Quantum++ v0.7.1

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Contents

1	Qua	ntum++														1	
2	Nam	nespace	Index													5	;
	2.1	Names	space List					 			 	 	 	 	 	. 5	,
3	Hier	archica	l Index													7	,
	3.1	Class I	Hierarchy					 			 	 	 	 	 	. 7	,
4	Clas	ss Index														9)
	4.1	Class I	List					 			 	 	 	 	 	. 9)
5	File	Index														11	
	5.1	File Lis	st					 		 	 	 	 	 	 	. 11	
6	Nam	nespace	Docume	ntatio	on											13	}
	6.1	qpp Na	amespace	Refe	rence			 		 	 	 	 	 	 	. 13	;
		6.1.1	Detailed	Desc	cription	n .		 			 	 	 	 	 	. 23	;
		6.1.2	Typedef	Docu	menta	ation		 			 	 	 	 	 	. 23	;
			6.1.2.1	bra				 		 	 	 	 	 	 	. 23	;
			6.1.2.2	cma	at			 		 	 	 	 	 	 	. 23	;
			6.1.2.3	cpl	x			 		 	 	 	 	 	 	. 24	Ļ
			6.1.2.4	dma	at			 			 	 	 	 	 	. 24	Ļ
			6.1.2.5	dyn	_col_	vect		 			 	 	 	 	 	. 24	Ļ
			6.1.2.6	dyn	_mat			 			 	 	 	 	 	. 24	Ļ
			6.1.2.7	dyn	_row_	_vect	t .	 			 	 	 	 	 	. 24	Ļ
			6.1.2.8	idx				 			 	 	 	 	 	. 24	ļ
			6.1.2.9	ket				 			 	 	 	 	 	. 24	Ļ
		6.1.3	Function	n Doci	ument	atior	١.	 			 	 	 	 	 	. 24	Ļ
			6.1.3.1	abs	sm			 			 	 	 	 	 	. 24	Ļ
			6.1.3.2	abs	sq			 			 	 	 	 	 	. 25	,
			6.1.3.3	abs	sq			 			 	 	 	 	 	. 25	,
			6.1.3.4		oint .												,
			6135	anti	icomm	า										25	

iv CONTENTS

apply	26
apply	26
apply	26
apply	27
apply	27
applyCTRL	27
applyCTRL	28
bloch2rho	28
choi2kraus	29
choi2super	29
comm	29
complement	29
compperm	30
concurrence	30
conjugate	30
contfrac2x	30
contfrac2x	31
cosm	31
cwise	31
det	31
dirsum	32
dirsum	32
dirsum	32
dirsum	33
dirsumpow	33
disp	33
disp	34
disp	34
disp	34
disp	35
eig	36
entanglement	36
entropy	36
entropy	37
evals	37
evects	37
expm	37
factors	38
funm	38
gcd	38
	apply apply apply apply apply applyCTRL applyCTRL bloch2rho choi2kraus choi2super comm complement comperm concurrence conjugate contfrac2x cosm cwise det dirsum dirsupow disp disp disp disp disp disp eriq disp eriq disp eriq eriq entanglement entropy entropy evals evects expm factors funm

CONTENTS

6.1.3.46	gcd	39
6.1.3.47	gconcurrence	39
6.1.3.48	grams	39
6.1.3.49	grams	39
6.1.3.50	grams	40
6.1.3.51	heig	40
6.1.3.52	hevals	40
6.1.3.53	hevects	41
6.1.3.54	inverse	41
6.1.3.55	invperm	41
6.1.3.56	isprime	41
6.1.3.57	kraus2choi	42
6.1.3.58	kraus2super	42
6.1.3.59	kron	42
6.1.3.60	kron	43
6.1.3.61	kron	43
6.1.3.62	kron	43
6.1.3.63	kronpow	44
6.1.3.64	lcm	44
6.1.3.65	lcm	44
6.1.3.66	load	44
6.1.3.67	loadMATLABmatrix	45
6.1.3.68	loadMATLABmatrix	45
6.1.3.69	loadMATLABmatrix	46
6.1.3.70	logdet	46
6.1.3.71	logm	46
6.1.3.72	lognegativity	46
6.1.3.73	measure	47
6.1.3.74	measure	47
6.1.3.75	measure	47
6.1.3.76	measure	48
6.1.3.77	measure	48
6.1.3.78	measure	49
6.1.3.79	measure	49
6.1.3.80	measure	50
6.1.3.81	measure	50
6.1.3.82	measure_seq	51
6.1.3.83	measure_seq	51
6.1.3.84	mket	51
6.1.3.85	mket	52

vi CONTENTS

6.1.3.86 modpow	 52
6.1.3.87 mprj	 52
6.1.3.88 mprj	 53
6.1.3.89 multiidx2n	 53
6.1.3.90 n2multiidx	 53
6.1.3.91 negativity	 53
6.1.3.92 norm	 54
6.1.3.93 omega	 54
6.1.3.94 operator"""_i	 54
6.1.3.95 operator"""_i	 54
6.1.3.96 powm	 54
6.1.3.97 prj	 55
6.1.3.98 prod	 55
6.1.3.99 prod	 55
6.1.3.100 prod	 55
6.1.3.101 ptrace	 56
6.1.3.102 ptrace	 56
6.1.3.103 ptrace1	 56
6.1.3.104 ptrace2	 58
6.1.3.105 ptranspose	 58
6.1.3.106 ptranspose	 58
6.1.3.107 qmutualinfo	 59
6.1.3.108 qmutualinfo	 59
6.1.3.109 rand	59
6.1.3.110 rand	 60
6.1.3.111 rand	 60
6.1.3.112 rand	60
6.1.3.113 randH	60
6.1.3.114 randidx	61
6.1.3.115 randket	61
6.1.3.116 randkraus	61
6.1.3.117 randn	61
6.1.3.118 randn	62
6.1.3.119 randn	62
6.1.3.120 randn	62
6.1.3.121 randperm	63
6.1.3.122 randrho	63
6.1.3.123 randU	63
6.1.3.124 randV	63
6.1.3.125 renyi	 64

CONTENTS vii

		6.1.3.126	3 renyi	64
		6.1.3.127	7 reshape	64
		6.1.3.128	3 rho2bloch	65
		6.1.3.129	9 rho2pure	65
		6.1.3.130	O save	65
		6.1.3.131	1 saveMATLABmatrix	66
		6.1.3.132	2 saveMATLABmatrix	66
		6.1.3.133	3 saveMATLABmatrix	66
		6.1.3.134	4 schatten	66
		6.1.3.135	5 schmidtA	67
		6.1.3.136	6 schmidtB	67
		6.1.3.137	7 schmidtcoeffs	67
		6.1.3.138	3 schmidtprobs	68
		6.1.3.139	9 sinm	68
		6.1.3.140	O spectralpowm	68
		6.1.3.141	1 sqrtm	69
		6.1.3.142	2 sum	69
		6.1.3.143	3 sum	69
		6.1.3.144	4 sum	69
		6.1.3.145	5 super2choi	70
		6.1.3.146	S svals	70
		6.1.3.147	7 svd	70
		6.1.3.148	3 svdU	70
		6.1.3.149	9 svdV	71
		6.1.3.150	O syspermute	71
		6.1.3.151	1 syspermute	71
		6.1.3.152	2 trace	71
		6.1.3.153	3 transpose	72
		6.1.3.154	4 tsallis	72
		6.1.3.155	5 tsallis	72
		6.1.3.156	5 x2contfrac	73
	6.1.4	Variable [Documentation	73
		6.1.4.1	chop	73
		6.1.4.2	ee	73
		6.1.4.3	eps	73
		6.1.4.4	infty	73
		6.1.4.5	maxn	73
		6.1.4.6	pi	73
6.2	qpp::ex	perimenta	al Namespace Reference	73
	6.2.1	Detailed I	Description	74

viii CONTENTS

	6.3	qpp::in	ternal Namespace Reference
		6.3.1	Detailed Description
		6.3.2	Function Documentation
			6.3.2.1 _check_cvector
			6.3.2.2 _check_dims
			6.3.2.3 _check_dims_match_cvect
			6.3.2.4 _check_dims_match_mat
			6.3.2.5 _check_dims_match_rvect
			6.3.2.6 _check_eq_dims
			6.3.2.7 _check_nonzero_size
			6.3.2.8 _check_perm
			6.3.2.9 _check_qubit_cvector
			6.3.2.10 _check_qubit_matrix
			6.3.2.11 _check_qubit_rvector
			6.3.2.12 _check_qubit_vector
			6.3.2.13 _check_rvector
			6.3.2.14 _check_square_mat
			6.3.2.15 _check_subsys_match_dims
			6.3.2.16 _check_vector
			6.3.2.17 _dirsum2
			6.3.2.18 _kron2
			6.3.2.19 _multiidx2n
			6.3.2.20 _n2multiidx
			6.3.2.21 variadic_vector_emplace
			6.3.2.22 variadic_vector_emplace
7	Clas	e Docu	mentation 77
•	7.1		odes Class Reference
		7.1.1	Detailed Description
		7.1.2	Member Enumeration Documentation
		7	7.1.2.1 Type
		7.1.3	Member Function Documentation
			7.1.3.1 codeword
		7.1.4	Friends And Related Function Documentation
			7.1.4.1 internal::Singleton < const Codes >
	7.2	app::E	xception Class Reference
		7.2.1	Detailed Description
		7.2.2	Member Enumeration Documentation
			7.2.2.1 Type
		7.2.3	Constructor & Destructor Documentation

CONTENTS

		7.2.3.1 Exception	81
		7.2.3.2 Exception	81
		7.2.3.3 ~Exception	82
	7.2.4	Member Function Documentation	82
		7.2.4.1 what	82
7.3	qpp::G	ates Class Reference	82
	7.3.1	Detailed Description	84
	7.3.2	Member Function Documentation	84
		7.3.2.1 CTRL	84
		7.3.2.2 expandout	85
		7.3.2.3 Fd	85
		7.3.2.4 ld	85
		7.3.2.5 Rn	86
		7.3.2.6 Xd	86
		7.3.2.7 Zd	86
	7.3.3	Friends And Related Function Documentation	87
		7.3.3.1 internal::Singleton< const Gates >	87
	7.3.4	Member Data Documentation	87
		7.3.4.1 CNOT	87
		7.3.4.2 CNOTba	87
		7.3.4.3 CZ	87
		7.3.4.4 FRED	87
		7.3.4.5 H	87
		7.3.4.6 ld2	87
		7.3.4.7 S	87
		7.3.4.8 SWAP	87
		7.3.4.9 T	87
		7.3.4.10 TOF	87
		7.3.4.11 X	87
		7.3.4.12 Y	88
		7.3.4.13 Z	88
7.4	qpp::In	t Class Reference	88
	7.4.1	Detailed Description	89
	7.4.2	Friends And Related Function Documentation	89
		7.4.2.1 internal::Singleton< const Init >	89
7.5	qpp::in	ernal::IOManipBase Class Reference	89
	7.5.1	Constructor & Destructor Documentation	89
		7.5.1.1 IOManipBase	89
		7.5.1.2 IOManipBase	90
		7.5.1.3 IOManipBase	90

CONTENTS

		7.5.1.4 ~IOManipBase	90
	7.5.2	Member Function Documentation	90
		7.5.2.1 operator=	90
		7.5.2.2 operator=	90
	7.5.3	Friends And Related Function Documentation	90
		7.5.3.1 operator<<	90
7.6	qpp::int	ternal::IOManipEigen Class Reference	90
	7.6.1	Constructor & Destructor Documentation	91
		7.6.1.1 IOManipEigen	91
		7.6.1.2 IOManipEigen	91
7.7	qpp::int	ternal::IOManipPointer< PointerType > Class Template Reference	91
	7.7.1	Constructor & Destructor Documentation	92
		7.7.1.1 IOManipPointer	92
		7.7.1.2 IOManipPointer	92
	7.7.2	Member Function Documentation	92
		7.7.2.1 operator=	92
7.8	qpp::int	ternal::IOManipRange< InputIterator > Class Template Reference	92
	7.8.1	Constructor & Destructor Documentation	93
		7.8.1.1 IOManipRange	93
7.9	qpp::Ra	andomDevices Class Reference	93
	7.9.1	Detailed Description	94
	7.9.2	Friends And Related Function Documentation	95
		7.9.2.1 internal::Singleton < RandomDevices >	95
	7.9.3	Member Data Documentation	95
		7.9.3.1 _rng	95
7.10	qpp::int	ternal::Singleton< T > Class Template Reference	95
	7.10.1	Detailed Description	95
	7.10.2	Constructor & Destructor Documentation	96
		7.10.2.1 Singleton	96
		7.10.2.2 Singleton	96
		7.10.2.3 ~Singleton	96
	7.10.3	Member Function Documentation	96
		7.10.3.1 get_instance	96
		7.10.3.2 get_thread_local_instance	96
		7.10.3.3 operator=	96
7.11	qpp::St	ates Class Reference	96
	7.11.1	Detailed Description	98
	7.11.2		98
		<u> </u>	98
	7.11.3	Member Data Documentation	98

CONTENTS xi

7.11.3.1 b00	98
7.11.3.2 b01	99
7.11.3.3 b10	99
7.11.3.4 b11	99
7.11.3.5 GHZ	99
7.11.3.6 pb00	99
7.11.3.7 pb01	99
7.11.3.8 pb10	99
7.11.3.9 pb11	99
7.11.3.10 pGHZ	99
7.11.3.11 pW	99
7.11.3.12 px0	99
7.11.3.13 px1	99
7.11.3.14 py0	100
7.11.3.15 py1	100
7.11.3.16 pz0	100
7.11.3.17 pz1	100
7.11.3.18 W	100
7.11.3.19 x0	100
7.11.3.20 x1	100
7.11.3.21 y0	100
7.11.3.22 y1	100
7.11.3.23 z0	100
7.11.3.24 z1	100
7.12 qpp::Timer Class Reference	101
7.12.1 Detailed Description	101
7.12.2 Constructor & Destructor Documentation	101
7.12.2.1 Timer	101
7.12.2.2 ~Timer	101
7.12.3 Member Function Documentation	101
7.12.3.1 seconds	101
7.12.3.2 tic	102
7.12.3.3 toc	102
7.12.4 Friends And Related Function Documentation	102
7.12.4.1 operator <<	102
7.12.5 Member Data Documentation	102
7.12.5.1 _end	102
7.12.5.2 _start	102
File Documentation	103

8

xii CONTENTS

8.1	classes/codes.h File Reference	103
	8.1.1 Detailed Description	103
8.2	classes/exception.h File Reference	103
	8.2.1 Detailed Description	104
8.3	classes/gates.h File Reference	104
	8.3.1 Detailed Description	105
8.4	classes/init.h File Reference	105
	8.4.1 Detailed Description	105
8.5	classes/random_devices.h File Reference	106
	8.5.1 Detailed Description	106
8.6	classes/states.h File Reference	106
	8.6.1 Detailed Description	107
8.7	classes/timer.h File Reference	107
	8.7.1 Detailed Description	107
8.8	constants.h File Reference	108
	8.8.1 Detailed Description	108
8.9	entanglement.h File Reference	109
	8.9.1 Detailed Description	110
8.10	entropies.h File Reference	110
	8.10.1 Detailed Description	111
8.11	experimental/test.h File Reference	111
	8.11.1 Detailed Description	111
8.12	functions.h File Reference	111
	8.12.1 Detailed Description	115
8.13	input_output.h File Reference	115
	8.13.1 Detailed Description	116
8.14	instruments.h File Reference	117
	8.14.1 Detailed Description	118
8.15	internal/classes/iomanip.h File Reference	118
	8.15.1 Detailed Description	119
8.16	internal/classes/singleton.h File Reference	119
	8.16.1 Detailed Description	119
8.17	internal/util.h File Reference	120
	8.17.1 Detailed Description	121
8.18	MATLAB/matlab.h File Reference	121
	8.18.1 Detailed Description	122
8.19	number_theory.h File Reference	122
	8.19.1 Detailed Description	123
8.20	operations.h File Reference	123
	8.20.1 Detailed Description	125

CONTENTS	xii

Index			13	31
	8.23.1	Detailed Description	. 12	29
8.23	types.h	n File Reference	. 12	28
	8.22.1	Detailed Description	. 12	28
8.22	random	n.h File Reference	. 12	27
	8.21.1	Detailed Description	. 12	27
8.21	qpp.h F	File Reference	. 12	25

Chapter 1

Quantum++

Version 0.7.1 - development

Development branch, use it at your own risk!

Switch to the master branch for the latest stable version.

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. It uses the Eigen 3 linear algebra library and, if available, the OpenMP multi-processing library. For additional Eigen 3 documentation see http://eigen.tuxfamily.org/dox/. For a simple Eigen 3 quick AS CII reference see http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance.

If you are interesting in contributing, please contact me. To contribute, you need to have a decent knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and some working experience with Eigen 3.

