Quantum++ v0.8.9-devel

Generated by Doxygen 1.8.10

Fri Oct 7 2016 10:07:18

# **Contents**

1	Qua	ntum++			1			
2	Nam	espace	Index					
	2.1	Names	pace List		5			
3	Hier	archical	Index		7			
	3.1	Class I	Hierarchy		7			
4	Clas	s Index			9			
	4.1	Class I	_ist		9			
5	File	Index			11			
	5.1	File Lis	t		11			
6	Nam	nespace	Documer	ntation	13			
	6.1	qpp Na	ımespace	Reference	13			
		6.1.1	Detailed	Description	25			
		6.1.2	Typedef [	Documentation	25			
			6.1.2.1	bigint	25			
			6.1.2.2	bra	25			
			6.1.2.3	cmat	25			
			6.1.2.4	cplx	25			
			6.1.2.5	dmat	25			
			6.1.2.6	dyn_col_vect	25			
			6.1.2.7	dyn_mat	25			
			6.1.2.8	dyn_row_vect	26			
			6.1.2.9	idx	26			
			6.1.2.10	ket	26			
			6.1.2.11	to_void	26			
		6.1.3	Function	Documentation	26			
			6.1.3.1	absm(const Eigen::MatrixBase< Derived > &A)	26			
			6.1.3.2	abssq(InputIterator first, InputIterator last)	26			

iv CONTENTS

6.1.3.3	abssq(const Container &c, typename std::enable_if< is_iterable< Container > ∴:value >::type *=nullptr)	26
6.1.3.4	abssq(const Eigen::MatrixBase< Derived > &A)	27
6.1.3.5	adjoint(const Eigen::MatrixBase< Derived > &A)	27
6.1.3.6	anticomm(const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)	27
6.1.3.7	apply(const Eigen::MatrixBase< Derived1 $>$ &state, const Eigen::MatrixBase< Derived2 $>$ &A, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	28
6.1.3.8	apply(const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)	28
6.1.3.9	${\it apply} ({\it const \ Eigen::} {\it MatrixBase} < {\it Derived} > \& {\it rho},  {\it const \ std::} {\it vector} < {\it cmat} > \& {\it Ks})$	28
6.1.3.10	apply(const Eigen::MatrixBase< Derived $>$ ρ, const std::vector< cmat $>$ &Ks, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	29
6.1.3.11	$apply(const\ Eigen::MatrixBase < Derived > \ρ, const\ std::vector < cmat > \&Ks, const\ std::vector < idx > \&subsys, idx\ d=2) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	30
6.1.3.12	applyCTRL(const Eigen::MatrixBase< Derived1 > &state, const Eigen::Matrix← Base< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, const std::vector< idx > &dims)	30
6.1.3.13	applyCTRL(const Eigen::MatrixBase< Derived1 > &state, const Eigen::Matrix← Base< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx d=2)	31
6.1.3.14	avg(const std::vector< double > &prob, const Container &X, typename std← ::enable_if< is_iterable< Container >::value >::type *=nullptr)	31
6.1.3.15	bloch2rho(const std::vector< double $> \&r)$	31
6.1.3.16	choi2kraus(const cmat &A)	32
6.1.3.17	choi2super(const cmat &A)	32
6.1.3.18	$\label{eq:comm} \mbox{comm(const Eigen::MatrixBase} < \mbox{Derived1} > \&\mbox{A, const Eigen::MatrixBase} < \mbox{Derived2} > \&\mbox{B)} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	32
6.1.3.19	$complement(std::vector < T > subsys, idx \; N) \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; .$	32
6.1.3.20	${\tt compperm}({\tt const\ std::vector} < {\tt idx} > {\tt \&perm}, {\tt const\ std::vector} < {\tt idx} > {\tt \σ})  .$	33
6.1.3.21	${\sf concurrence}({\sf const}\ {\sf Eigen::MatrixBase} {< \ \sf Derived} > \& {\sf A})\ .\ .\ .\ .\ .\ .\ .\ .\ .$	33
6.1.3.22	conjugate(const Eigen::MatrixBase< Derived > &A)	33
6.1.3.23	$contfrac2x(const\ std::vector< int>\&cf,\ idx\ n) \\ aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$	33
6.1.3.24	$contfrac2x(const\;std::\!vector\!\&cf)\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.$	34
6.1.3.25	cor(const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)	34
6.1.3.26	cosm(const Eigen::MatrixBase< Derived > &A)	34
6.1.3.27	cov(const dmat &probXY, const Container &X, const Container &Y, typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)	34
6.1.3.28	cwise(const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)( const type-name Derived::Scalar &))	35
6.1.3.29	det(const Eigen::MatrixBase< Derived > &A)	35
6.1.3.30	dirsum(const T &head)	35
6.1.3.31	dirsum(const T &head, const Args &tail)	36

CONTENTS

6.1.3.32	dirsum(const std::vector< Derived > &As)
6.1.3.33	dirsum(const std::initializer_list< Derived > &As)
6.1.3.34	dirsumpow(const Eigen::MatrixBase< Derived > &A, idx n) 3
6.1.3.35	disp(const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop) 3
6.1.3.36	disp(cplx z, double chop=qpp::chop)
6.1.3.37	disp(InputIterator first, InputIterator last, const std::string &separator, const std↔ ::string &start=""["", const std::string &end=""]"")
6.1.3.38	disp(const Container &c, const std::string &separator, const std::string &start=""["", const std::string &end=""]"", typename std::enable_if< is_iterable< Container >::value >::type *=nullptr)
6.1.3.39	disp(const PointerType *p, idx n, const std::string &separator, const std::string &start=""["", const std::string &end=""]"")
6.1.3.40	egcd(bigint m, bigint n)
6.1.3.41	eig(const Eigen::MatrixBase< Derived > &A)
6.1.3.42	entanglement(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
6.1.3.43	entropy(const Eigen::MatrixBase< Derived > &A)
6.1.3.44	entropy(const std::vector< double > &prob)
6.1.3.45	evals(const Eigen::MatrixBase< Derived > &A)
6.1.3.46	evects(const Eigen::MatrixBase< Derived > &A)
6.1.3.47	expm(const Eigen::MatrixBase< Derived > &A)
6.1.3.48	factors(bigint n)
6.1.3.49	$\label{eq:funm} \mbox{funm(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, cplx(*f)(const cplx \&))} \ . \ . \ . \ . \ 4$
6.1.3.50	gcd(bigint m, bigint n)
6.1.3.51	gcd(const std::vector< bigint > &ns)
6.1.3.52	gconcurrence(const Eigen::MatrixBase< Derived > &A)
6.1.3.53	grams(const std::vector< Derived > &Vs)
6.1.3.54	grams(const std::initializer_list< Derived > &Vs)
6.1.3.55	grams(const Eigen::MatrixBase< Derived > &A)
6.1.3.56	heig(const Eigen::MatrixBase< Derived > &A)
6.1.3.57	hevals(const Eigen::MatrixBase< Derived > &A)
6.1.3.58	hevects(const Eigen::MatrixBase< Derived > &A)
6.1.3.59	inverse(const Eigen::MatrixBase< Derived > &A)
6.1.3.60	invperm(const std::vector< idx > &perm)
6.1.3.61	ip(const Eigen::MatrixBase< Derived > φ, const Eigen::MatrixBase< Derived > ψ, const std::vector< idx > &subsys, const std::vector< idx > &dims) 4
6.1.3.62	ip(const Eigen::MatrixBase< Derived > φ, const Eigen::MatrixBase< Derived > ψ, const std::vector< idx > &subsys, idx d=2)
6.1.3.63	isprime(bigint n, idx k=80)
6.1.3.64	kraus2choi(const std::vector< cmat > &Ks)
6.1.3.65	kraus2super(const std::vector< cmat > &Ks)
6.1.3.66	kron(const T &head)

vi CONTENTS

6.1.3.67	kron(const T &head, const Args &tail)	46
6.1.3.68	$kron(const \ std::vector < Derived > \&As) \qquad . \qquad $	47
6.1.3.69	$kron(const\ std::initializer\_list < Derived > \&As) \ \dots \ \dots \ \dots \ \dots$	47
6.1.3.70	$kronpow(const\ Eigen::MatrixBase < Derived > \&A,\ idx\ n) \\  \ldots \ldots \ldots$	47
6.1.3.71	lcm(bigint m, bigint n)	48
6.1.3.72	lcm(const std::vector< bigint > &ns)	48
6.1.3.73	load(const std::string &fname)	48
6.1.3.74	loadMATLABmatrix(const std::string &, const std::string &)	49
6.1.3.75	loadMATLABmatrix(const std::string &mat_file, const std::string &var_name)	49
6.1.3.76	loadMATLABmatrix(const std::string &mat_file, const std::string &var_name)	49
6.1.3.77	logdet(const Eigen::MatrixBase< Derived > &A)	50
6.1.3.78	${\sf logm}({\sf const}\ {\sf Eigen} {\it ::} {\sf MatrixBase} {\it <}\ {\sf Derived} {\it >}\ \& {\sf A})\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\$	50
6.1.3.79	lognegativity(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	50
6.1.3.80	marginalX(const dmat &probXY)	50
6.1.3.81	marginalY(const dmat &probXY)	51
6.1.3.82	measure(const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)	51
6.1.3.83	measure(const Eigen::MatrixBase< Derived > &A, const std::initializer_list< cmat > &Ks)	51
6.1.3.84	$measure (const\ Eigen:: Matrix Base < Derived > \&A,\ const\ cmat\ \&U) \ \ . \ \ . \ \ . \ \ .$	51
6.1.3.85	$\label{lem:measure} \mbox{measure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const std::vector} < \mbox{cmat} > \&\mbox{Ks, const std::vector} < \mbox{idx} > \&\mbox{subsys, const std::vector} < \mbox{idx} > \&\mbox{dims}) $	52
6.1.3.86	$\label{lem:measure} \begin{tabular}{ll} measure(const Eigen::MatrixBase< Derived > \&A, const std::initializer_list< \\ cmat > \&Ks, const std::vector< idx > \&subsys, const std::vector< idx > \&dims) \\ \end{tabular}$	52
6.1.3.87	$\label{lem:measure} \mbox{measure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const std::vector} < \mbox{cmat} > \&\mbox{Ks, const std::vector} < \mbox{idx} > \&\mbox{subsys, idx d=2)} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	53
6.1.3.88	measure(const_Eigen::MatrixBase< Derived > &A, const_std::initializer_list< cmat > &Ks, const_std::vector< idx > &subsys, idx d=2)	53
6.1.3.89	measure(const Eigen::MatrixBase< Derived $>$ &A, const cmat &V, const std $\leftrightarrow$ ::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	54
6.1.3.90	$\label{lem:measure} \mbox{measure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const cmat \&V, const std} \leftrightarrow ::\mbox{vector} < \mbox{idx} > \&\mbox{subsys, idx d=2)} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	54
6.1.3.91	measure_seq(const Eigen::MatrixBase< Derived > &A, std::vector< idx > subsys, std::vector< idx > dims)	55
6.1.3.92	measure_seq(const Eigen::MatrixBase< Derived > &A, std::vector< idx > subsys, idx d=2)	55
6.1.3.93	$\label{eq:mket} \mbox{mket(const std::vector< idx > \&mask, const std::vector< idx > \&dims)} \ \ . \ \ . \ \ . \ \ . \ \ .$	56
6.1.3.94	mket(const std::vector< idx > &mask, idx d=2)	56
6.1.3.95	modinv(bigint a, bigint p)	56
6.1.3.96	modmul(bigint m, bigint n, bigint p)	56
6.1.3.97	modpow(bigint a, bigint n, bigint p)	57
6.1.3.98	mprj(const std::vector< idx > &mask, const std::vector< idx > &dims) $\dots$	57

CONTENTS vii

6.1.3.99	mprj(const std::vector $<$ idx $>$ &mask, idx d=2)	57
6.1.3.100	$\label{eq:multiidx2n} \text{multiidx2n} \\ \text{(const std::vector} \\ < \text{idx} > \\ \\ \\ \text{smidx}, \\ \\ \text{const std::vector} \\ < \\ \\ \\ \text{idx} > \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	58
6.1.3.101	n2multiidx(idx n, const std::vector< idx > &dims)	58
6.1.3.102	negativity(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	58
6.1.3.103	norm(const Eigen::MatrixBase< Derived > &A)	59
6.1.3.104	omega(idx D)	59
6.1.3.105	operator"""_i(unsigned long long int x) noexcept	59
6.1.3.106	operator"""_i(long double x) noexcept	59
6.1.3.107	$powm(const\ Eigen::MatrixBase < Derived > \&A, idx\ n)\ \ldots \ldots \ldots$	59
6.1.3.108	prj(const Eigen::MatrixBase< Derived $>$ &V)	60
6.1.3.109	prod(const Eigen::MatrixBase< Derived > &A)	60
6.1.3.110	prod(InputIterator first, InputIterator last)	60
6.1.3.111	prod(const Container &c, typename std::enable_if< is_iterable< Container > ::value >::type *=nullptr)	60
6.1.3.112	$\label{eq:ptrace}                                    $	61
6.1.3.113	ptrace(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)	61
6.1.3.114	$ptrace1(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&dims)$	61
6.1.3.115	$ptrace2(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&dims)$	63
6.1.3.116	ptranspose(const Eigen::MatrixBase< Derived $>$ &A, const std::vector< idx $>$ &subsys, const std::vector< idx $>$ &dims)	63
6.1.3.117	ptranspose(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)	63
6.1.3.118	$\label{lem:matrixBase} $$\operatorname{Derived} > \&A, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{subsysA}, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{subsysB}, \ \operatorname{const} \ \operatorname{std}:: \operatorname{vector} < \operatorname{idx} > \&\operatorname{dims})$$	64
6.1.3.119	qmutualinfo(const Eigen::MatrixBase< Derived $>$ &A, const std::vector< idx $>$ &subsysA, const std::vector< idx $>$ &subsysB, idx d=2)	64
6.1.3.120	rand(double a, double b)	64
6.1.3.121	rand(bigint a, bigint b)	65
6.1.3.122	rand(idx rows, idx cols, double a=0, double b=1)	66
6.1.3.123	rand(idx rows, idx cols, double a, double b)	66
6.1.3.124	rand(idx rows, idx cols, double a, double b)	66
6.1.3.125	$randH(idx\;D)\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots$	67
6.1.3.126	randidx(idx a=std::numeric_limits< idx >::min(), idx b=std::numeric_limits< idx >::max())	67
6.1.3.127	randket(idx D)	67
6.1.3.128	randkraus(idx N, idx D)	67
6.1.3.129	randn(idx rows, idx cols, double mean=0, double sigma=1)	68
6.1.3.130	randn(idx rows, idx cols, double mean, double sigma)	68
6.1.3.131	randn(idx rows, idx cols, double mean, double sigma)	68
6.1.3.132	randn(double mean=0, double sigma=1)	69

viii CONTENTS

6.1.3.133	randperm(idx n)	69
6.1.3.134	randprime(bigint a, bigint b, idx N=1000)	69
6.1.3.135	randrho(idx D)	69
6.1.3.136	randU(idx D)	70
6.1.3.137	randV(idx Din, idx Dout)	70
6.1.3.138	$renyi(const\ Eigen::MatrixBase < Derived > \&A,\ double\ alpha) \ \ \dots \ \ \dots$	70
6.1.3.139	$\label{eq:const_std} \textit{renyi}(\textit{const std::vector} < \textit{double} > \&\textit{prob}, \textit{double alpha})  .  .  .  .  .  .  .  .  .  $	70
6.1.3.140	$reshape (const \ Eigen:: Matrix Base < Derived > \&A, \ idx \ rows, \ idx \ cols) \\  \ldots  \ldots$	71
6.1.3.141	$\label{loch} \mbox{rho2bloch(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A}) \ \ . \ \ \ . \ \ \ \ \ . \ \ \ . \ \ \ . \ \ \ \ . \$	71
6.1.3.142	$\label{local_pure} \mbox{rho2pure(const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A})  .  .  .  .  .  .  .  .  .  $	71
6.1.3.143	$save(const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::string\ \&fname)  .  .  .$	72
6.1.3.144	saveMATLABmatrix(const Eigen::MatrixBase< Derived > &, const std::string &, const std::string &)	72
6.1.3.145	saveMATLABmatrix(const_Eigen::MatrixBase< dmat > &A, const_std::string &mat_file, const_std::string &var_name, const_std::string &mode)	72
6.1.3.146	saveMATLABmatrix(const_Eigen::MatrixBase< cmat > &A, const_std::string &mat_file, const_std::string &var_name, const_std::string &mode)	72
6.1.3.147	$schatten(const\ Eigen::MatrixBase < Derived > \&A,\ double\ p) \\  \dots \dots \\  \dots \\  \dots$	73
6.1.3.148	schmidtA(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	73
6.1.3.149	schmidtB(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)	73
6.1.3.150	$schmidt coeffs (const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&dims)$	73
6.1.3.151	$\label{lem:achmidtprobs} schmidtprobs(const \ Eigen::MatrixBase < Derived > \&A, \ const \ std::vector < idx > \&dims) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	74
6.1.3.152	sigma(const std::vector< double > &prob, const Container &X, typename std↔ ::enable_if< is_iterable< Container >::value >::type *=nullptr)	74
6.1.3.153	sinm(const Eigen::MatrixBase< Derived > &A)	74
6.1.3.154	$spectralpowm(const\ Eigen::MatrixBase < Derived > \&A,\ const\ cplx\ z)\ \ .\ \ .\ \ .$	75
6.1.3.155	$sqrtm(const\;Eigen :: MatrixBase < Derived > \& A) \ \ldots \ \ldots \ \ldots \ \ldots$	75
6.1.3.156	$sum(const\ Eigen::MatrixBase < Derived > \&A) \ \dots \ \dots \ \dots$	75
6.1.3.157	sum(InputIterator first, InputIterator last)	75
6.1.3.158	sum(const Container &c, typename std::enable_if< is_iterable< Container > ::value >::type *=nullptr)	76
6.1.3.159	super2choi(const cmat &A)	76
6.1.3.160	svals(const Eigen::MatrixBase< Derived > &A)	76
6.1.3.161	svd(const Eigen::MatrixBase< Derived > &A)	76
6.1.3.162	svdU(const Eigen::MatrixBase< Derived > &A)	77
6.1.3.163	svdV(const Eigen::MatrixBase< Derived > &A)	77
6.1.3.164	$syspermute (const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&perm,\ const\ std::vector < idx > \&dims)$	77

CONTENTS

		6.1.3.165	syspermute(const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)	77
		6.1.3.166	6 trace(const Eigen::MatrixBase< Derived > &A)	78
			' transpose(const Eigen::MatrixBase< Derived > &A)	78
			B tsallis(const Eigen::MatrixBase< Derived > &A, double q)	78
		6.1.3.169	tsallis(const std::vector< double > &prob, double q)	78
		6.1.3.170	uniform(idx N)	79
		6.1.3.171	var(const std::vector< double > &prob, const Container &X, typename std↔ ::enable_if< is_iterable< Container >::value >::type *=nullptr)	79
		6.1.3.172	2 x2contfrac(double x, idx n, idx cut=1e5)	79
	6.1.4	Variable I	Documentation	79
		6.1.4.1	chop	79
		6.1.4.2	ee	80
		6.1.4.3	eps	80
		6.1.4.4	infty	80
		6.1.4.5	maxn	80
		6.1.4.6	pi	80
6.2	qpp::ex	perimenta	Il Namespace Reference	80
	6.2.1	Detailed	Description	80
6.3	qpp::in	ternal Nam	nespace Reference	80
	6.3.1	Detailed	Description	81
	6.3.2	Function	Documentation	81
		6.3.2.1	$\verb _check_cvector(const Eigen::MatrixBase< Derived > \&A) $	81
		6.3.2.2	_check_dims(const std::vector< idx > &dims)	81
		6.3.2.3	$\_check\_dims\_match\_cvect(const \ std::vector< \ idx > \&dims, \ const \ Eigen:: \leftarrow \\ MatrixBase < Derived > \&V) \ $	82
		6.3.2.4	$\label{lem:check_dims_match_mat} $$\_ check\_dims\_match\_mat(const \ std::vector < idx > \&dims, \ const \ Eigen::Matrix \leftrightarrow Base < Derived > \&A) \ $	82
		6.3.2.5	$\label{lem:check_dims_match_rvect} $$\_\const \ std::vector< \ idx > \&dims, \ const \ Eigen::$$\longleftrightarrow $$MatrixBase< Derived > \&V) \ $	82
		6.3.2.6	_check_eq_dims(const std::vector< idx > &dims, idx dim) noexcept	82
		6.3.2.7	_check_matching_sizes(const T1 &lhs, const T2 &rhs) noexcept	82
		6.3.2.8	_check_nonzero_size(const T &x) noexcept	82
		6.3.2.9	$\_{check\_perm(const\ std::vector< idx>\&perm)\ .\ .\ .\ .\ .\ .}$	82
		6.3.2.10	$\_check\_qubit\_cvector(const\ Eigen::MatrixBase < Derived > \&V)\ noexcept  . \ . \ .$	82
		6.3.2.11	$\_check\_qubit\_matrix(const\ Eigen::MatrixBase < Derived > \&A)\ noexcept \ \ . \ \ . \ \ .$	82
		6.3.2.12	$\_check\_qubit\_rvector(const\ Eigen::MatrixBase < Derived > \&V)\ noexcept\ .\ .\ .\ .$	82
		6.3.2.13	$\_check\_qubit\_vector(const\ Eigen::MatrixBase < Derived > \&V)\ noexcept \ \ . \ \ . \ \ .$	82
		6.3.2.14	$\_check\_rvector(const\ Eigen::MatrixBase < Derived > \&A) \ \ . \ \ . \ \ . \ \ . \ \ .$	82
		6.3.2.15	$\_check\_square\_mat(const\ Eigen::MatrixBase < Derived > \&A)  .  .  .  .$	82
		6.3.2.16	$\_check\_subsys\_match\_dims(const std::vector< idx > \&subsys, const std \\ ::vector< idx > \&dims) \\ \dots $	82

CONTENTS

			6.3.2.17	_check_vector(const Eigen::MatrixBase< Derived > &A)	82
			6.3.2.18	_dirsum2(const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)	82
			6.3.2.19	_kron2(const Eigen::MatrixBase< Derived1 > &A, const Eigen::MatrixBase< Derived2 > &B)	82
			6.3.2.20	_multiidx2n(const idx *midx, idx numdims, const idx *dims) noexcept	82
			6.3.2.21	_n2multiidx(idx n, idx numdims, const idx *dims, idx *result) noexcept	82
			6.3.2.22	$\label{eq:variadic_vector_emplace} \mbox{variadic\_vector\_emplace} (\mbox{std::vector} < \mbox{T} > \&) \ . \ . \ . \ . \ . \ . \ . \ . \ . \ $	82
			6.3.2.23	$\label{eq:continuous} \mbox{variadic\_vector\_emplace(std::vector< T > \&v, First \&\&first, Args \&\&args)} \ . \ . \ .$	82
	6.4	qpp::in	ternal::_de	etails Namespace Reference	83
7	Clas	s Docu	mentation		85
	7.1	qpp::in	ternal::_de	etails::_Display_Impl Struct Reference	85
		7.1.1	Member	Function Documentation	85
			7.1.1.1	_display_impl(const T &_A, std::ostream &_os, double _chop=qpp::chop) const	85
	7.2	qpp::C	odes Class	s Reference	85
		7.2.1	Detailed	Description	87
		7.2.2	Member	Enumeration Documentation	87
			7.2.2.1	Type	87
		7.2.3	Construc	tor & Destructor Documentation	87
			7.2.3.1	Codes()	87
			7.2.3.2	~Codes()=default	87
		7.2.4	Member	Function Documentation	87
			7.2.4.1	codeword(Type type, idx i) const	87
		7.2.5	Friends A	And Related Function Documentation	88
			7.2.5.1	internal::Singleton< const Codes >	88
	7.3	qpp::E	xception C	lass Reference	88
		7.3.1	Detailed	Description	89
		7.3.2	Member	Enumeration Documentation	89
			7.3.2.1	Type	89
		7.3.3	Construc	tor & Destructor Documentation	90
			7.3.3.1	Exception(const std::string &where, const Type &type)	90
			7.3.3.2	Exception(const std::string &where, const std::string &custom)	91
		7.3.4	Member	Function Documentation	91
			7.3.4.1	_construct_exception_msg()	91
			7.3.4.2	what() const noexcept override	91
		7.3.5	Member	Data Documentation	91
			7.3.5.1	_custom	91
			7.3.5.2	msg	91
			7.3.5.3	type	91
			7.3.5.4	where	91
			-	_	

CONTENTS xi

7.4	qpp::G	ates Class	Reference	91	
	7.4.1	Detailed	Description	93	
	7.4.2	Construc	tor & Destructor Documentation	94	
		7.4.2.1	Gates()	94	
		7.4.2.2	$\sim$ Gates()=default	94	
	7.4.3	Member	Function Documentation	94	
		7.4.3.1	$\label{eq:ctrl}                                    $	94	
		7.4.3.2	$\label{eq:const_approx} \begin{split} & \text{expandout(const Eigen::MatrixBase} < \text{Derived} > \&\text{A, idx pos, const std::vector} < \\ & \text{idx} > \&\text{dims) const} \\ & \dots \dots$	94	
		7.4.3.3	Fd(idx D) const	95	
		7.4.3.4	Id(idx D) const	95	
		7.4.3.5	Rn(double theta, const std::vector< double $>$ &n) const $\dots \dots \dots$	95	
		7.4.3.6	Xd(idx D) const	96	
		7.4.3.7	Zd(idx D) const	96	
	7.4.4	Friends A	And Related Function Documentation	96	
		7.4.4.1	internal::Singleton < const Gates >	96	
	7.4.5	Member	Data Documentation	96	
		7.4.5.1	CNOT	96	
		7.4.5.2	CNOTba	96	
		7.4.5.3	CZ	96	
		7.4.5.4	FRED	96	
		7.4.5.5	$H \ldots \ldots$	97	
		7.4.5.6	ld2	97	
		7.4.5.7	s	97	
		7.4.5.8	SWAP	97	
		7.4.5.9	T	97	
		7.4.5.10	TOF	97	
		7.4.5.11	x	97	
		7.4.5.12	Y	97	
		7.4.5.13	z	97	
7.5	qpp::ID	isplay Cla	ss Reference	97	
	7.5.1	Detailed Description			
	7.5.2	Construc	tor & Destructor Documentation	99	
		7.5.2.1	IDisplay()=default	99	
		7.5.2.2	IDisplay(const IDisplay &)=default	99	
		7.5.2.3	IDisplay(IDisplay &&)=default	99	
		7.5.2.4	~IDisplay()=default	99	
	7.5.3	Member	Function Documentation	99	
		7.5.3.1	display(std::ostream &os) const =0	99	

xii CONTENTS

		7.5.3.2	operator=(const IDisplay &)=default	99
		7.5.3.3	operator=(IDisplay &&)=default	99
	7.5.4	Friends A	nd Related Function Documentation	99
		7.5.4.1	operator<<	99
7.6	qpp::In	it Class Re	eference	100
	7.6.1	Detailed I	Description	101
	7.6.2	Construct	tor & Destructor Documentation	101
		7.6.2.1	Init()	101
		7.6.2.2	$\sim$ Init()	101
	7.6.3	Friends A	nd Related Function Documentation	101
		7.6.3.1	${\it internal::} Singleton < const \ Init > $	101
7.7	qpp::in	ternal::ION	ManipEigen Class Reference	101
	7.7.1	Construct	tor & Destructor Documentation	102
		7.7.1.1	$IOManipEigen(const\ Eigen::MatrixBase < Derived > \&A,\ double\ chop=qpp::chop)$	102
		7.7.1.2	IOManipEigen(const cplx z, double chop=qpp::chop)	102
	7.7.2	Member I	Function Documentation	102
		7.7.2.1	display(std::ostream &os) const override	102
	7.7.3	Member I	Data Documentation	103
		7.7.3.1	_A	103
		7.7.3.2	_chop	103
7.8	qpp::in	ternal::IOM	ManipPointer         PointerType         Class Template Reference	103
	7.8.1	Construct	tor & Destructor Documentation	104
		7.8.1.1	$IOManipPointer(const\ PointerType\ *p,\ idx\ n,\ const\ std::string\ \&separator,\ const\ std::string\ \&start=""["",\ const\ std::string\ \&end=""]"")\$	104
		7.8.1.2	IOManipPointer(const IOManipPointer &)=default	104
	7.8.2	Member I	Function Documentation	104
		7.8.2.1	display(std::ostream &os) const override	104
		7.8.2.2	operator=(const IOManipPointer &)=default	104
	7.8.3	Member I	Data Documentation	104
		7.8.3.1	_end	104
		7.8.3.2	_n	104
		7.8.3.3	_p	104
		7.8.3.4	_separator	105
		7.8.3.5	_start	105
7.9	qpp::in	ternal::ION	fanipRange < InputIterator > Class Template Reference	105
	7.9.1	Construct	tor & Destructor Documentation	106
		7.9.1.1	IOManipRange(InputIterator first, InputIterator last, const std::string &separator, const std::string &start=""["", const std::string &end="""]"")	106
		7.9.1.2	IOManipRange(const IOManipRange &)=default	106
	7.9.2	Member I	Function Documentation	106

