Glueballs, gluon condensate, and pure glue QCD below T_c

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A quasiparticle description of pure glue QCD thermodynamics at $T \leq T_c$ is proposed and compared to recent lattice data. Given that a gas of glueballs with constant mass cannot quantitatively reproduce the early stages of the deconfinement phase transition, the problem is to identify a relevant mechanism leading to the observed sudden increase of the pressure, trace anomaly, *etc.* It is shown that the strong decrease of the gluon condensate near T_c combined with the increasing thermal width of the lightest glueballs might be the trigger of the phase transition.

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I. INTRODUCTION

The phenomenology related to the deconfinement phase transition from hadronic matter to quark-gluon plasma (QGP) at high enough temperatures or densities is a very active field of research since the beginnings of QCD, with pioneering works such as Refs. [1, 2]. On the experimental side, the QCD matter is currently studied in heavy-ion collisions at RHIC and SPS, and results from the LHC can also be hoped in the future [3]. The results obtained at RHIC so far suggest that a new phase has indeed been observed, that behaves like an almost perfect fluid rather than a weakly interacting gas, as expected from phenomenological approaches [4]. On a theoretical side, much work has been devoted to the understanding of the QGP, within several frameworks: Perturbative methods, quasiparticle models, AdS/CFT duality, lattice QCD, ... [5, 6].

Apart from its intrinsic interest, the knowledge of the QCD equation of state (EoS) has eventually applications in astrophysics, to decide for example whether the core of some neutron stars can consist in a QGP or not, or in cosmology in order to understand how and when the Universe hadronized. In principle, the most powerful nonperturbative technique for such a study is lattice QCD. The EoS of pure glue SU(3) QCD has been computed on the lattice more than a decade ago [7], while more detailed data have recently been obtained in Ref. [8], with gauge groups ranging from SU(3) to SU(8). Many other calculations of the QCD EoS have also been performed including quarks flavors, at zero chemical potential or not [6].

Among the various existing phenomenological approaches, quasiparticle models rely on the assumption that the QGP can be seen as a gas of deconfined quarks and gluons. Since pioneering works [9], they have been shown to be able to reproduce the various EoS computed in lattice QCD at $T \geq T_c$. It is thus tempting to apply such models to describe the QCD EoS below T_c. In that range however, the situation is less clear. Being in the confined phase, the QCD matter should rather be modeled by a hadron gas. Let us focus on the pure glue case, which already captures the essential physical features of the problem, without involving extra technical difficulties due to the presence of quarks. Then, the hadronic matter should intuitively be thought as an ideal glueball gas whose thermodynamical features are dominated by the lowest-lying glueballs: Bose-Einstein statistics tells us indeed that the contribution of a glueball of mass m_g and spin J_g is roughly proportional to $(2J_g + 1)\mathrm{e}^{-m_g/T}$. However, by naively putting in such a model the glueball masses obtained in lattice QCD [10], one fails to reproduce the strong increase of the thermodynamical variables near T_c, as already pointed out in Refs. [11, 12]. One way of getting a model in agreement with the lattice data is to assume a high-lying glueball of Hagedorn-type [11]. The exponential growth of the number of states with mass m_g then compensates the statistical suppression and brings a large enough contribution near T_c. But one has to accept that glueballs are excitations of a closed string, which is not so obvious, partly in view of the many models reproducing the lattice spectrum at zero temperature by assuming totally different frameworks [13].

In this work, a new way of understanding the early stages of the pure glue phase transition ($T \lesssim T_c$) is proposed. A Hagedorn glueball spectrum is not assumed. Therefore, only the lightest glueballs, that all models find to be the scalar and tensor ones, are assumed to bring a significant contribution to the EoS. As it will be shown, two ingredients

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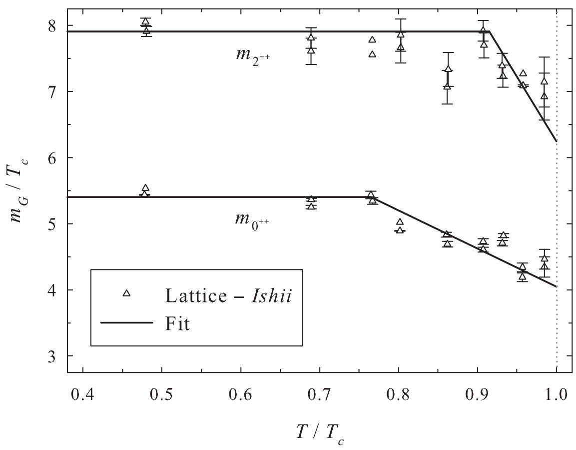

