ME200 Review 2.0

Friday, April 25, 2025 12:12

Unsteady Flow

$$\frac{dE}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h + \frac{v^2}{2} + gz\right)_i - \sum_e \dot{m}_e \left(h + \frac{v^2}{2} + gz\right)_e$$

Specific heat.

$C_v = \left(\frac{\partial u}{\partial T}\right)_v$ constant volume $\qquad C_p = \left(\frac{\partial h}{\partial T}\right)_p$ constant pressure.

$u = f(T, v)$ $\quad du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = C_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv.$

$h = f(T, p)$ $\quad dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp = c_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp.$

Saturation & Phase change $C_p \to \infty$

Liquid & Solid

$\frac{dh}{dT} \approx \frac{du}{dT}$ $\quad C_p \approx C_v \approx c$

Equation of States (EOS)

$pv = RT$.

$h = u + pv = u(T) + RT = h(T) \Rightarrow$ Specific internal energy $\propto T$.

Ideal Gas Law

$pv = RT$ $\quad R = \frac{R_u}{M} = \frac{8.314}{M} \Leftrightarrow PV = nR_uT$

$h = u + pv = u + RT \rightarrow C_p = C_v + R$ (Ideal Gas Only)

具体计算:

$\frac{\bar{c}_p}{R} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4 = f(T)$ $\quad \Delta h = R\int f(T)$, $\bar{c}_p = Rf(T)$

☆ The perfect gas specific heats.

$C_v =$ constant $\rightarrow C_p = C_v + R =$ constant.

$\frac{PV}{RT} = 1$ $\quad u_2 - u_1 = C_v(T_2 - T_1)$ $\quad h_2 - h_1 = c_p(T_2 - T_1)$

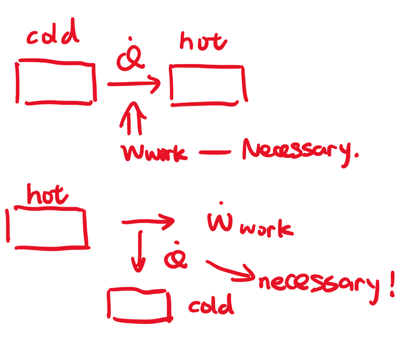
Reversibility

Second Law of Thermodynamics. $\rightarrow \eta < 10\%$

可逆过程 中间态、一定均匀, 与外界接触、一定是平衡过程.

Clausius' Consequence

K-P's Consequences



Equilibrium Process
平衡过程

$\Downarrow$

Hot $\xrightarrow[\dot{Q}]{}$ Cold

Reversible!

Reversible Power Cycle

$\dot{Q}_H$
$\Downarrow$
Engine $\Rightarrow \dot{W}_{cycle}$.
$\Downarrow$
$\dot{Q}_L$

$$\eta = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

$$\eta = \frac{\text{wanted energy}}{\text{Input energy}}$$

Work Consuming Cycles.

COP (Coefficient of performance)

$$COP = \frac{\dot{Q}_{desire}}{\dot{W}_{cycle}}$$

COP 可以 > 1

$\dot{Q}_H$
$\Uparrow$
Engine $\Leftarrow \dot{W}_{cycle}$
$\Uparrow$
$\dot{Q}_{cold}$

$COP_{refrigeration} = \beta = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L}$

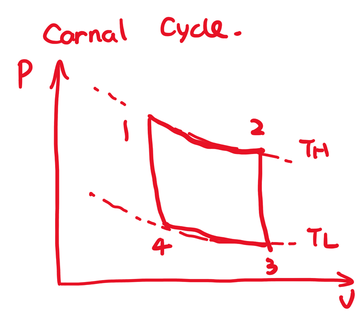
$COP_{heat\text{-}pump} = \gamma = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$

Carnot Cycle.

Isothermal Expansion $\rightarrow$ Adiabatic Expansion $\rightarrow$ Isothermal Compression $\rightarrow$ Adiabatic Compression.

等温 绝热

Carnot Cycle.



$\eta = \frac{W_{cycle}}{Q_H}$ ☆ 卡诺循环完全可逆.

1-2 $\quad \Delta E = 0 \Rightarrow Q_{12} = W_{12} = \int p dv = \int \frac{\dot{m}RT}{v} dv = \dot{m}RT \ln\frac{v_2}{v_1}$

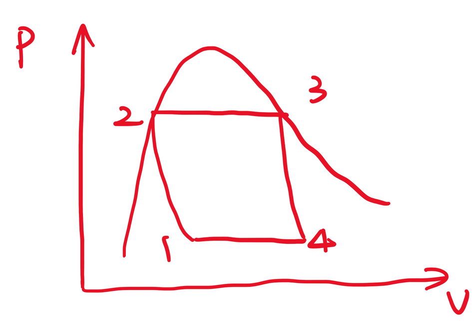
2-3 $\quad Q = 0 \quad \Delta U = -W_{23} \Rightarrow \int \dot{m} c_v dT = -\int_2^3 \frac{\dot{m}RT}{v} dv \Rightarrow C_v \ln\frac{T_3}{T_2} = -R\ln\frac{V_3}{V_2}$.

