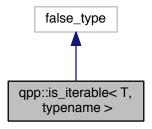
Checks whether *T* is compatible with an STL-like iterable container.

#include <traits.h>

Inheritance diagram for qpp::is\_iterable < T, typename >:



Collaboration diagram for qpp::is\_iterable < T, typename >:



# 7.11.1 Detailed Description

 $template < typename \ {\tt T}, \ typename \ {\tt = void} > {\tt struct \ qpp::is\_iterable} < \ {\tt T}, \ typename >$ 

Checks whether T is compatible with an STL-like iterable container.

Provides the member constant *value* which is equal to *true*, if *T* is compatible with an iterable container, i.e. provides at least *begin()* and *end()* member functions. Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

· traits.h

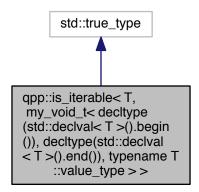
7.12 qpp::is\_iterable < T, my\_void\_t < decltype(std::declval < T >().begin()), decltype(std::declval < T >().end()), typename T::value\_type > > Struct Template Reference

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

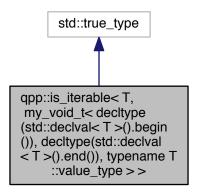
#include <traits.h>

 $Inheritance \ diagram \ for \ qpp::is\_iterable < T, \ my\_void\_t < \ decltype(std::declval < T > ().begin()), \ decltype(std::declval < T > ().begin()),$ 

:declval< T >().end()), typename T::value\_type > >:



Collaboration diagram for qpp::is\_iterable< T, my\_void\_t< decltype(std::declval< T >().begin()), decltype(std::declval< T >().begin()), typename T::value\_type > >:



# 7.12.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_iterable < T, \ my\_void\_t < \ decltype(std::declval < T > ().begin()), \ decltype(std::declval < T > ().end()), \ typename \ T::value\_type > >$ 

Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. The documentation for this struct was generated from the following file:

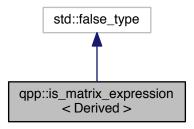
• traits.h

# 7.13 qpp::is\_matrix\_expression < Derived > Struct Template Reference

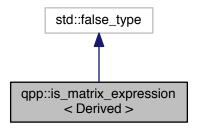
Checks whether the type is an Eigen matrix expression.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< Derived >:



Collaboration diagram for qpp::is\_matrix\_expression< Derived >:



#### 7.13.1 Detailed Description

 $template {<} typename \ Derived {>} struct \ qpp::is\_matrix\_expression {<} \ Derived {>}$ 

Checks whether the type is an Eigen matrix expression.

Provides the member constant *value* which is equal to *true*, if the type is an Eigen matrix expression of type *Eigen ∷MatrixBase Oerived >*. Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

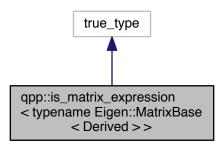
• traits.h

# 7.14 qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >> Struct Template Reference

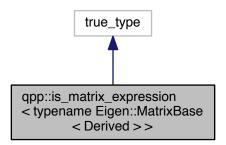
Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



Collaboration diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



#### 7.14.1 Detailed Description

 $template < typename\ Derived > struct\ qpp::is\_matrix\_expression < \ typename\ Eigen::MatrixBase < Derived > >$ 

 ${\it Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.}$ 

The documentation for this struct was generated from the following file:

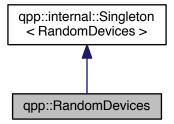
· traits.h

# 7.15 qpp::RandomDevices Class Reference

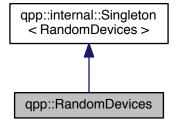
Singeleton class that manages the source of randomness in the library.

#include <classes/random\_devices.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



#### **Public Attributes**

std::mt19937 \_rng

Mersenne twister random number generator.

# **Private Member Functions**

• RandomDevices ()

Initializes and seeds the random number generators.

∼RandomDevices ()=default

Default destructor.

#### **Private Attributes**

```
    std::random_device _rd
    used to seed std::mt19937 _rng
```

#### **Friends**

class internal::Singleton < RandomDevices >

#### **Additional Inherited Members**

# 7.15.1 Detailed Description

Singeleton class that manages the source of randomness in the library.

Consists of a wrapper around an std::mt19937 Mersenne twister random number generator engine and an std⇔ ::random\_device engine. The latter is used to seed the Mersenne twister.

#### Warning

This class DOES NOT seed the standard C number generator used by Eigen::Matrix::Random(), since it is not thread safe. Do not use Eigen::Matrix::Random() or functions that depend on the C style random number engine, but use <a href="mailto:qpp::rand()">qpp::rand()</a> instead!

