Review for ECE211 2.0

Friday, April 18, 2025 11:40

Chapter 4.

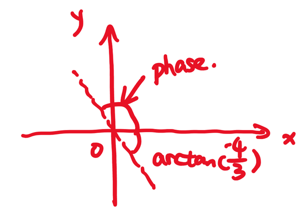
phasor. $f(t) = A\cos(\omega t+\theta) = \mathrm{Re}\{Ae^{j(\omega t+\theta)}\} = \mathrm{Re}\{Fe^{j\omega t}\}$

phasor $F = Ae^{j\theta} = \underset{\uparrow\ \text{magnitude}}{A}\angle\theta \leftarrow$ phase. 写成这个形式

$f(t) = Fe^{j\omega t}$ $\quad f'(t) = j\omega Fe^{j\omega t} = Ge^{j\omega t}$ $\quad G = j\omega F$

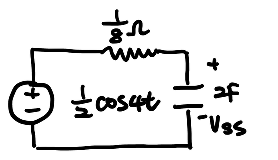
计算幅角: $Y = \dfrac{5j}{-3+4j} = Ae^{j\theta}$

$A = \dfrac{\sqrt{5^2}}{\sqrt{3^2+4^2}} = 1$ $\quad \dfrac{e^{j\frac{\pi}{5}}}{e^{j\theta}} = e^{j(\frac{\pi}{5}-\theta)}$



$\theta = \pi + \arctan(-\frac{4}{3})$

Steady-state. V_{ss}.



$\omega = 4.$

$V_s = \frac{1}{2}e^{j0}\cdot e^{j4t}$

$\frac{1}{2}e^{j0} - \frac{1}{8}\cdot 2\cdot\frac{dV_{ss}}{dt} - V_{ss} = 0 \Rightarrow \frac{1}{2} - \frac{1}{4}\cdot j\omega V_c - V_c = 0$

$V_c = \dfrac{\frac{1}{2}}{1+j} = Ae^{j\theta} = \frac{1}{2\sqrt{2}}e^{-j\frac{\pi}{4}} \Rightarrow V_{ss} = \frac{1}{2\sqrt{2}}\cos(4t-\frac{\pi}{4})$

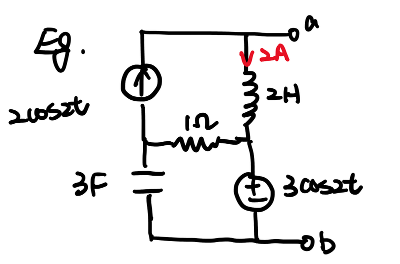
阻抗. Impedance

$R \to R$

$L \to j\omega L$ $\qquad L\frac{di}{dt} = j\omega L\cdot i(t)$

$C \to \frac{1}{j\omega C}$ $\qquad i = C\frac{dV}{dt} = j\omega C\,V(t)$

Eg.



$\omega = 2$

$Z_L = j\omega L = 4j$

$Z_C = \frac{1}{j\omega C} = \frac{1}{6j} = -\frac{j}{6}$

$V = 3$, $I = 2$

$V_T = 3 + 2\cdot 4j = 3+8j$

$Z_T = 4j$ short.

$I_N = \frac{V_T}{Z_T} = \frac{3+8j}{4j} = -\frac{3}{4}j + 2$

Average Power. $P_{avg} = \frac{1}{T}\int_T V(t)i(t)\,dt$ $\qquad V(t) = \mathrm{Re}\{Ve^{j\omega t}\}$ $\qquad i(t) = \mathrm{Re}\{ie^{j\omega t}\}$

$V(t)\cdot i(t) = \frac{1}{2}(Ve^{j\omega t} + V^*e^{-j\omega t})\cdot\frac{1}{2}(ie^{j\omega t} + i^*e^{-j\omega t})$

$= \frac{1}{4}(Vie^{j2\omega t} + V^*i + Vi^* + V^*i^*e^{-2j\omega t})$

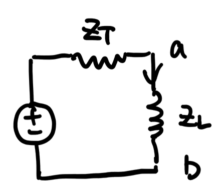
$= \frac{1}{2}\mathrm{Re}\{Vie^{j2\omega t}\} + \frac{1}{2}\mathrm{Re}\{Vi^*\}$

$P = \frac{1}{T}\int_T V(t)i(t)\,dt$

$= \frac{1}{T}\int_T \frac{1}{2}\mathrm{Re}\{Vie^{j2\omega t}\} + \frac{1}{2}\mathrm{Re}\{Vi^*\}$

$= \frac{1}{2T}\cdot T\cdot\mathrm{Re}\{Vi^*\} = \frac{1}{2}\mathrm{Re}\{Vi^*\}$

Available Power.