CONTENTS xiii

		7.9.2.1	display(std::ostream &os) const override	106
		7.9.2.2	operator=(const IOManipRange &)=default	106
	7.9.3	Member I	Data Documentation	106
		7.9.3.1	_end	106
		7.9.3.2	_first	106
		7.9.3.3	_last	106
		7.9.3.4	_separator	107
		7.9.3.5	_start	107
7.10	qpp::is_	_complex<	< T $>$ Struct Template Reference	107
	7.10.1	Detailed I	Description	107
7.11	qpp::is_	_complex<	< std::complex $<$ T $>$ $>$ Struct Template Reference	108
	7.11.1	Detailed I	Description	108
7.12	qpp::is_	_iterable<	T, typename $>$ Struct Template Reference	109
	7.12.1	Detailed I	Description	109
7.13		_	T, to_void< decltype(std::declval< T >().begin()), decltype(std::declval< T ame T::value_type > > Struct Template Reference	110
	7.13.1	Detailed I	Description	111
7.14	qpp::is_	_matrix_ex	xpression< Derived > Struct Template Reference	111
	7.14.1	Detailed I	Description	112
7.15	qpp::is_	_matrix_ex	${\sf cpression}{<}$ typename Eigen::MatrixBase ${<}$ Derived ${>}$ ${>}$ Struct Template Reference	112
	7.15.1	Detailed I	Description	113
7.16	qpp::Ra	andomDev	rices Class Reference	113
	7.16.1	Detailed I	Description	115
	7.16.2	Construct	tor & Destructor Documentation	115
		7.16.2.1	RandomDevices()	115
		7.16.2.2	$\sim\! RandomDevices() \!\!=\! default \ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	115
	7.16.3	Friends A	and Related Function Documentation	115
		7.16.3.1	$internal:: Singleton < Random Devices > \dots $	115
	7.16.4	Member I	Data Documentation	115
		7.16.4.1	_rd	115
		7.16.4.2	_rng	115
7.17	qpp::int	ternal::Sin	gleton < T > Class Template Reference	115
	7.17.1	Detailed I	Description	116
	7.17.2	Construct	tor & Destructor Documentation	116
		7.17.2.1	Singleton() noexcept=default	116
		7.17.2.2	Singleton(const Singleton &)=delete	116
		7.17.2.3	~Singleton()=default	116
	7.17.3	Member I	Function Documentation	117
		7.17.3.1	$get\_instance() \ noexcept(std::is\_nothrow\_constructible < T > ::value) \ . \ . \ . \ . \ .$	117
		7.17.3.2	$get\_thread\_local\_instance() \ noexcept(std::is\_nothrow\_constructible < T > ::value)$	117

XIV

	7.17.3.3 operator=(const Singleton &)=delete
7.18 qpp::S	tates Class Reference
7.18.1	Detailed Description
7.18.2	Constructor & Destructor Documentation
	7.18.2.1 States()
	7.18.2.2 ~States()=default
7.18.3	Friends And Related Function Documentation
	7.18.3.1 internal::Singleton< const States >
7.18.4	Member Data Documentation
	7.18.4.1 b00
	7.18.4.2 b01
	7.18.4.3 b10
	7.18.4.4 b11
	7.18.4.5 GHZ
	7.18.4.6 pb00
	7.18.4.7 pb01
	7.18.4.8 pb10
	7.18.4.9 pb11
	7.18.4.10 pGHZ
	7.18.4.11 pW
	7.18.4.12 px0
	7.18.4.13 px1
	7.18.4.14 py0
	7.18.4.15 py1
	7.18.4.16 pz0
	7.18.4.17 pz1
	7.18.4.18 W
	7.18.4.19 x0
	7.18.4.20 x1
	7.18.4.21 y0
	7.18.4.22 y1
	7.18.4.23 z0
	7.18.4.24 z1
7.19 qpp::T	imer< T, CLOCK_T > Class Template Reference
7.19.1	Detailed Description
7.19.2	Constructor & Destructor Documentation
	7.19.2.1 Timer() noexcept
	7.19.2.2 Timer(const Timer &)=default
	7.19.2.3 Timer(Timer &&)=default
	7.19.2.4 ~Timer()=default

CONTENTS xv

		7.19.3	Member Function Documentation	24
			7.19.3.1 display(std::ostream &os) const override	24
			7.19.3.2 get_duration() const noexcept	25
			7.19.3.3 operator=(const Timer &)=default	25
			7.19.3.4 operator=(Timer &&)=default	25
			7.19.3.5 tic() noexcept	25
			7.19.3.6 tics() const noexcept	25
			7.19.3.7 toc() noexcept	26
		7.19.4	Member Data Documentation	26
			7.19.4.1 _end	26
			7.19.4.2 _start	26
8	Tile I	Docume	ntation 12	٦,
0				
	8.1		/codes.h File Reference	
		8.1.1	Detailed Description	
	8.2		/exception.h File Reference	
		8.2.1	Detailed Description	
	8.3		/gates.h File Reference	
		8.3.1	Detailed Description	
	8.4		/idisplay.h File Reference	
		8.4.1	Detailed Description	
	8.5		/init.h File Reference	
		8.5.1	Detailed Description	
	8.6		/random_devices.h File Reference	
		8.6.1	Detailed Description	
	8.7	classes	/states.h File Reference	
		8.7.1	Detailed Description	31
	8.8	classes	timer.h File Reference	}2
		8.8.1	Detailed Description	}2
	8.9	constar	ts.h File Reference	}2
		8.9.1	Detailed Description	33
	8.10		ement.h File Reference	33
			Detailed Description	34
	8.11	entropi	s.h File Reference	34
			Detailed Description	35
	8.12	experin	ental/experimental.h File Reference	36
		8.12.1	Detailed Description	36
	8.13	function	s.h File Reference	36
		8.13.1	Detailed Description	10
	8.14	input_c	utput.h File Reference	10

xvi CONTENTS

	8.14.1 Detailed Description	141
8.15	instruments.h File Reference	141
	8.15.1 Detailed Description	143
8.16	internal/classes/iomanip.h File Reference	143
	8.16.1 Detailed Description	144
8.17	internal/classes/singleton.h File Reference	144
	8.17.1 Detailed Description	145
8.18	internal/util.h File Reference	145
	8.18.1 Detailed Description	146
8.19	macros.h File Reference	146
	8.19.1 Detailed Description	147
	8.19.2 Macro Definition Documentation	147
	8.19.2.1 ERROR	147
	8.19.2.2 ERRORLN	147
	8.19.2.3 PRINT	147
	8.19.2.4 PRINTLN	147
8.20	MATLAB/matlab.h File Reference	147
	8.20.1 Detailed Description	148
8.21	number_theory.h File Reference	148
	8.21.1 Detailed Description	149
8.22	operations.h File Reference	149
	8.22.1 Detailed Description	151
8.23	qpp.h File Reference	151
	8.23.1 Detailed Description	153
	8.23.2 Macro Definition Documentation	153
	8.23.2.1 _QPP_UNUSED	153
8.24	random.h File Reference	153
	8.24.1 Detailed Description	154
8.25	statistics.h File Reference	154
	8.25.1 Detailed Description	155
8.26	traits.h File Reference	156
	8.26.1 Detailed Description	156
8.27	types.h File Reference	157
	8.27.1 Detailed Description	158

Index

159

## Chapter 1

## Quantum++

### Version 0.8.9 - development

Quantum++ is a C++11 general purpose quantum computing library, composed solely of template header files. Quantum++ is written in standard C++11 and has very low external dependencies, using only the Eigen 3 linear algebra header-only template library and, if available, the OpenMP multi-processing library.

Quantum++ is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes. The main design factors taken in consideration were the ease of use, high portability, and high performance.

If you are interesting in contributing, please contact me. To contribute, you need to have a good knowledge of C++ (preferably C++11), including templates and the standard library, a basic knowledge of quantum computing and linear algebra, and working experience with Eigen 3.

For additional Eigen 3 documentation see http://eigen.tuxfamily.org/dox/. For a simple Eigen 3 quick ASCII reference see http://eigen.tuxfamily.org/dox/AsciiQuickReference.txt.

Copyright (c) 2013 - 2016 Vlad Gheorghiu, vgheorgh AT gmail DOT com.

Quantum++ is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation, either version 3 of the License, or (at your option) any later version.

Quantum++ is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

You should have received a copy of the GNU General Public License along with Quantum++. If not, see http-://www.gnu.org/licenses/.

#### **Building instructions**

#### Configuration:

- Compiler: q++ version 4.8.2 or later (for good C++11 support)
- Eigen 3 library located in \$HOME/eigen
- Quantum++ library located in \$HOME/qpp
- MATLAB compiler include header files: /Applications/MATLAB\_R2016a.app/extern/include
- MATLAB compiler shared library files: /Applications/MATLAB\_R2016a.app/bin/maci64

2 Quantum++

#### Building without a build system

- Example file: \$HOME/qpp/examples/minimal.cpp
- Output executable: \$HOME/qpp/examples/minimal
- Must run the commands below from inside the directory \$HOME/qpp/examples

### Release version (without ${\tt MATLAB}$ support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

#### Debug version (without MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    minimal.cpp -o minimal
```

#### Release version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -03 -DNDEBUG -DEIGEN_NO_DEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I/Applications/MATLAB_R2016a.app/extern/include \
    -L/Applications/MATLAB_R2016a.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

#### Debug version (with MATLAB support):

```
g++ -pedantic -std=c++11 -Wall -Wextra -Weffc++ -fopenmp \
    -g3 -DDEBUG \
    -isystem $HOME/eigen -I $HOME/qpp/include \
    -I /Applications/MATLAB_R2016a.app/extern/include \
    -L /Applications/MATLAB_R2016a.app/bin/maci64 \
    -lmx -lmat minimal.cpp -o minimal
```

#### Building using cmake (version 3.0 or later)

The current version of the repository has a ./CMakeLists.txt configuration file for building examples using cmake. To build an example using cmake, I recommend an out-of-source build, i.e., from the root of the project (where ./include is located), type

```
mkdir ./build
cd ./build
cmake ..
make
```

The above commands build the relase version (default) executable qpp, from the source file ./examples/minimal.cpp, without MATLAB support (default), inside the directory ./build. To build a different configuration, e.g. debug version with MATLAB support, type from the root of the project

```
cd ./build  \begin{tabular}{ll} $\sf cd ./build \\ $\sf rm -rf * \\ $\sf cmake -DCMAKE\_BUILD\_TYPE=Debug -DWITH\_MATLAB=ON .. \\ $\sf make \\ \end{tabular}
```

#### Or, to disable OpenMP support (enabled by default), type

```
cd ./build
rm -rf *
cmake -DWITH_OPENMP=OFF ..
make
```

To change the name of the example file, the location of the Eigen 3 library or the location of MATLAB installation, edit the ./CMakeLists.txt file. See also ./CMakeLists.txt for additional options. Do not forget to remove everything from the ./build directory before a fresh build!

#### Additional remarks

- The C++ compiler must be C++11 compliant.
- If using Windows, I recommend compiling under cygwin via cmake and g++. See also http←://stackoverflow.com/questions/28997206/cygwin-support-for-c11-in-g4-9-2 for a bug related to lack of support for some C++11 math functions, and how to fix it. Quick fix: patch the standard library header file <cmath> using the provided patch./cmath\_cygwin.patch.
- If your compiler does not support OpenMP (as it is the case e.g with clang++), disable OpenMP in your build, as otherwise the linker may not find the gomp library.
- If you run the program on OS X with MATLAB support, make sure that the environment variable DYLD\_L → IBRARY\_PATH is set to point to the MATLAB compiler library location, see the run\_OSX\_MATLAB script. Otherwise, you will get a runtime error like dyld: Library not loaded: @rpath/libmat. ← dylib.

```
* I recommend running via a script, as otherwise setting the 
'DYLD_LIBRARY_PATH' globally may interfere with 
[macports] (https://www.macports.org/)' [cmake] (http://www.cmake.org/) 
installation (in case you use [cmake] (http://www.cmake.org/) from 
[macports] (https://www.macports.org/)). If you use a script, 
then the environment variable is local to the script and 
does not interfere with the rest of the system.

* Example of script, assumed to be located in the root directory 
of Quantum++:

#!/bin/sh

MATLAB=/Applications/MATLAB_R2016a.app 
export DYLD_LIBRARY_PATH=$DYLD_LIBRARY_PATH:$MATLAB/bin/maci64
./build/gpp
```

• If you build a debug version with g++ under OS X and use gdb to step inside template functions you may want to add -fno-weak compiler flag. See http://stackoverflow.← com/questions/23330641/gnu-gdb-can-not-step-into-template-functions-os-x-mavericks for more details about this problem.

Quantum++

# Chapter 2

# Namespace Index

## 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

op	
Quantum++ main namespace	13
pp::experimental	
Experimental/test functions/classes, do not use or modify	30
pp::internal	
Internal utility functions, do not use/modify	30
pp::internal:: details	33

6 Namespace Index

# **Chapter 3**

# **Hierarchical Index**

## 3.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:
qpp::internal::_details::_Display_Impl
qpp::internal::IOManipEigen
std::exception
qpp::Exception
false_type
qpp::is_complex< T >
qpp::is_iterable < T, typename >
qpp::is_matrix_expression< Derived >
qpp::IDisplay
qpp::internal::IOManipEigen
qpp::internal::IOManipPointer< PointerType >
qpp::internal::IOManipRange< InputIterator >
qpp::Timer < T, CLOCK_T >
qpp::internal::Singleton < T >
qpp::internal::Singleton < const Codes >
qpp::Codes
qpp::internal::Singleton < const Gates >
qpp::Gates
qpp::internal::Singleton < const Init >
qpp::Init
qpp::internal::Singleton < const States >
gpp::States
qpp::internal::Singleton < RandomDevices >
qpp::RandomDevices
" '
true_type  qpp::is complex < std::complex < T >>
qpp:.is_complex< std.:complex< r >>
appis_iterable<
gnn: is matrix expression< typename Figen: MatrixBase< Derived >>

8 **Hierarchical Index** 

# **Chapter 4**

# **Class Index**

## 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:	
<pre>qpp::internal::_details::_Display_Impl</pre>	85
Const Singleton class that defines quantum error correcting codes	85
qpp::Exception	
Generates custom exceptions, used when validating function parameters	88
qpp::Gates	
Const Singleton class that implements most commonly used gates	91
qpp::IDisplay	
Abstract class (interface) that mandates the definition of virtual std::ostream& display(std←	
::ostream& os) const	97
qpp::Init	
Const Singleton class that performs additional initializations/cleanups	
qpp::internal::IOManipEigen	
qpp::internal::IOManipPointer< PointerType >	
qpp::internal::IOManipRange< InputIterator >	105
qpp::is_complex < T >	107
Checks whether the type is a complex type	107
<pre>qpp::is_complex&lt; std::complex&lt; T &gt;&gt;     Checks whether the type is a complex number type, specialization for complex types</pre>	100
qpp::is iterable < T, typename >	100
Checks whether <i>T</i> is compatible with an STL-like iterable container	100
qpp::is_iterable < T, to_void < decltype(std::declval < T >().begin()), decltype(std::declval < T >().end()),	100
typename T::value type >>	
Checks whether <i>T</i> is compatible with an STL-like iterable container, specialization for STL-like	
iterable containers	110
qpp::is matrix expression< Derived >	
Checks whether the type is an Eigen matrix expression	111
qpp::is_matrix_expression< typename Eigen::MatrixBase< Derived > >	
Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expres-	
sions	112
qpp::RandomDevices	
Singeleton class that manages the source of randomness in the library	113
qpp::internal::Singleton< T >	
Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously	
recurring template pattern)	115
qpp::States	
Const Singleton class that implements most commonly used states	117

10 Class Index

<pre>qpp::Timer&lt; T, CLOCK_</pre>	_T >	>																		
Chronometer																			1	21

# **Chapter 5**

# File Index

## 5.1 File List

Here is a list of all files with brief descriptions:

constants.h	
Constants	132
entanglement.h	
Entanglement functions	130
entropies.h	
Entropy functions	134
functions.h	
Generic quantum computing functions	136
input_output.h	
Input/output functions	140
instruments.h	
Measurement functions	14 <sup>-</sup>
macros.h	
Preprocessor macros	140
number_theory.h	4.4
Number theory functions	148
operations.h  Quantum operation functions	
·	
qpp.h Quantum++ main header file, includes all other necessary headers	
random.h	
Randomness-related functions	
statistics.h	
Statistics functions	
traits.h	
Type traits	150
types.h	
Type aliases	
classes/codes.h	
Quantum error correcting codes	
classes/exception.h	
Exceptions	
classes/gates.h	
Quantum gates	128
classes/idisplay.h	
Display interface via the non-virtual interface (NVI)	129
classes/init.h	
Initialization	130

12 File Index

lasses/random_devices.h	
Random devices	130
lasses/states.h	
Quantum states	131
lasses/timer.h	
Timing	132
xperimental/experimental.h	
Experimental/test functions/classes	136
nternal/util.h	
Internal utility functions	145
nternal/classes/iomanip.h	
Input/output manipulators	143
nternal/classes/singleton.h	
Singleton pattern via CRTP	144
IATLAB/matlab.h	
Input/output interfacing with MATLAB	147

## **Chapter 6**

## **Namespace Documentation**

### 6.1 qpp Namespace Reference

Quantum++ main namespace.

### **Namespaces**

· experimental

Experimental/test functions/classes, do not use or modify.

internal

Internal utility functions, do not use/modify.

### Classes

class Codes

const Singleton class that defines quantum error correcting codes

class Exception

Generates custom exceptions, used when validating function parameters.

• class Gates

const Singleton class that implements most commonly used gates

class IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

class Init

const Singleton class that performs additional initializations/cleanups

· struct is\_complex

Checks whether the type is a complex type.

struct is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

• struct is\_iterable

Checks whether T is compatible with an STL-like iterable container.

struct is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().end()), typename T::value\_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

· struct is matrix expression

Checks whether the type is an Eigen matrix expression.

struct is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

· class RandomDevices

Singeleton class that manages the source of randomness in the library.

· class States

const Singleton class that implements most commonly used states

· class Timer

Chronometer.

### **Typedefs**

```
template<typename... > 
using to_void = void
```

Alias template that implements the proposal for void\_t.

• using idx = std::size\_t

Non-negative integer index.

• using bigint = long long int

Big integer.

using cplx = std::complex < double >

Complex number in double precision.

• using ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

• using bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

template<typename Scalar >

```
using dyn_mat = Eigen::Matrix < Scalar, Eigen::Dynamic, Eigen::Dynamic >
```

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

```
using dyn_col_vect = Eigen::Matrix< Scalar, Eigen::Dynamic, 1 >
```

Dynamic Eigen column vector over the field specified by Scalar.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$ 

```
using dyn row vect = Eigen::Matrix < Scalar, 1, Eigen::Dynamic >
```

Dynamic Eigen row vector over the field specified by Scalar.

### **Functions**

constexpr cplx operator""\_i (unsigned long long int x) noexcept

User-defined literal for complex  $i=\sqrt{-1}$  (integer overload)

• constexpr cplx operator""\_i (long double x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

• cplx omega (idx D)

D-th root of unity.

template<typename Derived >

dyn\_col\_vect< double > schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
&dims)

Schmidt coefficients of the bi-partite pure state A.

template<typename Derived >

```
cmat schmidtA (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
```

Schmidt basis on Alice side.

```
• template<typename Derived >
  cmat schmidtB (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Schmidt basis on Bob side.

    template<typename Derived >

  std::vector< double > schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx >
  &dims)
     Schmidt probabilities of the bi-partite pure state A.
• template<typename Derived >
  double entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Entanglement of the bi-partite pure state A.

    template<typename Derived >

  double gconcurrence (const Eigen::MatrixBase< Derived > &A)
     G-concurrence of the bi-partite pure state A.
template<typename Derived >
  double negativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Negativity of the bi-partite mixed state A.

    template<typename Derived >

  double lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
     Logarithmic negativity of the bi-partite mixed state A.

    template<typename Derived >

  double concurrence (const Eigen::MatrixBase< Derived > &A)
      Wootters concurrence of the bi-partite qubit mixed state A.

    template<typename Derived >

  double entropy (const Eigen::MatrixBase< Derived > &A)
      von-Neumann entropy of the density matrix A

    double entropy (const std::vector< double > &prob)

     Shannon entropy of the probability distribution prob.
• template<typename Derived >
  double renyi (const Eigen::MatrixBase< Derived > &A, double alpha)
      Renyi- \alpha entropy of the density matrix A, for \alpha \geq 0.

    double renyi (const std::vector< double > &prob, double alpha)

     Renyi- \alpha entropy of the probability distribution prob, for \alpha \geq 0.

    template<typename Derived >

  double tsallis (const Eigen::MatrixBase< Derived > &A, double q)
      Tsallis- q entropy of the density matrix A, for q \geq 0.

    double tsallis (const std::vector< double > &prob, double q)

      Tsallis- q entropy of the probability distribution prob, for q \geq 0.

    template<typename Derived >

  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector< idx > &subsysB, const std::vector< idx > &dims)
      Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  double qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const
  std::vector < idx > &subsysB, idx d=2)
     Quantum mutual information between 2 subsystems of a composite system.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > transpose (const Eigen::MatrixBase< Derived > &A)
      Transpose.
• template<typename Derived >
  dyn mat< typename Derived::Scalar > conjugate (const Eigen::MatrixBase< Derived > &A)
      Complex conjugate.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > adjoint (const Eigen::MatrixBase< Derived > &A)
```

```
Adjoint.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > inverse (const Eigen::MatrixBase< Derived > &A)
     Inverse.

    template<typename Derived >

  Derived::Scalar trace (const Eigen::MatrixBase< Derived > &A)
      Trace.

    template<typename Derived >

  Derived::Scalar det (const Eigen::MatrixBase< Derived > &A)
     Determinant.

    template<typename Derived >

  Derived::Scalar logdet (const Eigen::MatrixBase< Derived > &A)
     Logarithm of the determinant.
template<typename Derived >
  Derived::Scalar sum (const Eigen::MatrixBase< Derived > &A)
     Element-wise sum of A.

    template<typename Derived >

  Derived::Scalar prod (const Eigen::MatrixBase< Derived > &A)
     Element-wise product of A.

    template<typename Derived >

  double norm (const Eigen::MatrixBase< Derived > &A)
     Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > eig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition.
 template<typename Derived >
  dyn_col_vect< cplx > evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.
• template<typename Derived >
  cmat evects (const Eigen::MatrixBase< Derived > &A)
     Eigenvectors.
• template<typename Derived >
  std::pair< dyn_col_vect< double >, cmat > heig (const Eigen::MatrixBase< Derived > &A)
     Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > hevals (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvalues.

    template<typename Derived >

  cmat hevects (const Eigen::MatrixBase< Derived > &A)
     Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn_col_vect< double >, cmat > svd (const Eigen::MatrixBase< Derived > &A)
     Full singular value decomposition.

    template<typename Derived >

  dyn_col_vect< double > svals (const Eigen::MatrixBase< Derived > &A)
     Singular values.

    template<typename Derived >

  cmat svdU (const Eigen::MatrixBase< Derived > &A)
     Left singular vectors.
• template<typename Derived >
```

cmat svdV (const Eigen::MatrixBase< Derived > &A)

Right singular vectors.

```
• template<typename Derived >
  cmat funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
     Functional calculus f(A)
• template<typename Derived >
  cmat sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.
template<typename Derived >
  cmat absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.

    template<typename Derived >

  cmat expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.
• template<typename Derived >
  cmat logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.

    template<typename Derived >

  double schatten (const Eigen::MatrixBase< Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn mat< OutputScalar > cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)( const type-
  name Derived::Scalar &))
     Functor.
• template<typename T >
  dyn_mat< typename T::Scalar > kron (const T &head)
     Kronecker product.

    template<typename T, typename... Args>

  dyn_mat< typename T::Scalar > kron (const T &head, const Args &...tail)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > kron (const std::initializer_list< Derived > &As)
     Kronecker product.
template<typename Derived >
  dyn_mat< typename Derived::Scalar > kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.

    template<typename T >

  dyn_mat< typename T::Scalar > dirsum (const T &head)
     Direct sum.
```

```
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > dirsum (const T &head, const Args &...tail)
     Direct sum.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > dirsum (const std::vector< Derived > &As)

    template<typename Derived >

  dyn mat< typename Derived::Scalar > dirsum (const std::initializer list< Derived > &As)
     Direct sum.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > reshape (const Eigen::MatrixBase< Derived > &A, idx rows, idx
  cols)
     Reshape.

    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > comm (const Eigen::MatrixBase< Derived1 > &A, const Eigen::⊷
  MatrixBase< Derived2 > &B)
     Commutator.

    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > prj (const Eigen::MatrixBase< Derived > &V)
     Projector.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > grams (const std::initializer_list< Derived > &Vs)
      Gram-Schmidt orthogonalization.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.

    std::vector< idx > n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

     Multi-index to non-negative integer index.

    ket mket (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Multi-partite qudit ket.

    ket mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat mprj (const std::vector < idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
  std::vector< double > abssq (InputIterator first, InputIterator last)
```

Computes the absolute values squared of an STL-like range of complex numbers.

template<typename Container >

std::vector< double > abssq (const Container &c, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Computes the absolute values squared of an STL-like container.

template<typename Derived >

```
std::vector< double > abssq (const Eigen::MatrixBase< Derived > &A)
```

Computes the absolute values squared of an Eigen expression.

• template<typename InputIterator >

```
std::iterator traits< InputIterator >::value type sum (InputIterator first, InputIterator last)
```

Element-wise sum of an STL-like range.

template<typename Container >

Container::value\_type sum (const Container &c, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Element-wise sum of the elements of an STL-like container.

template<typename InputIterator >

```
std::iterator traits < InputIterator >::value type prod (InputIterator first, InputIterator last)
```

Element-wise product of an STL-like range.

• template<typename Container >

Container::value\_type prod (const Container &c, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Element-wise product of the elements of an STL-like container.

• template<typename Derived >

```
dyn_col_vect< typename Derived::Scalar > rho2pure (const Eigen::MatrixBase< Derived > &A)
```

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

template<typename T >

```
std::vector< T > complement (std::vector< T > subsys, idx N)
```

Constructs the complement of a subsystem vector.

template<typename Derived >

```
std::vector< double > rho2bloch (const Eigen::MatrixBase< Derived > &A)
```

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

template<typename Derived >

```
internal::IOManipEigen disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
```

Eigen expression ostream manipulator.

internal::IOManipEigen disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

 $\bullet \ \ \text{template}{<} \text{typename InputIterator} >$ 

internal::IOManipRange < InputIterator > disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >

internal::IOManipRange< typename Container::const\_iterator > disp (const Container &c, const std::string &separator, const std::string &start="[", const std::string &end="]", typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

• template<typename PointerType >

internal::IOManipPointer< PointerType > disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

ullet template<typename Derived >

```
\mbox{void save (const Eigen::MatrixBase} < \mbox{Derived} > \&\mbox{A, const std::string \&fname)}
```

Saves Eigen expression to a binary file (internal format) in double precision.

• template<typename Derived >

dyn mat< typename Derived::Scalar > load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::← MatrixBase< Derived > &psi, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Generalized inner product.

• template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::← MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple < idx, std::vector < double >, std::vector < cmat > > measure (const Eigen::MatrixBase < Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const std::initializer list< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > measure (const Eigen::MatrixBase< Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

• template<typename Derived >

std::tuple< std::vector< idx >, double, cmat > measure\_seq (const Eigen::MatrixBase< Derived > &A, std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >

Derived loadMATLABmatrix (const std::string &, const std::string &)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat loadMATLABmatrix (const std::string &mat file, const std::string &var name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

• template<>

cmat loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

• template<typename Derived >

void saveMATLABmatrix (const Eigen::MatrixBase< Derived > &, const std::string &, const std::string &, const std::string &)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

• template<>

void saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<</li>

void saveMATLABmatrix (const Eigen::MatrixBase < cmat > &A, const std::string &mat\_file, const std::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

std::vector< int > x2contfrac (double x, idx n, idx cut=1e5)

Simple continued fraction expansion.

• double contfrac2x (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• bigint gcd (bigint m, bigint n)

Greatest common divisor of two integers.

bigint gcd (const std::vector< bigint > &ns)

Greatest common divisor of a list of integers.

bigint lcm (bigint m, bigint n)

Least common multiple of two integers.

bigint lcm (const std::vector< bigint > &ns)

Least common multiple of a list of integers.

std::vector< idx > invperm (const std::vector< idx > &perm)

Inverse permutation.

std::vector< idx > compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)

Compose permutations.

std::vector< bigint > factors (bigint n)

Prime factor decomposition.

• bigint modmul (bigint m, bigint n, bigint p)

Modular multiplication without overflow.

bigint modpow (bigint a, bigint n, bigint p)

Fast integer power modulo p based on the SQUARE-AND-MULTIPLY algorithm.

std::tuple < bigint, bigint, bigint > egcd (bigint m, bigint n)

Extended greatest common divisor of two integers.

bigint modinv (bigint a, bigint p)

Modular inverse of a mod p.

• bool isprime (bigint n, idx k=80)

Primality test based on the Miller-Rabin's algorithm.

bigint randprime (bigint a, bigint b, idx N=1000)

Generates a random big prime uniformly distributed in the interval [a, b].

template < typename Derived1, typename Derived2 >

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > applyCTRL (const Eigen::MatrixBase< Derived1 > &state, const
Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys, idx
d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

• template<typename Derived1 , typename Derived2 >

```
dyn_mat< typename Derived1::Scalar > apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen ← ::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)
```

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived</li>

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)
```

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

• template<typename Derived >

```
cmat apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

• template<typename Derived >

```
\label{lem:const} \begin{array}{l} \text{cmat apply (const Eigen::} MatrixBase < Derived > \& rho, const std::vector < cmat > \& Ks, const std::vector < idx > \& subsys, idx d=2) \end{array}
```

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho

cmat kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

• template<typename Derived >

```
dyn_mat< typename Derived::Scalar > ptrace1 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector < idx > &dims)
```

Partial trace.

• template<typename Derived >

