W1	Learning Area	MATHEMATICS	Grade Level	9
	Quarter	3^{RD}	Date	

I. LESSON TITLE	PARALLELOGRAMS
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	• Determines the conditions that make a quadrilateral a parallelogram. **M9GE-IIIa-2** • Uses properties to find measures of angles, sides and other quantities involving parallelograms. **M9GE-IIIb-1**
III. CONTENT/CORE CONTENT	**Parallelogram** – a quadrilateral with two pairs of parallel sides. Any two of its opposite sides are congruent. Any two of its opposite angles are congruent. Any two consecutive angles are supplementary. Its diagonals bisect each other. And its diagonal forms two congruent triangles.

IV. LEARNING PHASES	**Suggested Timeframe**	**Learning Activities**
A. Introduction	40 minutes	In this lesson, we shall focus on quadrilaterals that are parallelograms, their properties, and different theorems regarding relationships about its sides and angles. But first, we need to remember the Quadrilateral family tree. From this, we can see the relationships between all the quadrilaterals.

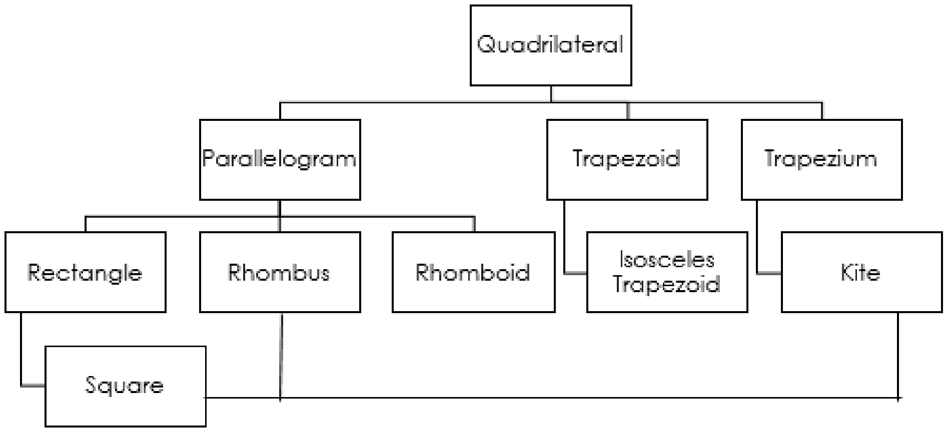
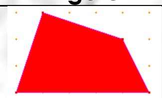
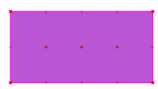
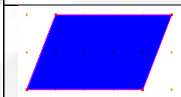
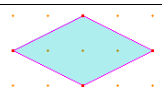
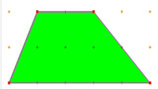
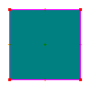
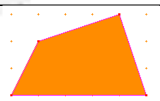
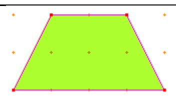
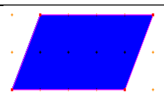
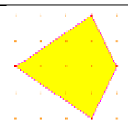
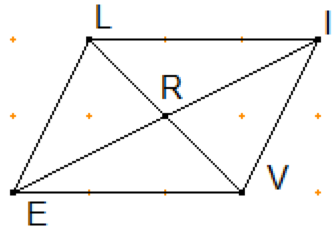


Figure	Description	Figure	Description
	**Quadrilaterals** -Four-sided polygon and sum of all angles is equal to 360°		**Rectangles** -All angles are equal
	**Parallelograms** -2 pairs of parallel sides		**Rhombus** -All sides are equal
	**Trapezoids** -1 pair of parallel sides		**Squares** -All sides and all angles are equal
	**Trapeziums** -No pair of parallel sides -same properties as quadrilaterals		**Isosceles Trapezoids** -a pair of legs are congruent
	**Rhomboids** -same properties as parallelograms		**Kite** -2 pairs of non-opposite congruent sides

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities
		We need to define a parallelogram. A parallelogram is a quadrilateral with two pairs of parallel sides. When naming quadrilaterals like this parallelogram in the right, we can call this Parallelogram LIVE or Parallelogram LEVI, and **NOT** Parallelogram LVIE or Parallelogram IEVL. They need to be named either clockwise or counterclockwise starting at any vertex sequentially.
B. Development	60 minutes	See below



Properties of Parallelogram

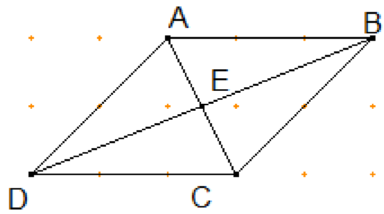
- In a parallelogram, any two opposite sides are congruent.
  $\overline{LE} \cong \overline{IV}$ and $\overline{LI} \cong \overline{EV}$
- In a parallelogram, any two opposite angles are congruent.
  $\angle L \cong \angle V$ and $\angle I \cong \angle E$
- In a parallelogram, any two consecutive angles are supplementary
  $m\angle L + m\angle I = 180^o$, $m\angle L + m\angle E = 180^o$, $m\angle V + m\angle I = 180^o$, and $m\angle V + m\angle E = 180^o$
- The diagonals of a parallelogram bisect each other.
  $\overline{LR} \cong \overline{VR}$ and $\overline{ER} \cong \overline{IR}$
- A diagonal of a parallelogram forms two congruent triangles.
  $\Delta LEV \cong \Delta VIL$ and $\Delta ELI \cong \Delta IVE$

Conditions for Parallelograms

- If one pair of opposite sides of a quadrilateral are parallel and congruent, then that quadrilateral is a parallelogram
  If $\overline{LE} \cong \overline{IV}$ and $\overline{LE} // \overline{IV}$, then Quadrilateral LIVE is a parallelogram.

Solving Problems on the Properties of Parallelograms

Below is a parallelogram ABCD. Consider each given information and answer the questions that follow



1. Given m$\overline{AB}$ = (3x – 5) cm, m$\overline{BC}$ = (2y – 7) cm, m$\overline{CD}$ = (x + 7) cm and m$\overline{AD}$ = (y + 3) cm.
   a. How long is m$\overline{AB}$?

$\overline{AB} \cong \overline{CD}$	In a parallelogram, any two opposite sides are congruent.
3x – 5 = x + 7	Substitution Property
3x – x – 5 + 5 = x – x + 7 + 5	Addition Property of Equality
$\frac{2x}{2} = \frac{12}{2}$	Multiplication Property of Equality
x = 6	Division Property
m$\overline{AB}$ = (3x – 5) cm	Given
m$\overline{AB}$ = (3(6) – 5) cm	Substitution Property
m$\overline{AB}$ = (18 – 5) cm	Multiplication Property
m$\overline{AB}$ = 13 cm	Subtraction Property

   b. How long is m$\overline{AD}$?

$\overline{BC} \cong \overline{AD}$	In a parallelogram, any two opposite sides are congruent.
2y – 7 = y + 3	Substitution Property
2y – y – 7 + 7 = y – y + 3 + 7	Addition Property of Equality
y = 10	Subtraction and Addition Property
m$\overline{AD}$ = (y + 3) cm	Given
m$\overline{AD}$ = (10 + 3) cm	Substitution Property
m$\overline{AD}$ = 13 cm	Addition Property

   c. What is the perimeter of Parallelogram ABCD?

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities

Perimeter = $\overline{AB}$ + $\overline{BC}$ + $\overline{CD}$ + $\overline{AD}$	Given
Perimeter = (3x – 5) + (2y – 7) +(x +7) + (y+3) cm	Substitution Property
Perimeter = (3(6) – 5) + (2(10) – 7) +((6) +7) + ((10+3) cm	Substitution Property
Perimeter = 52 cm	Addition Property

2. $\angle BAD$ measures $(2a + 25)^0$ while $\angle BCD$ measures $(3a - 15)^0$.
   a. What is the value of a?

$\angle BAD \cong \angle BCD$	In a parallelogram, any two opposite angles are congruent.
$(2a + 25)^0 = (3a - 15)^0$	Substitution Property
$(2a - 2a + 25 + 15)^0 = (3a - 2a - 15 + 15)^0$	Addition Property of Equality
$a = 40^0$	Subtraction and Addition Property

   b. What is m$\angle BAD$?

m$\angle BAD$ = $(2a + 25)^0$	Given
m$\angle BAD$ = $(2(40) + 25)^0$	Substitution Property
m$\angle BAD$ = $(80 + 25)^0$	Multiplication Property
m$\angle BAD$ = 105^0	Addition Property

   c. What is m$\angle CBA$?

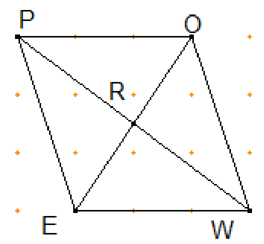
m$\angle CBA$ = 180^0 – m$\angle BAD$	In a parallelogram, any two consecutive angles are supplementary
m$\angle CBA$ = $180^0 - 105^0$	Substitution Property
m$\angle CBA$ = 75^0	Subtraction Property

3. Diagonals AC and BD meet at E. DE is 8 cm and AC is 3 cm.
   a. How long is BD? 16 cm since in a parallelogram, the diagonals bisect each other
   b. How long is AE? 1.5 cm since in a parallelogram, the diagonals bisect each other

C. Engagement — 60 minutes

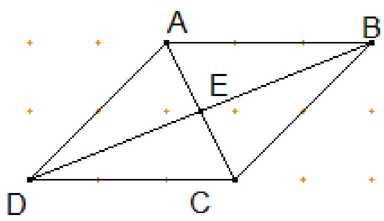
Learning Activity 1: Directions: Refer to the figure on the right and answer the following:
Given: Parallelogram POWE

1. $\overline{PO} \cong$_____
2. $\angle O \cong$_____
3. $m\angle W + m\angle E =$_____
4. $\overline{PR} \cong$_____
5. $\Delta OPE \cong$_____



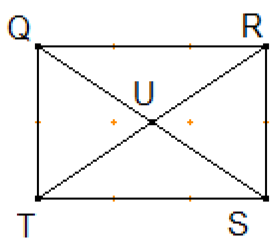
Learning Activity 2: Directions: Refer to the figure on the right and answer the following:
Given: Parallelogram ABCD

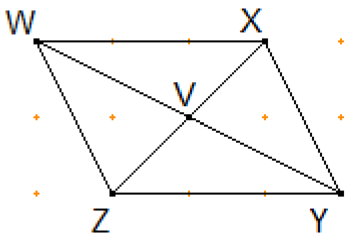
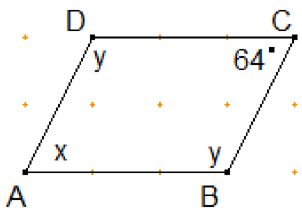
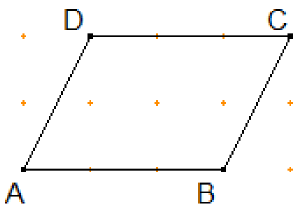
1. If m$\overline{AB}$ = 13 cm, then m$\overline{DC}$ = _____
2. If m$\angle B$ = 42°, then m$\angle D$ = _____
3. If m$\angle B$ = 58°, then m$\angle C$ = _____
4. If m$\overline{AE}$ = 7 cm, then m$\overline{CE}$ = _____
5. If m$\overline{BD}$ = 24 cm, then m$\overline{BE}$ = _____



Learning Activity 3: Directions: Refer to the figure on the right and answer the following:
Given: Parallelogram QRST

1. If m$\overline{QR}$ = 3x – 5 cm and m$\overline{TS}$ = 2x + 5 cm, then what is m$\overline{QR}$ and m$\overline{TS}$?
2. If m$\angle Q$ = (6x + 12)° and m$\angle S$ = (7x – 1)°, then what is m$\angle Q$ and m$\angle S$?
3. If m$\angle T$ = (8x + 11)° and m$\angle S$ = (3x + 4)°, then what is m$\angle T$ and m$\angle S$?



