Unification of Clustering, Concept Formation, Categorization, and Analogy Making

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Abstract

The mechanisms listed in the title of this article are usually considered separately, examined by different lines of research in cognitive science. (Clustering is typically a topic in artificial intelligence.) The idea presented here is that abstract clustering leads to concept formation, which is the basis of categorization, which in turn is the basis of analogy making; and all these functions are supported by the same underlying mechanism, which is examined in some detail. It is suggested that the listed functions appeared among animals in that temporal order, for evolutionary reasons. The spontaneous nature of the occurrence of analogy making is also explained.

Keywords: analogies; analogy making; categorization; core; concepts; concept formation; clustering; object individuation.

Object Individuation as Simple Clustering

Suppose an observer is given the visual input in Figure 1(a):

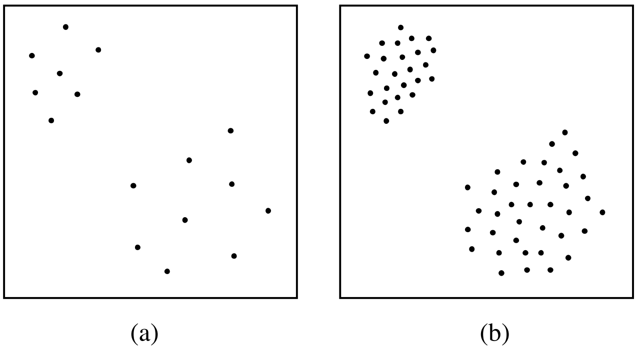

Figure 1: Clusters (or groups) of dots.

One quite likely answer to the question: "What does Figure 1(a) show?" is: "Two groups of dots." If the observer answers simply: "Some dots," thus showing no sign of perceiving two clusters, we can imagine the dots packed more densely, as in Figure 1(b), in which case the existence of two clusters becomes quite obvious. Bringing this idea to its logical limit, we can make the dots so dense that their in-between spaces are hard to discern. In that case the observer has no choice but see two contiguous dark regions. Thus, starting with dots distributed sparsely within two clusters, we ended up with two concrete *objects*.

The dots of Figure 1 are 2-dimensional. We can imagine adding dimensions to them in various ways. For example, the dots can be colored, thus resembling pixels of a realistic picture. Motion can be added, so dots that move together in one direction will be perceived as belonging to the same object, even if colored differently. Depth is a possibility as well, turning the clusters into solid regions in 3-dimensional space. Thus we obtain a multi-dimensional abstract space — impossible to depict on a printed page, yet no less real — in which multi-dimensional dots allow the perceiving agent to *individuate objects*.

Concept Formation as Abstract Clustering

We shall now abstract the previously-described multi-dimensional space of dots. Suppose the observer is an infant, between one and three years of age, observing various objects that are really fruits and vegetables; but the infant does not know the words "fruit" and "vegetable" yet, which are used only rarely in the infant's environment. Still, the infant is able to form the two concepts mentally, even without the help of linguistic labels. How is this act of concept formation made possible? Before seeing some well-known answers we can focus on only two of the many dimensions (or features) that objects such as vegetables and fruits have. Specifically, suppose we focus on "sweetness" and "hardness of skin", measuring them objectively in some way. (E.g., the former by the percent of sugars, and the latter by the force required to puncture the object.) If we make a 2d-plot putting "sweetness" on the x-axis and "hardness of skin" on the y-axis, and represent each kind of fruit or vegetable with a single dot, we obtain a diagram similar to the one in Figure 2.

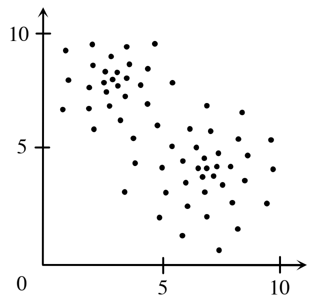

Figure 2: Vegetables (upper-left) and fruits (lower-right).

It should be noted that here each dot represents not one object (*this* apple), but an entire category of objects ("apples"); i.e., each dot stands for the *type,* not the token. Still, arranging dots of types according to the two given dimensions we get two clusters of dots: one corresponding to "vegetables" (due to their generally harder skin and fewer sugars) and one to "fruits" (due to their generally softer skin and more sugars). There will be some members that cannot be easily categorized (e.g., "tomato", "olive"), but on average there will be more members near a central region for "vegetables", and also near another central region for "fruits". This is a mere consequence of the fact that any natural property, such as sweetness and hardness of skin, has a normal (or normal-like) distribution.