3-4 $\quad \Delta E = 0 \Rightarrow Q_{34} = \dot{m}RT_L \ln\frac{V_4}{V_3}$ $\qquad Q_L = Q_{34}$ $\quad Q_H = Q_{12}$ $\qquad \frac{Q_L}{Q_H} = \frac{T_L}{T_H}\frac{\ln\frac{V_4}{V_3}}{\ln\frac{V_2}{V_1}}$ $\qquad \eta = 1 - \left|\frac{Q_L}{Q_H}\right| = 1 - \left(\frac{T_L}{T_H}\right)$

4-1 $\quad C_v \ln\frac{T_1}{T_4} = -R\ln\frac{V_1}{V_4}$

不可逆的 power cycle 效率更低.

☆ 有2个热源, 何可逆供能循环, 拥有相同的效率 $\eta = 1 - \frac{T_L}{T_H}$



2-3 Isothermal $\quad W_{2\text{-}3} = P(V_3 - V_2) = \dot{m} P_2 (v_3 - v_2)$ $\qquad \dot{Q}_H = \dot{Q}_{23} = \Delta U + \dot{W}_{23} = (U_3 - U_2) + \dot{W}_{23}$

$\dot{w}_{2\text{-}3} = \frac{\dot{W}_{23}}{\dot{m}} = P_2(v_3 - v_2)$

3-4 Adiabatic

$Q_{34} = 0$ $\quad u_4 - u_3 = -\dot{W}_{34} \Rightarrow u_4 = u_3 - \dot{W}_{34}$

4-1 Isothermal $\quad \left|\frac{Q_L}{Q_H}\right| = \frac{T_L}{T_H} \Rightarrow Q_L = \frac{T_L}{T_H}|Q_H|$

$u_1 - u_4 = \dot{Q}_{4\text{-}1} - \dot{W}_{4\text{-}1}$ $\quad \dot{W}_{4\text{-}1} = -p(v_4 - v_1)$

1-2 $\quad \dot{Q}_{12} = 0$, $\quad u_2 - u_1 = -\dot{w}_{12}$

☆ $\Delta U = -W_{adiabatic}$
First Law.

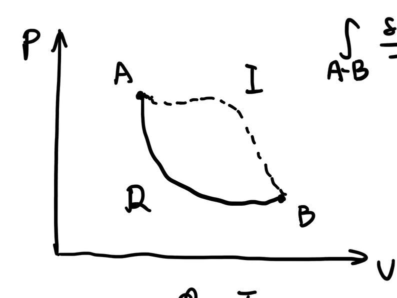
Entropy

$$\Delta S = \int_R \frac{\delta Q_{rev}}{T}$$

Second Law

Increased entropy principle

Eg.

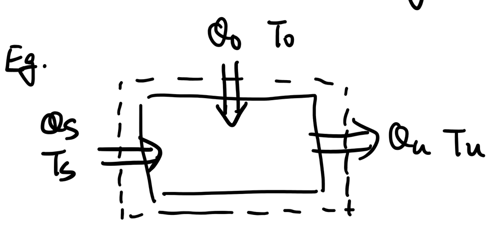


$\int_{A\text{-}B} \frac{\delta Q}{T} + \int_{B\text{-}A} \frac{\delta Q_R}{T} + \sigma_s = 0$
$\Downarrow$
Irreversible.

$\sigma_s \geq 0$ Entropy production $\rightarrow$ 对不可逆过程.

$S_B - S_A = \int_{A\text{-}B} \frac{\delta Q}{T} + \sigma_s$ ($\sigma_s \geq 0$) $\Rightarrow \sigma_s = \Delta S$ 若 < 0 则无法发生.
查表.

Eg.



$Q_u = Q_0 + Q_s$

$\frac{Q_s}{T_s} + \frac{Q_0}{T_0} - \frac{Q_u}{T_u} + \sigma_s = 0$

$\sigma_s = -Q_u\left(\frac{1}{T_0} - \frac{1}{T_u}\right) - Q_s\left(\frac{1}{T_s} - \frac{1}{T_0}\right) \geq 0$

$Q_u \leq Q_s\left(\frac{1}{T_0} - \frac{1}{T_s}\right) / \left(\frac{1}{T_0} - \frac{1}{T_u}\right)$

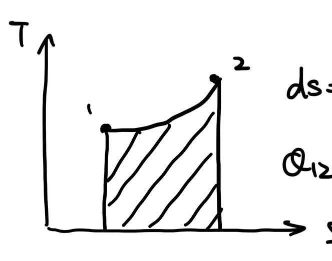
Entropy balance. $\quad$ Closed System $\quad S_2 - S_1 \geq \int_{1\text{-}2} \frac{\delta Q}{T}$

Isolated System $\quad S_2 - S_1 \geq 0$

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \underbrace{\left|\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e\right|}_{\text{边界上的熵转移}} + \dot{\sigma}_s$$

$\dot{\sigma}_s$: 熵增常数.