IV. LEARNING PHASES	Suggested Timeframe	Learning Activities
		4. If $m\overline{QU}$ = 2x – 8 cm and $m\overline{US}$ = x + 3 cm, then what is $m\overline{QS}$? 5. If $\overline{RU}$ = 5x – 6 cm, and $\overline{RT}$ = 6x cm, then what is $m\overline{UT}$?
D. Assimilation	20 minutes	Directions: Refer to the figure on the right and answer the following: Given: Parallelogram WXYZ 1. If $m\overline{WX}$ = 6x cm, $m\overline{XY}$ = 5y – 4 cm, $m\overline{YZ}$ = x + 35 cm and $m\overline{WZ}$ = y + 17 cm, then what is the perimeter of parallelogram WXYZ? 2. If $m\angle XWY$ = (3x + 11)° and $m\angle ZYW$ = (7x – 1)°, when $m\angle ZWY$ = (2y + 2)° and $m\angle XYW$ = (3x – 2)°, then what is $m\angle W$ and $m\angle Y$? 
V. ASSESSMENT (Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6)	20 minutes	For numbers 1 – 5, choose the letter of the best answer. 1. Which of the following is sufficient to guarantee that a quadrilateral is a parallelogram? A. The diagonals are perpendicular B. Two consecutive angles are congruent C. Pair of adjacent sides are congruent D. The diagonals bisect each other 2. What values of x and y guarantee that Quadrilateral ABCD is a parallelogram? A. x = 64°, y = 116° B. x = 32°, y = 116° C. x = 64°, y = 64° D. x = 32°, y = 64°  3. In the same figure above, If AD = 2x – 10 cm and BC = x + 30 cm, then BC = A. 50 cm B. 60 cm C. 70 cm D. 80 cm 4. Quadrilateral ABCD is a parallelogram. If $m\angle B$ = (x + 40)° and $m\angle D$ = (2x + 20)°, what is $m\angle B$? A. 50° B. 60° C. 70° D. 80° 5. Quadrilateral ABCD is a parallelogram. If $m\angle A$ = (3x – 10)° and $m\angle D$ = (2x + 40)°, what is $m\angle A$? A. 50° B. 60° C. 70° D. 80° 
VI. REFLECTION	20 minutes	• The learner communicates the explanation of their personal assessment as indicated in the Learner's Assessment Card. • The learner, in their notebook, will write their personal insights about the lesson using the prompts below. I understand that __________________. I realize that ________________________. I need to learn more about ___________.

Prepared by:	Wilson Ray G. Anzures	**Checked by:**	Ma. Filipina M. Drio/ Reymark R. Queaño

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

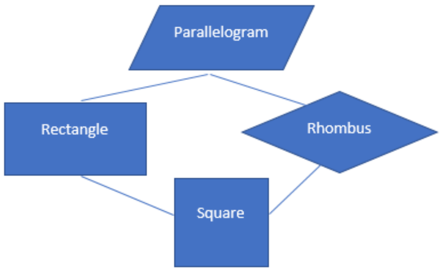
✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

W2	Learning Area	MATHEMATICS	Grade Level	9
	Quarter	3RD	Date	

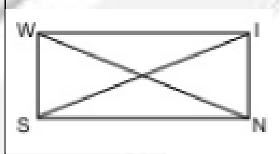
I. LESSON TITLE	RECTANGLES, RHOMBI and SQUARES
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	• Proves theorems on the different kinds of parallelogram (rectangle, rhombus, square). **M9GE-IIIc-1**
III. CONTENT/CORE CONTENT	**Rectangle –** an equiangular parallelogram. All angles are right angles, and its diagonals are congruent. **Rhombus –** an equilateral parallelogram. All sides are equal. Its diagonals are perpendicular. Each diagonal bisects opposite angles. **Square -** an equiangular and equilateral quadrilateral. All the properties of parallelogram, rectangle and rhombus

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities
A. Introduction	10 minutes	In this lesson, we shall focus on parallelograms that are rectangle, rhombus, and squares, their properties, and different theorems regarding relationships about their sides and angles. We need to remember is that rectangles, rhombi, and squares have all the properties of a parallelogram that have been discussed in the previous activity sheets. 
B. Development	60 minutes	see below

Properties of Rectangle

- All the properties of Parallelogram
- If a parallelogram has one right angle, then it has four right angles. and the parallelogram is a rectangle.
- The diagonals of a rectangle are congruent.

Given: Rectangle WINS with diagonals $\overline{WN}$ and $\overline{SI}$
Prove: $\overline{WN} \cong \overline{SI}$



Statements	Reasons
1. Rectangle WINS with diagonals $\overline{WN}$ and $\overline{SI}$	Given
2. $\overline{WS} \cong \overline{IN}$	In a parallelogram, any two opposite sides are congruent.
3. $\angle WSN$ and $\angle INS$ are right angles	If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.
4. $\angle S \cong \angle N$	All right angles are congruent.
5. $\overline{SN} \cong \overline{NS}$	Reflexive Property
6. $\Delta WSN \cong \Delta INS$	SAS Postulate
7. $\overline{WN} \cong \overline{SI}$	CPCTC

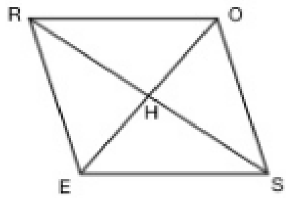
Properties of Rhombus

- All the properties of Parallelogram
- Diagonals of a rhombus are perpendicular.
- Each diagonal of a rhombus bisects opposite angles.

Given: Rhombus ROSE with diagonals $\overline{RS}$ and $\overline{OE}$

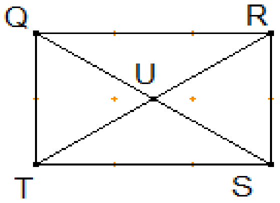
Statements	Reasons
1. Rhombus ROSE diagonals $\overline{RS}$ and $\overline{OE}$	Given

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities

Prove: $\overline{RS} \perp \overline{OE}$ 	2. $\overline{OS} \cong \overline{RO}$	Definition of a Rhombus
	3. $\overline{RS}$ and $\overline{OE}$ bisect each other	The diagonals of a parallelogram bisect each other.
	4. H is the midpoint of $\overline{RS}$ and $\overline{OE}$	Definition of a midpoint
	5. $\overline{OH} \cong \overline{OH}$	Reflexive Property
	6. $\Delta ROH \cong \Delta SOH$	SSS Postulate
	7. $\angle RHO \cong \angle SHO$	CPCTC
	8. $\angle RHO$ and $\angle SHO$ are right angles	$\angle RHO$ and $\angle SHO$ form a linear pair and are congruent
	9. $\overline{RS} \perp \overline{OE}$	Perpendicular lines meet to form a right angle

Properties of Square

- All the properties of Parallelogram
- All the properties of Rectangle
- All the properties of Rhombus



On the right is a rectangle QRST. Consider each given information and answer the questions that follow.

1. If m$\angle QRT$ = 25°, find m$\angle TRS$, m$\angle QSR$, and m$\angle RTS$.

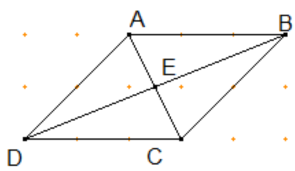
m$\angle QRT$ = 25⁰	Given
m$\angle QRS$ = 90⁰	Definition of a Rectangle
m$\angle QRS$ = m$\angle QRT$ + m$\angle TRS$	Angle Addition Postulate
90⁰ = 25⁰ + m$\angle TRS$	Substitution
90⁰ – 25⁰ = 25⁰ – 25⁰+ m$\angle TRS$	Addition Property of Equality
m$\angle TRS$ = 65⁰	Subtraction Property
$\overline{QS} \cong \overline{RT}$	The diagonals of a rectangle are congruent
$\overline{US} \cong \overline{UR}$	The diagonals of a parallelogram bisect each other
$\angle URS \cong \angle USR$	Converse of Isosceles Triangle Theorem
m$\angle TRS$ = 65⁰	Base angles of an isosceles triangles are congruent.
$\angle QRT \cong \angle RTS$	Alternate Interior Angle Theorem
m$\angle RTS$ = 65⁰	Substitution

2. If m$\overline{QS}$ = 5x – 14 cm, and m$\overline{RT}$ = 4x + 6 cm, then what is x, m$\overline{QS}$ and m$\overline{RT}$?

$\overline{QS} \cong \overline{RT}$	The diagonals of a rectangle are congruent
5x – 14 = 4x + 6 cm	Substitution Property
5x – 4x – 14 + 14 = 4x – 4x + 6 + 14	Addition Property of Equality
x = 20	Division Property
m$\overline{QS}$ = (5x – 14) cm	Given
m$\overline{QS}$ = (5(20) – 14) cm	Substitution Property
m$\overline{QS}$ = (100 – 14) cm	Multiplication Property
m$\overline{QS}$ and m$\overline{RT}$ = 86 cm	Subtraction Property

On the right is a rhombus ABCD. Consider each given information and answer the questions that follow

3. If m$\angle ABD$ = 25°, find m$\angle DBC$, m$\angle BCD$, and m$\angle BCA$.



IV. LEARNING PHASES	Suggested Timeframe	Learning Activities

m$\angle ABD$ = 25^0	Given
$\angle ABD \cong \angle CBD$	Each diagonal of a rhombus bisects opposite angles
m$\angle DBC$ = 25^0	Substitution
m$\angle ABC$ = m$\angle ABD$ + m$\angle CBD$	Angle Addition Postulate
m$\angle ABC$ = 25^0 + 25^0	Addition Property of Equality
m$\angle ABC$ = 50^0	Addition Property
m$\angle ABC$ + m$\angle BCD$ = 180°	In a parallelogram, any two consecutive angles are supplementary
50^0 + m$\angle BCD$ = 180°	Substitution
m$\angle BCD$ = 130^0	Transposition
m$\angle BCD$ = 65^0	Each diagonal of a rhombus bisects opposite angles

4. If m$\overline{AE}$ = x + 2 cm, m$\overline{BE}$ = 4x + 4 cm, and m$\overline{AB}$ = 5x cm then what is x, m$\overline{AE}$, m$\overline{BE}$ and m$\overline{AB}$?

(m$\overline{AB}$)^2 = (m$\overline{AE}$)^2 + (m$\overline{BE}$)^2	Diagonals of a rhombus are perpendicular
$(5x)^2 = (x + 2)^2 + (4x + 4)^2$	Substitution Property
$25x^2 = x^2 + 4x + 4 + 16x^2 + 32x + 16$	Square of a binomial
$25x^2 = 17x^2 + 36x + 20$	Combining like terms
$25x^2 – 17x^2 – 36x – 20 = 17x^2 – 17x^2 + 36x – 36x + 20 – 20$	Addition Property of Equality
$8x^2 – 36x – 20 = 0$	Subtraction Property
$4(x – 5)(2x + 1) = 0$	Factoring
x = 5	Zero product property (x = -1/2 is not considered)
m$\overline{AE}$ = 7 cm, m$\overline{BE}$ = 24 cm and m$\overline{AB}$ = 25 cm	Substitution Property

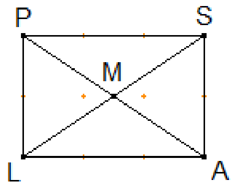
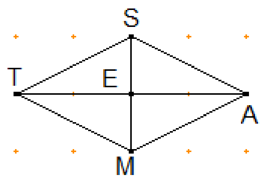
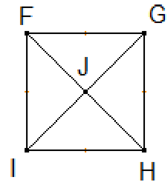
C. Engagement

60 minutes

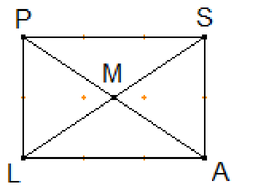
Learning Activity 1: Direction: Choosing only among rectangle, rhombus, and square, name all parallelograms that have the following property.
1. All sides and angles are congruent.
2. All sides are congruent.
3. Diagonals are equal.
4. Diagonals are perpendicular.
5. Opposite sides are congruent and parallel.

Learning Activity 2: Direction: Determine if the statement is Always true, Sometimes True, or Never True.
1. A square is an equiangular rhombus.
2. A rectangle is a rhombus.
3. All rhombi are squares.
4. All rectangles are parallelograms.
5. All parallelograms are squares.

Learning Activity 3: Direction: Find the measure of the unknown angles and sides of the given parallelogram as shown in the figure below. Show your solution.