Instead of saying that the observer perceived two categories in the previous example, we can say that the observer *formed two concepts*. Naturally, concepts are multi-dimensional: unlike the true dots of Figure 1, the "dots" in Figure 2 — properly called "exemplars" — have multiple dimensions (e.g., volume, weight, variety of colors, brightness of colors, quantity of pits, way of consumption, and a host of other ones). Another difference between the two figures is that in Figure 1 all dots are simultaneously present, whereas in Figure 2 each exemplar is perceived at some time, and a long time might pass until the perception of another exemplar. At no time are all the exemplars in Figure 2 simultaneously present and available for re-examination. Therefore, concept formation is *incremental,* happening in a one-exemplar-at-a-time fashion.

The previous example can be generalized in every case of concept formation, even when concepts are abstract and lack linguistic labels. For instance, meeting various people who live in a country we might form "facial types" in our minds. Such concepts often have no words associated with them, but when we see a new face we know it belongs to "that" type (category) of faces. The formed concepts can also belong to a different modality, such as audition: we can form categories of music, such as classic music, jazz, rock, country, etc.; and even sub-categories, such as baroque, symphonic, opera, etc., all sub-categories within "classic music". A more abstract example is categories of characters of people: we may form the concepts "arrogant", "modest", "gullible", "rational", "irritable", "insensitive", and so on, with the dimensions and exemplars in this space being of entirely abstract nature.

The Generalized Context Model for Categorization

How do people decide to which category (or concept — the two words will be used interchangeably in what follows) an exemplar belongs? Psychologists have modeled the process of categorization by observing the behavior of subjects under controlled laboratory conditions, and one of the most successful models has been the Generalized Context Model (GCM) (Nosofsky, 1984; Nosofsky, 1986), an elaboration of the earlier Context Model (Medin & Schaffer, 1978). The GCM defines first the distance d_{ij} between exemplars x_i, x_j:

$$d_{ij} = \sqrt[r]{\sum_{k=1}^{n} w_k \left| x_{ik} - x_{jk} \right|^r}$$

Figure 3: Formula for distance d between exemplars.

In the above formula, (x_{i1}, ..., x_{in}) are the n coordinates of exemplar x_i; similarly for exemplar x_j. The w_k's are weights for each coordinate with the requirement that their sum be 1. Each w_k models *priming* along dimension k.

The formula in Figure 3 defines a weighted Minkowskian metric. When $r = 2$ we have a weighted Euclidean distance, which is often the metric of choice in experimental settings. With $r = 1$ we have what is known as "Manhattan distance", used often in computer science.

Given the distance d between two exemplars x_i x_j, the GCM computes the similarity s between them:

$$s_{ij} = e^{-(cd_{ij})^q}$$

Figure 4: Formula for similarity s between exemplars.

Thus, the greater the distance d, the smaller the similarity s. Higher values for parameter c correspond to perceiving more categories. Finally, with $q = 1$ we have a simulated-annealing-like decay function, whereas with $q = 2$ we get a Gaussian-like decay function.

The GCM includes a third formula giving the probability $P(J \mid i)$ that a given new exemplar x_i is categorized in an already formed category J:

$$P(J \mid i) = \sum_{j \in J} s_{ij} \Bigg/ \left(\sum_{K} \sum_{k \in K} s_{ik} \right)$$

Figure 5: Formula for probability $P(J \mid i)$ of categorization.

Thus, the numerator sums the similarity of exemplar x_i to each exemplar in the tested category J, and the denominator sums the similarity of exemplar x_i to every known exemplar.

The GCM formula given in Figure 5 is the basic one. Over the years, several parameters have been added to it, accounting for various effects in experimental results (Ashby & Maddox, 1993; Nosofsky & Johansen, 2000). Such additions, however, are justified only if the values of the parameters are determined before the experimental session, and remain unchanged in all future experiments. If parameter values are tweaked to match experimental results with a 20/20 hindsight, then the GCM does not represent an *invariant* among models of categorization.