```
\frac{dyn\_mat<}{typename} \ Derived::Scalar > ptrace2 \ (const \ Eigen::MatrixBase< \ Derived > \&A, \ const \ std \\ \because vector < idx > \&dims)
```

Partial trace.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector<
idx > &subsys, const std::vector< idx > &dims)

Partial trace.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptrace (const Eigen::MatrixBase< Derived > &A, const std::vector
idx > &subsys, idx d=2)

Partial trace.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std↔ ::vector< idx > &subsys, const std::vector< idx > &dims)

Partial transpose.

• template<typename Derived >

dyn\_mat< typename Derived::Scalar > ptranspose (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &subsys, idx d=2)

Partial transpose.

template<typename Derived >

Subsystem permutation.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > syspermute (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &perm, idx d=2)

Subsystem permutation.

· double rand (double a, double b)

Generates a random real number uniformly distributed in the interval [a, b)

• bigint rand (bigint a, bigint b)

Generates a random big integer uniformly distributed in the interval [a, b].

• idx randidx (idx a=std::numeric\_limits < idx >::min(), idx b=std::numeric\_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

template<typename Derived >

Derived rand (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>

dmat rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

• template<typename Derived >

Derived randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

template<>

cmat randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double randn (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

· cmat randU (idx D)

Generates a random unitary matrix.

cmat randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > randkraus (idx N, idx D)

Generates a set of random Kraus operators.

· cmat randH (idx D)

Generates a random Hermitian matrix.

ket randket (idx D)

Generates a random normalized ket (pure state vector)

cmat randrho (idx D)

Generates a random density matrix.

std::vector< idx > randperm (idx n)

Generates a random uniformly distributed permutation.

std::vector< double > uniform (idx N)

Uniform probability distribution vector.

std::vector< double > marginalX (const dmat &probXY)

Marginal distribution.

std::vector< double > marginalY (const dmat &probXY)

Marginal distribution.

• template<typename Container >

double avg (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Average.

• template<typename Container >

double cov (const dmat &probXY, const Container &X, const Container &Y, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Covariance.

template<typename Container >

double var (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Variance.

• template<typename Container >

double sigma (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Standard deviation.

• template<typename Container >

double cor (const dmat &probXY, const Container &X, const Container &Y, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Correlation.

## **Variables**

• constexpr double chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

constexpr idx maxn = 64

Maximum number of allowed qu(d)its (subsystems)

constexpr double pi = 3.141592653589793238462643383279502884

 $\pi$ 

constexpr double ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

constexpr double infty = std::numeric\_limits < double > ::infinity()
 Used to denote infinity in double precision.

## 6.1.1 Detailed Description

Quantum++ main namespace.

## 6.1.2 Typedef Documentation

6.1.2.1 using qpp::bigint = typedef long long int

Big integer.

6.1.2.2 using qpp::bra = typedef Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

6.1.2.3 using qpp::cmat = typedef Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

6.1.2.4 using qpp::cplx = typedef std::complex < double >

Complex number in double precision.

6.1.2.5 using qpp::dmat = typedef Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

6.1.2.6 template<typename Scalar > using qpp::dyn\_col\_vect = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, 1>

Dynamic Eigen column vector over the field specified by Scalar.

Example:

```
// type of colvect is Eigen::Matrix<float, Eigen::Dynamic, 1>
auto colvect = dyn_col_vect<float>(2);
```

6.1.2.7 template<typename Scalar > using qpp::dyn\_mat = typedef Eigen::Matrix<Scalar, Eigen::Dynamic, Eigen::Dynamic>

Dynamic Eigen matrix over the field specified by Scalar.

Example:

```
// type of mat is Eigen::Matrix<float, Eigen::Dynamic, Eigen::Dynamic>
auto mat = dyn_mat<float>(2,3);
```

6.1.2.8 template < typename Scalar > using qpp::dyn\_row\_vect = typedef Eigen::Matrix < Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

## Example:

```
// type of rowvect is Eigen::Matrix<float, 1, Eigen::Dynamic>
auto rowvect = dyn_row_vect<float>(3);
```

6.1.2.9 using qpp::idx = typedef std::size\_t

Non-negative integer index.

6.1.2.10 using qpp::ket = typedef Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

6.1.2.11 template < typename... > using qpp::to\_void = typedef void

Alias template that implements the proposal for void\_t.

See also

```
http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2014/n3911
```

## 6.1.3 Function Documentation

6.1.3.1 template<typename Derived > cmat qpp::absm ( const Eigen::MatrixBase< Derived > & A )

Matrix absolut value.

**Parameters** 

Α	Eigen expression
---	------------------

### Returns

Matrix absolut value of A

6.1.3.2 template<typename InputIterator > std::vector<double> qpp::abssq ( InputIterator first, InputIterator last )

Computes the absolute values squared of an STL-like range of complex numbers.

## **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

### Returns

Real vector consisting of the range absolut values squared

6.1.3.3 template<typename Container > std::vector<double> qpp::abssq ( const Container & c, typename std::enable\_if< is iterable< Container >::value >::type \* = nullptr )

Computes the absolute values squared of an STL-like container.

С	STL-like container
---	--------------------

### Returns

Real vector consisting of the container's absolut values squared

6.1.3.4 template < typename Derived > std::vector < double > qpp::abssq ( const Eigen::MatrixBase < Derived > & A )

Computes the absolute values squared of an Eigen expression.

**Parameters** 

Α	Eigen expression
---	------------------

### Returns

Real vector consisting of the absolut values squared

6.1.3.5 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::adjoint ( const Eigen::MatrixBase< Derived > & A )

## Adjoint.

### **Parameters**

Α	Eigen expression

### Returns

Adjoint (Hermitian conjugate) of A, as a dynamic matrix over the same scalar field as A

6.1.3.6 template<typename Derived1 , typename Derived2 >  $dyn_mat$ <typename Derived1::Scalar>  $dyn_mat$ <typename Derived1

Anti-commutator.

## See also

qpp::comm()

Anti-commutator  $\{A,B\} = AB + BA$ . Both A and B must be Eigen expressions over the same scalar field.

# **Parameters**

Α	Eigen expression
В	Eigen expression

## Returns

Anti-commutator AB + BA, as a dynamic matrix over the same scalar field as A

6.1.3.7 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > dpp::apply ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims )$ 

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

### Returns

Gate A applied to the part subsys of state

6.1.3.8 template < typename Derived1 , typename Derived2 >  $dyn_mat$  < typename Derived1::Scalar > dx < Eigen::MatrixBase < Derived1 > dx < tools template < dx < tools

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

Note

The dimension of the gate A must match the dimension of subsys

## **Parameters**

state	Eigen expression
Α	Eigen expression
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

### Returns

Gate A applied to the part subsys of state

6.1.3.9 template<typename Derived > cmat qpp::apply ( const Eigen::MatrixBase< Derived > & rho, const std::vector< cmat > & Ks )

Applies the channel specified by the set of Kraus operators *Ks* to the density matrix *rho*.

## **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators

## Returns

Output density matrix after the action of the channel

6.1.3.10 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Applies the channel specified by the set of Kraus operators *Ks* to the part *subsys* of the multi-partite density matrix *rho*.

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
dims	Dimensions of the multi-partite system

## Returns

Output density matrix after the action of the channel

6.1.3.11 template < typename Derived > cmat qpp::apply ( const Eigen::MatrixBase < Derived > & rho, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho

### **Parameters**

rho	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes where the Kraus operators Ks are applied
d	Subsystem dimensions

### Returns

Output density matrix after the action of the channel

6.1.3.12 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::applyCTRL ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

## See also

qpp::Gates::CTRL()

## Note

The dimension of the gate *A* must match the dimension of *subsys*. Also, all control subsystems in *ctrl* must have the same dimension.

## **Parameters**

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
dims	Dimensions of the multi-partite system

## Returns

CTRL-A gate applied to the part subsys of state

6.1.3.13 template < typename Derived1 , typename Derived2 >  $dyn_mat < typename Derived1::Scalar > dpp::applyCTRL ( const Eigen::MatrixBase < Derived1 > & state, const Eigen::MatrixBase < Derived2 > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, idx d = 2 )$ 

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

### See also

qpp::Gates::CTRL()

### Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

state	Eigen expression
Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
d	Subsystem dimensions

### Returns

CTRL-A gate applied to the part subsys of state

6.1.3.14 template<typename Container > double qpp::avg ( const std::vector< double > & prob, const Container & X, typename std::enable\_if< is\_iterable< Container >::value >::type \* = nullptr )

## Average.

## Parameters

proi	Real probability vector representing the probability distribution of X
	Random variable values represented by an STL-like container

## Returns

Average of X

6.1.3.15 cmat qpp::bloch2rho ( const std::vector< double > & r ) [inline]

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

### See also

qpp::rho2bloch()

## **Parameters**

r	3-dimensional real vector

## Returns

Qubit density matrix

6.1.3.16 std::vector<cmat> qpp::choi2kraus ( const cmat & A ) [inline]

Orthogonal Kraus operators from Choi matrix.

See also

qpp::kraus2choi()

Extracts a set of orthogonal (under Hilbert-Schmidt operator norm) Kraus operators from the Choi matrix A

Note

The Kraus operators satisfy  $Tr(K_i^{\dagger}K_j) = \delta_{ij}$  for all  $i \neq j$ 

### **Parameters**

A Cho	choi matrix

Returns

Set of orthogonal Kraus operators

6.1.3.17 cmat qpp::choi2super ( const cmat & A ) [inline]

Converts Choi matrix to superoperator matrix.

See also

qpp::super2choi()

### **Parameters**

Α	Choi matrix
---	-------------

Returns

Superoperator matrix

6.1.3.18 template < typename Derived1 , typename Derived2 > dyn\_mat < typename Derived1::Scalar > qpp::comm ( const Eigen::MatrixBase < Derived1 > & A, const Eigen::MatrixBase < Derived2 > & B)

Commutator.

See also

qpp::anticomm()

Commutator [A, B] = AB - BA. Both A and B must be Eigen expressions over the same scalar field.

## **Parameters**

A	Eigen expression
В	Eigen expression

### Returns

Commutator AB - BA, as a dynamic matrix over the same scalar field as A

6.1.3.19 template < typename T > std::vector < T > qpp::complement ( std::vector < T > subsys, idx N )

Constructs the complement of a subsystem vector.

subsys	Subsystem vector
N	Total number of systems

## Returns

The complement of *subsys* with respect to the set  $\{0, 1, \dots, N-1\}$ 

6.1.3.20 std::vector < idx > & perm, const std::vector < idx > & sigma ) [inline]

Compose permutations.

## **Parameters**

perm	Permutation
sigma	Permutation

### Returns

Composition of the permutations  $perm \circ sigma = perm(sigma)$ 

6.1.3.21 template < typename Derived > double qpp::concurrence ( const Eigen::MatrixBase < Derived > & A )

Wootters concurrence of the bi-partite qubit mixed state A.

### **Parameters**

A   Eigen expression	
----------------------	--

## Returns

Wootters concurrence

6.1.3.22 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::conjugate ( const Eigen::MatrixBase< Derived > & A )

Complex conjugate.

**Parameters** 

Α	Eigen expression

## Returns

Complex conjugate of A, as a dynamic matrix over the same scalar field as A

6.1.3.23 double qpp::contfrac2x ( const std::vector < int > & cf, idx n ) [inline]

Real representation of a simple continued fraction.

## See also

qpp::x2contfrac()

cf	Integer vector containing the simple continued fraction expansion
n	Number of terms considered in the continued fraction expansion. If <i>n</i> is greater than the size
	of <i>cf</i> ,then all terms in <i>cf</i> are considered.

## Returns

Real representation of the simple continued fraction

6.1.3.24 double qpp::contfrac2x ( const std::vector < int > & cf ) [inline]

Real representation of a simple continued fraction.

See also

qpp::x2contfrac()

### **Parameters**

cr   Integer vector containing the simple continued traction expansion
--

### Returns

Real representation of the simple continued fraction

6.1.3.25 template<typename Container > double qpp::cor ( const dmat & probXY, const Container & X, const Container & Y, typename std::enable\_if< is\_iterable< Container >::type \* = nullptr )

## Correlation.

## **Parameters**

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)
X	Random variable values represented by an STL-like container
Y	Random variable values represented by an STL-like container

## Returns

Correlation of X and Y

6.1.3.26 template < typename Derived > cmat qpp::cosm ( const Eigen::MatrixBase < Derived > & A )

Matrix cos.

**Parameters** 

Α	Eigen expression

## Returns

Matrix cosine of A

6.1.3.27 template<typename Container > double qpp::cov ( const dmat & probXY, const Container & X, const Container & Y, typename std::enable\_if< is\_iterable< Container >::value >::type \* = nullptr )

## Covariance.

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)
X	Random variable values represented by an STL-like container
Y	Random variable values represented by an STL-like container

### Returns

Covariance of X and Y

6.1.3.28 template<typename OutputScalar , typename Derived > dyn\_mat<OutputScalar> qpp::cwise ( const Eigen::MatrixBase< Derived > & A, OutputScalar(\*)( const typename Derived::Scalar &) f )

### Functor.

## **Parameters**

Α	Eigen expression
f	Pointer-to-function from scalars of A to OutputScalar

### Returns

Component-wise f(A), as a dynamic matrix over the *OutputScalar* scalar field

6.1.3.29 template<typename Derived > Derived::Scalar qpp::det ( const Eigen::MatrixBase< Derived > & A )

## Determinant.

## **Parameters**

Α	Eigen expression

## Returns

Determinant of A, as a scalar over the same scalar field as A. Returns  $\pm \infty$  when the determinant overflows/underflows.

 $6.1.3.30 \quad template < typename \ T > dyn\_mat < typename \ T :: Scalar > qpp:: dirsum \ ( \ const \ T \ \& \ head \ )$ 

Direct sum.

See also

qpp::dirsumpow()

Used to stop the recursion for the variadic template version of qpp::dirsum()

### **Parameters**

head	Eigen expression

## Returns

Its argument head

6.1.3.31 template < typename T, typename... Args > dyn\_mat < typename T::Scalar > qpp::dirsum ( const T & head, const Args &... tail )

Direct sum.

See also

qpp::dirsumpow()

### **Parameters**

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

### Returns

Direct sum of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.32 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::dirsum ( const std::vector< Derived > & As )

Direct sum.

See also

qpp::dirsumpow()

### **Parameters**

As std::vector of Eigen expressions
-------------------------------------

## Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.33 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::dirsum ( const std::initializer\_list < Derived > & As )

Direct sum.

See also

qpp::dirsumpow()

## **Parameters**

As	std::initializer list of Eigen expressions, such as {A1, A2,, Ak}

### Returns

Direct sum of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.34 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::dirsumpow ( const Eigen::MatrixBase< Derived > & A, idx n )

Direct sum power.

See also

qpp::dirsum()

### **Parameters**

Α	Eigen expression
n	Non-negative integer

## Returns

Direct sum of A with itself n times  $A^{\oplus n}$ , as a dynamic matrix over the same scalar field as A

6.1.3.35 template<typename Derived > internal::IOManipEigen qpp::disp ( const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop )

Eigen expression ostream manipulator.

### **Parameters**

Α	Eigen expression
chop	Set to zero the elements smaller in absolute value than <i>chop</i>

## Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.36 internal::IOManipEigen qpp::disp(cplx z, double chop = qpp::chop) [inline]

Complex number ostream manipulator.

### **Parameters**

Z	Complex number (or any other type implicitly cast-able to std::complex <double>)</double>
chop	Set to zero the elements smaller in absolute value than chop

### Returns

Instance of qpp::internal::internal::IOManipEigen

6.1.3.37 template < typename InputIterator > internal::IOManipRange < InputIterator > qpp::disp ( InputIterator first, InputIterator last, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

Range ostream manipulator.

**Parameters** 

first	Iterator to the first element of the range
last	Iterator to the last element of the range
separator	Separator
start	Left marking
end	Right marking

## Returns

Instance of qpp::internal::internal::IOManipRange

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

#### **Parameters**

С	Container
separator	Separator
start	Left marking
end	Right marking

### Returns

Instance of qpp::internal::internal::IOManipRange

6.1.3.39 template<typename PointerType > internal::IOManipPointer<PointerType> qpp::disp ( const PointerType \* p, idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " )

C-style pointer ostream manipulator.

## **Parameters**

р	Pointer to the first element
n	Number of elements to be displayed
separator	Separator
start	Left marking
end	Right marking

### Returns

Instance of qpp::internal::iOManipPointer

6.1.3.40 std::tuple<br/>bigint, bigint> qpp::egcd ( bigint m, bigint n ) [inline]

Extended greatest common divisor of two integers.

### See also

qpp::gcd()

m	Integer
n	Integer

## Returns

Tuple of: 1. Integer a, 2. Integer b, and 3. Non-negative integer gcd(a,b) such that am + bn = gcd(a,b)

6.1.3.41 template<typename Derived > std::pair<dyn\_col\_vect < cplx>, cmat> qpp::eig ( const Eigen::MatrixBase< Derived > & A )

Full eigen decomposition.

See also

qpp::heig()

### **Parameters**

Α	Eigen expression

### Returns

Pair of: 1. Eigenvalues of *A*, as a complex dynamic column vector, and 2. Eigenvectors of *A*, as columns of a complex dynamic matrix

6.1.3.42 template < typename Derived > double qpp::entanglement ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Entanglement of the bi-partite pure state A.

Defined as the von-Neumann entropy of the reduced density matrix of one of the subsystems

See also

qpp::entropy()

### **Parameters**

Γ	Α	Eigen expression
	dims	Dimensions of the bi-partite system

## Returns

Entanglement, with the logarithm in base 2

6.1.3.43 template < typename Derived > double qpp::entropy ( const Eigen::MatrixBase < Derived > & A )

von-Neumann entropy of the density matrix A

A Eigen expression

Returns

von-Neumann entropy, with the logarithm in base 2

6.1.3.44 double qpp::entropy ( const std::vector < double > & prob ) [inline]

Shannon entropy of the probability distribution prob.

**Parameters** 

prob Real probability vector

Returns

Shannon entropy, with the logarithm in base 2

6.1.3.45 template<typename Derived > dyn\_col\_vect<cplx> qpp::evals ( const Eigen::MatrixBase< Derived > & A )

Eigenvalues.

See also

qpp::hevals()

**Parameters** 

A Eigen expression

Returns

Eigenvalues of A, as a complex dynamic column vector

6.1.3.46 template < typename Derived > cmat qpp::evects ( const Eigen::MatrixBase < Derived > & A )

Eigenvectors.

See also

qpp::hevects()

**Parameters** 

A | Eigen expression

Returns

Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.47 template<typename Derived > cmat qpp::expm ( const Eigen::MatrixBase< Derived > & A )

Matrix exponential.

Α	Eigen expression
---	------------------

Returns

Matrix exponential of A

**6.1.3.48** std::vector<br/>bigint> qpp::factors(bigint n) [inline]

Prime factor decomposition.

Note

Runs in  $\mathcal{O}(\sqrt{n})$  time complexity

### **Parameters**

n	Integer different from 0, 1 or -1

## Returns

Integer vector containing the factors

6.1.3.49 template < typename Derived > cmat qpp::funm ( const Eigen::MatrixBase < Derived > & A, cplx(\*)(const cplx &) f

Functional calculus f(A)

# **Parameters**

Α	Eigen expression
f	Pointer-to-function from complex to complex

## Returns

f(A)

6.1.3.50 bigint qpp::gcd (bigint m, bigint n) [inline]

Greatest common divisor of two integers.

See also

qpp::lcm()

## **Parameters**

m	Integer
n	Integer

## Returns

Greatest common divisor of *m* and *n* 

6.1.3.51 bigint qpp::gcd ( const std::vector < bigint > & ns ) [inline]

Greatest common divisor of a list of integers.

See also

qpp::lcm()

### **Parameters**

ns	List of integers

## Returns

Greatest common divisor of all numbers in ns

6.1.3.52 template < typename Derived > double qpp::gconcurrence ( const Eigen::MatrixBase < Derived > & A )

G-concurrence of the bi-partite pure state A.

Note

Both local dimensions must be equal

Uses qpp::logdet() to avoid overflows

See also

qpp::logdet()

## **Parameters**

## Returns

G-concurrence

6.1.3.53 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const std::vector< Derived > & Vs )

Gram-Schmidt orthogonalization.

## **Parameters**

Vs	std::vector of Eigen expressions as column vectors
----	--

### Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.54 template < typename Derived >  $dyn_mat$  < typename Derived::Scalar > qpp::grams ( const std::initializer\_list < Derived > & Vs)

Gram-Schmidt orthogonalization.

Vs std::initializer\_list of Eigen expressions as column vectors

Returns

Gram-Schmidt vectors of Vs as columns of a dynamic matrix over the same scalar field as its arguments

6.1.3.55 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::grams ( const Eigen::MatrixBase< Derived > & A )

Gram-Schmidt orthogonalization.

**Parameters** 

A | Eigen expression, the input vectors are the columns of A

Returns

Gram-Schmidt vectors of the columns of A, as columns of a dynamic matrix over the same scalar field as A

6.1.3.56 template<typename Derived > std::pair<dyn\_col\_vect < double>, cmat> qpp::heig ( const Eigen::MatrixBase< Derived > & A )

Full eigen decomposition of Hermitian expression.

See also

qpp::eig()

**Parameters** 

A Eigen expression

Returns

Pair of: 1. Eigenvalues of A, as a real dynamic column vector, and 2. Eigenvectors of A, as columns of a complex dynamic matrix

6.1.3.57 template<typename Derived > dyn\_col\_vect<double> qpp::hevals ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvalues.

See also

qpp::evals()

**Parameters** 

A Eigen expression

Returns

Eigenvalues of Hermitian A, as a real dynamic column vector

6.1.3.58 template<typename Derived > cmat qpp::hevects ( const Eigen::MatrixBase< Derived > & A )

Hermitian eigenvectors.

See also

qpp::evects()

### **Parameters**

Α	Eigen expression

Returns

Eigenvectors of Hermitian A, as columns of a complex matrix

6.1.3.59 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::inverse ( const Eigen::MatrixBase< Derived > & A )

Inverse.

**Parameters** 

Α	Eigen expression

## Returns

Inverse of A, as a dynamic matrix over the same scalar field as A

6.1.3.60 std::vector<idx> qpp::invperm(const std::vector< idx > & perm) [inline]

Inverse permutation.

**Parameters** 

perm	Permutation
------	-------------

## Returns

Inverse of the permutation perm

6.1.3.61 template<typename Derived > dyn\_col\_vect<typename Derived::Scalar> qpp::ip ( const Eigen::MatrixBase< Derived > & phi, const Eigen::MatrixBase< Derived > & psi, const std::vector< idx > & subsys, const std::vector< idx > & dims )

Generalized inner product.

### **Parameters**

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined

dims	Dimensions of the multi-partite system

## Returns

The inner product  $\langle \phi_{subsys} | \psi \rangle$ , as a scalar or column vector over the remaining Hilbert space

6.1.3.62 template < typename Derived > dyn\_col\_vect < typename Derived::Scalar > qpp::ip ( const Eigen::MatrixBase < Derived > & phi, const Eigen::MatrixBase < Derived > & psi, const std::vector < idx > & subsys, idx d = 2 )

Generalized inner product.

## **Parameters**

phi	Column vector Eigen expression
psi	Column vector Eigen expression
subsys	Subsystem indexes over which <i>phi</i> is defined
d	Subsystem dimensions

## Returns

The inner product  $\langle \phi_{subsys} | \psi \rangle$ , as a scalar or column vector over the remaining Hilbert space

6.1.3.63 bool qpp::isprime (bigint n, idx k = 80) [inline]

Primality test based on the Miller-Rabin's algorithm.

#### **Parameters**

n	Integer different from 0, 1 or -1
k	Number of iterations. The probability of a false positive is $2^{-k}$ .

## Returns

True if the number is (most-likely) prime, false otherwise

6.1.3.64 cmat qpp::kraus2choi ( const std::vector < cmat > & Ks ) [inline]

Choi matrix.

### See also

qpp::choi2kraus()

Constructs the Choi matrix of the channel specified by the set of Kraus operators  $\mathit{Ks}$  in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

## Note

The superoperator matrix S and the Choi matrix C are related by  $S_{ab,mn}=C_{ma,nb}$ 

Ks Set of Kraus operators

Returns

Choi matrix

6.1.3.65 cmat qpp::kraus2super ( const std::vector < cmat > & Ks ) [inline]

Superoperator matrix.

Constructs the superoperator matrix of the channel specified by the set of Kraus operators Ks in the standard operator basis  $\{|i\rangle\langle j|\}$  ordered in lexicographical order, i.e.  $|0\rangle\langle 0|$ ,  $|0\rangle\langle 1|$  etc.

**Parameters** 

```
Ks Set of Kraus operators
```

Returns

Superoperator matrix

6.1.3.66 template<typename T > dyn\_mat<typename T::Scalar> qpp::kron ( const T & head )

Kronecker product.

See also

qpp::kronpow()

Used to stop the recursion for the variadic template version of <a href="app::kron()">app::kron()</a>

Parameters

nead   Eigen expression
-------------------------

Returns

Its argument head

6.1.3.67 template<typename T , typename... Args> dyn\_mat<typename T::Scalar> qpp::kron ( const T & head, const Args &... tail )

Kronecker product.

See also

qpp::kronpow()

**Parameters** 

head	Eigen expression
tail	Variadic Eigen expression (zero or more parameters)

## Returns

Kronecker product of all input parameters, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.68 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::kron ( const std::vector< Derived > & As )

Kronecker product.

See also

qpp::kronpow()

## **Parameters**

As	std::vector of Eigen expressions

### Returns

Kronecker product of all elements in *As*, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

Kronecker product.

See also

qpp::kronpow()

### **Parameters**

As	std::initializer_list of Eigen expressions, such as {A1, A2, ,Ak}
----	---

## Returns

Kronecker product of all elements in As, evaluated from left to right, as a dynamic matrix over the same scalar field as its arguments

6.1.3.70 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::kronpow ( const Eigen::MatrixBase< Derived > & A, idx n )

Kronecker power.

See also

qpp::kron()

Α	Eigen expression
n	Non-negative integer

## Returns

Kronecker product of A with itself n times  $A^{\otimes n}$ , as a dynamic matrix over the same scalar field as A

```
6.1.3.71 bigint qpp::lcm (bigint m, bigint n) [inline]
```

Least common multiple of two integers.

### See also

qpp::gcd()

### **Parameters**

т	Integer
n	Integer

### Returns

Least common multiple of m and n

```
6.1.3.72 bigint qpp::lcm ( const std::vector < bigint > & ns ) [inline]
```

Least common multiple of a list of integers.

See also

qpp::gcd()

## Parameters

ns	List of integers

### Returns

Least common multiple of all numbers in ns

6.1.3.73 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::load ( const std::string & fname )

Loads Eigen matrix from a binary file (internal format) in double precision.

## See also

```
qpp::save()
```

The template parameter cannot be automatically deduced and must be explicitly provided, depending on the scalar field of the matrix that is being loaded.

## Example:

```
// loads a previously saved Eigen dynamic complex matrix from "input.bin" auto mat = load<cmat>("input.bin");
```

fname	Output file name
-------	------------------

6.1.3.74 template < typename Derived > Derived qpp::loadMATLABmatrix ( const std::string & , const std::string & )

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

See also

```
qpp::saveMATLABmatrix()
```

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be loaded)

```
6.1.3.75 template<> dmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// loads a previously saved Eigen dynamic double matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<dmat>("input.mat");
```

Note

If var\_name is a complex matrix, only the real part is loaded

### **Parameters**

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

### Returns

Eigen double dynamic matrix (qpp::dmat)

```
6.1.3.76 template <> cmat qpp::loadMATLABmatrix ( const std::string & mat_file, const std::string & var_name )
[inline]
```

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

See also

```
qpp::saveMATLABmatrix()
```

The template parameter cannot be automatically deduced and must be explicitly provided

### Example:

```
// loads a previously saved Eigen dynamic complex matrix
// from the MATLAB file "input.mat"
auto mat = loadMATLABmatrix<cmat>("input.mat");
```

mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be loaded

## Returns

Eigen complex dynamic matrix (qpp::cmat)

6.1.3.77 template<typename Derived > Derived::Scalar qpp::logdet ( const Eigen::MatrixBase< Derived > & A )

Logarithm of the determinant.

Useful when the determinant overflows/underflows

**Parameters** 

Α	Eigen expression

## Returns

Logarithm of the determinant of A, as a scalar over the same scalar field as A

6.1.3.78 template < typename Derived > cmat qpp::logm ( const Eigen::MatrixBase < Derived > & A )

Matrix logarithm.

## **Parameters**

Α	Eigen expression
	0 1

## Returns

Matrix logarithm of A

6.1.3.79 template < typename Derived > double qpp::lognegativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Logarithmic negativity of the bi-partite mixed state A.

## **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Logarithmic negativity, with the logarithm in base 2

6.1.3.80 std::vector<double> qpp::marginalX ( const dmat & probXY ) [inline]

Marginal distribution.

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

### Returns

Real vector consisting of the marginal distribution of X

6.1.3.81 std::vector<double> qpp::marginalY ( const dmat & probXY ) [inline]

Marginal distribution.

### **Parameters**

probXY	Real matrix representing the joint probability distribution of X and Y in lexicographical order
	(X labels the rows, Y labels the columns)

### Returns

Real vector consisting of the marginal distribution of Y

6.1.3.82 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

## **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.83 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks)

Measures the state A using the set of Kraus operators Ks.

## **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.84 template<typename Derived > std::tuple<idx, std::vector<double>, std::vector<cmat> > qpp::measure ( const Eigen::MatrixBase< Derived > & A, const cmat & U )

Measures the state A in the orthonormal basis specified by the unitary matrix *U*.

Α	Eigen expression
U	Unitary matrix whose columns represent the measurement basis vectors

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.85 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

### See also

qpp::measure\_seq()

### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.86 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

### See also

qpp::measure\_seq()

## Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.87 template < typename Derived > std::tuple < idx, std::vector < double > , std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::vector < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

### See also

qpp::measure\_seq()

#### Note

The dimension of all *Ks* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

#### **Parameters**

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.88 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const std::initializer\_list < cmat > & Ks, const std::vector < idx > & subsys, idx d = 2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

## See also

qpp::measure\_seq()

## Note

The dimension of all Ks must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
Ks	Set of Kraus operators
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.89 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Measures the part *subsys* of the multi-partite state vector or density matrix *A* in the orthonormal basis or rank-1 POVM specified by the matrix *V*.

#### See also

qpp::measure\_seq()

#### Note

The dimension of *V* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

## **Parameters**

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

## Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

6.1.3.90 template < typename Derived > std::tuple < idx, std::vector < double >, std::vector < cmat > > qpp::measure ( const Eigen::MatrixBase < Derived > & A, const cmat & V, const std::vector < idx > & subsys, idx d = 2)

Measures the part *subsys* of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

## See also

qpp::measure\_seq()

## Note

The dimension of *V* must match the dimension of *subsys*. The measurement is destructive, i.e. the measured subsystems are traced away.

Α	Eigen expression
V	Matrix whose columns represent the measurement basis vectors or the bra parts of the rank-1
	POVM
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

### Returns

Tuple of: 1. Result of the measurement, 2. Vector of outcome probabilities, and 3. Vector of post-measurement normalized states

```
6.1.3.91 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, std::vector < idx > dims )
```

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

#### See also

qpp::measure()

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes that are measured
dims	Dimensions of the multi-partite system

## Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