#### 7.15.2 Constructor & Destructor Documentation

```
7.15.2.1 qpp::RandomDevices::RandomDevices() [inline], [private]
```

Initializes and seeds the random number generators.

```
7.15.2.2 qpp::RandomDevices::~RandomDevices() [private], [default]
```

Default destructor.

#### 7.15.3 Friends And Related Function Documentation

```
7.15.3.1 friend class internal::Singleton < RandomDevices > [friend]
```

#### 7.15.4 Member Data Documentation

```
7.15.4.1 std::random_device qpp::RandomDevices::_rd [private]
```

used to seed std::mt19937 rng

7.15.4.2 std::mt19937 qpp::RandomDevices::\_rng

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

classes/random\_devices.h

# 7.16 qpp::internal::Singleton < T > Class Template Reference

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

#### **Static Public Member Functions**

- static T & get\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)
- static thread\_local T & get\_thread\_local\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)

#### **Protected Member Functions**

- Singleton () noexcept=default
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete
- virtual ∼Singleton ()=default

# 7.16.1 Detailed Description

template<typename T>class qpp::internal::Singleton< T>

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from qpp::internal::Singleton, make qpp::internal::Singleton a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function qpp::internal::Singleton::get\_instance() (qpp::internal::Singleton::get\_thread\_local\_cinstance()), which returns a reference (thread\_local reference) to your newly created singleton (thread-safe in C++11).

#### Example:

#### See also

Code of qpp::Codes, qpp::Gates, qpp::Init, qpp::RandomDevices, qpp::States or qpp.h for real world examples of usage.

#### 7.16.2 Constructor & Destructor Documentation

```
7.16.2.1 template<typename T> qpp::internal::Singleton< T>::Singleton( ) [protected], [default], [noexcept]
```

```
7.16.2.2 template<typename T> qpp::internal::Singleton< T>::Singleton ( const Singleton< T> & ) [protected], [delete]
```

7.16.2.3 template virtual qpp::internal::Singleton< T>::
$$\sim$$
Singleton( ) [protected], [virtual], [default]

#### 7.16.3 Member Function Documentation

The documentation for this class was generated from the following file:

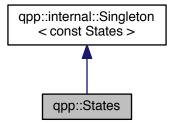
• internal/classes/singleton.h

# 7.17 qpp::States Class Reference

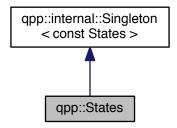
const Singleton class that implements most commonly used states

```
#include <classes/states.h>
```

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



#### **Public Attributes**

```
    ket x0 {ket::Zero(2)}
```

Pauli Sigma-X 0-eigenstate |+>

ket x1 {ket::Zero(2)}

Pauli Sigma-X 1-eigenstate |->

ket y0 {ket::Zero(2)}

Pauli Sigma-Y 0-eigenstate | y+>

ket y1 {ket::Zero(2)}

Pauli Sigma-Y 1-eigenstate | y->

ket z0 {ket::Zero(2)}

Pauli Sigma-Z 0-eigenstate | 0>

ket z1 {ket::Zero(2)}

Pauli Sigma-Z 1-eigenstate | 1>

cmat px0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.

• cmat px1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.

• cmat py0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 0-eigenstate  $|y+\rangle < y+|$ .

cmat py1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 1-eigenstate | y-><y-|.

cmat pz0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.

cmat pz1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 1-eigenstate | 1><1|.

ket b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

ket b01 {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

ket b10 {ket::Zero(4)}

Bell-10 state (following the convention in Nielsen and Chuang)

ket b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

cmat pb00 {cmat::Zero(4, 4)}

Projector onto the Bell-00 state.

cmat pb01 {cmat::Zero(4, 4)}

Projector onto the Bell-01 state.

cmat pb10 {cmat::Zero(4, 4)}

Projector onto the Bell-10 state.

cmat pb11 {cmat::Zero(4, 4)}

Projector onto the Bell-11 state.

ket GHZ {ket::Zero(8)}

GHZ state.

ket W {ket::Zero(8)}

W state.

cmat pGHZ {cmat::Zero(8, 8)}

Projector onto the GHZ state.

cmat pW {cmat::Zero(8, 8)}

Projector onto the W state.

# **Private Member Functions**

- States ()
- $\sim$ States ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const States >

# **Additional Inherited Members**

#### 7.17.1 Detailed Description

const Singleton class that implements most commonly used states

#### 7.17.2 Constructor & Destructor Documentation

```
7.17.2.1 qpp::States::States( ) [inline],[private]
```

Initialize the states

```
7.17.2.2 qpp::States::~States() [private], [default]
```

Default destructor.