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Building instructions

Configuration:

- Compiler: g++ version 4.8 or later (for good C++11 support)
- Eigen 3 library located in \$HOME/eigen
- Quantum++ library located in \$HOME/qpp
- MATLAB compiler include header files: /Applications/MATLAB_R2014b.app/extern/include
- MATLAB compiler shared library files: /Applications/MATLAB_R2014b.app/bin/maci64

2 Quantum++

Building without a build system

- Example file: \$HOME/qpp/examples/example.cpp
- Output executable: \$HOME/qpp/examples/example
- Must run the commands below from inside the directory \$HOME/qpp/examples

Release version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

Debug version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    example.cpp -o example
```

Release version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2014b.app/extern/include \
    -L/Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

Debug version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2014b.app/extern/include \
    -L /Applications/MATLAB_R2014b.app/bin/maci64 \
    -lmx -lmat example.cpp -o example
```

Building using cmake

The current version of the repository has a ./CMakeLists.txt configuration file for building examples using cmake. To build an example using cmake, I recommend an out-of-source build, i.e., from the root of the project (where ./include is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the relase version (default) executable qpp, from the source file ./examples/example.cpp, without MATLAB support (default), inside the directory ./build. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build  \begin{tabular}{ll} $\sf cd ./build \\ $\sf rm -rf * \\ $\sf cmake -DCMAKE\_BUILD\_TYPE=Debug -DWITH\_MATLAB=ON .. \\ $\sf make \\ \end{tabular}
```

Or, to disable OpenMP support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the ./CMakeLists.txt file. See also ./CMakeLists.txt for additional options. Do not forget to remove everything from the ./build directory before a fresh build!

Additional remarks

- The C++ compiler must be C++11 compliant.
- If using Windows, I recommend compiling under cygwin via cmake and g++. See also http←://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2 for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file cmath using the patch./cmath_cygwin.patch.
- If your compiler does not support OpenMP (as it is the case e.g with clang++), disable OpenMP in your build, as otherwise the linker may not find the gomp library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD_L← IBRARY_PATH is set to point to the MATLAB compiler library location, see the run_OSX_MATLAB script. Otherwise, you will get a runtime error like dyld: Library not loaded: @rpath/libmat.← dylib.

```
* I recommend running via a script, as otherwise setting the 
'DYLD_LIBRARY_PATH' globally may interfere with 
[macports] (https://www.macports.org/)' [cmake] (http://www.cmake.org/) 
installation (in case you use [cmake] (http://www.cmake.org/) from 
[macports] (https://www.macports.org/)). If you use a script, 
then the environment variable is local to the script and 
does not interfere with the rest of the system.

* Example of running script, run from inside the directory where 
the executable 'qpp' is located:

#!/bin/sh # Run Quantum++ under OS X with MATLAB support

export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:"/Applications/MATLAB_R2014b.app/bin/maci64"
./qpp
```

• If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See http://stackoverflow.← com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks for more details about this problem.

Quantum++

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

qpp		
	Quantum++ main namespace	13
qpp::exp	erimental	
	Experimental/test functions/classes, do not use or modify	73
qpp::inte	rnal	
	Internal utility functions, do not use/modify	74

6 Namespace Index

Chapter 3

Hierarchical Index

3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

8 **Hierarchical Index**

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

qpp::Codes	
Const Singleton class that defines quantum error correcting codes	77
qpp::Exception	
Generates custom exceptions, used when validating function parameters	79
qpp::Gates	
Const Singleton class that implements most commonly used gates	82
qpp::Init	
Const Singleton class that performs additional initializations/cleanups	88
qpp::internal::IOManipBase	89
qpp::internal::IOManipEigen	90
qpp::internal::IOManipPointer< PointerType >	91
qpp::internal::IOManipRange< InputIterator >	92
qpp::RandomDevices	
Singeleton class that manages the source of randomness in the library	93
qpp::internal::Singleton< T >	
Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously	
recurring template pattern)	95
qpp::States	
Const Singleton class that implements most commonly used states	96
qpp::Timer	
Measures time	101

10 Class Index

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

constants.h
Constants
entanglement.h
Entanglement functions
entropies.h
Entropy functions
functions.h
Generic quantum computing functions
input_output.h
Input/output functions
instruments.h
Measurement functions
number_theory.h
Number theory functions
operations.h
Quantum operation functions
qpp.h
Quantum++ main header file, includes all other necessary headers
random.h
Randomness-related functions
types.h
Type aliases
classes/codes.h
Quantum error correcting codes
classes/exception.h
Exceptions
classes/gates.h
Quantum gates
classes/init.h
Initialization
classes/random_devices.h
Random devices
classes/states.h
Quantum states
classes/timer.h Timing
· · · · · · · · · · · · · · · · · · ·
experimental/test.h
Experimental/test functions/classes

12 File Index

internal/util.h										
Internal utility functions		 	 	 						120
internal/classes/iomanip.h										
Input/output manipulators		 	 	 			 			118
internal/classes/singleton.h										
Singleton pattern via CRTP		 	 	 			 			119
MATLAB/matlab.h										
Input/output interfacing with MATLA	В.	 	 	 			 			121

Chapter 6

Namespace Documentation

6.1 qpp Namespace Reference

Quantum++ main namespace.

Namespaces

· experimental

Experimental/test functions/classes, do not use or modify.

interna

Internal utility functions, do not use/modify.

Classes

· class Codes

const Singleton class that defines quantum error correcting codes

class Exception

Generates custom exceptions, used when validating function parameters.

· class Gates

const Singleton class that implements most commonly used gates

class Init

const Singleton class that performs additional initializations/cleanups

• class RandomDevices

Singeleton class that manages the source of randomness in the library.

class States

const Singleton class that implements most commonly used states

class Timer

Measures time.

Typedefs

```
• using idx = std::size_t
```

Non-negative integer index.

using cplx = std::complex < double >

Complex number in double precision.

• using ket = Eigen::VectorXcd

Namespace Documentation Complex (double precision) dynamic Eigen column vector. using bra = Eigen::RowVectorXcd Complex (double precision) dynamic Eigen row vector. using cmat = Eigen::MatrixXcd Complex (double precision) dynamic Eigen matrix. using dmat = Eigen::MatrixXd Real (double precision) dynamic Eigen matrix. template<typename Scalar > using dyn_mat = Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic > Dynamic Eigen matrix over the field specified by Scalar. template<typename Scalar > using dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 > Dynamic Eigen column vector over the field specified by Scalar. template<typename Scalar > using dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic > Dynamic Eigen row vector over the field specified by Scalar. **Functions** constexpr cplx operator""_i (unsigned long long int x) noexcept User-defined literal for complex $i = \sqrt{-1}$ (integer overload) constexpr cplx operator""_i (long double x) noexcept User-defined literal for complex $i = \sqrt{-1}$ (real overload) cplx omega (idx D) D-th root of unity. • template<typename Derived > dyn_col_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt coefficients of the bi-partite pure state A. ullet template<typename Derived >cmat schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt basis on Alice side. • template<typename Derived > cmat schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims) Schmidt basis on Bob side. template<typename Derived > &dims)

std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >

Schmidt probabilities of the bi-partite pure state A.

• template<typename Derived >

double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Entanglement of the bi-partite pure state A.

template<typename Derived >

double gconcurrence (const Eigen::MatrixBase< Derived > &A)

G-concurrence of the bi-partite pure state A.

template<typename Derived >

double negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Negativity of the bi-partite mixed state A.

template<typename Derived >

double lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Logarithmic negativity of the bi-partite mixed state A.

```
• template<typename Derived >
  double concurrence (const Eigen::MatrixBase< Derived > &A)
      Wootters concurrence of the bi-partite qubit mixed state A.
template<typename Derived >
  double entropy (const Eigen::MatrixBase< Derived > &A)
      von-Neumann entropy of the density matrix A

    double entropy (const std::vector< double > &prob)

     Shannon entropy of the probability distribution prob.
template<typename Derived >
  double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)
     Renyi- \alpha entropy of the density matrix A, for \alpha \geq 0.

    double renyi (const std::vector< double > &prob, double alpha)

     Renyi- \alpha entropy of the probability distribution prob, for \alpha > 0.
• template<typename Derived >
  double tsallis (const Eigen::MatrixBase< Derived > &A, double q)
      Tsallis- q entropy of the density matrix A, for q \ge 0.

    double tsallis (const std::vector< double > &prob, double q)

      Tsallis- q entropy of the probability distribution prob, for q \geq 0.
• template<typename Derived >
  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, const std::vector< idx > &dims)
     Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, idx d=2)
      Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)
     Adjoint.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)
      Trace.

    template<typename Derived >

  Derived::Scalar det (const Eigen::MatrixBase < Derived > &A)
      Determinant.
• template<typename Derived >
  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum of A.
template<typename Derived >
  Derived::Scalar prod (const Eigen::MatrixBase < Derived > &A)
```

Element-wise product of A.

```
• template<typename Derived >
  double norm (const Eigen::MatrixBase< Derived > &A)
     Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition.
template<typename Derived >
  dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)
     Eigenvalues.

    template<typename Derived >

  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  std::pair< dyn col vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition of Hermitian expression.
• template<typename Derived >
  dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.

    template<typename Derived >

  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.
template<typename Derived >
  std::tuple< cmat, dyn_col_vect< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.
ullet template<typename Derived >
  dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.
• template<typename Derived >
  cmat svdU (const Eigen::MatrixBase< Derived > &A)
     Left singular vectors.

    template<typename Derived >

  cmat svdV (const Eigen::MatrixBase< Derived > &A)
     Right singular vectors.

    template<typename Derived >

  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)

    template<typename Derived >

  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.
• template<typename Derived >
  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.

    template<typename Derived >

  cmat cosm (const Eigen::MatrixBase< Derived > &A)
```

```
Matrix cos.
• template<typename Derived >
  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.
template<typename Derived >
  dyn_mat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.

    template<typename Derived >

  double schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn mat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const type-
  name Derived::Scalar &))
     Functor.

    template<typename T >

  dyn_mat< typename T::Scalar > kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn mat< typename T::Scalar > kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > kron (const std::vector< Derived > &As)
     Kronecker product.
template<typename Derived >
  dyn mat< typename Derived::Scalar > kron (const std::initializer list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.

    template<typename T >

  dyn_mat< typename T::Scalar > dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > dirsum (const T &head, const Args &...tail)
     Direct sum.
template<typename Derived >
  dyn_mat< typename Derived::Scalar > dirsum (const std::vector< Derived > &As)
     Direct sum.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > dirsum (const std::initializer_list< Derived > &As)
     Direct sum.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
```

```
Direct sum power.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx
  cols)
     Reshape.

    template<typename Derived1 , typename Derived2 >

  dyn mat< typename Derived1::Scalar > comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::←
  MatrixBase < Derived2 > &B)
     Commutator.
```

```
    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &V)
      Projector.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > grams (const std::vector< Derived > &Vs)
      Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > grams (const std::initializer_list< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)
      Gram-Schmidt orthogonalization.

    std::vector < idx > n2multiidx (idx n, const std::vector < idx > &dims)

     Non-negative integer index to multi-index.

    idx multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

     Multi-index to non-negative integer index.

    ket mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

     Multi-partite qudit ket.

    ket mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
  std::vector< double > abssq (InputIterator first, InputIterator last)
      Computes the absolute values squared of a range of complex numbers.

    template<typename Derived >

  std::vector< double > abssq (const Eigen::MatrixBase< Derived > &V)
     Computes the absolute values squared of a column vector.

    template<typename InputIterator >

  std::iterator_traits< InputIterator >::value_type sum (InputIterator first, InputIterator last)
      Element-wise sum of an STL-like range.

    template<typename Container >

  Container::value_type sum (const Container &c)
      Element-wise sum of the elements of an STL-like container.

    template<typename InputIterator >

  std::iterator traits< InputIterator >::value type prod (InputIterator first, InputIterator last)
     Element-wise product of an STL-like range.

    template<typename Container >

  Container::value_type prod (const Container &c)
     Element-wise product of the elements of an STL-like container.
• template<typename Derived >
  dyn_col_vect< typename Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)
     Finds the pure state representation of a matrix proportional to a projector onto a pure state.
template<typename T >
  std::vector< T > complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

template < typename Derived >
 std::vector < double > rho2bloch (const Eigen::MatrixBase < Derived > &A)

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$

internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

Eigen expression ostream manipulator.

internal::IOManipEigen disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

• template<typename InputIterator >

internal::IOManipRange< InputIterator > disp (const InputIterator &first, const InputIterator &last, const std ::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

• template<typename Container >

internal::IOManipRange< typename Container::const_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >

internal::IOManipPointer< PointerType > disp (const PointerType *p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

• template<typename Derived >

void save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple < idx, std::vector < double >, std::vector < cmat > > measure (const Eigen::MatrixBase < Derived > &A, const std::initializer_list < cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

```
std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

```
std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)
```

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template < typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, std::vector < idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

std::tuple < std::vector < idx >, double, cmat > measure_seq (const Eigen::MatrixBase < Derived > &A, std::vector < idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

• template<typename Derived >

Derived loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

cmat loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$

void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat_file, const std
::string &var name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

template<>

void saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

• template<>

void saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

std::vector< int > x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

• double contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

- double $\operatorname{contfrac2x}$ (const std ::vector< $\operatorname{int} > \operatorname{\&cf}$)

Real representation of a simple continued fraction.

unsigned long long int gcd (unsigned long long int m, unsigned long long int n)

Greatest common divisor of two non-negative integers.

unsigned long long int gcd (const std::vector< unsigned long long int > &ns)

Greatest common divisor of a list of non-negative integers.

• unsigned long long int lcm (unsigned long long int m, unsigned long long int n)

Least common multiple of two positive integers.

unsigned long long int lcm (const std::vector< unsigned long long int > &ns)

Least common multiple of a list of positive integers.

std::vector< idx > invperm (const std::vector< idx > &perm)

Inverse permutation.

std::vector< idx > compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< unsigned long long int > factors (unsigned long long int n)

Prime factor decomposition.

• bool isprime (unsigned long long int n)

Primality test.

- unsigned long long int modpow (unsigned long long int a, unsigned long long int n, unsigned long long int p)

 Integer power modulo p.
- template<typename Derived1 , typename Derived2 >
 dyn_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
 Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const
 std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

 $\frac{dyn_mat}{dyn_mat} < typename \ Derived1::Scalar > \frac{dyn_mat}{dx} < Derived1 > &state, \ const \ Eigen::MatrixBase < Derived2 > &A, \ const \ std::vector < \frac{idx}{dx} > &ctrl, \ const \ std::vector < \frac{idx}{dx} > &subsys, \ \frac{idx}{dx} < d=2)$

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen↔ ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)
```

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

ullet template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector<
idx > &subsys, idx d=2)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

 $dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std <math>\leftarrow ::vector < idx > &dims)$

Partial trace.

ullet template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace2 (const Eigen::MatrixBase< Derived > &A, const std↔ ::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &subsys, const std::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector
idx > &subsys, idx d=2)

Partial trace.

• template<typename Derived >

Partial transpose.

template<typename Derived >

dyn_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial transpose.

• template<typename Derived >

Subsystem permutation.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std ← ::vector < idx > &perm, idx d=2)

Subsystem permutation.

double rand (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

idx randidx (idx a=std::numeric_limits < idx >::min(), idx b=std::numeric_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

template<typename Derived >

Derived rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

• template<>

cmat rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

• template<typename Derived >

Derived randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

• template<>

dmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

template<>

cmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

· cmat randU (idx D)

Generates a random unitary matrix.

cmat randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat randH (idx D)

Generates a random Hermitian matrix.

ket randket (idx D)

Generates a random normalized ket (pure state vector)

cmat randrho (idx D)

Generates a random density matrix.

std::vector< idx > randperm (idx n)

Generates a random uniformly distributed permutation.

Variables

• constexpr double chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx maxn = 64

Maximum number of allowed qu(d)its (subsystems)

constexpr double pi = 3.141592653589793238462643383279502884

 π

• constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

constexpr double infty = std::numeric limits<double>::infinity()

Used to denote infinity in double precision.

6.1.1 Detailed Description

Quantum++ main namespace.

6.1.2 Typedef Documentation

6.1.2.1 using qpp::bra = typedef Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

6.1.2.2 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.2.3 using qpp::cplx = typedef std::complex < double>

Complex number in double precision.

6.1.2.4 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.2.5 template < typename Scalar > using qpp::dyn_col_vect = typedef Eigen::Matrix < Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by Scalar.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

6.1.2.6 template<typename Scalar > using qpp::dyn_mat = typedef Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

6.1.2.7 template < typename Scalar > using qpp::dyn_row_vect = typedef Eigen::Matrix < Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

6.1.2.8 using qpp::idx = typedef std::size_t

Non-negative integer index.

6.1.2.9 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

- 6.1.3 Function Documentation
- 6.1.3.1 template<typename Derived > cmat qpp::absm (const Eigen::MatrixBase< Derived > & A)

Matrix absolut value.

Α	Eigen expression
---	------------------

Returns

Matrix absolut value of A

6.1.3.2 template<typename InputIterator > std::vector<double> qpp::abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of a range of complex numbers.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Real vector consisting of the range absolut values squared

6.1.3.3 template<typename Derived > std::vector<double> qpp::abssq (const Eigen::MatrixBase< Derived > & V)

Computes the absolute values squared of a column vector.

Parameters

V	Eigen expression

Returns

Real vector consisting of the absolut values squared

6.1.3.4 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::adjoint (const Eigen::MatrixBase< Derived > & A)

Adjoint.

Parameters

Α	Eigen expression

Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field as A

6.1.3.5 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::anticomm (const Eigen::MatrixBase< Derived1 > & A, const Eigen::MatrixBase< Derived2 > & B)

Anti-commutator.