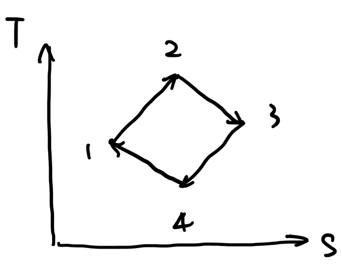
T-S Diagram.



$ds = \frac{\delta Q_{rev}}{T}$

$Q_{12} = \int T ds = S_{面积}$

Only for reversible process

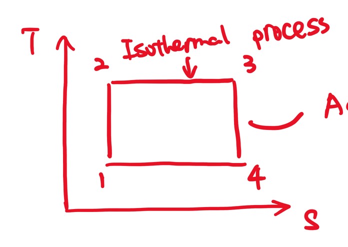


1-2 $\quad Q > 0$
2-3 $\quad Q > 0$
3-4 $\quad Q < 0$
4-1 $\quad Q < 0$

$W_{cyc} = Q_{in} - Q_{out}$

$\eta = \frac{W_{cyc}}{Q_{in}} = \frac{\text{T-S enclosed area}}{\text{T-S area under 1-3}}$

Carnot Cycle T-S diagram



T-ds Equations.

First Law for a closed system $\quad \Delta U = \delta Q_{rev} - \delta W_{rev}$

Simple chemical system & Arbitrary reversible process $\quad dU = \delta Q_{rev} - pdV$

Simple chemical system: $\quad dQ = Tds$

Gibbs Equations.

$H = U + PV \rightarrow dH = du + dpv + pdv \rightarrow du + pdv = dH - dp \cdot v$

$Tds = du + pdv \Rightarrow Tds = du + pdv$

$Tds = c_v dT + pdv \rightarrow ds = \frac{c_v}{T}dT + \frac{P}{T}dv$ $\qquad S_2 - S_1 = \int \frac{c_v}{T}dT + \frac{P}{T}dv$

$Tds = C_p dT - vdp \rightarrow ds = \frac{C_p}{T}dT - \frac{V}{T}dp$ $\qquad S_2 - S_1 = \int \frac{C_p}{T}dT - \frac{V}{T}dp$

Ideal Gas Case: $\quad pv = RT \Rightarrow \frac{P}{T} = \frac{R}{V}$, $\frac{V}{T} = \frac{R}{P}$,

$S - S_0 = \int_{T_0}^{T} \frac{c_v}{T}dT + \int_{V_0}^{V_1} \frac{R}{V}dv$ $\qquad S - S_0 = C_v \ln\frac{T}{T_0} + R\ln\frac{V}{V_0}$

☆ For Ideal Gas $\quad C_p = C_v + R$

For Perfect Gas $\quad C_v \rightarrow$ constant
$C_p = C_v + R$

Perfect Gas: $\quad C_p = C_v + R \qquad C_v \rightarrow$ constant

$S - S_0 = C_p \ln\frac{T}{T_0} - R\ln\frac{P}{P_0}$

Entropy Change for Ideal Gas

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{C_p(T)}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dp$$

$\int_{T_1}^{T_2} C_p \frac{dT}{T} = \int_0^{T_2} C_p \frac{dT}{T} - \int_0^{T_1} C_p \frac{dT}{T} = S^0(T_2) - S^0(T_1)$ Absolute entropy.

$\Rightarrow S(T_2, P_2) - S(T_1, P_1) = S^0(T_2) - S^0(T_1) - R\ln\frac{P_2}{P_1}$

Entropy Equation

$$\frac{dS}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \dot{m}(S_{in} - S_{out}) + \dot{\sigma}_s$$

Isentropic for ideal gas.

Adiabatic + Reversible

$\dot{Q} = 0 \qquad \Delta S = 0 \Rightarrow \frac{ds}{dt} = 0$

$\Delta S = S^0(T_2) - S^0(T_1) - R\ln\frac{P_2}{P_1}$ $\qquad \frac{P_2}{P_1} = \frac{\exp\left(\frac{S^0(T_2)}{R}\right)}{\exp\left(\frac{S^0(T_1)}{R}\right)} = \frac{P_{r_2}}{P_{r_1}}$ relative pressure.

$v = \frac{RT}{P}$ $\qquad \frac{V_2}{V_1} = \frac{RT_2}{P_2} \cdot \frac{P_1}{RT_1} = \frac{RP_{r_1}}{RP_{r_2}} \cdot \frac{T_2}{T_1} = \frac{V_{r_2}}{V_{r_1}}$ $\qquad V_r = \frac{RT}{P_r}$ relative volume.

Constant Specific Heat Assumption.

$\Delta h = \Delta u + p\Delta v + v\Delta p$

$\Delta h - v\Delta p = \Delta u + p\Delta v$

Reversible process.

$0 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$ $\qquad K = \frac{C_p}{C_v}$ $\quad C_p = \frac{KR}{K-1}$

$0 = C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$ $\qquad C_v = \frac{R}{K-1}$

$\Downarrow$

$\Delta S = 0$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$

$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{K-1}$ $\qquad \Leftarrow C_p$ Constant

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{K}$