```
6.1.3.92 template < typename Derived > std::tuple < std::vector < idx > , double, cmat > qpp::measure_seq ( const Eigen::MatrixBase < Derived > & A, std::vector < idx > subsys, idx d = 2)
```

Sequentially measures the part *subsys* of the multi-partite state vector or density matrix *A* in the computational basis.

### See also

qpp::measure()

## Parameters

Α	Eigen expression
subsys	Subsystem indexes that are measured
d	Subsystem dimensions

## Returns

Tuple of: 1. Vector of outcome results of the measurement (ordered in increasing order with respect to *subsys*, i.e. first measurement result corresponds to the subsystem with the smallest index), 2. Outcome probability, and 3. Post-measurement normalized state

6.1.3.93 ket qpp::mket ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|\text{mask}\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

#### Parameters

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

### Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.94 ket qpp::mket ( const std::vector < idx > & mask, idx d = 2 ) [inline]

Multi-partite qudit ket.

Constructs the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension *d. mask* is a std::vector of non-negative integers, and each element in *mask* has to be strictly smaller than *d*.

### **Parameters**

mask	std::vector of non-negative integers
d	Subsystem dimensions

### Returns

Multi-partite qudit state vector, as a complex dynamic column vector

6.1.3.95 bigint qpp::modinv (bigint a, bigint p) [inline]

Modular inverse of a mod p.

See also

qpp::egcd()

Note

a and p must be co-prime

## **Parameters**

а	Non-negative integer
р	Non-negative integer

### Returns

Modular inverse  $a^{-1}$  of  $a \mod p$ 

6.1.3.96 bigint qpp::modmul (bigint m, bigint n, bigint p) [inline]

Modular multiplication without overflow.

Computes  $mn \bmod p$  without overflow

m	Integer
n	Integer
р	Positive integer

## Returns

 $m n \mathrm{mod} p$  avoiding overflow

6.1.3.97 bigint qpp::modpow (bigint a, bigint n, bigint p) [inline]

Fast integer power modulo *p* based on the SQUARE-AND-MULTIPLY algorithm.

Note

Uses qpp::modmul() that avoids overflows

Computes  $a^n \bmod p$ 

#### **Parameters**

а	Non-negative integer
n	Non-negative integer
р	Strictly positive integer

#### Returns

 $a^n \bmod p$ 

6.1.3.98 cmat qpp::mprj ( const std::vector < idx > & mask, const std::vector < idx > & dims ) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , where mask is a std::vector of non-negative integers. Each element in mask has to be smaller than the corresponding element in dims.

## **Parameters**

mask	std::vector of non-negative integers
dims	Dimensions of the multi-partite system

## Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.99 cmat qpp::mprj (const std::vector < idx > & mask, idx d = 2) [inline]

Projector onto multi-partite qudit ket.

Constructs the projector onto the multi-partite qudit ket  $|mask\rangle$ , all subsystem having equal dimension d. mask is a std::vector of non-negative integers, and each element in mask has to be strictly smaller than d.

mask	std::vector of non-negative integers
d	Subsystem dimensions

## Returns

Projector onto multi-partite qudit state vector, as a complex dynamic matrix

6.1.3.100 idx qpp::multiidx2n ( const std::vector < idx > & midx, const std::vector < idx > & dims ) [inline]

Multi-index to non-negative integer index.

#### See also

qpp::n2multiidx()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

## **Parameters**

midx	Multi-index
dims	Dimensions of the multi-partite system

## Returns

Non-negative integer index

6.1.3.101 std::vector<idx> qpp::n2multiidx ( idx n, const std::vector< idx> & dims ) [inline]

Non-negative integer index to multi-index.

## See also

qpp::multiidx2n()

Uses standard lexicographical order, i.e. 00...0, 00...1 etc.

## **Parameters**

n	Non-negative integer index
dims	Dimensions of the multi-partite system

## Returns

Multi-index of the same size as dims

6.1.3.102 template < typename Derived > double qpp::negativity ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Negativity of the bi-partite mixed state A.

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Negativity

6.1.3.103 template < typename Derived > double qpp::norm ( const Eigen::MatrixBase < Derived > & A )

Frobenius norm.

**Parameters** 

Α	Eigen expression

Returns

Frobenius norm of A

```
6.1.3.104 cplx qpp::omega (idx D) [inline]
```

D-th root of unity.

**Parameters** 

D Non-negative integer
------------------------

## Returns

D-th root of unity  $\exp(2\pi i/D)$ 

```
6.1.3.105 constexpr cplx qpp::operator""_i ( unsigned long long int x ) [inline], [noexcept]
```

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

## Example:

```
auto z = 4_i; // type of z is std::complex<double>
```

**6.1.3.106** constexpr cplx qpp::operator""\_i ( long double x ) [inline], [noexcept]

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

## Example:

```
auto z = 4.5_i; // type of z is std::complex<double>
```

6.1.3.107 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::powm ( const Eigen::MatrixBase < Derived > & A, idx n )

Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.

See also

qpp::spectralpowm()

Explicitly multiplies the matrix A with itself n times. By convention  $A^0=I$ .

Α	Eigen expression
n	Non-negative integer

## Returns

Matrix power  $A^n$ , as a dynamic matrix over the same scalar field as A

6.1.3.108 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::prj ( const Eigen::MatrixBase< Derived > & V )

## Projector.

Normalized projector onto state vector

#### **Parameters**

V	Eigen expression

#### Returns

Projector onto the state vector V, or the matrix Zero if V has norm zero (i.e. smaller than qpp::eps), as a dynamic matrix over the same scalar field as A

6.1.3.109 template < typename Derived > Derived::Scalar qpp::prod ( const Eigen::MatrixBase < Derived > & A )

Element-wise product of A.

#### **Parameters**

Α	Eigen expression
---	------------------

## Returns

Element-wise product of A, as a scalar over the same scalar field as A

6.1.3.110 template<typename InputIterator > std::iterator\_traits<InputIterator>::value\_type qpp::prod ( InputIterator *first*, InputIterator *last* )

Element-wise product of an STL-like range.

## **Parameters**

first	Iterator to the first element of the range
last	Iterator to the last element of the range

## Returns

Element-wise product of the range, as a scalar over the same scalar field as the range

Element-wise product of the elements of an STL-like container.

С	STL-like container
---	--------------------

## Returns

Element-wise product of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.112 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase< Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims )

Partial trace.

#### See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

#### **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

#### Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.113 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::ptrace ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2)

Partial trace.

See also

```
qpp::ptrace1(), qpp::ptrace2()
```

Partial trace of the multi-partite state vector or density matrix over a list of subsystems

## **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

## Returns

Partial trace  $Tr_{subsys}(\cdot)$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.114 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptrace1 ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Partial trace.

See also

qpp::ptrace2()

Partial trace over the first subsystem of bi-partite state vector or density matrix

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

#### Returns

Partial trace  $Tr_A(\cdot)$  over the first subsystem A in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.115 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptrace2 ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims)

Partial trace.

See also

qpp::ptrace1()

Partial trace over the second subsystem of bi-partite state vector or density matrix

## **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system (must be a std::vector with 2 elements)

## Returns

Partial trace  $Tr_B(\cdot)$  over the second subsytem B in a bi-partite system  $A\otimes B$ , as a dynamic matrix over the same scalar field as A

6.1.3.116 template < typename Derived >  $dyn_mat < typename Derived::Scalar > qpp::ptranspose ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsys, const std::vector < idx > & dims )$ 

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

## **Parameters**

Α	Eigen expression
subsys	Subsystem indexes
dims	Dimensions of the multi-partite system

## Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.117 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::ptranspose ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsys, idx d = 2 )

Partial transpose.

Partial transpose of the multi-partite state vector or density matrix over a list of subsystems

Α	Eigen expression
subsys	Subsystem indexes
d	Subsystem dimensions

## Returns

Partial transpose  $(\cdot)^{T_{subsys}}$  over the subsytems *subsys* in a multi-partite system, as a dynamic matrix over the same scalar field as A

6.1.3.118 template < typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & subsysA, const std::vector < idx > & subsysB, const std::vector < idx > & dims )

Quantum mutual information between 2 subsystems of a composite system.

## **Parameters**

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
dims	Dimensions of the multi-partite system

## Returns

Mutual information between the 2 subsystems

6.1.3.119 template<typename Derived > double qpp::qmutualinfo ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & subsysA, const std::vector< idx > & subsysB, idx d = 2)

Quantum mutual information between 2 subsystems of a composite system.

## **Parameters**

Α	Eigen expression
subsysA	Indexes of the first subsystem
subsysB	Indexes of the second subsystem
d	Subsystem dimensions

## Returns

Mutual information between the 2 subsystems

**6.1.3.120** double qpp::rand ( double a, double b ) [inline]

Generates a random real number uniformly distributed in the interval [a, b)

## **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

## Returns

Random real number (double) uniformly distributed in the interval [a, b)

**6.1.3.121** bigint qpp::rand (bigint a, bigint b) [inline]

Generates a random big integer uniformly distributed in the interval [a, b].

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

#### Returns

Random big integer uniformly distributed in the interval [a, b]

```
6.1.3.122 template < typename Derived > Derived qpp::rand ( idx rows, idx cols, double a = 0, double b = 1 )
```

Generates a random matrix with entries uniformly distributed in the interval [a, b)

If complex, then both real and imaginary parts are uniformly distributed in [a, b)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

```
6.1.3.123 template <> dmat qpp::rand (idx rows, idx cols, double a, double b) [inline]
```

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries uniformly distributed in [-1,1)
auto mat = rand<dmat>(3, 3, -1, 1);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

## Returns

Random real matrix

```
6.1.3.124 template<> cmat qpp::rand ( idx rows, idx cols, double a, double b ) [inline]
```

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) uniformly distributed in [-1,1) auto mat = rand<cmat>(3, 3, -1, 1);
```

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
а	Beginning of the interval, belongs to it
b	End of the interval, does not belong to it

## Returns

Random complex matrix

6.1.3.125 cmat qpp::randH(idx D) [inline]

Generates a random Hermitian matrix.

#### **Parameters**

	D	Dimension of the Hilbert space
--	---	--------------------------------

#### Returns

Random Hermitian matrix

Generates a random index (idx) uniformly distributed in the interval [a, b].

## **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it

## Returns

Random index (idx) uniformly distributed in the interval [a, b]

6.1.3.127 **ket qpp::randket(idx D)** [inline]

Generates a random normalized ket (pure state vector)

## **Parameters**

D	Dimension of the Hilbert space

## Returns

Random normalized ket

6.1.3.128 std::vector<cmat> qpp::randkraus ( idx N, idx D ) [inline]

Generates a set of random Kraus operators.

Note

The set of Kraus operators satisfy the closure condition  $\sum_i K_i^\dagger K_i = I$ 

N	Number of Kraus operators
D	Dimension of the Hilbert space

#### Returns

Set of N Kraus operators satisfying the closure condition

6.1.3.129 template < typename Derived > Derived qpp::randn ( idx rows, idx cols, double mean = 0, double sigma = 1 )

Generates a random matrix with entries normally distributed in N(mean, sigma)

If complex, then both real and imaginary parts are normally distributed in N(mean, sigma)

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat

6.1.3.130 template<> dmat qpp::randn ( idx rows, idx cols, double mean, double sigma ) [inline]

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// generates a 3 x 3 random Eigen::MatrixXd,
// with entries normally distributed in N(0,2)
auto mat = randn<dmat>(3, 3, 0, 2);
```

#### **Parameters**

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

## Returns

Random real matrix

6.1.3.131 template<> cmat qpp::randn(idx rows, idx cols, double mean, double sigma) [inline]

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

The template parameter cannot be automatically deduced and must be explicitly provided

## Example:

```
// generates a 3 x 3 random Eigen::MatrixXcd, 
// with entries (both real and imaginary) normally distributed in N(0,2) auto mat = randn<cmat>(3, 3, 0, 2);
```

rows	Number of rows of the random generated matrix
cols	Number of columns of the random generated matrix
mean	Mean
sigma	Standard deviation

## Returns

Random complex matrix

6.1.3.132 double qpp::randn ( double mean = 0, double sigma = 1 ) [inline]

Generates a random real number (double) normally distributed in N(mean, sigma)

#### **Parameters**

mean	Mean
sigma	Standard deviation

## Returns

Random real number normally distributed in N(mean, sigma)

6.1.3.133 std::vector<idx> qpp::randperm(idx n) [inline]

Generates a random uniformly distributed permutation.

Uses Knuth shuffle method (as implemented by std::shuffle), so that all permutations are equally probable

## **Parameters**

n	Size of the permutation
---	-------------------------

## Returns

Random permutation of size n

6.1.3.134 bigint qpp::randprime (bigint a, bigint b, idx N = 1000) [inline]

Generates a random big prime uniformly distributed in the interval [a, b].

## **Parameters**

а	Beginning of the interval, belongs to it
b	End of the interval, belongs to it
N	Maximum number of candidates

## Returns

Random big integer uniformly distributed in the interval [a, b]

6.1.3.135 cmat qpp::randrho(idx D) [inline]

Generates a random density matrix.

D	Dimension of the Hilbert space

#### Returns

Random density matrix

6.1.3.136 cmat qpp::randU(idx D) [inline]

Generates a random unitary matrix.

#### **Parameters**

D	Dimension of the Hilbert space

#### Returns

Random unitary

6.1.3.137 cmat qpp::randV (idx Din, idx Dout) [inline]

Generates a random isometry matrix.

## **Parameters**

Din	Size of the input Hilbert space
Dout	Size of the output Hilbert space

## Returns

Random isometry matrix

6.1.3.138 template < typename Derived > double qpp::renyi ( const Eigen::MatrixBase < Derived > & A, double alpha )

Renyi-  $\alpha$  entropy of the density matrix  ${\it A}$ , for  $\alpha \geq 0$ .

Note

When  $\alpha \to 1$  the Renyi entropy converges to the von-Neumann entropy, with the logarithm in base 2

## **Parameters**

A	Eigen expression
alpha	Non-negative real number, use qpp::infty for $\alpha=\infty$

## Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

**6.1.3.139** double qpp::renyi ( const std::vector < double > & prob, double alpha ) [inline]

Renyi-  $\alpha$  entropy of the probability distribution *prob*, for  $\alpha \geq 0$ .

Note

When  $\alpha \to 1$  the Renyi entropy converges to the Shannon entropy, with the logarithm in base 2

prob	Real probability vector
alpha	Non-negative real number, use qpp::infty for $\alpha = \infty$

#### Returns

Renyi-  $\alpha$  entropy, with the logarithm in base 2

6.1.3.140 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::reshape ( const Eigen::MatrixBase< Derived > & A, idx rows, idx cols )

## Reshape.

Uses column-major order when reshaping (same as MATLAB)

#### **Parameters**

Α	Eigen expression
rows	Number of rows of the reshaped matrix
cols	Number of columns of the reshaped matrix

## Returns

Reshaped matrix with rows rows and cols columns, as a dynamic matrix over the same scalar field as A

6.1.3.141 template < typename Derived > std::vector < double > qpp::rho2bloch ( const Eigen::MatrixBase < Derived > & A )

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

## See also

qpp::bloch2rho()

## Note

It is implicitly assumed that the density matrix is Hermitian

#### **Parameters**

Α	Eigen expression

## Returns

3-dimensional Bloch vector

6.1.3.142 template<typename Derived > dyn\_col\_vect<typename Derived::Scalar> qpp::rho2pure ( const Eigen::MatrixBase< Derived > & A )

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

## Note

No purity check is done, the input state A must have rank one, otherwise the function returns the first non-zero eigenvector of A

Α	Eigen expression, assumed to be proportional to a projector onto a pure state, i.e. A is
	assumed to have rank one

## Returns

The unique non-zero eigenvector of A, as a dynamic column vector over the same scalar field as A

6.1.3.143 template<typename Derived > void qpp::save ( const Eigen::MatrixBase< Derived > & A, const std::string & fname )

Saves Eigen expression to a binary file (internal format) in double precision.

## See also

qpp::load()

#### **Parameters**

Α	Eigen expression
fname	Output file name

6.1.3.144 template < typename Derived > void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < Derived > & , const std::string & , const std::string & )

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

## See also

## qpp::loadMATLABmatrix()

This is the generic version that always throws qpp::Exception::Type::UNDEFINED\_TYPE. It is specialized only for qpp::dmat and qpp::cmat (the only matrix types that can be saved)

6.1.3.145 template <> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < dmat > & A, const std::string & mat\_file, const std::string & war\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

#### See also

qpp::loadMATLABmatrix()

## **Parameters**

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.146 template <> void qpp::saveMATLABmatrix ( const Eigen::MatrixBase < cmat > & A, const std::string & mat\_file, const std::string & war\_name, const std::string & mode ) [inline]

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

#### See also

qpp::loadMATLABmatrix()

#### **Parameters**

Α	Eigen expression over the complex field
mat_file	MATALB .mat file
var_name	Variable name in the .mat file representing the matrix to be saved
mode	Saving mode (append, overwrite etc.), see MATLAB <i>matOpen()</i> documentation for details

6.1.3.147 template < typename Derived > double qpp::schatten ( const Eigen::MatrixBase < Derived > & A, double p )

Schatten matrix norm.

## **Parameters**

Α	Eigen expression
р	Real number, greater or equal to 1, use qpp::infty for $p=\infty$

#### Returns

Schatten-p matrix norm of A

6.1.3.148 template<typename Derived > cmat qpp::schmidtA ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt basis on Alice side.

## **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

#### Returns

Unitary matrix  ${\cal U}$  whose columns represent the Schmidt basis vectors on Alice side.

6.1.3.149 template<typename Derived > cmat qpp::schmidtB ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt basis on Bob side.

## **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Unitary matrix V whose columns represent the Schmidt basis vectors on Bob side.

6.1.3.150 template<typename Derived > dyn\_col\_vect<double> qpp::schmidtcoeffs ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & dims )

Schmidt coefficients of the bi-partite pure state A.

Note

The sum of the squares of the Schmidt coefficients equals 1

## See also

qpp::schmidtprobs()

#### **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Schmidt coefficients of A, as a real dynamic column vector

6.1.3.151 template < typename Derived > std::vector < double > qpp::schmidtprobs ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & dims )

Schmidt probabilities of the bi-partite pure state A.

Defined as the squares of the Schmidt coefficients. The sum of the Schmidt probabilities equals 1.

#### See also

qpp::schmidtcoeffs()

## **Parameters**

Α	Eigen expression
dims	Dimensions of the bi-partite system

## Returns

Real vector consisting of the Schmidt probabilites of A

6.1.3.152 template < typename Container > double qpp::sigma ( const std::vector < double > & prob, const Container & X, typename std::enable\_if < is\_iterable < Container >::value >::type \* = nullptr)

Standard deviation.

## **Parameters**

prob	Real probability vector representing the probability distribution of X
X	Random variable values represented by an STL-like container

## Returns

Standard deviation of X

6.1.3.153 template < typename Derived > cmat qpp::sinm ( const Eigen::MatrixBase < Derived > & A )

Matrix sin.

A Eigen expression

Returns

Matrix sine of A

6.1.3.154 template < typename Derived > cmat qpp::spectralpowm ( const Eigen::MatrixBase < Derived > & A, const cplx z )

Matrix power.

See also

qpp::powm()

Uses the spectral decomposition of A to compute the matrix power. By convention  $A^0 = I$ .

#### **Parameters**

Α	Eigen expression
Z	Complex number

## Returns

Matrix power  $A^z$ 

6.1.3.155 template<typename Derived > cmat qpp::sqrtm ( const Eigen::MatrixBase< Derived > & A )

Matrix square root.

**Parameters** 

Α	Eigen expression

Returns

Matrix square root of A

6.1.3.156 template < typename Derived > Derived::Scalar qpp::sum ( const Eigen::MatrixBase < Derived > & A )

Element-wise sum of A.

**Parameters** 

A Eigen expression
--------------------

Returns

Element-wise sum of A, as a scalar over the same scalar field as A

6.1.3.157 template < typename InputIterator > std::iterator\_traits < InputIterator > ::value\_type qpp::sum ( InputIterator first, InputIterator last )

Element-wise sum of an STL-like range.

first	Iterator to the first element of the range
last	Iterator to the last element of the range

## Returns

Element-wise sum of the range, as a scalar over the same scalar field as the range

6.1.3.158 template < typename Container > Container::value\_type qpp::sum ( const Container & c, typename std::enable\_if < is\_iterable < Container >::value >::type \* = nullptr )

Element-wise sum of the elements of an STL-like container.

#### **Parameters**

С	STL-like container

## Returns

Element-wise sum of the elements of the container, as a scalar over the same scalar field as the container

6.1.3.159 cmat qpp::super2choi( const cmat & A ) [inline]

Converts superoperator matrix to Choi matrix.

#### See also

qpp::choi2super()

## **Parameters**

Α	Superoperator matrix

## Returns

Choi matrix

6.1.3.160 template < typename Derived > dyn\_col\_vect < double > qpp::svals ( const Eigen::MatrixBase < Derived > & A )

Singular values.

# **Parameters**

Α	Eigen expression
---	------------------

## Returns

Singular values of  $\emph{A}$ , ordered in decreasing order, as a real dynamic column vector

6.1.3.161 template < typename Derived > std::tuple < cmat, dyn\_col\_vect < double >, cmat > qpp::svd ( const Eigen::MatrixBase < Derived > & A )

Full singular value decomposition.

Α	Eigen expression
---	------------------

## Returns

Tuple of: 1. Left sigular vectors of *A*, as columns of a complex dynamic matrix, 2. Singular values of *A*, ordered in decreasing order, as a real dynamic column vector, and 3. Right singular vectors of *A*, as columns of a complex dynamic matrix

6.1.3.162 template<typename Derived > cmat qpp::svdU ( const Eigen::MatrixBase< Derived > & A )

Left singular vectors.

#### **Parameters**

Α	Eigen expression

#### Returns

Complex dynamic matrix, whose columns are the left singular vectors of A

6.1.3.163 template < typename Derived > cmat qpp::svdV ( const Eigen::MatrixBase < Derived > & A )

Right singular vectors.

#### **Parameters**

Α	Eigen expression

## Returns

Complex dynamic matrix, whose columns are the right singular vectors of A

6.1.3.164 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::syspermute ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & perm, const std::vector < idx > & perm < idx > & perm

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit perm[i] is permuted to the location i.

## **Parameters**

Α	Eigen expression	
perm	Permutation	
dims	dims Dimensions of the multi-partite system	

## Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.165 template<typename Derived >  $dyn_mat$ <typename Derived::Scalar> qpp::syspermute ( const Eigen::MatrixBase< Derived > & A, const std::vector< idx > & perm, idx d = 2 )

Subsystem permutation.

Permutes the subsystems of a state vector or density matrix. The qubit *perm[i]* is permuted to the location *i*.

Α	Eigen expression	
perm	Permutation	
d	d Subsystem dimensions	

## Returns

Permuted system, as a dynamic matrix over the same scalar field as A

6.1.3.166 template<typename Derived > Derived::Scalar qpp::trace ( const Eigen::MatrixBase< Derived > & A )

Trace.

#### **Parameters**

Λ	Figur averagion
I A	Eigen expression

### Returns

Trace of A, as a scalar over the same scalar field as A

6.1.3.167 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::transpose ( const Eigen::MatrixBase< Derived > & A )

Transpose.

#### **Parameters**

A Eigen expression
--------------------

## Returns

Transpose of A, as a dynamic matrix over the same scalar field as A

6.1.3.168 template<typename Derived > double qpp::tsallis ( const Eigen::MatrixBase< Derived > & A, double q )

Tsallis- q entropy of the density matrix A, for  $q \ge 0$ .

Note

When  $q \to 1$  the Tsallis entropy converges to the von-Neumann entropy, with the logarithm in base e

# **Parameters**

Α	Eigen expression	
q	Non-negative real number	

## Returns

Tsallis- q entropy

6.1.3.169 double qpp::tsallis ( const std::vector < double > & prob, double q ) [inline]

Tsallis- q entropy of the probability distribution prob, for  $q \geq 0$ .

Note

When  $q \to 1$  the Tsallis entropy converges to the Shannon entropy, with the logarithm in base e

prob	pb Real probability vector	
q Non-negative real number		

### Returns

Tsallis- q entropy

6.1.3.170 std::vector<double> qpp::uniform(idx N) [inline]

Uniform probability distribution vector.

#### **Parameters**

#### Returns

Real vector consisting of a uniform distribution of size N

6.1.3.171 template < typename Container > double qpp::var ( const std::vector < double > & prob, const Container & X, typename std::enable\_if < is\_iterable < Container >::value >::type \* = nullptr)

## Variance.

#### **Parameters**

prob	Real probability vector representing the probability distribution of X
X Random variable values represented by an STL-like container	

## Returns

Variance of X

6.1.3.172 std::vector<int> qpp::x2contfrac ( double x, idx n, idx cut = 1e5 ) [inline]

Simple continued fraction expansion.

#### See also

qpp::contfrac2x()

## **Parameters**

X	Real number			
n	Maximum number of terms in the expansion			
cut Stop the expansion when the next term is greater than cut				

## Returns

Integer vector containing the simple continued fraction expansion of x. If there are m less than n terms in the expansion, a shorter vector with m components is returned.

## 6.1.4 Variable Documentation

6.1.4.1 constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

6.1.4.2 constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

6.1.4.3 constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

Example:

```
if(std::abs(x) < qpp::eps) // x is zero</pre>
```

6.1.4.4 constexpr double qpp::infty = std::numeric\_limits < double >::infinity()

Used to denote infinity in double precision.

6.1.4.5 constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

Used internally to allocate arrays on the stack (for speed reasons)

6.1.4.6 constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

# 6.2 **qpp::experimental Namespace Reference**

Experimental/test functions/classes, do not use or modify.

## 6.2.1 Detailed Description

Experimental/test functions/classes, do not use or modify.

# 6.3 qpp::internal Namespace Reference

Internal utility functions, do not use/modify.

## **Namespaces**

· \_details

## **Classes**

- class IOManipEigen
- class IOManipPointer
- class IOManipRange
- class Singleton

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

## **Functions**

```
    void n2multiidx (idx n, idx numdims, const idx *dims, idx *result) noexcept

• idx _multiidx2n (const idx *midx, idx numdims, const idx *dims) noexcept
template<typename Derived >
  bool check square mat (const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
  bool <u>_check_vector</u> (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

 bool <u>_check_rvector</u> (const Eigen::MatrixBase< Derived > &A)
template<typename Derived >
  bool check cvector (const Eigen::MatrixBase< Derived > &A)
template<typename T >
  bool check nonzero size (const T &x) noexcept

    template<typename T1 , typename T2 >

  bool _check_matching_sizes (const T1 &lhs, const T2 &rhs) noexcept

    bool <u>_check_dims</u> (const std::vector< idx > &dims)

    template<typename Derived >

  bool _check_dims_match_mat (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &A)
• template<typename Derived >
 bool _check_dims_match_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

    template<typename Derived >

  bool _check_dims_match_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase< Derived > &V)

    bool <u>_check_eq_dims</u> (const std::vector< idx > &dims, idx dim) noexcept

    bool <u>_check_subsys_match_dims</u> (const std::vector < idx > &subsys, const std::vector < idx > &dims)

    template<typename Derived >

  bool _check_qubit_matrix (const Eigen::MatrixBase< Derived > &A) noexcept

    template < typename Derived >

  bool <u>_check_qubit_cvector</u> (const Eigen::MatrixBase< Derived > &V) noexcept
template<typename Derived >
  bool _check_qubit_rvector (const Eigen::MatrixBase< Derived > &V) noexcept
template<typename Derived >
  bool check qubit vector (const Eigen::MatrixBase< Derived > &V) noexcept

    bool check perm (const std::vector < idx > &perm)

    template<typename Derived1 , typename Derived2 >

  dyn_mat< typename Derived1::Scalar > kron2 (const Eigen::MatrixBase< Derived1 > &A, const Eigen ←
  ::MatrixBase< Derived2 > &B)
• template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > _dirsum2 (const Eigen::MatrixBase< Derived1 > &A, const
```

## template<typename T > void variadic vector

void variadic\_vector\_emplace (std::vector< T > &)

Eigen::MatrixBase< Derived2 > &B)

template<typename T, typename First, typename... Args>
 void variadic\_vector\_emplace (std::vector< T > &v, First &&first, Args &&...args)

## 6.3.1 Detailed Description

Internal utility functions, do not use/modify.

## 6.3.2 Function Documentation

- 6.3.2.1 template < typename Derived > bool qpp::internal::\_check\_cvector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.2 bool qpp::internal::\_check\_dims ( const std::vector < idx > & dims ) [inline]

- 6.3.2.3 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_cvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.3.2.4 template < typename Derived > bool qpp::internal::\_check\_dims\_match\_mat ( const std::vector < idx > & dims, const Eigen::MatrixBase < Derived > & A)
- 6.3.2.5 template<typename Derived > bool qpp::internal::\_check\_dims\_match\_rvect ( const std::vector< idx > & dims, const Eigen::MatrixBase< Derived > & V )
- 6.3.2.6 bool qpp::internal::\_check\_eq\_dims ( const std::vector < idx > & dims, idx dim ) [inline], [noexcept]
- 6.3.2.7 template<typename T1 , typename T2 > bool qpp::internal::\_check\_matching\_sizes ( const T1 & *lhs*, const T2 & *rhs* ) [noexcept]
- 6.3.2.8 template < typename T > bool qpp::internal::\_check\_nonzero\_size ( const T & x ) [noexcept]
- 6.3.2.9 bool qpp::internal::\_check\_perm ( const std::vector < idx > & perm ) [inline]
- $\textbf{6.3.2.10} \quad \textbf{template} < \textbf{typename Derived} > \textbf{bool qpp::internal::\_check\_qubit\_cvector} \ \, \textbf{( const Eigen::MatrixBase} < \textbf{Derived} > \textbf{\& V} \\ \, \textbf{( noexcept]}$
- 6.3.2.12 template < typename Derived > bool qpp::internal::\_check\_qubit\_rvector ( const Eigen::MatrixBase < Derived > & V ) [noexcept]
- 6.3.2.13 template < typename Derived > bool qpp::internal::\_check\_qubit\_vector ( const Eigen::MatrixBase < Derived > & V ) [noexcept]
- 6.3.2.14 template < typename Derived > bool qpp::internal::\_check\_rvector ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.15 template < typename Derived > bool qpp::internal::\_check\_square\_mat ( const Eigen::MatrixBase < Derived > & A )
- 6.3.2.16 bool qpp::internal::\_check\_subsys\_match\_dims ( const std::vector < idx > & subsys, const std::vector < idx > & dims ) [inline]
- 6.3.2.17 template < typename Derived > bool qpp::internal:: check\_vector ( const Eigen::MatrixBase < Derived > & A )
- $\begin{array}{ll} \textbf{6.3.2.19} & \textbf{template} < \textbf{typename Derived1} \ , \ \textbf{typename Derived2} > \textbf{dyn\_mat} < \textbf{typename Derived1::Scalar} > \textbf{qpp::internal::\_kron2} \ ( \\ \textbf{const Eigen::MatrixBase} < \textbf{Derived1} > \& \textit{\textbf{A}}, \ \textbf{const Eigen::MatrixBase} < \textbf{Derived2} > \& \textit{\textbf{B}} \ ) \end{array}$
- 6.3.2.21 void qpp::internal::\_n2multiidx ( idx n, idx n numdims, const idx \* dims, idx \* result ) [inline], [noexcept]
- 6.3.2.22 template < typename T > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > & )
- 6.3.2.23 template < typename T , typename First , typename... Args > void qpp::internal::variadic\_vector\_emplace ( std::vector < T > &  $\nu$ , First && first, Args &&... args )

# 6.4 qpp::internal::\_details Namespace Reference

# Classes

• struct \_Display\_Impl

Names	pace	Docur	mentatior

# **Chapter 7**

# **Class Documentation**

# 7.1 qpp::internal::\_details::\_Display\_Impl Struct Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::\_details::\_Display\_Impl:



## **Public Member Functions**

template<typename T >
 std::ostream & \_display\_impl (const T &\_A, std::ostream &\_os, double \_chop=qpp::chop) const

## 7.1.1 Member Function Documentation

The documentation for this struct was generated from the following file:

· internal/classes/iomanip.h

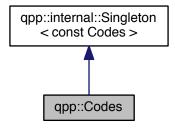
# 7.2 qpp::Codes Class Reference

const Singleton class that defines quantum error correcting codes

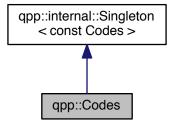
86 Class Documentation

#include <classes/codes.h>

Inheritance diagram for qpp::Codes:



## Collaboration diagram for qpp::Codes:



# **Public Types**

enum Type { Type::FIVE\_QUBIT = 1, Type::SEVEN\_QUBIT\_STEANE, Type::NINE\_QUBIT\_SHOR }
 Code types, add more codes here if needed.

## **Public Member Functions**

ket codeword (Type type, idx i) const
 Returns the codeword of the specified code type.

## **Private Member Functions**

· Codes ()

Default constructor.

Codes ()=default

Default destructor.

## **Friends**

class internal::Singleton < const Codes >

## **Additional Inherited Members**

## 7.2.1 Detailed Description

const Singleton class that defines quantum error correcting codes

## 7.2.2 Member Enumeration Documentation

```
7.2.2.1 enum qpp::Codes::Type [strong]
```

Code types, add more codes here if needed.

See also

```
qpp::Codes::codeword()
```

#### Enumerator

```
FIVE_QUBIT [[5,1,3]] qubit code

SEVEN_QUBIT_STEANE [[7,1,3]] Steane qubit code

NINE_QUBIT_SHOR [[9,1,3]] Shor qubit code
```

## 7.2.3 Constructor & Destructor Documentation

```
7.2.3.1 qpp::Codes::Codes( ) [inline],[private]
```

Default constructor.

