with no hair, bad eyes, false teeth, and flat feet he knew he would hardly be of use as a soldier. But having witnessed at first hand Hitler's coming to power in Germany, he was entirely serious about doing what he best could to contribute to the defeat of Hitler's Germany. He began spending his summers and half of most academic-year weeks at the Army's Aberdeen Proving Ground, doing immediately useful research in exterior ballistics.

To complete that part of the story: McShane took a leave of absence from the University of Virginia for the period 1943–45 to continue full-time his work at Aberdeen Proving Ground in a section headed at that time by astronomer Edwin Hubble, now memorialized by the Hubble Space Telescope. By 1944 he had succeeded in bringing to join him in that work J. L. Kelley, a brilliant 1940 University of Virginia Ph.D. in topology. In 1953, well after World War II was over, there appeared *Exterior Ballistics*, McShane's joint book with Kelley and Aberdeen's Frank Reno.

—Truman Botts
Arlington, VA

(Received February 28, 1998)

Teach Calculus with Big *O*

I am pleased to see so much serious attention being given to improvements in the way calculus has traditionally been taught, but I'm surprised that nobody has been discussing the kinds of changes that I personally believe would be most valuable. If I were responsible for teaching calculus to college undergraduates and advanced high school students today and if I had the opportunity to deviate from the existing textbooks, I would certainly make major changes by emphasizing several notational improvements that advanced mathematicians have been using for more than a hundred years.

The most important of these changes would be to introduce the O notation and related ideas at an early stage. This notation, first used by Bachmann in 1894 and later popularized by Landau, has the great virtue that it makes calculations simpler, so it simplifies many parts of the subject, yet it is highly intuitive and easily learned. The key idea is to be able to deal with quantities that are only partly specified and to use them in the midst of formulas.

I would begin my ideal calculus course by introducing a simpler "A notation", which means "absolutely at most". For example, $A(2)$ stands for a quantity whose absolute value is less than or equal to 2. This notation has a natural connection with decimal numbers: Saying that π is approximately 3.14 is equivalent to saying that $\pi = 3.14 + A(.005)$. Students will easily discover how to calculate with A:

$$\begin{aligned} &10^{A(2)} = A(100);\\ &(3.14 + A(.005))\,(1 + A(0.01))\\ &\quad = 3.14 + A(.005) + A(0.0314)\\ &\qquad + A(.00005)\\ &\quad = 3.14 + A(0.03645)\\ &\quad = 3.14 + A(.04)\,. \end{aligned}$$

I would of course explain that the equality sign is not symmetric with respect to such notations; we have $3 = A(5)$ and $4 = A(5)$ but not $3 = 4$, nor can we say that $A(5) = 4$. We can, however, say that $A(0) = 0$.

The A notation applies to variable quantities as well as to constant ones. For example,

$$\begin{aligned} \sin x &= A(1);\\ x &= A(x);\\ A(x) &= xA(1);\\ A(x) \pm A(y) &= A(x + y)\\ &\qquad \text{if } x \ge 0 \text{ and } y \ge 0;\\ (1 + A(t))^2 &= 1 + 3A(t)\\ &\qquad \text{if } t = A(1). \end{aligned}$$

Once students have caught on to the idea of A notation, they are ready for O notation, which is even less specific. In its simplest form, $O(x)$ stands for something that is $CA(x)$ for some constant C, but we don't say what C is. We also define side conditions on the variables that appear in the formulas. For example, if n is a positive integer, we can say that any cubic polynomial in n is $O(n^3)$.

I would define the derivative by first defining what might be called a "strong derivative": The function f has a strong derivative $f'(x)$ at point x if

$$f(x + \epsilon) = f(x) + f'(x)\epsilon + O(\epsilon^2)$$

whenever ϵ is sufficiently small. The vast majority of all functions that arise in practical work have strong derivatives, so I believe this definition best captures the intuition I want students to have about derivatives.