		
Rectangle PSAL has a diagonal m$\overline{PA}$ = 16 cm. Find: 1. m$\overline{PM}$ = 2. m$\overline{SL}$ =	Rhombus SAMT with m$\angle STM$ = 60°. Find: 3. m$\angle MTA$ = 4. m$\angle TSM$ =	Square FGHI Find: 5. m$\angle GJH$ =

<table>
<tr><th>IV. LEARNING PHASES</th><th>Suggested Timeframe</th><th>Learning Activities</th></tr>
<tr><td>D. Assimilation</td><td>30 minutes</td><td>
Direction: Find the measure of the unknown angles and sides of the given parallelogram as shown in the figure below. Show your solution.
<table>
<tr><td>



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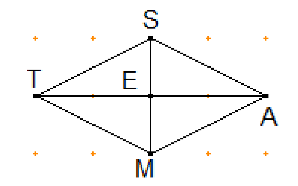

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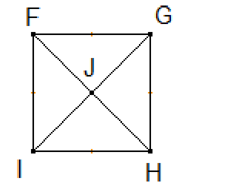
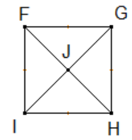
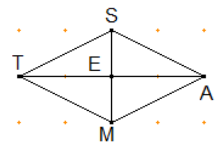


</td></tr>
<tr><td>Rectangle PSAL has a diagonal m$\overline{PA}$ = 5x – 14 cm and m$\overline{LS}$ = 4x + 6 cm. Find:
1. m$\overline{PM}$ =
2. m$\overline{SL}$ =</td><td>Rhombus SAMT with m$\angle STM$ = (3x + 11)$^{\circ}$. Find:
3. m$\angle MTA$ =
4. m$\angle TSM$ =</td><td>Square FGHI
Find:
5. m$\angle JFG$ =</td></tr>
</table>
</td></tr>
<tr><td>V. ASSESSMENT
(Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6)</td><td>30 minutes</td><td>
Directions: Read each of the following carefully. Choose the letter that corresponds to the correct answer.

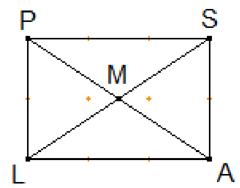


1. Square FGHI. If m∠FJG = (5x + 10)$^{\circ}$, find the value of x.
A. 7 B. 8 C. 16 D. 20



2. In rhombus SAMT, what is the measure of $\angle ASM$, if m∠SAT = 35°?
A. 35º B. 55º C. 70º D. 110º

3. What is the measure of m∠STM?
A. 35º B. 55º C. 70º D. 110º



4. In a rectangle PSAL, the length of diagonal m$\overline{PA}$ = 30 cm. Find the length of side m$\overline{SM}$.
A. 15 cm B. 20 cm C. 25cm D. 30cm

5. If m∠PLS = 60º, what is m∠ALS?
A. 30º B. 45º C. 60º D. 65º
</td></tr>
<tr><td>VI. REFLECTION</td><td>20 minutes</td><td>
• The learner communicates the explanation of their personal assessment as indicated in the Learner's Assessment Card.
• The learner, in their notebook, will write their personal insights about the lesson using the prompts below.
I understand that ________________.
I realize that ____________________.
I need to learn more about __________.
</td></tr>
</table>

Prepared by:	Wilson Ray G. Anzures	**Checked by:**	Ma. Filipina M. Drio/ Reymark R. Queño

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

W3	Learning Area	MATHEMATICS	Grade Level	9
	Quarter	3RD	Date	

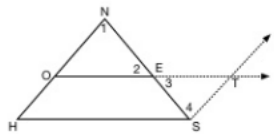
I. LESSON TITLE	MIDLINE THEOREM, TRAPEZOIDS and KITES
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	• proves the Midline Theorem. **M9GE-IIId-1** • proves theorems on trapezoids and kites. **M9GE-IIId-2**
III. CONTENT/CORE CONTENT	**Midline Theorem** – The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long. **Trapezoid** – a quadrilateral with one pair of parallel sides. Where the median of a trapezoid is parallel to each base and its length is one-half the sum of the lengths of the bases. **Isosceles Trapezoid** – a trapezoid with a pair of legs that are congruent. The base angles are congruent. Opposite angles are supplementary. Diagonals are congruent. **Kite** – a quadrilateral with two pairs of adjacent sides that are congruent, a rhombus is a special kind of kite. The perpendicular bisector of at least one is the other diagonal. The area is half the product of the lengths of its diagonals.

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities
A. Introduction	5 minutes	In this lesson, we shall focus on other forms of quadrilaterals, but before we start the discussion, we will talk about the proof of the triangle midline theorem. It will be essential in discussing the median of a trapezoid and its proof. And then, we will look at the proofs of different special quadrilaterals and their properties.
B. Development	60 minutes	(see below)

Midline Theorem – The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long.

Given: ΔHNS, O is the midpoint of $\overline{HN}$ and E is the midpoint of $\overline{NS}$

Prove: $\overline{OE} // \overline{HS}$ and $\overline{OE} = \frac{1}{2}\overline{HS}$

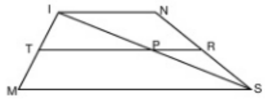


Statements	Reasons
1. ΔHNS, O is the midpoint of $\overline{HN}$ and E is the midpoint of $\overline{NS}$	Given
2. In $\overrightarrow{OE}$, there is a point T such that $\overline{OE} \cong \overline{ET}$	Line Postulate
3. $\overline{NE} \cong \overline{ES}$	Definition of a midpoint
4. $\angle 2 \cong \angle 3$	Vertical Angles Theorem
5. $\Delta NEO \cong \Delta SET$	SAS Postulate
6. $\angle 1 \cong \angle 4$	CPCTC
7. $\overline{HN} // \overline{ST}$	Converse of Alternate Interior Angles Theorem
8. $\overline{OH} \cong \overline{ON}$	Definition of a midpoint
9. $\overline{ON} \cong \overline{TS}$	CPCTC
10. $\overline{OH} \cong \overline{TS}$	Transitive Property
11. Quadrilateral HOTS is a parallelogram	If opposite sides of a quadrilateral are congruent and parallel, then it is a parallelogram.
12. $\overline{OE} // \overline{HS}$	Definition of a parallelogram
13. $\overline{OE} + \overline{ET} = \overline{OT}$	Segment Addition Postulate
14. $\overline{OE} + \overline{OE} = \overline{OT}$	Substitution Property
15. $2\overline{OE} = \overline{OT}$	Addition Property
16. $\overline{HS} \cong \overline{OT}$	In a parallelogram, any two opposite sides are congruent.
17. $2\overline{OE} = \overline{HS}$	Substitution Property
18. $\overline{OE} = \frac{1}{2}\overline{HS}$	Divide both sides by two

Definition of Trapezoid – a quadrilateral with one pair of parallel sides.

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities

Midsegment Theorem of Trapezoid – The median of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

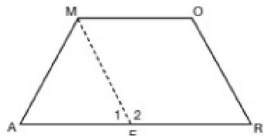


Given: Trapezoid MINS with median $\overline{TR}$ that intersects the diagonal $\overline{IS}$ at P.
Prove: $\overline{MS}//\overline{TR}//\overline{IN}$ and $\overline{TR} = \frac{1}{2}(\overline{MS} + \overline{IN})$

Statements	Reasons
1. Trapezoid MINS with median $\overline{TR}$ that intersects the diagonal $\overline{IS}$ at P.	Given
2. $\overline{TP} + \overline{PR} = \overline{TR}$	Segment Addition Postulate
3. $\overline{TP}//\overline{MS}$ and $\overline{TP} = \frac{1}{2}\overline{MS}$ $\overline{RP}//\overline{IN}$ and $\overline{RP} = \frac{1}{2}\overline{IN}$	Midline Theorem
4. $\overline{MS}//\overline{IN}$	Definition of Trapezoid
5. $\overline{MS}//\overline{TP} + \overline{RP}//\overline{IN}$	Transitive Property
6. $\overline{MS}//\overline{TR}//\overline{IN}$	Addition Property
7. $\overline{TR} = \frac{1}{2}\overline{MS} + \frac{1}{2}\overline{IN}$	Substitution Property
8. $\overline{TR} = \frac{1}{2}(\overline{MS} + \overline{IN})$	Factoring

Definition of Isosceles Trapezoid – a trapezoid with opposite sides that are congruent.

Properties of Isosceles Trapezoid



- The base angles of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid AMOR with $\overline{MO}//\overline{AR}$
Prove: $\angle A \cong \angle R$ and $\angle AMO \cong \angle O$

Statements	Reasons
1. Isosceles Trapezoid AMOR with $\overline{MO}//\overline{AR}$.	Given
2. $\overline{AM} \cong \overline{OR}$	Definition of Isosceles Trapezoid
3. From M, draw $\overline{ME}//\overline{OR}$ where E lies on $\overline{AR}$	Two points determine a line
4. Quadrilateral MORE is a parallelogram	Definition of Parallelogram
5. $\overline{ME} \cong \overline{OR}$	In a parallelogram, any two opposite sides are congruent.
6. $\overline{MA} \cong \overline{ME}$	Transitive Property
7. ΔAME is an isosceles triangle	Definition of Isosceles Triangle
8. $\angle A \cong \angle 1$	Base angles of an isosceles triangles are congruent.
9. $\angle 1 \cong \angle R$	Corresponding Angles Theorem
10. $\angle A \cong \angle R$	Transitive Property
11. $\angle A$ and $\angle AMO$ are supplementary angles. $\angle O$ and $\angle R$ are supplementary angles.	Same Side Interior Angle Theorem
12. $\angle AMO \cong \angle O$	Supplements of congruent angles are also congruent

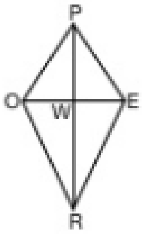
- Opposite angles of an isosceles trapezoid are supplementary.
- Diagonals of an isosceles trapezoid are congruent.

Definition of Kite – a quadrilateral with two pairs of adjacent sides that are congruent, a rhombus is a special kind of kite.

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities

Properties of Kite

- In a kite, the perpendicular bisector of at least one is the other diagonal.
- The area of a kite is half the product of the lengths of its diagonals.



Given: Kite ROPE with diagonals $\overline{PR}$ and $\overline{OE}$ intersect at point W.

Prove: Area of kite ROPE = $\frac{1}{2}(\overline{UE})(\overline{PR})$.

Statements	Reasons
1. Kite ROPE with diagonals $\overline{PR}$ and $\overline{OE}$ intersect at point W.	Given
2. $\overline{PR} \perp \overline{OE}$	Diagonals of a kite are perpendicular to each other.
3. Area of kite ROPE = Area of ΔOPE + Area of ΔORE	Area Addition Postulate
4. Area of $\Delta OPE = \frac{1}{2}(\overline{OE})(\overline{PW})$ and Area of $\Delta ORE = \frac{1}{2}(\overline{OE})(\overline{WR})$	Formula for Area of Triangles
5. Area of kite ROPE = $=\frac{1}{2}(\overline{OE})(\overline{PW} + \overline{WR})$	Factoring
6. $\overline{PW} + \overline{WR} = \overline{PR}$	Segment Addition Postulate
7. Area of Kite ROPE = $\frac{1}{2}(\overline{UE})(\overline{PR})$	Substitution
8. $\overline{CX} \cong \overline{TX}$	CPCTC

C. Engagement

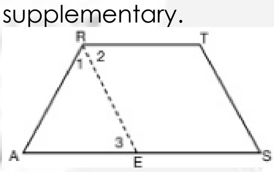
60 minutes

Learning Activity 1: Directions: Complete the two-column proof.

- Opposite angles of an isosceles trapezoid are supplementary.

Given: Isosceles Trapezoid ARTS with $\overline{RT} // \overline{AS}$

Prove: $\angle ARS$ and $\angle S$ are supplementary. $\angle A$ and $\angle T$ are supplementary.