The formula in Figure 5 requires that all exemplars seen so far be stored in memory so that they are compared against the new exemplar x_i. (Hence, the GCM is called a lossless, exemplar-based model.) However, for purposes of computing efficiency, the following modification can be made: as long as exemplars in category J are few and do not form a statistically significant sample, all of them are stored individually, together with an estimated mean value μ and standard deviation σ; but when the sample acquires a statistically significant size, then the probability $P(J|i)$ is not found by Formula 5 anymore, but by applying standard statistical methods that compute the probability for a datum to belong to a population of estimated μ and σ. This yields a *hybrid* of exemplar- and prototype-based (Rosch, 1973; Rosch, 1975; Rosch & Mervis, 1975) models of categorization.

Also noteworthy is that the GCM — being a model of categorization, and not of concept formation — does not determine how categories are formed in the first place; it rather assumes that some categories exist, and answers how to assign new members to those categories. To solve the problem of how to form different categories one could postulate that when the maximum P over all known categories is below a fixed threshold then the new exemplar creates a new category. Better yet, one may employ ideas from any of a large number of methods of clustering (e.g.: Jain, Murty, & Flynn, 1999), or from other, cognitively compatible procedures (Papari & Petkov, 2005; this author, to appear). The existence of a wealth of algorithms & formulas indicates that concept formation can be simulated by computers and thus is not an exclusively human ability.

Analogy Making as Complex Categorization

The term "analogy making" typically evokes puzzles of the form "A : B :: C : D" in the layperson's mind. An example could be: "a sock is to foot as a ____ is to hand", with the obvious answer "glove" filling in the blanks. But this is only a special form of the general concept of analogy making. To witness a truly spectacular example, which will be used as a litmus test in this article, we shall present one that was experienced by Douglas Hofstadter (1995a). Here it is, as recounted by him:

"My daughter Monica, then a bit over a year old, was sitting on our playroom floor, pushing the on–off button of a Dustbuster (a hand-held battery-operated vacuum cleaner), which she loved to do because of the buzzing noise it made. At one point, she noticed a differently-shaped button on a different part of the Dustbuster, so of course she tried pushing that one. Nothing happened. She tried several times, and then gave up. The reason it did nothing was that this was the release button for the lid that holds the trashbag inside the machine, and pushing it does nothing. You have to slide it, and even then, all that happens is that the lid flips open. That was way beyond her, but the feeling of disappointment was not.

"When I saw Monica trying that second button and getting nowhere, I went over and showed her what it did. All of a sudden, completely out of the blue, there flashed to my mind an experience from my own childhood. As a child, I always loved mathematics. Something that excited me no end was the operation of exponentiation. I made table after table of squares, cubes, and higher powers of many integers, and I compared the powers and studied their patterns and so on. I was just enchanted by them. One day, when I was about eight, I happened to see one of my father's physics papers lying around on a table in our house, and I looked at the equations. Of course, they were way beyond my grasp, but I did notice a salient feature of the notation: the ubiquitous use of subscripts. Of course I knew that superscripts represented the beautiful, endlessly deep operation of exponentiation, so I jumped to the conclusion that subscripts, looking so much like superscripts, must likewise represent some kind of marvelously deep mathematical concept, so I asked my father. To my surprise, he said that subscripts were simply used to distinguish one variable from another, and that no arithmetical calculation whatsoever was symbolized by putting a subscript of '3', say, on the letter 'x'. Thus were dashed my childish hopes of finding some new mathematical treasure."

And Hofstadter continues:

"This was the memory that flashed into my mind when little Monica failed to make a new noise by pushing the second button on the Dustbuster. Monica was me, I was my Dad, the first button was superscripts, the second button was subscripts, the buzzing noise was the thrill of exponentiation, the lack of noise was the meaninglessness of subscripts… When you hear about it, it makes perfect sense — the two events map onto each other very elegantly — fathers, children, disappointment, and all. But how was it that this retrieval occurred? How did the eight-year-old boy store the original memory? How did the adult fish it out, some forty years later, triggered by the event involving his baby daughter?"

Those are indeed deep questions that remain unanswered in cognitive science. In what follows, an answer will be given to the first and last question.

