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Steady-state superradiance with alkaline earth atoms

D. Meiser and M. J. Holland

JILA and Department of Physics, The University of Colorado, Boulder, Colorado 80309-0440, USA

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Earth-alkaline-like atoms with ultra-narrow transitions open the door to a new regime of cavity quantum electrodynamics. That regime is characterized by a critical photon number that is many orders of magnitude smaller than what can be achieved in conventional systems. We show that it is possible to achieve superradiance in steady state with such systems. We discuss the basic underlying mechanisms as well as the key experimental requirements.

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Superradiance, first introduced by Dicke over 50 years ago [1], is one of the pillars of cavity quantum electrodynamics (CQED). Superradiance occurs due to the constructive interference of the probability amplitudes for spontaneous decay of several atoms. Due to its generality and conceptual simplicity it is a paradigm system for collective behavior. Usually superradiance is transient; atoms initially prepared in the excited state relax to the ground state rapidly and the collective emission terminates. To date superradiance has not been achieved in a continuous fashion. The goal of this letter is to show that steady-state superradiance can be achieved with ultracold alkaline earth atoms in high finesse cavities. Such systems are experimentally available in the form of optical lattice clocks [2, 3]. Recently Bose Einstein condensates of such atoms have also become available with Ca [4] and Sr atoms [5, 6].

Atoms with a two-electron level structure possess narrow inter-combination lines that, due to selection rules, are dipole-forbidden, and typically have lifetimes many orders of magnitude longer than dipole-allowed transitions. The long lived excited state is essential for superradiance in steady-state because it allows the buildup of population inversion even as the population of the excited state is drained by the collective decay. For dipole-allowed transitions, on the other hand, superradiant decay is so rapid that it would exhaust the supply of excited state atoms before it could be replenished by repumping, and consequently the superradiant emission must cease.

Besides being of fundamental importance, steady state superradiant systems are also interesting because of their potential applications. The most immediate application is the possibility to build *active* optical clocks where the light serving as a frequency standard is derived directly from the atoms [7, 8]. Such systems have the potential to improve the stability of the best clocks by about two orders of magnitude. Another area of application is to strongly-correlated physics. The atoms in this system evolve into exotic many-particle states due to their collective interaction with the light field. These states could be of interest for quantum information purposes as well as for the exploration and study of many-particle phases of condensed matter.

The model that we consider is depicted in Fig. 1(a). N two level atoms with excited state $|e\rangle$ and ground state $|g\rangle$ decay with rate $\Gamma_c = \mathcal{C}\Gamma$ through the mode of a cavity, where $\mathcal{C} = g^2/(\Gamma\kappa)$ is the single atom cooperativity parameter. The transition from $|e\rangle$ to $|g\rangle$ is assumed to be a narrow inter-combination line. The atomic free space spontaneous emission rate is Γ, the single photon Rabi frequency is g, and the cavity decay rate is κ. For simplicity we assume that all atoms couple identically to the cavity mode. The rate Γ_c is assumed to be much larger than the rates for non-collective decay processes and dephasing. That approximation requires that $N\mathcal{C} \gg 1$. At the same time the atoms are being *non-collectively* repumped to the excited state with an effective rate w. This could be achieved by resonantly driving a transition to a third state with Rabi frequency Ω that decays rapidly with rate γ to $|e\rangle$.

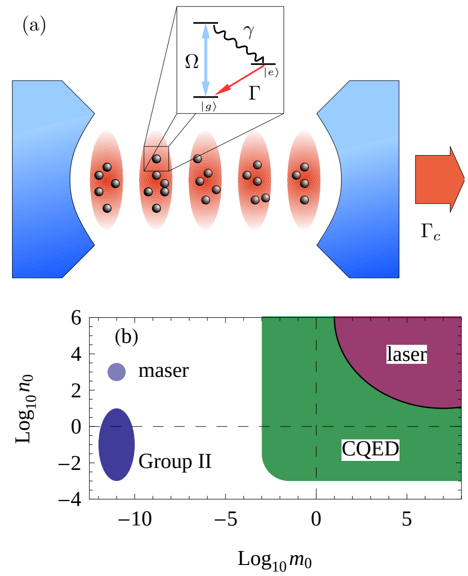

FIG. 1: (Color online) (a) Schematic of a coupled atom-cavity system leading to steady state superradiance. (b) The parameter space of CQED spanned by the critical atom number n_0 and the critical photon number m_0. The collective behavior of the coupled atom-cavity system undergoes a cross-over from stimulated emission dominated (laser-like) at large m_0 to collective spontaneous emission dominated (superradiance-like) for small m_0.