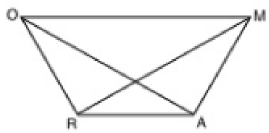
Statements	Reasons
1. (1)	Given
2. $\overline{AR} \cong \overline{ST}$	(2)
3. (3) and $\angle ART \cong \angle S$	The base angles of an isosceles trapezoid are congruent.
4. $\angle A + \angle ART = 180^0$ and $\angle S + \angle T = 180^0$	Same Side Interior Angle Theorem
5. $\angle A + \angle T = 180^0$ and $\angle S + \angle ART = 180^0$	(4)
6. (5)	Definition of Supplementary Angles

Learning Activity 2: Directions: Complete the two-column proof.

- Diagonals of an isosceles trapezoid are congruent.

Given: Isosceles Trapezoid ROMA with diagonals $\overline{RM}$ and $\overline{AO}$

Prove: $\overline{RM} \cong \overline{AO}$



Statements	Reasons
1. Isosceles Trapezoid ROMA with diagonals $\overline{RM}$ and $\overline{AO}$.	(1)
2. (2)	Definition of Isosceles Trapezoid
3. $\angle ORA \cong \angle MAR$	(3)
4. $\overline{RA} \cong \overline{AR}$	(4)
5. (5)	SAS Postulate
6. $\overline{RM} \cong \overline{AO}$	CPCTC

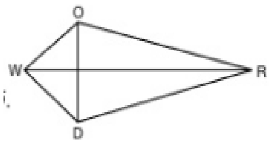
Learning Activity 3: Directions: Complete the two-column proof.

- A diagonal of a kite is an angle bisector of a pair of opposite angles.

	Statements	Reasons

IV. LEARNING PHASES	Suggested Timeframe	Learning Activities
		(see proof below)

Given: Kite WORD with diagonals $\overline{OD}$ and $\overline{WR}$.
Prove: $\overline{WR}$ is angle bisector of $\angle OWD$ and $\angle ORD$.

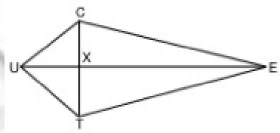


1.	Kite WORD with diagonals $\overline{OD}$ and $\overline{WR}$.	(1)
2.	$\overline{WO} \cong \overline{WD}$ and $\overline{RO} \cong \overline{RD}$	(2)
3.	$\overline{WR} \cong \overline{WR}$	Reflexive Property
4.	(3)	SSS Postulate
5.	(4)	(5)
6.	$\overline{WR}$ is angle bisector of $\angle OWD$ and $\angle ORD$.	Definition of Angle Bisector

D. Assimilation — 20 minutes

Directions: Complete the two-column proof.

Given: Kite CUTE with diagonals $\overline{UE}$ and $\overline{CT}$ intersect at point X.
Prove: $\overline{UE}$ is the perpendicular bisector of $\overline{CT}$.

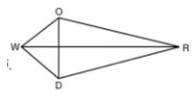


Statements	Reasons
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

V. ASSESSMENT
(Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6) — 20 minutes

Directions: Read each of the following carefully. Choose the letter that corresponds to the correct answer.

1. What theorem states that, "The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long"?
   A. Similarity Theorem　　C. Pythagorean Theorem
   B. Triangle Midline Theorem　　D. Alternate Interior Angles Theorem
2. According to the median theorem of a trapezoid, the median is equal to one-half of the ___________ of the bases.
   A. sum　B. difference　C. product　D. quotient
3. A property of an isosceles trapezoid in which it has congruent diagonals is the same property of ___________.
   A. kite　B. rectangle　C. rhombus　D. parallelogram
4. Given kite WORD, which angle pair are congruent?
   A. ∠W and ∠R　　C. ∠W and ∠D
   B. ∠W and ∠O　　D. ∠O and ∠D



5. Given kite WORD, which angle pair are bisected by a diagonal?
   A. ∠W and ∠R　　C. ∠W and ∠D
   B. ∠W and ∠O　　D. ∠O and ∠D

VI. REFLECTION — 20 minutes

- The learner communicates the explanation of their personal assessment as indicated in the Learner's Assessment Card.
- The learner, in their notebook, will write their personal insights about the lesson using the prompts below.
  I understand that ________________.
  I realize that ____________________.
  I need to learn more about __________.

Prepared by:	Wilson Ray G. Anzures	Checked by:	Ma. Filipina M. Drio/Reymark R. Queaño

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.
✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.
? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

W4	Learning Area	Mathematics	Grade Level	Nine
	Quarter	Third	Date	

I. LESSON TITLE	Solving Problems Involving Parallelograms, Trapezoids, and Kites
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	Solves problems involving parallelograms, trapezoids and kites **M9GE-IIIe-1**
III. CONTENT/CORE CONTENT	

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
A. Introduction	5 minutes	In this lesson, we shall focus on solving problems involving the relationship of sides and angles in parallelograms, trapezoids, and kites using their properties and different theorems. We need to remember all the definitions, properties, and theorems that we have already discussed regarding parallelograms, trapezoids, and kites in the previous lessons.
B. Development	90 minutes	SOLVING PROBLEMS INVOLVING PARALLELOGRAMS, TRAPEZOIDS, AND KITES

Steps in Geometric Problem Solving:

1. Read the problem carefully.
2. Recognize the relationship of the given figure.
3. Pay attention to the labels.
4. Use appropriate definition, property, postulate, or theorem.
5. Answer the question.

SOLVING PROBLEMS INVOLVING PARALLELOGRAMS, TRAPEZOIDS, AND KITES

1. Given: Quadrilateral WISH is a parallelogram

a. If $m\angle W = (x + 15)^0$ and $m\angle S = (2x + 5)^0$, what is $m\angle W$?

$m\angle W = m\angle S$	In a parallelogram, any two opposite angles are congruent.
$(x + 15)^0 = (2x + 5)^0$	Substitution
$(x - x + 15 - 5)^0 = (2x - x + 5 - 5)^0$	Addition Property of Equality
$x = 10^0$	Subtraction and Addition Property
$m\angle W = ((10) + 15)^0$	Substitution
$m\angle W = 25^0$	Addition Property

b. If $\overline{WI} = 3y + 3$ and $\overline{HS} = y + 13$, how long is $\overline{HS}$?

$\overline{WI} \cong \overline{HS}$	In a parallelogram, any two opposite sides are congruent.
$3y + 3 = y + 13$	Substitution
$3y - y + 3 - 3 = y - y + 13 - 3$	Addition Property of Equality
$2y = 10$	Subtraction and Addition Property
$y = 5$	Dividing both sides by 2
$\overline{HS} = (5) + 13$	Substitution
$\overline{HS} = 18$	Addition Property

c. Quadrilateral WISH is a rectangle, and its perimeter is 56 cm. One side is 5 cm less than twice the other side. What are the dimensions and how large is its area?

Perimeter of Rectangle = 2L + 2W	Formula for Perimeter of Rectangle
56 = 2L + 2(2L – 5) cm	Substitution
56 = 2L + 4L – 10 cm	Distributive Property
56 + 10 = 6L – 10 + 10 cm	Addition Property of Equality
6L = 66 cm	Addition Property
L = 11 cm	Dividing both sides by 6
56 = 2(11) + 2W cm	Substitution
56 = 22 + 2W cm	Multiplication Property
56 – 22 = 22 – 22 + 2W cm	Addition Property of Equality
2W = 34 cm	Subtraction Property
W = 17 cm	Dividing both sides by 2
Area of Rectangle = LW	Formula for Area of Rectangle
Area of Rectangle = 11 cm * 17 cm	Substitution
Area of Rectangle = 187 cm^2	Multiplication Property

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
		d. What is the perimeter and the area of the largest square that can be formed from Rectangle WISH from the previous question?

L = 11 cm	Determine the smaller number from the length and width of the rectangle
Area of Square = s^2	Formula for Area of Square
Area of Square = $(11\text{ cm})^2$	Substitution
Area of Square = 121 cm^2	Multiplication Property

2. Given: Isosceles trapezoid POST with $\overline{OS}//\overline{PT}$ and $\overline{ER}$ is its median.

a. If $\overline{OS}$ = 3x – 2, $\overline{PT}$ = 2x + 10 and $\overline{ER}$ = 14, how long is each base?

$\overline{ER} = \frac{1}{2}(\overline{OS} + \overline{PT})$	Formula for length of median
$14 = \frac{1}{2}((3x - 2) + (2x + 10))$	Substitution
$28 = 5x + 8$	Combining like terms and simplifying
$28 - 8 = 5x + 8 - 8$	Addition Property of Equality
$5x = 20$	Subtraction Property
$x = 4$	Dividing both sides by 5
$\overline{OS} = 3(4) - 2$	Substitution
$\overline{OS} = 10$	Multiplication and Subtraction Property
$\overline{PT} = 2(4) + 10$	Substitution
$\overline{PT} = 18$	Multiplication and Addition Property

b. If $m\angle P = (2x + 5)^0$ and $m\angle O = (3x - 10)^0$, what is $m\angle T$?

$m\angle P$ and $m\angle O$ are supplementary	Same Side Interior Angles are Supplementary
$(2x + 5)^0 + (3x - 10)^0 = 180^0$	Substitution
$(5x - 5)^0 = 180^0$	Addition and Subtraction Property
$(5x - 5 + 5)^0 = (180 + 5)^0$	Addition Property of Equality
$5x = 185^0$	Addition Property
$x = 37^0$	Divide both sides by 5
$m\angle P$ and $m\angle T$ are congruent	In a isosceles trapezoid, base angles are congruent
$m\angle T = (2(37) + 5)^0$	Substitution
$m\angle T = 79^0$	Simplify

c. One base is twice the other and $\overline{ER}$ is 6 cm long. If its perimeter is 27 cm, how long is each leg?

$\overline{ER} = \frac{1}{2}(\overline{OS} + \overline{PT})$	Formula for length of median
$6 = \frac{1}{2}((x) + (2x))$	Substitution
$12 = 3x$	Combining like terms and simplifying
$x = 4$	Dividing both sides by 3
Perimeter of Isosceles Trapezoid = $2L + B_1 + B_2$	Formula of Perimeter of Isosceles Trapezoid
$27 = 2L + 4 + 2(4)$	Substitution
$27 = 2L + 12$	Multiplication and Addition Property
$27 - 12 = 2L + 12 - 12$	Addition Property of Equality
$2L = 15$	Multiplication and Addition Property
L = 7.5 cm	Dividing both sides by 2

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

d. $\overline{ER}$ is 8.5 inches long and one leg measures 9 inches. What is its perimeter if one of the bases is 3 inches more than the other?

$\overline{ER} = \frac{1}{2}(\overline{OS} + \overline{PT})$	Formula for length of median
$8.5 = \frac{1}{2}((x) + (x + 3))$	Substitution
$17 = 2x + 3$	Combining like terms and simplifying
17 – 3 = 2x + 3 – 3	Addition Property of Equality
2x = 14	
$x = 7$	Dividing both sides by 2
Perimeter of Isosceles Trapezoid = $2L + B_1 + B_2$	Formula of Perimeter of Isosceles Trapezoid
Perimeter of Isosceles Trapezoid = 2(9) + 7 + (7 + 3)	Substitution
Perimeter of Isosceles Trapezoid = 18 + 7 + 10	Multiplication and Addition Property
Perimeter of Isosceles Trapezoid = 35 inches	Addition Property

3. **Given: Quadrilateral LIKE is a kite with $\overline{LI} \cong \overline{IK}$ and $\overline{LE} \cong \overline{KE}$.**

a. $\overline{LE}$ is twice $\overline{LI}$. If its perimeter is 21 cm, how long is $\overline{LE}$?

Perimeter of Kite = $2S_1 + 2S_2$	Formula of Perimeter of Kite
21 = $2\overline{LI} + 2(2\overline{LI})$	Substitution
21 = $2S_1 + 4\overline{LI}$	Multiplication Property
21 = $6\overline{LI}$	Combining like terms
$\overline{LI}$ = 3.5 cm	Dividing both sides by 6
$\overline{LE}$ = 7 cm	$\overline{LE}$ is twice $\overline{LI}$

b. What is the area if one of the diagonals is 4 more than the other and $\overline{IE} + \overline{LK}$ = 16 inches?