See also

qpp::comm()

Anti-commutator $\{A,B\} = AB + BA$. Both A and B must be Eigen expressions over the same scalar field.

A	Eigen expression
В	Eigen expression

Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field as A

6.1.3.6 template < typename Derived1 , typename Derived2 > $dyn_mat < typename Derived1::Scalar > dpp::apply (const Eigen::MatrixBase < Derived2 > & A, const std::vector < <math>idx > & subsys$, const std::vector < idx > & dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

Parameters

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

Returns

Gate A applied to the part subsys of state

6.1.3.7 template < typename Derived1 , typename Derived2 > $dyn_mat < typename Derived1::Scalar > dpp::apply (const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & subsys, idx d = 2)$

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

Parameters

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

Returns

Gate A applied to the part subsys of state

6.1.3.8 template<typename Derived > cmat qpp::apply (const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks)

Applies the channel specified by the set of Kraus operators *Ks* to the density matrix *rho*.

rho	Eigen expression
Ks	Set of Kraus operators

Returns

Output density matrix after the action of the channel

6.1.3.9 template<typename Derived > cmat qpp::apply (const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const std::vector< idx > & dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

Parameters

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
dims	Dimensions of the multi-partite system

Returns

Output density matrix after the action of the channel

6.1.3.10 template < typename Derived > cmat qpp::apply (const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

Parameters

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
d	Subsystem dimensions

Returns

Output density matrix after the action of the channel

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

See also

qpp::Gates::CTRL()

Note

The dimension of the gate A must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

Returns

CTRL-A gate applied to the part subsys of state

6.1.3.12 template<typename Derived1 , typename Derived2 > dyn_mat<typename Derived1::Scalar> qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > & state, const Eigen::MatrixBase< Derived2 > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx d = 2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

See also

qpp::Gates::CTRL()

Note

The dimension of the gate A must match the dimension of subsys

Parameters

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

Returns

CTRL-A gate applied to the part subsys of state

6.1.3.13 cmat qpp::bloch2rho (const std::vector< double > & r) [inline]

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

See also

qpp::rho2bloch()

Parameters

r	3-dimensional real vector

Returns

Qubit density matrix

6.1.3.14 std::vector<cmat> qpp::choi2kraus(const cmat & A) [inline]

Orthogonal Kraus operators from Choi matrix.

See also

qpp::kraus2choi()

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix A

Note

The Kraus operators satisfy $Tr(K_i^{\dagger}K_j) = \delta_{ij}$ for all $i \neq j$

Parameters

Α	Choi matrix

Returns

Set of orthogonal Kraus operators

6.1.3.15 cmat qpp::choi2super (const cmat & A) [inline]

Converts Choi matrix to superoperator matrix.

See also

qpp::super2choi()

Parameters

Α	Choi matrix
---	-------------

Returns

Superoperator matrix

6.1.3.16 template < typename Derived1 , typename Derived2 > dyn_mat < typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B)

Commutator.

See also

qpp::anticomm()

Commutator [A, B] = AB - BA. Both A and B must be Eigen expressions over the same scalar field.

Parameters

Α	Eigen expression
В	Eigen expression

Returns

Commutator AB - BA, as a dynamic matrix over the same scalar field as A

6.1.3.17 template < typename T > std::vector < T > qpp::complement (std::vector < T > subsys, idx N)

Constructs the complement of a subsystem vector.

subsys	Subsystem vector
N	Total number of systems

Returns

The complement of *subsys* with respect to the set $\{0, 1, \dots, N-1\}$

6.1.3.18 std::vector < idx > & perm, const std::vector < idx > & sigma) [inline]

Compose permutations.

Parameters

perm	Permutation
sigma	Permutation

Returns

Composition of the permutations $perm \circ sigma = perm(sigma)$

6.1.3.19 template < typename Derived > double qpp::concurrence (const Eigen::MatrixBase < Derived > & A)

Wootters concurrence of the bi-partite qubit mixed state A.

Parameters

A Eigen expression	
--------------------	--

Returns

Wootters concurrence

6.1.3.20 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::conjugate (const Eigen::MatrixBase< Derived > & A)

Complex conjugate.

Parameters

Α	Eigen expression

Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field as A

6.1.3.21 double qpp::contfrac2x (const std::vector < int > & cf, idx n) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

cf	Integer vector containing the simple continued fraction expansion
n	Number of terms considered in the continued fraction expansion. If n is greater than the size
	of <i>cf</i> ,then all terms in <i>cf</i> are considered.

Returns

Real representation of the simple continued fraction

6.1.3.22 double qpp::contfrac2x (const std::vector < int > & cf) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

Parameters

cf	Integer vector containing the simple continued fraction expansion

Returns

Real representation of the simple continued fraction

6.1.3.23 template < typename Derived > cmat qpp::cosm (const Eigen::MatrixBase < Derived > & A)

Matrix cos.

Parameters

Α	Eigen expression

Returns

Matrix cosine of A

6.1.3.24 template<typename OutputScalar , typename Derived > dyn_mat <OutputScalar> qpp::cwise (const Eigen::MatrixBase< Derived > & A, OutputScalar(*)(const typename Derived::Scalar &) f)

Functor.

Parameters

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

Returns

Component-wise f(A), as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.25 template<typename Derived > Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > & A)

Determinant.

Α	Eigen expression
---	------------------

Returns

Determinant of A, as a scalar in the same scalar field as A. Returns $\pm \infty$ when the determinant overflows/underflows.

6.1.3.26 template < typename T > dyn_mat < typename T::Scalar > qpp::dirsum (const T & head)

Direct sum.

See also

qpp::dirsumpow()

Used to stop the recursion for the variadic template version of qpp::dirsum()

Parameters

head	Eigen expression

Returns

Its argument head

6.1.3.27 template < typename T , typename ... Args > dyn_mat < typename T::Scalar > qpp::dirsum (const T & head, const Args &... tail)

Direct sum.

See also

qpp::dirsumpow()

Parameters

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.28 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::dirsum (const std::vector< Derived > & As)

Direct sum.

See also

qpp::dirsumpow()

As	std::vector of Eigen expressions
----	----------------------------------

Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Direct sum.

See also

qpp::dirsumpow()

Parameters

As	std::initializer_list of Eigen expressions, such as {A1, A2,, Ak}
----	---

Returns

Direct sum of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.30 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::dirsumpow (const Eigen::MatrixBase< Derived > & A, idx n)

Direct sum power.

See also

qpp::dirsum()

Parameters

Α	Eigen expression
n	Non-negative integer

Returns

Direct sum of A with itself n times $A^{\oplus n}$, as a dynamic matrix over the same scalar field as A

6.1.3.31 template<typename Derived > internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop)

Eigen expression ostream manipulator.

Parameters

Α	Eigen expression
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.32 internal::IOManipEigen qpp::disp(cplx z, double chop = qpp::chop) [inline]

Complex number ostream manipulator.

Parameters

Z	Complex number (or any other type implicitly cast-able to std::complex <double>)</double>
chop	Set to zero the elements smaller in absolute value than chop

Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.33 template<typename InputIterator > internal::IOManipRange<InputIterator> qpp::disp (const InputIterator & first, const InputIterator & last, const std::string & separator, const std::string & start = " [", const std::string & end = "] ")

Range ostream manipulator.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range
separator	Separator
start	Left marking
end	Right marking

Returns

Instance of qpp::internal::internal::IOManipRange

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

Parameters

	С	Container
	separator	Separator
	start	Left marking
Ì	end	Right marking

Returns

Instance of qpp::internal::internal::IOManipRange

6.1.3.35 template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp (const PointerType * p, idx n, const std::string & separator, const std::string & start = " [", const std::string & end = "] ")

C-style pointer ostream manipulator.

р	Pointer to the first element
n	Number of elements to be displayed
separator	Separator
start	Left marking
end	Right marking

Returns

Instance of qpp::internal::IOManipPointer

6.1.3.36 template<typename Derived > std::pair<dyn_col_vect<cplx>, cmat> qpp::eig (const Eigen::MatrixBase< Derived > & A)

Full eigen decomposition.

See also

qpp::heig()

Parameters

Α	Eigen expression

Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.37 template < typename Derived > double qpp::entanglement (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

See also

qpp::entropy()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Entanglement, with the logarithm in base 2

6.1.3.38 template<typename Derived > double qpp::entropy (const Eigen::MatrixBase< Derived > & A)

von-Neumann entropy of the density matrix A

A Eigen expression

Returns

von-Neumann entropy, with the logarithm in base 2

6.1.3.39 double qpp::entropy (const std::vector < double > & prob) [inline]

Shannon entropy of the probability distribution prob.

Parameters

prob Real probability vector

Returns

Shannon entropy, with the logarithm in base 2

6.1.3.40 template<typename Derived > dyn_col_vect<cplx> qpp::evals (const Eigen::MatrixBase< Derived > & A)

Eigenvalues.

See also

qpp::hevals()

Parameters

A Eigen expression

Returns

Eigenvalues of A, as a complex dynamic column vector

6.1.3.41 template<typename Derived > cmat qpp::evects (const Eigen::MatrixBase< Derived > & A)

Eigenvectors.

See also

qpp::hevects()

Parameters

A Eigen expression

Returns

Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.42 template<typename Derived > cmat qpp::expm (const Eigen::MatrixBase< Derived > & A)

Matrix exponential.

Α	Eigen expression
---	------------------

Returns

Matrix exponential of A

6.1.3.43 std::vector < unsigned long long int > qpp::factors (unsigned long long int n) [inline]

Prime factor decomposition.

Note

Runs in $\mathcal{O}(\sqrt{n})$ time complexity

Parameters

n	Integer strictly greater than 1

Returns

Integer vector containing the factors

6.1.3.44 template<typename Derived > cmat qpp::funm (const Eigen::MatrixBase< Derived > & A, cplx(*)(const cplx &) f

Functional calculus f(A)

Parameters

A	Eigen expression
f	Pointer-to-function from complex to complex

Returns

f(A)

6.1.3.45 unsigned long long int qpp::gcd (unsigned long long int *m*, unsigned long long int *n*) [inline]

Greatest common divisor of two non-negative integers.

See also

qpp::lcm()

Parameters

т	Non-negative integer
n	Non-negative integer

Returns

Greatest common divisor of m and n

6.1.3.46 unsigned long long int qpp::gcd (const std::vector < unsigned long long int > & ns) [inline]

Greatest common divisor of a list of non-negative integers.

See also

qpp::lcm()

Parameters

ns	List of non-negative integers

Returns

Greatest common divisor of all numbers in ns

6.1.3.47 template < typename Derived > double qpp::gconcurrence (const Eigen::MatrixBase < Derived > & A)

G-concurrence of the bi-partite pure state A.

Note

Both local dimensions must be equal

Uses qpp::logdet() to avoid overflows

See also

qpp::logdet()

Parameters

Α	Eigen expression
---	------------------

Returns

G-concurrence

6.1.3.48 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams (const std::vector< Derived > & Vs)

Gram-Schmidt orthogonalization.

Parameters

Vs	std::vector of Eigen expressions as column vectors
----	--

Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.49 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::grams (const std::initializer_list < Derived > & Vs)

Gram-Schmidt orthogonalization.

Vs std::initializer_list of Eigen expressions as column vectors

Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.50 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::grams (const Eigen::MatrixBase< Derived > & A)

Gram-Schmidt orthogonalization.

Parameters

A Eigen expression, the input vectors are the columns of A

Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field as A

6.1.3.51 template<typename Derived > std::pair<dyn_col_vect<double>, cmat> qpp::heig (const Eigen::MatrixBase< Derived > & A)

Full eigen decomposition of Hermitian expression.

See also

qpp::eig()

Parameters

Α	Eigen expression

Returns

Pair of: 1. Eigenvalues of A, as a real dynamic column vector, and 2. Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.52 template<typename Derived > dyn_col_vect<double> qpp::hevals (const Eigen::MatrixBase< Derived > & A)

Hermitian eigenvalues.

See also

qpp::evals()

Parameters

Α	Eigen expression

Returns

Eigenvalues of Hermitian A, as a real dynamic column vector

6.1.3.53 template<typename Derived > cmat qpp::hevects (const Eigen::MatrixBase< Derived > & A)

Hermitian eigenvectors.

See also

qpp::evects()

Parameters

A | Eigen expression

Returns

Eigenvectors of Hermitian A, as columns of a complex matrix

6.1.3.54 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::inverse (const Eigen::MatrixBase< Derived > & A)

Inverse.

Parameters

A | Eigen expression

Returns

Inverse of A, as a dynamic matrix over the same scalar field as A

6.1.3.55 std::vector<idx> qpp::invperm(const std::vector<idx> & perm) [inline]

Inverse permutation.

Parameters

perm | Permutation

Returns

Inverse of the permutation perm

6.1.3.56 bool qpp::isprime (unsigned long long int *n*) [inline]

Primality test.

Note

Runs in $\mathcal{O}(\sqrt{n})$ time complexity

Parameters

n Integer strictly greater than 1

Returns

True if the number is prime, false otherwise

6.1.3.57 cmat qpp::kraus2choi (const std::vector < cmat > & Ks) [inline]

Choi matrix.

See also

qpp::choi2kraus()

Constructs the Choi matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis $\{|i\rangle\langle j|\}$ ordered in lexicographical order, i.e. $|0\rangle\langle 0|$, $|0\rangle\langle 1|$ etc.

Note

The superoperator matrix S and the Choi matrix C are related by $S_{ab,mn}=C_{ma,nb}$

Parameters

Ks Set of Kraus operators

Returns

Choi matrix

6.1.3.58 cmat qpp::kraus2super (const std::vector < cmat > & Ks) [inline]

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis $\{|i\rangle\langle j|\}$ ordered in lexicographical order, i.e. $|0\rangle\langle 0|$, $|0\rangle\langle 1|$ etc.

Parameters

Ks	Set of Kraus operators

Returns

Superoperator matrix

6.1.3.59 template<typename T > dyn_mat<typename T::Scalar> qpp::kron (const T & head)

Kronecker product.

See also

qpp::kronpow()

Used to stop the recursion for the variadic template version of qpp::kron()

Parameters

head | Eigen expression

Returns

Its argument head

6.1.3.60 template<typename T , typename... Args> dyn_mat<typename T::Scalar> qpp::kron (const T & head, const Args &... tail)

Kronecker product.

See also

qpp::kronpow()

Parameters

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.61 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::kron (const std::vector< Derived > & As)

Kronecker product.

See also

qpp::kronpow()

Parameters

As	std::vector of Eigen expressions
----	----------------------------------

Returns

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker product.

See also

qpp::kronpow()

Parameters

As	std::initializer_list of Eigen expressions, such as {A1, A2, ,Ak}
----	---

Returns

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.63 template < typename Derived > dyn_mat < typename Derived::Scalar > dyn_mat < typename Derived > & A, dx n)

Kronecker power.

See also

qpp::kron()

Parameters

Α	Eigen expression
n	Non-negative integer

Returns

Kronecker product of A with itself n times $A^{\otimes n}$, as a dynamic matrix over the same scalar field as A

6.1.3.64 unsigned long long int qpp::lcm (unsigned long long int m, unsigned long long int n) [inline]

Least common multiple of two positive integers.

See also

qpp::gcd()

Parameters

m	Positive integer
n	Positive integer

Returns

Least common multiple of m and n

6.1.3.65 unsigned long long int qpp::lcm (const std::vector < unsigned long long int > & ns) [inline]

Least common multiple of a list of positive integers.

See also

qpp::gcd()

Parameters

ns	List of positive integers

Returns

Least common multiple of all numbers in ns

6.1.3.66 template < typename Derived > dyn_mat < typename Derived::Scalar > qpp::load (const std::string & fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

See also

```
qpp::save()
```

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin"
auto mat = load<cmat>("input.bin");
```

Parameters

Α	Eigen expression
fname	Output file name

6.1.3.67 template < typename Derived > Derived qpp::loadMATLABmatrix (const std::string & mat_file, const std::string & var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

See also

```
qpp::saveMATLABmatrix()
```

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be loaded)

```
6.1.3.68 template <> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

Note

If var_name is a complex matrix, only the real part is loaded

Parameters

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen double dynamic matrix (qpp::dmat)

6.1.3.69 template<> cmat qpp::loadMATLABmatrix (const std::string & mat_file, const std::string & var_name)
[inline]

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

qpp::saveMATLABmatrix()

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

Parameters

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

Returns

Eigen complex dynamic matrix (qpp::cmat)

6.1.3.70 template<typename Derived > Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > & A)

Logarithm of the determinant.

Useful when the determinant overflows/underflows

Parameters

Α	Eigen expression
---	------------------

Returns

Logarithm of the determinant of A, as a scalar in the same scalar field as A

6.1.3.71 template<typename Derived > cmat qpp::logm (const Eigen::MatrixBase< Derived > & A)

Matrix logarithm.

Parameters

Α	Eigen expression

Returns

Matrix logarithm of A

6.1.3.72 template < typename Derived > double qpp::lognegativity (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Logarithmic negativity of the bi-partite mixed state A.