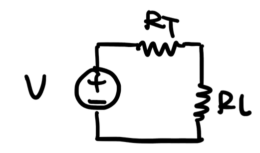


$I_L = \frac{V_T}{Z_T+Z_L}$ $\qquad V_L = I_L\cdot Z_L = V_T\cdot\frac{Z_L}{Z_T+Z_L}$

$P_L = \frac{1}{2}\mathrm{Re}\{VI^*\} = \frac{1}{2}\mathrm{Re}\{V_T\cdot\frac{Z_L}{Z_T+Z_L}\cdot(\frac{V_T}{Z_T+Z_L})^*\}$

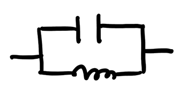
$= \frac{1}{2}\mathrm{Re}\{Z_L|V_T|^2\cdot\frac{1}{|Z_T+Z_L|^2}\} = \frac{1}{2}\cdot\mathrm{Re}\{Z_L\}\cdot|V_T|^2\cdot\frac{1}{|R_T+R_L+jX_T+jX_L|^2}$ (RL)

$X_T + X_L = 0$ $\quad R_L = R_T$ $\Rightarrow P_L = \frac{1}{2}\cdot R_T\cdot|V_T|^2\cdot\frac{1}{4R_T^2} = \frac{1}{8}\frac{|V_T|^2}{R_T} = P_{max}$



$P_{max} = \frac{V^2}{4R_T}$

Resonance 若 $\omega = \frac{1}{\sqrt{LC}} \Rightarrow$ $Z_T = 0$

$\Rightarrow$  $Z_T = \infty$

Chapter 5.

Frequency Response. $H(\omega) \Rightarrow$ 输入: $V_i(t)$ 输出 $V_o(t)$ $\qquad H(\omega)$ 把输入 V_i 和 V_o 联系起来.

$H(\omega)$ 与常系数独立.

$V_o(t) = H(\omega)V_i(t)$, $H(\omega) = \frac{V_o(t)}{V_i(t)}$

First Order Filter.

Low pass $|H(\omega)| \doteq \frac{1}{\sqrt{1+\omega^2}}$

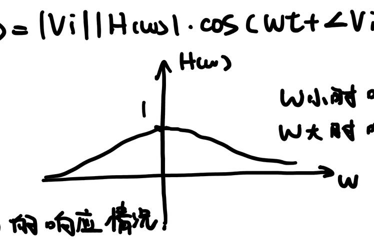
High pass $|H(\omega)| \doteq \frac{\omega}{\sqrt{1+\omega^2}}$

Band Pass $|H(\omega)| \doteq \frac{\omega}{\sqrt{\omega^2+(1-\omega^2)^2}} \Rightarrow H(0) = 0$, $H(\infty) = 0$

$V_o(t) = H(\omega)V_i(t) = |V_i||H(\omega)|\cdot\cos(\omega t + \angle V_i + \angle H(\omega))$

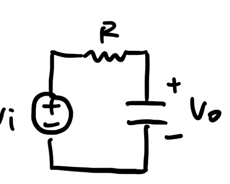
$H(\omega) - \omega$ 图

通过 $H(\omega)-\omega$ 图能看出输出对于不同 ω 的响应情况



ω小时响应大
ω大时响应小

$\Rightarrow$ 低通滤波 $\Leftarrow$ V_i



Conjugate Symmetry $H(-\omega) = H^*(\omega)$

Even amplitude response. $|H(-\omega)| = |H(\omega)|$

Odd phase response $\angle H(-\omega) = -\angle H(\omega)$

Real DC response $H(0) = H^*(0)$ real value

Steady Response to $e^{j\omega t}$ $\qquad e^{j\omega t} \to \mathrm{LTI} \to H(\omega)e^{j\omega t}$

Multiple Frequencies. $f(t) = \sum_n C_n\cos(\omega_n t+\theta_n) + \sum_k b_k\sin(\omega_k t+\phi_k) + \sum_m F_m e^{j\omega_m t}$

$y(t) = \sum_n C_n|H(\omega_n)|\cos(\omega_n t+\theta_n+\angle H(\omega_n))$

$+ \sum_k b_k|H(\omega_k)|\sin(\omega_k t+\phi_k+\angle H(\omega_k))$

$+ \sum_m F_n|H(\omega_m)|e^{j\omega_m t}$

Decibel Amplitude Response.

