TEXAS TECH UNIVERSITY

CS4379: Parallel and Concurrent Programming
CS5379: Parallel Processing

Lecture 4

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Course Info

- **Lecture Time**: TR, 12:30-1:50
- **Lecture Location**: ECE 217
- **Sessions**: CS4379-001, CS4379-002, CS5379-001, CS5379-D01
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Outline

- Questions?

- Interconnection networks for parallel computers (cont.)
  - Linear arrays, meshes, and generalized meshes
  - Tree-based networks
- Evaluating network topologies
- Quiz #1

Linear Arrays

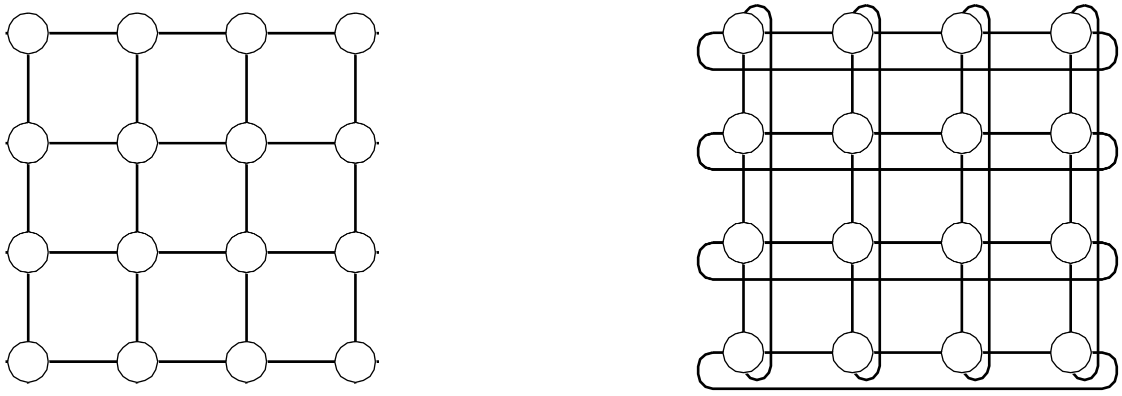
- In a linear array, each node has two neighbors, one to its left and one to its right
- If the nodes at either end are connected, we refer to it as a 1-D torus or a ring.

(a) (b)

Linear arrays: (a) with no wraparound links; (b) with wraparound link.

Meshes

- A generalization of linear array to 2 dimensions has nodes with 4 neighbors, to the north, south, east, and west.



(a) (b)

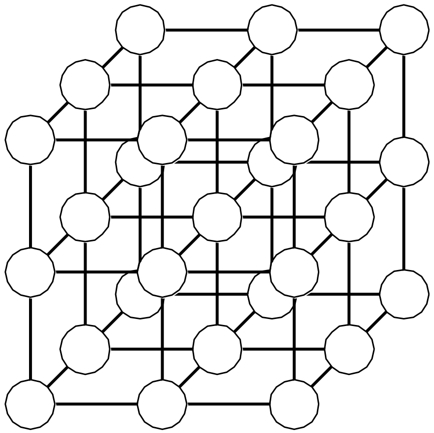
Two dimensional meshes: (a) 2-D mesh with no wraparound; (b) 2-D mesh with wraparound link (2-D torus)

Properties of 2-D Meshes

- Each dimension has sqrt(*p*) nodes
- Each node can be identified with a two-tuple (*i*, *j*)
- Every node (except those on the periphery) is connected to four neighbors whose indices differ in any dimension by one
  - 2-D torus: each node has exactly four neighbors
- Can be laid out in 2-D space, attractive from wiring standpoint
- Widely used, a variety of regularly structured computations can be mapped naturally to a 2-D mesh

Generalized Meshes

- A further generalization to *d* dimensions has nodes with *2d* neighbors (except nodes on the periphery)
- 3-D mesh

