Hill Climbing and Simulated Annealing. Finding global minima

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Abstract

In this paper we will analyze the results and efficiency of three algorithms (Hill Climbing First Improvement, Hill Climbing Best Improvement, Simulated Annealing) with different parameters. The final result shows that Simulated Annealing is the most accurate and efficient algorithm.

1 Introduction

1.1 Short Introduction

In this paper we will analyze three heuristic methods for finding the global minimum of four chosen functions. We will analyze the efficiency and accuracy of the results on several dimensions. In the case of the Hill Climbing algorithm, I used first fit and best fit.

In the first section of this paper, the introduction and motivation are presented. The second part covers the methods and the description of the algorithms. In the third part, the functions and parameters of the algorithms are presented. In the fourth section I analyzed the results and finally the conclusion of the experiments is presented.

1.1.1 Motivation

The motivation of this work is to see which of the three algorithms finds the best result and which is more efficient in terms of time. We will also analyze how the results change when we modify certain parameters.

2 Method

Both Hill Climbing and Simulated Annealing work with bit strings, not base 10 numbers. The functions I used for experiments are: De Jong1, Schwefel's function 7, Rastrigin's function 6 and Michalewicz's function 12.

I studied the functions in 5, 10 and 30 dimensions. In order to obtain the best possible results, I iterated the algorithms 1000 times for 5 and 10 dimensions, and for time reasons, I iterated 100 times for 30 dimensions. Each set of experiments consists of 30 such runs, and the results have a precision of 5 decimals.

These algorithms initially generate a random bitstring that is evaluated. After that we explore the neighborhood of the current bitstring. Neighbouring consists in changing a single bit from the bitstring. Each algorithm has a different way of choosing which neighbour bitstring to explore. This process is repeated a large number of times until we find a result as close as possible to the global minimum.

2.1 Parametrs

The Hill Climbing algorithm receives as parameters a function, an interval [a,b], the maximum number of iterations (100 or 1000) and the number of dimensions D of of the function $D \in \{5, 10, 30\}$. Simulated Annealing also receives a temperature initial T equal to 500 and a cooling function g(T,t) by which T is multiplied by a subunit value equal to 0.95.
For Simulated Annealing maximum number of iterations is 10000.

2.2 Hill Climbing

Hill Climbing is a mathematical optimization technique which belongs to the family of local search. It is an iterative algorithm that starts with an arbitrary solution to a problem, then attempts to find a better solution by making an incremental change to the solution. If the change produces a better solution, another incremental change is made to the new solution, and so on until no further improvements can be found.

The difference between best fit and first fit is that best fit looks for the best solution in the neighborhood, while first fit looks for the first solution better than the current solution

2.3 Simulated Annealing

Simulated annealing (SA) is a probabilistic technique for approximating the global optimum of a given function. Specifically, it is a metaheuristic to approximate global optimization in a large search space for an optimization problem.

3 Experiment Description

The precision with which we find the solutions is 5 decimal places. I tested the 2 algorithms on 4 functions. In the case of Hill Climbing I used both first improve- ment, as well as best improvement.

De Jong's function 1

$$f_1(x) = \sum_{i=1}^{n} x_i^2$$

$-5.12 \leq x_i \leq 5.12$
global minima: $f(x) = 0$, $i = 1 : n$.

Schwefel's function 7

$$f_7(x) = \sum_{i=1}^{n} -x_i \cdot \sin\left(\sqrt{|x_i|}\right)$$

$-500 \leq x_i \leq 500$
global minima: $f(x) = -n \cdot 418.9829$; $i = 1 : n$.

Rastrigin's function 6

$$f_6(x) = 10 \cdot n + \sum_{i=1}^{n} (x_i^2 - 10 \cdot \cos(2 \cdot \pi \cdot x_i))$$

$-5.12 \leq x_i \leq 5.12$
global minima: $f(x) = 0$, $i = 1 : n$.

Michalewicz's function 12

$$f_{12}(x) = -\sum_{i=1}^{n} \sin(x_i) \cdot \left(\sin \frac{i \cdot x_i^2}{\pi}\right)^{2 \cdot m}$$

$i = 1 : n.$, $m = 10$, $0 \leq x_i \leq \pi$
global minima: f(x)=-4.687 (n=5); f(x)=-9.66 (n=10);

For 5 and 10 dimensions I ran the algorithms 1000 times, and for for 30 dimensions I ran run 100 times.