$\overline{IE} + \overline{LK}$ = 16	Given
x + (x + 4) = 16	Substitution
2x + 4 = 16	Combining like terms
2x + 4 – 4 = 16 – 4	Addition Property of Equality
2x = 12	Subtraction Property
x = 6	Dividing both sides by 2
Area of Kite = $\frac{1}{2}D_1D_2$	Formula of Area of Kite
Area of Kite = $\frac{1}{2}(\overline{IE})(\overline{LK})$	Substitution
Area of Kite = $\frac{1}{2}(6)(10)$	Substitution
Area of Kite = 30 inches^2	Multiplication Property

c. $\overline{IE}$ = (x – 1) ft and $\overline{LK}$ = (x + 2) ft. If its area is 44 ft^2, how long are $\overline{IE}$ and $\overline{LK}$?

Area of Kite = $\frac{1}{2}D_1D_2$	Formula of Area of Kite
Area of Kite = $\frac{1}{2}(\overline{IE})(\overline{LK})$	Substitution
44= $\frac{1}{2}(x-1)(x+2)$	Substitution
88 = $x^2 + x - 2$	Simplifying
$x^2 + x - 2 - 88 = 0$	Transposition
$x^2 + x - 90 = 0$	Subtraction Property
(x – 9)(x + 10) = 0	Factoring
x = 9 or -10	Zero Product Rule but only consider 9 since there is no negative measure.
$\overline{IE}$ = ((9) – 1) ft and $\overline{LK}$ = ((9)+ 2) ft	Substitution
$\overline{IE}$ = 8 ft and $\overline{LK}$ = 11 ft	Addition and Subtraction Property

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
C. Engagement	30 minutes	Directions: Illustrate and solve the following problems: 1. Two consecutive sides of a parallelogram measure 4 *m* and 9 *m*, respectively. What is the perimeter of the parallelogram? 2. One diagonal of a square measure (2x + 4) *in*. If the other diagonal measures 16 *in*, what is x? 3. Given trapezoid QRST with $\overline{QR}//\overline{TS}$ and $\overline{UV}$ as the median. If m$\overline{QR}$ = 12 cm and m$\overline{UV}$ = 24 cm, what is m$\overline{UV}$? 4. An isosceles trapezoid with a diagonal that measures 42 cm and one leg measures 23 cm. What is the length of the other diagonal? 5. Given kite HOPE with diagonals m$\overline{HP}$ = 10 cm and m$\overline{OE}$ = 18 *cm*. What is the area of the kite?
D. Assimilation	30 minutes	Directions: Solve the following problems. Show your complete solutions. 1. A table cloth is cut into a parallelogram in which two opposite angles measure (8x – 33)º and (5x + 15)º? Find the measures of all the angles. 2. One lateral face of the roof of the school building is trapezoid in shape. One of the bases of this trapezoid is 6 *m* longer than the other base. Find the length of the two bases if the median measures 19 *m*. 3. A rectangular garden has a perimeter of 56 *ft*. Its length is 5 *ft* less than twice the width. What is the area of the garden? 4. A tabletop is an isosceles trapezoid in shape. The median is 5.5 *dm,* and one of its legs measures 2.5 *dm*. If one of the tabletop bases is 1 *dm* more than the other, find its perimeter. 5. The area of the paper used by William in the making of his kite is 60 square inches, and one of its diagonals is 2 inches less than the other diagonal. Find the lengths of the two diagonals.
V. ASSESSMENT (Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6)	30 minutes	Directions: Illustrate the following and solve for what is required. Show your complete solution. 1. One side of a rectangle is 3 *m* more than the other. If the perimeter of the rectangle is 30 *m*, what are its dimensions? a. L = 4 m and W = 7 m c. L = 6 m and W = 9 m b. L = 5 m and W = 8 m d L = 7 m and W = 10 m 2. A rhombus with a perimeter of 60 *in* has a side with a length of (8x) *in*. Find x. a. 1.675 b. 1.875 c. 2.275 d. 7.5 3. One base of a trapezoid is 4 *cm* less than twice the other. If the median measures 13 *cm*, what is the length of the longer base? a. 10 b. 12 c. 16 d. 20 4. Isosceles trapezoid POST with $\overline{OS}//\overline{PT}$. If m∠O = (10x + 20)^{O} and m∠P= 8x – 2)^{O} , what is x? a. 9 b. 10 c. 11 d. 12 5. Given kite LOVE which m$\overline{LO}$ = 8 *in* and m$\overline{VE}$ =20 *in*. What is the perimeter of the kite? a. 28 in b. 34 in c. 56 in d. 80 in
VI. REFLECTION	20 minutes	• The learners communicate the explanation of their personal assessment as indicated in the Learner's Assessment Card. • The learner will write their personal insights about the lesson in their notebook using the prompts below: I understand that ___________________. I realize that ________________________. I need to learn more about __________.

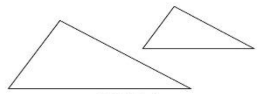
Prepared by:	Wilson Ray G. Anzures	**Checked by:**	Ma. Filipina M. Drio

W5	Learning Area	Mathematics	Grade Level	Nine
	Quarter	Third	Date	

I. LESSON TITLE	Proportion and Application of Fundamental Theorems of Proportionality
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	Lesson 1: Describes a proportion **M9GE-IIIf-1** Lesson 2: Applies the fundamental theorems of proportionality to solve problems involving proportions **M9GE-IIIf-2**
III. CONTENT/CORE CONTENT	Describing a proportion and solving problems involving proportion

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
A. Introduction	30 minutes	

Let's find out what you already know about proportion.
Answer the following.
A. Which of the following pair of pictures is an example of proportion?


Pair No. 1


Pair No. 2

B. There are different sets of ingredients in preparing buko pie. Given below are the ingredients for pie filling which is good for eight persons.

Pie Filling

$^{1}/_{3}$ cup	cornstarch
$^{1}/_{2}$ cup	coconut water
$^{1}/_{2}$ cup	all-purpose cream
$^{3}/_{4}$ cup	sugar
4 cups	young coconut meat

If you are asked to prepare for 24 persons, how much coconut water is needed in preparing the filling?

C. Is there a possible way to find the height of the flag of the Philippines raised at San Pablo City Plaza without directly measuring it?

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
B. Development	60 minutes	

PROPORTION

When we recall the definition of ratio of two numbers, it is the comparison of two quantities. For any two numbers, x and y, $y \neq 0$ the ratio is the quotient obtained by dividing x and y. The two numbers are called the terms. The ratio can be written in the following form: $^{x}/_{y}$(fraction form), x:y(read as "x is to y"),x to y.

The following ratios can be reduced to the same value: $\frac{6}{9}$, $\frac{30}{45}$, 4 : 6. Their simplest form is 2 : 3 or $\frac{2}{3}$. Ratios that can be reduced to the same value are called equivalent ratios.

Example

Given: $x = 6$, $y = 18$, $z = 15$.
Give each ratio in simplest form.

a. $\frac{x}{y}$ b. y to z c. $x + z : y$

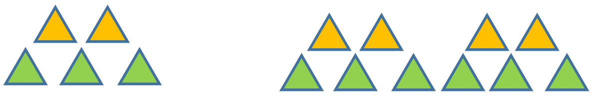
Solution

a. $\frac{x}{y} = \frac{6}{18} = \frac{1}{3}$
b. y to z is 18 to 15 or 6 to 5
c. $x + z : y$ is 21 : 18 or 7 : 6

The equation stating that two ratios are equal is called a proportion. In symbols, $\frac{a}{b} = \frac{c}{d}$, where b and $d \neq 0$, or $a : b = c : \mathbf{d}$ (read as "a is to b as c is to d").

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Example 1



$\frac{2}{5} = \frac{4}{10}$

So 2 out 5 is equal to 4 out 10. They are in proportion.

Example 2

When four meters of cable wire costs 90 pesos, then:

- 10 meters of that cable wire costs 225 pesos
- 12 meters of that cable wire costs 270 pesos

All these ratios: $\frac{4}{90}$, $\frac{10}{222}$, and $\frac{12}{270}$ can be simplified as $\frac{2}{45}$.

Thus, the following are proportions:

$$\frac{4}{90} = \frac{10}{222} = \frac{12}{270}$$

The ratio and proportion have many uses or relationship in our everyday life such as dealing with the measures of the ingredients in cooking recipes, the amount of profit earned per sale, enlarging or reducing the size of a drawing, measuring the height of an object without directly measuring it, and so many others.

APPLICATION OF FUNDAMENTAL THEOREMS OF PROPORTIONALITY

In geometry, we used proportion to compare lengths of segments. To solve for unknown length, we often used the properties of proportion.

Properties of Proportion

If $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$, and $a, b, c,$ and $d \neq 0$, then each of the following is true:

- ✓ $ad = cb$
- ✓ $\frac{a}{c} = \frac{b}{d}$ or $\frac{a}{c} = \frac{b}{d}$
- ✓ $\frac{b}{a} = \frac{d}{c}$
- ✓ $\frac{a+b}{b} = \frac{c+d}{d}$
- ✓ $\frac{a-b}{b} = \frac{c-d}{d}$

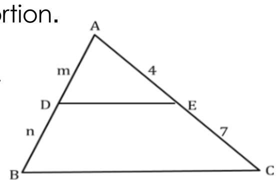
If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, and b, d and $f \neq 0$, then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f} = k$.

Here are some examples on how to apply the fundamental theorems of proportionality to solve problems involving proportions.

Example 1

Use the proportion $\frac{m}{n} = \frac{4}{7}$ to complete each proportion.

a. $\frac{n}{7} =$ ____ b. $\frac{4}{m} =$ ____ c. $\frac{n}{m} =$ ____ d. $\frac{n+m}{m} =$ ____



Solution

a. $\frac{n}{7} = \frac{m}{4}$ b. $\frac{4}{m} = \frac{7}{n}$ c. $\frac{n}{m} = \frac{7}{4}$ d. $\frac{n+m}{m} = \frac{11}{4}$

Example 2

Find the value of x in the following proportions.

a. $\frac{9}{x} = \frac{15}{20}$ b. $x:6 = 15:18$ c. $\frac{x+3}{4} = \frac{9}{2}$ d. $\frac{x+2}{3} = \frac{4x}{6}$

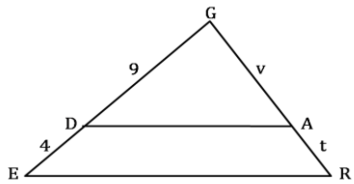
Solution

a. $\frac{9}{x} = \frac{15}{20} \rightarrow 15 \cdot x = 9 \cdot 20 \rightarrow 15x = 180 \rightarrow \left(\frac{1}{15}\right)15x = \left(\frac{1}{15}\right)180 \rightarrow x = 12$

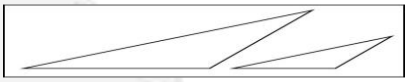
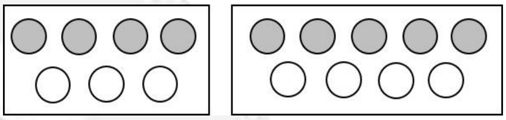
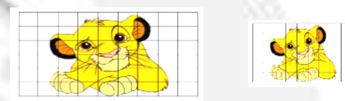
b. $x:6 = 15:18 \rightarrow 6 \cdot 15 = 18 \cdot x \rightarrow 90 = 18x \rightarrow \left(\frac{1}{18}\right)90 = \left(\frac{1}{18}\right)18x \rightarrow 5 = x$ or $x = 5$

c. $\frac{x+3}{4} = \frac{9}{2} \rightarrow 4 \cdot 9 = 2\,(x+3) \rightarrow 36 = 2x + 6 \rightarrow 36 - (6) = 2x + 6 - (6) \rightarrow 30 = 2x$
$\rightarrow \left(\frac{1}{2}\right) 30 = \left(\frac{1}{2}\right)(2x) \rightarrow 15 = x$ or $x = 15$