# 7.17.3 Friends And Related Function Documentation

**7.17.3.1** friend class internal::Singleton < const States > [friend]

#### 7.17.4 Member Data Documentation

7.17.4.1 ket qpp::States::b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

```
7.17.4.2 ket qpp::States::b01 {ket::Zero(4)}
Bell-01 state (following the convention in Nielsen and Chuang)
7.17.4.3 ket qpp::States::b10 {ket::Zero(4)}
Bell-10 state (following the convention in Nielsen and Chuang)
7.17.4.4 ket qpp::States::b11 {ket::Zero(4)}
Bell-11 state (following the convention in Nielsen and Chuang)
7.17.4.5 ket qpp::States::GHZ {ket::Zero(8)}
GHZ state.
7.17.4.6 cmat qpp::States::pb00 {cmat::Zero(4, 4)}
Projector onto the Bell-00 state.
7.17.4.7 cmat qpp::States::pb01 {cmat::Zero(4, 4)}
Projector onto the Bell-01 state.
7.17.4.8 cmat qpp::States::pb10 {cmat::Zero(4, 4)}
Projector onto the Bell-10 state.
7.17.4.9 cmat qpp::States::pb11 {cmat::Zero(4, 4)}
Projector onto the Bell-11 state.
7.17.4.10 cmat qpp::States::pGHZ {cmat::Zero(8, 8)}
Projector onto the GHZ state.
7.17.4.11 cmat qpp::States::pW {cmat::Zero(8, 8)}
Projector onto the W state.
7.17.4.12 cmat qpp::States::px0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.
7.17.4.13 cmat qpp::States::px1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.
```

```
7.17.4.14 cmat qpp::States::py0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 0-eigenstate |y+><y+|.
7.17.4.15 cmat qpp::States::py1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.
7.17.4.16 cmat qpp::States::pz0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.
7.17.4.17 cmat qpp::States::pz1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.
7.17.4.18 ket qpp::States::W {ket::Zero(8)}
W state.
7.17.4.19 ket qpp::States::x0 {ket::Zero(2)}
Pauli Sigma-X 0-eigenstate |+>
7.17.4.20 ket qpp::States::x1 {ket::Zero(2)}
Pauli Sigma-X 1-eigenstate |->
7.17.4.21 ket qpp::States::y0 {ket::Zero(2)}
Pauli Sigma-Y 0-eigenstate |y+>
7.17.4.22 ket qpp::States::y1 {ket::Zero(2)}
Pauli Sigma-Y 1-eigenstate |y->
7.17.4.23 ket qpp::States::z0 {ket::Zero(2)}
Pauli Sigma-Z 0-eigenstate |0>
7.17.4.24 ket qpp::States::z1 {ket::Zero(2)}
Pauli Sigma-Z 1-eigenstate |1>
The documentation for this class was generated from the following file:
```

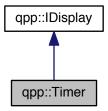
classes/states.h

# 7.18 qpp::Timer Class Reference

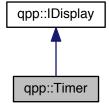
Chronometer.

#include <classes/timer.h>

Inheritance diagram for qpp::Timer:



Collaboration diagram for qpp::Timer:



#### **Public Member Functions**

• Timer () noexcept

Constructs an instance with the current time as the starting point.

• void tic () noexcept

Resets the chronometer.

• const Timer & toc () noexcept

Stops the chronometer.

· double seconds () const noexcept

Time passed in seconds.

• Timer (const Timer &)=default

Default copy constructor.

• Timer (Timer &&)=default

Default move constructor.

• Timer & operator= (const Timer &)=default

Default copy assignment operator.

• Timer & operator= (Timer &&)=default

Default move assignment operator.

virtual ∼Timer ()=default

Default virtual destructor.

#### **Protected Attributes**

```
std::chrono::steady_clock::time_point _start
```

```
• std::chrono::steady_clock::time_point _end
```

#### **Private Member Functions**

 std::ostream & display (std::ostream &os) const override *qpp::IDisplay::display()* override

#### 7.18.1 Detailed Description

Chronometer.

Uses a std::chrono::steady\_clock. It is not affected by wall clock changes during runtime.

#### 7.18.2 Constructor & Destructor Documentation

```
7.18.2.1 qpp::Timer::Timer() [inline], [noexcept]
```

Constructs an instance with the current time as the starting point.

```
7.18.2.2 qpp::Timer::Timer(const Timer & ) [default]
```

Default copy constructor.

```
7.18.2.3 qpp::Timer::Timer(Timer && ) [default]
```

Default move constructor.

```
7.18.2.4 virtual qpp::Timer:: \sim Timer( ) [virtual], [default]
```

Default virtual destructor.