4 Rezultate Experimentale

For all iterations in Simulated Annealing T = 500, the decrease rate of temperature = 0.95

De Jong's function 1

Dimensiuni	Algoritm	global min	$f(x)_{min}$	$f(x)_{max}$	$f(x)_{mediu}$	St_{dev}	t_{min}	t_{max}
5	HC First	0	0	0	0	0	0.2118	0.248223
	HC best		0	0	0	0	0.395843	0.520077
	SA		0.00000	0.00013	0.00006	0.00003	0.70904	0.83243
10	HC first	0	0	0	0	0	1.58141	1.69059
	HC best		0	0	0	0	3.01573	3.18758
	SA		0.00007	0.00016	0.000115	0.00003	2.69955	2.89203
30	HC first	0	0	0	0	0	44.627	45.4418
	HC best		0	0	0	0	80.5437	87.4715
	SA		0.00020	0.00063	0.00036	0.0001	24.80901	26.99770

Schwefel's function 7

Dimensiuni	Algoritm	global min	$f(x)_{min}$	$f(x)_{max}$	$f(x)_{mediu}$	St_{dev}	t_{min}
5	HC First	-2094.9145	-2094.81	-2026.24	-2070.11167	19.78838	5.21495
	HC best		-2094.91	-2094.6	-2094.79267	0.08493	9.39046
	SA		-2094.91428	-1845.45714	-2005.42469	91.16911	1.47378
10	HC first	-4189.829	-4155.18	-3802.68	-3910.00967	87.25599	40.5118
	HC best		-4162.32	-3975.83	-4084.10433	62.30153	71.7419
	SA		-4189.82814	-3787.59268	-4056.24513	124.10864	5.80003
30	HC first	-12569.487	-10697	-10254.8	-10439.7714	143.57572	106.851
	HC best		-11315.5	-10892.3	-11046.4429	156.93537	196.591
	SA		-12465.94593	-11567.92672	-11948.1595	258.05182	49.57727

Rastrigin's function 6

Dimensiuni	Algoritm	global min	f(x)$_{min}$	f(x)$_{max}$	f(x)$_{mediu}$	St$_{dev}$	t$_{min}$	t$_{max}$
5	HC First	0	0.994961	2.2308	1.14961	0.28647	2.02391	2.78926
	HC best		0.994961	2.36502	1.58865	0.69051	3.48498	3.9313
	SA		0.00001	9.91851	3.72135	3.37989	0.80148	0.94118
10	HC first	0	3.70751	7.93328	5.76274	1.38482	15.9735	17.3813
	HC best		2.2308	5.21568	3.31083	0.95844	26.3769	28.0574
	SA		1.99000	15.12415	7.13123	3.96537	3.05357	3.35341
30	HC first	0	37.4072	49.8073	43.70897	3.75739	39.0448	42.0615
	HC best		28.9757	39.2327	35.47829	3.10688	70.1411	74.5854
	SA		5.21600	28.44792	17.72855	7.31128	26.30931	28.36853

Michalewicz's function 12

Dimensiuni	Algoritm	global min	f(x)$_{min}$	f(x)$_{max}$	f(x)$_{mediu}$	St$_{dev}$	t$_{min}$	t$_{max}$
5	HC First	-4.687	-4.68765	-4.68146	-4.68523	0.00195	2.47741	2.66352
	HC best		-4.68766	-4.6841	-4.68681	0.00097	4.41537	4.62606
	SA		-4.68707	-4.20237	-4.52008	0.12613	1.08329	1.20677
10	HC first	-9.66	-9.50669	-9.09083	-9.26461	0.10880	18.4873	19.6796
	HC best		-9.52974	-9.16455	-9.363739	0.08401	31.9508	33.2136
	SA		-9.59564	-8.46337	-9.11268	0.27797	4.31905	4.91260
30	HC first	-29.63088	-27.1615	-24.6143	-25.43349	0.56259	43.0619	52.2509
	HC best		-26.8392	-25.9418	-26.33364	0.22927	80.0825	90.7012
	SA		-28.76496	-27.12881	-27.78552	0.47401	39.87190	41.15500

5 Conclusion

In conclusion, Hill Climbing Best Fit is more accurate than First Fit, but almost twice slower. For a small number of dimensions I would choose hill climbing first fit because it is much faster and almost as good as best fit, but for a big number of dimensions I would choose best fit because is more accurate.

Simulated annealing is much faster than Hill Climbing and is very accurate. Simulated Annealing seems to be the best algorithm among all three.

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