<table>
<tr><th>IV. LEARNING PHASES</th><th>Suggested Time Frame</th><th>Learning Activities</th></tr>
<tr><td></td><td></td><td>
d. $\frac{x+2}{3} = \frac{4x}{6} \rightarrow 3 \cdot 4x = 6(x+2) \rightarrow 12x = 6x + 12 \rightarrow 12x - 6x = 6x - 6x + 12 \rightarrow 6x = 12$
$\rightarrow \left(\frac{1}{6}\right)6x = \left(\frac{1}{6}\right)12 \rightarrow x = 2$
Example 3
Determine the value/s of indicated quantities from the given proportions.
a. If $u : v = 3 : 2$, find $4u - 3v : 4u + v$.
b. Find the ratio a : b, if $2a^2 - ab - 3b^2 = 0$ where a and $b \neq 0$.
c. If $x : y : z = 1 : 3 : 5$ where x, y and $z > 0$, find the values of x, y and z when $x^2 - y^2 + z^2 = 68$.
Solution
a. $\frac{u}{v} = \frac{3}{2} \rightarrow u = \frac{3v}{2}$
In the ratio $\frac{4u - 3v}{4u + v}$, plug in the value of u in terms of v.
$\frac{4\left(\frac{3v}{2}\right) - 3v}{4\left(\frac{3v}{2}\right) + v} = \frac{6v - 3v}{6v + v} = \frac{3v}{7v} = \frac{3}{7}$
Thus, $4u - 3v : 4u + v = 3 : 7$.
b. $2a^2 - ab - 3b^2 = 0$
$(2a - 3b)(a + b) = 0$
$2a - 3b = 0 \quad ; \quad a + b = 0$
$2a - 3b + 3b = 0 + 3b \quad ; \quad a + b - b = 0 - b$
$2a = 3b \quad ; \quad a = -b$
$\frac{2a}{2b} = \frac{3b}{2b} \quad ; \quad \frac{a}{b} = \frac{-b}{b}$
$\frac{a}{b} = \frac{3}{2} \quad ; \quad \frac{a}{b} = \frac{-1}{1}$
Therefore, a : b = 3 : 2 or -1 : 1.
c. Let $\frac{x}{1} = \frac{y}{3} = \frac{z}{5} = k,\ k \neq 0$.
Hence, $x = k, y = 3k$, and $z = 5k$.
Plug in the obtained value of x, y and z in $x^2 - y^2 + z^2 = 68$.
$(k)^2 - (3k)^2 + (5k)^2 = 68$
$k^2 - 9k^2 + 25k^2 = 68$
$26k^2 - 9k^2 = 68$
$17k^2 = 68$
$\frac{17k^2}{17} = \frac{68}{17}$
$k^2 = 4k^2 = 2^2$ or $(-2)^2$
$k = 2$ or -2 Disregard -2 since the $x, y,$ and $z > 0$.
So, $x = 2$; $y = 3k = 3(2) = 6$; and $z = 5k = 5(2) = 10$.
</td></tr>
<tr><td>C. Engagement</td><td>60 minutes</td><td>
Learning Task 1
Directions: Answer the following accordingly.
1. How can you say that the enlarged piece of drawing is proportional to its original size?
2. How can we relate the number of liters of fuel we put in the car tank and the cost we will pay? If 1L of gas costs 44 pesos, how much do you think is 4.5L?
3. Four out of 18 male students and three out of 21 female students failed on one of the weekly online tests. Are the ratios of male and female students who failed this test proportional? Why or why not?



Learning Task 2
Directions: Solve the following.
1. Use the proportion $\frac{v}{t} = \frac{9}{4}$ to complete each proportion.
a. $\frac{v}{9} =$ ___   b. $\frac{4}{t} =$ ___   c. $\frac{t}{v} =$ ___   d. $\frac{v-t}{t} =$ ___
2. Find the value of y in the following proportions.
</td></tr>
</table>

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
		a. $\frac{y}{21}=\frac{28}{49}$ b. $\frac{y+5}{12}=\frac{9}{4}$ c. $\frac{y+4}{6}=\frac{7y}{18}$ d. $\frac{2y-3}{3}=\frac{3y-7}{2}$ 3. If $m:n=5:3$, find $3m+4n:6m-2n$. 4. Find the ratio $e:f$, if $5e^2-13ef-6f^2=0$ where e and $f\neq 0$.
D. Assimilation	20 minutes	Directions: Answer the following accordingly. 1. How would you describe proportion? 2. Cite an example where you can apply proportion in your everyday life. Describe how you can you apply the proportion in that situation. 3. If $a:b:c=5:3:2$ where a, b and $c>0$, find the values of a, b and c when $a^2-b^2-c^2=108$.
V. ASSESSMENT (Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6)	20 minutes	Directions: Choose the letter of the correct answer. 1. The following describe a proportion EXCEPT letter ___. a. 3 : 7 = 18 : 42 b.  c.  d.  2. If $\frac{m}{n}=\frac{h}{k}$, which of the following is not true? a. $\frac{n}{m}=\frac{k}{h}$ c. $\frac{m}{n}=\frac{k}{h}$ b. $km=hn$ d. $\frac{m}{h}=\frac{n}{k}$ 3. Find the value of x in $\frac{5x+4}{10}=\frac{3x}{5}$. a. 5 b. 4 c. 3 d. 2 4. Find the ratio $x:y$ if $4x^2-8xy-5y^2=0$ where x and $y\neq 0$. a. -1 : 2 or 5 : 2 c. -1 : 1 or 5 : 4 b. 1 : 2 or -5 : 2 d. 1 : 1 or -5 : 4 5. The length and width of a rectangle whose perimeter is 60 cm are in the ratio 3 : 2. What is the area of the rectangle? a. 108 sq. cm c. 360 sq. cm b. 216 sq. cm d. 600 sq. cm
VI. REFLECTION	20 minutes	• The learners communicate the explanation of their personal assessment as indicated in the Learner's Assessment Card. • The learners will write their personal insights about the lesson in their notebook using the prompts below: I understand that ________________. I realize that ____________________. I need to learn more about __________.

Prepared by:	Edgar V. Tuico	Checked by:	MA. FILIPINA M. DRIO

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

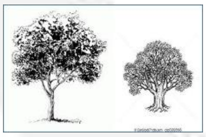
Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

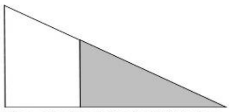
W6-7	Learning Area	Mathematics	Grade Level	Nine
	Quarter	Third	Date	

I. LESSON TITLE	Similarity and Triangle Similarity Theorem
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	Lesson 1: Illustrates similarity of figures **(M9GE-IIIg-1)** Lesson 2: Proves the conditions for similarity of triangles. 1.1 SAS Similarity Theorem 1.2 SSS Similarity Theorem 1.3 AA Similarity Theorem 1.4 Right Triangle Similarity Theorem 1.5 Special Right Triangle Theorem **(M9GE-IIIg-h-1)**
III. CONTENT/CORE CONTENT	Similar Figures and the Proof of Triangle Similarity Theorems

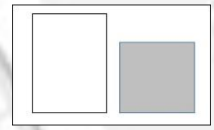
<table>
<tr><th>IV. LEARNING PHASES</th><th>Suggested Time Frame</th><th>Learning Activities</th></tr>
<tr><td>A. Introduction</td><td>15 minutes</td><td>

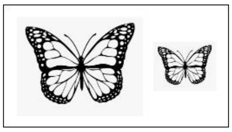
Can you identify which of the following pairs are similar?

a. 

c. 

The small and big triangles

b. 

d. 

How can you say that the pair of figures or image are similar?

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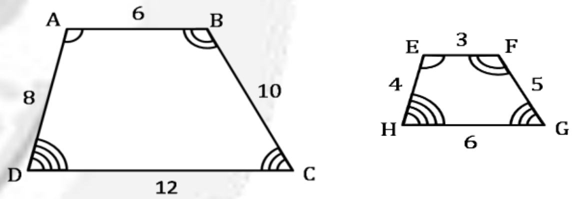
<tr><td>B. Development</td><td>180 minutes</td><td>

Similarity

In this lesson, you will learn that there are triangles and other polygons that have the same shape but do not necessarily have the same size. The illustrative example below will give you an idea on how we can say that the given figures are similar.



If you will observe, trapezoid ABCD and trapezoid EFGH have the same shape. When you pair the corresponding vertices, the angles coincide. It shows that their corresponding angles are congruent: ∠A ≅∠E ; ∠B ≅ ∠F ; ∠C ≅ ∠G ; and ∠D ≅ ∠H.

Another thing is that the ratios of the measure of the lengths of their corresponding sides are equal.

Thus, in EFGH to ABCD, $\frac{EF}{AB}=\frac{3}{6}=\frac{FG}{BC}=\frac{5}{10}=\frac{GH}{CD}=\frac{6}{12}=\frac{EH}{AD}=\frac{4}{8}=\frac{1}{2}$. Here, the scale factor k is $^1/_2$. We could also turn it around as ABCD to EFGH where $\frac{AB}{EF}=\frac{6}{3}=\frac{BC}{FG}=\frac{10}{5}=\frac{CD}{GH}=\frac{12}{6}=\frac{AD}{EH}=\frac{8}{4}=\frac{2}{1}=2$. Now here, the scale factor k is 2.

Based on the illustrative example, two polygons are similar (the symbol is ~) if their vertices can be paired so that corresponding angles are congruent and the lengths of their corresponding sides are proportional.

To indicate that trapezoid ABCD is similar to trapezoid EFGH, you can write ABCD ~ EFGH. If you use this notation, write the corresponding vertices on the same order.

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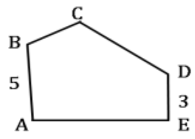
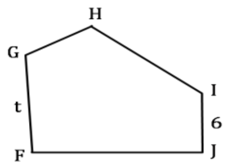
IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Example:





Complete the following statement.

a. If ABCDE ~ FGHIJ, then

$\angle B \cong$ ___ ; $\angle J \cong$ ___ ; $\frac{CD}{HI}$ = ___ ; t = ___

b. The scale factor of FGHIJ ~ ABCDE is ___.

Solution

a. $\angle B \cong \angle G$; $\angle J \cong \angle E$; $\frac{CD}{HI} = \frac{DE}{IJ} = \frac{3}{6} = \frac{1}{2}$;

$\frac{AB}{FG} = \frac{DE}{IJ} \rightarrow \frac{5}{t} = \frac{3}{6} \rightarrow 3t = 30 \rightarrow \left(\frac{1}{3}\right)3t = \left(\frac{1}{3}\right)30 \rightarrow t = 10$

b. The scale factor of FGHIJ ~ ABCDE is 2.

Triangle Similarity Theorems

In this lesson, we are only going to focus on the similarity of two triangles. We will apply our prior knowledge on the definition of similar polygons to understand the postulates and theorems in proving the similarity of triangles.

To prove the similarity of two triangles using the definition of similarity, we must establish that the three corresponding angles are congruent and that the three ratios of the lengths of corresponding sides are equal.

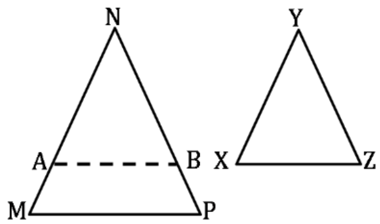
If the three corresponding angles of two triangles are congruent, then we can conclude that the triangles are similar. We call this as AAA Similarity Theorem.

Illustration

Given: $\triangle MNP \leftrightarrow \triangle XYZ$, $\angle M \cong \angle X$, $\angle N \cong \angle Y$ and $\angle P \cong \angle Z$

Prove: $\triangle MNP \sim \triangle XYZ$

Proof:



Statement	Reason
Construct $\overline{AB}$, such that $\overline{AN} \cong \overline{XY}$; $\overline{NB} \cong \overline{YZ}$.	By construction
$\angle N \cong \angle Y$	Given
$\triangle ANB \cong \triangle XYZ$	SAS Congruence Theorem
$\angle NAB \cong \angle X$, $\angle NBA \cong \angle Z$	Corresponding parts of congruent triangle are congruent. (CPCTC)
$\angle NAB \cong \angle M$, $\angle NBA \cong \angle P$	Transitive Property
$m\angle MNP = m\angle ANB$	Reflexive Property
$\angle MNP \cong \angle ANB$	Definition of congruent angles
$\overline{AB} \parallel \overline{MP}$	If two lines are cut by a transversal, the corresponding angles are congruent and the two lines are parallel.
$\frac{NA}{NM} = \frac{NB}{NP}$	Basic Proportionality Theorem
NA = YX ; NB = YZ	Congruent segments have equal measures.
$\triangle ANB \sim \triangle MNP$	Definition of similar triangles
$\therefore \triangle MNP \sim \triangle XYZ$	Transitive Property

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Here are the other Triangle Similarity Theorems.

