25/03/2020 TD 5

HLIN602

Logique 2

TD5

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Exo 1 :

1. $(\forall x. P(x)) \Rightarrow \exists y. P(y)$

$= \exists x. (P(x) \Rightarrow \exists y. P(y))$

$= \exists x. \exists y. (P(x) \Rightarrow P(y))$

2. $(\forall x. \exists y. R(x,y)) \Rightarrow \exists x. \forall y. R(x,y)$

$= (\forall x. \exists y. R(x,y)) \Rightarrow \exists a. \forall b. R(a,b)$

$= \exists x. ((\exists y. R(x,y)) \Rightarrow \exists a. \forall b. R(a,b))$

$= \exists x (\forall y. (R(x,y) \Rightarrow \exists a. \forall b. R(a,b)))$

$= \exists x. (\forall y. (\exists a. (R(x,y) \Rightarrow \forall b. R(a,b))))$

$= \exists x. (\forall y. (\exists a. (\forall b. (R(x,y) \Rightarrow R(a,b)))))$

3. $(\exists x. \forall y. R(x,y)) \Rightarrow \exists x. \forall y. R(x,y)$

$= \forall x. \exists y. R(x,y) \Rightarrow \exists z. \forall t. R(z,t)$

$= \forall x. \exists y. \exists z. \forall t. R(x,y) \Rightarrow R(z,t)$

4. $(P(x) \Rightarrow \forall x. Q(x)) \Rightarrow ((\exists x. P(x)) \Rightarrow \forall x. Q(x))$

$= (P(x) \Rightarrow \forall y. Q(y)) \Rightarrow ((\exists z. P(z)) \Rightarrow \forall t. Q(t))$

$= (P(x) \Rightarrow \forall y. Q(y)) \Rightarrow (\forall z. \forall t. P(z) \Rightarrow Q(t))$

$= (\forall y. P(x) \Rightarrow Q(y)) \Rightarrow (\forall z. \forall t. P(z) \Rightarrow Q(t))$

$= \exists y. P(x) \Rightarrow Q(y) \Rightarrow (\forall z. \forall t. P(z) \Rightarrow Q(t))$

$= \exists y. \forall z. \forall t. P(x) \Rightarrow Q(y) \Rightarrow P(z) \Rightarrow Q(t)$

5. $(\exists x. \forall y. (\exists z. S(x,y,z)) \wedge R(x,y)) \Rightarrow \exists y. (\forall x. S(x,y,z)) \wedge \exists x. R(x,y)$

$= (\exists x. \forall y. (\exists z. S(x,y,z)) \wedge R(x,y)) \Rightarrow \exists b. \forall a. S(a,b,z) \wedge \exists c. R(c,b)$

$= (\exists x. \forall y. (\exists z. S(x,y,z)) \wedge R(x,y)) \Rightarrow \exists b. \forall a. \exists c. S(a,b,z) \wedge R(c,b)$

$= \exists x. \forall y. \exists z. S(x,y,z) \wedge R(x,y) \Rightarrow \exists b. \forall a. \exists c. S(a,b,z) \wedge R(c,b)$

$= \exists x. \forall y. \exists z. \exists b. \forall a. \exists c. S(x,y,z) \wedge R(x,y) \Rightarrow S(a,b,z) \wedge R(c,b)$

Exo 2:

1. $((\forall x. \phi) \Rightarrow \phi') \Rightarrow (\exists x. \phi \Rightarrow \phi')$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\phi \vdash \phi, \phi'}\,ax}{\phi \vdash \forall x. \phi, \phi'}\,\forall r \qquad \dfrac{}{\phi', \phi \vdash \phi'}\,ax}{(\forall x. \phi) \Rightarrow \phi', \phi \vdash \phi'}\,\Rightarrow g}{(\forall x. \phi) \Rightarrow \phi' \vdash \phi \Rightarrow \phi'}\,\Rightarrow d}{(\forall x. \phi) \Rightarrow \phi' \vdash \exists x. \phi \Rightarrow \phi'}\,\exists d}{\vdash ((\forall x. \phi) \Rightarrow \phi') \Rightarrow (\exists x. \phi \Rightarrow \phi')}\,\Rightarrow d$$

2. $(\exists x. \phi \Rightarrow \phi') \Rightarrow ((\forall x. \phi) \Rightarrow \phi')$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\phi \vdash \phi, \phi'}\,ax \qquad \dfrac{}{\phi, \phi' \vdash \phi'}\,ax}{\phi \Rightarrow \phi', \phi \vdash \phi'}\,\Rightarrow l}{\phi \Rightarrow \phi', (\forall x). \phi \vdash \phi'}\,\forall l}{\phi \Rightarrow \phi' \vdash (\forall x. \phi) \Rightarrow \phi'}\,\Rightarrow r}{\exists x. \phi \Rightarrow \phi' \vdash (\forall x. \phi) \Rightarrow \phi'}\,\exists l}{\vdash (\exists x. \phi \Rightarrow \phi') \Rightarrow ((\forall x. \phi) \Rightarrow \phi')}\,\Rightarrow r$$

Exo 4:

