MECH4450 Aerospace Propulsion

Major Assignment - Part 2

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1 Methodology

Combustion was modelled using the same model that was developed in part 1, however the temperature was calculated at each step along the solution from the heat released during combustion, rather than assuming a temperature profile ahead of time.

In order to calculate the temperature needed by the chemistry model, and also to calculate thrust, the total temperature and Mach number must be solved alongside the chemistry model. The additional equations to be solved are Eq. 1 and 2.

$$\frac{\partial M^2}{\partial x} = M^2 \frac{1 + \frac{\gamma_b - 1}{2} M^2}{1 - M^2} \left(-\frac{2}{A}\frac{\partial A}{\partial x} + \frac{1 + \gamma_b M^2}{T_t}\frac{\partial T_t}{\partial x} + \frac{4\gamma_b M^2 C_f}{D} \right) \tag{1}$$

$$\frac{\partial T_t}{\partial x} = -\frac{1}{c_{pb}} \sum_i \frac{\partial Y_i}{\partial x} h^\circ_{f,i} \tag{2}$$

Where

$$\frac{\partial Y_i}{\partial x} = \mathcal{M}_i \left\{ \frac{1}{\sum_j [X_j]\,\mathcal{M}_j} \frac{\partial [X_i]}{\partial x} - \frac{[X_i]}{\left(\sum_j [X_j]\,\mathcal{M}_j\right)^2} \sum_j \mathcal{M}_j \frac{\partial [X_j]}{\partial x} \right\} \tag{3}$$

and

$$\frac{\partial [X_j]}{\partial x} = \frac{1}{v(x)} \frac{\partial [X_j]}{\partial t} \tag{4}$$

Where $\frac{\partial [X_j]}{\partial t}$ is calculated using the chemistry model, and $v(x)$ is the velocity of the gas x m along the combustion chamber. This converts the time derivatives computed by the chemistry model into spatial derivatives needed to model the combustion. The velocity can be found using the Mach number of the flow at x.

$$v(x) = M(x)\sqrt{\gamma_b R_b T(x)} \tag{5}$$

Since $T_t(x)$ and $M(x)$ is solved for at each time step, $T(x)$ can be calculated using isentropic flow relations (Eq. 6). This is needed by the chemistry model, and to calculate $v(x)$.

$$T(x) = T_t(x) \left[1 + \frac{\gamma_b - 1}{2} M(x)^2 \right]^{-1} \tag{6}$$

Thus, using all the above equations, a system of ordinary differential equations can be solved to give the mass fraction of each species, total temperature and

Mach number at each point along the combustor.

$$\frac{\partial}{\partial x}\begin{bmatrix}[C_2\,H_4]\\ [O_2]\\ [H_2\,O]\\ [CO]\\ [CO_2]\\ [N_2]\\ T_t\\ M^2\end{bmatrix}=\begin{bmatrix}f_0\left(T_t,M^2,[X_i]\right)\\ f_1\left(T_t,M^2,[X_i]\right)\\ f_2\left(T_t,M^2,[X_i]\right)\\ f_3\left(T_t,M^2,[X_i]\right)\\ f_4\left(T_t,M^2,[X_i]\right)\\ f_5\left(T_t,M^2,[X_i]\right)\\ f_6\left(T_t,M^2,[X_i]\right)\\ f_7\left(T_t,M^2,[X_i]\right)\end{bmatrix}\tag{7}$$

Additionally, the density of the flow at each point along the combustor can be calculated from conservation of mass.

$$\rho(x)=\frac{\dot{m}}{v(x)A(x)}\tag{8}$$

This density can then be used to calculate the pressure at each point in the combustor using the ideal gas law.

$$P(x)=\rho(x)RT(x)\tag{9}$$

2 Results and Discussion

2.1 Inlet Modelling

The flight conditions of the scramjet can be used to calculate properties of the gas entering the scramjet. Given values for the freestream flight conditions are listed in Table. 2.1.

Property	**Symbol**	**Value**
Mach Number	M_0	10
Static Temperature	T_0	220 K
Dynamic Pressure	q_0	50 kPa
Specific Gas Constant	R	287 J/(kg K)

Table 2.1: Freestream flight conditions

Using the static temperature to calculate the speed of sound $a_0^2=\gamma RT_0$, and the flight Mach number, the velocity of gas entering the scramjet is given as:

$$v_0=M_0\sqrt{\gamma RT_0}=2973\,\mathrm{m/s}\tag{10}$$

The vehicle operates at constant dynamic pressure, given by the relation $q_0=\frac{1}{2}\rho_0 v_0^2$. With velocity known, this can be rearranged to give the inlet density, and subsequently pressure from the ideal gas law.

$$\rho_0=\frac{2q_0}{v_0^2}=0.011\,\mathrm{kg/m^3}\tag{11}$$

$$p_0=\rho_0 RT=714.4\,\mathrm{Pa}\tag{12}$$

The specific heat at constant pressure for the air fuel mixture can be calculated from the ratio of specific heats and gas constant determined from the CFD modelling of the inlet.

$$c_{pb} = \frac{\gamma_b}{\gamma_b - 1} Rb \tag{13}$$

$$= 1.188\,45\,\mathrm{kJ/(kg\,K)} \tag{14}$$

The density of the air fuel mixture at the inlet to the combustor can be calculated using the ideal gas law.