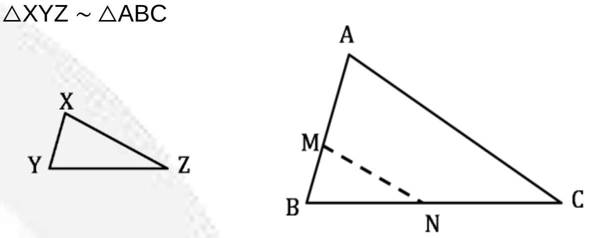
1.1 <u>SAS Similarity Theorem</u>

If an angle of one triangle is congruent to an angle of another triangle and if the lengths of the sides including these angles are proportional, then the triangles are similar.

Illustration

Given: $\triangle XYZ \leftrightarrow \triangle ABC$, $\angle Y \cong \angle B$ and $\frac{YX}{BA} = \frac{YZ}{BC}$

Prove: $\triangle XYZ \sim \triangle ABC$

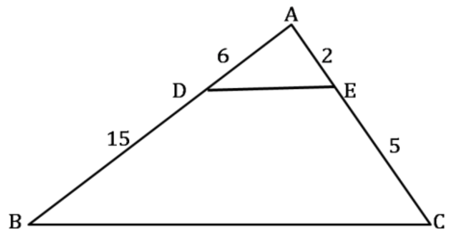


Proof:

Statements	Reasons
Draw $\overline{MN}$ such that $\overline{BM} \cong \overline{YX}$ and $\overline{BN} \cong \overline{YZ}$.	By construction
$\angle Y \cong \angle B$	Given
$\triangle XYZ \cong \triangle MBN$	SAS Congruence Theorem
$\overline{BM} \cong \overline{YX}$ and $\overline{BN} \cong \overline{YZ}$	CPCTC
$\frac{YX}{BA} = \frac{YZ}{BC}$	Given
$\frac{BM}{BA} = \frac{BN}{BC}$	By substitution
MN ∥ AC	Converse of Basic Proportionality Theorem
$\angle BMN \cong \angle BAC$ and $\angle BNM \cong \angle BCA$	If two parallel lines are cut by a transversal, corresponding angles are congruent.
$\angle B \cong \angle B$	Reflexive Property
$\triangle ABC \sim \triangle MBN$	AAA Similarity Theorem
$\therefore \triangle XYZ \sim \triangle ABC$	Transitive Property

Example 1

Show that the triangles ABC and ADE in the figure on the right are similar.

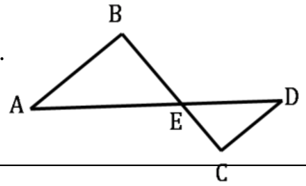


Solution

- $\angle BAC \cong \angle DAE$ by Reflexive Property
- Since the measures of the lengths of the sides are given, calculate the ratios of the corresponding sides. $\frac{AB}{AD} = \frac{21}{6} = \frac{7}{2}$ and $\frac{AC}{AE} = \frac{7}{2}$
- The length of the corresponding side of the two triangles are proportional and the incuded angles are congruent, therefore, $\angle ABC \sim \angle ADE$.

Example 2

Given $\frac{AE}{DE} = \frac{BE}{CE}$, prove that $\triangle BEA \sim \triangle CED$.



IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Solution

Proof:

Statements	Reasons
$\frac{AE}{DE}=\frac{BE}{CE}$	Given or by hypothesis
∠BEA and ∠CED are vertical angles.	Definition of vertical angles
∠BEA ≅ ∠CED	Vertical angles are congruent.
△BEA ~ △CED	SAS Similarity Theorem

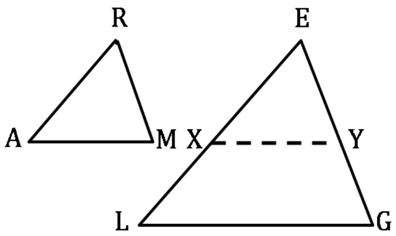
1.2 <u>SSS Similarity Theorem</u>

If the three corresponding sides of two triangles are proportional, then the two triangles are similar.

Illustration

Given: △ARM ↔ △LEG, $\frac{AR}{LE}=\frac{RM}{EG}=\frac{AM}{LG}$

Prove: △ARM ~ △LEG

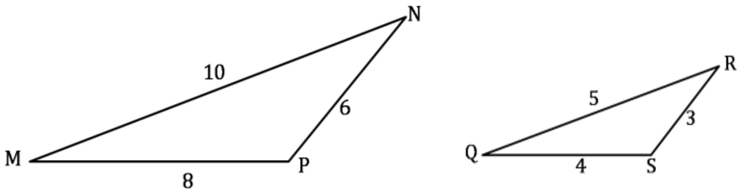


Proof:

Statements	Reasons
Draw $\overline{XY}$ such that $\overline{XE} \cong AR$ and $\overline{EY} \cong \overline{RM}$.	By construction
XE = AR and EY = RM	Congruent segments have equal measures.
$\frac{AR}{LE}=\frac{RM}{EG}=\frac{AM}{LG}$	Given
$\frac{XE}{LE}=\frac{EY}{EG}$	By substitution
∠E ≅ ∠E	Reflexive Property
△LEG ~ △XEY	SAS Similarity Theorem
$\frac{XY}{LG}=\frac{XE}{LE}$	Definition of similar triangles
$\frac{XY}{LG}=\frac{AR}{LE}$	By substitution (XE = AR)
XY = LG$\left(\frac{AR}{LE}\right)$; AM = LG$\left(\frac{AR}{LE}\right)$	Multiplication Property
XY = AM	Transitive Property
△ARM ≅ △XEY	SSS Congruece Theorem
∴ △ARM ~ △LEG	Transitive Property

Example 1

Show that the triangles MNP and QRS in the figure below are similar.



Solution

- Since the measures of the lengths of the sides are given, calculate the ratios of the corresponding sides. $\frac{MN}{QR}=\frac{10}{5}=\frac{2}{1}=2$; $\frac{MP}{QS}=\frac{8}{4}=2$; $\frac{NP}{RS}=\frac{6}{3}=2$
- The ratios of the lengths of the three corresponding sides of the two triangles are equal, thus △MNP ~△QRS.

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Example 2

a. Given: △CAR and △PET. State the proportions that must be true if △CAR ~△PET by SSS Similarity.

b. Given the statement that shows the proportionality of the three corresponding sides of the two triangles, $\frac{DO}{KE} = \frac{ON}{EY} = \frac{DN}{KY}$, name the two similar triangles.

Solution

a. $\frac{CA}{PE} = \frac{AR}{ET} = \frac{CR}{PT}$

b. △DON and △KEY

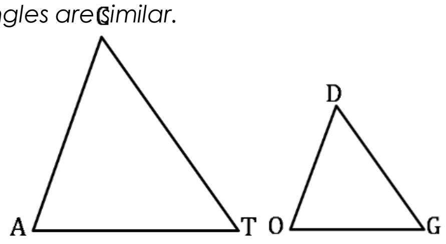
1.3 <u>AA Similarity Theorem</u>

If two angles of one triangle are congruent respectively to two angles of another triangle, then the triangles are similar.

Illustration

Given:∠A ≅∠O ; ∠C ≅∠D

Prove:△CAT ~△DOG



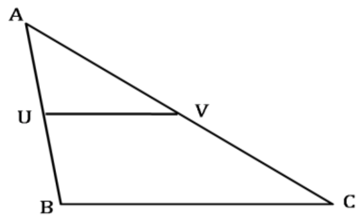
Proof:

Statements	Reasons
∠A ≅∠O ; ∠C ≅∠D	Given
m∠A ≅ m∠O ; m∠C ≅ m∠D	Definition of Congruent Angles
m∠A + m∠C = m∠O ≅ m∠D	Addition Property
m∠A + m∠C + m∠T= 180 m∠O + m∠D + m∠G = 180	The sum of the measures of the interior angles of a triangle is 180.
m∠A + m∠C + m∠T= m∠O + m∠D + m∠G	Transitive Property
m∠T = m∠G	Addition Property
∠T = ∠G	Definition of congruent angles
∴△CAT ~ △DOG	AAA Similarity Theorem

Example1

Given: $\overline{UV} \parallel \overline{BC}$

Prove : △ABC ~ △AUV by AA Similarity Theorem



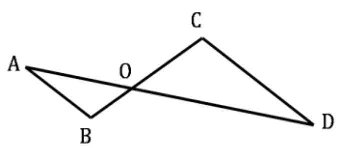
Solution

Proof:

Statements	Reasons
$\overline{UV} \parallel \overline{BC}$	Given
$\angle AUV \cong \angle ABC$	If two parallel lines are cut by a transversal, corresponding angles are congruent.
m∠BAC ≅ $m\angle$UAV	Reflexive Property
∠BAC ≅ ∠UAV	Definition of congruent angles
∴ △ABC ~ △AUV	AA Similarity Theorem

Example 2

Given: $\overline{AB} \parallel DC$. Name at least two pairs of corresponding angles that are congruent to prove that △AOB ~ △DOC by AA Similarity Theorem.



IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

Solution

- If $\overline{AB}$ and $\overline{DC}$ are parallel and cut by transversal AD, then the congruent alternate interior angles are ∠BAO and ∠CDO.
- If $\overline{AB}$ and $\overline{DC}$ are parallel and cut by transversal BC, then the congruent alternate interior angles are ∠ABO and ∠DCO.
- Vertical angles are congruent , hence ∠AOB and ∠DOC are congruent.

1.4 Right Triangle Similarity Theorem

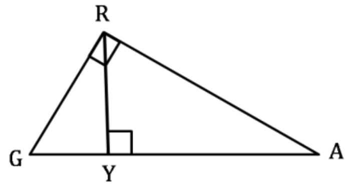
In a right triangle, the altitude to the hypotenuse divides the triangle into similar triangles, each similar to the original triangle.

Illustration

Given: △GRA is a right triangle with ∠GRA as right angle, $\overline{GA}$ as the hypotenuse and $\overline{RY}$ is the altitude to the hypotenuse of △GRA.

Prove: △GRA ~ △RYG ~ △RYA

Proof:



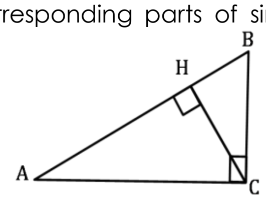
Statements	Reasons
GRA is a right triangle with ∠GRA as right angle, $\overline{GA}$ as the hypotenuse and $\overline{RY}$ as the altitude to the hypotenuse of △GRA.	Given
$\overline{RY} \perp \overline{GA}$	Definition of Altitude
∠RYG and ∠RYA are right angles	Definition of Perpendicular Lines
∠RYG ≅ ∠RYA ≅ ∠GRA	Definition of Right Angles
∠YGR ≅ ∠RGA ; ∠YAR ≅ ∠RAG	Reflexive Property
∴ △GRA ~ △RYG ~ △RYA	AA Similarity Theorem

Example

- Using the figure on the right, name the three similar triangles.
- Write the proportions that exist among corresponding parts of similar triangles.

Solution

- △ABC, △ACH and △CBH
- $\frac{AB}{AC} = \frac{BC}{CH} = \frac{AC}{AH}$; $\frac{AC}{CB} = \frac{CH}{BH} = \frac{AH}{CH}$; $\frac{AB}{CB} = \frac{BC}{BH} = \frac{AC}{CH}$



1.5 Special Right Triangle Theorem

We have two theorems under the special triangle:

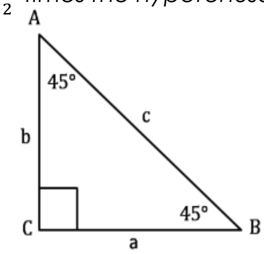
1.5.1 The Isosceles Right Triangle Theorem or the 45°-45°-90° Right Triangle Theorem

- *The length of the hypotenuse of a 45°-45°-90°triangle is $\sqrt{2}$ times the length of the leg and each leg is $\frac{\sqrt{2}}{2}$ times the hypotenuse.*

Illustration

Given:△ABC is a 45°-45°-90° triangle.