I'm sure it would be a pleasure for both students and teacher if calculus were taught in this way. The extra time needed to introduce O notation is amply repaid by the simplifications that occur later. In fact, there probably will be time to introduce the "o notation", which is equivalent to the taking of limits, and to give the general definition of a not necessarily strong derivative:

$$f(x + \epsilon) = f(x) + f'(x)\epsilon + o(\epsilon)\,.$$

But I would not mind leaving a full exploration of such things to a more advanced course, when it will easily be picked up by anyone who has learned the basics with O alone.

Students will be motivated to use O notation for two important reasons. First, it significantly simplifies calculations because it allows us to be sloppy—but in a satisfactorily controlled way. Second, it appears in the power series calculations of symbolic algebra systems like Maple and Mathematica, which today's students will surely be using.

For more than twenty years I have dreamed of writing a calculus text entitled *O Calculus*, in which the subject would be taught along the lines sketched above. Perhaps my ideas are preposterous, but I'm hoping that this letter will catch the attention of people who are much more capable than I of writing calculus texts for the new millenium. And I hope that some of these now-classical ideas will prove to be at least half as fruitful for students of the next generation as they have been for me.

Further details appear at http://www-cs-faculty.stanford.edu/~knuth/calc/.

—Donald E. Knuth
Stanford University

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Conference to Honor Memory of Pontryagin

Readers may have noticed that a conference is scheduled in Moscow August 31–September 5, 1998, to honor the memory of the late Russian mathematician L. S. Pontryagin. What may be less well known are the reasons why this conference has been the subject of considerable controversy.

In late January I was invited to speak at the conference. Soon after, I noted the name of I. R. Shafarevich as a member of the Organizing Committee. I found that troubling, because he is the well-known author of the extreme right-wing and anti-Semitic polemic "Russophobia", a former editor of the anti-Semitic daily *Deyn*, and at the present time one of the most outspoken political activists in the extreme nationalistic and anti-Semitic Pamyat Party. On making inquiries, I learned that S. Novikov had resigned from the Organizing and Program Committees in November. In a letter to the organizers of the conference he explained his reasons. Professor Novikov has given me permission to quote from his letter:

> I respect Pontryagin and was ready to give his memory proper respect; however I already warned you that this occasion may be used for making an antisemitic shabash. I certainly refuse to participate jointly with Shafarevich, who has nothing in common with Pontryagin scientifically.

Upon further inquiry I received a letter from a member of the Program Committee in my area, who wrote that "good mathematicians who agreed to come take only about 1/3 of places, and many weak persons with suspicious past and strange science have applied."

I am writing to alert other would-be participants to the actual state of affairs. The decision as to whether to boycott this meeting or to participate in it is not a simple matter. One does not wish to lend one's support, even indirectly, to abhorrent ideas, but on the other hand one does not wish to turn one's back on those among our colleagues who are trying hard to keep the focus of this meeting on mathematics. After some consideration I have decided not to participate. I urge those who do participate to make every effort to disassociate themselves publicly from political events having nothing to do with the mathematics of Pontryagin. Those who decide not to participate because of the prominence of Shafarevich in the Organizing Committee are urged to send a letter to the organizers giving their reasons.

—Joan S. Birman
Barnard College and
Columbia University

(Received March 23, 1998)

Remarks about Birman's Letter

Professor J. Birman, in her "Letter to the Editor" concerning the Pontryagin memorial conference, gives the following quotation from my private letter: "…good mathematicians who agreed to come take only about 1/3 of places, and many weak persons with suspicious past and strange science have applied."

I find it necessary to make two contextual remarks, which I consider as extremely important for adequate reading of this sentence.

1. In this part of the letter I analyzed only the situation with applications to the topology section of the conference; in the other three sections it was much better. Indeed, a preceding part of the letter was as follows: "I am afraid that if nothing changes then our section can be the weakest among all four sections….Because good…etc."

2. This letter was written in February, about two months before the deadline for applications. Since then something has changed: many good topologists and geometers have agreed to come; thus my fears that many places would remain for weak mathematicians (who, as usual, also have applied) were not realized.

Independently of these remarks, I use this opportunity to confirm my deep respect for Professor J. Birman and also my very negative estimation of the national theories of Professor I. R. Shafarevich.

—V. A. Vassiliev
Independent Moscow University and
Steklov Mathematical Institute

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