$|H(\omega)|_{dB} = 10\log_{10}(|H(\omega)|^2) = 20\log_{10}|H(\omega)|$

$|H(\omega)| = \begin{cases} \frac{1}{\sqrt{2}} & |H(\omega)|_{dB} = -3\text{dB} \\ 1 & |H(\omega)|_{dB} = 0\text{dB} \\ \sqrt{2} & |H(\omega)|_{dB} = 3\text{dB} \\ 2 & |H(\omega)|_{dB} = 6\text{dB} \end{cases}$

Chapter 6.

Periodic Signals. $f(t) = \cos t + \cos 2t \Rightarrow \omega_0 = 1$, $T = 2\pi$

$\frac{\omega_1}{\omega_2}$ 有理数 $\Rightarrow$ periodic $\qquad T = \mathrm{LCM}(T_1, T_2)$ 最小公倍数. $\quad \omega_0 = \mathrm{GCD}(\omega_1,\omega_2)$ 最大公约数

Fourier Series. $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$ $\quad F_n \to$ independent of time.

$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}\,dt$ $\quad F_0 \to$ DC component.

傅利叶变换存在条件: 1) $\int_T |f(t)|\,dt < \infty$ 2) 有限个断点 3) 最大最小值不发散.

Eg. $f(t) = \begin{cases} 1 & t\in[0,1) \\ 0 & t\in[1,2) \end{cases}$

$\omega_0 = \frac{2\pi}{T} = \pi$ $\qquad F_n = \frac{1}{T}(\int_0^1 1\cdot e^{-jn\omega_0 t}dt + \int_1^2 0\cdot e^{-jn\omega_0 t}dt)$

$= \frac{1}{2}\cdot\frac{1}{-jn\omega_0}(e^{-jn\omega_0} - 1) = \begin{cases} 0 & n \text{ even} \\ \frac{1}{jn\omega_0} & n \text{ odd.} \end{cases}$

$F_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{2}$

$f(t) = \frac{1}{2} + \sum_{-\infty,\ n\ \text{odd}}^{\infty}\frac{1}{jn\pi}e^{jn\omega_0 t}$

Fourier series forms.

1) $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$

2) $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t))$ $\quad \begin{cases} a_0 = 2F_0 \\ F_0 = \frac{1}{2}a_0 \end{cases}$ $\quad a_n = F_n + F_{-n}$, $F_n = \frac{a_n - jb_n}{2}$ $\quad b_n = j(F_n - F_{-n})$, $F_{-n} = \frac{a_n + jb_n}{2}$

Compact Fourier Series. 3) $f(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta_n)$

$C_0 = 2F_0$ $\quad C_n = 2|F_n|$ $\quad \theta_n = \angle F_n$, $F_n = \frac{C_n}{2}e^{j\theta_n} = F_{-n}^*$

Eg. $f(t) = \begin{cases} 1 & t\in[0,1) \\ 0 & t\in[1,2) \end{cases} \Rightarrow F_n = \begin{cases} \frac{1}{jn\pi} & n \text{ odd} \\ \frac{1}{2} & n=0 \\ 0 & n \text{ even}, n\neq 0 \end{cases}$

$\omega_0 = \pi$

Exp form: $f(t) = \frac{1}{2} + \sum_{-\infty,\ n\ \text{is odd}}^{\infty}\frac{1}{jn\pi}e^{jn\pi t} = \frac{1}{2} + \sum_{n=1,\ \text{odd}}^{\infty}(\frac{1}{jn\pi}e^{jn\pi t} + \frac{1}{j(-n)\pi}e^{-jn\pi t})$

$a_n = 0$ for $n \geq 1$, $a_0 = 1$ $\qquad b_n = \begin{cases} 0 & \text{even} \\ \frac{2}{n\pi} & \text{odd} \end{cases}$ $\quad = \frac{1}{2} + \sum_{n=1,\ \text{odd}}^{\infty}(\frac{1}{n\pi}\cdot\frac{e^{jn\pi t} - e^{-jn\pi t}}{j}) = \boxed{\frac{1}{2} + \sum_{n=1,\ \text{odd}}^{\infty}\frac{2}{n\pi}\sin(n\pi t)}$

$\Downarrow$

Compact form: $\frac{1}{2} + \sum_{n=1,\ \text{odd}}^{\infty}|\frac{2}{n\pi}|\cos(n\pi t - \frac{\pi}{2})$

Scaling property. $f(t) \to F_n$ $\qquad kf(t) \to kF_n$.