There are two key experimental requirements for realizing the physics discussed here. The most important requirement is that the collective decay be dominant over all other decay and decoherence processes, *i.e.* $N\mathcal{C}\Gamma \gg \Gamma, 1/T_2$, where T_2 is the spin dephasing time. The second requirement is that one must be able to repopulate the atomic excited state at a rate equal to the collective decay rate, *i.e.* it must be possible to achieve $w \sim N\mathcal{C}\Gamma$ without significant atomic losses.

The model under consideration is similar to typical many-atom CQED systems that are studied theoretically and experimentally, except that the atomic dipole moment is many orders of magnitude smaller. Nevertheless, the small dipole moment can lead to profound consequences. These can be characterized by the critical atom number $n_0 = (\kappa\Gamma)/(g^2)$ and the critical photon number $m_0 = (\gamma^2)/(g^2)$ that tell us to what degree quantum effects are important: $n_0 < 1$ means that a single atom can substantially affect the cavity field and $m_0 < 1$ means that the electrical field corresponding to a single photon in the cavity can saturate the atomic transition. For ultra-narrow inter-combination lines in earth-alkaline-like atoms, critical photon numbers as small as $m_0 \sim 10^{-12}$ can realistically be achieved. These systems reside in an exotic region of parameter space that has previously been inaccessible. For example for cavity QED systems with alkali atoms the records are in the $m_0, n_0 \sim 0.01 - 0.001$ range. It is impossible to substantially improve upon these values as can be seen by rewriting the critical photon number as $n_0 = 2\pi A/(F\sigma)$ and $m_0 = 4\pi^2 V_{\text{eff}}/(Q\lambda_0^3)$ where A is the cross section of the cavity mode, $\sigma = 3\lambda_0^2/(2\pi)$ is the resonant cross section of the atoms, λ_0 is the wavelength of the resonant light, and F and Q are the finesse of the cavity and the quality factor of the atomic transition, respectively. Since the dimensionless $V_{\text{eff}}/(\lambda_0^3)$ must be at least unity on fundamental grounds, and is typically orders of magnitude larger, the atomic resonance Q must be extraordinarily high to reach $m_0 \sim 10^{-12}$, and the values of Q that are possible for the optically allowed dipole transitions in alkali atoms and similar systems are insufficient. Incidentally, masers, which operate in the microwave domain, have critical photon numbers similar to the narrow linewidth atoms, but are classical due to their rather large critical atom number, and have photon energies that are many orders of magnitude smaller than is characteristic in the optical domain.

The evolution of the system shown in Fig. 1 is given by the master equation,

$$\frac{d\hat{\rho}}{dt} = -\frac{\Gamma_c}{2}\left(\hat{J}_+\hat{J}_-\hat{\rho} + \hat{\rho}\hat{J}_+\hat{J}_- - 2\hat{J}_-\hat{\rho}\hat{J}_+\right) - \frac{w}{2}\sum_{j=1}^{N}\left(\hat{\sigma}_-^{(j)}\hat{\sigma}_+^{(j)}\hat{\rho} + \hat{\rho}\hat{\sigma}_-^{(j)}\hat{\sigma}_+^{(j)} - 2\hat{\sigma}_+^{(j)}\hat{\rho}\hat{\sigma}_k-^{(j)}\right). \tag{1}$$