Before proceeding we should note that the idea of analogy making is often perceived differently among researchers in that domain. Often, two structures that we already know are analogous are given (usually to software that simulates the researcher's theory), and the "puzzle" is to "discover" how the two structures map onto each other. The words "puzzle" and "discover" are put into quotes here because once we know that an analogy exists between the two structures to find the mapping between them is algorithmically no more impressive than computing the prime factors of an integer.[1]

In a slightly less obviously boring case of "discovery", one structure is given, but the mapping and the analogous structure must be found among a possibly large number of other structures stored in memory. What makes this problem cognitively unrealistic is that analogies such as the aforementioned Monica-Dustbuster-Hofstadter-Subscripts analogy (henceforth: the MDHS) do not occur at the press of a button. Nobody asked Hofstadter: "Please sir, do come up with an analogy *now*!" — which is analogous to what goes on in several lines of research in our times.[2] [3]

The hard problem is to explain the *spontaneity* by which analogies such as the MDHS occur. The answer can be understood by realizing that analogy making, especially in its most astonishing and puzzling manifestations, is a case of complex categorization. It involves a process that we did not describe yet, which we call "core extraction".

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[1] Surely there are complex algorithms to find the prime factors of integers, but the problem is *not cognitively interesting*.

[2] No explicit references will be given here out of tactfulness. For an early — now defunct — system that worked just as described, see "Bacon" in the AI literature.

[3] In argumentation, phrases such as: "But what you're saying is like…" indicate a consciously-driven and highly creative search for analogy, but not among fixed and stored structures in memory.

Core Extraction

As with concept formation, the process of core extraction has "humble roots" that reach all the way to visual input, but can also be abstract and apply in analogy making, including its most amazing cases such as the MDHS. We start by examining the simplest form of core extraction, which occurs in input of visual form. Consider Figure 6.

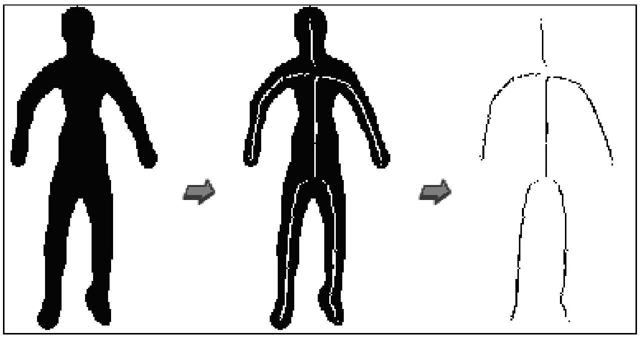

Figure 6: Core extraction in visual input.

Suppose the original visual input (either to a program or to the human eye) is the shadowed outline of a human, as shown in Figure 6 on the left. In the middle, we see that some of the pixels near the "center" of the human figurine are missing, and the same pixels have been singled out and shown on the right. Those extracted pixels are not arbitrary but the result of an algorithmic process, called "thinning" in visual input processing in computer science. It works by successively eliminating pixels at the border of the figure until only the pixels that have a maximal distance from the original border remain. The survivors are called the "median pixels" in the relevant literature, but here we adopt the term "core pixels", for reasons of consistency with our terms.

One might wonder if a "stick figure" like the one made of core pixels is natural or even possible in human cognition. Evidence that it is at least possible is that children draw humans and animals initially as stick figures, which they do spontaneously, without any prior training. Children seem to *abstract* the input, ridding it — quite justifiably for their purposes — of what they consider as "extraneous" or "useless" details. The real-world input is actually much richer than what is shown in Figure 6 on the left. Thus, abstraction through extraction of core pixels does not require the higher faculties of an adult intelligence, but is possible at a very early stage in cognitive development.

Why is the process of core extraction of pixels important? Because it allows the viewer to match, for example, the figurine in Figure 6 with similar ones and determine that they are all "the same" figurine. This is depicted in Figure 7, where the figurine on the right has rather low pixel-to-pixel relation with the figurine on the left, if superimposed on it. However, the "black pixels" are not important — and even a child seems to know that. (After all, the non-core pixels could be colored differently, have random holes among them, and small irregularities at their borders.) What is important is the *structure* of the figurine, and the structure is represented by the core pixels, if they are parsed as lines that intersect and meet each other. Parsed as lines, the core pixels of the two figurines form two *analogous structures* with homologous parts: two "arms", two "legs", a "torso", and a "head". Thus an analogy can be perceived, and the two inputs can be seen as similar.

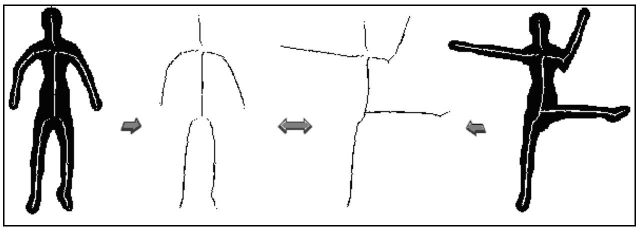

Figure 7: Visual analogy-making with core extraction.