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Logarithmic negativity, with the logarithm in base 2

 $6.1.3.73 \quad template < typename \ Derived > std::tuple < idx, \ std::vector < double >, \ std::vector < cmat > > \ qpp::measure (\ const \ Eigen::MatrixBase < Derived > & \textit{A}, \ const \ std::vector < cmat > & \textit{Ks} \)$

Measures the state A using the set of Kraus operators Ks.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.74 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.75 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const cmat & U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

Parameters

	Α	Eigen expression
ſ	U	Unitary matrix whose columns represent the measurement basis vectors

Returns

6.1.3.76 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.77 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system
Ks	Set of Kraus operators

Returns

6.1.3.78 template<typename Derived > std::tuple<idx, std::vector < double>, std::vector<cmat> > qpp::measure (const Eigen::MatrixBase< Derived > & A, const std::vector< cmat > & Ks, const std::vector< idx > & subsys, const idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
Ks	Set of Kraus operators

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.79 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const std::initializer_list < cmat > & Ks, const std::vector < idx > & subsys, const idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

See also

qpp::measure_seq()

Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
Ks	Set of Kraus operators

Returns

6.1.3.80 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure (const Eigen::MatrixBase < Derived > & A, const cmat & U, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis specified by the unitary matrix *U*.

See also

```
qpp::measure_seq()
```

Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

A	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system
U	Unitary matrix whose columns represent the measurement basis vectors

Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

Measures the part *subsys* of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

See also

```
qpp::measure_seq()
```

Note

The dimension of *U* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions
U	Unitary matrix whose columns represent the measurement basis vectors

Returns

6.1.3.82 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq (const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, std::vector < idx > dims)

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

qpp::measure()

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.83 template < typename Derived > std::tuple < std::vector < idx>, double, cmat> qpp::measure_seq (const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, idx d = 2)

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

See also

qpp::measure()

Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.84 ket qpp::mket (const std::vector < idx > & mask, const std::vector < idx > & dims) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket $|\text{mask}\rangle$, where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

Parameters

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.85 ket qpp::mket (const std::vector < idx > & mask, idx d = 2) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket $|\text{mask}\rangle$, all subsystem having equal dimension *d. mask* is a std::vector of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

Parameters

mask	std::vector of non-negative integers
d	Subsystem dimensions

Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.86 unsigned long long int qpp::modpow (unsigned long long int a, unsigned long long int n, unsigned long long int p)

[inline]

Integer power modulo p.

Computes $a^n \bmod p$

Parameters

а	Non-negative integer
n	Non-negative integer
р	Strictly positive integer

Returns

 $a^n \bmod p$

6.1.3.87 cmat qpp::mprj (const std::vector < idx > & mask, const std::vector < idx > & dims) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket $|mask\rangle$, where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

Parameters

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.88 cmat qpp::mprj (const std::vector < idx > & mask, idx d = 2) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket $|mask\rangle$, all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

Parameters

mask	std::vector of non-negative integers
d	Subsystem dimensions

Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.89 idx qpp::multiidx2n(const std::vector < idx > & midx, const std::vector < idx > & dims) [inline]

Multi-index to non-negative integer index.

See also

qpp::n2multiidx()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

Parameters

midx	Multi-index
dims	Dimensions of the multi-partite system

Returns

Non-negative integer index

6.1.3.90 std::vector<idx> qpp::n2multiidx(idx n, const std::vector< idx > & dims) [inline]

Non-negative integer index to multi-index.

See also

qpp::multiidx2n()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

Parameters

n	Non-negative integer index
dims	Dimensions of the multi-partite system

Returns

Multi-index of the same size as dims

6.1.3.91 template < typename Derived > double qpp::negativity (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Negativity of the bi-partite mixed state A.

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Negativity

6.1.3.92 template < typename Derived > double qpp::norm (const Eigen::MatrixBase < Derived > & A)

Frobenius norm.

Parameters

Α	Eigen expression

Returns

Frobenius norm of A

```
6.1.3.93 cplx qpp::omega (idx D) [inline]
```

D-th root of unity.

Parameters

D Non-negative integer

Returns

D-th root of unity $\exp(2\pi i/D)$

```
6.1.3.94 constexpr cplx qpp::operator""_i ( unsigned long long int x ) [inline], [noexcept]
```

User-defined literal for complex $i = \sqrt{-1}$ (integer overload)

Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

```
6.1.3.95 constexpr cplx qpp::operator""_i(long double x) [inline], [noexcept]
```

User-defined literal for complex $i = \sqrt{-1}$ (real overload)

Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.96 template < typename Derived > dyn_mat < typename Derived::Scalar > dyn_mat < typename Derived > dyn_mat < ty

Matrix power.

See also

qpp::spectralpowm()

Explicitly multiplies the matrix A with itself n times. By convention $A^0 = I$.

Α	Eigen expression
n	Non-negative integer

Returns

Matrix power A^n , as a dynamic matrix over the same scalar field as A

6.1.3.97 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::prj (const Eigen::MatrixBase< Derived > & V)

Projector.

Normalized projector onto state vector

Parameters

V	Eigen expression

Returns

Projector onto the state vector V, or the matrix Zero if V has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field as A

6.1.3.98 template < typename Derived > Derived::Scalar qpp::prod (const Eigen::MatrixBase < Derived > & A)

Element-wise product of A.

Parameters

Α	Eigen expression
---	------------------

Returns

Element-wise product of A, as a scalar in the same scalar field as A

6.1.3.99 template < typename InputIterator > std::iterator_traits < InputIterator > ::value_type qpp::prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Element-wise product of the range, as a scalar in the same scalar field as the range

6.1.3.100 template < typename Container > Container::value_type qpp::prod (const Container & c)

Element-wise product of the elements of an STL-like container.

c STL-like containe	
---------------------	--

Returns

Element-wise product of the elements of the container, as a scalar in the same scalar field as the container

6.1.3.101 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace (const Eigen::MatrixBase< Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims)

Partial trace.

See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

Returns

Partial trace $Tr_{subsys}(\cdot)$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.102 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::ptrace (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2)

Partial trace.

See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

Returns

Partial trace $Tr_{subsys}(\cdot)$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.103 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptrace1 (const Eigen::MatrixBase < Derived > & A, const std::vector< idx > & dims)

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

Returns

Partial trace $Tr_A(\cdot)$ over the first subsytem A in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.104 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptrace2 (const Eigen::MatrixBase < Derived > & A, const std::vector< idx > & dims)

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

Returns

Partial trace $Tr_B(\cdot)$ over the second subsystem B in a bi-partite system $A\otimes B$, as a dynamic matrix over the same scalar field as A

6.1.3.105 template < typename Derived > $dyn_mat < typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims)$

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

Returns

Partial transpose $(\cdot)^{T_{subsys}}$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.106 template<typename Derived > dyn_mat <typename Derived::Scalar> qpp::ptranspose (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2)

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Parameters

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

Returns

Partial transpose $(\cdot)^{T_{subsys}}$ over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.107 template < typename Derived > double qpp::qmutualinfo (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsysA, const std::vector < idx > & subsysB, const std::vector < idx > & dims)

Quantum mutual information between 2 subsystems of a composite system.

Parameters

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
dims	Dimensions of the multi-partite system

Returns

Mutual information between the 2 subsystems

6.1.3.108 template < typename Derived > double qpp::qmutualinfo (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsysA, const std::vector < idx > & subsysB, idx d = 2)

Quantum mutual information between 2 subsystems of a composite system.

Parameters

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
d	Subsystem dimensions

Returns

Mutual information between the 2 subsystems

6.1.3.109 double qpp::rand (double a = 0, double b = 1) [inline]

Generates a random real number uniformly distributed in the interval [a, b)

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random real number (double) uniformly distributed in the interval [a, b)

6.1.3.110 template < typename Derived > Derived qpp::rand (idx rows, idx cols, double a = 0, double b = 1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat

```
6.1.3.111 template <> dmat qpp::rand (idx rows, idx cols, double a, double b) [inline]
```

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random real matrix

```
6.1.3.112 template<> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) uniformly distributed in [-1,1) auto mat = rand<cmat>(3, 3, -1, 1);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

Returns

Random complex matrix

```
6.1.3.113 cmat qpp::randH(idx D) [inline]
```

Generates a random Hermitian matrix.

Parameters

7	Discouries of the Ullbook source
D	Dimension of the Hilbert space
_	- monorous and a most opened

Returns

Random Hermitian matrix

Generates a random index (idx) uniformly distributed in the interval [a, b].

Parameters

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

Returns

Random index (idx) uniformly distributed in the interval [a, b]

```
6.1.3.115 ket qpp::randket(idx D) [inline]
```

Generates a random normalized ket (pure state vector)

Parameters

	Discounting of the Little at the control of the con
1)	Dimension of the Hilbert space
	Billionological of the Linbert opace

Returns

Random normalized ket

```
6.1.3.116 std::vector<cmat> qpp::randkraus(idx N, idx D) [inline]
```

Generates a set of random Kraus operators.

Note

The set of Kraus operators satisfy the closure condition $\sum_i K_i^\dagger K_i = I$

Parameters

N	Number of Kraus operators
D	Dimension of the Hilbert space

Returns

Set of N Kraus operators satisfying the closure condition

6.1.3.117 template < typename Derived > Derived qpp::randn (idx rows, idx cols, double mean = 0, double sigma = 1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat

6.1.3.118 template<> dmat qpp::randn(idx rows, idx cols, double mean, double sigma) [inline]

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

Returns

Random real matrix

```
6.1.3.119 template <> cmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) normally distributed in N(0,2) auto mat = randn<cmat>(3, 3, 0, 2);
```

Parameters

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

Returns

Random complex matrix

```
6.1.3.120 double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]
```

Generates a random real number (double) normally distributed in N(mean, sigma)

Parameters

mean	Mean

sigma Standard deviation

Returns

Random real number normally distributed in N(mean, sigma)

6.1.3.121 std::vector<idx> qpp::randperm(idx n) [inline]

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by std::shuffle), so that all permutations are equally probable Parameters

n Size of the permutation

Returns

Random permutation of size n

6.1.3.122 cmat qpp::randrho(idx D) [inline]

Generates a random density matrix.

Parameters

D Dimension of the Hilbert space

Returns

Random density matrix

6.1.3.123 cmat qpp::randU(idx D) [inline]

Generates a random unitary matrix.

Parameters

D Dimension of the Hilbert space

Returns

Random unitary

6.1.3.124 cmat qpp::randV (idx Din, idx Dout) [inline]

Generates a random isometry matrix.

Parameters

Din Size of the input Hilbert space

Dout	Size of the output Hilbert space

Returns

Random isometry matrix

6.1.3.125 template < typename Derived > double qpp::renyi (const Eigen::MatrixBase < Derived > & A, double alpha)

Renyi- α entropy of the density matrix A, for $\alpha \geq 0$.

Note

When $\alpha \to 1$ the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

Parameters

Α	Eigen expression
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

Returns

Renyi- α entropy, with the logarithm in base 2

6.1.3.126 double qpp::renyi (const std::vector < double > & prob, double alpha) [inline]

Renyi- α entropy of the probability distribution *prob*, for $\alpha \geq 0$.

Note

When $\alpha \to 1$ the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

Parameters

prob	Real probability vector
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

Returns

Renyi- α entropy, with the logarithm in base 2

6.1.3.127 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::reshape (const Eigen::MatrixBase< Derived > & A, idx rows, idx cols)

Reshape.

Uses column-major order when reshaping (same as MATLAB)

Parameters

Α	Eigen expression
rows	Number of rows of the reshaped matrix

cols Number of columns of the reshaped matrix

Returns

Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field as A

6.1.3.128 template<typename Derived > std::vector<double> qpp::rho2bloch (const Eigen::MatrixBase< Derived > & A)

Computes the 3-dimensional real Bloch vector corresponding to the gubit density matrix A.

See also

qpp::bloch2rho()

Note

It is implicitly assumed that the density matrix is Hermitian

Parameters

Α	Eigen expression

Returns

3-dimensional Bloch vector

6.1.3.129 template<typename Derived > dyn_col_vect<typename Derived::Scalar> qpp::rho2pure (const Eigen::MatrixBase< Derived > & A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

Note

No purity check is done, the input state A must have rank one, otherwise the function returns the first non-zero eigenvector of A

Parameters

Α	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. A is
	assumed to have rank one

Returns

The unique non-zero eigenvector of A, as a dynamic column vector over the same scalar field as A

6.1.3.130 template<typename Derived > void qpp::save (const Eigen::MatrixBase< Derived > & A, const std::string & fname)

Saves Eigen expression to a binary file (internal format) in double precision.

See also

qpp::load()

Parameters

Α	Eigen expression
fname	Output file name

6.1.3.131 template<typename Derived > void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

See also

qpp::loadMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be saved)

6.1.3.132 template <> void qpp::saveMATLABmatrix (const Eigen::MatrixBase < dmat > & A, const std::string & mat_file, const std::string & var_name, const std::string & mode) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

qpp::loadMATLABmatrix()

Parameters

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.133 template <> void qpp::saveMATLABmatrix (const Eigen::MatrixBase < cmat > & A, const std::string & mat_file, const std::string & war_name, const std::string & mode) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

qpp::loadMATLABmatrix()

Parameters

	Α	Eigen expression over the complex field
	mat_file	MATALB .mat file
	var_name	Variable name in the .mat file representing the matrix to be saved
Ī	mode	Saving mode (append, overwrite etc.), see MATLAB matOpen() documentation for details

 $6.1.3.134 \quad template < typename \ Derived > double \ qpp::schatten \ (\ const \ Eigen::Matrix Base < Derived > \& \ \textit{A}, \ double \ \textit{p} \)$

Schatten matrix norm.

Parameters

Α	Eigen expression
р	Real number, greater or equal to 1, use qpp::infty for $p=\infty$

Returns

Schatten-p matrix norm of A

6.1.3.135 template < typename Derived > cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt basis on Alice side.

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Unitary matrix ${\cal U}$ whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.136 template < typename Derived > cmat qpp::schmidtB (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt basis on Bob side.

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Unitary matrix ${\cal V}$ whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.137 template<typename Derived > dyn_col_vect<double> qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims)

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

See also

qpp::schmidtprobs()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Schmidt coefficients of A, as a real dynamic column vector

6.1.3.138 template < typename Derived > std::vector < double > qpp::schmidtprobs (const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

See also

qpp::schmidtcoeffs()

Parameters

Α	Eigen expression
dims	Dimensions of the bi-partite system

Returns

Real vector consisting of the Schmidt probabilites of A

6.1.3.139 template < typename Derived > cmat qpp::sinm (const Eigen::MatrixBase < Derived > & A)

Matrix sin.

Parameters

Α	Eigen expression

Returns

Matrix sine of A

6.1.3.140 template < typename Derived > cmat qpp::spectralpowm (const Eigen::MatrixBase < Derived > & A, const cplx z)

Matrix power.

See also

qpp::powm()

Uses the spectral decomposition of A to compute the matrix power. By convention $A^0 = I$.

Parameters

Α	Eigen expression
Z	Complex number

Returns

Matrix power A^z

6.1.3.141 template<typename Derived > cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > & A)

Matrix square root.

Parameters

Α	Eigen expression
	gon_expression

Returns

Matrix square root of A

6.1.3.142 template < typename Derived > Derived::Scalar qpp::sum (const Eigen::MatrixBase < Derived > & A)

Element-wise sum of A.

Parameters

Α	Eigen expression
---	------------------

Returns

Element-wise sum of A, as a scalar in the same scalar field as A

6.1.3.143 template < typename InputIterator > std::iterator_traits < InputIterator >::value_type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

Parameters

first	Iterator to the first element of the range
last	Iterator to the last element of the range

Returns

Element-wise sum of the range, as a scalar in the same scalar field as the range

6.1.3.144 template<typename Container > Container::value_type qpp::sum (const Container & c)

Element-wise sum of the elements of an STL-like container.

Parameters

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c STL-like container

Returns

Element-wise sum of the elements of the container, as a scalar in the same scalar field as the container

6.1.3.145 cmat qpp::super2choi(const cmat & A) [inline]

Converts superoperator matrix to Choi matrix.

See also

qpp::choi2super()

Parameters

A Superoperator matrix

Returns

Choi matrix

6.1.3.146 template<typename Derived > dyn_col_vect<double> qpp::svals (const Eigen::MatrixBase< Derived > & A)

Singular values.

Parameters

Α	Eigen expression

Returns

Singular values of A, ordered in decreasing order, as a real dynamic column vector

6.1.3.147 template<typename Derived > std::tuple<cmat, dyn_col_vect<double>, cmat> qpp::svd (const Eigen::MatrixBase< Derived > & A)

Full singular value decomposition.

Parameters

Α	Eigen expression

Returns

Tuple of: 1. Left sigular vectors of A, as columns of a complex dynamic matrix, 2. Singular values of A, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of A, as columns of a complex dynamic matrix

6.1.3.148 template<typename Derived > cmat qpp::svdU (const Eigen::MatrixBase< Derived > & A)

Left singular vectors.

Parameters

Α	Eigen expression
---	------------------

Returns

Complex dynamic matrix, whose columns are the left singular vectors of A

6.1.3.149 template < typename Derived > cmat qpp::svdV (const Eigen::MatrixBase < Derived > & A)

Right singular vectors.

Parameters

Α	Eigen expression

Returns

Complex dynamic matrix, whose columns are the right singular vectors of A

6.1.3.150 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, const std::vector< idx > & perm

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit *perm[i]* is permuted to the location *i*.

Parameters

Α	Eigen expression
perm	Permutation
dims	Dimensions of the multi-partite system

Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.151 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::syspermute (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2)

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit *perm[i]* is permuted to the location *i*.

Parameters

Α	Eigen expression
perm	Permutation
d	Subsystem dimensions

Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.152 template<typename Derived > Derived::Scalar qpp::trace (const Eigen::MatrixBase< Derived > & A)

Trace.

Parameters

Α	Eigen expression
---	------------------

Returns

Trace of A, as a scalar in the same scalar field as A

6.1.3.153 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::transpose (const Eigen::MatrixBase < Derived > & A)

Transpose.

Parameters

Α	Eigen expression

Returns

Transpose of A, as a dynamic matrix over the same scalar field as A

6.1.3.154 template < typename Derived > double qpp::tsallis (const Eigen::MatrixBase < Derived > & A, double q)

Tsallis- q entropy of the density matrix A, for $q \ge 0$.

Note

When $q \to 1$ the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base e

Parameters

Α	Eigen expression
q	Non-negative real number

Returns

Tsallis- q entropy

6.1.3.155 double qpp::tsallis (const std::vector < double > & prob, double q) [inline]

Tsallis- q entropy of the probability distribution *prob*, for $q \ge 0$.

Note

When $q \to 1$ the Tsallis entropy converges to the Shannon entropy, with the logarithm in base e

Parameters

prob	Real probability vector
q	Non-negative real number

Returns

Tsallis-q entropy

6.1.3.156 std::vector<int> qpp::x2contfrac (double x, idx n, idx cut = 1e5) [inline]

Simple continued fraction expansion.

See also

qpp::contfrac2x()

Parameters

X	Real number
n	Number of terms in the expansion
cut	Stop the expansion when the next term is greater than <i>cut</i>

Returns

Integer vector containing the simple continued fraction expansion of x. If there are m less than n terms in the expansion, a shorter vector with m components is returned.

6.1.4 Variable Documentation

6.1.4.1 constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

6.1.4.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.4.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::eps) // x is zero</pre>
```

6.1.4.4 constexpr double qpp::infty = std::numeric_limits < double >::infinity()

Used to denote infinity in double precision.