FIG. 1: Scalar and tensor glueball masses computed through a pole-mass fit in Ref. [15] (symbols), compared to a fit of the form (2), with (in units of T_c) $T_{0++} = 0.765$, $m^0_{0++} = 5.404$, $b_{0++} = 21.191$, $T_{2++} = 0.915$, $m^0_{2++} = 7.904$, and $b_{2++} = 72.040$. A standard value $T_c = 272$ MeV is used.

are needed: The nontrivial contribution of the gluon condensate to the trace anomaly and a significant reduction of the glueball masses near T_c. Since that point is the most counterintuitive one at first sight, let us analyze it first.

II. GLUEBALL MASS REDUCTION

The behavior of glueballs at finite temperature has been first investigated on the lattice in Refs. [14, 15]. The basic conclusion of those works is twofold, following the procedure used to analyze the results. On one hand, if the temporal glueball correlator is fitted assuming glueball states with zero width, then the 0^{++} and 2^{++} pole masses are found to significantly decrease for $T \lesssim T_c$. On the other hand, a Breit-Wigner fit can be applied to that correlator; then the glueball masses are found to be roughly constant from $T = 0$ to T_c, with a thermal width which is nearly zero at low temperature but then increases rather linearly near T_c. That conclusion is quite intuitive and might be a general feature of glueballs near T_c: The glueballs should progressively be "dissolved" in the medium when approaching the deconfinement temperature, and their width should increase accordingly. Notice that such a constant mass is expected from calculations relying on effective low-energy lagrangians [16]. It is moreover shown in Ref. [15] that the pole mass, $m_g(T)$, and the Breit-Wigner mass, $\bar{m}_g(T)$, and thermal width, $\Gamma_g(T)$, are linked as follows:

$$m_g(T) \approx \bar{m}_g(T) - 2T + \sqrt{4T^2 - \Gamma_g(T)^2}. \tag{1}$$

Only the glueball mass appears in the computation of the EoS [see for example Eq. (3) below]. Consequently, the pole mass $m_g(T)$ is the most relevant one to use in a quasiparticle framework because it includes the thermal width effects in an effective way.

As shown in Fig. 1, the pole masses computed in Ref. [15] are well described below T_c by the form

$$\begin{aligned} m_g(T) &= m^0_g & T &\le T_g \\ &= m^0_g - b_g\,(T - T_g) & T_g &< T < T_c, \end{aligned} \tag{2}$$

which, compared to Eq. (1), qualitatively encodes the results of the Breit-Wigner fit performed in Ref. [15]: a constant mass $\bar{m}_g(T) = m^0_g$ and a thermal width growing as $\Gamma_g(T) \propto (T - T_g)$ at T larger than some T_g. Remark that the thermal broadening of the glueballs generates a pole-mass reduction; interestingly such a glueball mass reduction near the phase transition has also been predicted by computations within the dual Ginzburg-Landau theory [17]. We point out that the value $T_c = 272$ MeV will be assumed in the rest of this paper. Such a value is located in the typical range $260 - 280$ MeV for the critical temperature in pure glue QCD [7].

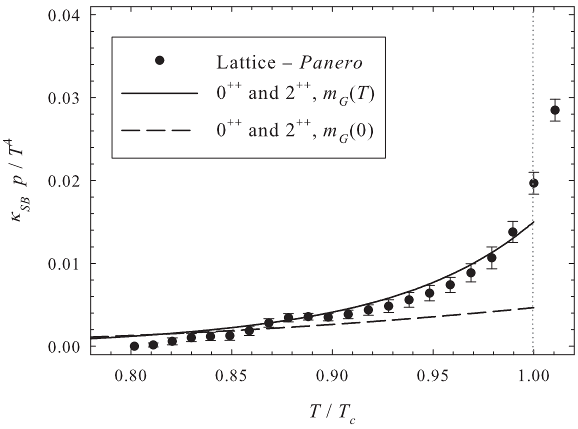

FIG. 2: Pressure computed in pure glue SU(3) lattice QCD and normalized to the Stefan-Boltzmann factor $\kappa_{SB} = 45/8\pi^2$, taken from Ref. [8] (full circles). The lattice data are compared to Eq. (4) using either $m_g = m_g^0$ (dashed line) or $m_g = m_g(T)$ given by Eq. (2) (solid line).