1. $\forall x. P(x) \Rightarrow \exists y. \forall x. R(x,y)$

$s(\forall x. P(x) \Rightarrow \exists y. \forall z. R(z,y))$

$= s(P(x) \Rightarrow \exists y. \forall z. R(z,y))$

$= h(P(x)) \Rightarrow s(\exists y. \forall z. R(z,y))$

$= P(x) \Rightarrow s(\forall z. R(z,y))[c/y]$

$= P(x) \Rightarrow s(R(z,y))[c/y]$

$= P(x) \Rightarrow R(z,y)\,[c/y]$

$= \forall x. \forall z. P(x) \Rightarrow R(z,c)$

$S = \{\neg P(x) \vee R(z,c)\}$

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Exo 4 :

2. $(\exists x. \forall y. R(x,y)) \Rightarrow \forall y. \exists x. R(x,y)$

$s((\exists x. \forall y. R(x,y)) \Rightarrow \forall b. \exists a. R(a,b))$

$= h(\exists x. \forall y. R(x,y)) \Rightarrow s(\forall b. \exists a. R(a,b))$

$= h(\forall y. R(x,y)) \Rightarrow s(\exists a. R(a,b))$

$= h(R(x,y))[f(x)/y] \Rightarrow s(R(a,b))[g(b)/a]$

$= h(R(x,f(x))) \Rightarrow s(R(g(b),b))$

$= R(x,f(x)) \Rightarrow R(g(b),b)$

$S = \{\neg R(x,f(x)) \vee R(g(b),b)\}$

3. $((\exists x. P(x) \Rightarrow Q(x)) \vee \forall y. P(y)) \wedge \forall x. \exists y. Q(y) \Rightarrow P(x)$

$s(((\exists x. P(x) \Rightarrow Q(x)) \vee \forall y. P(y)) \wedge \forall a. \exists b. Q(b) \Rightarrow P(a))$

$= s(((\exists x. P(x) \Rightarrow Q(x)) \vee \forall y. P(y))) \wedge s(\forall a. \exists b. Q(b) \Rightarrow P(a))$

$= [s((\exists x. P(x) \Rightarrow Q(x)) \vee s(\forall y. P(y))] \wedge s(\exists b. Q(b) \Rightarrow P(a))$

$= [s(\exists x. P(x) \Rightarrow Q(x)) \vee s(P(y))] \wedge s(Q(b) \Rightarrow P(a))[k/b]$

$= [s(P(x) \Rightarrow Q(x))[c/x] \vee P(y)] \wedge s(Q(k) \Rightarrow P(a))$

$= [s(P(c) \Rightarrow Q(c)) \vee P(y)] \wedge h(Q(k)) \Rightarrow s(P(a))$

$= [s(P(c)) \Rightarrow s(Q(c)) \vee P(y)] \wedge Q(k) \Rightarrow P(a)$

$= [P(c) \Rightarrow Q(c) \vee P(y)] \wedge Q(k) \Rightarrow P(a)$

$[P(c) \Rightarrow Q(c) \vee P(y)] \wedge (Q(k) \Rightarrow P(a))$

$= [\neg P(c) \vee Q(c) \vee P(y)] \wedge (\neg Q(k) \vee P(a))$

$S = \{(\neg P(c) \vee Q(c) \vee P(y)), (\neg Q(k) \vee P(a))\}$

Exo 6 :

1. $F = \{g(f(x), f(y)), g(f(f(a)), f(z))\}$

$\{g(f(x), f(y)), g(f(f(a)), f(z))\}$ → décompose

$\{f(x) = f(f(a)), f(y) = f(z)\}$ → décompose

$\{x = f(a), y = z\}$

$mgu(F) = [f(a)/x, z/y]$

2. $F = \{h(x, f(a), x), h(h(a,b,y), f(y), h(a,b,a))\}$

$\{h(x, f(a), x), h(h(a,b,y), f(y), h(a,b,a))\}$ → décompose

$\{x = h(a,b,y), f(a) = f(y), x = h(a,b,a)\}$ → décompose

$\{x = h(a,b,y), a = y, x = h(a,b,a)\}$ → swap

$\{x = h(a,b,y), y = a, x = h(a,b,a)\}$ → eliminate

$\{x = h(a,b,a), y = a\}$

$mgu(F) = [h(a,b,a)/x, a/y]$

3. $F = \{g(y, f(f(a))), g(f(a), y)\}$

$\{g(y, f(f(x))), g(f(a), y)\}$ → décompose

$\{y = f(a), f(f(x)) = y\}$ → eliminate

$\{y = f(a), f(f(x)) = f(a)\}$ → decompose

$\{y = f(a), f(x) = a\}$ → eliminate

$\{y = a, f(x) = a\}$

$mgu(F) = [a/y, a/f(x)]$

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Exo 6 :

4. $F = \{h(a, x, f(x)), h(a, y, y)\}$

$\{h(a, x, f(x)), h(a, y, y)\}$ → decompose

$\{a = a, x = y, f(x) = y\}$ → delete

$\{x = y, f(x) = y\}$ → swap

$\{x = y, y = f(x)\}$ → eliminate

$\{x = y, \underline{y = f(y)}\}$ → check

$\perp$ car $y \in f(y)$

5. $F = \{g(x, g(y, z)), g(g(a, b), x), g(x, g(a, x))\}$

$\{g(x, g(y, z)), g(g(a, b), x), g(x, g(a, x))\}$ → decompose

$\{g(x, g(y, z)), g(a, b) = x, \underline{x} = g(a, \underline{x})\}$ → check

$\perp$