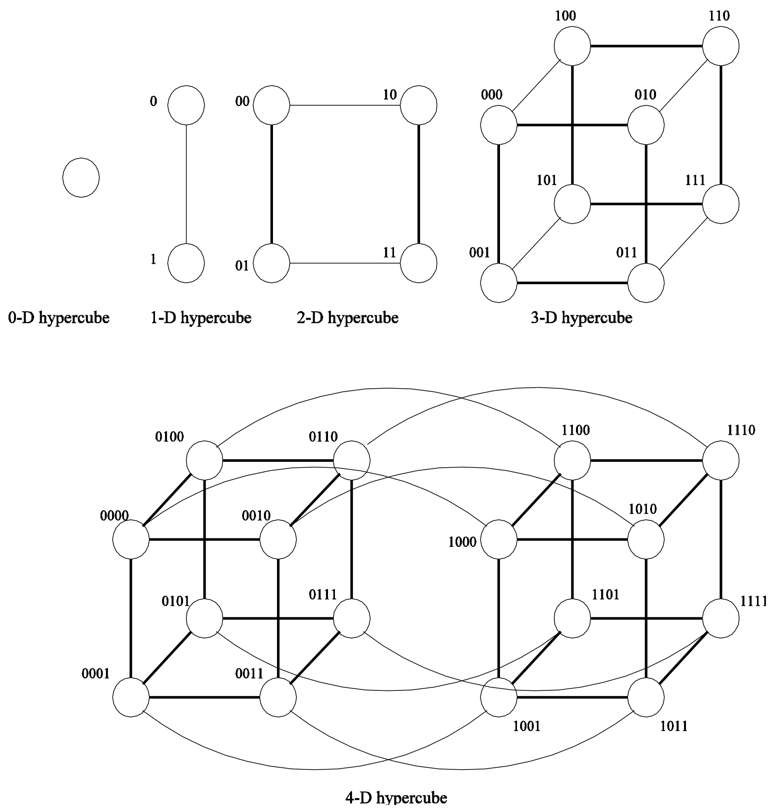


A 3-D mesh with no wraparound.

  - A variety of computations, e.g. physical simulations, can be mapped to 3-D topologies
    - 3-D weather modeling, structural modeling, etc.
    - Commonly used, e.g. Cray T3E, Jaguar/Titan machine at ORNL
- How is 3-D torus constructed?

Hypercubes and their Construction

- A special case of a *d*-dimensional mesh is a hypercube.
  - Two nodes along each dimension
  - With dimensions $d = \log p$, where p is the total number of nodes.



A *d*-dimensional hypercube is constructed by connecting corresponding nodes of two (*d*-1) dimensional hypercubes

Numbering mechanism can be used to tell the distance

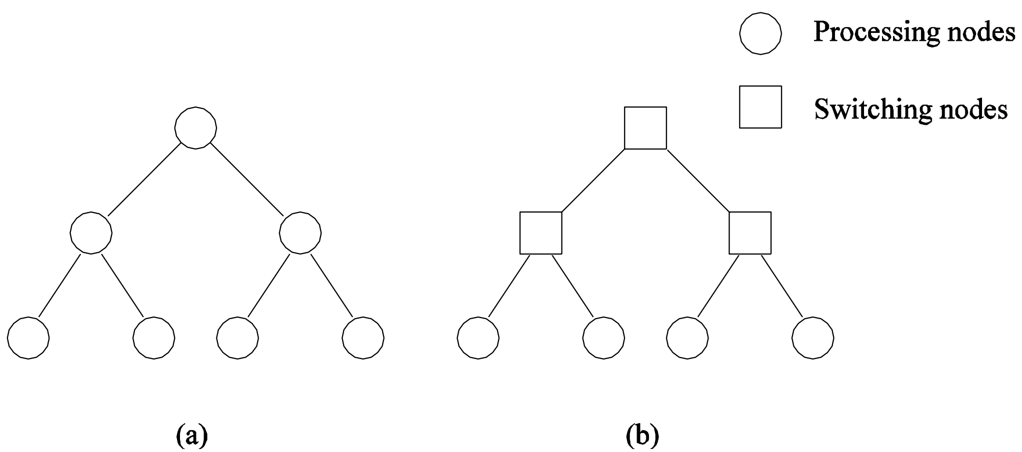
Construction of hypercubes from hypercubes of lower dimension.

Properties of Hypercubes

- The distance between any two nodes is at most *log p*.
- Each node has *log p* neighbors.
- The distance between two nodes is given by the number of bit positions at which the two nodes differ.

Tree-Based Networks

- Tree network is one in which only one path b.t. any pair of nodes
  - Linear arrays and star-connected networks are special cases
- Static tree network: processing element at each node
- Dynamic tree network: nodes at intermediate level are switching nodes
- How is a message routed?
  - Route up to the root of the smallest subtree then routes down