III. GLUEBALL GAS PRESSURE

The pressure of an ideal gas of glueballs with mass m_g and spin J_g simply corresponds to that of an ideal Bose-Einstein gas (see *e.g.* [18]):

$$p_g = -\frac{(2J_g+1)T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - \mathrm{e}^{-\sqrt{k^2+m_g^2}/T}\right). \tag{3}$$

The pressure of the QCD matter below T_c should then be given by $p = \sum_g p_g$, the sum running over all the glueball states. But, since a Hagedorn spectrum is not assumed, only the lowest-lying glueballs will significantly contribute and we take in a first approximation

$$p \approx p_{0^{++}} + p_{2^{++}}. \tag{4}$$

Results obtained from this last equation are shown in Fig. 2 and compared to the lattice data of Ref. [8]. The conclusions are the following. First, using a glueball gas with constant masses $m_g = m_g^0$ fails to reproduce the observed increase of pressure near T_c, even by taking lighter values than the fitted ones, given in Fig. 1. Second, using the temperature-dependent masses, $m_g = m_g(T)$, fitted from Ref. [15] greatly improves the agreement with lattice QCD and is a first argument in favor of the scenario proposed here. Remark that $p(T_c)$ is slightly underestimated; it is probably due to the neglect of higher-lying glueballs, the lightest of which is the 0^{-+} one, whose finite-temperature properties remain to be clarified.

IV. GLUON CONDENSATE AND TRACE ANOMALY

The next step is now the computation of the trace anomaly, *a priori* straightforwardly defined from the pressure (4) by

$$\bar{\Delta} = T^5 \partial_T \left(\frac{p}{T^4}\right). \tag{5}$$

A look at Fig. 3 clearly shows that our model with $m_g(T)$, although successfully reproducing the lattice pressure, severely underestimates the trace anomaly near the phase transition. This seemingly paradoxical situation can be clarified as follows: Only the glueballs are able to contribute to the pressure, but when speaking of the trace anomaly, the nontrivial role of the gluon condensate has to be taken into account through its nonzero energy density. That argument has already been proposed when studying the QCD matter at $T > 1.2\,T_c$ [19]. Let us now apply it below T_c.

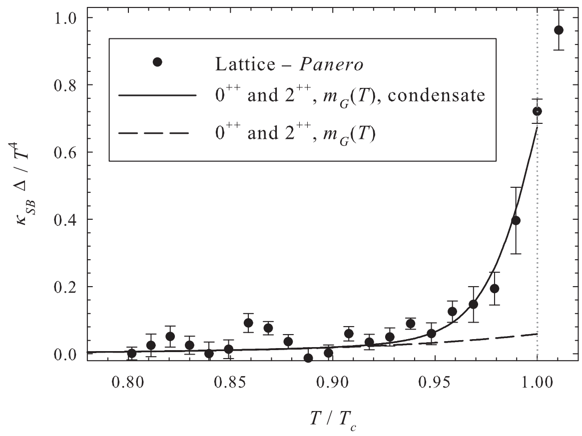

FIG. 3: Same as Fig. 2 for the trace anomaly. The dashed line is the glueball contribution (5), while the solid line comes from Eq. (6), where $\Delta_{G^2} = \tilde{\Delta}_{G^2}$ following Eq. (8).