#### 7.18.3 Member Function Documentation

qpp::IDisplay::display() override

**Parameters** 

ſ		•
- 1	os l	()utnut etream
- 1	05	Output stream
- 1		

#### Returns

Writes to the output stream the number of seconds that passed between the instantiation/reset and invocation of qpp::Timer::toc().

Implements qpp::IDisplay.

```
7.18.3.2 Timer& qpp::Timer::operator=( const Timer & ) [default]
```

Default copy assignment operator.

```
7.18.3.3 Timer& qpp::Timer::operator=( Timer && ) [default]
```

Default move assignment operator.

```
7.18.3.4 double qpp::Timer::seconds ( ) const [inline], [noexcept]
```

Time passed in seconds.

#### Returns

Number of seconds that passed between the instantiation/reset and invocation of qpp::Timer::toc()

```
7.18.3.5 void qpp::Timer::tic( ) [inline], [noexcept]
```

Resets the chronometer.

Resets the starting/ending point to the current time

```
7.18.3.6 const Timer& qpp::Timer::toc( ) [inline], [noexcept]
```

Stops the chronometer.

Set the current time as the ending point

Returns

Current instance

#### 7.18.4 Member Data Documentation

```
7.18.4.1 std::chrono::steady_clock::time_point qpp::Timer::_end [protected]
```

```
7.18.4.2 std::chrono::steady_clock::time_point qpp::Timer::_start [protected]
```

The documentation for this class was generated from the following file:

· classes/timer.h

# **Chapter 8**

# **File Documentation**

# 8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



# Classes

• class qpp::Codes

const Singleton class that defines quantum error correcting codes

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.1.1 Detailed Description

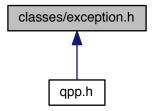
Quantum error correcting codes.

# 8.2 classes/exception.h File Reference

Exceptions.

122 File Documentation

This graph shows which files directly or indirectly include this file:



#### **Classes**

• class qpp::Exception

Generates custom exceptions, used when validating function parameters.

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.2.1 Detailed Description

Exceptions.

# 8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



#### Classes

class qpp::Gates

const Singleton class that implements most commonly used gates

#### **Namespaces**

qpp

Quantum++ main namespace.

# 8.3.1 Detailed Description

Quantum gates.

# 8.4 classes/idisplay.h File Reference

Display interface via the non-virtual interface (NVI)

This graph shows which files directly or indirectly include this file:



#### **Classes**

· class qpp::IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.4.1 Detailed Description

Display interface via the non-virtual interface (NVI)

124 File Documentation

# 8.5 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Init

const Singleton class that performs additional initializations/cleanups

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.5.1 Detailed Description

Initialization.

# 8.6 classes/random\_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



# **Classes**

• class qpp::RandomDevices

Singeleton class that manages the source of randomness in the library.

# **Namespaces**

qpp

Quantum++ main namespace.

# 8.6.1 Detailed Description

Random devices.

# 8.7 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



# Classes

• class qpp::States

const Singleton class that implements most commonly used states

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.7.1 Detailed Description

Quantum states.

126 File Documentation

# 8.8 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Timer Chronometer.

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.8.1 Detailed Description

Timing.

# 8.9 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

qpp

Quantum++ main namespace.

#### **Functions**

• constexpr cplx qpp::operator""\_i (unsigned long long int x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

• constexpr cplx qpp::operator""\_i (long double x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

• cplx qpp::omega (idx D)

D-th root of unity.

#### **Variables**

• constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

• constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

• constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

• constexpr double qpp::infty = std::numeric\_limits<double>::infinity()

Used to denote infinity in double precision.

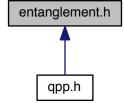
# 8.9.1 Detailed Description

Constants.

# 8.10 entanglement.h File Reference

Entanglement functions.

This graph shows which files directly or indirectly include this file:



128 File Documentation

### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 dyn\_col\_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector<
 idx > &dims)

Schmidt coefficients of the bi-partite pure state A.

template < typename Derived >
 cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)

template<typename Derived >
 cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Schmidt basis on Bob side.

Schmidt basis on Alice side.

template<typename Derived >
 std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx
 > &dims)

Schmidt probabilities of the bi-partite pure state A.

template < typename Derived >
 double qpp::entanglement (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)
 Entanglement of the bi-partite pure state A.

template<typename Derived >
 double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)

G-concurrence of the bi-partite pure state A.

template < typename Derived >
 double qpp::negativity (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)
 Negativity of the bi-partite mixed state A.

template<typename Derived >
 double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

template<typename Derived >
 double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)

Wootters concurrence of the bi-partite qubit mixed state A.