$$\rho_{3b} = \frac{P_{3b}}{RT_{3b}} \tag{15}$$

$$= \frac{70.09 \times 10^3}{288.45 \times 1400} \tag{16}$$

$$= 0.174\,\mathrm{kg/m^3} \tag{17}$$

The area of the inlet to the combustor, A_3, can be calculated from conservation of mass applied to the fully mixed air-fuel mixture at the combustor inlet.

$$A_3 = \frac{\dot{m}}{\rho_{3b} v_{3b}} \tag{18}$$

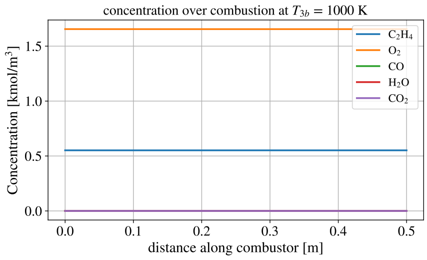
CALCULATE THE REMAINING FLOW STATES

2.2 Combustor Modelling

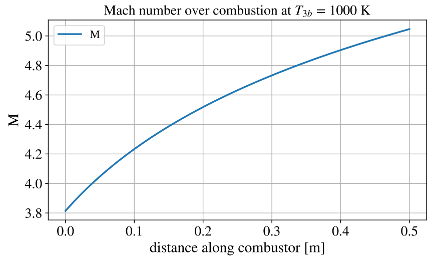
The above system of equations was solved in python for three different inflow temperatures; 1000 K, 1237.63 K, and 1400 K. Plots of properties along the combustor are plotted in Figures

2.3 Nozzle Modelling

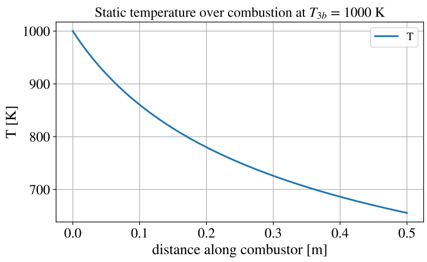
2.4 Overall Performance and Discussion



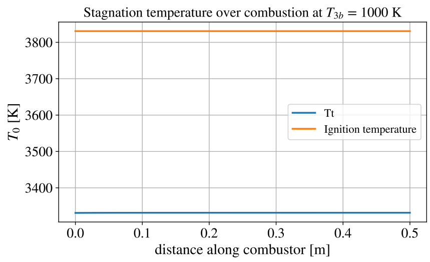
(a) Species Concentration



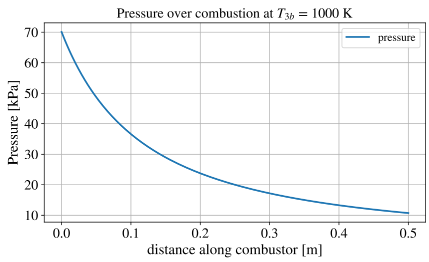
(b) Mach number



(c) Static temperature



(d) Stagnation temperature



(e) Pressure

Figure 2.1: Properties over combustion at 1000 K

(a) Species Concentration

part_2_img/mach_1237.pdf

(b) Mach number

part_2_img/static_temp_1237.pdf

(c) Static temperature

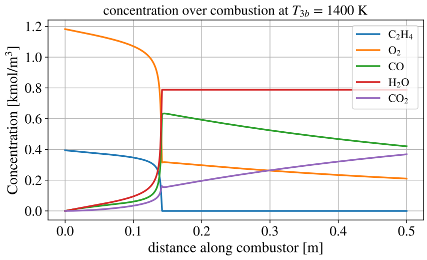
part_2_img/stag_temp_1237.pdf

(d) Stagnation temperature

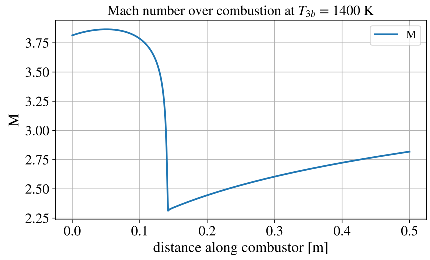
part_2_img/pressure_1237.pdf

(e) Pressure

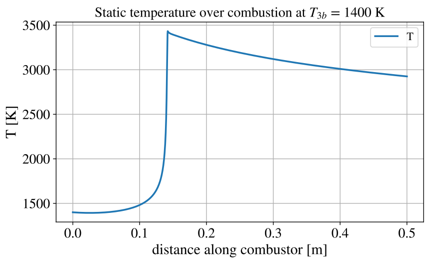
Figure 2.2: Properties over combustion at 1237 K



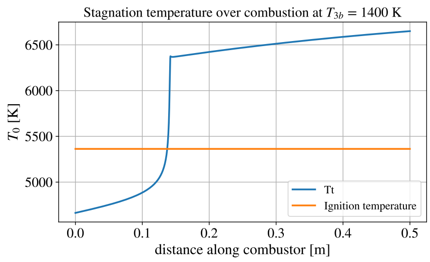
(a) Species Concentration



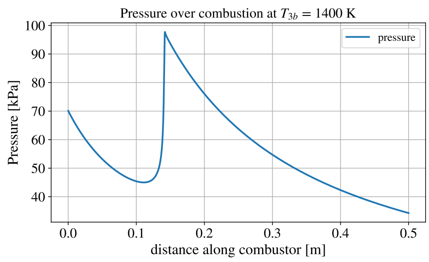
(b) Mach number



(c) Static temperature



(d) Stagnation temperature



(e) Pressure

Figure 2.3: Properties over combustion at 1400 K