It is known that the gluon condensate at temperature T, defined by $\left\langle G^2\right\rangle_T = -\left\langle \frac{\beta}{g} G^a_{\mu\nu} G^{\mu\nu}_a(T)\right\rangle$, contributes to the QCD trace anomaly as $\Delta_{G^2} = \left\langle G^2\right\rangle_0 - \left\langle G^2\right\rangle_T$ [19, 20]. Thus the total trace anomaly, Δ, should rather be

$$\Delta = \bar{\Delta} + \Delta_{G^2}. \tag{6}$$

The gluon condensate can moreover be written as the sum of a magnetic and an electric part, *i.e.* $\left\langle G^2\right\rangle_T = \left\langle G^2_e\right\rangle_T + \left\langle G^2_m\right\rangle_T$ in euclidean space. It appears from lattice QCD simulations that $\left\langle G^2_m\right\rangle_T \approx \left\langle G^2_m\right\rangle_0$ on one hand, and that $\left\langle G^2_e\right\rangle_T$ is such $\left\langle G^2_e\right\rangle_0 \approx \left\langle G^2\right\rangle_0/2$ but then falls very quickly near T_c to reach a zero value just after the phase transition [21]. Consequently, one expects [19]

$$\Delta_{G^2} \;=\; \frac{\left\langle G^2\right\rangle_0}{2}\left[1 - c_e(T)\right], \text{where } c_e(T) = \frac{\left\langle G^2_e\right\rangle_T}{\left\langle G^2_e\right\rangle_0} \tag{7}$$

can be known from lattice computations [21].

Since $c_e(T)$ is only known at a few temperatures from the lattice, it is better to compute first the values of Δ_{G^2} that fit the trace anomaly obtained in Ref. [8]. The curve obtained is very accurately fitted by the form

$$\tilde{\Delta}_{G^2} = f_1\, T_c^4 \left[(T/T_c)^2 - f_2\right] \left[1 - \frac{1}{\left(1 + \mathrm{e}^{(T/T_c - f_3)/f_4}\right)^2}\right], \tag{8a}$$

with

$$f_1 = 3.477,\ f_2 = 0.358,\ f_3 = 1.014,\ f_4 = 0.014. \tag{8b}$$

As shown in Fig. 3, the trace anomaly (6) computed with $\Delta_{G^2} = \tilde{\Delta}_{G^2}$ fits the lattice data very well. The key observation is now that the fitted term (8) is in good agreement with that obtained by putting the values of $c_e(T)$ available from the lattice study [21] in the theoretical estimate (7). A typical value $\left\langle G^2\right\rangle_0$ =0.030 GeV^4 has been used as a mean value of various results obtained so far in the literature [22]. Such an agreement between the needed value of the gluon condensate and the one theoretically expected is a relevant check of the mechanism presented here describing the phase transition.

Since in Ref. [8], to which our model is compared, the energy density as well as the entropy density are obtained as linear combinations of the pressure and trace anomaly following standard thermodynamics, it is enough for our purpose to have considered p and Δ. We nevertheless mention for completeness that the entropy density, $s = \partial_T p$, has been independently computed on the lattice in Ref. [11] between $0.7\,T_c$ and T_c. The present model actually underestimates the dimensionless ratio s/T^3 obtained in this last work, but it is worth saying that the numerical values of the parameters have been here chosen to reach an optimal agreement with Ref. [8]. For example, taking $T_c = 280$ MeV instead of 272 MeV leads to an entropy density which is as large as the one of Ref. [11] at the critical temperature.

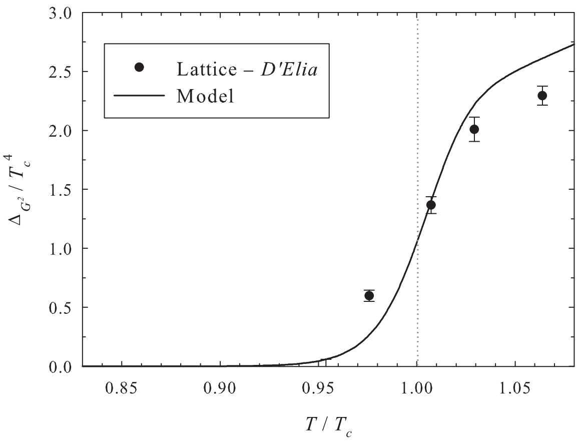

FIG. 4: Trace anomaly contribution $\tilde{\Delta}_{G^2}$ needed to fit the lattice data presented in Fig. 3 (solid line). The curve agrees with formula (7) in which $c_e(T)$ has been taken from the lattice study [21], together with the typical value $\left\langle G^2 \right\rangle_0$ =0.030 GeV^4 =5.48 T_c^4 (full circles).

