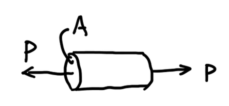
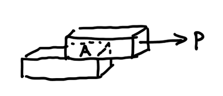
Review

Saturday, March 8, 2025 11:47

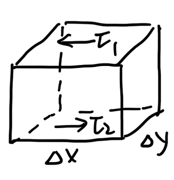
Normal Stress $\sigma_z = \lim\limits_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$  $\sigma = \frac{P}{A}$ 拉伸为正，压缩为负

$\sigma_{avg} = \frac{F}{A}$

Shear Stress $\tau_{zx} = \lim\limits_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$  $\tau = \frac{P}{A}$ 都为正，A为断裂面面积

$\tau_{avg} = \frac{F}{A}$

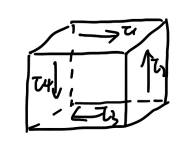
Shear Stress Equilibrium



$\vec{F_1} = \vec{\tau_1} \cdot \Delta x \cdot \Delta y$

$\vec{F_2} = \vec{\tau_2} \cdot \Delta x \cdot \Delta y$

$\vec{F_1} + \vec{F_2} = 0$



Equilibrium:

$\tau_1 = \tau_2 = \tau_3 = \tau_4$

① τ 对应的外力合力平衡

② 合力产生的 couple moment 平衡.

Factor of Safety: $F.S. = \frac{F_{fail}}{F_{allow}} = \frac{\sigma_{fail}}{\sigma_{allow}} = \frac{\tau_{fail}}{\tau_{allow}}$

Load factor (γ)

$R = \sum \gamma \cdot F$ Dead load 自重 Live Load 外力

Resistance factors ϕ (phi)

Design Criteria

LRFD criterion $\phi P_n \geqslant \sum \gamma_i R_i$ P_n: nominal strength.

Normal strain

$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$, $\varepsilon = \lim \frac{\Delta s' - \Delta s}{\Delta s}$

Shear strain

$\gamma = \frac{\pi}{2} - \lim \theta'$

Nominal / Engineering strain: $\varepsilon = \frac{\delta}{L_0}$

proportional limit σ_{pl}: 维持弹性形变的最大 normal stress

plastic deformation 塑形形变 → 对应 yield stress / point σ_y

↳ cause permanent deformation

回弹按弹性回弹.

Strain Hardening → σ 再增加，但 σ ~ ε 不是线性.

最大的 σ 称为 ultimate stress σ_u

Necking → 快断裂的状态.. σ 下降，断裂的 σ 称为 fracture stress σ_f

Ductile material → No well-defined yield point.

↳ Yield point: 0.2% ε → paralleled to elastic process

交点为 σ_{ys}

Brittle material: 对压力表现好.

杨氏模量，$\sigma = E\varepsilon$ → Elastic process.

Strain energy: $\Delta U = \frac{1}{2}\sigma\varepsilon\,\Delta V$ ⇒ $\sigma S = F$ $\varepsilon L = \Delta L$ → $\Delta U = \frac{1}{2} F \cdot \Delta L$

Strain energy density: $u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\varepsilon$

If elastic: $u = \frac{1}{2}\frac{\sigma^2}{E}$

Modulus of resilience: $u_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}$ largest strain energy density

Modulus of toughness: U_{max} before it fractures.

poisson's Ratio: $\varepsilon_{long} = \frac{\delta}{L}$ $\varepsilon_{lat} = \frac{\delta'}{r}$. $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

Shear modulus of elasticity / the modulus of rigidity

$\boxed{G}$ $\tau = G\gamma$, $G = \frac{E}{2(1+\nu)}$

Creep: 蠕变 完全崩溃前持续的形变

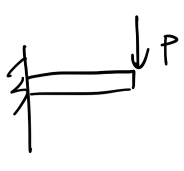
Fatigue: 疲劳.

Saint-Venant's principle: 足够远处的负载可被等效.

对截面变化的 Axially loaded member

$d\delta = \frac{P(x)\,dx}{A(x)\,E(x)} \iff \sigma = E(x)\varepsilon$ $\frac{P(x)}{A(x)} = E(x)\frac{d\delta}{dx}$

$\delta = \int_0^L \frac{P(x)\,dx}{A(x)\,E(x)}$ (P 拉伸为正)

principle of superposition  不是线性变化

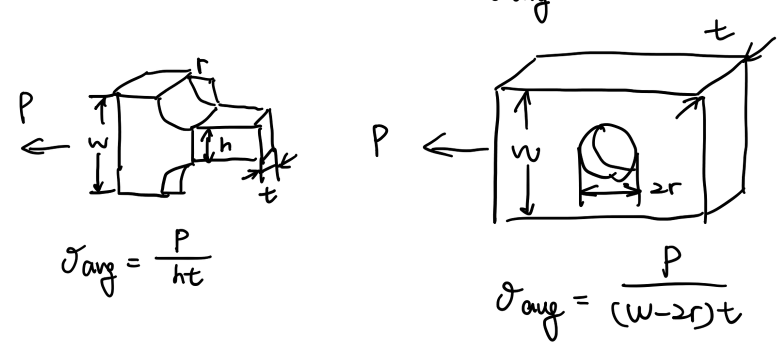
Statically indeterminate 静态不定

例子. $\delta_{A/B} = 0 \Rightarrow \frac{F_A L_A}{AE} = \frac{F_B L_B}{AE}$ $F_A + F_B = P \Rightarrow$ $F_A = P\frac{L_{CB}}{L}$ $F_B = P\frac{L_{AC}}{L}$