Here, $\hat{\sigma}_-^{(j)} = (\hat{\sigma}_+^{(j)})^\dagger = |g\rangle\langle e|$ is the lowering operator for atom j and $\hat{J}_- = (\hat{J}_+)^\dagger = \sum_{j=1}^{N}\hat{\sigma}_-^{(j)}$ is the collective decay operator brought about by the coupling of the atoms to the rapidly decaying cavity field. In deriving this master equation we have assumed that the cavity field decays so rapidly that the mean photon number is much smaller than unity, and thus the field has been adiabatically eliminated. This approximation is valid for atoms with an extremely weak dipole moment such as those that we are interested in. The collective decay part occurs in the superradiance master equation first introduced by Bonifacio *et. al.* [9]; see also [10]. The repumping can be thought of as spontaneous "absorption" from the ground state to the excited state.

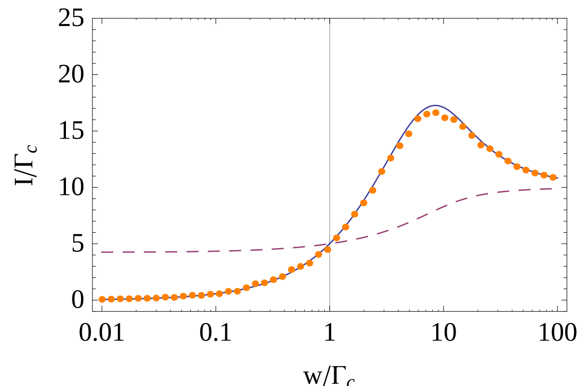

FIG. 2: (Color online) Emission rate as a function of repump rate for 10 atoms. The orange dots are the expectation values extracted from the Monte-Carlo wave function simulations. The blue line shows the semi-classical result obtained from Eqs.(3-5), and the purple dashed line is the emission rate for uncorrelated particles, $N_e\Gamma_c$.

We unravel the master equation into Monte Carlo wavefunction trajectories [11], and from the trajectories $|\psi(t)\rangle$ we extract expectation values $\langle\hat{O}(t)\rangle = \langle\psi(t)|\hat{O}|\psi(t)\rangle$ of system observables $\hat{O}$. Steady state expectation values $\langle\hat{O}\rangle_{\text{SS}}$ are then obtained by calculating time averages, $\langle\hat{O}\rangle_{\text{SS}} = \frac{1}{T}\int_{t_1}^{t_1+T}\mathrm{d}t\,\langle\hat{O}(t)\rangle$, where t_1 is chosen large enough to allow the system to settle to steady state, and T is chosen long enough for statistical errors to be controlled. We have empirically checked that the steady state does not depend on the choice of initial conditions.

In Fig. 2 we show that Eq. (1) leads to sustained superradiance by investigating the mean photon emission rate

$$I = \Gamma_c\langle\hat{J}_+\hat{J}_-\rangle_{\text{SS}}\,, \tag{2}$$

together with the emission rate $N_e\Gamma_c$ that one would expect for uncorrelated atoms, with N_e the population of the excited state. Three qualitatively different regimes can be distinguished: strong pumping with $w > N\Gamma_c$, intermediate pumping with $\Gamma_c < w < N\Gamma_c$, and weak pumping $w < \Gamma_c$. For very strong pumping, $w \gg N\Gamma_c$ the emission rate approaches the maximum possible emission rate for uncorrelated atoms, $N\Gamma_c$. As the pump