```
7.2.3.2 qpp::Codes::~Codes( ) [private],[default]
```

Default destructor.

## 7.2.4 Member Function Documentation

```
7.2.4.1 ket qpp::Codes::codeword ( Type type, idx i ) const [inline]
```

Returns the codeword of the specified code type.

See also

```
qpp::Codes::Type
```

## **Parameters**

type	Code type
i	Codeword index

## Returns

i-th codeword of the code type

88 Class Documentation

## 7.2.5 Friends And Related Function Documentation

**7.2.5.1** friend class internal::Singleton < const Codes > [friend]

The documentation for this class was generated from the following file:

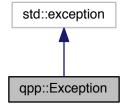
classes/codes.h

# 7.3 qpp::Exception Class Reference

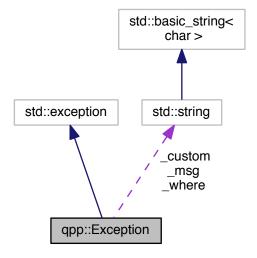
Generates custom exceptions, used when validating function parameters.

#include <classes/exception.h>

Inheritance diagram for qpp::Exception:



Collaboration diagram for qpp::Exception:



## **Public Types**

enum Type {

Type::UNKNOWN\_EXCEPTION = 1, Type::ZERO\_SIZE, Type::MATRIX\_NOT\_SQUARE, Type::MATRIX\_← NOT\_CVECTOR,

Type::MATRIX\_NOT\_RVECTOR, Type::MATRIX\_NOT\_VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_C↔ VECTOR, Type::MATRIX\_NOT\_SQUARE\_OR\_RVECTOR,

Type::MATRIX\_NOT\_SQUARE\_OR\_VECTOR, Type::MATRIX\_MISMATCH\_SUBSYS, Type::DIMS\_INVA← LID, Type::DIMS\_NOT\_EQUAL,

Type::DIMS\_MISMATCH\_MATRIX, Type::DIMS\_MISMATCH\_CVECTOR, Type::DIMS\_MISMATCH\_RVE  $\leftarrow$  CTOR, Type::DIMS\_MISMATCH\_VECTOR,

Type::SUBSYS\_MISMATCH\_DIMS, Type::PERM\_INVALID, Type::PERM\_MISMATCH\_DIMS, Type::NOT ← QUBIT\_MATRIX,

Type::NOT\_QUBIT\_CVECTOR, Type::NOT\_QUBIT\_RVECTOR, Type::NOT\_QUBIT\_VECTOR, Type::NO← T QUBIT\_SUBSYS,

Type::NOT\_BIPARTITE, Type::NO\_CODEWORD, Type::OUT\_OF\_RANGE, Type::TYPE\_MISMATCH, Type::SIZE MISMATCH, Type::UNDEFINED TYPE, Type::CUSTOM EXCEPTION }

Exception types, add more here if needed.

## **Public Member Functions**

• Exception (const std::string &where, const Type &type)

Constructs an exception.

• Exception (const std::string &where, const std::string &custom)

Constructs an exception.

virtual const char \* what () const noexcept override

Overrides std::exception::what()

## **Private Member Functions**

• void \_construct\_exception\_msg ()

Constructs the exception description from its type.

## **Private Attributes**

- · std::string \_where
- std::string msg
- Type \_type
- · std::string \_custom

## 7.3.1 Detailed Description

Generates custom exceptions, used when validating function parameters.

Customize this class if more exceptions are needed

## 7.3.2 Member Enumeration Documentation

**7.3.2.1 enum qpp::Exception::Type** [strong]

Exception types, add more here if needed.

90 Class Documentation

#### See also

qpp::Exception::\_construct\_exception\_msg()

#### Enumerator

UNKNOWN\_EXCEPTION Unknown exception

ZERO\_SIZE Zero sized object, e.g. empty Eigen::Matrix or std::vector<> with no elements

MATRIX\_NOT\_SQUARE Eigen::Matrix is not square

MATRIX\_NOT\_CVECTOR Eigen::Matrix is not a column vector

MATRIX\_NOT\_RVECTOR Eigen::Matrix is not a row vector

MATRIX\_NOT\_VECTOR Eigen::Matrix is not a row/column vector

MATRIX\_NOT\_SQUARE\_OR\_CVECTOR Eigen::Matrix is not square nor a column vector

MATRIX\_NOT\_SQUARE\_OR\_RVECTOR Eigen::Matrix is not square nor a row vector

MATRIX\_NOT\_SQUARE\_OR\_VECTOR Eigen::Matrix is not square nor a row/column vector

MATRIX\_MISMATCH\_SUBSYS Matrix size mismatch subsystem sizes (e.g. in qpp::apply())

**DIMS\_INVALID** std::vector<idx> of dimensions has zero size or contains zeros

**DIMS\_NOT\_EQUAL** Local/global dimensions are not equal

**DIMS\_MISMATCH\_MATRIX** Product of the elements of std::vector<idx> of dimensions is not equal to the number of rows of Eigen::Matrix (assumed to be a square matrix)

**DIMS\_MISMATCH\_CVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a column vector)

**DIMS\_MISMATCH\_RVECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row vector)

**DIMS\_MISMATCH\_VECTOR** Product of the elements of std::vector<idx> of dimensions is not equal to the number of elements of Eigen::Matrix (assumed to be a row/column vector)

**SUBSYS\_MISMATCH\_DIMS** std::vector<idx> of subsystem labels has duplicates, or has entries that are larger than the size of the std::vector<idx> of dimensions

**PERM\_INVALID** std::vector<idx> does note represent a valid permutation

**PERM\_MISMATCH\_DIMS** Size of the std::vector<idx> representing the permutation is different from the size of the std::vector<idx> of dimensions

NOT\_QUBIT\_MATRIX Eigen::Matrix is not 2 x 2

NOT\_QUBIT\_CVECTOR Eigen::Matrix is not 2 x 1

NOT\_QUBIT\_RVECTOR Eigen::Matrix is not 1 x 2

NOT\_QUBIT\_VECTOR Eigen::Matrix is not 1 x 2 nor 2 x 1

NOT\_QUBIT\_SUBSYS Subsystems are not 2-dimensional

NOT\_BIPARTITE std::vector<idx> of dimensions has size different from 2

**NO\_CODEWORD** Codeword does not exist, thrown when calling qpp::Codes::codeword() with invalid index i

OUT\_OF\_RANGE Parameter out of range

TYPE\_MISMATCH Scalar types do not match

SIZE\_MISMATCH Sizes do not match

UNDEFINED\_TYPE Templated specialization not defined for this type

CUSTOM\_EXCEPTION Custom exception, user must provide a custom message

## 7.3.3 Constructor & Destructor Documentation

7.3.3.1 qpp::Exception::Exception ( const std::string & where, const Type & type ) [inline]

Constructs an exception.

where	Text representing where the exception occured
type	Exception type, defined in qpp::Exception::Type

7.3.3.2 qpp::Exception::Exception ( const std::string & where, const std::string & custom ) [inline]

Constructs an exception.

This is an overloaded member function, provided for convenience. It differs from the above function only in what argument(s) it accepts.

#### **Parameters**

where	Text representing where the exception occured
custom	Exception description

## 7.3.4 Member Function Documentation

7.3.4.1 void qpp::Exception::\_construct\_exception\_msg() [inline], [private]

Constructs the exception description from its type.

See also

qpp::Exception::Type

Must modify the code of this function if more exceptions are added

7.3.4.2 virtual const char\* qpp::Exception::what( ) const [inline], [override], [virtual], [noexcept]

Overrides std::exception::what()

Returns

**Exception** description

## 7.3.5 Member Data Documentation

**7.3.5.1** std::string qpp::Exception::\_custom [private]

**7.3.5.2** std::string qpp::Exception::\_msg [private]

**7.3.5.3 Type qpp::Exception::\_type** [private]

**7.3.5.4 std::string qpp::Exception::\_where** [private]

The documentation for this class was generated from the following file:

· classes/exception.h

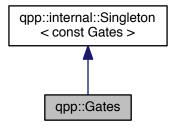
# 7.4 qpp::Gates Class Reference

const Singleton class that implements most commonly used gates

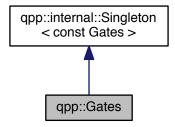
92 Class Documentation

#include <classes/gates.h>

Inheritance diagram for qpp::Gates:



Collaboration diagram for qpp::Gates:



## **Public Member Functions**

- cmat Rn (double theta, const std::vector< double > &n) const
   Qubit rotation of theta about the 3-dimensional real (unit) vector n.
- cmat Zd (idx D) const

Generalized Z gate for qudits.

• cmat Fd (idx D) const

Fourier transform gate for qudits.

• cmat Xd (idx D) const

Generalized X gate for qudits.

template<typename Derived = Eigen::MatrixXcd>
 Derived Id (idx D) const

Identity gate.

• template<typename Derived >

 $\frac{dyn\_mat}{dx} < typename\ Derived::Scalar > CTRL\ (const\ Eigen::MatrixBase < Derived > \&A,\ const\ std::vector < idx > \&ctrl,\ const\ std::vector < idx > \&subsys,\ idx\ n,\ idx\ d=2)\ const$ 

Generates the multi-partite multiple-controlled-A gate in matrix form.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > expandout (const Eigen::MatrixBase< Derived > &A, idx pos, const std::vector< idx > &dims) const
 Expands out.

#### **Public Attributes**

cmat Id2 {cmat::Identity(2, 2)}
 Identity gate.

cmat H {cmat::Zero(2, 2)}

Hadamard gate.

cmat X {cmat::Zero(2, 2)}

Pauli Sigma-X gate.

cmat Y {cmat::Zero(2, 2)}

Pauli Sigma-Y gate.

cmat Z {cmat::Zero(2, 2)}

Pauli Sigma-Z gate.

• cmat S {cmat::Zero(2, 2)}

S gate.

cmat T {cmat::Zero(2, 2)}

T gate.

cmat CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

cmat CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

• cmat CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

• cmat SWAP {cmat::Identity(4, 4)}

SWAP gate.

• cmat TOF {cmat::ldentity(8, 8)}

Toffoli gate.

• cmat FRED {cmat::Identity(8, 8)}

Fredkin gate.

#### **Private Member Functions**

• Gates ()

Initializes the gates.

∼Gates ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const Gates >

#### **Additional Inherited Members**

#### 7.4.1 Detailed Description

const Singleton class that implements most commonly used gates

#### 7.4.2 Constructor & Destructor Documentation

```
7.4.2.1 qpp::Gates::Gates() [inline], [private]
```

Initializes the gates.

```
7.4.2.2 qpp::Gates::∼Gates() [private], [default]
```

Default destructor.

#### 7.4.3 Member Function Documentation

7.4.3.1 template < typename Derived > dyn\_mat < typename Derived::Scalar > qpp::Gates::CTRL ( const Eigen::MatrixBase < Derived > & A, const std::vector < idx > & ctrl, const std::vector < idx > & subsys, idx n, idx d = 2 ) const [inline]

Generates the multi-partite multiple-controlled-A gate in matrix form.

See also

```
qpp::applyCTRL()
```

Note

The dimension of the gate A must match the dimension of subsys

#### **Parameters**

Α	Eigen expression
ctrl	Control subsystem indexes
subsys	Subsystem indexes where the gate A is applied
n	Total number of subsystems
d	Subsystem dimensions

#### Returns

CTRL-A gate, as a matrix over the same scalar field as A

7.4.3.2 template<typename Derived > dyn\_mat<typename Derived::Scalar> qpp::Gates::expandout ( const Eigen::MatrixBase< Derived > & A, idx pos, const std::vector< idx > & dims ) const [inline]

Expands out.

See also

qpp::kron()

Expands out A as a matrix in a multi-partite system. Faster than using qpp::kron(I, I, ..., I, A, I, ..., I).

#### **Parameters**

Α	Eigen expression

pos	Position
dims	Dimensions of the multi-partite system

#### Returns

Tensor product  $I\otimes\cdots\otimes I\otimes A\otimes I\otimes\cdots\otimes I$ , with A on position pos, as a dynamic matrix over the same scalar field as A

7.4.3.3 cmat qpp::Gates::Fd(idx D) const [inline]

Fourier transform gate for qudits.

Note

Defined as 
$$F = \sum_{jk} \exp(2\pi i j k/D) |j\rangle\langle k|$$

#### **Parameters**

D	Dimension of the Hilbert space
---	--------------------------------

#### Returns

Fourier transform gate for qudits

7.4.3.4 template<typename Derived = Eigen::MatrixXcd> Derived qpp::Gates::Id ( idx D ) const [inline]

Identity gate.

Note

Can change the return type from complex matrix (default) by explicitly specifying the template parameter

#### **Parameters**

D	Dimension of the Hilbert space

#### Returns

Identity gate

7.4.3.5 cmat qpp::Gates::Rn ( double theta, const std::vector < double > & n ) const [inline]

Qubit rotation of *theta* about the 3-dimensional real (unit) vector *n*.

#### **Parameters**

theta	Rotation angle
n	3-dimensional real (unit) vector

#### Returns

Rotation gate

7.4.3.6 cmat qpp::Gates::Xd(idx D)const [inline]

Generalized X gate for qudits.

Note

Defined as  $X=\sum_{j}|j\oplus 1\rangle\langle j|$ , i.e. raising operator  $X|j\rangle=|j\oplus 1\rangle>$ 

#### **Parameters**

D Dimension of the Hilbert space

Returns

Generalized X gate for qudits

7.4.3.7 cmat qpp::Gates::Zd(idx D) const [inline]

Generalized Z gate for qudits.

Note

Defined as  $Z = \sum_{j} \exp(2\pi i j/D) |j\rangle\langle j|$ 

**Parameters** 

D Dimension of the Hilbert space

Returns

Generalized Z gate for qudits

- 7.4.4 Friends And Related Function Documentation
- 7.4.4.1 friend class internal::Singleton < const Gates > [friend]
- 7.4.5 Member Data Documentation
- 7.4.5.1 cmat qpp::Gates::CNOT {cmat::Identity(4, 4)}

Controlled-NOT control target gate.

7.4.5.2 cmat qpp::Gates::CNOTba {cmat::Zero(4, 4)}

Controlled-NOT target control gate.

7.4.5.3 cmat qpp::Gates::CZ {cmat::Identity(4, 4)}

Controlled-Phase gate.

7.4.5.4 cmat qpp::Gates::FRED {cmat::Identity(8, 8)}

Fredkin gate.

```
7.4.5.5 cmat qpp::Gates::H {cmat::Zero(2, 2)}
Hadamard gate.
7.4.5.6 cmat qpp::Gates::ld2 {cmat::ldentity(2, 2)}
Identity gate.
7.4.5.7 cmat qpp::Gates::S {cmat::Zero(2, 2)}
S gate.
7.4.5.8 cmat qpp::Gates::SWAP {cmat::Identity(4, 4)}
SWAP gate.
7.4.5.9 cmat qpp::Gates::T {cmat::Zero(2, 2)}
T gate.
7.4.5.10 cmat qpp::Gates::TOF {cmat::Identity(8, 8)}
Toffoli gate.
7.4.5.11 cmat qpp::Gates::X {cmat::Zero(2, 2)}
Pauli Sigma-X gate.
7.4.5.12 cmat qpp::Gates::Y {cmat::Zero(2, 2)}
Pauli Sigma-Y gate.
7.4.5.13 cmat qpp::Gates::Z {cmat::Zero(2, 2)}
Pauli Sigma-Z gate.
The documentation for this class was generated from the following file:

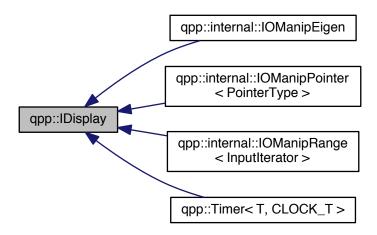
    classes/gates.h
```

# 7.5 qpp::IDisplay Class Reference

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

#include <classes/idisplay.h>

Inheritance diagram for qpp::IDisplay:



#### **Public Member Functions**

• IDisplay ()=default

Default constructor.

• IDisplay (const IDisplay &)=default

Default copy constructor.

• IDisplay (IDisplay &&)=default

Default move constructor.

• IDisplay & operator= (const IDisplay &)=default

Default copy assignment operator.

• IDisplay & operator= (IDisplay &&)=default

Default move assignment operator.

virtual ∼IDisplay ()=default

Default virtual destructor.

#### **Private Member Functions**

• virtual std::ostream & display (std::ostream &os) const =0

Must be overridden by all derived classes.

#### **Friends**

• std::ostream & operator<< (std::ostream &os, const IDisplay &rhs)

Overloads the extraction operator.

#### 7.5.1 Detailed Description

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

This class defines friend inline std::ostream& operator<< (std::ostream& os, const qpp::IDisplay& rhs). The latter delegates the work to the pure private virtual function qpp::IDisplay::display() which has to be overridden by all derived classes.

#### 7.5.2 Constructor & Destructor Documentation

```
7.5.2.1 qpp::IDisplay::IDisplay() [default]
```

Default constructor.

```
7.5.2.2 qpp::IDisplay::IDisplay ( const IDisplay & ) [default]
```

Default copy constructor.

```
7.5.2.3 qpp::IDisplay::IDisplay ( IDisplay && ) [default]
```

Default move constructor.

```
7.5.2.4 virtual qpp::IDisplay::~IDisplay( ) [virtual], [default]
```

Default virtual destructor.

#### 7.5.3 Member Function Documentation

```
7.5.3.1 virtual std::ostream& qpp::IDisplay::display ( std::ostream & os ) const [private], [pure virtual]
```

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs).

Implemented in qpp::internal::IOManipEigen, qpp::internal::IOManipPointer< PointerType >, qpp::internal::IO $\leftarrow$  ManipRange< InputIterator >, and qpp::Timer< T, CLOCK\_T >.

```
7.5.3.2 IDisplay& qpp::IDisplay::operator=( const IDisplay & ) [default]
```

Default copy assignment operator.

```
7.5.3.3 IDisplay& qpp::IDisplay::operator=(IDisplay&&) [default]
```

Default move assignment operator.

#### 7.5.4 Friends And Related Function Documentation

```
7.5.4.1 std::ostream& operator<< ( std::ostream & os, const | Display & rhs ) [friend]
```

Overloads the extraction operator.

Delegates the work to the virtual function qpp::IDisplay::display()

The documentation for this class was generated from the following file:

· classes/idisplay.h

# 7.6 qpp::Init Class Reference

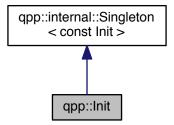
const Singleton class that performs additional initializations/cleanups

```
#include <classes/init.h>
```

Inheritance diagram for qpp::Init:



Collaboration diagram for qpp::Init:



## **Private Member Functions**

• Init ()

Additional initializations.

• ∼Init ()

Cleanups.

#### **Friends**

• class internal::Singleton< const Init >

#### **Additional Inherited Members**

#### 7.6.1 Detailed Description

const Singleton class that performs additional initializations/cleanups

#### 7.6.2 Constructor & Destructor Documentation

```
7.6.2.1 qpp::Init::Init( ) [inline],[private]
```

Additional initializations.

```
7.6.2.2 qpp::Init::~Init() [inline], [private]
```

Cleanups.

#### 7.6.3 Friends And Related Function Documentation

**7.6.3.1** friend class internal::Singleton < const lnit > [friend]

The documentation for this class was generated from the following file:

· classes/init.h

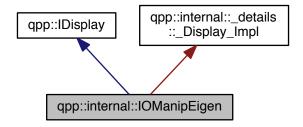
# 7.7 qpp::internal::IOManipEigen Class Reference

```
#include <internal/classes/iomanip.h>
```

Inheritance diagram for qpp::internal::IOManipEigen:



Collaboration diagram for qpp::internal::IOManipEigen:



#### **Public Member Functions**

- template<typename Derived >
   IOManipEigen (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)
- IOManipEigen (const cplx z, double chop=qpp::chop)

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- · cmat A
- double \_chop

#### 7.7.1 Constructor & Destructor Documentation

- 7.7.1.1 template<typename Derived > qpp::internal::IOManipEigen::IOManipEigen ( const Eigen::MatrixBase< Derived > & A, double chop = qpp::chop ) [inline], [explicit]
- 7.7.1.2 qpp::internal::IOManipEigen::IOManipEigen ( const cplx z, double chop = qpp::chop ) [inline], [explicit]

#### 7.7.2 Member Function Documentation

7.7.2.1 std::ostream& qpp::internal::IOManipEigen::display( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

#### 7.7.3 Member Data Documentation

7.7.3.1 cmat qpp::internal::IOManipEigen::\_A [private]

**7.7.3.2** double qpp::internal::IOManipEigen::\_chop [private]

The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

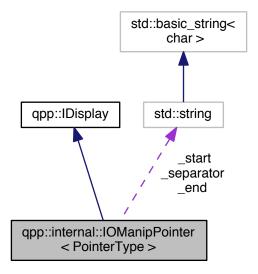
# 7.8 qpp::internal::IOManipPointer< PointerType > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipPointer< PointerType >:



Collaboration diagram for qpp::internal::IOManipPointer< PointerType >:



#### **Public Member Functions**

- IOManipPointer (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipPointer (const IOManipPointer &)=default
- IOManipPointer & operator= (const IOManipPointer &)=default

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- const PointerType \* \_p
- idx n
- · std::string separator
- · std::string \_start
- · std::string \_end

#### 7.8.1 Constructor & Destructor Documentation

- 7.8.1.1 template < typename PointerType > qpp::internal::IOManipPointer < PointerType >::IOManipPointer ( const PointerType \* p, idx n, const std::string & separator, const std::string & start = " [ ", const std::string & end = " ] " ) [inline], [explicit]
- 7.8.1.2 template<typename PointerType> qpp::internal::IOManipPointer< PointerType>::IOManipPointer( const IOManipPointer<< PointerType> & ) [default]
- 7.8.2 Member Function Documentation

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.8.2.2 template<typename PointerType> IOManipPointer& qpp::internal::IOManipPointer< PointerType
  >::operator=( const IOManipPointer< PointerType > & ) [default]
- 7.8.3 Member Data Documentation
- 7.8.3.1 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_end [private]
- 7.8.3.2 template<typename PointerType> idx qpp::internal::IOManipPointer< PointerType>::\_n [private]
- 7.8.3.3 template<typename PointerType> const PointerType\* qpp::internal::IOManipPointer< PointerType >::\_p [private]

- 7.8.3.4 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType>::\_separator [private]
- 7.8.3.5 template<typename PointerType> std::string qpp::internal::IOManipPointer< PointerType >::\_start [private]

The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

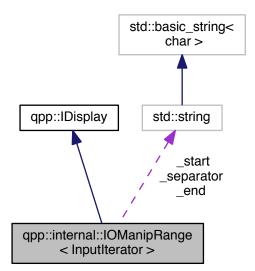
# 7.9 qpp::internal::IOManipRange < InputIterator > Class Template Reference

#include <internal/classes/iomanip.h>

Inheritance diagram for qpp::internal::IOManipRange< InputIterator >:



Collaboration diagram for qpp::internal::IOManipRange< InputIterator >:



#### **Public Member Functions**

- IOManipRange (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")
- IOManipRange (const IOManipRange &)=default
- IOManipRange & operator= (const IOManipRange &)=default

#### **Private Member Functions**

std::ostream & display (std::ostream &os) const override
 Must be overridden by all derived classes.

#### **Private Attributes**

- · InputIterator first
- · InputIterator last
- · std::string \_separator
- std::string \_start
- · std::string \_end

#### 7.9.1 Constructor & Destructor Documentation

- 7.9.1.2 template<typename InputIterator> qpp::internal::IOManipRange< InputIterator>::IOManipRange ( const IOManipRange < InputIterator > & ) [default]
- 7.9.2 Member Function Documentation
- 7.9.2.1 template<typename InputIterator> std::ostream& qpp::internal::IOManipRange< InputIterator >::display ( std::ostream & os ) const [inline], [override], [private], [virtual]

Must be overridden by all derived classes.

The actual stream extraction processing is performed by the overriden member function in the derived class. This function is automatically invoked by friend inline std::ostream& operator<<(std::ostream& os, const IDisplay& rhs). Implements qpp::IDisplay.

- 7.9.2.2 template<typename InputIterator> IOManipRange& qpp::internal::IOManipRange< InputIterator >::operator= ( const IOManipRange< InputIterator > & ) [default]
- 7.9.3 Member Data Documentation
- **7.9.3.1 template**<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_end [private]
- 7.9.3.2 template<typename InputIterator> InputIterator qpp::internal::IOManipRange< InputIterator>::\_first [private]
- 7.9.3.3 template<typename InputIterator> InputIterator qpp::internal::IOManipRange< InputIterator>::\_last [private]

- 7.9.3.4 template<typename InputIterator> std::string qpp::internal::IOManipRange< InputIterator >::\_separator [private]
- $\textbf{7.9.3.5} \quad \textbf{template} < \textbf{typename InputIterator} > \textbf{std::string qpp::internal::IOManipRange} < \textbf{InputIterator} > \textbf{::\_start} \\ [\texttt{private}]$

The documentation for this class was generated from the following file:

• internal/classes/iomanip.h

# 7.10 qpp::is\_complex < T > Struct Template Reference

Checks whether the type is a complex type.

#include <traits.h>

Inheritance diagram for qpp::is\_complex< T >:



Collaboration diagram for qpp::is\_complex< T >:



# 7.10.1 Detailed Description

template < typename T > struct qpp::is\_complex < T >

Checks whether the type is a complex type.

Provides the member constant *value* which is equal to *true*, if the type is a complex type, i.e. *std::complex<T>* 

The documentation for this struct was generated from the following file:

· traits.h

# 7.11 qpp::is\_complex < std::complex < T > > Struct Template Reference

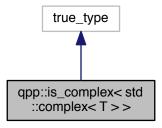
Checks whether the type is a complex number type, specialization for complex types.