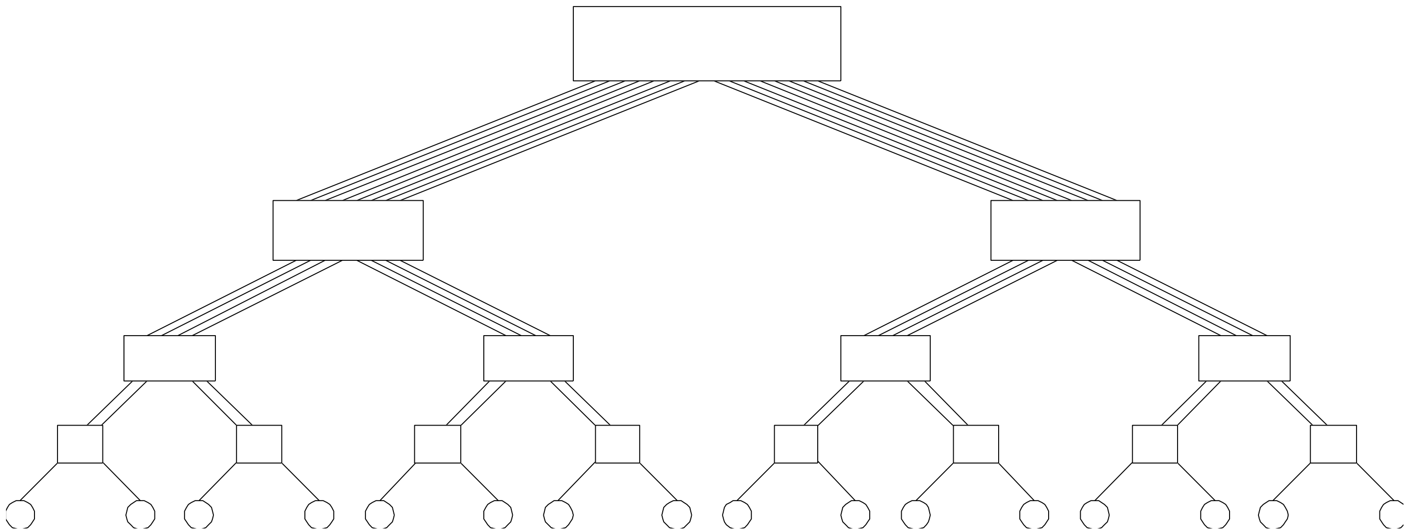


Complete binary tree networks: (a) a static tree network; and (b) a dynamic tree network.

Tree Properties

- The distance between any two nodes is no more than *2logp*.
- Links higher up the tree potentially carry more traffic than those at the lower levels.
- For this reason, a variant called a fat-tree, fattens the links as we go up the tree.
- Trees can be laid out in 2D with no wire crossings. This is an attractive property of trees.

Fat Trees



A fat tree network of 16 processing nodes.

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Diameter

- Diameter: the maximum distance between any two nodes
  - The distance between two processing nodes is defined as the shortest path (in terms of number of links) between them

- The diameter of a linear array is?
  - $p - 1$
- Mesh?
  - $2(\sqrt{p} - 1)$
- Tree (complete binary tree)?
  - $2 \log ((p+1)/2)$ or $2 (\log(p+1)-1)$
- Hypercube?
  - $\log p$
- Completely connected network?
  - 1

Arc Connectivity

- Connectivity: a measure of the multiplicity of paths between any two processing nodes
  - A network with high connectivity is desirable, because it lowers contention for communication resources

- Arc connectivity: One measure of connectivity is the minimum number of arcs that must be removed from the network to break it into two disconnected networks

Arc Connectivity

- Linear array?
  - 1
- Ring?
  - 2
- Mesh?
  - 2
- 2-D torus?
  - 4
- Tree?
  - 1
- Hypercube?
  - *log p*
- Completely connected network?
  - *p-1*

Bisection Width

- Bisection Width: The minimum number of links you must cut to divide the network into two equal parts

- The bisection width of a linear array and ring, respectively?
  - 1, 2
- Mesh, 2D-torus?
  - $\sqrt{p}$ $2\sqrt{p}$
- Tree?
  - 1
- Hypercube?
  - $p/2$
- Completely connected network?
  - $p^2/4$

Cost

- Cost: many criteria can be used to evaluate the cost of a network
- One way of defining the cost of a network is in terms of the number of communication links or the number of wires required by the network
- However, a number of other factors, such as the ability to layout the network, the length of wires, etc., also factor in to the cost.

Cost (No. of links)

- Linear array, ring?
  - *p-1, p*
- 2-D mesh, 2-D torus
  - $2(p - \sqrt{p})$ *2p*
- Tree (complete binary tree)?
  - *p-1*
- Hypercube?
  - *(p log p)/2*
- Completely connected network?
  - *p(p-1)/2*

Evaluating Network Topologies

Network	Diameter	Bisection Width	Arc Connectivity	Cost (No. of links)
Completely-connected	1	$p^2/4$	$p-1$	$p(p-1)/2$
Star	2	1	1	$p-1$
Complete binary tree	$2\log((p+1)/2)$	1	1	$p-1$
Linear array	$p-1$	1	1	$p-1$
2-D mesh, no wraparound	$2(\sqrt{p}-1)$	$\sqrt{p}$	2	$2(p-\sqrt{p})$
2-D wraparound mesh	$2\lfloor\sqrt{p}/2\rfloor$	$2\sqrt{p}$	4	$2p$
Hypercube	$\log p$	$p/2$	$\log p$	$(p\log p)/2$
Wraparound *k*-ary *d*-cube	$d\lfloor k/2\rfloor$	$2k^{d-1}$	$2d$	dp

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Readings

- Reference book ITPC, Chapter 2
  - 2.4
  - 2.8
- Foster, DBPP, 3.7
  - https://www.mcs.anl.gov/~itf/dbpp/text/node33.html#SECTION02472000000000000000

Questions?

Questions/Suggestions/Comments are always welcome!

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If you write me an email for this class, please start the email subject with [CS4379] or [CS5379].