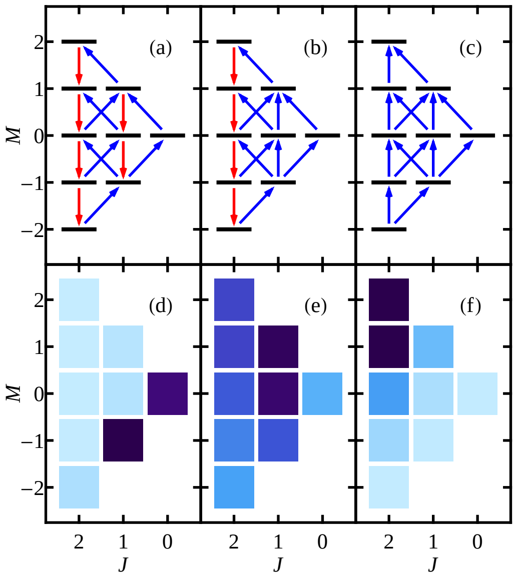

FIG. 3: (Color online) Transition rates between J and M eigenspaces (a-c) and populations $P_{M,J}$ of these subspaces (d-f) for $w = 0.1\,\Gamma_c$ (a, d), $w = 2.0\,\Gamma_c$ (b, e), $w = 10.0\,\Gamma_c$ (c, f), and $N = 4$ atoms. Red arrows indicate decay dominated transitions and blue arrows indicate repump dominated transitions. The transition rates are calculated by averaging over degenerate initial states and summing over final states. In (d-f) darker shades indicate larger population.

rate decreases, the emission rate *increases* beyond $N\Gamma_c$. This implies the presence of correlations between different atoms, resulting in a collective enhancement of the emission rate. The behavior of the system in the weak pumping regime, $w < \Gamma_c$ is surprising and counter-intuitive in two ways. First, nearly half the atoms remain in the excited state even as $w/\Gamma_c \to 0$. Second, the emission rate is greatly suppressed below the value for uncorrelated atoms, *i.e.* the atoms are subradiant. Subradiance, like superradiance, implies the presence of correlations between different atoms.

The qualitative behavior of the emission rate can be understood if we look at the dynamics of the system in the collective basis $|J, M, \xi\rangle$ where J characterizes the total angular momentum, the magnetic quantum number M characterizes the inversion, and the multiplicity quantum number ξ enumerates the degenerate manifold at given J and M. Figures 3(a-c) show the subspaces corresponding to well defined J and M along with the net transition rates, for $N = 4$ atoms. Panels (d-f) show the steady state probabilities, $P_{M,J} = \langle \hat{P}_{M,J}\rangle_{\rm ss}$ where $\hat{P}_{M,J} = \sum_\xi |J, M, \xi\rangle\langle J, M, \xi|$, for the system to be in the various subspaces. The right hand column (panel c and f) of Fig. 3 shows the strong pumping limit where repumping dominates the collective decay and consequently a large fraction of the atoms accumulate in the excited state.

In the intermediate pumping regime, $w \lesssim N\Gamma_c$, shown in the central column (panel b and e), there is a non-trivial competition between collective decay and repumping. It is apparent that for states where $J \sim \mathcal{O}(N/2)$ and is therefore close to its maximum possible value, the collective decay dominates, whereas for states with small J the non-collective repumping dominates. This leads to a cycle in which the decay and repumping balance with an enhanced average value for $\langle J_+ J_-\rangle$, *i.e.* an enhanced emission rate. Note that it is important that the repumping is non-collective for superradiance to occur in steady state. If the repumping also preserves J, as is the situation considered in Refs. [12, 13, 14], there is no mechanism to balance the non-collective decay and decoherence processes that tend to decrease J and are unavoidable in experiment.

In the weak pumping regime, shown in the left hand column (panel a and d), the collective emission drives the system into states with $M = -J$ that cannot decay by means of $\hat{J}_-$. For $M < 0$ the repumping predominantly drives transitions $J \to J-1$ while transitions from $J \to J+1$ are rare. Thus the system evolves along a dynamical pathway with smaller and smaller J, eventually reaching $J \sim 1$ where it becomes trapped. This steady state has an almost equal number of atoms in the ground and excited state, and yet has a greatly suppressed emission rate. This is due to subradiance arising from strong atom-atom correlations. The requirement for this subradiance to occur is $\mathcal{C} \gg 1$, which is much more stringent than the condition for superradiance.

