1 Task 1

1.1

Finding the first order price sensitivity of Lower Bound of Arithmetic and Geometric Asian options

$$
\begin{aligned}
LB_n &= e^{-rT}\left(\frac{1}{n}\sum_{k=1}^{n} S_k - K\right)\mathbb{1}_{\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} > K} \\
G_n &= e^{-rT} max\left(\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} - K, 0\right) \\
g\left(S_i|S_{i-1}\right) &= \frac{1}{S_i\sigma\sqrt{t_i - t_{i-1}}}\Phi\left(\xi\left(S_i|S_{i-1}\right)\right) \\
\mu'_{LB,S_0} &= \mathbb{E}_{S_0}\left(e^{-rT}\left(\frac{1}{n}\sum_{k=1}^{n} S_k - K\right)\mathbb{1}_{\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} > K}\cdot\frac{\xi\left(S_i|S_{i-1}\right)}{S_0\sigma\sqrt{t_1}}\right) \\
&= \mathbb{E}_{S_0}\left(e^{-rT}\left(\frac{1}{n}\sum_{k=1}^{n} S_k - K\right)\mathbb{1}_{\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} > K}\cdot\frac{ln\frac{S_1}{S_0} - \left(r - \frac{1}{2}\sigma^2\right)\left(t_1 - t_0\right)}{\sigma\sqrt{t_1 - t_0}S_0\sigma\sqrt{t_1}}\right)
\end{aligned}
$$

$$
\mu'_{G,S_0} = \mathbb{E}_{S_0}\left(e^{-rT}\left(\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} - K\right)^{+}\cdot\frac{\xi\left(S_i|S_{i-1}\right)}{S_0\sigma\sqrt{t_1}}\right)
$$

No, as shown above, the lower bound payoff is discontinuous at $\left(\prod_{k=1}^{n} S_k\right)^{\frac{1}{n}} = K$*. Such that it is not possible to obtain the PW estimator. LR first order sensitivity estimator should be used instead.*

1.2

$$
\begin{aligned}
\frac{d\,\mathbb{E}\left(LB_n\right)}{dS_0} &= \frac{e^{-rT}}{n}\sum_{k=1}^{n} e^{\mu_k+\sigma_k^2}\cdot\mathcal{N}\left(b + a_k\right) + \frac{S_0e^{-rT}}{n}\sum_{k=1}^{n} e^{\mu_k+\sigma_k^2}\cdot\phi\left(b + a_k\right)\cdot\frac{1}{S_0}\frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&- Ke^{-rT}\cdot\phi\left(b\right)\cdot\frac{1}{S_0}\frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&= \frac{e^{-rT}}{n}\sum_{k=1}^{n} e^{\mu_k+\sigma_k^2}\cdot\mathcal{N}\left(b + a_k\right) + \frac{e^{-rT}}{n}\sum_{k=1}^{n} e^{\mu_k+\sigma_k^2}\cdot\phi\left(b + a_k\right)\cdot\frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&- Ke^{-rT}\cdot\phi\left(b\right)\cdot\frac{1}{S_0}\frac{1}{\hat{\sigma}\sqrt{\hat{T}}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dG_n}{dS_0} &= e^{\left(r-\frac{\sigma^2}{2}+\frac{\hat{\sigma}^2}{2}\right)\hat{T}-rT} \cdot \mathcal{N}\left(d\right) + S_0 e^{\left(r-\frac{\sigma^2}{2}+\frac{\hat{\sigma}^2}{2}\right)\hat{T}-rT} \phi\left(d\right) \frac{1}{S_0} \frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&- Ke^{-rT} \cdot \phi\left(d - \hat{\sigma}\sqrt{\hat{T}}\right) \cdot \frac{1}{S_0} \frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&= e^{\left(r-\frac{\sigma^2}{2}+\frac{\hat{\sigma}^2}{2}\right)\hat{T}-rT} \cdot \mathcal{N}\left(d\right) + e^{\left(r-\frac{\sigma^2}{2}+\frac{\hat{\sigma}^2}{2}\right)\hat{T}-rT} \phi\left(d\right) \frac{1}{\hat{\sigma}\sqrt{\hat{T}}} \\
&- Ke^{-rT} \cdot \phi\left(d - \hat{\sigma}\sqrt{\hat{T}}\right) \cdot \frac{1}{S_0} \frac{1}{\hat{\sigma}\sqrt{\hat{T}}}
\end{aligned}
$$