Logarithmic negativity of the bi-partite mixed state A.

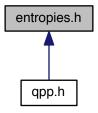
# 8.10.1 Detailed Description

Entanglement functions.

# 8.11 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 double qpp::entropy (const Eigen::MatrixBase< Derived > &A)

von-Neumann entropy of the density matrix A

double qpp::entropy (const std::vector< double > &prob)

Shannon entropy of the probability distribution prob.

• template<typename Derived >

double <a href="mailto:qpp::renyi">qpp::renyi</a> (const Eigen::MatrixBase< Derived > &A, double alpha)

Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .

double qpp::renyi (const std::vector< double > &prob, double alpha)

Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .

• template<typename Derived >

double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)

Tsallis- q entropy of the density matrix A, for  $q \geq 0$ .

double qpp::tsallis (const std::vector< double > &prob, double q)

Tsallis- q entropy of the probability distribution prob, for  $q \geq 0$ .

• template<typename Derived >

double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)

Quantum mutual information between 2 subsystems of a composite system.

template<typename Derived >

 $\label{lem:double qpp::qmutualinfo} double \ qpp:: qmutualinfo \ (const \ Eigen:: Matrix Base < Derived > \&A, const \ std:: vector < idx > \&subsysA, const \ std:: vector < idx > \&subsysB, idx \ d=2)$ 

Quantum mutual information between 2 subsystems of a composite system.

# 8.11.1 Detailed Description

Entropy functions.

130 File Documentation

# 8.12 experimental/test.h File Reference

Experimental/test functions/classes.

#### **Namespaces**

• qpp

Quantum++ main namespace.

• qpp::experimental

Experimental/test functions/classes, do not use or modify.

# 8.12.1 Detailed Description

Experimental/test functions/classes.

# 8.13 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



#### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

- template < typename Derived >
   dyn\_mat < typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase < Derived > &A)
   Transpose.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)
   Complex conjugate.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)
   Adjoint.

```
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar qpp::trace (const Eigen::MatrixBase < Derived > &A)
      Trace
• template<typename Derived >
  Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)
      Determinant.
\bullet \ \ {\it template}{<} {\it typename Derived} >
  Derived::Scalar <a href="mailto:qpp::logdet">qpp::logdet</a> (const Eigen::MatrixBase</a> Derived > &A)
      Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::sum">qpp::sum</a> (const Eigen::MatrixBase< Derived > &A)
      Element-wise sum of A.
• template<typename Derived >
  Derived::Scalar <a href="mailto:open:prod">open:prod</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise product of A.

    template<typename Derived >

  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase< Derived > &A)
      Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition.
\bullet \ \ {\it template}{<} {\it typename Derived} >
  dyn_col_vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.

    template<typename Derived >

  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
      Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvalues.

    template<typename Derived >

  cmat qpp::hevects (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn col vect< double >, cmat > qpp::svd (const Eigen::MatrixBase< Derived > &A)
      Full singular value decomposition.
• template<typename Derived >
  dyn_col_vect< double > qpp::svals (const Eigen::MatrixBase< Derived > &A)
      Singular values.

    template<typename Derived >

  cmat qpp::svdU (const Eigen::MatrixBase< Derived > &A)
      Left singular vectors.

    template<typename Derived >

  cmat qpp::svdV (const Eigen::MatrixBase< Derived > &A)
      Right singular vectors.

    template<typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
```

```
Functional calculus f(A)
• template<typename Derived >
  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.
template<typename Derived >
  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat qpp::logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.
• template<typename Derived >
  double qpp::schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn_mat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const
  typename Derived::Scalar &))
     Functor.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)
```

Direct sum.

```
• template<typename Derived >
  dyn mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows,
  idx cols)
     Reshape.
- template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Commutator.

    template < typename Derived1, typename Derived2 >

  dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &Vs)
     Gram-Schmidt orthogonalization.
template<typename Derived >
  dyn mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.

    std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

     Multi-index to non-negative integer index.

    ket qpp::mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

     Multi-partite qudit ket.

    ket qpp::mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat qpp::mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
  std::vector< double > qpp::abssq (InputIterator first, InputIterator last)
     Computes the absolute values squared of an STL-like range of complex numbers.

    template<typename Container >

  std::vector< double > qpp::abssq (const Container &c, typename std::enable_if< is_iterable< Container
  >::value >::type *=nullptr)
```

Computes the absolute values squared of an STL-like container.

template < typename Derived >
 std::vector < double > qpp::abssq (const Eigen::MatrixBase < Derived > &A)

Computes the absolute values squared of an Eigen expression.