The Monte Carlo simulations provide a clear picture of the dynamical interference, and are complete in that they allow us to include a full description of the Hilbert space evolution. Due to the exponential scaling of the dimension of the Hilbert space of the system with the number of particles we are however limited to relatively small numbers of particles (of order 20). To consider mesoscopic particle numbers, we use a semi-classical approximation that consists of keeping only pair correlations. This approximation is validated by the excellent agreement with the Monte-Carlo results for small particle numbers. Mathematically, the semi-classical approximation is implemented by expanding expectation values of the system operators $\{\hat{\sigma}_z^{(j)}, \hat{\sigma}_+^{(j)}, \hat{\sigma}_-^{(j)}\}$ in terms of cumulants $\langle \ldots \rangle_c$, where $\hat{\sigma}_z^{(j)} = |e\rangle\langle e| - |g\rangle\langle g|$. We use that all expectation values are symmetrical with respect to particle exchange, *e.g.* $\langle \hat{\sigma}_+^{(i)} \hat{\sigma}_-^{(j)}\rangle = \langle \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)}\rangle$ for all $i \neq j$. All non-zero cumulants up to second order can be expressed in terms of $\langle \hat{\sigma}_z^{(1)}\rangle_c = \langle \hat{\sigma}_z^{(1)}\rangle$, $\langle \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)}\rangle_c = \langle \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)}\rangle$, and $\langle \hat{\sigma}_z^{(1)} \hat{\sigma}_z^{(2)}\rangle_c = \langle \hat{\sigma}_z^{(1)} \hat{\sigma}_z^{(2)}\rangle - \langle \hat{\sigma}_z^{(1)}\rangle^2$ and their equations of motion are

$$\begin{aligned}\frac{d\langle \hat{\sigma}_z^{(1)}\rangle_c}{dt} &= -(w + \Gamma_c)\langle \hat{\sigma}_z^{(1)}\rangle_c \\ &\quad - 2\Gamma_c(N-1)\langle \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)}\rangle_c\,,\end{aligned} \tag{3}$$

$$\frac{d\langle\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(2)}\rangle_c}{dt} = -(w+\Gamma_c)\langle\hat{\sigma}_+^{(1)}\sigma_-^{(2)}\rangle_c + \frac{\Gamma_c}{2}\left(\langle\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\rangle_c + \langle\hat{\sigma}_z^{(1)}\rangle_c\right) + \Gamma_c(N-2)\Big(\langle\hat{\sigma}_z^{(1)}\rangle_c\langle\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(2)}\rangle_c + \langle\hat{\sigma}_z^{(1)}\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(3)}\rangle_c\Big)\,, \quad (4)$$

$$\frac{d\langle\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\rangle}{dt} = -2(w+\Gamma_c)\langle\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\rangle_c + 4\Gamma_c\Big(\langle\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(2)}\rangle_c(1+\langle\hat{\sigma}_z^{(1)}\rangle_c) - (N-2)\langle\hat{\sigma}_z^{(1)}\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(3)}\rangle_c\Big)\,. \quad (5)$$

By dropping the small third order cumulant $\langle\hat{\sigma}_z^{(1)}\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(3)}\rangle_c$ we obtain a closed set of equations. The steady state is found by setting the time derivatives to zero. The resulting cubic equations can be solved exactly and these solutions are the basis of the analytical curve in Fig. (2). The expressions one obtains are however very complicated. Simple expressions may be obtained in the regime $\Gamma_c \ll w \sim N\Gamma_c$ in which collective emission occurs. By introducing the rescaled operators $\hat{j}_z = \hat{J}_z/N$ and $\hat{j}_\pm = \hat{J}_\pm/N$ where $\hat{J}_z = 1/2\sum_j \hat{\sigma}_z^{(j)}$, we find that to leading order in $1/N$ the only non-zero expectation values are $\langle\hat{j}_z\rangle_c$ and $\langle\hat{j}_+\hat{j}_-\rangle_c$ and they evolve according to,

$$\frac{d\langle\hat{j}_z\rangle_c}{dt} = -w(\langle\hat{j}_z\rangle_c - 1/2) - N\Gamma_c\langle\hat{j}_+\hat{j}_-\rangle_c \quad (6)$$

$$\frac{d\langle\hat{j}_+\hat{j}_-\rangle_c}{dt} = -w\langle\hat{j}_+\hat{j}_-\rangle_c + 2N\Gamma_c\langle\hat{j}_z\rangle_c\langle\hat{j}_+\hat{j}_-\rangle_c. \quad (7)$$