```
#include <traits.h>
```

Inheritance diagram for qpp::is\_complex < std::complex < T > >:



Collaboration diagram for qpp::is\_complex< std::complex< T > :



#### 7.11.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_complex < std::complex < T > >$ 

Checks whether the type is a complex number type, specialization for complex types.

The documentation for this struct was generated from the following file:

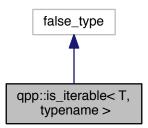
· traits.h

# 7.12 qpp::is\_iterable < T, typename > Struct Template Reference

Checks whether *T* is compatible with an STL-like iterable container.

#include <traits.h>

Inheritance diagram for qpp::is\_iterable < T, typename >:



Collaboration diagram for qpp::is\_iterable < T, typename >:



#### 7.12.1 Detailed Description

 $template < typename \ {\tt T}, typename \ {\tt = void} > {\tt struct \ qpp::is\_iterable} < \ {\tt T}, typename >$ 

Checks whether *T* is compatible with an STL-like iterable container.

Provides the member constant *value* which is equal to *true*, if *T* is compatible with an iterable container, i.e. provides at least *begin()* and *end()* member functions. Otherwise, *value* is equal to *false*.

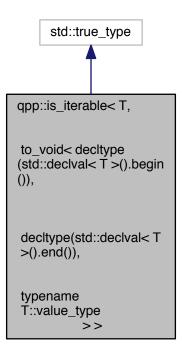
The documentation for this struct was generated from the following file:

• traits.h

7.13 qpp::is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().end()), typename T::value\_type > > Struct Template Reference

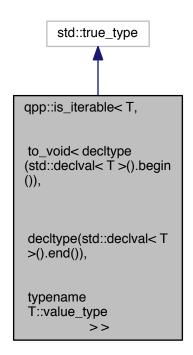
Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. #include <traits.h>

Inheritance diagram for qpp::is\_iterable < T, to\_void < decltype(std::declval < T >().begin()), decltype(std::declval < T >().end()), typename T::value\_type > >:



 $Collaboration \ \ diagram \ \ for \ \ qpp::is\_iterable < \ \ T, \ \ to\_void < \ \ decltype(std::declval < \ T \ >().begin()), \ \ decltype(std::declval < \ T \ >().begin())$ 

 $: declval < T > ().end()), \ typename \ T:: value\_type > >:$ 



#### 7.13.1 Detailed Description

 $template < typename \ T > struct \ qpp::is\_iterable < \ T, \ to\_void < \ decltype(std::declval < \ T > ().begin()), \ decltype(std::declval < \ T > ().end()), \ typename \ T::value\_type > >$ 

Checks whether *T* is compatible with an STL-like iterable container, specialization for STL-like iterable containers. The documentation for this struct was generated from the following file:

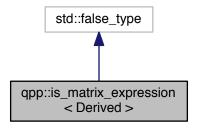
• traits.h

# 7.14 qpp::is\_matrix\_expression < Derived > Struct Template Reference

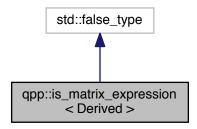
Checks whether the type is an Eigen matrix expression.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< Derived >:



Collaboration diagram for qpp::is\_matrix\_expression< Derived >:



#### 7.14.1 Detailed Description

 $template {<} typename \ Derived {>} struct \ qpp::is\_matrix\_expression {<} \ Derived {>}$ 

Checks whether the type is an Eigen matrix expression.

Provides the member constant *value* which is equal to *true*, if the type is an Eigen matrix expression of type *Eigen* :: *MatrixBase* < *Derived* >. Otherwise, *value* is equal to *false*.

The documentation for this struct was generated from the following file:

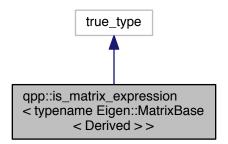
· traits.h

# 7.15 qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >> Struct Template Reference

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

#include <traits.h>

Inheritance diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



Collaboration diagram for qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived >>:



#### 7.15.1 Detailed Description

 $template < typename\ Derived > struct\ qpp::is\_matrix\_expression < typename\ Eigen::MatrixBase < Derived > > typename\ Derived > typename\ Derived > typename\ Derived > typename\ Derived > > typename\ Derived > > typename\ Derived > > typename\ Derived >$ 

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions. The documentation for this struct was generated from the following file:

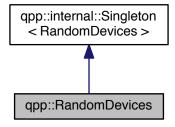
• traits.h

# 7.16 qpp::RandomDevices Class Reference

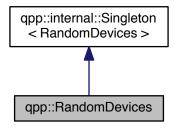
Singeleton class that manages the source of randomness in the library.

#include <classes/random\_devices.h>

Inheritance diagram for qpp::RandomDevices:



Collaboration diagram for qpp::RandomDevices:



#### **Public Attributes**

std::mt19937 \_rng

Mersenne twister random number generator.

## **Private Member Functions**

• RandomDevices ()

Initializes and seeds the random number generators.

•  $\sim$ RandomDevices ()=default

Default destructor.

#### **Private Attributes**

std::random\_device \_rd
 used to seed std::mt19937 \_rng

#### **Friends**

class internal::Singleton < RandomDevices >

#### **Additional Inherited Members**

#### 7.16.1 Detailed Description

Singeleton class that manages the source of randomness in the library.

Consists of a wrapper around an std::mt19937 Mersenne twister random number generator engine and an std⇔ ::random\_device engine. The latter is used to seed the Mersenne twister.

#### Warning

This class DOES NOT seed the standard C number generator used by Eigen::Matrix::Random(), since it is not thread safe. Do not use Eigen::Matrix::Random() or functions that depend on the C style random number engine, but use <a href="mailto:qpp::rand()">qpp::rand()</a> instead!

#### 7.16.2 Constructor & Destructor Documentation

```
7.16.2.1 qpp::RandomDevices::RandomDevices() [inline], [private]
```

Initializes and seeds the random number generators.

```
7.16.2.2 qpp::RandomDevices::~RandomDevices() [private], [default]
```

Default destructor.

#### 7.16.3 Friends And Related Function Documentation

```
7.16.3.1 friend class internal::Singleton < RandomDevices > [friend]
```

#### 7.16.4 Member Data Documentation

```
7.16.4.1 std::random_device qpp::RandomDevices::_rd [private]
```

used to seed std::mt19937 \_rng

```
7.16.4.2 std::mt19937 qpp::RandomDevices::_rng
```

Mersenne twister random number generator.

The documentation for this class was generated from the following file:

• classes/random\_devices.h

# 7.17 qpp::internal::Singleton< T > Class Template Reference

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

```
#include <internal/classes/singleton.h>
```

#### **Static Public Member Functions**

- static T & get\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)
- static T & get\_thread\_local\_instance () noexcept(std::is\_nothrow\_constructible < T >::value)

#### **Protected Member Functions**

- Singleton () noexcept=default
- Singleton (const Singleton &)=delete
- Singleton & operator= (const Singleton &)=delete
- virtual ∼Singleton ()=default

#### 7.17.1 Detailed Description

template < typename T> class qpp::internal::Singleton < T>

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

To implement a singleton, derive your class from qpp::internal::Singleton, make qpp::internal::Singleton a friend of your class, then declare the constructor and destructor of your class as private. To get an instance, use the static member function qpp::internal::Singleton::get\_instance() (qpp::internal::Singleton::get\_thread\_local\_← instance()), which returns a reference (thread\_local\_reference) to your newly created singleton (thread-safe in C++11).

#### Example:

#### See also

Code of qpp::Codes, qpp::Gates, qpp::Init, qpp::RandomDevices, qpp::States or qpp.h for real world examples of usage.

#### 7.17.2 Constructor & Destructor Documentation

```
7.17.2.1 template<typename T> qpp::internal::Singleton< T>::Singleton( ) [protected], [default], [noexcept]
```

```
7.17.2.2 template<typename T> qpp::internal::Singleton< T>::Singleton ( const Singleton< T> & ) [protected], [delete]
```

```
7.17.2.3 template<typename T> virtual qpp::internal::Singleton< T>::\simSingleton( ) [protected], [virtual], [default]
```

#### 7.17.3 Member Function Documentation

- 7.17.3.3 template<typename T> Singleton& qpp::internal::Singleton< T>::operator=( const Singleton< T>& ) [protected], [delete]

The documentation for this class was generated from the following file:

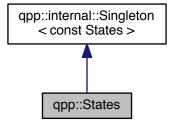
• internal/classes/singleton.h

# 7.18 qpp::States Class Reference

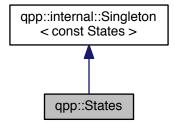
const Singleton class that implements most commonly used states

#include <classes/states.h>

Inheritance diagram for qpp::States:



Collaboration diagram for qpp::States:



#### **Public Attributes**

```
    ket x0 {ket::Zero(2)}

      Pauli Sigma-X 0-eigenstate |+>

    ket x1 {ket::Zero(2)}

      Pauli Sigma-X 1-eigenstate |->

    ket y0 {ket::Zero(2)}

      Pauli Sigma-Y 0-eigenstate | y+>

    ket y1 {ket::Zero(2)}

      Pauli Sigma-Y 1-eigenstate | y->

    ket z0 {ket::Zero(2)}

      Pauli Sigma-Z 0-eigenstate | 0>

    ket z1 {ket::Zero(2)}

      Pauli Sigma-Z 1-eigenstate | 1>
• cmat px0 {cmat::Zero(2, 2)}
      Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.

    cmat px1 {cmat::Zero(2, 2)}

      Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.

    cmat py0 {cmat::Zero(2, 2)}

      Projector onto the Pauli Sigma-Y 0-eigenstate |y+>< y+|.

    cmat py1 {cmat::Zero(2, 2)}

      Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.

    cmat pz0 {cmat::Zero(2, 2)}

      Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.

    cmat pz1 {cmat::Zero(2, 2)}

      Projector onto the Pauli Sigma-Z 1-eigenstate | 1><1|.

    ket b00 {ket::Zero(4)}

      Bell-00 state (following the convention in Nielsen and Chuang)
ket b01 {ket::Zero(4)}
      Bell-01 state (following the convention in Nielsen and Chuang)

    ket b10 {ket::Zero(4)}

      Bell-10 state (following the convention in Nielsen and Chuang)
ket b11 {ket::Zero(4)}
      Bell-11 state (following the convention in Nielsen and Chuang)

    cmat pb00 {cmat::Zero(4, 4)}

      Projector onto the Bell-00 state.

    cmat pb01 {cmat::Zero(4, 4)}

      Projector onto the Bell-01 state.

    cmat pb10 {cmat::Zero(4, 4)}

      Projector onto the Bell-10 state.

    cmat pb11 {cmat::Zero(4, 4)}

      Projector onto the Bell-11 state.
ket GHZ {ket::Zero(8)}
      GHZ state.
ket W {ket::Zero(8)}
      W state.
cmat pGHZ {cmat::Zero(8, 8)}
      Projector onto the GHZ state.
cmat pW {cmat::Zero(8, 8)}
```

Projector onto the W state.

#### **Private Member Functions**

- States ()
- ∼States ()=default

Default destructor.

#### **Friends**

class internal::Singleton < const States >

#### **Additional Inherited Members**

#### 7.18.1 Detailed Description

const Singleton class that implements most commonly used states

#### 7.18.2 Constructor & Destructor Documentation

```
7.18.2.1 qpp::States::States( ) [inline],[private]
```

Initialize the states

```
7.18.2.2 qpp::States::~States() [private], [default]
```

Default destructor.

#### 7.18.3 Friends And Related Function Documentation

```
7.18.3.1 friend class internal::Singleton < const States > [friend]
```

#### 7.18.4 Member Data Documentation

```
7.18.4.1 ket qpp::States::b00 {ket::Zero(4)}
```

Bell-00 state (following the convention in Nielsen and Chuang)

```
7.18.4.2 ket qpp::States::b01 {ket::Zero(4)}
```

Bell-01 state (following the convention in Nielsen and Chuang)

```
7.18.4.3 ket qpp::States::b10 {ket::Zero(4)}
```

Bell-10 state (following the convention in Nielsen and Chuang)

7.18.4.4 ket qpp::States::b11 {ket::Zero(4)}

Bell-11 state (following the convention in Nielsen and Chuang)

```
7.18.4.5 ket qpp::States::GHZ {ket::Zero(8)}
GHZ state.
7.18.4.6 cmat qpp::States::pb00 {cmat::Zero(4, 4)}
Projector onto the Bell-00 state.
7.18.4.7 cmat qpp::States::pb01 {cmat::Zero(4, 4)}
Projector onto the Bell-01 state.
7.18.4.8 cmat qpp::States::pb10 {cmat::Zero(4, 4)}
Projector onto the Bell-10 state.
7.18.4.9 cmat qpp::States::pb11 {cmat::Zero(4, 4)}
Projector onto the Bell-11 state.
7.18.4.10 cmat qpp::States::pGHZ {cmat::Zero(8, 8)}
Projector onto the GHZ state.
7.18.4.11 cmat qpp::States::pW {cmat::Zero(8, 8)}
Projector onto the W state.
7.18.4.12 cmat qpp::States::px0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 0-eigenstate |+><+|.
7.18.4.13 cmat qpp::States::px1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-X 1-eigenstate |-><-|.
7.18.4.14 cmat qpp::States::py0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 0-eigenstate |y+>< y+|.
7.18.4.15 cmat qpp::States::py1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Y 1-eigenstate |y-><y-|.
7.18.4.16 cmat qpp::States::pz0 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 0-eigenstate |0><0|.
```

```
7.18.4.17 cmat qpp::States::pz1 {cmat::Zero(2, 2)}
Projector onto the Pauli Sigma-Z 1-eigenstate |1><1|.
7.18.4.18 ket qpp::States::W {ket::Zero(8)}
W state.
7.18.4.19 ket qpp::States::x0 {ket::Zero(2)}
Pauli Sigma-X 0-eigenstate |+>
7.18.4.20 ket qpp::States::x1 {ket::Zero(2)}
Pauli Sigma-X 1-eigenstate |->
7.18.4.21 ket qpp::States::y0 {ket::Zero(2)}
Pauli Sigma-Y 0-eigenstate |y+>
7.18.4.22 ket qpp::States::y1 {ket::Zero(2)}
Pauli Sigma-Y 1-eigenstate |y->
7.18.4.23 ket qpp::States::z0 {ket::Zero(2)}
Pauli Sigma-Z 0-eigenstate |0>
7.18.4.24 ket qpp::States::z1 {ket::Zero(2)}
Pauli Sigma-Z 1-eigenstate |1>
The documentation for this class was generated from the following file:

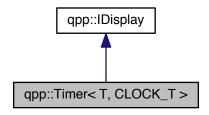
    classes/states.h
```

# 7.19 qpp::Timer < T, CLOCK\_T > Class Template Reference

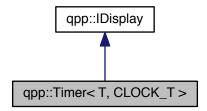
#### Chronometer.

#include <classes/timer.h>

Inheritance diagram for qpp::Timer< T, CLOCK\_T >:



Collaboration diagram for qpp::Timer< T, CLOCK\_T >:



#### **Public Member Functions**

• Timer () noexcept

Constructs an instance with the current time as the starting point.

· void tic () noexcept

Resets the chronometer.

• const Timer & toc () noexcept

Stops the chronometer.

• double tics () const noexcept

Time passed in the duration specified by T.

• template<typename U = T>

U get\_duration () const noexcept

Duration specified by U.

• Timer (const Timer &)=default

Default copy constructor.

• Timer (Timer &&)=default

Default move constructor.

• Timer & operator= (const Timer &)=default

Default copy assignment operator.

Timer & operator= (Timer &&)=default

Default move assignment operator.

virtual ∼Timer ()=default

Default virtual destructor.

#### **Protected Attributes**

- CLOCK\_T::time\_point \_start
- CLOCK\_T::time\_point \_end

#### **Private Member Functions**

• std::ostream & display (std::ostream &os) const override

qpp::IDisplay::display() override

#### 7.19.1 Detailed Description

 $template < typename \ T = std::chrono::duration < double >, \ typename \ CLOCK\_T = std::chrono::steady\_clock > class \ qpp::Timer < T, CLOCK\_T >$ 

#### Chronometer.

**Template Parameters** 

T	Tics duration, default is std::chrono::duration <double, 1="">, i.e. seconds in double</double,>
	precision
CLOCK_T	Clock's type, default is std::chrono::steady_clock, not affected by wall clock
	changes during runtime

#### 7.19.2 Constructor & Destructor Documentation

Constructs an instance with the current time as the starting point.

```
7.19.2.2 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> qpp::Timer< T, CLOCK_T > ::Timer( const Timer< T, CLOCK_T > & ) [default]
```

Default copy constructor.

```
7.19.2.3 template < typename T = std::chrono::duration < double >, typename CLOCK_T = std::chrono::steady_clock > qpp::Timer < T, CLOCK_T >::Timer ( T, CLOCK_T > && ) [default]
```

Default move constructor.

Default virtual destructor.

### 7.19.3 Member Function Documentation

qpp::IDisplay::display() override

#### **Parameters**

OS	Output stream
----	---------------

#### Returns

Writes to the output stream the number of tics (specified by T) that passed between the instantiation/reset and invocation of <a href="mailto:qpp::Timer::toc()">qpp::Timer::toc()</a>.

Implements qpp::IDisplay.

Duration specified by U.

#### **Template Parameters**

U	Duration, default is T, which defaults to std::chrono::duration <double, 1="">, i.e.</double,>
	seconds in double precision

#### Returns

Duration that passed between the instantiation/reset and invocation of qpp::Timer::toc()

```
7.19.3.3 template < typename T = std::chrono::duration < double >, typename CLOCK_T = std::chrono::steady_clock > Timer & qpp::Timer < T, CLOCK_T > ::operator = ( const Timer < T, CLOCK_T > & ) [ default ]
```

Default copy assignment operator.

7.19.3.4 template<typename T = std::chrono::duration<double>, typename CLOCK\_T = std::chrono::steady\_clock> Timer& qpp::Timer< T, CLOCK\_T > ::operator=( Timer< T, CLOCK\_T > && ) [default]

Default move assignment operator.

```
7.19.3.5 template<typename T = std::chrono::duration<double>, typename CLOCK_T = std::chrono::steady_clock> void qpp::Timer< T, CLOCK_T >::tic() [inline], [noexcept]
```

Resets the chronometer.

Resets the starting/ending point to the current time

Time passed in the duration specified by T.

#### Returns

Number of tics (specified by T) that passed between the instantiation/reset and invocation of qpp::Timer::toc()

Stops the chronometer.

Set the current time as the ending point

Returns

Current instance

- 7.19.4 Member Data Documentation
- 7.19.4.2 template<typename T = std::chrono::duration<double>, typename CLOCK\_T = std::chrono::steady\_clock> CLOCK\_T::time\_point qpp::Timer< T, CLOCK\_T >::\_start [protected]

The documentation for this class was generated from the following file:

· classes/timer.h

# **Chapter 8**

# **File Documentation**

# 8.1 classes/codes.h File Reference

Quantum error correcting codes.

This graph shows which files directly or indirectly include this file:



#### Classes

• class qpp::Codes

const Singleton class that defines quantum error correcting codes

### **Namespaces**

• qpp

Quantum++ main namespace.

### 8.1.1 Detailed Description

Quantum error correcting codes.

# 8.2 classes/exception.h File Reference

Exceptions.

128 File Documentation

This graph shows which files directly or indirectly include this file:



#### **Classes**

• class qpp::Exception

Generates custom exceptions, used when validating function parameters.

#### **Namespaces**

• qpp

Quantum++ main namespace.

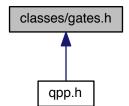
### 8.2.1 Detailed Description

Exceptions.

# 8.3 classes/gates.h File Reference

Quantum gates.

This graph shows which files directly or indirectly include this file:



### Classes

class qpp::Gates

const Singleton class that implements most commonly used gates

### **Namespaces**

qpp

Quantum++ main namespace.

# 8.3.1 Detailed Description

Quantum gates.

# 8.4 classes/idisplay.h File Reference

Display interface via the non-virtual interface (NVI)

This graph shows which files directly or indirectly include this file:



### **Classes**

· class qpp::IDisplay

Abstract class (interface) that mandates the definition of virtual std::ostream& display(std::ostream& os) const.

# **Namespaces**

• qpp

Quantum++ main namespace.

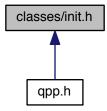
# 8.4.1 Detailed Description

Display interface via the non-virtual interface (NVI)

# 8.5 classes/init.h File Reference

Initialization.

This graph shows which files directly or indirectly include this file:



### Classes

• class qpp::Init

const Singleton class that performs additional initializations/cleanups

# **Namespaces**

• qpp

Quantum++ main namespace.

# 8.5.1 Detailed Description

Initialization.

# 8.6 classes/random\_devices.h File Reference

Random devices.

This graph shows which files directly or indirectly include this file:



# **Classes**

• class qpp::RandomDevices

Singeleton class that manages the source of randomness in the library.

# **Namespaces**

qpp

Quantum++ main namespace.

# 8.6.1 Detailed Description

Random devices.

# 8.7 classes/states.h File Reference

Quantum states.

This graph shows which files directly or indirectly include this file:



# Classes

• class qpp::States

const Singleton class that implements most commonly used states

# **Namespaces**

• qpp

Quantum++ main namespace.

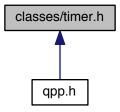
# 8.7.1 Detailed Description

Quantum states.

# 8.8 classes/timer.h File Reference

Timing.

This graph shows which files directly or indirectly include this file:



### Classes

class qpp::Timer < T, CLOCK\_T >
 Chronometer.

# **Namespaces**

qpp

Quantum++ main namespace.

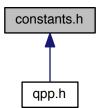
# 8.8.1 Detailed Description

Timing.

# 8.9 constants.h File Reference

Constants.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

qpp

Quantum++ main namespace.

### **Functions**

• constexpr cplx qpp::operator""\_i (unsigned long long int x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (integer overload)

• constexpr cplx qpp::operator""\_i (long double x) noexcept

User-defined literal for complex  $i = \sqrt{-1}$  (real overload)

• cplx qpp::omega (idx D)

D-th root of unity.

#### **Variables**

• constexpr double qpp::chop = 1e-10

Used in qpp::disp() for setting to zero numbers that have their absolute value smaller than qpp::chop.

• constexpr double qpp::eps = 1e-12

Used to decide whether a number or expression in double precision is zero or not.

• constexpr idx qpp::maxn = 64

Maximum number of allowed qu(d)its (subsystems)

• constexpr double qpp::pi = 3.141592653589793238462643383279502884

 $\pi$ 

• constexpr double qpp::ee = 2.718281828459045235360287471352662497

Base of natural logarithm, e.

• constexpr double qpp::infty = std::numeric\_limits<double>::infinity()

Used to denote infinity in double precision.

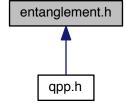
# 8.9.1 Detailed Description

Constants.

# 8.10 entanglement.h File Reference

Entanglement functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 dyn\_col\_vect< double > qpp::schmidtcoeffs (const Eigen::MatrixBase< Derived > &A, const std::vector<
 idx > &dims)

Schmidt coefficients of the bi-partite pure state A.

template < typename Derived >
 cmat qpp::schmidtA (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)

 $\begin{tabular}{ll} \bullet & template < typename \ Derived > \\ \bullet & cmat \ qpp::schmidtB \ (const \ Eigen::MatrixBase < Derived > \&A, \ const \ std::vector < idx > \&dims) \\ \end{tabular}$ 

Schmidt basis on Bob side.

Schmidt basis on Alice side.

template<typename Derived >
 std::vector< double > qpp::schmidtprobs (const Eigen::MatrixBase< Derived > &A, const std::vector< idx
 > &dims)

Schmidt probabilities of the bi-partite pure state A.

template<typename Derived >
 double qpp::entanglement (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)
 Entanglement of the bi-partite pure state A.

template<typename Derived >
 double qpp::gconcurrence (const Eigen::MatrixBase< Derived > &A)

G-concurrence of the bi-partite pure state A.

template < typename Derived >
 double qpp::negativity (const Eigen::MatrixBase < Derived > &A, const std::vector < idx > &dims)
 Negativity of the bi-partite mixed state A.

template<typename Derived >
 double qpp::lognegativity (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &dims)

template<typename Derived >
 double qpp::concurrence (const Eigen::MatrixBase< Derived > &A)

Wootters concurrence of the bi-partite qubit mixed state A.

Logarithmic negativity of the bi-partite mixed state A.

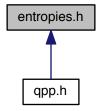
# 8.10.1 Detailed Description

Entanglement functions.

# 8.11 entropies.h File Reference

Entropy functions.

This graph shows which files directly or indirectly include this file:



# **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 double qpp::entropy (const Eigen::MatrixBase< Derived > &A)

von-Neumann entropy of the density matrix A

double qpp::entropy (const std::vector< double > &prob)

Shannon entropy of the probability distribution prob.

• template<typename Derived >

double <a href="mailto:qpp::renyi">qpp::renyi</a> (const Eigen::MatrixBase< Derived > &A, double alpha)

Renyi-  $\alpha$  entropy of the density matrix A, for  $\alpha \geq 0$ .

double qpp::renyi (const std::vector< double > &prob, double alpha)

Renyi-  $\alpha$  entropy of the probability distribution prob, for  $\alpha \geq 0$ .

• template<typename Derived >

double qpp::tsallis (const Eigen::MatrixBase< Derived > &A, double q)

Tsallis- q entropy of the density matrix A, for  $q \geq 0$ .

double qpp::tsallis (const std::vector< double > &prob, double q)

Tsallis- q entropy of the probability distribution prob, for  $q \geq 0$ .

• template<typename Derived >

double qpp::qmutualinfo (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, const std::vector< idx > &dims)

Quantum mutual information between 2 subsystems of a composite system.

template<typename Derived >

double <a href="mailto:qpp::qmutualinfo">qpp::qmutualinfo</a> (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsysA, const std::vector< idx > &subsysB, idx d=2)

Quantum mutual information between 2 subsystems of a composite system.

### 8.11.1 Detailed Description

Entropy functions.

# 8.12 experimental/experimental.h File Reference

Experimental/test functions/classes.

### **Namespaces**

• qpp

Quantum++ main namespace.

• qpp::experimental

Experimental/test functions/classes, do not use or modify.

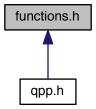
# 8.12.1 Detailed Description

Experimental/test functions/classes.

# 8.13 functions.h File Reference

Generic quantum computing functions.

This graph shows which files directly or indirectly include this file:



# **Namespaces**

• qpp

Quantum++ main namespace.

### **Functions**

- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::transpose (const Eigen::MatrixBase< Derived > &A)
   Transpose.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::conjugate (const Eigen::MatrixBase< Derived > &A)
   Complex conjugate.
- template<typename Derived >
   dyn\_mat< typename Derived::Scalar > qpp::adjoint (const Eigen::MatrixBase< Derived > &A)
   Adjoint.

```
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::inverse (const Eigen::MatrixBase< Derived > &A)

    template<typename Derived >

  Derived::Scalar qpp::trace (const Eigen::MatrixBase < Derived > &A)
      Trace
• template<typename Derived >
  Derived::Scalar qpp::det (const Eigen::MatrixBase< Derived > &A)
      Determinant.
\bullet \ \ {\it template}{<} {\it typename Derived}>
  Derived::Scalar <a href="mailto:qpp::logdet">qpp::logdet</a> (const Eigen::MatrixBase</a> Derived > &A)
      Logarithm of the determinant.

    template<typename Derived >

  Derived::Scalar <a href="mailto:qpp::sum">qpp::sum</a> (const Eigen::MatrixBase< Derived > &A)
      Element-wise sum of A.
• template<typename Derived >
  Derived::Scalar <a href="mailto:open:prod">open:prod</a> (const Eigen::MatrixBase</a> Derived > &A)
      Element-wise product of A.

    template<typename Derived >

  double <a href="mailto:qpp::norm">qpp::norm</a> (const Eigen::MatrixBase< Derived > &A)
      Frobenius norm.

    template<typename Derived >

  std::pair< dyn_col_vect< cplx >, cmat > qpp::eig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition.
\bullet \ \ {\it template}{<} {\it typename Derived}>
  dyn_col_vect< cplx > qpp::evals (const Eigen::MatrixBase< Derived > &A)
      Eigenvalues.

    template<typename Derived >

  cmat qpp::evects (const Eigen::MatrixBase< Derived > &A)
      Eigenvectors.

    template<typename Derived >

  std::pair< dyn_col_vect< double >, cmat > qpp::heig (const Eigen::MatrixBase< Derived > &A)
      Full eigen decomposition of Hermitian expression.

    template<typename Derived >

  dyn_col_vect< double > qpp::hevals (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvalues.

    template<typename Derived >

  cmat qpp::hevects (const Eigen::MatrixBase< Derived > &A)
      Hermitian eigenvectors.

    template<typename Derived >

  std::tuple< cmat, dyn col vect< double >, cmat > qpp::svd (const Eigen::MatrixBase< Derived > &A)
      Full singular value decomposition.
• template<typename Derived >
  dyn_col_vect< double > qpp::svals (const Eigen::MatrixBase< Derived > &A)
      Singular values.

    template<typename Derived >

  cmat qpp::svdU (const Eigen::MatrixBase< Derived > &A)
      Left singular vectors.

    template<typename Derived >

  cmat qpp::svdV (const Eigen::MatrixBase< Derived > &A)
      Right singular vectors.

    template<typename Derived >

  cmat qpp::funm (const Eigen::MatrixBase< Derived > &A, cplx(*f)(const cplx &))
```

```
Functional calculus f(A)
• template<typename Derived >
  cmat qpp::sqrtm (const Eigen::MatrixBase< Derived > &A)
     Matrix square root.

    template<typename Derived >

  cmat qpp::absm (const Eigen::MatrixBase< Derived > &A)
     Matrix absolut value.
template<typename Derived >
  cmat qpp::expm (const Eigen::MatrixBase< Derived > &A)
     Matrix exponential.

    template<typename Derived >

  cmat qpp::logm (const Eigen::MatrixBase< Derived > &A)
     Matrix logarithm.

    template<typename Derived >

  cmat qpp::sinm (const Eigen::MatrixBase< Derived > &A)
     Matrix sin.
• template<typename Derived >
  cmat qpp::cosm (const Eigen::MatrixBase< Derived > &A)
     Matrix cos.

    template<typename Derived >

  cmat qpp::spectralpowm (const Eigen::MatrixBase< Derived > &A, const cplx z)
     Matrix power.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::powm (const Eigen::MatrixBase< Derived > &A, idx n)
     Fast matrix power based on the SQUARE-AND-MULTIPLY algorithm.

    template<typename Derived >

  double <a href="mailto:qpp::schatten">qpp::schatten</a> (const Eigen::MatrixBase</a> Derived > &A, double p)
     Schatten matrix norm.
• template<typename OutputScalar , typename Derived >
  dyn_mat< OutputScalar > qpp::cwise (const Eigen::MatrixBase< Derived > &A, OutputScalar(*f)( const
  typename Derived::Scalar &))
     Functor.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::kron (const T &head)
     Kronecker product.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::kron (const T &head, const Args &...tail)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::vector< Derived > &As)
     Kronecker product.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::kron (const std::initializer_list< Derived > &As)
     Kronecker product.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::kronpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Kronecker power.
• template<typename T >
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head)
     Direct sum.
• template<typename T , typename... Args>
  dyn_mat< typename T::Scalar > qpp::dirsum (const T &head, const Args &...tail)
```

Direct sum.