• template<typename InputIterator >

std::iterator traits < InputIterator >::value type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

• template<typename Container >

Container::value\_type qpp::sum (const Container &c)

Element-wise sum of the elements of an STL-like container.

• template<typename InputIterator >

std::iterator traits< InputIterator >::value type qpp::prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

• template<typename Container >

Container::value type <a href="mailto:open">opp::prod</a> (const Container &c)

Element-wise product of the elements of an STL-like container.

• template<typename InputIterator >

 $std::conditional < is\_complex < typename \ std::iterator\_traits < InputIterator >::value\_type >::value, \ cplx, \ double >::type \ qpp::avg \ (InputIterator \ first, \ InputIterator \ last)$ 

Average of the elements of an STL-like range.

• template<typename Container >

std::conditional< is\_complex< typename Container::value\_type >::value, cplx, double >::type qpp::avg (const Container &c)

Average of the elements of an STL-like container.

template<typename Derived >

std::conditional< is\_complex< typename Derived::Scalar >::value, cplx, double >::type qpp::avg (const Eigen::MatrixBase< Derived > &A)

Average of the elements of an Eigen expression.

• template<typename Derived >

```
dyn_col_vect< typename Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)
```

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

```
std::vector< T > qpp::complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

• template<typename Derived >

```
std::vector< double > qpp::rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat qpp::bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

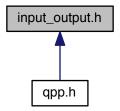
# 8.13.1 Detailed Description

Generic quantum computing functions.

# 8.14 input\_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
 Eigen expression ostream manipulator.

• internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

template<typename InputIterator >
 internal::IOManipRange< InputIterator > qpp::disp (const InputIterator &first, const InputIterator &last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >
 internal::IOManipRange< typename Container::const\_iterator > qpp::disp (const Container &c, const std
 ::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >
 internal::IOManipPointer< PointerType > qpp::disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

• template<typename Derived >

void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > qpp::load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

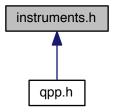
# 8.14.1 Detailed Description

Input/output functions.

#### 8.15 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

 $\bullet \ \ {\sf template}{<} {\sf typename \ Derived} >$ 

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

ullet template<typename Derived >

std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > &A, const std::vector < cmat > &Ks, const std::vector < idx > &subsys, const std::vector < idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >

 $std::tuple < std::vector < idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase < Derived > \&A, std::vector < idx > subsys, std::vector < idx > dims)$ 

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

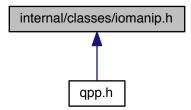
#### 8.15.1 Detailed Description

Measurement functions.

# 8.16 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



## Classes

- class qpp::internal::IOManipRange
   InputIterator >
- class qpp::internal::IOManipPointer< PointerType >
- class qpp::internal::IOManipEigen

# **Namespaces**

• qpp

Quantum++ main namespace.

• qpp::internal

Internal utility functions, do not use/modify.

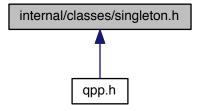
# 8.16.1 Detailed Description

Input/output manipulators.

# 8.17 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



# Classes

class qpp::internal::Singleton< T >

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

# **Namespaces**

• qpp

Quantum++ main namespace.

· qpp::internal

Internal utility functions, do not use/modify.

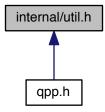
# 8.17.1 Detailed Description

Singleton pattern via CRTP.

# 8.18 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

## **Functions**

- void qpp::internal::\_n2multiidx (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx qpp::internal::\_multiidx2n (const idx \*midx, idx numdims, const idx \*dims) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_square\_mat (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   bool qpp::internal::\_check\_vector (const Eigen::MatrixBase< Derived > &A)
- template < typename Derived >
   bool qpp::internal::\_check\_rvector (const Eigen::MatrixBase < Derived > &A)
- template<typename Derived >
   bool qpp::internal::\_check\_cvector (const Eigen::MatrixBase< Derived > &A)
- template<typename T >
   bool qpp::internal::\_check\_nonzero\_size (const T &x) noexcept
- bool qpp::internal::\_check\_dims (const std::vector< idx > &dims)
- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase
   Derived > &V)
- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase
   Derived > &V)
- bool qpp::internal::\_check\_eq\_dims (const std::vector< idx > &dims, idx dim) noexcept
- bool qpp::internal::\_check\_subsys\_match\_dims (const std::vector < idx > &subsys, const std::vector < idx > &dims)

```
    template<typename Derived >
        bool qpp::internal::_check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept
```

template<typename Derived >
 bool qpp::internal::\_check\_qubit\_cvector (const Eigen::MatrixBase< Derived > &V) noexcept

template<typename Derived >
 bool qpp::internal::\_check\_qubit\_rvector (const Eigen::MatrixBase< Derived > &V) noexcept

template<typename Derived >
 bool qpp::internal::\_check\_qubit\_vector (const Eigen::MatrixBase< Derived > &V) noexcept