V. CONCLUDING REMARKS

A new way of understanding the pure glue QCD equation of state below T_c has been proposed. The basic idea is that pure glue hadronic matter consists in both a glueball gas and a gluon condensate part, the latter bringing no contribution to pressure but rather to the trace anomaly *via* its nonzero energy density. It can be intuitively expected that the thermal width of glueballs tends to increase when approaching the deconfinement phase transition. Such a behavior is taken into account in our formalism through a decrease of the glueball masses near T_c. Moreover, that effect has to be combined with the vanishing of the gluon condensate at the critical temperature. That general mechanism allows to reproduce the sudden increase in pressure and trace anomaly observed in recent pure glue lattice data.

Some final remarks can be done about the case $N_c > 3$, that has not been considered here up to now. By definition, $\kappa_{SB} \propto 1/(N_c^2 - 1)$. Above T_c, the dominant contribution to the equation of state should come from deconfined gluons, whose $(N_c^2 - 1)$ color degrees of freedom cancel the κ_{SB} factor, leading to globally constant thermodynamical properties with respect to N_c as observed in Ref. [8]. Below T_c however, all glueballs are in a color singlet and their mass scales as N_c^0. The thermodynamical observables normalized according to κ_{SB} should thus tend to zero at large N_c. This is suggested by the results of Ref. [8], showing that the increase of the trace anomaly near T_c is more and more peaked with increasing N_c, leading to a phase transition which is more and more of first-order type, in agreement with what is expected from the present approach. It can be hoped that future studies of glueball and glueball condensate properties at finite-temperature with arbitrary N_c will allow a more accurate validation of the ideas developed in this work.

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[1] J. C. Collins and M. J. Perry, Phys. Rev. Lett. **34**, 1353 (1975).
[2] E. V. Shuryak, Phys. Rep. **61**, 71 (1980).
[3] D. G. d'Enterria, J. Phys. G **34**, S53 (2007).
[4] A. Peshier and W. Cassing, Phys. Rev. Lett. **94**, 172301 (2005).
[5] U. Kraemmer and A. Rebhan, Rept. Prog. Phys. **67**, 351 (2004); H. Satz, J. Phys. G **32**, R25 (2006); E. Shuryak, J. Phys. G **35**, 104044 (2008).
[6] C. DeTar and U. M. Heller, Eur. Phys. J. A **41**, 405 (2009).

[7] G. Boyd *et al.*, Phys. Rev. Lett. **75**, 4169 (1995); Nucl. Phys. B **469**, 419 (1995).
[8] M. Panero, arXiv:0907.3719.
[9] V. Goloviznin and H. Satz, Z. Phys. C **57**, 671 (1993); P. Lévai and U. Heinz, Phys. Rev. C **57**, 1879 (1998).
[10] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60**, 034509 (1999).
[11] H. B. Meyer, Phys. Rev. D **80**, 051502 (2009).
[12] N. Ishii and H. Suganuma, Eur. Phys. J. A **17**, 77 (2003).
[13] V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E **18**, 1 (2009).
[14] N. Ishii, H. Suganuma and H. Matsufuru, Phys. Rev. D **66**, 014507 (2002).
[15] N. Ishii, H. Suganuma and H. Matsufuru, Phys. Rev. D **66**, 094506 (2002).
[16] J. Sollfrank and U. W. Heinz, Z. Phys. C **65**, 111 (1995); B. J. Schaefer, O. Bohr and J. Wambach, Phys. Rev. D **65**, 105008 (2002).
[17] H. Ichie, H. Suganuma and H. Toki, Phys. Rev. D **52**, 2944 (1995).
[18] J. D. Walecka, *Fundamentals of Statistical Mechanics* (Imperial College Press and World Scientific Publishing Co. Pte. Ltd., 2000).
[19] P. Castorina and M. Mannarelli, Phys. Rev. C **75**, 054901 (2007).
[20] H. Leutwyler, Lecture given at the Workshop on Effective Field Theories, Dobogoko, Hungary, 22-26 August 1991.
[21] M. D'Elia, A. Di Giacomo and E. Meggiolaro, Phys. Rev. D **67**, 114504 (2003).
[22] P. E. L. Rakow, PoS **LAT2005**, 284 (2006), and references therein.