Time-shift property. $f(t) \to F_n$ $\qquad f(t-t_0) \to F_n\cdot e^{-jn\omega_0 t_0} \Rightarrow H_n$

Addition property $f(t) \to F_n$, $y(t) \to Y_n$ $\qquad x(t) = f(t)+y(t) \to X_n = F_n + Y_n$

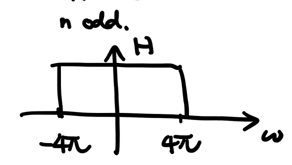
Eg. $g(t) = \begin{cases} 2 & t\in[0,1) \\ 0 & t\in[1,2) \end{cases}$ $\qquad G_n = \begin{cases} \frac{2}{jn\pi} & n \text{ odd} \\ 1 & n=0 \\ 0 & n \text{ even} \end{cases}$

$x(t) = \begin{cases} 1 & t\in[0,1) \\ -1 & t\in[1,2) \end{cases}$ $\qquad x(t) = g(t) - 1$ (改变 DC offset) $\Rightarrow X_n = \begin{cases} \frac{2}{jn\pi} & n \text{ odd} \\ 1-1=0 & n=0 \\ 0 & n \text{ even.} \end{cases}$

Average power. $P = \frac{1}{T}\int_T |f(t)|^2\,dt = \sum_{n=-\infty}^{\infty}|F_n|^2$

Eg. $f(t) = \begin{cases} 1 & t\in[0,1) \\ 0 & t\in[1,2) \end{cases} = \frac{1}{2} + \sum_{n=-\infty,\ n\ \text{odd.}}^{\infty}\frac{1}{jn\pi}e^{jn\pi t}$

$H(\omega) = \mathrm{rect}(\frac{\omega}{8\pi}) \to$



LTI system $e^{j\omega t} \to \boxed{\mathrm{LTI}} \to H(\omega)e^{j\omega t}$

$\cos(\omega t+\theta) \to \boxed{\mathrm{LTI}} \to |H(\omega)|\cos(\omega t+\theta+\angle H(\omega))$

$f(t) = \sum_{n=-\infty}^{\infty}F_n e^{jn\omega_0 t} \to \boxed{\mathrm{LTI}} \to \sum_{n=-\infty}^{\infty}H(n\omega_0)F_n e^{jn\omega_0 t}$

$f(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty}C_n\cos(n\omega_0 t+\theta_n) \to \boxed{\mathrm{LTI}} \to \frac{C_0}{2}H(0) + \sum_{n=1}^{\infty}|H(n\omega_0)|C_n\cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

$y(t) = \frac{1}{2}\cdot H(0) + \sum_{n=-\infty}^{\infty}H(n\omega_0)\cdot\frac{1}{jn\pi}e^{jn\pi t}$ $\qquad \begin{cases} H(0) = 1 \\ H(\pm\pi) = 1 \\ H(\pm 3\pi) = 1 \end{cases}$

$= \frac{1}{2} + \frac{1}{j\pi}e^{j\pi t} + \frac{1}{-j\pi}e^{-j\pi t} + \frac{1}{j3\pi}e^{j3\pi t} + \frac{1}{j(-3\pi)}e^{-j3\pi t}$

$= \frac{1}{2} + \frac{2}{\pi}\sin(\pi t) + \frac{2}{3\pi}\sin(3\pi t)$

THD. $f(t) = \cos(\omega_0 t) \to \boxed{\text{not}-\mathrm{LTI}} \to \frac{C_0}{2} + C_1\cos(\omega_0 t) + C_2\cos(2\omega_0 t) \cdots$

$\mathrm{THD} = \dfrac{\text{power in harmonics}}{\text{power in }\omega_0} = \dfrac{\frac{C_2^2}{2} + \frac{C_3^2}{2}\cdots}{\frac{C_1^2}{2}} = \dfrac{P_{avg} - \frac{C_0^2}{4} - \frac{C_1^2}{2}}{\frac{C_1^2}{2}}$ $\qquad P_{avg} = \frac{1}{T}\int_T|f(t)|^2dt$

$P_{y(t)} = (\frac{1}{2})^2 + \frac{1}{2}\cdot(\frac{2}{\pi})^2 + \frac{1}{2}\cdot(\frac{2}{3\pi})^2 \doteq 0.48.$