• bool qpp::internal::\_check\_perm (const std::vector< idx > &perm)

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::internal::\_kron2 (const Eigen::MatrixBase< Derived1 > &A,
 const Eigen::MatrixBase< Derived2 > &B)

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::internal::\_dirsum2 (const Eigen::MatrixBase< Derived1 > &A,
 const Eigen::MatrixBase< Derived2 > &B)

template < typename T > void qpp::internal::variadic\_vector\_emplace (std::vector < T > &)

template<typename T, typename First, typename... Args>
 void qpp::internal::variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)

## 8.18.1 Detailed Description

Internal utility functions.

## 8.19 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

# **Namespaces**

• qpp

Quantum++ main namespace.

# **Functions**

template < typename Derived >
 Derived qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

dmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

Loads an Eigen dynamic matrix from a wirt End .mat me, specialization for double matrices (qpp..dme

cmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

• template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std ::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std ::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

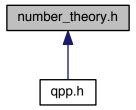
## 8.19.1 Detailed Description

Input/output interfacing with MATLAB.

# 8.20 number\_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

## **Functions**

std::vector< int > qpp::x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

double <a href="mailto:qpp::contfrac2x">qpp::contfrac2x</a> (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double <a href="mailto:qpp::contfrac2x">qpp::contfrac2x</a> (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• ubigint qpp::gcd (ubigint m, ubigint n)

Greatest common divisor of two non-negative integers.

ubigint qpp::gcd (const std::vector< ubigint > &ns)

Greatest common divisor of a list of non-negative integers.

• ubigint qpp::lcm (ubigint m, ubigint n)

Least common multiple of two positive integers.

ubigint qpp::lcm (const std::vector< ubigint > &ns)

Least common multiple of a list of positive integers.

std::vector< idx > qpp::invperm (const std::vector< idx > &perm)

Inverse permutation.

• std::vector< idx > qpp::compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< ubigint > qpp::factors (ubigint n)

Prime factor decomposition.

bool qpp::isprime (ubigint n)

Primality test.

• ubigint <a href="mailto:qpp::modpow">qpp::modpow</a> (ubigint a, ubigint n, ubigint p)

Integer power modulo p.

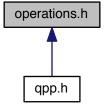
## 8.20.1 Detailed Description

Number theory functions.

# 8.21 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



# Namespaces

qpp

Quantum++ main namespace.

## **Functions**

template<typename Derived1 , typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 const std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 idx d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat qpp::kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat qpp::kraus2choi (const std::vector< cmat > &Ks)

Choi matrix

std::vector< cmat > qpp::choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat qpp::choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat qpp::super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

 $\label{lem:dyn_mat} $$ dyn_mat< typename\ Derived::Scalar > qpp::ptrace1\ (const\ Eigen::MatrixBase< Derived > &A,\ const\ std $$ ::vector< idx > &dims)$$ 

Partial trace.

template<typename Derived >

 $dyn_mat < typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase < Derived > &A, const std <math>\leftrightarrow$  ::vector < idx > &dims)

Partial trace.

• template<typename Derived >

 $\label{lem:dyn_mat} $$ dyn_mat< typename\ Derived::Scalar > qpp::ptrace\ (const\ Eigen::MatrixBase< Derived > &A,\ const\ std::vector< idx > &subsys,\ const\ std::vector< idx > &dims) $$$ 

Partial trace.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial trace.