6.1.4.5 constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 π

6.2 qpp::experimental Namespace Reference

Experimental/test functions/classes, do not use or modify.

6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

Classes

- class IOManipBase
- class IOManipEigen
- class IOManipPointer
- class IOManipRange
- · class Singleton

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

Functions

- void _n2multiidx (idx n, idx numdims, const idx *dims, idx *result) noexcept
- idx _multiidx2n (const idx *midx, idx numdims, const idx *dims) noexcept
- template<typename Derived >

bool _check_square_mat (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool <u>_check_vector</u> (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ {\sf template}{<} {\sf typename \ Derived}>$

bool <u>_check_rvector</u> (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool <u>_check_cvector</u> (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ \text{template}{<} \text{typename T} >$

bool _check_nonzero_size (const T &x) noexcept

- bool check dims (const std::vector < idx > &dims)
- template<typename Derived >

bool _check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

 $bool_check_dims_match_cvect \ (const \ std::vector < idx > \&dims, \ const \ Eigen::MatrixBase < Derived > \&V)$

template<typename Derived >

bool <u>_check_dims_match_rvect</u> (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

- bool _check_eq_dims (const std::vector < idx > &dims, idx dim) noexcept
- bool _check_subsys_match_dims (const std::vector < idx > &subsys, const std::vector < idx > &dims)
- template<typename Derived >

bool _check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept

 $\bullet \ \ \mathsf{template}{<} \mathsf{typename} \ \mathsf{Derived} >$

 ${\tt bool_check_qubit_cvector} \; ({\tt const} \; {\tt Eigen::MatrixBase} < {\tt Derived} > \& {\tt V}) \; {\tt noexcept}$

 $\bullet \ \ \mathsf{template} \mathord{<} \mathsf{typename} \ \mathsf{Derived} >$

bool _check_qubit_rvector (const Eigen::MatrixBase< Derived > &V) noexcept

• template<typename Derived >

 $bool_check_qubit_vector~(const~Eigen::MatrixBase < Derived > \&V)~noexcept$

- bool <u>_check_perm</u> (const std::vector< idx > &perm)
- template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > _kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen
 ::MatrixBase< Derived2 > &B)

- template<typename Derived1 , typename Derived2 >
 dyn_mat< typename Derived1::Scalar > _dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >
 void variadic_vector_emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
 void variadic_vector_emplace (std::vector< T > &v, First &&first, Args &&...args)

6.3.1 Detailed Description

Internal utility functions, do not use/modify.

6.3.2 Function Documentation

- 6.3.2.1 template < typename Derived > bool qpp::internal::_check_cvector (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.2 bool qpp::internal::_check_dims (const std::vector < idx > & dims) [inline]
- 6.3.2.3 template<typename Derived > bool qpp::internal::_check_dims_match_cvect (const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V)
- 6.3.2.4 template < typename Derived > bool qpp::internal::_check_dims_match_mat (const std::vector < idx > & dims, const Eigen::MatrixBase < Derived > & A)
- 6.3.2.5 template<typename Derived > bool qpp::internal::_check_dims_match_rvect (const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V)
- 6.3.2.6 bool qpp::internal::_check_eq_dims (const std::vector < idx > & dims, idx dim) [inline], [noexcept]
- 6.3.2.7 template < typename T > bool qpp::internal::_check_nonzero_size(const T & x) [noexcept]
- 6.3.2.8 bool qpp::internal::_check_perm (const std::vector < idx > & perm) [inline]
- 6.3.2.9 template<typename Derived > bool qpp::internal::_check_qubit_cvector (const Eigen::MatrixBase< Derived > & V) [noexcept]
- $\begin{array}{ll} \textbf{6.3.2.10} & \textbf{template} < \textbf{typename Derived} > \textbf{bool qpp::internal::_check_qubit_matrix (const Eigen::MatrixBase} < \textbf{Derived} > \textbf{\& A)} \\ & [\texttt{noexcept}] \end{array}$
- 6.3.2.11 template < typename Derived > bool qpp::internal::_check_qubit_rvector (const Eigen::MatrixBase < Derived > & V) [noexcept]
- 6.3.2.12 template < typename Derived > bool qpp::internal::_check_qubit_vector (const Eigen::MatrixBase < Derived > & V) [noexcept]
- 6.3.2.13 template < typename Derived > bool qpp::internal::_check_rvector (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.14 template < typename Derived > bool qpp::internal:: check square mat (const Eigen::MatrixBase < Derived > & A)
- 6.3.2.15 bool qpp::internal::_check_subsys_match_dims (const std::vector < idx > & subsys, const std::vector < idx > & dims) [inline]
- $6.3.2.16 \quad template < typename \ Derived > bool \ qpp::internal::_check_vector \ (\ const \ Eigen::MatrixBase < \ Derived > \& \ A \)$

- 6.3.2.18 template < typename Derived1 , typename Derived2 > dyn_mat < typename Derived1::Scalar > qpp::internal::_kron2 (const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B)
- 6.3.2.19 idx qpp::internal::_multiidx2n (const idx * midx, idx numdims, const idx * dims) [inline], [noexcept]
- 6.3.2.20 void qpp::internal::_n2multiidx (idx n, idx n numdims, const idx * dims, idx * result) [inline], [noexcept]
- 6.3.2.21 template < typename T > void qpp::internal::variadic_vector_emplace (std::vector < T > &)
- 6.3.2.22 template < typename T , typename First , typename... Args > void qpp::internal::variadic_vector_emplace (std::vector < T > & v, First && first, Args &&... args)

Chapter 7

Class Documentation

7.1 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

#include <classes/codes.h>

Inheritance diagram for qpp::Codes:



Collaboration diagram for qpp::Codes:



Public Types

enum Type { Type::FIVE_QUBIT = 1, Type::SEVEN_QUBIT_STEANE, Type::NINE_QUBIT_SHOR }
 Code types, add more codes here if needed.

Public Member Functions

ket codeword (Type type, idx i) const
 Returns the codeword of the specified code type.

Friends

class internal::Singleton < const Codes >

Additional Inherited Members

7.1.1 Detailed Description

const Singleton class that defines quantum error correcting codes

7.1.2 Member Enumeration Documentation

```
7.1.2.1 enum qpp::Codes::Type [strong]
```

Code types, add more codes here if needed.

See also

```
qpp::Codes::codeword()
```

Enumerator

```
FIVE_QUBIT [[5,1,3]] qubit code

SEVEN_QUBIT_STEANE [[7,1,3]] Steane qubit code

NINE_QUBIT_SHOR [[9,1,3]] Shor qubit code
```

7.1.3 Member Function Documentation

```
7.1.3.1 ket qpp::Codes::codeword ( Type type, idx i ) const [inline]
```

Returns the codeword of the specified code type.

See also

```
qpp::Codes::Type
```

Parameters

type	Code type

Returns

i-th codeword of the code type

7.1.4 Friends And Related Function Documentation

7.1.4.1 friend class internal::Singleton< **const Codes** > [friend]

The documentation for this class was generated from the following file:

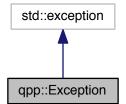
classes/codes.h

7.2 qpp::Exception Class Reference

Generates custom exceptions, used when validating function parameters.

#include <classes/exception.h>

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



Public Types

enum Type {

Type::UNKNOWN_EXCEPTION = 1, Type::ZERO_SIZE, Type::MATRIX_NOT_SQUARE, Type::MATRIX_← NOT_CVECTOR,

Type::MATRIX_NOT_RVECTOR, Type::MATRIX_NOT_VECTOR, Type::MATRIX_NOT_SQUARE_OR_C ∨ VECTOR, Type::MATRIX_NOT_SQUARE_OR_RVECTOR,

Type::MATRIX_NOT_SQUARE_OR_VECTOR, Type::MATRIX_MISMATCH_SUBSYS, Type::DIMS_INVA← LID, Type::DIMS_NOT_EQUAL,

Type::DIMS_MISMATCH_MATRIX, Type::DIMS_MISMATCH_CVECTOR, Type::DIMS_MISMATCH_RVE← CTOR, Type::DIMS MISMATCH VECTOR,

Type::SUBSYS_MISMATCH_DIMS, Type::PERM_INVALID, Type::PERM_MISMATCH_DIMS, Type::NOT ← QUBIT_MATRIX,

Type::NOT_QUBIT_CVECTOR, Type::NOT_QUBIT_RVECTOR, Type::NOT_QUBIT_VECTOR, Type::NO← T QUBIT_SUBSYS,

Type::NOT_BIPARTITE, Type::NO_CODEWORD, Type::OUT_OF_RANGE, Type::TYPE_MISMATCH, Type::UNDEFINED_TYPE, Type::CUSTOM_EXCEPTION }

Exception types, add more here if needed.

Public Member Functions

• Exception (const std::string &where, const Type &type)

Constructs an exception.

• Exception (const std::string &where, const std::string &custom)

Constructs an exception.

· virtual const char * what () const noexceptoverride

Overrides std::exception::what()

virtual ~Exception ()=default

Default virtual destructor.

7.2.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

7.2.2 Member Enumeration Documentation

```
7.2.2.1 enum qpp::Exception::Type [strong]
```

Exception types, add more here if needed.

See also

```
qpp::Exception::_construct_exception_msg()
```

Enumerator

UNKNOWN EXCEPTION Unknown exception

ZERO_SIZE Zero sized object, e.g. empty Eigen::Matrix or std::vector<> with no elements

MATRIX_NOT_SQUARE Eigen::Matrix is not square

MATRIX_NOT_CVECTOR Eigen::Matrix is not a column vector

MATRIX_NOT_RVECTOR Eigen::Matrix is not a row vector

MATRIX_NOT_VECTOR Eigen::Matrix is not a row/column vector

MATRIX_NOT_SQUARE_OR_CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX_NOT_SQUARE_OR_RVECTOR Eigen::Matrix is not square nor a row vector

MATRIX_NOT_SQUARE_OR_VECTOR Eigen::Matrix is not square nor a row/column vector

MATRIX_MISMATCH_SUBSYS Matrix size mismatch subsystem sizes (e.g. in qpp::apply())

DIMS_INVALID std::vector<idx> of dimensions has zero size or contains zeros

DIMS_NOT_EQUAL Local/global dimensions are not equal

DIMS_MISMATCH_MATRIX Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

DIMS_MISMATCH_CVECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

DIMS_MISMATCH_RVECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

DIMS_MISMATCH_VECTOR Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

SUBSYS_MISMATCH_DIMS std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

PERM_INVALID std::vector<idx> does note represent a valid permutation

PERM_MISMATCH_DIMS Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

NOT_QUBIT_MATRIX Eigen::Matrix is not 2 x 2

NOT_QUBIT_CVECTOR Eigen::Matrix is not 2 x 1

NOT_QUBIT_RVECTOR Eigen::Matrix is not 1 x 2

NOT_QUBIT_VECTOR Eigen::Matrix is not 1 x 2 nor 2 x 1

NOT_QUBIT_SUBSYS Subsystems are not 2-dimensional

NOT_BIPARTITE std::vector<idx> of dimensions has size different from 2

NO_CODEWORD Codeword does not exist, thrown when calling qpp::Codes::codeword() with invalid index i

OUT_OF_RANGE Parameter out of range

TYPE_MISMATCH Scalar types do not match

UNDEFINED_TYPE Templated specialization not defined for this type

CUSTOM_EXCEPTION Custom exception, user must provide a custom message

7.2.3 Constructor & Destructor Documentation

7.2.3.1 qpp::Exception::Exception (const std::string & where, const Type & type) [inline]

Constructs an exception.

Parameters

where	Text representing where the exception occured
type	Exception type, defined in qpp::Exception::Type

7.2.3.2 qpp::Exception::Exception (const std::string & where, const std::string & custom) [inline]

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

Parameters

where	Text representing where the exception occured
custom	Exception description

7.2.3.3 virtual qpp::Exception::~Exception() [virtual], [default]

Default virtual destructor.

7.2.4 Member Function Documentation

7.2.4.1 virtual const char* qpp::Exception::what () const [inline], [override], [virtual], [noexcept]

Overrides std::exception::what()

Returns

Exception description

The documentation for this class was generated from the following file:

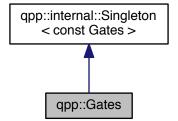
· classes/exception.h

7.3 qpp::Gates Class Reference

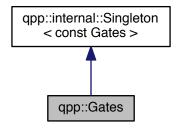
const Singleton class that implements most commonly used gates

#include <classes/gates.h>

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



Public Member Functions

• cmat Rn (double theta, const std::vector< double > &n) const

Rotation of theta about the 3-dimensional real unit vector n.

• cmat Zd (idx D) const

Generalized Z gate for qudits.

• cmat Fd (idx D) const

Fourier transform gate for qudits.

• cmat Xd (idx D) const

Generalized X gate for qudits.

• template<typename Derived = Eigen::MatrixXcd>

Derived Id (idx D) const

Identity gate.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > CTRL (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &ctrl, const std::vector< idx > &subsys, idx n, idx d=2) const

Generates the multi-partite multiple-controlled-A gate in matrix form.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const
std::vector< idx > &dims) const

Expands out.

Public Attributes

• cmat Id2 {cmat::Identity(2, 2)}

Identity gate.

cmat H {cmat::Zero(2, 2)}

Hadamard gate.

cmat X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

cmat Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

cmat Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

cmat S {cmat::Zero(2, 2)}

S gate.

• cmat T {cmat::Zero(2, 2)}

T gate.

cmat CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

• cmat CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

• cmat CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

• cmat SWAP {cmat::Identity(4, 4)}

SWAP gate.

cmat TOF {cmat::Identity(8, 8)}

Toffoli gate.

• cmat FRED {cmat::Identity(8, 8)}

Fredkin gate.

Friends

class internal::Singleton < const Gates >

Additional Inherited Members

7.3.1 Detailed Description

const Singleton class that implements most commonly used gates

7.3.2 Member Function Documentation

7.3.2.1 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::CTRL (const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & ctrl, const std::vector< idx > & subsys, idx n, idx d = 2) const [inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

See also

qpp::applyCTRL()

Note

The dimension of the gate A must match the dimension of subsys

Parameters

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
n	Total number of subsystems

d	Subsystem dimensions

Returns

CTRL-A gate, as a matrix over the same scalar field as A

7.3.2.2 template<typename Derived > dyn_mat<typename Derived::Scalar> qpp::Gates::expandout (const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims) const [inline]

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

Parameters

Α	Eigen expression
pos	Position
dims	Dimensions of the multi-partite system

Returns

Tensor product $I \otimes \cdots \otimes I \otimes A \otimes I \otimes \cdots \otimes I$, with A on position pos, as a dynamic matrix over the same scalar field as A

7.3.2.3 cmat qpp::Gates::Fd(idx D) const [inline]

Fourier transform gate for qudits.

Note

Defined as
$$F = \sum_{jk} \exp(2\pi i j k/D) |j\rangle\langle k|$$

Parameters

D Dimension of the Hilbert space

Returns

Fourier transform gate for qudits

7.3.2.4 template<typename Derived = Eigen::MatrixXcd> Derived qpp::Gates::Id (idx D) const [inline]

Identity gate.

Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

Parameters

D	Dimension of the Hilbert space
---	--------------------------------

Returns

Identity gate

7.3.2.5 cmat qpp::Gates::Rn (double theta, const std::vector < double > & n) const [inline]

Rotation of *theta* about the 3-dimensional real unit vector *n*.

Parameters

theta	Rotation angle
n	3-dimensional real unit vector

Returns

Rotation gate

7.3.2.6 cmat qpp::Gates::Xd(idx D)const [inline]

Generalized X gate for qudits.