```
• template<typename Derived >
  dyn mat< typename Derived::Scalar > qpp::dirsum (const std::vector< Derived > &As)

    template < typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::dirsum (const std::initializer_list< Derived > &As)

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::dirsumpow (const Eigen::MatrixBase< Derived > &A, idx n)
     Direct sum power.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::reshape (const Eigen::MatrixBase< Derived > &A, idx rows,
  idx cols)
     Reshape.
- template<typename Derived1 , typename Derived2 >
  dyn_mat< typename Derived1::Scalar > qpp::comm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Commutator.

    template < typename Derived1, typename Derived2 >

  dyn_mat< typename Derived1::Scalar > qpp::anticomm (const Eigen::MatrixBase< Derived1 > &A, const
  Eigen::MatrixBase< Derived2 > &B)
     Anti-commutator.

    template<typename Derived >

  dyn mat< typename Derived::Scalar > qpp::prj (const Eigen::MatrixBase< Derived > &V)
     Projector.

    template<typename Derived >

  dyn_mat< typename Derived::Scalar > qpp::grams (const std::vector< Derived > &Vs)
     Gram-Schmidt orthogonalization.
• template<typename Derived >
  dyn_mat< typename Derived::Scalar > qpp::grams (const std::initializer_list< Derived > &Vs)
     Gram-Schmidt orthogonalization.
template<typename Derived >
  dyn mat< typename Derived::Scalar > qpp::grams (const Eigen::MatrixBase< Derived > &A)
     Gram-Schmidt orthogonalization.

    std::vector< idx > qpp::n2multiidx (idx n, const std::vector< idx > &dims)

     Non-negative integer index to multi-index.

    idx qpp::multiidx2n (const std::vector< idx > &midx, const std::vector< idx > &dims)

     Multi-index to non-negative integer index.

    ket qpp::mket (const std::vector< idx > &mask, const std::vector< idx > &dims)

     Multi-partite qudit ket.

    ket qpp::mket (const std::vector< idx > &mask, idx d=2)

     Multi-partite qudit ket.

    cmat qpp::mprj (const std::vector < idx > &mask, const std::vector < idx > &dims)

     Projector onto multi-partite qudit ket.

    cmat qpp::mprj (const std::vector< idx > &mask, idx d=2)

     Projector onto multi-partite qudit ket.
• template<typename InputIterator >
  std::vector< double > qpp::abssq (InputIterator first, InputIterator last)
     Computes the absolute values squared of an STL-like range of complex numbers.

    template<typename Container >

  std::vector< double > qpp::abssq (const Container &c, typename std::enable_if< is_iterable< Container
  >::value >::type *=nullptr)
```

Computes the absolute values squared of an STL-like container.

template < typename Derived >
 std::vector < double > qpp::abssq (const Eigen::MatrixBase < Derived > &A)

Computes the absolute values squared of an Eigen expression.

template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type qpp::sum (InputIterator first, InputIterator last)

Element-wise sum of an STL-like range.

• template<typename Container >

Container::value\_type qpp::sum (const Container &c, typename std::enable\_if< is\_iterable< Container >--:value >::type \*=nullptr)

Element-wise sum of the elements of an STL-like container.

• template<typename InputIterator >

std::iterator\_traits< InputIterator >::value\_type qpp::prod (InputIterator first, InputIterator last)

Element-wise product of an STL-like range.

template<typename Container >

Container::value\_type qpp::prod (const Container &c, typename std::enable\_if< is\_iterable< Container >
::value >::type \*=nullptr)

Element-wise product of the elements of an STL-like container.

template<typename Derived >

dyn\_col\_vect< typename Derived::Scalar > qpp::rho2pure (const Eigen::MatrixBase< Derived > &A)

Finds the pure state representation of a matrix proportional to a projector onto a pure state.

• template<typename T >

std::vector< T > qpp::complement (std::vector< T > subsys, idx N)

Constructs the complement of a subsystem vector.

template<typename Derived >

std::vector< double > qpp::rho2bloch (const Eigen::MatrixBase< Derived > &A)

Computes the 3-dimensional real Bloch vector corresponding to the qubit density matrix A.

cmat qpp::bloch2rho (const std::vector< double > &r)

Computes the density matrix corresponding to the 3-dimensional real Bloch vector r.

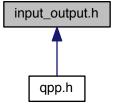
# 8.13.1 Detailed Description

Generic quantum computing functions.

# 8.14 input\_output.h File Reference

Input/output functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

qpp

Quantum++ main namespace.

#### **Functions**

template<typename Derived >
 internal::IOManipEigen qpp::disp (const Eigen::MatrixBase< Derived > &A, double chop=qpp::chop)

Eigen expression ostream manipulator.

• internal::IOManipEigen qpp::disp (cplx z, double chop=qpp::chop)

Complex number ostream manipulator.

template<typename InputIterator >
 internal::IOManipRange< InputIterator > qpp::disp (InputIterator first, InputIterator last, const std::string &separator, const std::string &start="[", const std::string &end="]")

Range ostream manipulator.

template<typename Container >
 internal::IOManipRange< typename Container::const\_iterator > qpp::disp (const Container &c, const std
 ::string &separator, const std::string &start="[", const std::string &end="]", typename std::enable\_if< is\_←
 iterable< Container >::value >::type \*=nullptr)

Standard container ostream manipulator. The container must support std::begin(), std::end() and forward iteration.

template<typename PointerType >
 internal::IOManipPointer< PointerType > qpp::disp (const PointerType \*p, idx n, const std::string &separator, const std::string &start="[", const std::string &end="]")

C-style pointer ostream manipulator.

template < typename Derived > void qpp::save (const Eigen::MatrixBase < Derived > &A, const std::string &fname)

Saves Eigen expression to a binary file (internal format) in double precision.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::load (const std::string &fname)

Loads Eigen matrix from a binary file (internal format) in double precision.

# 8.14.1 Detailed Description

Input/output functions.

### 8.15 instruments.h File Reference

Measurement functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

### **Functions**

template<typename Derived >
 dyn\_col\_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, const std::vector< idx > &dims)
 Generalized inner product.

 $\bullet \;\; {\sf template}{<} {\sf typename \; Derived} >$ 

dyn\_col\_vect< typename Derived::Scalar > qpp::ip (const Eigen::MatrixBase< Derived > &phi, const Eigen::MatrixBase< Derived > &psi, const std::vector< idx > &subsys, idx d=2)

Generalized inner product.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$ 

 $std::tuple < idx, \ std::vector < \ double >, \ std::vector < \ cmat > > \ qpp::measure \ (const \ Eigen::MatrixBase < \ Derived > &A, \ const \ std::vector < \ cmat > &Ks)$ 

Measures the state A using the set of Kraus operators Ks.

template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::initializer\_list< cmat > &Ks)

Measures the state A using the set of Kraus operators Ks.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const cmat &U)

Measures the state A in the orthonormal basis specified by the unitary matrix U.

• template<typename Derived >

std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase< Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

• template<typename Derived >

 $std::tuple < idx, \ std::vector < double >, \ std::vector < cmat > > qpp::measure \ (const \ Eigen::MatrixBase < Derived > &A, \ const \ std::initializer_list < cmat > &Ks, \ const \ std::vector < idx > &subsys, \ const \ std::vector < idx > &dims)\\$ 

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::vector< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const std::initializer\_list< cmat > &Ks, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A using the set of Kraus operators Ks.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::tuple< idx, std::vector< double >, std::vector< cmat > > qpp::measure (const Eigen::MatrixBase
 Derived > &A, const cmat &V, const std::vector< idx > &subsys, idx d=2)

Measures the part subsys of the multi-partite state vector or density matrix A in the orthonormal basis or rank-1 POVM specified by the matrix V.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, std::vector< idx > dims)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

template<typename Derived >
 std::tuple< std::vector< idx >, double, cmat > qpp::measure\_seq (const Eigen::MatrixBase< Derived > &A,
 std::vector< idx > subsys, idx d=2)

Sequentially measures the part subsys of the multi-partite state vector or density matrix A in the computational basis.

### 8.15.1 Detailed Description

Measurement functions.

# 8.16 internal/classes/iomanip.h File Reference

Input/output manipulators.

This graph shows which files directly or indirectly include this file:



### Classes

- struct qpp::internal::\_details::\_Display\_Impl
- class qpp::internal::IOManipRange< InputIterator >
- class qpp::internal::IOManipPointer< PointerType >
- class qpp::internal::IOManipEigen

# **Namespaces**

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

• qpp::internal::\_details

# 8.16.1 Detailed Description

Input/output manipulators.

# 8.17 internal/classes/singleton.h File Reference

Singleton pattern via CRTP.

This graph shows which files directly or indirectly include this file:



### Classes

class qpp::internal::Singleton< T >

Singleton policy class, used internally to implement the singleton pattern via CRTP (Curiously recurring template pattern)

### **Namespaces**

qpp

Quantum++ main namespace.

· qpp::internal

Internal utility functions, do not use/modify.

### 8.17.1 Detailed Description

Singleton pattern via CRTP.

### 8.18 internal/util.h File Reference

Internal utility functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

qpp::internal

Internal utility functions, do not use/modify.

### **Functions**

- void qpp::internal::\_n2multiidx (idx n, idx numdims, const idx \*dims, idx \*result) noexcept
- idx qpp::internal::\_multiidx2n (const idx \*midx, idx numdims, const idx \*dims) noexcept
- template < typename Derived >

 $bool\ qpp::internal::\_check\_square\_mat\ (const\ Eigen::MatrixBase < Derived > \&A)$ 

template<typename Derived >

bool qpp::internal::\_check\_vector (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$ 

bool qpp::internal:: check rvector (const Eigen::MatrixBase< Derived > &A)

• template<typename Derived >

bool qpp::internal::\_check\_cvector (const Eigen::MatrixBase< Derived > &A)

 $\bullet \ \ template {<} typename \ T >$ 

bool qpp::internal::\_check\_nonzero\_size (const T &x) noexcept

• template<typename T1 , typename T2 >

bool qpp::internal::\_check\_matching\_sizes (const T1 &lhs, const T2 &rhs) noexcept

- bool qpp::internal:: check dims (const std::vector < idx > &dims)
- template<typename Derived >

 $bool\ qpp::internal::\_check\_dims\_match\_mat\ (const\ std::vector < idx > \&dims,\ const\ Eigen::MatrixBase < Derived > \&A)$ 

template<typename Derived >
 bool qpp::internal::\_check\_dims\_match\_cvect (const std::vector< idx > &dims, const Eigen::MatrixBase
 Derived > &V)

- template<typename Derived >
   bool qpp::internal::\_check\_dims\_match\_rvect (const std::vector< idx > &dims, const Eigen::MatrixBase
   Derived > &V)
- bool qpp::internal::\_check\_eq\_dims (const std::vector< idx > &dims, idx dim) noexcept
- bool qpp::internal::\_check\_subsys\_match\_dims (const std::vector< idx > &subsys, const std::vector< idx > &dims)
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_matrix (const Eigen::MatrixBase< Derived > &A) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_cvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_rvector (const Eigen::MatrixBase< Derived > &V) noexcept
- template<typename Derived >
   bool qpp::internal::\_check\_qubit\_vector (const Eigen::MatrixBase< Derived > &V) noexcept
- bool qpp::internal::\_check\_perm (const std::vector< idx > &perm)
- template<typename Derived1 , typename Derived2 >
   dyn\_mat< typename Derived1::Scalar > qpp::internal::\_kron2 (const Eigen::MatrixBase< Derived1 > &A,
   const Eigen::MatrixBase< Derived2 > &B)
- template<typename Derived1, typename Derived2 >
   dyn\_mat< typename Derived1::Scalar > qpp::internal::\_dirsum2 (const Eigen::MatrixBase< Derived1 > &A,
   const Eigen::MatrixBase< Derived2 > &B)
- template<typename T >
   void qpp::internal::variadic vector emplace (std::vector< T > &)
- template<typename T, typename First, typename... Args>
   void qpp::internal::variadic vector emplace (std::vector< T > &v, First &&first, Args &&...args)

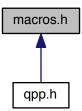
### 8.18.1 Detailed Description

Internal utility functions.

# 8.19 macros.h File Reference

Preprocessor macros.

This graph shows which files directly or indirectly include this file:



### **Macros**

- #define PRINT(x)
- #define PRINTLN(x)

```
• #define ERROR(x)
```

• #define ERRORLN(x)

### 8.19.1 Detailed Description

Preprocessor macros.

### 8.19.2 Macro Definition Documentation

```
8.19.2.1 #define ERROR( x )
```

Prints an error message to std::cerr

```
8.19.2.2 #define ERRORLN( x )
```

Prints an error message to std::cerr and adds a new line

```
8.19.2.3 #define PRINT( x )
```

Prints a message

```
8.19.2.4 #define PRINTLN( x )
```

Prints a message and adds a new line

### 8.20 MATLAB/matlab.h File Reference

Input/output interfacing with MATLAB.

```
#include "mat.h"
#include "mex.h"
```

### **Namespaces**

• qpp

Quantum++ main namespace.

### **Functions**

template < typename Derived >
 Derived qpp::loadMATLABmatrix (const std::string &, const std::string &)

Loads an Eigen dynamic matrix from a MATLAB .mat file, generic version.

template<>

dmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

cmat qpp::loadMATLABmatrix (const std::string &mat\_file, const std::string &var\_name)

Loads an Eigen dynamic matrix from a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

template<typename Derived >
 void qpp::saveMATLABmatrix (const Eigen::MatrixBase< Derived > &, const std::string &, const std::string &)

Saves an Eigen dynamic matrix to a MATLAB .mat file, generic version.

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< dmat > &A, const std::string &mat\_file, const std 
::string &var name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for double matrices (qpp::dmat)

template<>

void qpp::saveMATLABmatrix (const Eigen::MatrixBase< cmat > &A, const std::string &mat\_file, const std ::string &var\_name, const std::string &mode)

Saves an Eigen dynamic matrix to a MATLAB .mat file, specialization for complex matrices (qpp::cmat)

# 8.20.1 Detailed Description

Input/output interfacing with MATLAB.

# 8.21 number\_theory.h File Reference

Number theory functions.

This graph shows which files directly or indirectly include this file:



# **Namespaces**

• qpp

Quantum++ main namespace.

Simple continued fraction expansion.

### **Functions**

- std::vector < int > qpp::x2contfrac (double x, idx n, idx cut=1e5)
- double <a href="mailto:qpp::contfrac2x">qpp::contfrac2x</a> (const std::vector< int > &cf, idx n)

Real representation of a simple continued fraction.

double qpp::contfrac2x (const std::vector< int > &cf)

Real representation of a simple continued fraction.

• bigint qpp::gcd (bigint m, bigint n)

Greatest common divisor of two integers.

bigint qpp::gcd (const std::vector< bigint > &ns)

Greatest common divisor of a list of integers.

• bigint qpp::lcm (bigint m, bigint n)

Least common multiple of two integers.

bigint qpp::lcm (const std::vector< bigint > &ns)

Least common multiple of a list of integers.

std::vector< idx > qpp::invperm (const std::vector< idx > &perm)

Inverse permutation.

std::vector< idx > qpp::compperm (const std::vector< idx > &perm, const std::vector< idx > &sigma)
 Compose permutations.

std::vector< bigint > qpp::factors (bigint n)

Prime factor decomposition.

• bigint qpp::modmul (bigint m, bigint n, bigint p)

Modular multiplication without overflow.

• bigint qpp::modpow (bigint a, bigint n, bigint p)

Fast integer power modulo p based on the SQUARE-AND-MULTIPLY algorithm.

std::tuple < bigint, bigint, bigint > qpp::egcd (bigint m, bigint n)

Extended greatest common divisor of two integers.

• bigint qpp::modinv (bigint a, bigint p)

Modular inverse of a mod p.

• bool qpp::isprime (bigint n, idx k=80)

Primality test based on the Miller-Rabin's algorithm.

• bigint qpp::randprime (bigint a, bigint b, idx N=1000)

Generates a random big prime uniformly distributed in the interval [a, b].

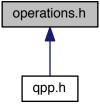
### 8.21.1 Detailed Description

Number theory functions.

# 8.22 operations.h File Reference

Quantum operation functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

qpp

Quantum++ main namespace.

### **Functions**

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 const std::vector< idx > &dims)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1, typename Derived2 >
 dyn\_mat< typename Derived1::Scalar > qpp::applyCTRL (const Eigen::MatrixBase< Derived1 > &state,
 const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &ctrl, const std::vector< idx > &subsys,
 idx d=2)

Applies the controlled-gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

template<typename Derived1 , typename Derived2 >

dyn\_mat< typename Derived1::Scalar > qpp::apply (const Eigen::MatrixBase< Derived1 > &state, const Eigen::MatrixBase< Derived2 > &A, const std::vector< idx > &subsys, idx d=2)

Applies the gate A to the part subsys of the multi-partite state vector or density matrix state.

 $\bullet \ \ {\it template}{<} {\it typename Derived} >$ 

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks)

Applies the channel specified by the set of Kraus operators Ks to the density matrix rho.

template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, const std::vector< idx > &dims)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

ullet template<typename Derived >

cmat qpp::apply (const Eigen::MatrixBase< Derived > &rho, const std::vector< cmat > &Ks, const std ::vector< idx > &subsys, idx d=2)

Applies the channel specified by the set of Kraus operators Ks to the part subsys of the multi-partite density matrix rho.

cmat qpp::kraus2super (const std::vector< cmat > &Ks)

Superoperator matrix.

cmat qpp::kraus2choi (const std::vector< cmat > &Ks)

Choi matrix.

std::vector< cmat > qpp::choi2kraus (const cmat &A)

Orthogonal Kraus operators from Choi matrix.

cmat qpp::choi2super (const cmat &A)

Converts Choi matrix to superoperator matrix.

cmat qpp::super2choi (const cmat &A)

Converts superoperator matrix to Choi matrix.

template<typename Derived >

dyn\_mat< typename Derived::Scalar > qpp::ptrace1 (const Eigen::MatrixBase< Derived > &A, const std ← ::vector< idx > &dims)

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptrace2 (const Eigen::MatrixBase< Derived > &A, const std
 ::vector< idx > &dims)

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std
 ::vector< idx > &subsys, const std::vector< idx > &dims)

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptrace (const Eigen::MatrixBase< Derived > &A, const std
 ::vector< idx > &subsys, idx d=2)

Partial trace.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, const std::vector< idx > &dims)

Partial transpose.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::ptranspose (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &subsys, idx d=2)

Partial transpose.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, const std::vector< idx > &dims)

Subsystem permutation.

template<typename Derived >
 dyn\_mat< typename Derived::Scalar > qpp::syspermute (const Eigen::MatrixBase< Derived > &A, const std::vector< idx > &perm, idx d=2)

Subsystem permutation.

### 8.22.1 Detailed Description

Quantum operation functions.

# 8.23 qpp.h File Reference

Quantum++ main header file, includes all other necessary headers.

```
#include <algorithm>
#include <cassert>
#include <chrono>
#include <cmath>
#include <complex>
#include <cstdlib>
#include <cstring>
#include <ctime>
#include <exception>
#include <fstream>
#include <functional>
#include <initializer_list>
#include <iomanip>
#include <iostream>
#include <iterator>
#include <limits>
#include <numeric>
#include <ostream>
#include <random>
#include <sstream>
#include <stdexcept>
#include <string>
#include <tuple>
#include <type_traits>
#include <utility>
#include <vector>
#include <Eigen/Dense>
#include <Eigen/SVD>
#include "macros.h"
#include "types.h"
#include "classes/exception.h"
#include "constants.h"
#include "traits.h"
#include "classes/idisplay.h"
#include "internal/util.h"
#include "internal/classes/iomanip.h"
#include "input_output.h"
#include "internal/classes/singleton.h"
#include "classes/init.h"
#include "functions.h"
#include "classes/codes.h"
#include "classes/gates.h"
#include "classes/states.h"
#include "classes/random_devices.h"
#include "statistics.h"
#include "operations.h"
#include "entropies.h"
#include "entanglement.h"
#include "random.h"
#include "classes/timer.h"
#include "instruments.h"
#include "number_theory.h"
```

#### **Namespaces**

qpp

Quantum++ main namespace.

### **Macros**

• #define \_QPP\_UNUSED\_

### 8.23.1 Detailed Description

Quantum++ main header file, includes all other necessary headers.

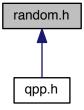
### 8.23.2 Macro Definition Documentation

8.23.2.1 #define \_QPP\_UNUSED\_

# 8.24 random.h File Reference

Randomness-related functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

qpp

Quantum++ main namespace.

### **Functions**

• double <a href="mailto:qpp::rand">qpp::rand</a> (double a, double b)

Generates a random real number uniformly distributed in the interval [a, b)

bigint qpp::rand (bigint a, bigint b)

Generates a random big integer uniformly distributed in the interval [a, b].

idx qpp::randidx (idx a=std::numeric\_limits < idx >::min(), idx b=std::numeric\_limits < idx >::max())

Generates a random index (idx) uniformly distributed in the interval [a, b].

 $\bullet \ \ {\it template}{<} {\it typename Derived}>$ 

Derived <a href="mailto:qpp::rand">qpp::rand</a> (idx rows, idx cols, double a=0, double b=1)

Generates a random matrix with entries uniformly distributed in the interval [a, b)

template<>
 dmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random real matrix with entries uniformly distributed in the interval [a, b), specialization for double matrices (qpp::dmat)

template<>

cmat qpp::rand (idx rows, idx cols, double a, double b)

Generates a random complex matrix with entries (both real and imaginary) uniformly distributed in the interval [a, b), specialization for complex matrices (qpp::cmat)

template<typename Derived >

Derived qpp::randn (idx rows, idx cols, double mean=0, double sigma=1)

Generates a random matrix with entries normally distributed in N(mean, sigma)

template<>

dmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random real matrix with entries normally distributed in N(mean, sigma), specialization for double matrices (qpp::dmat)

• template<>

cmat qpp::randn (idx rows, idx cols, double mean, double sigma)

Generates a random complex matrix with entries (both real and imaginary) normally distributed in N(mean, sigma), specialization for complex matrices (qpp::cmat)

• double <a href="mailto:qpp::randn">qpp::randn</a> (double mean=0, double sigma=1)

Generates a random real number (double) normally distributed in N(mean, sigma)

cmat qpp::randU (idx D)

Generates a random unitary matrix.

cmat qpp::randV (idx Din, idx Dout)

Generates a random isometry matrix.

std::vector< cmat > qpp::randkraus (idx N, idx D)

Generates a set of random Kraus operators.

cmat qpp::randH (idx D)

Generates a random Hermitian matrix.

ket gpp::randket (idx D)

Generates a random normalized ket (pure state vector)

cmat qpp::randrho (idx D)

Generates a random density matrix.

• std::vector< idx > qpp::randperm (idx n)

Generates a random uniformly distributed permutation.

# 8.24.1 Detailed Description

Randomness-related functions.

### 8.25 statistics.h File Reference

Statistics functions.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

#### **Functions**

std::vector< double > qpp::uniform (idx N)

Uniform probability distribution vector.

std::vector< double > qpp::marginalX (const dmat &probXY)

Marginal distribution.

std::vector< double > qpp::marginalY (const dmat &probXY)

Marginal distribution.

• template<typename Container >

double qpp::avg (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Average.

• template<typename Container >

double qpp::cov (const dmat &probXY, const Container &X, const Container &Y, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Covariance.

• template<typename Container >

double qpp::var (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Variance.

• template<typename Container >

double qpp::sigma (const std::vector< double > &prob, const Container &X, typename std::enable\_if< is\_← iterable< Container >::value >::type \*=nullptr)

Standard deviation.

• template<typename Container >

double qpp::cor (const dmat &probXY, const Container &X, const Container &Y, typename std::enable\_if< is\_iterable< Container >::value >::type \*=nullptr)

Correlation.

### 8.25.1 Detailed Description

Statistics functions.

# 8.26 traits.h File Reference

Type traits.

This graph shows which files directly or indirectly include this file:



### **Classes**

struct qpp::is\_iterable < T, typename >

Checks whether T is compatible with an STL-like iterable container.

struct qpp::is\_iterable< T, to\_void< decltype(std::declval< T >().begin()), decltype(std::declval< T >().complete end()), typename T::value\_type >>

Checks whether T is compatible with an STL-like iterable container, specialization for STL-like iterable containers.

struct qpp::is\_matrix\_expression< Derived >

Checks whether the type is an Eigen matrix expression.

struct qpp::is\_matrix\_expression< typename Eigen::MatrixBase< Derived > >

Checks whether the type is an Eigen matrix expression, specialization for Eigen matrix expressions.

struct qpp::is\_complex< T >

Checks whether the type is a complex type.

struct qpp::is\_complex< std::complex< T >>

Checks whether the type is a complex number type, specialization for complex types.

# **Namespaces**

• qpp

Quantum++ main namespace.

### **Typedefs**

```
template<typename... >
using qpp::to_void = void
```

Alias template that implements the proposal for void\_t.

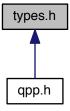
# 8.26.1 Detailed Description

Type traits.

# 8.27 types.h File Reference

Type aliases.

This graph shows which files directly or indirectly include this file:



### **Namespaces**

• qpp

Quantum++ main namespace.

# **Typedefs**