<ul> <li>template<typename derived="">         dyn_mat&lt; typename Derived::Scalar &gt; qpp::ptranspose (const Eigen::MatrixBase&lt; Derived &gt; &amp;A, std::vector&lt; idx &gt; &amp;subsys, const std::vector&lt; idx &gt; &amp;dims)</typename></li> </ul>	const
Partial transpose.	
<ul> <li>template<typename derived="">         dyn_mat&lt; typename Derived::Scalar &gt; qpp::ptranspose (const Eigen::MatrixBase&lt; Derived &gt; &amp;A, std::vector&lt; idx &gt; &amp;subsys, idx d=2)</typename></li> </ul>	const
Partial transpose.	
<ul> <li>template<typename derived="">         dyn_mat&lt; typename Derived::Scalar &gt; qpp::syspermute (const Eigen::MatrixBase&lt; Derived &gt; &amp;A, std::vector&lt; idx &gt; &amp;perm, const std::vector&lt; idx &gt; &amp;dims)</typename></li> </ul>	const
Subsystem permutation.	
<ul> <li>template<typename derived="">         dyn_mat&lt; typename Derived::Scalar &gt; qpp::syspermute (const Eigen::MatrixBase&lt; Derived &gt; &amp;A, std::vector&lt; idx &gt; &amp;perm, idx d=2)</typename></li> </ul>	const
Subsystem permutation.	
8.21.1 Detailed Description	
Quantum operation functions.	
8.22 qpp.h File Reference	

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "traits.h"
#include "classes/idisplay.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

## Namespaces

• qpp

Quantum++ main namespace.

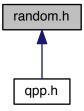
# 8.22.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

## 8.23 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



## **Namespaces**

• qpp

Quantum++ main namespace.

## **Functions**

double <a href="mailto:qpp::rand">qpp::rand</a> (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

- bigint qpp::rand (bigint a=std::numeric\_limits< bigint >::min(), bigint b=std::numeric\_limits< bigint >::max())

  Generates a random big integer uniformly distributed in the interval [a, b].
- ubigint qpp::rand (ubigint a=std::numeric\_limits< ubigint >::min(), ubigint b=std::numeric\_limits< ubigint > \cdot ::max())

Generates a random non-negative big integer uniformly distributed in the interval [a, b].

idx qpp::randidx (idx a=std::numeric\_limits < idx >::min(), idx b=std::numeric\_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

• template<typename Derived >

Derived qpp::rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

8.24 traits.h File Reference 147

• template<typename Derived >

Derived qpp::randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

• template<>

dmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

template<>

cmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double <a href="mailto:qpp::randn">qpp::randn</a> (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat qpp::randU (idx D)

Generates a random unitary matrix.

cmat qpp::randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > qpp::randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat qpp::randH (idx D)

Generates a random Hermitian matrix.

ket qpp::randket (idx D)

Generates a random normalized ket (pure state vector)

cmat qpp::randrho (idx D)

Generates a random density matrix.

std::vector< idx > qpp::randperm (idx n)

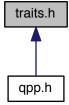
Generates a random uniformly distributed permutation.

## 8.23.1 Detailed Description

Randomness-related functions.

## 8.24 traits.h File Reference

This graph shows which files directly or indirectly include this file:



## Classes

struct qpp::is\_iterable < T, typename >

Checks whether T is compatible with an STL-like iterable container.

• struct qpp::is\_iterable< T, my\_void\_t< decltype(std::declval< T >().begin()), decltype(std::declval< T >().← end()), typename T::value\_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

struct qpp::is\_matrix\_expression< Derived >

Checks whether the type is an Eigen matrix expression.

struct qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived > >

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

struct qpp::is\_complex< T >

Checks whether the type is a complex type.

struct qpp::is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

## **Namespaces**

• qpp

Quantum++ main namespace.

# **Typedefs**

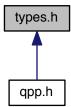
template < class... > using qpp::my\_void\_t = void

Alias template that implements the proposal for void\_t.

# 8.25 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



# **Namespaces**

• qpp

Quantum++ main namespace.

# **Typedefs**

```
using qpp::idx = std::size_t
     Non-negative integer index.

    using qpp::bigint = long long int

     Big integer.
• using qpp::ubigint = unsigned long long int
     Non-negative big integer.
• using qpp::cplx = std::complex< double >
      Complex number in double precision.
• using qpp::ket = Eigen::VectorXcd
      Complex (double precision) dynamic Eigen column vector.
• using <a href="mailto:qpp::bra">qpp::bra</a> = Eigen::RowVectorXcd
      Complex (double precision) dynamic Eigen row vector.
• using qpp::cmat = Eigen::MatrixXcd
      Complex (double precision) dynamic Eigen matrix.
using qpp::dmat = Eigen::MatrixXd
      Real (double precision) dynamic Eigen matrix.
• template<typename Scalar >
  using <a href="mailto:qpp::dyn_mat">qpp::dyn_mat</a> = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >
     Dynamic Eigen matrix over the field specified by Scalar.
template<typename Scalar >
  using qpp::dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
     Dynamic Eigen column vector over the field specified by Scalar.
template<typename Scalar >
  using qpp::dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
     Dynamic Eigen row vector over the field specified by Scalar.
```

# 8.25.1 Detailed Description

Type aliases.

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