The steady state solutions are

$$\langle\hat{j}_z\rangle_{\rm SS} = \begin{cases} \frac{1}{2}\frac{w}{N\Gamma_c}\,, & w < N\Gamma_c \\ \frac{1}{2}\,, & w \geq N\Gamma_c \end{cases}\,, \quad (8)$$

and

$$\langle\hat{j}_+\hat{j}_-\rangle_{\rm SS} = \begin{cases} \frac{1}{2}\frac{w}{N\Gamma_c}\left(1 - \frac{w}{N\Gamma_c}\right)\,, & w < N\Gamma_c \\ 0 & w \geq N\Gamma_c \end{cases}\,. \quad (9)$$

The fact that $\langle\hat{j}_+\hat{j}_-\rangle_{\rm SS}$ is of order unity for $w \lesssim N\Gamma_c$ indicates the presence of strong correlations between atoms. From Eq. (9) we can extract the maximum intensity,

$$I_{\max} = N^2\Gamma_c/8, \quad (10)$$

obtained at $w = N\Gamma_c/2$. The scaling of that intensity with N^2 underlines the collective nature of the light emission. Remarkably, $I_{\max}$ is only a factor 1/2 smaller than the maximum possible emission rate $\Gamma_c\langle J,0|\hat{J}_+\hat{J}_-|J,0\rangle$.

Not only is this source of light bright, however, it also has a long coherence time—a property shared with lasers. In lasers, the coherence time of the light field is much longer than the cavity-ring-down time because of the fact that the field is macroscopically occupied [15]. To find the coherence time of the atoms in the superradiant system considered in this paper, we study the two-time correlation function of the atomic dipole $\langle\hat{\sigma}_+^{(1)}(t+\tau)\hat{\sigma}_-^{(2)}(t)\rangle$. Using the quantum regression theorem we find the equations of motion

$$\frac{d\langle\hat{\sigma}_+^{(1)}(\tau)\hat{\sigma}_-^{(2)}(0)\rangle}{d\tau} = -\frac{1}{2}\left(w + \Gamma_c - (N-2)\langle\hat{\sigma}_z^{(1)}\rangle_{\rm SS}\right) \times \langle\hat{\sigma}_+^{(1)}(\tau)\hat{\sigma}_-^{(2)}(0)\rangle\,, \quad (11)$$

where we have factorized $\langle\hat{\sigma}_+^{(1)}(\tau)\hat{\sigma}_z^{(2)}(\tau)\hat{\sigma}_-^{(3)}(0)\rangle \approx \langle\hat{\sigma}_+^{(1)}\rangle_{\rm SS}\langle\hat{\sigma}_z^{(2)}(\tau)\hat{\sigma}_-^{(3)}(0)\rangle$ and we have assumed that the system is in steady state. Inserting $\langle\hat{\sigma}_z^{(1)}\rangle_{\rm SS} = 2\langle\hat{j}_z\rangle_{\rm SS}$ from above we find that the spins of different atoms remain coherent with each other for $t_{\rm coh} = N/(N\Gamma_c + 2w)$. In the superradiant regime where $w \sim N\Gamma_c$ the coherence time is thus indeed a factor $\sim N$ larger than that for just two atoms. The phase of the atomic dipole diffuses slowly which is analogous to the slow phase diffusion of the field of a laser. This result is significant especially with an eye to possible applications as an ultrastable local oscillator or frequency reference.

In summary, we have shown that steady state superradiance can be achieved with atoms with an ultra-narrow transition in a cavity. Alkaline-earth-like atoms are prime candidates for realizing such systems and open up a new regime of CQED characterized by an extremely small critical photon number. In future work we will study the noise properties of the emitted light and the correlated atomic state. It will be intruiging to consider the crossover from steady-state superradiance to lasing that occurs as the system goes from the bad cavity limit to the good cavity limit.

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