Can the above-described process of core extraction by successive elimination of peripheral pixels apply similarly to something as abstract as the MDHS analogy? It surely can, by a process of successive attrition of detail, i.e., loss of most specific information first, less specific information later, and so on, until only the most abstract one remains, which is the "core" of the situation. For example, in the Monica–Dustbuster situation, the color of the Dustbuster is of very little relevance, and will be among the first pieces of information that will be ignored[4]; ditto for the loudness of its sound; the other toys that were spread on the floor; the weather conditions on that day; and so on. Likewise, in the Hofstadter–Subscripts situation, too specific information includes things like the specific style of hand-writing, annotations that Hofstadter's father might have added on some papers, desktop objects that were around, and so on. After ignoring all irrelevant details and abstracting the remaining information, one can come up with the "core" that is common in both situations, shown in Figure 8.

There is a father–child relation, a toy with a single feature with which the child has had fun playing, a second similar feature of the toy suddenly discovered by the child, an expectation by the child that this second feature might be as enjoyable as the first, and a disappointment after the child is informed by the father that the second feature does nothing very interesting.

Figure 8: Core of the MDHS analogy.

The core in Figure 8 might appear as if it resulted from deliberate pondering after carefully examining the two situations. But no claim is made here that there is anything like a conscious effort of core extraction by the perceiver. The process must be as subconscious and automatic as the one described earlier in the visual domain, evinced by children drawing stick figures. Besides, the core extraction process is ubiquitous, as the following examples suggest.

[4] Here, "ignored" does not mean "forgotten from memory"; it means "ignored for the purposes of reaching core information".

For instance, the reader most probably does not remember the exact words used in the last paragraph of the previous page, nor that there were exactly four sentences in it. Most probably, however (and hopefully), the reader remembers that it was about the subconscious manner in which the core is reached. The *gist* of a story is its core.

Or, consider a musical piece. Most of us lack the feeling that we can "play back" in our minds entire symphonic pieces, remembering even minor instruments that participate in the orchestra. But we can always play the melody of the piece on the piano with a single finger (or recognize the melody if someone else does so). All that the core of a melody retains is the right pitch of the notes, their duration, and their order within the temporal sequence.

Spontaneity of Analogy Making

Equipped with the core-extraction process, it is not hard to outline the way in which the MDHS analogy occurred — an outline that should apply in all spontaneous analogies:

- First, some input is perceived. "Input" can be anything: from a concrete object to an entire event or situation. There is no case of analogy yet, nor is there a need to make one, either at once or in the foreseeable future.
- The cognitive agent stores in memory not just literally the raw input, but successive abstractions of its parts. Its most abstract and generalized parts comprise its core. Just as any other piece of information, the core has some *features,* i.e., dimensions. It is represented here by a dot, as in the diagrams shown in the first part of this article.
- An indefinite period of time might pass (four decades in the MDHS case). During this time the stored memory may fade out, losing the connections to its most specific information or distorting its contents, yet retaining best its most abstract elements, near the core.
- At some unexpected future time new input is perceived. The cognitive agent does what was just described above: makes successive layers of abstraction, which include the core. This core, consisting of features (dimensions) is another "dot" that belongs to a conceptual space (the square in Fig. 9). As such, it has the fate of every "dot": it is *categorized* in the conceptual space. Categorization models, such as the GCM, describe how this is done.
- Then and only then an analogy *might* be spotted: when the core is positioned in the conceptual space, it might find itself "near" (in the sense of a psychological metric as in Figures 3–4) the core of an old structure. The new core *activates* the old one and an analogy is spotted.

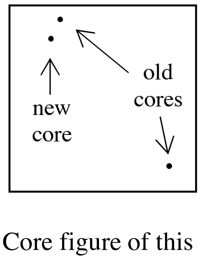

Figure 9: Core figure of this article.

How can the new core "activate" the old one, which is required at the last step for a spontaneous analogy to occur? Simply, categorization implies that when a new "dot" (core) is placed in a conceptual space it is not just added there as an inert object, but is *categorized,* i.e., "informs" a category that it belongs to it, and the category activates its members — the nearer, the higher. All this is part of categorization, as described in section "Concept Formation".