Note

Defined as $X = \sum_j |j \oplus 1\rangle\langle j|$

Parameters

D	Dimension of the Hilbert space

Returns

Generalized X gate for qudits

7.3.2.7 cmat qpp::Gates::Zd(idx D) const [inline]

Generalized Z gate for qudits.

Note

Defined as $Z = \sum_{j} \exp(2\pi i j/D) |j\rangle\langle j|$

Parameters

D	Dimension of the Hilbert space

Returns

Generalized Z gate for qudits

```
7.3.3 Friends And Related Function Documentation
7.3.3.1 friend class internal::Singleton < const Gates > [friend]
7.3.4 Member Data Documentation
7.3.4.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}
Controlled-NOT control target gate.
7.3.4.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}
Controlled-NOT target control gate.
7.3.4.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}
Controlled-Phase gate.
7.3.4.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}
Fredkin gate.
7.3.4.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}
Hadamard gate.
7.3.4.6 cmat qpp::Gates::ld2 {cmat::Identity(2, 2)}
Identity gate.
7.3.4.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}
S gate.
7.3.4.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}
SWAP gate.
7.3.4.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}
T gate.
7.3.4.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}
Toffoli gate.
7.3.4.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}
Pauli Sigma-X gate.
```

7.3.4.12 cmat qpp::Gates::Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

7.3.4.13 cmat qpp::Gates::Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

The documentation for this class was generated from the following file:

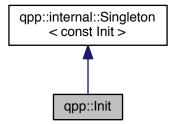
classes/gates.h

7.4 qpp::Init Class Reference

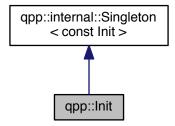
const Singleton class that performs additional initializations/cleanups

#include <classes/init.h>

Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



Friends

• class internal::Singleton< const Init >

Additional Inherited Members

7.4.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

7.4.2 Friends And Related Function Documentation

7.4.2.1 friend class internal::Singleton < const lnit > [friend]

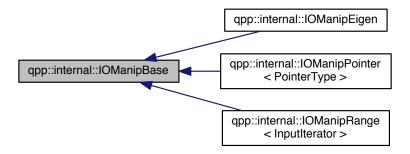
The documentation for this class was generated from the following file:

· classes/init.h

7.5 qpp::internal::IOManipBase Class Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipBase:



Public Member Functions

- IOManipBase ()=default
- IOManipBase (const IOManipBase &)=default
- IOManipBase (IOManipBase &&)=default
- IOManipBase & operator= (const IOManipBase &)=default
- IOManipBase & operator= (IOManipBase &&)=default
- virtual ~IOManipBase ()=default

Friends

std::ostream & operator<< (std::ostream &os, const IOManipBase &rhs)

7.5.1 Constructor & Destructor Documentation

7.5.1.1 qpp::internal::IOManipBase::IOManipBase() [default]

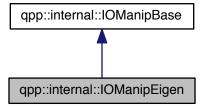
```
7.5.1.2 qpp::internal::IOManipBase::IOManipBase ( const IOManipBase & ) [default]
7.5.1.3 qpp::internal::IOManipBase::IOManipBase ( IOManipBase && ) [default]
7.5.1.4 virtual qpp::internal::IOManipBase::~IOManipBase ( ) [virtual], [default]
7.5.2 Member Function Documentation
7.5.2.1 IOManipBase& qpp::internal::IOManipBase::operator= ( const IOManipBase & ) [default]
7.5.2.2 IOManipBase& qpp::internal::IOManipBase::operator= ( IOManipBase && ) [default]
7.5.3 Friends And Related Function Documentation
7.5.3.1 std::ostream& operator<< ( std::ostream & os, const IOManipBase & rhs ) [friend]</p>
```

The documentation for this class was generated from the following file:

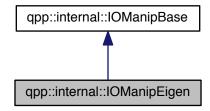
• internal/classes/iomanip.h

7.6 qpp::internal::IOManipEigen Class Reference

#include <internal/classes/iomanip.h>
Inheritance diagram for qpp::internal::IOManipEigen:



Collaboration diagram for qpp::internal::IOManipEigen:



Public Member Functions

- template<typename Derived >
 IOManipEigen (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
- IOManipEigen (const cplx z, double chop=qpp::chop)

7.6.1 Constructor & Destructor Documentation

- 7.6.1.1 template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen (const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop) [inline], [explicit]
- 7.6.1.2 qpp::internal::IOManipEigen::IOManipEigen (const cplx z, double chop = qpp::chop) [inline], [explicit]

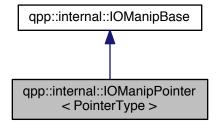
The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

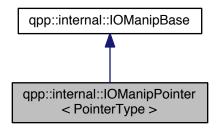
7.7 qpp::internal::IOManipPointer < PointerType > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipPointer< PointerType >:



Collaboration diagram for qpp::internal::IOManipPointer< PointerType >:



Public Member Functions

- IOManipPointer (const PointerType *p, const idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipPointer (const IOManipPointer &)=default
- IOManipPointer & operator= (const IOManipPointer &)=default

7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer (const PointerType * p, const idx n, const std::string & separator, const std::string & start = "[", const std::string & end = "]") [inline], [explicit]
- 7.7.1.2 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer (const IOManipPointer < PointerType > &) [default]

7.7.2 Member Function Documentation

7.7.2.1 template<typename PointerType> IOManipPointer& qpp::internal::IOManipPointer< PointerType
>::operator=(const IOManipPointer< PointerType > &) [default]

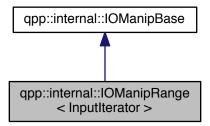
The documentation for this class was generated from the following file:

· internal/classes/iomanip.h

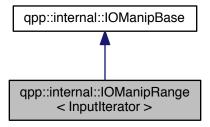
7.8 qpp::internal::IOManipRange < InputIterator > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipRange< InputIterator >:



Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



Public Member Functions

• IOManipRange (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

7.8.1 Constructor & Destructor Documentation

The documentation for this class was generated from the following file:

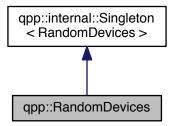
· internal/classes/iomanip.h

7.9 qpp::RandomDevices Class Reference

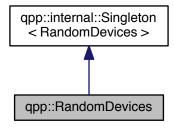
Singeleton class that manages the source of randomness in the library.

#include <classes/random_devices.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



Public Attributes

std::mt19937 _rng

Mersenne twister random number generator.

Friends

class internal::Singleton < RandomDevices >

Additional Inherited Members

7.9.1 Detailed Description

Singeleton class that manages the source of randomness in the library.

Consists of a wrapper around an std::mt19937 Mersenne twister random number generator engine and an std⇔ ::random_device engine. The latter is used to seed the Mersenne twister.

Warning

This class DOES NOT seed the standard C number generator used by Eigen::Matrix::Random(), since it is not thread safe. Do not use Eigen::Matrix::Random() or functions that depend on the C style random number engine, but use qpp::rand() instead!

7.9.2 Friends And Related Function Documentation

7.9.2.1 friend class internal::Singleton < RandomDevices > [friend]

7.9.3 Member Data Documentation

7.9.3.1 std::mt19937 qpp::RandomDevices::_rng

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

· classes/random devices.h

7.10 qpp::internal::Singleton < T > Class Template Reference

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

Static Public Member Functions

- static T & get_instance () noexcept(std::is_nothrow_constructible < T >::value)
- static thread_local T & get_thread_local_instance () noexcept(std::is_nothrow_constructible < T >::value)

Protected Member Functions

- Singleton () noexcept=default
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete
- virtual ∼Singleton ()=default

7.10.1 Detailed Description

template<typename T>class qpp::internal::Singleton<T>

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from qpp::internal::Singleton, make qpp::internal::Singleton a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function qpp::internal::Singleton::get_instance() (qpp::internal::Singleton::get_thread_local_← instance()), which returns a reference (thread_local reference) to your newly created singleton (thread-safe in C++11).

Example:

96 Class Documentation

See also

Code of qpp::Codes, qpp::Gates, qpp::Init, qpp::RandomDevices, qpp::States or qpp.h for real world examples of usage.

7.10.2 Constructor & Destructor Documentation

```
7.10.2.1 template<typename T> qpp::internal::Singleton< T>::Singleton( ) [protected], [default], [noexcept]
```

```
7.10.2.2 template<typename T> qpp::internal::Singleton< T>::Singleton ( const Singleton< T> & ) [protected], [delete]
```

```
7.10.2.3 template<typename T> virtual qpp::internal::Singleton< T>::\simSingleton( ) [protected], [virtual], [default]
```

7.10.3 Member Function Documentation

```
7.10.3.1 template<typename T> static T& qpp::internal::Singleton< T>::get_instance( ) [inline], [static], [noexcept]
```

```
7.10.3.2 template<typename T> static thread_local T& qpp::internal::Singleton< T>::get_thread_local_instance( ) [inline], [static], [noexcept]
```

```
7.10.3.3 template<typename T> Singleton& qpp::internal::Singleton< T>::operator=( const Singleton< T> & ) [protected], [delete]
```

The documentation for this class was generated from the following file:

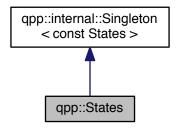
• internal/classes/singleton.h

7.11 qpp::States Class Reference

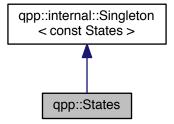
const Singleton class that implements most commonly used states

```
#include <classes/states.h>
```

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



Public Attributes

```
    ket x0 {ket::Zero(2)}
    Pauli Sigma-X 0-e
```

Pauli Sigma-X 0-eigenstate |+>

ket x1 {ket::Zero(2)}

Pauli Sigma-X 1-eigenstate |->

ket y0 {ket::Zero(2)}

Pauli Sigma-Y 0-eigenstate |y+>

ket y1 {ket::Zero(2)}

Pauli Sigma-Y 1-eigenstate | y->

ket z0 {ket::Zero(2)}

Pauli Sigma-Z 0-eigenstate |0>

ket z1 {ket::Zero(2)}

Pauli Sigma-Z 1-eigenstate | 1>

cmat px0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.

cmat px1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.

cmat py0 {cmat::Zero(2, 2)}

98 Class Documentation

Projector onto the Pauli Sigma-Y 0-eigenstate |y+><y+|.

cmat py1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.

cmat pz0 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.

cmat pz1 {cmat::Zero(2, 2)}

Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.

ket b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

ket b01 {ket::Zero(4)}

Bell-01 state (following the convention in Nielsen and Chuang)

ket b10 {ket::Zero(4)}

Bell-10 state (following the convention in Nielsen and Chuang)

ket b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

cmat pb00 {cmat::Zero(4, 4)}

Projector onto the Bell-00 state.

cmat pb01 {cmat::Zero(4, 4)}

Projector onto the Bell-01 state.

cmat pb10 {cmat::Zero(4, 4)}

Projector onto the Bell-10 state.

cmat pb11 {cmat::Zero(4, 4)}

Projector onto the Bell-11 state.

ket GHZ {ket::Zero(8)}

GHZ state.

ket W {ket::Zero(8)}

W state.

cmat pGHZ {cmat::Zero(8, 8)}

Projector onto the GHZ state.

cmat pW {cmat::Zero(8, 8)}

Projector onto the W state.

Friends

class internal::Singleton < const States >

Additional Inherited Members

7.11.1 Detailed Description

const Singleton class that implements most commonly used states

7.11.2 Friends And Related Function Documentation

7.11.2.1 friend class internal::Singleton < const States > [friend]

7.11.3 Member Data Documentation

7.11.3.1 ket qpp::States::b00 {ket::Zero(4)}

Bell-00 state (following the convention in Nielsen and Chuang)

```
7.11.3.2 ket qpp::States::b01 {ket::Zero(4)}
Bell-01 state (following the convention in Nielsen and Chuang)
7.11.3.3 ket qpp::States::b10 {ket::Zero(4)}
Bell-10 state (following the convention in Nielsen and Chuang)
7.11.3.4 ket qpp::States::b11 {ket::Zero(4)}
Bell-11 state (following the convention in Nielsen and Chuang)
7.11.3.5 ket qpp::States::GHZ {ket::Zero(8)}
GHZ state.
7.11.3.6 cmat qpp::States::pb00 {cmat::Zero(4, 4)}
Projector onto the Bell-00 state.
7.11.3.7 cmat qpp::States::pb01 {cmat::Zero(4, 4)}
Projector onto the Bell-01 state.
7.11.3.8 cmat qpp::States::pb10 {cmat::Zero(4, 4)}
Projector onto the Bell-10 state.
7.11.3.9 cmat qpp::States::pb11 {cmat::Zero(4, 4)}
Projector onto the Bell-11 state.
7.11.3.10 cmat qpp::States::pGHZ {cmat::Zero(8, 8)}
Projector onto the GHZ state.
7.11.3.11 cmat qpp::States::pW {cmat::Zero(8, 8)}
Projector onto the W state.
7.11.3.12 cmat qpp::States::px0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.
7.11.3.13 cmat qpp::States::px1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.
```

100 Class Documentation

```
7.11.3.14 cmat qpp::States::py0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 0-eigenstate |y+><y+|.
7.11.3.15 cmat qpp::States::py1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.
7.11.3.16 cmat qpp::States::pz0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.
7.11.3.17 cmat qpp::States::pz1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.
7.11.3.18 ket qpp::States::W {ket::Zero(8)}
W state.
7.11.3.19 ket qpp::States::x0 {ket::Zero(2)}
Pauli Sigma-X 0-eigenstate |+>
7.11.3.20 ket qpp::States::x1 {ket::Zero(2)}
Pauli Sigma-X 1-eigenstate |->
7.11.3.21 ket qpp::States::y0 {ket::Zero(2)}
Pauli Sigma-Y 0-eigenstate |y+>
7.11.3.22 ket qpp::States::y1 {ket::Zero(2)}
Pauli Sigma-Y 1-eigenstate |y->
7.11.3.23 ket qpp::States::z0 {ket::Zero(2)}
Pauli Sigma-Z 0-eigenstate |0>
7.11.3.24 ket qpp::States::z1 {ket::Zero(2)}
Pauli Sigma-Z 1-eigenstate |1>
The documentation for this class was generated from the following file:
```

classes/states.h

7.12 qpp::Timer Class Reference

Measures time.

```
#include <classes/timer.h>
```

Public Member Functions

· Timer () noexcept

Constructs an instance with the current time as the starting point.

• void tic () noexcept

Resets the chronometer.

• const Timer & toc () noexcept

Stops the chronometer.

• double seconds () const noexcept

Time passed in seconds.

virtual ∼Timer ()=default

Default virtual destructor.

Protected Attributes

- std::chrono::steady_clock::time_point _start
- · std::chrono::steady_clock::time_point_end

Friends

std::ostream & operator<< (std::ostream &os, const Timer &rhs)
 Overload for std::ostream operators.

7.12.1 Detailed Description

Measures time.

Uses a std::chrono::steady_clock. It is not affected by wall clock changes during runtime.

7.12.2 Constructor & Destructor Documentation

```
7.12.2.1 qpp::Timer::Timer( ) [inline],[noexcept]
```

Constructs an instance with the current time as the starting point.

```
7.12.2.2 virtual qpp::Timer::~Timer() [virtual], [default]
```

Default virtual destructor.

7.12.3 Member Function Documentation

```
7.12.3.1 double qpp::Timer::seconds( ) const [inline], [noexcept]
```

Time passed in seconds.

102 Class Documentation

Returns

Number of seconds that passed between the instantiation/reset and invocation of qpp::Timer::toc()

```
7.12.3.2 void qpp::Timer::tic() [inline], [noexcept]
```

Resets the chronometer.

Resets the starting/ending point to the current time

```
7.12.3.3 const Timer& qpp::Timer::toc( ) [inline], [noexcept]
```

Stops the chronometer.

Set the current time as the ending point

Returns

Current instance

7.12.4 Friends And Related Function Documentation

7.12.4.1 std::ostream& operator<<(std::ostream & os, const Timer & rhs) [friend]

Overload for std::ostream operators.

Parameters

os	Output stream
rhs	Timer instance

Returns

Writes to the output stream the number of seconds that passed between the instantiation/reset and invocation of qpp::Timer::toc().

7.12.5 Member Data Documentation

```
7.12.5.1 std::chrono::steady_clock::time_point qpp::Timer::_end [protected]
```

 $\textbf{7.12.5.2} \quad \textbf{std::chrono::steady_clock::time_point qpp::Timer::_start} \quad \texttt{[protected]}$

The documentation for this class was generated from the following file:

classes/timer.h

Chapter 8

File Documentation

8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Codes const Singleton class that defines quantum error correcting codes

Namespaces

• qpp

Quantum++ main namespace.

8.1.1 Detailed Description

Quantum error correcting codes.

8.2 classes/exception.h File Reference

Exceptions.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::Exception

Generates custom exceptions, used when validating function parameters.

Namespaces

• qpp

Quantum++ main namespace.

8.2.1 Detailed Description

Exceptions.

8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



Classes

· class qpp::Gates

const Singleton class that implements most commonly used gates

Namespaces

qpp

Quantum++ main namespace.

8.3.1 Detailed Description

Quantum gates.

8.4 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



Classes

· class qpp::Init

const Singleton class that performs additional initializations/cleanups

Namespaces

• qpp

Quantum++ main namespace.

8.4.1 Detailed Description

Initialization.