```
• using qpp::idx = std::size t
```

Non-negative integer index.

• using qpp::bigint = long long int

Big integer.

• using qpp::cplx = std::complex< double >

Complex number in double precision.

using qpp::ket = Eigen::VectorXcd

Complex (double precision) dynamic Eigen column vector.

• using qpp::bra = Eigen::RowVectorXcd

Complex (double precision) dynamic Eigen row vector.

• using qpp::cmat = Eigen::MatrixXcd

Complex (double precision) dynamic Eigen matrix.

• using qpp::dmat = Eigen::MatrixXd

Real (double precision) dynamic Eigen matrix.

 $\bullet \ \ \text{template}{<} \text{typename Scalar} >$ 

using <a href="mailto:qpp::dyn\_mat">qpp::dyn\_mat</a> = Eigen::Matrix< Scalar, Eigen::Dynamic, Eigen::Dynamic >

Dynamic Eigen matrix over the field specified by Scalar.

• template<typename Scalar >

using qpp::dyn\_col\_vect = Eigen::Matrix < Scalar, Eigen::Dynamic, 1 >

Dynamic Eigen column vector over the field specified by Scalar.

• template<typename Scalar >

using <a href="mailto:qpp::dyn\_row\_vect">qpp::dyn\_row\_vect</a> = Eigen::Matrix< Scalar, 1, Eigen::Dynamic >

Dynamic Eigen row vector over the field specified by Scalar.

Type aliases.

# Index

_A	qpp::Timer, 126
qpp::internal::IOManipEigen, 103	qpp::internal::IOManipPointer, 104
_QPP_UNUSED_	qpp::internal::IOManipRange, 106
qpp.h, 153	_first
_check_cvector	qpp::internal::IOManipRange, 106
qpp::internal, 81	_kron2
_check_dims	qpp::internal, 82
qpp::internal, 81	_last
_check_dims_match_cvect	qpp::internal::IOManipRange, 106
qpp::internal, 81	_msg
_check_dims_match_mat	qpp::Exception, 91
qpp::internal, 82	_multiidx2n
_check_dims_match_rvect	qpp::internal, 82
qpp::internal, 82	_n
_check_eq_dims	qpp::internal::IOManipPointer, 104
qpp::internal, 82	_n2multiidx
_check_matching_sizes	qpp::internal, 82
qpp::internal, 82	_p
_check_nonzero_size	qpp::internal::IOManipPointer, 104
qpp::internal, 82	_rd
_check_perm	qpp::RandomDevices, 115
qpp::internal, 82	_rng
_check_qubit_cvector	qpp::RandomDevices, 115
qpp::internal, 82	_separator
_check_qubit_matrix	qpp::internal::IOManipPointer, 104
qpp::internal, 82	qpp::internal::IOManipRange, 106
_check_qubit_rvector	_start
qpp::internal, 82	qpp::Timer, 126
_check_qubit_vector	qpp::internal::IOManipPointer, 105
qpp::internal, 82	qpp::internal::IOManipRange, 107
_check_rvector	_type
qpp::internal, 82	qpp::Exception, 91
_check_square_mat	_where
qpp::internal, 82	qpp::Exception, 91
_check_subsys_match_dims	$\sim$ Codes
qpp::internal, 82	qpp::Codes, 87
_check_vector	$\sim$ Gates
qpp::internal, 82	qpp::Gates, 94
_chop	$\sim$ IDisplay
qpp::internal::IOManipEigen, 103	qpp::IDisplay, 99
_construct_exception_msg	$\sim$ Init
qpp::Exception, 91	qpp::Init, 101
_custom	$\sim$ RandomDevices
qpp::Exception, 91	qpp::RandomDevices, 115
_dirsum2	$\sim$ Singleton
qpp::internal, 82	qpp::internal::Singleton, 116
_display_impl	$\sim$ States
qpp::internal::_details::_Display_Impl, 85	qpp::States, 119
end	~Timer

qpp::Timer, 123	qpp::Codes, 87
,	codeword
absm	qpp::Codes, 87
qpp, 26	comm
abssq	qpp, <mark>32</mark>
qpp, 26, 27	complement
adjoint	qpp, 32
qpp, 27	compperm
anticomm	qpp, 33
qpp, 27	concurrence
apply	qpp, 33
qpp, 27, 28, 30	conjugate
applyCTRL	qpp, <mark>33</mark>
qpp, 30	constants.h, 132
avg	contfrac2x
qpp, 31	qpp, 33, 34
	cor
b00	qpp, 34
qpp::States, 119	cosm
b01	qpp, 34
qpp::States, 119	COV
b10	qpp, <mark>34</mark>
qpp::States, 119	cplx
b11	qpp, <mark>25</mark>
qpp::States, 119	cwise
bigint	qpp, 35
qpp, 25	-11-1-7
bloch2rho	DIMS_INVALID
qpp, 31	qpp::Exception, 90
bra	DIMS_MISMATCH_CVECTOR
qpp, 25	qpp::Exception, 90
	DIMS_MISMATCH_MATRIX
CNOT	qpp::Exception, 90
qpp::Gates, 96	DIMS MISMATCH RVECTOR
CNOTba	qpp::Exception, 90
qpp::Gates, 96	DIMS MISMATCH VECTOR
CTRL	qpp::Exception, 90
qpp::Gates, 94	DIMS_NOT_EQUAL
CUSTOM_EXCEPTION	qpp::Exception, 90
qpp::Exception, 90	det
CZ	qpp, 35
qpp::Gates, 96	dirsum
choi2kraus	qpp, 35, 36
qpp, 31	dirsumpow
choi2super	qpp, 36
qpp, 32	disp
chop	qpp, 37, 38
qpp, 79	display
classes/codes.h, 127	qpp::IDisplay, 99
classes/exception.h, 127	qpp::Timer, 124
classes/gates.h, 128	qpp::internal::IOManipEigen, 102
classes/idisplay.h, 129	qpp::internal::IOManipPointer, 104
classes/init.h, 130	qpp::internal::IOManipFointer, 104
classes/random_devices.h, 130	dmat dmatIOManiphange, 106
classes/states.h, 131	
classes/timer.h, 132	qpp, 25
	dyn_col_vect
cmat	qpp, 25
qpp, 25 Codes	dyn_mat
Oudes	qpp, 25

di	
dyn_row_vect	grams
qpp, 25	qpp, 42, 43
EDDOD	11
ERROR	Н
macros.h, 147	qpp::Gates, 96
ERRORLN	heig
macros.h, 147	qpp, <del>43</del>
ee	hevals
qpp, 79	qpp, 43
egcd	hevects
qpp, 38	qpp, 43
eig	
qpp, 39	IDisplay
entanglement	qpp::IDisplay, 99
-	IOManipEigen
qpp, 39	
entanglement.h, 133	qpp::internal::IOManipEigen, 102
entropies.h, 134	IOManipPointer
entropy	qpp::internal::IOManipPointer, 104
qpp, 39, 40	IOManipRange
eps	qpp::internal::IOManipRange, 106
qpp, 80	ld
evals	qpp::Gates, 95
qpp, 40	ld2
evects	qpp::Gates, 97
qpp, 40	idx
Exception	qpp, 26
	infty
qpp::Exception, 90, 91	qpp, 80
expandout	Init
qpp::Gates, 94	
experimental/experimental.h, 136	app::Init, 101
expm	input_output.h, 140
qpp, 40	instruments.h, 141
	internal/classes/iomanip.h, 143
FIVE_QUBIT	internal/classes/singleton.h, 144
qpp::Codes, 87	internal/util.h, 145
FRED	internal::Singleton< const Codes >
qpp::Gates, 96	qpp::Codes, 88
factors	internal::Singleton < const Gates >
qpp, 41	qpp::Gates, 96
Fd	internal::Singleton< const Init >
qpp::Gates, 95	gpp::Init, 101
functions.h, 136	internal::Singleton< const States >
funm	qpp::States, 119
	internal::Singleton< RandomDevices >
qpp, 41	_
GHZ	qpp::RandomDevices, 115
	inverse
qpp::States, 119	qpp, 44
Gates	invperm
qpp::Gates, 94	qpp, 44
gcd	ip
qpp, 41	qpp, 44, 45
gconcurrence	isprime
qpp, 42	qpp, 45
get_duration	n
qpp::Timer, 125	ket
get_instance	qpp, 26
qpp::internal::Singleton, 117	kraus2choi
get_thread_local_instance	qpp, 45
qpp::internal::Singleton, 117	kraus2super
appmemaiomgieton, 117	ινιαμοσουμοι

qpp, 46	mprj
kron	qpp, 57
qpp, 46, 47	multiidx2n
kronpow	qpp, 58
qpp, 47	
	n2multiidx
lcm	qpp, 58
qpp, 48	NINE_QUBIT_SHOR
load	qpp::Codes, 87
qpp, 48	NO_CODEWORD
loadMATLABmatrix	qpp::Exception, 90
qpp, 49	NOT_BIPARTITE
logdet	qpp::Exception, 90
qpp, 50	NOT_QUBIT_CVECTOR
logm	qpp::Exception, 90
qpp, 50	NOT_QUBIT_MATRIX
lognegativity	qpp::Exception, 90
qpp, 50	NOT_QUBIT_RVECTOR
MATI ADV. III. I 447	qpp::Exception, 90
MATLAB/matlab.h, 147	NOT_QUBIT_SUBSYS
MATRIX_MISMATCH_SUBSYS	qpp::Exception, 90
qpp::Exception, 90	NOT_QUBIT_VECTOR
MATRIX_NOT_CVECTOR	qpp::Exception, 90
qpp::Exception, 90	negativity
MATRIX_NOT_RVECTOR	qpp, <mark>58</mark>
qpp::Exception, 90	norm
MATRIX_NOT_SQUARE	qpp, <del>59</del>
qpp::Exception, 90	number_theory.h, 148
MATRIX_NOT_SQUARE_OR_CVECTOR	OUT OF BANGE
<pre>qpp::Exception, 90 MATRIX_NOT_SQUARE_OR_RVECTOR</pre>	OUT_OF_RANGE
qpp::Exception, 90	qpp::Exception, 90
MATRIX NOT SQUARE OR VECTOR	omega
qpp::Exception, 90	qpp, 59 operations.h, 149
MATRIX_NOT_VECTOR	operator<<
qpp::Exception, 90	qpp::IDisplay, 99
macros.h, 146	operator=
ERROR, 147	qpp::IDisplay, 99
ERRORLN, 147	qpp::Timer, 125
PRINT, 147	qpp::internal::IOManipPointer, 104
PRINTLN, 147	qpp::internal::IOManipRange, 106
marginalX	qpp::internal::Singleton, 117
qpp, 50	operator""_i
marginalY	qpp, 59
qpp, 51	<b>Чрр, 33</b>
maxn	PERM INVALID
qpp, 80	qpp::Exception, 90
measure	PERM_MISMATCH_DIMS
qpp, 51–54	qpp::Exception, 90
measure_seq	pGHZ
qpp, 55	qpp::States, 120
mket	PRINT
qpp, 55, 56	macros.h, 147
modinv	PRINTLN
qpp, 56	macros.h, 147
modmul	pW
qpp, 56	qpp::States, 120
modpow	pb00
qpp, 57	qpp::States, 120

pb01		
pb10	•	,
qpp::States, 120         cwise, 35           pb11         def, 35           qpp, 80         dirsum, 35, 36           powm         disp, 37, 38           qpp, 59         dyn_col_vect, 25           pj         dyn_col_vect, 25           qpp, 60         dyn_row_vect, 25           prod         ee, 79           qpp, 60         egcd, 33           ptrace         eig, 39           qpp, 61         eps, 80           ptrace1         entropy, 39, 40           qpp, 63         evals, 40           qpp, 63         evects, 40           px1         gcd, 41           px1         gconcurrence, 42           qpp, 63         factors, 41           px1         gconcurrence, 42           qpp::States, 120         grams, 42, 43           py0         heyals, 43           py1         hevals, 43           py1         hevals, 43           py2         pp::States, 120         inty, 80           pp::States, 120         inty, 80           pp::States, 120         inty, 80           qpp::States, 120         inty, 80           pp::At, 45         inty, 80           qpp::States, 120		
pb111	•	•
pp::States, 120 pi	• •	
pi	•	
qpp, 80 powm		
powm	•	•
qpp, 59         dyn_col_vect, 25           prj         dyn_ramt, 25           qpp, 60         dyn_row_vect, 25           prod         ep, 60           qpp, 61         egd, 38           ptrace         eig, 39           qpp, 61         entropy, 39, 40           qpp, 63         entropy, 39, 40           pp, 63         evects, 40           pp, 63         factors, 41           px0         funm, 41           qpp::States, 120         gcd, 41           px1         gconcurrence, 42           qpp::States, 120         heig, 43           hevects, 43         idx, 26           idx, 26         infty, 80           pp::States, 120         inverse, 44           pp::States, 120         inverse, 44           pp::States, 120         inverse, 44           qpp::States, 120         inverse, 44           pp::States, 120         inverse, 44           pp::States, 120         inverse, 44           pp::States, 120         inverse, 44           qpp::States, 120         inverse, 44           pp::States, 120         inverse, 44           pp::States, 120         inverse, 44           inverse, 44         inverse, 44		•
prj	•	
qpp, 60 prod		
prod	• •	•
ptrace		•
ptrace qpp, 61 entanglement, 39 entropy, 39, 40 eps, 80 evels, 40 eps, 80 evects, 40 eps, 63 evects, 40 eps, 63 evects, 41 funm, 41 gcd, 41 gpp::States, 120 gcd, 41 gconcurrence, 42 grams, 42, 43 heig, 43 hevels, 43 hevels, 43 epp::States, 120 py0 hevels, 43 idx, 26 infty, 80	•	
qpp, 61 ptrace1		<del>-</del>
ptrace1	•	
qpp, 61 ptrace2		<del>-</del>
ptrace2	qpp, 61	• •
qpp, 63 ptranspose	• •	•
ptranspose         expm, 40           qx0         factors, 41           qx0         funm, 41           qpp::States, 120         gcd, 41           py0         heig, 43           qpp::States, 120         hevals, 43           py1         hevects, 43           qpp::States, 120         infty, 80           pz0         inverse, 44           qpp::States, 120         inverse, 44           pz1         inverse, 44           qpp::States, 120         ip, 44, 45           qmutualinfo         isprime, 45           qpp, 64         ket, 26           qpp, 13         kraus2cuper, 46           absm, 26         kron, 46, 47           absm, 26         kron, 46, 47           adjoint, 27         lom, 48           apply, 27, 28, 30         load, 48           apply, 25         log, 50           bioth2rho, 31         lognegativity, 50           bra, 25         marginalY, 51	qpp, 63	
qpp, 63	ptranspose	
px0     qpp::States, 120     px1     qpp::States, 120     py0     qpp::States, 120     py0     qpp::States, 120     py1     qpp::States, 120     inverse, 44     invperm, 44     ip, 44, 45     isprime, 45     ket, 26     kraus2super, 46     kraus2super, 46     kraus2super, 46     kraus2super, 46     kraus2super, 46     kraus2super, 48     load/ATALABmatrix, 49     load/ATLABmatrix, 49     load/ATLABmatrix, 49     logdet, 50     lognegativity, 50     marginalX, 50     marginalY, 51     maxn, 80     marginalY,	qpp, 63	·
qpp::States, 120 px1 qpp::States, 120 py0 qpp::States, 120 py1 qpp::States, 120 py1 qpp::States, 120 py20 qpp::States, 120 py20 qpp::States, 120 qpp::States, 120 py21 qpp::States, 120 py21 qpp::States, 120 py21 qpp::States, 120 py21 qpp::States, 120 qidx, 25 qp	px0	•
px1	qpp::States, 120	
qpp::States, 120  py0  qpp::States, 120  pp1  qpp::States, 120  pp20  qpp::States, 120  pp20  qpp::States, 120  pp21  qpp::States, 120  pp21  qpp::States, 120  pp3::States, 120  pp4, 64  qpp, 64  qpp, 64  qpp, 13  absm, 26  absm, 26  absan, 26, 27  adjoint, 27  anticomm, 27  anticomm, 27  apply, 27, 28, 30  applyCTRL, 3	px1	=
qpp::States, 120 py1	qpp::States, 120	•
app::States, 120 py1 app::States, 120 pp20 app::States, 120 app::States, 14 app::At	py0	
qpp::States, 120 qpp::States, 120 infty, 80 inverse, 44 invperm, 44 ip, 44, 45 ip, 44, 45 qmutualinfo qpp, 64 qpp, 13 absm, 26 abssq, 26, 27 adjoint, 27 anticomm, 27 apply, 27, 28, 30 applyCTRL, 30 applyCTRL, 30 ayg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 complement, 32 comperm, 33 concurrence, 33 conjugate, 33 contfrac2x, 33, 34  inverse, 44 invperm, 44 ip, 44, 45 isprime, 45 ket, 26 ket, 26 kraus2cuper, 46 kraus	qpp::States, 120	
pz0		hevects, 43
qpp::States, 120 pz1 qpp::States, 120 inverse, 44 invperm, 44 ip, 44, 45 isprime, 45 qpp, 64 qpp, 64 qpp, 13 absm, 26 absm, 26 abssq, 26, 27 adjoint, 27 anticomm, 27 anticomm, 27 apply, 27, 28, 30 applyCTRL, 30 avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 complement, 32 comperm, 33 concurrence, 33 concurrence, 33 concurrence, 33 concurrence, 33 concurrence, 33 contfrac2x, 33, 34	• •	idx, 26
pz1 inverse, 44 invperm, 44 invperm, 44 invperm, 44 invperm, 44 invperm, 45 isprime, 45 ket, 26 ket, 26 kraus2choi, 45 kraus2super, 46 kraus2choi, 45 kraus2super, 46 kron, 46, 47 kronpow, 47 lcm, 48 load, 48 load, 48 load, 48 load, 48 load, 48 load, 48 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalX, 50 marginalY, 51 choi2super, 32 chop, 79 measure, 51–54 measure_seq, 55 complement, 32 comperm, 33 concurrence, 33 conjugate, 33 contfrac2x, 33, 34 multiidx2n, 58	•	infty, 80
qpp::States, 120  qmutualinfo		inverse, 44
qmutualinfo qpp, 64 ket, 26 qpp, 13 kraus2choi, 45 kat, 26 absm, 26 kraus2choi, 47 kronpow, 47 lcm, 48 load, 48 load, 48 load, 48 load, 48 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 complement, 32 comperm, 33 concurrence, 33 conjugate, 33 contfrac2x, 33, 34 load, 48 lispring, 45 lognegativity, 56 lognegativity, 56 lognegativity, 50 marginalX, 50 marginalX, 50 marginalY, 51 maxin, 80 modpow, 57 multiidx2n, 58	•	invperm, 44
qpp, 64 qpp, 13     absm, 26     abssq, 26, 27     adjoint, 27     anticomm, 27     anticomm, 27     apply, 27, 28, 30     applyCTRL, 30     avg, 31     bigint, 25     bloch2rho, 31     bra, 25     choi2kraus, 31     choi2super, 32     chop, 79     cmat, 25     complement, 32     compperm, 33     concurrence, 33     contfrac2x, 33, 34     kraus2choi, 45     kraus2super, 46     kron, 46, 47     kronpow, 47     lcm, 48     load, 48     loadMATLABmatrix, 49     logdet, 50     logme, 50     logme, 50     lognegativity, 50     marginalX, 50     marginalY, 51     measure, 51–54     measure_seq, 55     complement, 32     compown, 33     concurrence, 33     concurrence, 33     contfrac2x, 33, 34     multiidx2n, 58	qpp::States, 120	ip, 44, 45
qpp, 64       ket, 26         qpp, 13       kraus2choi, 45         absm, 26       kraus2super, 46         abssq, 26, 27       kron, 46, 47         adjoint, 27       kronpow, 47         anticomm, 27       lom, 48         apply, 27, 28, 30       load, 48         applyCTRL, 30       loadMATLABmatrix, 49         avg, 31       logdet, 50         bigint, 25       logm, 50         bloch2rho, 31       lognegativity, 50         bra, 25       marginalX, 50         choi2kraus, 31       marginalY, 51         choi2super, 32       maxn, 80         chop, 79       measure, 51–54         cmat, 25       measure_seq, 55         comm, 32       mket, 55, 56         complement, 32       modinv, 56         compperm, 33       modmul, 56         concurrence, 33       modpow, 57         conjugate, 33       mprj, 57         contfrac2x, 33, 34       multiidx2n, 58	amutualinfo	isprime, 45
qpp, 13       kraus2choi, 45         absm, 26       kraus2super, 46         abssq, 26, 27       kron, 46, 47         adjoint, 27       kronpow, 47         anticomm, 27       lcm, 48         apply, 27, 28, 30       load, 48         applyCTRL, 30       loadMATLABmatrix, 49         avg, 31       logdet, 50         bigint, 25       logm, 50         bloch2rho, 31       lognegativity, 50         bra, 25       marginalX, 50         choi2kraus, 31       marginalY, 51         choi2super, 32       maxn, 80         chop, 79       measure, 51–54         cmat, 25       measure_seq, 55         comm, 32       mket, 55, 56         complement, 32       modinv, 56         comperm, 33       modmul, 56         concurrence, 33       modpow, 57         conjugate, 33       morpi, 57         contfrac2x, 33, 34       multiidx2n, 58	•	
absm, 26 abssq, 26, 27 adjoint, 27 anticomm, 27 apply, 27, 28, 30 applyCTRL, 30 avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 complement, 32 complement, 32 concurrence, 33 concurrence, 33 contfrac2x, 33, 34  kraus2super, 46 kron, 46, 47 kronpow, 47 lom, 48 load, 48 loadMATLABmatrix, 49 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modpow, 57 modpow, 57 modpow, 57 modpow, 57 multiidx2n, 58		kraus2choi, 45
adjoint, 27 anticomm, 27 apply, 27, 28, 30 applyCTRL, 30 avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 complement, 32 complement, 32 concurrence, 33 concurrence, 33 confrac2x, 33, 34  kronpow, 47 lcm, 48 load, 48 loadMATLABmatrix, 49 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modmul, 56 modpow, 57 modpow, 57 modpow, 57 modpow, 57 modpow, 58		kraus2super, 46
anticomm, 27 apply, 27, 28, 30 applyCTRL, 30 avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 concurrence, 33 concurrence, 33 contfrac2x, 33, 34  load, 48 load, 48 load, 48 loadMATLABmatrix, 49 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modpow, 57 modpow, 57 modpow, 57 morj, 57 confirac2x, 33, 34	abssq, 26, 27	
apply, 27, 28, 30 applyCTRL, 30 applyCTRL, 30 loadMATLABmatrix, 49 avg, 31 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 complement, 32 concurrence, 33 concurrence, 33 contfrac2x, 33, 34 loadMATLABmatrix, 49 loadMATLABmatrix, 49 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modinv, 56 modpow, 57 modpow, 57 morj, 57 conjugate, 33 multiidx2n, 58	adjoint, 27	kronpow, 47
applyCTRL, 30 avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 complement, 32 concurrence, 33 concurrence, 33 contfrac2x, 33, 34  loadMATLABmatrix, 49 logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modpow, 57 modpow, 57 morj, 57 contfrac2x, 33, 34	anticomm, 27	lcm, 48
avg, 31 bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 complement, 32 concurrence, 33 concurrence, 33 contfrac2x, 33, 34  logdet, 50 logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 modinv, 56 modinv, 56 modpow, 57 mprj, 57 contfrac2x, 33, 34	apply, 27, 28, 30	,
bigint, 25 bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 complement, 32 complement, 32 conjugate, 33 contfrac2x, 33, 34  logm, 50 lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 mket, 55, 56 modinv, 56 modpow, 57 mprj, 57 contfrac2x, 33, 34	applyCTRL, 30	•
bloch2rho, 31 bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 compperm, 33 concurrence, 33 contfrac2x, 33, 34  lognegativity, 50 marginalX, 50 marginalY, 51 maxn, 80 measure, 51–54 measure_seq, 55 mket, 55, 56 modinv, 56 modmul, 56 modpow, 57 modpow, 57 mprj, 57 multiidx2n, 58	avg, 31	•
bra, 25 choi2kraus, 31 choi2super, 32 chop, 79 cmat, 25 comm, 32 complement, 32 compperm, 33 concurrence, 33 conturence, 34 conturence, 35 conturence, 35 conturence, 36 conturence, 37 co	bigint, 25	<del>-</del>
choi2kraus, 31 marginalY, 51 choi2super, 32 maxn, 80 chop, 79 measure, 51–54 cmat, 25 measure_seq, 55 comm, 32 mket, 55, 56 complement, 32 modiny, 56 compperm, 33 modmul, 56 concurrence, 33 modpow, 57 conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58		
choi2super, 32       maxn, 80         chop, 79       measure, 51–54         cmat, 25       measure_seq, 55         comm, 32       mket, 55, 56         complement, 32       modinv, 56         compperm, 33       modmul, 56         concurrence, 33       modpow, 57         conjugate, 33       mprj, 57         contfrac2x, 33, 34       multiidx2n, 58		•
chop, 79 chop, 79 cmat, 25 comm, 32 complement, 32 compperm, 33 concurrence, 33 conjugate, 33 contfrac2x, 33, 34  measure, 51–54 measure_seq, 55 mket, 55, 56 modinv, 56 modmul, 56 modpow, 57 mprj, 57 multiidx2n, 58		<del>-</del>
cmat, 25 comm, 32 mket, 55, 56 complement, 32 compperm, 33 modmul, 56 concurrence, 33 conjugate, 33 contfrac2x, 33, 34 measure_seq, 55 mket, 55, 56 modinv, 56 modinv, 56 modpow, 57 modpow, 57 mprj, 57 multiidx2n, 58	·	*
comm, 32 mket, 55, 56 complement, 32 modinv, 56 compperm, 33 modmul, 56 concurrence, 33 modpow, 57 conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58	·	
complement, 32 modinv, 56 compperm, 33 modmul, 56 concurrence, 33 modpow, 57 conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58		_ ·
compperm, 33 modmul, 56 concurrence, 33 modpow, 57 conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58		, ,
concurrence, 33 modpow, 57 conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58	·	
conjugate, 33 mprj, 57 contfrac2x, 33, 34 multiidx2n, 58	• •	
contfrac2x, 33, 34 multiidx2n, 58		•
	· ·	• •
001, 34 HZIIIululux, 38		
	ooi, o <del>r</del>	nzmumux, Ju

negativity, 58	codeword, 87
norm, 59	FIVE_QUBIT, 87
omega, 59	internal::Singleton< const Codes >, 88
operator""_i, 59	NINE_QUBIT_SHOR, 87
pi, 80	SEVEN_QUBIT_STEANE, 87
powm, 59	Type, 87
prj, 60	qpp::Exception, 88
prod, 60	_construct_exception_msg, 91
ptrace, 61	_custom, 91
ptrace1, 61	_msg, 91
ptrace2, 63	_type, 91
ptranspose, 63	_where, 91
qmutualinfo, 64	CUSTOM_EXCEPTION, 90
rand, 64, 66	DIMS_INVALID, 90
randH, 67	DIMS_MISMATCH_CVECTOR, 90
randU, 70	DIMS_MISMATCH_MATRIX, 90
randV, 70	DIMS_MISMATCH_RVECTOR, 90
randidx, 67	DIMS_MISMATCH_VECTOR, 90
randket, 67	DIMS_NOT_EQUAL, 90
randkraus, 67	Exception, 90, 91
randn, 68, 69	MATRIX_MISMATCH_SUBSYS, 90
randperm, 69	MATRIX_NOT_CVECTOR, 90
randprime, 69	MATRIX_NOT_RVECTOR, 90
randrho, 69	MATRIX_NOT_SQUARE, 90
renyi, 70	MATRIX_NOT_SQUARE_OR_CVECTOR, 90
reshape, 71	MATRIX_NOT_SQUARE_OR_RVECTOR, 90
rho2bloch, 71	MATRIX_NOT_SQUARE_OR_VECTOR, 90
rho2pure, 71	MATRIX_NOT_VECTOR, 90
save, 72	NO_CODEWORD, 90
saveMATLABmatrix, 72	NOT_BIPARTITE, 90
schatten, 73	NOT_QUBIT_CVECTOR, 90
schmidtA, 73	NOT_QUBIT_MATRIX, 90
schmidtB, 73	NOT_QUBIT_RVECTOR, 90
schmidtcoeffs, 73	NOT_QUBIT_SUBSYS, 90
schmidtprobs, 74	NOT_QUBIT_VECTOR, 90
sigma, 74	OUT_OF_RANGE, 90
sinm, 74	PERM_INVALID, 90
spectralpowm, 75	PERM_MISMATCH_DIMS, 90
sqrtm, 75	SIZE_MISMATCH, 90
sum, 75, 76	SUBSYS_MISMATCH_DIMS, 90
super2choi, 76	TYPE_MISMATCH, 90
svals, 76	Type, 89
svd, 76	UNDEFINED_TYPE, 90
svdU, 77	UNKNOWN_EXCEPTION, 90
svdV, 77	what, 91
syspermute, 77	ZERO_SIZE, 90
to_void, 26	qpp::Gates, 91
trace, 78	$\sim$ Gates, 94
transpose, 78	CNOT, 96
tsallis, 78	CNOTba, 96
uniform, 79	CTRL, 94
var, 79	CZ, 96
x2contfrac, 79	expandout, 94
qpp.h, 151	FRED, 96
_QPP_UNUSED_, 153	Fd, 95
qpp::Codes, 85	Gates, 94
$\sim$ Codes, 87	H, 96
Codes, 87	ld, 95

ld2, 97	_start, 126
internal::Singleton< const Gates >, 96	∼Timer, 123
Rn, 95	display, 124
S, 97	get_duration, 125
SWAP, 97	operator=, 125
T, 97	tic, 125
TOF, 97	tics, 125
X, 97	Timer, 123
Xd, 95	toc, 125
Y, 97	qpp::Timer< T, CLOCK_T >, 121
Z, 97	qpp::experimental, 80
Zd, 96	qpp::internal, 80
qpp::IDisplay, 97	_check_cvector, 81
$\sim$ IDisplay, 99	_check_dims, 81
display, 99	_check_dims_match_cvect, 81
IDisplay, 99	_check_dims_match_mat, 82
operator<<, 99	_check_dims_match_rvect, 82
operator=, 99	_check_eq_dims, 82
qpp::Init, 100	_check_matching_sizes, 82
$\sim$ Init, 101	_check_nonzero_size, 82
Init, 101	_check_perm, 82
internal::Singleton< const Init >, 101	_check_qubit_cvector, 82
qpp::RandomDevices, 113	_check_qubit_matrix, 82
_rd, 115	_check_qubit_rvector, 82
_rng, 115	_check_qubit_vector, 82
$\sim$ RandomDevices, 115	_check_rvector, 82
internal::Singleton< RandomDevices >, 115	_check_square_mat, 82
RandomDevices, 115	_check_subsys_match_dims, 82
qpp::States, 117	_check_vector, 82
$\sim$ States, 119	_dirsum2, 82
b00, 119	_kron2, 82
b01, 119	_multiidx2n, 82
b10, 119	_n2multiidx, 82
b11, 119	variadic_vector_emplace, 82
GHZ, 119	qpp::internal::_details, 83
internal::Singleton < const States >, 119	qpp::internal::_details::_Display_Impl, 85
pGHZ, 120	_display_impl, 85
pW, 120	qpp::internal::IOManipEigen, 101
pb00, 120	_A, 103
pb01, 120	_chop, 103
pb10, 120	display, 102
pb11, 120	IOManipEigen, 102
px0, 120	qpp::internal::IOManipPointer
px1, 120	_end, 104
py0, 120	_n, 104
py1, 120	_p, 104
pz0, 120	_separator, 104
pz1, 120	_start, 105
States, 119	display, 104
W, 121	IOManipPointer, 104
x0, 121	operator=, 104
x1, 121	qpp::internal::IOManipPointer< PointerType >, 103
y0, 121	qpp::internal::IOManipRange
y1, 121	_end, 106
z0, 121	_first, 106
z1, 121	_last, 106
qpp::Timer	_separator, 106
_end, 126	_start, 107

display, 106	SEVEN_QUBIT_STEANE
IOManipRange, 106 operator=, 106	qpp::Codes, 87 SIZE MISMATCH
qpp::internal::IOManipRange< InputIterator >, 105	qpp::Exception, 90
qpp::internal::Singleton	SUBSYS_MISMATCH_DIMS
∼Singleton, 116	qpp::Exception, 90
get_instance, 117	SWAP
get_thread_local_instance, 117	qpp::Gates, 97
operator=, 117	save
Singleton, 116	qpp, 72
qpp::internal::Singleton< T >, 115	saveMATLABmatrix
qpp::is_complex < std::complex < T > >, 108	qpp, 72
qpp::is_complex< T >, 107	schatten
qpp::is_iterable < T, to_void < decltype(std::declval < T	qpp, 73
>().begin()), decltype(std::declval< T >().	schmidtA
end()), typename T::value_type > >, 110 qpp::is_iterable< T, typename >, 109	qpp, 73 schmidtB
qpp::is_matrix_expression< Derived >, 111	qpp, 73
qpp::is_matrix_expression< between ≥, 111 qpp::is_matrix_expression< typename Eigen::Matrix←	schmidtcoeffs
Base < Derived > >, 112	qpp, 73
5400 ( 5611104 ) 7, 112	schmidtprobs
rand	qpp, 74
qpp, 64, 66	sigma
randH	qpp, 74
qpp, 67	Singleton
randU	qpp::internal::Singleton, 116
qpp, 70	sinm
randV	qpp, 74
qpp, 70	spectralpowm
randidx	qpp, 75
qpp, 67	sqrtm
randket	qpp, 75
qpp, 67 randkraus	States
qpp, 67	qpp::States, 119
randn	statistics.h, 154 sum
qpp, 68, 69	qpp, 75, 76
random.h, 153	super2choi
RandomDevices	qpp, 76
qpp::RandomDevices, 115	svals
randperm	qpp, 76
qpp, 69	svd
randprime	qpp, 76
qpp, 69	svdU
randrho	qpp, <b>77</b>
qpp, 69	svdV
renyi	qpp, 77
qpp, 70	syspermute
reshape	qpp, 77
qpp, 71 rho2bloch	Т
	qpp::Gates, 97
qpp, 71 rho2pure	TOF
qpp, 71	qpp::Gates, 97
Rn	TYPE_MISMATCH
qpp::Gates, 95	qpp::Exception, 90
-11-12	tic
S	qpp::Timer, 125
qpp::Gates, 97	tics

```
qpp::Timer, 125
                                                      ZERO_SIZE
Timer
                                                           qpp::Exception, 90
    qpp::Timer, 123
                                                      Zd
                                                           qpp::Gates, 96
to_void
    qpp, 26
toc
    qpp::Timer, 125
trace
    qpp, 78
traits.h, 156
transpose
    qpp, 78
tsallis
    qpp, 78
Type
    qpp::Codes, 87
    qpp::Exception, 89
types.h, 157
UNDEFINED TYPE
    qpp::Exception, 90
UNKNOWN_EXCEPTION
    qpp::Exception, 90
uniform
    qpp, 79
var
    qpp, 79
variadic_vector_emplace
    qpp::internal, 82
W
    qpp::States, 121
what
    qpp::Exception, 91
Χ
    qpp::Gates, 97
x0
    qpp::States, 121
х1
    qpp::States, 121
x2contfrac
    qpp, 79
Xd
    qpp::Gates, 95
Υ
    qpp::Gates, 97
y0
    qpp::States, 121
у1
    qpp::States, 121
Ζ
    qpp::Gates, 97
z0
    qpp::States, 121
z1
    qpp::States, 121
```