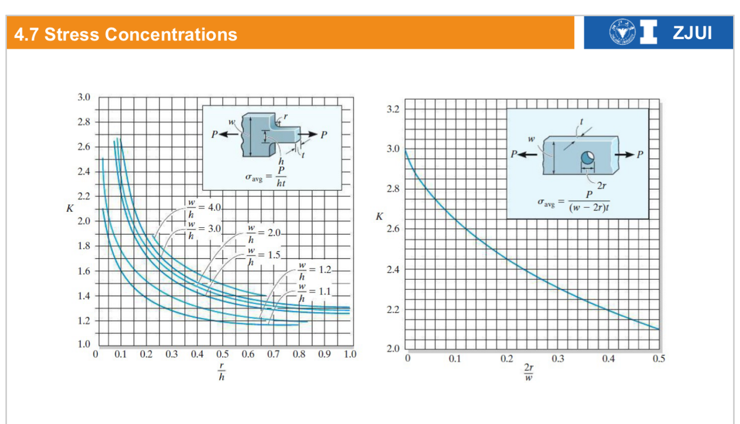
Thermal Stress: $\delta_T = \alpha \Delta T L$

Stress Concentration: P 必须作用在体积中心

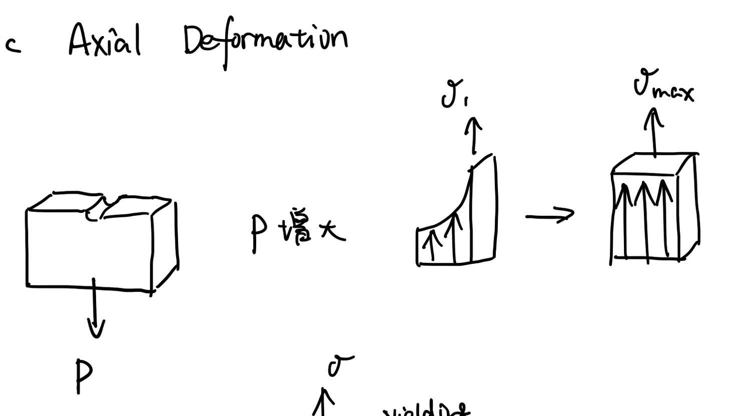
Stress-Concentration factor $K = \frac{\sigma_{max}}{\sigma_{avg}}$ → 不均匀分布的最大 strain



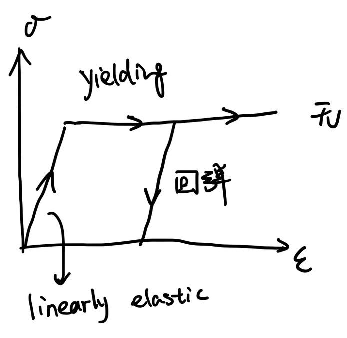
具体计算要查表.



Inelastic Axial Deformation

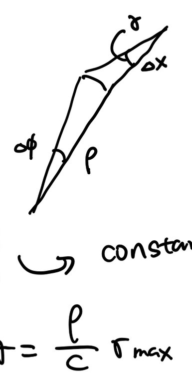


Residual Stress



Torsion

$\gamma = \frac{\pi}{2} - \theta'$



$BD = \rho\Delta\phi = \Delta x\gamma$

$\gamma = \rho\frac{\Delta\phi}{\Delta x} = \rho\frac{d\phi}{dx}$ ↳ constant

$\gamma_{max} = c\frac{d\phi}{dx}$, $\gamma = \frac{\rho}{c}\gamma_{max}$ $\tau = G\gamma \Rightarrow \tau = \frac{\rho}{c}\tau_{max}$

$T = \int_A \rho(\tau dA) = \int_A \rho \cdot \frac{\rho}{c}\tau_{max}\,dA = \frac{\tau_{max}}{c}\int_A \rho^2 dA$ → polar moment of inertia

力矩

$P = \frac{T\,d\theta}{dt} = T\omega = 2\pi f T$ (frequency) $T = \frac{P}{2\pi f}$ $\frac{J}{c} = \frac{T}{\tau_{max}} = \frac{T}{\tau_{allow}}$

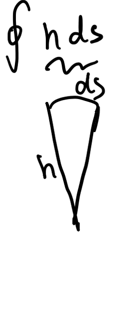
Angel of twist. $d\phi = \gamma\frac{dx}{\rho} = \frac{\rho\,\tau_{max}}{cG\rho}dx = \frac{T(x)\,dx}{J(x)\,G(x)}$, $\phi = \int_0^L \frac{T(x)\,dx}{J(x)\,G(x)} \iff \delta = \int \frac{P(x)\,dx}{A(x)\,E(x)}$ $\left(\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} = \frac{P}{AE}\right)$

If T, L, G are constant, then $\phi = \frac{TL}{JG}$ → $\phi = \sum\frac{TL}{JG}$

逆时针为正.

Shear flow. $q = \tau_{avg} t$ $dF_A = \tau_A(t_A dx)$ $dF_B = \tau_B(t_B dx)$ ⇒ $dF_A = dF_B$ $\tau_A t_A = \tau_B t_B$

$dT = h(dF) = h(\tau_{avg} t\,ds)$ ⇒ $T = \oint h\tau_{avg} t\,ds$ → $\tau_{avg} t \oint h\,ds = 2\tau_{avg} t \oint dA_m = 2\tau_{avg} t \cdot A_m$



横截面

Hence $\tau_{avg} = \frac{T}{2tA_m}$ $q = \frac{T}{2A_m}$

边界包络面积.

$\phi = \int\frac{T\,dx}{J(x)G(x)} = \frac{TL}{4A_m^2 G}\oint\frac{ds}{t}$ ⇒ $\frac{TL}{GJ} = \frac{TL}{4A_m^2 G}\oint\frac{ds}{t}$ $J = \frac{1}{4A_m^2}\oint\frac{ds}{t}$

Stress Concentration: $\tau_{max} = K\frac{Tc}{J}$