8.5 classes/random_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::RandomDevices

Singeleton class that manages the source of randomness in the library.

Namespaces

• qpp

Quantum++ main namespace.

8.5.1 Detailed Description

Random devices.

8.6 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



Classes

• class qpp::States

const Singleton class that implements most commonly used states

Namespaces

qpp

Quantum++ main namespace.

8.6.1 Detailed Description

Quantum states.

8.7 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



Classes

· class qpp::Timer

Measures time.

Namespaces

• qpp

Quantum++ main namespace.

8.7.1 Detailed Description

Timing.

8.8 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

• constexpr cplx qpp::operator""_i (unsigned long long int x) noexcept

User-defined literal for complex $i=\sqrt{-1}$ (integer overload)

• constexpr cplx qpp::operator""_i (long double x) noexcept

User-defined literal for complex $i = \sqrt{-1}$ (real overload)

• cplx qpp::omega (idx D)

D-th root of unity.

Variables

• constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

• constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

• constexpr double qpp::pi = 3.141592653589793238462643383279502884

7

constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

• constexpr double qpp::infty = std::numeric_limits<double>::infinity()

Used to denote infinity in double precision.

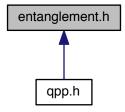
8.8.1 Detailed Description

Constants.

8.9 entanglement.h File Reference

Entanglement functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

template<typename Derived >
 dyn_col_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector<
 idx > &dims)

Schmidt coefficients of the bi-partite pure state A.

• template<typename Derived >

cmat qpp::schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Schmidt basis on Alice side.

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$

cmat qpp::schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Schmidt basis on Bob side.

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$

std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Schmidt probabilities of the bi-partite pure state A.

template<typename Derived >

double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Entanglement of the bi-partite pure state A.

• template<typename Derived >

double qpp::gconcurrence (const Eigen::MatrixBase Derived > &A)

G-concurrence of the bi-partite pure state A.

• template<typename Derived >

 $\label{eq:const_equation} \mbox{double qpp::negativity (const Eigen::MatrixBase < Derived > \&A, const std::vector < idx > \&dims)}$

Negativity of the bi-partite mixed state A.

ullet template<typename Derived >

double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

Logarithmic negativity of the bi-partite mixed state A.

template<typename Derived >
 double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)
 Wootters concurrence of the bi-partite qubit mixed state A.

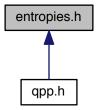
8.9.1 Detailed Description

Entanglement functions.

8.10 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



Namespaces

dbb

Quantum++ main namespace.

Functions

- template < typename Derived >
 double qpp::entropy (const Eigen::MatrixBase < Derived > &A)
 von-Neumann entropy of the density matrix A
- double qpp::entropy (const std::vector< double > &prob)

Shannon entropy of the probability distribution prob.

Renyi- α entropy of the density matrix A, for $\alpha \geq 0$.

- template<typename Derived >
 double qpp::renyi (const Eigen::MatrixBase< Derived > &A, double alpha)
- double qpp::renyi (const std::vector< double > &prob, double alpha)

Renyi- α entropy of the probability distribution prob, for $\alpha \geq 0$.

template<typename Derived >
 double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)

Tsallis- q entropy of the density matrix A, for $q \geq 0$.

double qpp::tsallis (const std::vector< double > &prob, double q)

Tsallis- q entropy of the probability distribution prob, for $q \geq 0$.

template<typename Derived >
 double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)

Quantum mutual information between 2 subsystems of a composite system.

template<typename Derived >
 double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)

Quantum mutual information between 2 subsystems of a composite system.

8.10.1 Detailed Description

Entropy functions.

8.11 experimental/test.h File Reference

Experimental/test functions/classes.

Namespaces

• qpp

Quantum++ main namespace.

· qpp::experimental

Experimental/test functions/classes, do not use or modify.

8.11.1 Detailed Description

Experimental/test functions/classes.

8.12 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Hermitian eigenvalues.

Functions

```
    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > gpp::conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)
      Inverse.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::trace">qpp::trace</a> (const Eigen::MatrixBase</a> Derived > &A)
      Trace.

    template<typename Derived >

  Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)
      Determinant.

    template<typename Derived >

  Derived::Scalar qpp::logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::sum">qpp::sum</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise sum of A.

    template<typename Derived >

  Derived::Scalar <a href="mailto:open:prod">open:prod</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise product of A.
• template<typename Derived >
  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase</a> Derived > &A)
      Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition.

    template<typename Derived >

  dyn col vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.

    template<typename Derived >

  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition of Hermitian expression.
• template<typename Derived >
  dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)
```

```
• template<typename Derived >
  cmat qpp::hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.
• template<typename Derived >
  std::tuple< cmat, dyn_col_vect< double >, cmat > qpp::svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.
• template<typename Derived >
  dyn_col_vect< double > qpp::svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.

    template<typename Derived >

  cmat qpp::svdU (const Eigen::MatrixBase< Derived > &A)
     Left singular vectors.

    template<typename Derived >

  cmat qpp::svdV (const Eigen::MatrixBase< Derived > &A)
     Right singular vectors.

    template < typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
• template<typename Derived >
  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.
• template<typename Derived >
  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.
• template<typename Derived >
  cmat qpp::logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.
template<typename Derived >
  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Matrix power.

    template<typename Derived >

  double qpp::schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn mat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)(const
  typename Derived::Scalar &))
     Functor.
template<typename T >
  dyn_mat< typename T::Scalar > qpp::kron (const T &head)
     Kronecker product.
```

```
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)
     Kronecker product.
\bullet \ \ {\it template}{<} {\it typename Derived}>
  dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn mat< typename T::Scalar > qpp::dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.
ullet template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows,
  idx cols)
     Reshape.

    template < typename Derived1 , typename Derived2 >

  dyn mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Commutator.
• template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.
• std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)
```

Non-negative integer index to multi-index.

idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

Multi-index to non-negative integer index.

ket qpp::mket (const std::vector < idx > &mask, const std::vector < idx > &dims)

Multi-partite qudit ket.

ket qpp::mket (const std::vector< idx > &mask, idx d=2)

Multi-partite qudit ket.

• cmat qpp::mprj (const std::vector< idx > &mask, const std::vector< idx > &dims)

Projector onto multi-partite qudit ket.

cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)

Projector onto multi-partite qudit ket.

template<typename InputIterator >

std::vector< double > qpp::abssq (InputIterator first, InputIterator last)

Computes the absolute values squared of a range of complex numbers.

template<typename Derived >

std::vector< double > qpp::abssq (const Eigen::MatrixBase< Derived > &V)

Computes the absolute values squared of a column vector.

• template<typename InputIterator >

std::iterator_traits< InputIterator >::value_type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

• template<typename Container >

Container::value_type qpp::sum (const Container &c)

Element-wise sum of the elements of an STL-like container.

• template<typename InputIterator >

std::iterator_traits< InputIterator >::value_type qpp::prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

• template<typename Container >

Container::value_type qpp::prod (const Container &c)

Element-wise product of the elements of an STL-like container.

template<typename Derived >

dyn_col_vect< typename Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

```
std::vector< T > qpp::complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

• template<typename Derived >

```
std::vector< double > qpp::rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat qpp::bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

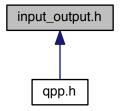
8.12.1 Detailed Description

Generic quantum computing functions.

8.13 input_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

template<typename Derived >
 internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
 Eigen expression ostream manipulator.

• internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

• template<typename InputIterator > internal::IOManipRange< InputIterator > qpp::disp (const InputIterator &first, const InputIterator &last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >
 internal::IOManipRange< typename Container::const_iterator > qpp::disp (const Container &c, const std
 ::string &separator, const std::string &start="[", const std::string &end="]")

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >
 internal::IOManipPointer< PointerType > qpp::disp (const PointerType *p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

• template<typename Derived >

void qpp::save (const Eigen::MatrixBase< Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

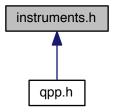
8.13.1 Detailed Description

Input/output functions.

8.14 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

 $\bullet \ \ {\sf template}{<} {\sf typename \ Derived} >$

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

ullet template<typename Derived >

 $std::tuple < idx, \ std::vector < double >, \ std::vector < cmat > > qpp::measure \ (const \ Eigen::MatrixBase < Derived > &A, \ const \ std::vector < cmat > &Ks, \ const \ std::vector < idx > &subsys, \ const \ std::vector < idx > &dims)\\$

 $\textit{Measures the part subsys of the multi-partite state vector or density matrix A using the set of \textit{Kraus operators Ks}.$

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U, const std::vector< idx > &subsys, const idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A\ in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

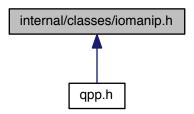
8.14.1 Detailed Description

Measurement functions.

8.15 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



Classes

- class qpp::internal::IOManipBase
- class qpp::internal::IOManipRange< InputIterator >
- class qpp::internal::IOManipPointer< PointerType >
- class qpp::internal::IOManipEigen

Namespaces

• qpp

Quantum++ main namespace.

• qpp::internal

Internal utility functions, do not use/modify.

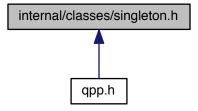
8.15.1 Detailed Description

Input/output manipulators.

8.16 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



Classes

class qpp::internal::Singleton< T >

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

Namespaces

• qpp

Quantum++ main namespace.

· qpp::internal

Internal utility functions, do not use/modify.

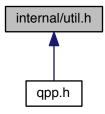
8.16.1 Detailed Description

Singleton pattern via CRTP.

8.17 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

Functions

- void qpp::internal::_n2multiidx (idx n, idx numdims, const idx *dims, idx *result) noexcept
- idx qpp::internal::_multiidx2n (const idx *midx, idx numdims, const idx *dims) noexcept
- template<typename Derived >
 bool qpp::internal::_check_square_mat (const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::_check_vector (const Eigen::MatrixBase< Derived > &A)
- template < typename Derived >
 bool qpp::internal::_check_rvector (const Eigen::MatrixBase < Derived > &A)
- template<typename Derived >
 bool qpp::internal::_check_cvector (const Eigen::MatrixBase< Derived > &A)
- template<typename T >
 bool qpp::internal::_check_nonzero_size (const T &x) noexcept
- bool qpp::internal::_check_dims (const std::vector< idx > &dims)
- template<typename Derived >
 bool qpp::internal::_check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
- template<typename Derived >
 bool qpp::internal::_check_dims_match_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase
 Derived > &V)
- template<typename Derived >
 bool qpp::internal::_check_dims_match_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase
 Derived > &V)
- bool qpp::internal::_check_eq_dims (const std::vector< idx > &dims, idx dim) noexcept
- bool qpp::internal::_check_subsys_match_dims (const std::vector < idx > &subsys, const std::vector < idx > &dims)

- template < typename Derived >
 bool qpp::internal::_check_qubit_matrix (const Eigen::MatrixBase < Derived > &A) noexcept
- template<typename Derived >
 bool qpp::internal::_check_qubit_cvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
 bool qpp::internal::_check_qubit_rvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
 bool qpp::internal::_check_qubit_vector (const Eigen::MatrixBase< Derived > &V) noexcept
- bool qpp::internal::_check_perm (const std::vector< idx > &perm)
- template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::internal::_kron2 (const Eigen::MatrixBase< Derived1 > &A,
 const Eigen::MatrixBase< Derived2 > &B)
- template < typename Derived1, typename Derived2 >
 dyn_mat < typename Derived1::Scalar > qpp::internal::_dirsum2 (const Eigen::MatrixBase < Derived1 > &A,
 const Eigen::MatrixBase < Derived2 > &B)
- template < typename T > void qpp::internal::variadic vector emplace (std::vector < T > &)
- template<typename T, typename First, typename... Args>
 void qpp::internal::variadic vector emplace (std::vector< T > &v, First &&first, Args &&...args)

8.17.1 Detailed Description

Internal utility functions.

8.18 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

Namespaces

• qpp

Quantum++ main namespace.

Functions

- template < typename Derived >
 Derived qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)
- Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

dmat qpp::loadMATLABmatrix (const std::string &mat_file, const std::string &var_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

• template<>

cmat gpp::loadMATLABmatrix (const std::string &mat file, const std::string &var name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &A, const std::string &mat_file, const std::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

• template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat_file, const std ::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat_file, const std ::string &var_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

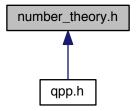
8.18.1 Detailed Description

Input/output interfacing with MATLAB.

8.19 number_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

Functions

std::vector< int > qpp::x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

double qpp::contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double qpp::contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• unsigned long long int qpp::gcd (unsigned long long int m, unsigned long long int n)

Greatest common divisor of two non-negative integers.

• unsigned long long int qpp::gcd (const std::vector< unsigned long long int > &ns)

Greatest common divisor of a list of non-negative integers.

• unsigned long long int qpp::lcm (unsigned long long int m, unsigned long long int n)

Least common multiple of two positive integers.

unsigned long long int qpp::lcm (const std::vector< unsigned long long int > &ns)

Least common multiple of a list of positive integers.

std::vector< idx > qpp::invperm (const std::vector< idx > &perm)

Inverse permutation.

std::vector< idx > qpp::compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)
 Compose permutations.

• std::vector< unsigned long long int > qpp::factors (unsigned long long int n)

Prime factor decomposition.

• bool qpp::isprime (unsigned long long int n)

Primality test.

• unsigned long long int qpp::modpow (unsigned long long int a, unsigned long long int n, unsigned long long int p)

Integer power modulo p.

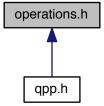
8.19.1 Detailed Description

Number theory functions.

8.20 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 const std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 idx d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

dyn_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template < typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat qpp::kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat qpp::kraus2choi (const std::vector< cmat > &Ks)

Choi matrix

std::vector< cmat > qpp::choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat qpp::choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat qpp::super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::ptrace1 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase< Derived > &A, const std↔ ::vector< idx > &dims)

Partial trace.

• template<typename Derived >

Partial trace.

template<typename Derived >

dyn_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial trace.

 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, con</typename> 	ıs ^ı
std::vector< idx > &subsys, const std::vector< idx > &dims)	
Partial transpose.	
 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, constd::vector< idx > &subsys, idx d=2)</typename> 	າຣາ
Partial transpose.	
 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, constd::vector< idx > &perm, const std::vector< idx > &dims)</typename> 	າຣາ
Subsystem permutation.	
 template<typename derived=""> dyn_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, constd::vector< idx > &perm, idx d=2)</typename> 	າຣາ
Subsystem permutation.	
8.20.1 Detailed Description	
Quantum operation functions.	

8.21 qpp.h File Reference

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random devices.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

Namespaces

qpp

Quantum++ main namespace.

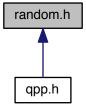
8.21.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

8.22 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



Namespaces

• qpp

Quantum++ main namespace.

Functions

• double qpp::rand (double a=0, double b=1)

Generates a random real number uniformly distributed in the interval [a, b)

idx qpp::randidx (idx a=std::numeric_limits< idx >::min(), idx b=std::numeric_limits< idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

• template<typename Derived >

Derived qpp::rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

• template<>

cmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

• template<typename Derived >

Derived qpp::randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

• template<>

cmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

double qpp::randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat qpp::randU (idx D)

Generates a random unitary matrix.

• cmat qpp::randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > qpp::randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat qpp::randH (idx D)

Generates a random Hermitian matrix.

ket qpp::randket (idx D)

Generates a random normalized ket (pure state vector)

cmat qpp::randrho (idx D)

Generates a random density matrix.

std::vector< idx > qpp::randperm (idx n)

Generates a random uniformly distributed permutation.

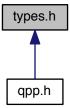
8.22.1 Detailed Description

Randomness-related functions.

8.23 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



Namespaces

qpp

Quantum++ main namespace.

using qpp::idx = std::size_t

Typedefs

```
Non-negative integer index.
using qpp::cplx = std::complex< double >
     Complex number in double precision.
using qpp::ket = Eigen::VectorXcd
     Complex (double precision) dynamic Eigen column vector.
• using qpp::bra = Eigen::RowVectorXcd
     Complex (double precision) dynamic Eigen row vector.
• using qpp::cmat = Eigen::MatrixXcd
     Complex (double precision) dynamic Eigen matrix.
using qpp::dmat = Eigen::MatrixXd
     Real (double precision) dynamic Eigen matrix.
• template<typename Scalar >
  using qpp::dyn mat = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >
     Dynamic Eigen matrix over the field specified by Scalar.
• template<typename Scalar >
  using qpp::dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
     Dynamic Eigen column vector over the field specified by Scalar.
template<typename Scalar >
 using qpp::dyn_row_vect = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >
     Dynamic Eigen row vector over the field specified by Scalar.
```

8.23.1 Detailed Description

Type aliases.