Figure 9 summarizes the given steps in analogy making. It explains why analogy making is spontaneous and appears so effortless: because it is no more than an act of categorization — a categorization of cores of concepts in a conceptual space. Because it involves the process of core extraction, it is also called "complex categorization" here.

Analogy-making does not end with spotting the similarity of cores. It often continues with a conscious effort to find more analogous parts than those suggested originally by the cores, and if found, the importance of the analogy is raised; or it might be marked as a false analogy after all. However, such analysis is beyond the scope of the present article.

Another important point is that the previous discussion might give the impression that there is always a unique core to every situation. This is far from true. Not only different people, but even the same person can perceive different cores at two different times, or in two different contexts. This can be modeled — at least up to some extent — by the weights w_k of the formula in Figure 3. By modifying the w_k's according to context even the same person can perceive another core, suppressing some dimensions and magnifying others, akin to "moving the dot" in the conceptual space.

Underpinnings in Prior Research and Thought

The idea that concepts and situations form cores when processed and stored in memory is not new. Hofstadter has described it in technical reports and books since the 1980's (the "core–halo" structure; best in 1995a). Rosch's even earlier prototype theory of concepts is a step in the same direction, as is later research in psychology regarding the "typicality" of concepts (e.g., Barsalou, 1987). This idea is exemplified in Figure 2: some fruits are more typical than others, forming the swarm of dots around the center, i.e., the core area of the concept "fruit"; ditto for any other concept.

If we want to be fair, however, we should note that this idea has roots that reach all the way back to antiquity. Let us ponder for a moment what the essence of Plato's "Theory of Essences" (or "Forms") really is. According to Plato (e.g., 1992), a geometrical triangle is not the object drawn with a stick on dirt — which contains "impurities", such as curved lines, thick lines, uneven surface, etc. — but an *abstraction* in one's mind. How do we arrive at that abstraction? By looking at the real, "imperfect" triangles drawn on dirt and removing the impurities: we imagine lines to be perfectly straight, of zero width, and so on. In other words, our minds arrive at the *core* of the concept "triangle". Thus, the first known theory of conceptual cores was in reality proposed by Plato in the 4th C. BC (although he attributed it to his teacher Socrates).

The idea that analogy making is one of the fundamental mechanisms of thought was proposed and explored also by Hofstadter (1995b; 2001), at a time when, to many cognitive scientists, "analogy" meant simply "A : B :: C : D". In the 1995b publication, in particular, the connection between categorization and analogy making is made clear: when we see a letter "A" we make an analogy between it and our concept of "letter A" in long-term memory (LTM), which is nothing but an act of categorization. A publication in the same spirit is that of Sander (2000). The idea that even the simpler act of *object individuation* (the understanding that "there is an object here", preceding object recognition) is the evolutionary origin of the same mechanism has not appeared elsewhere, to the best of our knowledge.

There is at least one other publication that focuses on the unification of categorization and analogy making (Dietrich, 2010). Its scope, however, is narrower, as Dietrich states: "we have the unification of a certain class of analogies and a certain class of categorization." (p. 342). More important, seeking an analogy between a source s in working memory and a target t_i in LTM (where i ranges among all possible targets in LTM), Dietrich proposes a "rapid abstraction" process on t_i (reminiscent of our "core extraction"), but which acts *after* s and t_i become candidates for analogy making. This begs the question of what it is that selects t_i as a target of analogy in the first place.

Evolutionary Aspects

The fact that children draw stick figures of objects, which hints at a core extraction process, together with the well-established observation that development "recapitulates" (in an abstract way) evolution, suggest that core extraction (a vital step in analogy making) is probably not an exclusively human ability. Chimpanzees, for example, are known to "fish" termites using sticks; to succeed in doing so they cannot be seeing the sticks as what they really are but must be abstracting them to their core nature: long, thin, sturdy objects. All great apes, as well as other mammals and birds, are known to play with *toys*. A toy is really an abstraction, standing for something other than what it actually is.

Other cognitively advanced animals seem to have a basic categorization ability, which is most probably restricted to the visual domain of "here and now", not reaching abstract concept formation. As for object individuation — the "low tech" end of the unified mechanism — it is probably present in the simplest of "cognitively enabled" creatures, such as frogs and fish. These observations suggest that the unified mechanism described here did not appear suddenly in our species, nor is it our exclusive province, but emerged tens of millions of years ago. It passed through successive stages of sophistication before it reached the human-only stage of analogy making, and thus evolved into arguably the most fundamental "core" of cognition.

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