Homework 03 Part I: Formal methods

Problem 1 (selected from ACST, Exercise 2.3.1. Total: 30 point)

Given a Kripke structure *(W, I, J)*, The function *J* can be extended to work over arbitrary principal expressions via the following definition:

$$J(P \,\&\, Q) = J(P) \cup J(Q),$$
$$J(P \mid Q) = J(P) \circ J(Q).$$

Follow this definition to complete

a) (10 point) *part a.* of Exercise 2.3.1.
b) (10 point) *part d.* of Exercise 2.3.1.
c) (10 point) *part f.* of Exercise 2.3.1.

The description for Exercise 2.3.1 is given below for your reference.

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Exercise 2.3.1 *Recall the Kripke structure* $\langle W_0, I_0, J_0 \rangle$ *from Example 2.7, and further suppose that*

$$J_0(Ida) = \{(sw, sc), (sc, sw), (ns, sc), (ns, ns)\}.$$

Calculate the following relations:

a. $J_0(Hal \,\&\, Gil)$

b. $J_0(Gil \mid Hal)$

c. $J_0(Flo \,\&\, Ida)$

d. $J_0(Hal \mid Ida)$

e. $J_0(Ida \mid Hal)$

f. $J_0(Hal \,\&\, (Ida \mid Hal))$

g. $J_0(Hal \mid (Ida \,\&\, Hal))$

Problem 2 (selected from ACST, Exercise 2.3.3. Total: 40 point)

This question refers to the Kripke model given in ACST, Exercise 2.3.3.

a) (10 point) Give a graphical representation of the given Kripke model ***M***. That is:

   i. Draw the directed graph which represents the relation *J*.
   ii. Label each node with the corresponding name in *W*.
   iii. For each proposition *p* in the set *PropVar*, put its name at each node where *p* is true in the world represented by the node.

   Continue to complete the following parts of Exercise 2.3.3

b) (10 point) *part b* of Exercise 2.3.3.
c) (10 point) *part d* of Exercise 2.3.3.
d) (10 point) *part e* of Exercise 2.3.3.

The description for Exercise 2.3.3 is given below for your reference.

Exercise 2.3.3 *Let $\mathcal{M}$ be the Kripke structure $\langle W, I, J\rangle$, where W, I, and J are defined as follows:*

- $W = \{t, u, v, x, y, z\}$
- $I : \textbf{PropVar} \to 2^W$ *given by:*

$$I(p) = \{x, y, z\}$$
$$I(q) = \{x, y, t\}$$
$$I(r) = \{y, t, u, z\}$$

- $J : \textbf{PName} \to 2^{W \times W}$ *given by:*

$$J(A) = \{(w, w) \mid w \in W\} \cup \{(x, y), (x, z), (z, t), (y, v), (v, y), (v, x)\}$$
$$J(B) = \{(x, w) \mid w \in W\} \cup \{(y, t), (z, t), (t, v)\}.$$

Calculate each of the following sets.

a. $\mathcal{E}_{\mathcal{M}}[\![(p \supset q) \supset r]\!]$

b. $\mathcal{E}_{\mathcal{M}}[\![A \text{ says } (p \supset r)]\!]$

c. $\mathcal{E}_{\mathcal{M}}[\![A \text{ says } (B \text{ says } q)]\!]$

d. $\mathcal{E}_{\mathcal{M}}[\![B \text{ says } (B \text{ says } q)]\!]$

e. $\mathcal{E}_{\mathcal{M}}[\![A \text{ controls } (B \text{ says } q)]\!]$

f. $\mathcal{E}_{\mathcal{M}}[\![A \text{ controls } (B \text{ controls } q)]\!]$

Problem 3 (Selected from ACST, Section 3.2. Total: 50 point)

a. (15 point) *Give a formal proof* as required in Exercise 3.2.2.
   (10 point) *Draw a proof tree* with respect to the proof you gave.

b. (15 point) *Give a formal proof* as required in Exercise 3.2.4.
   (10 point) *Draw a proof tree* with respect to the proof you gave.