Index

ahaak ayaatar	annicintarnal (IOManin Paga 00
_check_cvector	qpp::internal::IOManipBase, 90
qpp::internal, 75	~Singleton
_check_dims	qpp::internal::Singleton, 96
qpp::internal, 75	~Timer
_check_dims_match_cvect	qpp::Timer, 101
qpp::internal, 75	aham
_check_dims_match_mat	absm
qpp::internal, 75	qpp, 24
_check_dims_match_rvect	abssq
qpp::internal, 75	qpp, 25
_check_eq_dims	adjoint
qpp::internal, 75	qpp, 25
_check_nonzero_size	anticomm
qpp::internal, 75	qpp, 25
_check_perm	apply
qpp::internal, 75	qpp, 26, 27
_check_qubit_cvector	applyCTRL
qpp::internal, 75	qpp, 27, 28
_check_qubit_matrix	
qpp::internal, 75	b00
_check_qubit_rvector	qpp::States, 98
qpp::internal, 75	b01
_check_qubit_vector	qpp::States, 98
qpp::internal, 75	b10
_check_rvector	qpp::States, 99
	b11
qpp::internal, 75	qpp::States, 99
_check_square_mat	bloch2rho
qpp::internal, 75	qpp, 28
_check_subsys_match_dims	bra
qpp::internal, 75	qpp, 23
_check_vector	
qpp::internal, 75	CNOT
_dirsum2	qpp::Gates, 87
qpp::internal, 75	CNOTba
_end	qpp::Gates, 87
qpp::Timer, 102	CTRL
_kron2	qpp::Gates, 84
qpp::internal, 76	CUSTOM_EXCEPTION
_multiidx2n	qpp::Exception, 81
qpp::internal, 76	CZ
_n2multiidx	qpp::Gates, 87
qpp::internal, 76	choi2kraus
_rng	qpp, 28
qpp::RandomDevices, 95	choi2super
start	qpp, 29
qpp::Timer, 102	chop
~Exception	qpp, 73
qpp::Exception, 82	classes/codes.h, 103
~IOManipBase	classes/exception.h, 103
·	

classes/gates.h, 104	qpp, 73
classes/init.h, 105	eig
classes/random_devices.h, 106	qpp, <mark>36</mark>
classes/states.h, 106	entanglement
classes/timer.h, 107	qpp, <mark>36</mark>
cmat	entanglement.h, 109
qpp, 23	entropies.h, 110
codeword	entropy
qpp::Codes, 78	qpp, 36, 37
comm	eps
qpp, 29	qpp, 73
complement	evals
qpp, 29	qpp, 37
compperm	evects
qpp, 30	qpp, 37
concurrence	Exception
qpp, 30	qpp::Exception, 81
conjugate	expandout
qpp, 30	qpp::Gates, 85
constants.h, 108	experimental/test.h, 111
contfrac2x	expm
qpp, 30, 31	qpp, 37
cosm	qpp, σ ^γ
qpp, 31	FIVE QUBIT
cplx	qpp::Codes, 78
•	FRED
qpp, 23 cwise	qpp::Gates, 87
	factors
qpp, 31	
DIMS_INVALID	qpp, 38 Fd
qpp::Exception, 81	
DIMS_MISMATCH_CVECTOR	qpp::Gates, 85
qpp::Exception, 81	functions.h, 111
DIMS MISMATCH MATRIX	funm
qpp::Exception, 81	qpp, 38
DIMS_MISMATCH_RVECTOR	GHZ
qpp::Exception, 81 DIMS_MISMATCH_VECTOR	qpp::States, 99
qpp::Exception, 81	gcd
	qpp, 38
DIMS_NOT_EQUAL	gconcurrence
qpp::Exception, 81	qpp, 39
det	get_instance
qpp, 31	qpp::internal::Singleton, 96
dirsum	get_thread_local_instance
qpp, 32, 33	qpp::internal::Singleton, 96
dirsumpow	grams
qpp, 33	qpp, 39, 40
disp	
qpp, 33, 34	Н
dmat	qpp::Gates, 87
qpp, 24	heig
dyn_col_vect	qpp, 40
qpp, 24	hevals
dyn_mat	qpp, 40
qpp, 24	hevects
dyn_row_vect	qpp, 40
qpp, 24	
	IOManipBase
ee	qpp::internal::IOManipBase, 89, 90

IOManipEigen	MATLAB/matlab.h, 121
qpp::internal::IOManipEigen, 91	MATRIX_MISMATCH_SUBSYS
IOManipPointer	qpp::Exception, 81
qpp::internal::IOManipPointer, 92	MATRIX_NOT_CVECTOR
IOManipRange	qpp::Exception, 80
qpp::internal::IOManipRange, 93	MATRIX_NOT_RVECTOR
ld	qpp::Exception, 80
qpp::Gates, 85	MATRIX_NOT_SQUARE
ld2	qpp::Exception, 80
qpp::Gates, 87	MATRIX_NOT_SQUARE_OR_CVECTOR
idx	qpp::Exception, 80
qpp, 24	MATRIX_NOT_SQUARE_OR_RVECTOR
infty	qpp::Exception, 81
qpp, 73	MATRIX_NOT_SQUARE_OR_VECTOR
input_output.h, 115	qpp::Exception, 81
instruments.h, 117	MATRIX_NOT_VECTOR
internal/classes/iomanip.h, 118	qpp::Exception, 80
internal/classes/singleton.h, 119	maxn
internal/util.h, 120	qpp, 73
internal::Singleton < const Codes >	measure
qpp::Codes, 79	qpp, 47–50
internal::Singleton< const Gates >	measure_seq
qpp::Gates, 87	qpp, 50, 51
internal::Singleton< const Init >	mket
qpp::Init, 89	qpp, 51, 52
internal::Singleton < const States >	modpow
qpp::States, 98	qpp, 52
internal::Singleton < Random Devices >	mprj
qpp::RandomDevices, 95	qpp, <u>52</u>
inverse	multiidx2n
qpp, 41	qpp, 53
invperm	
qpp, 41	n2multiidx
isprime	qpp, 53
qpp, 41	NINE_QUBIT_SHOR
	qpp::Codes, 78
ket	NO_CODEWORD
qpp, 24	qpp::Exception, 81
kraus2choi	NOT_BIPARTITE
qpp, 41	qpp::Exception, 81
kraus2super	NOT_QUBIT_CVECTOR
qpp, 42	qpp::Exception, 81
kron	NOT_QUBIT_MATRIX
qpp, 42, 43	qpp::Exception, 81
kronpow	NOT_QUBIT_RVECTOR
qpp, 43	qpp::Exception, 81
lcm	NOT_QUBIT_SUBSYS
qpp, 44	qpp::Exception, 81
load	NOT_QUBIT_VECTOR
qpp, 44	qpp::Exception, 81
loadMATLABmatrix	negativity
qpp, 45	qpp, 53
logdet	norm
qpp, 46	qpp, 54
logm	number_theory.h, 122
qpp, 46	OUT_OF_RANGE
lognegativity	qpp::Exception, 81
qpp, 46	omega
4PP, TO	omoga

ann 54	ann	10
qpp, 54	qpp	
operations.h, 123		absm, 24
operator<<		abssq, 25
qpp::Timer, 102		adjoint, 25
qpp::internal::IOManipBase, 90		anticomm, 25
operator=		apply, 26, 27
qpp::internal::IOManipBase, 90		applyCTRL, 27, 28
qpp::internal::IOManipPointer, 92		bloch2rho, 28
qpp::internal::Singleton, 96		bra, <mark>23</mark>
operator""_i		choi2kraus, 28
qpp, 54		choi2super, 29
		chop, 73
PERM_INVALID		cmat, 23
qpp::Exception, 81		comm, 29
PERM_MISMATCH_DIMS		complement, 29
qpp::Exception, 81		compperm, 30
pGHZ		concurrence, 30
qpp::States, 99		conjugate, 30
Wq		
qpp::States, 99		contfrac2x, 30, 31
pb00		cosm, 31
qpp::States, 99		cplx, 23
pb01		cwise, 31
qpp::States, 99		det, 31
pb10		dirsum, 32, 33
•		dirsumpow, 33
qpp::States, 99		disp, 33, 34
pb11		dmat, 24
qpp::States, 99		dyn_col_vect, 24
pi 70		dyn_mat, 24
qpp, 73		dyn_row_vect, 24
powm		ee, 73
qpp, 54		eig, 36
prj		entanglement, 36
qpp, 5 5		entropy, 36, 37
prod		eps, 73
qpp, 55		•
ptrace		evals, 37
qpp, 56		evects, 37
ptrace1		expm, 37
qpp, 56		factors, 38
ptrace2		funm, 38
qpp, 58		gcd, 38
ptranspose		gconcurrence, 39
qpp, 58		grams, 39, 40
px0		heig, 40
qpp::States, 99		hevals, 40
px1		hevects, 40
qpp::States, 99		idx, 24
py0		infty, 73
qpp::States, 99		inverse, 41
• •		invperm, 41
py1		isprime, 41
qpp::States, 100		ket, 24
pz0		kraus2choi, 41
qpp::States, 100		kraus2super, 42
pz1		kron, 42, 43
qpp::States, 100		
amutu alinfa		kronpow, 43
qmutualinfo		lcm, 44
qpp, 59		load, 44

loadMATLABmatrix, 45	tsallis, 72
logdet, 46	x2contfrac, 72
logm, 46	qpp.h, 125
lognegativity, 46	qpp::Codes, 77
maxn, 73	codeword, 78
measure, 47–50	FIVE_QUBIT, 78
measure_seq, 50, 51	internal::Singleton< const Codes >, 79
mket, 51, 52	NINE_QUBIT_SHOR, 78
modpow, 52	SEVEN_QUBIT_STEANE, 78
mprj, 52	Type, 78
multiidx2n, 53	qpp::Exception, 79
n2multiidx, 53	~Exception, 82
negativity, 53	CUSTOM_EXCEPTION, 81
norm, 54	DIMS_INVALID, 81
omega, 54	DIMS_MISMATCH_CVECTOR, 81
operator""_i, 54	DIMS_MISMATCH_MATRIX, 81
pi, 73	DIMS_MISMATCH_RVECTOR, 81
powm, 54 prj, 55	DIMS_MISMATCH_VECTOR, 81
	DIMS_NOT_EQUAL, 81
prod, 55	Exception, 81
ptrace, 56 ptrace1, 56	MATRIX_MISMATCH_SUBSYS, 81 MATRIX_NOT_CVECTOR, 80
•	MATRIX_NOT_CVECTOR, 80 MATRIX_NOT_RVECTOR, 80
ptrace2, 58 ptranspose, 58	MATRIX NOT SQUARE, 80
qmutualinfo, 59	MATRIX_NOT_SQUARE_OR_CVECTOR, 80
rand, 59, 60	MATRIX_NOT_SQUARE_OR_RVECTOR, 81
randH, 60	MATRIX_NOT_SQUARE_OR_VECTOR, 81
randU, 63	MATRIX NOT VECTOR, 80
randV, 63	NO_CODEWORD, 81
randidx, 61	NOT_BIPARTITE, 81
randket, 61	NOT QUBIT CVECTOR, 81
randkraus, 61	NOT QUBIT MATRIX, 81
randn, 61, 62	NOT QUBIT RVECTOR, 81
randperm, 63	NOT QUBIT SUBSYS, 81
randrho, 63	NOT_QUBIT_VECTOR, 81
renyi, 64	OUT_OF_RANGE, 81
reshape, 64	PERM_INVALID, 81
rho2bloch, 65	PERM_MISMATCH_DIMS, 81
rho2pure, 65	SUBSYS MISMATCH DIMS, 81
save, 65	TYPE_MISMATCH, 81
saveMATLABmatrix, 66	Type, 80
schatten, 66	UNDEFINED_TYPE, 81
schmidtA, 67	UNKNOWN_EXCEPTION, 80
schmidtB, 67	what, 82
schmidtcoeffs, 67	ZERO_SIZE, 80
schmidtprobs, 68	qpp::Gates, 82
sinm, 68	CNOT, 87
spectralpowm, 68	CNOTba, 87
sqrtm, 69	CTRL, 84
sum, 69	CZ, 87
super2choi, 70	expandout, 85
svals, 70	FRED, 87
svd, 70	Fd, 85
svdU, 70	H, 87
svdV, 71	ld, 85
syspermute, 71	ld2, 87
trace, 71	internal::Singleton $<$ const Gates $>$, 87
transpose, 72	Rn, 86

C 07	abady narm 75
S, 87	_check_perm, 75
SWAP, 87	_check_qubit_cvector, 75
T, 87 TOF, 87	_check_qubit_matrix, 75 _check_qubit_rvector, 75
	_check_qubit_vector, 75
X, 87 Xd, 86	_check_rvector, 75
Y, 87	_check_square_mat, 75
Z, 88	_check_subsys_match_dims, 75
Zd, 86	_check_vector, 75
qpp::Init, 88	_dirsum2, 75
internal::Singleton< const Init >, 89	_kron2, 76
qpp::RandomDevices, 93	_multiidx2n, 76
_rng, 95	_n2multiidx, 76
internal::Singleton< RandomDevices >, 95	variadic_vector_emplace, 76
qpp::States, 96	qpp::internal::IOManipBase, 89
b00, 98	∼IOManipBase, 90
b01, 98	IOManipBase, 89, 90
b10, 99	operator<<, 90
b11, 99	operator=, 90
GHZ, 99	qpp::internal::IOManipEigen, 90
	IOManipEigen, 91
internal::Singleton< const States >, 98 pGHZ, 99	gpp::internal::IOManipPointer
pW, 99	IOManipPointer, 92
•	operator=, 92
pb00, 99	qpp::internal::IOManipPointer< PointerType >, 91
pb01, 99	qpp::internal::IOManipRange
pb10, 99	IOManipRange, 93
pb11, 99	qpp::internal::IOManipRange< InputIterator >, 92
px0, 99	qpp::internal::Singleton
px1, 99	∼Singleton, 96
py0, 99	get_instance, 96
py1, 100	get_thread_local_instance, 96
pz0, 100	operator=, 96
pz1, 100	Singleton, 96
W, 100	qpp::internal::Singleton< T >, 95
x0, 100	
x1, 100	rand
y0, 100	qpp, 59, 60
y1, 100	randH
z0, 100	qpp, 60
z1, 100	randU
qpp::Timer, 101	qpp, 63
_end, 102	randV
_start, 102	qpp, 63
∼Timer, 101	randidx
operator<<, 102	qpp, 61
seconds, 101	randket
tic, 102	qpp, 61
Timer, 101	randkraus
toc, 102	qpp, 61
qpp::experimental, 73	randn
qpp::internal, 74	qpp, 61, 62
_check_cvector, 75	random.h, 127
_check_dims, 75	randperm
_check_dims_match_cvect, 75	qpp, 63
_check_dims_match_mat, 75	randrho
_check_dims_match_rvect, 75	qpp, 63
_check_eq_dims, 75	renyi
_check_nonzero_size, 75	qpp, 64

reshape	
	qpp::Gates, 87
qpp, 64	TYPE MISMATCH
rho2bloch	qpp::Exception, 81
qpp, 65	tic
rho2pure	qpp::Timer, 102
qpp, 65	Timer
Rn	qpp::Timer, 101
_	
qpp::Gates, 86	toc
	qpp::Timer, 102
S	trace
qpp::Gates, 87	qpp, 71
SEVEN_QUBIT_STEANE	transpose
qpp::Codes, 78	
	qpp, 72
SUBSYS_MISMATCH_DIMS	tsallis
qpp::Exception, 81	qpp, 72
SWAP	Туре
qpp::Gates, 87	
	qpp::Codes, 78
save	qpp::Exception, 80
qpp, 65	types.h, 128
saveMATLABmatrix	
qpp, 66	UNDEFINED_TYPE
schatten	qpp::Exception, 81
	UNKNOWN EXCEPTION
qpp, 66	-
schmidtA	qpp::Exception, 80
qpp, 67	variadia vaatar amplaaa
schmidtB	variadic_vector_emplace
	qpp::internal, 76
qpp, 67	147
schmidtcoeffs	W
qpp, 67	qpp::States, 100
schmidtprobs	what
qpp, 68	qpp::Exception, 82
	4pp.:=x00ption, 02
seconds	
	X
qpp::Timer, 101	X gpp::Gates 87
	qpp::Gates, 87
qpp::Timer, 101 Singleton	qpp::Gates, 87
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96	qpp::Gates, 87
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm	qpp::Gates, 87
<pre>qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68</pre>	qpp::Gates, 87 x0 qpp::States, 100 x1
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87
qpp::Timer, 101 Singleton	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87
qpp::Timer, 101 Singleton	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70 svdU	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70 svdU qpp, 70	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70 svdU	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svdU qpp, 70 svdV	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svdU qpp, 70 svdU qpp, 70 svdV qpp, 71	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70 svdU qpp, 70 svdV qpp, 71 syspermute	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svdU qpp, 70 svdU qpp, 70 svdV qpp, 71	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1 qpp::States, 100
qpp::Timer, 101 Singleton	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1 qpp::States, 100 ZERO_SIZE
qpp::Timer, 101 Singleton qpp::internal::Singleton, 96 sinm qpp, 68 spectralpowm qpp, 68 sqrtm qpp, 69 sum qpp, 69 super2choi qpp, 70 svals qpp, 70 svd qpp, 70 svdU qpp, 70 svdV qpp, 71 syspermute	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1 qpp::States, 100 ZERO_SIZE qpp::Exception, 80
qpp::Timer, 101 Singleton	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1 qpp::States, 100 ZERO_SIZE
qpp::Timer, 101 Singleton	qpp::Gates, 87 x0 qpp::States, 100 x1 qpp::States, 100 x2contfrac qpp, 72 Xd qpp::Gates, 86 Y qpp::Gates, 87 y0 qpp::States, 100 y1 qpp::States, 100 Z qpp::Gates, 88 z0 qpp::States, 100 z1 qpp::States, 100 ZERO_SIZE qpp::Exception, 80