Prove: c = a$\sqrt{2}$



Proof:

△ABC is a 45°-45°-90° triangle. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$), $a^2 + a^2 = c^2$. Simplifying, it follows that $c^2 = 2a^2$, $c = \sqrt{2a^2}$, and $c = a\sqrt{2}$.

<table>
<tr><th>IV. LEARNING PHASES</th><th>Suggested Time Frame</th><th>Learning Activities</th></tr>
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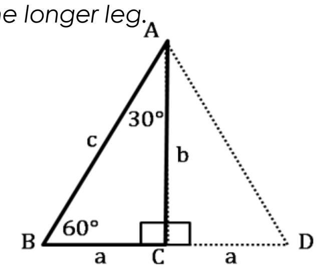
1.5.2 The 30°-60°-90° Right Triangle Theorem

- *In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.On the other hand, the shorter leg is $\frac{1}{2}$ the hypotenuse or $\frac{\sqrt{3}}{3}$ times the longer leg.*

Illustration

Given:△ABC is a 30°-60°-90° triangle.

*Prove:*c = 2a and b = a$\sqrt{3}$



Proof:

Draw △ADC so that △ABC ≅ △ADC. m∠BAC + m∠DAC = m∠BAD = 60°. m∠B = m∠D = m∠BAD = 60°. This shows that △ABDis equiangular, and hence, equilateral. It follows that c = 2a. Using Pythagorean Theorem, $a^2 + b^2 = (2a)^2 = 4a^2$. When simplified, $b^2 = 3a^2$ or b = a$\sqrt{3}$.

</td></tr>
<tr><td>C. Engagement</td><td>60 minutes</td><td>

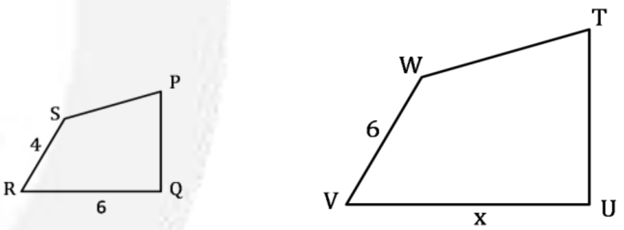
Learning Task

Directions: Answer each of the following.

1. How do you find the scale factor of similar polygons?

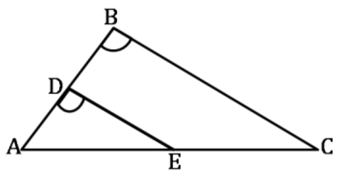
2. Illustrate or draw: △ART ~△PEN. Then, complete each statement: ∠A ≅___ ; ∠R ≅ ___ ; ∠T ≅ ___ ; $\frac{AR}{PE}$ = ___ = ___

3. Complete the following statement.

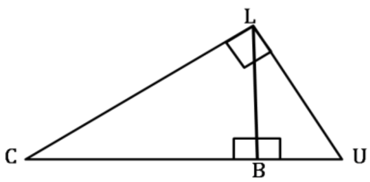


a. If PQRS~ TUVW, then
∠R≅ ___ ; ∠Q≅ ___ ; $\frac{PS}{TW}$ = ___ ; x = ___
b. The scale factor of PQRS ~ TUVW is ___.

4. In the given figure, △ADE ~ △ABC. Which triangle similarity theorem justifies this similarity? Show proof toyour answer.



5. a. Using the figure below, name the three similar triangles.
b. Write the proportions that exist among corresponding parts of similar triangles.



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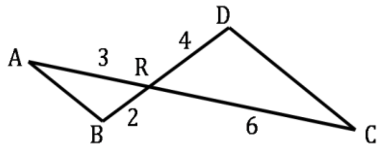
</table>

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
D. Assimilation	60 minutes	see below

D. Assimilation — 60 minutes

Directions: Answer each of the following accordingly.
1. How do you find similar polygons?
2. Are all squares similar? Explain your answer.
3. Using the figure on the right, are the two triangles similar?
   If so, state the triangle similarity theorem and justify your answer.

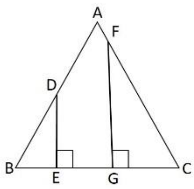


4. a. Given: △CUP and △JAR. State the proportions that must be true if △ CUP ~ △JAR by SSS Similarity.
   b. Given the statement that shows the proportionality of the three corresponding sides of the two triangles, $\frac{TR}{OU} = \frac{RY}{UT} = \frac{TY}{OT}$, name the two similar triangles.
5. How do you solve a 30°-60°-90° right triangle given only the hypotenuse?

V. ASSESSMENT (Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6) — 60 minutes

Directions: Answer each of the following accordingly.
1. Complete each statement.
   a. If for two polygons corresponding angles are ___ and corresponding sides are ___, then the polygons are similar.
   b. If the scale factor between two similar triangles is one, then the triangles are___.
   c. To find the length of the hypotenuse of a 45°-45°-90° triangle, multiply the length of one of the legs by ___.
2. Fill in the statements and reasons that are left blank in proving the proportionality of the given triangles.

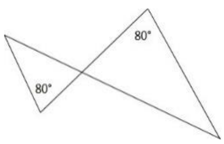
Given: ΔABC is isosceles with base $\overline{BC}$. $\overline{DE} \perp \overline{BC}, \overline{FG} \perp \overline{BC}$.
Prove: $\frac{DE}{FG} = \frac{BE}{CG}$

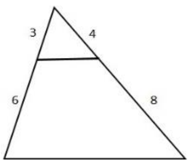


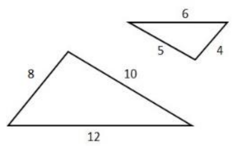
Proof:

Statements	Reasons
1. ΔABC is isosceles	1.
2.	2. Base angles of an isosceles triangle are congruent.
3. $\overline{DE} \perp \overline{BC}, \overline{FG} \perp \overline{BC}$	3.
4. ∠BED and ∠CGF are right angles.	4.
5. ΔBED and ΔCGF are right triangles.	5.
6.	6. Right Triangle Similarity Theorem
7. ∴ $\frac{DE}{FG} = \frac{BE}{CG}$	7.

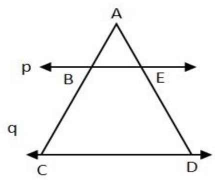
3. The following pairs of triangles are similar. State a theorem that supports your answer.

a. 

b. 

c. 

4. *Given: p // q*
Which of the following is not necessarily true?
   a. AB : AC = AE : AD
   b. ∠ACD ≅ ∠ABE
   c. AB : BC = AE : ED
   d. AB : ED = AE = BC
   e. m∠BED + m∠CDA = 180



IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
VI. REFLECTION	20 minutes	• The learners communicate the explanation of their personal assessment as indicated in the Learner's Assessment Card. • The learners will write their personal insights about the lesson in their notebook using the prompts below: I understand that ___________________. I realize that ________________________. I need to learn more about __________.

Prepared by:	Edgar V. Tuico	Checked by:	MA. FILIPINA M. DRIO

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

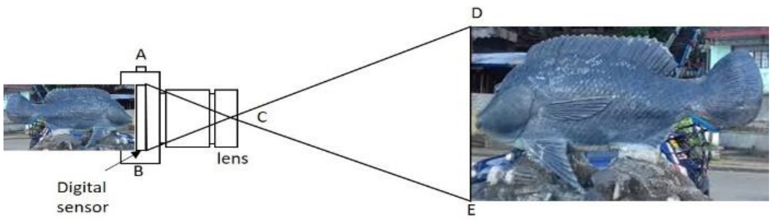
✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

W8	Learning Area	Mathematics	Grade Level	Nine
	Quarter	Third	Date	

I. LESSON TITLE	The Application of Similar Triangles Theorems and Proof of Pythagorean Theorem, and Solving Related Problems.
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	Lesson 1: Applies the theorems to show that given triangles are similar **(M9GE-IIIi-1)** Lesson 2: Proves the Pythagorean Theorem **(M9GE-IIIi-2)** Lesson 3: Solves problems that involve triangle similarity and right triangles **(M9GE-IIIj-1)**
III. CONTENT/CORE CONTENT	Applying the Similar Triangles Theorems and the Proof of Pythagorean Theorem in Solving Related Problems.

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
A. Introduction	10 minutes	 By observing the photo of Casa San Pablo, can you identify some geometric figures? Can you identify figures with the same shape but different in sizes? The concept of similarity has a wide range of applications such as in engineering, architecture, surveying, visual arts like painting, photography, and many others. In photography, when a photograph is taken, the image formed on the digital sensor is similar to the object being photographed. The illustration below will explain to us how similar triangles are being formed. This will also give us an idea on how we can apply the triangle similarity theorems in solving real-life mathematical problems that are related to this lesson.  Here, $\Delta ACB \cong \Delta ECD$.

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
B. Development	80 minutes	Follow these examples on how to apply similar triangle theorems.

Application of similar triangle theorems

1. Given $\Delta BRY \sim \Delta ANT$. What is the measure of $\overline{BR}$ and $\overline{BY}$?

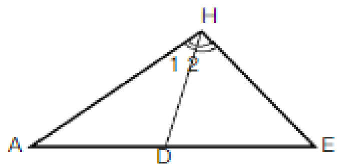
a. Solving for $\overline{BR}$

$\frac{\overline{NT}}{\overline{RY}} = \frac{\overline{AN}}{\overline{BR}}$	Formula for Proportionality
$\frac{10}{30} = \frac{15}{\overline{BR}}$	Substitution
$\overline{BR} = \frac{15(30)}{10}$	Multiplication Property of Equality
$\overline{BR} = 45$	Multiplication and Division Property

b. Solving for $\overline{BY}$

$\frac{\overline{NT}}{\overline{RY}} = \frac{\overline{AT}}{\overline{BY}}$	Formula for Proportionality
$\frac{10}{30} = \frac{15}{\overline{BY}}$	Substitution
$\overline{BY} = \frac{18(30)}{10}$	Multiplication Property of Equality
$\overline{BY} = 54$	Multiplication and Division Property

2. Given: $\overline{HD}$ bisects $\angle AHE$. Complete the following table below using Triangle Bisector Theorem.



Set	$\overline{AH}$	$\overline{HE}$	$\overline{AD}$	$\overline{DE}$	$\overline{AE}$
1	18	?	?	25	40
2	8	$y-2$	$\frac{9}{2}$	$\frac{y}{2}$	?

a. Solving for set 1 value of $\overline{HE}$ and $\overline{AD}$

$\overline{AD} + \overline{DE} = \overline{AE}$	Segment Addition Postulate
$\overline{AD} + 25 = 40$	Substitution
$\overline{AD} + 25 - 25 = 40 - 25$	Addition Property of Equality
$\overline{AD} = 15$	Subtraction
$\frac{\overline{AH}}{\overline{AD}} = \frac{\overline{HE}}{\overline{DE}}$	Triangle Bisector Theorem
$\frac{18}{15} = \frac{\overline{HE}}{25}$	Substitution
$\overline{HE} = \frac{18(25)}{15}$	Multiplication Property of Equality
$\overline{HE} = 30$	Multiplication and Division Property

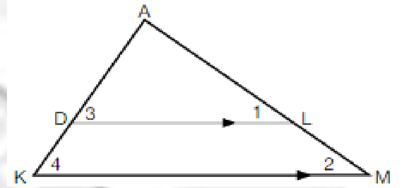
b. Solving for set 3 values of $\overline{HE}$ and $\overline{DE}$

$\frac{\overline{AH}}{\overline{AD}} = \frac{\overline{HE}}{\overline{DE}}$	Triangle Bisector Theorem
$\frac{8}{\frac{9}{2}} = \frac{y-2}{\frac{y}{2}}$	Substitution

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

$8(\frac{y}{2}) = \frac{9}{2}(y-2)$	Multiplication Property of Equality
$8y = 9(y-2)$	Multiply both sides by two
$8y = 9y - 18$	Distributive Property
$8y - 8y + 18 = 9y - 8y - 18 + 18$	Addition Property of Equality
$y = 18$	Addition and Subtraction Property
$\overline{HE} = (18) - 2$	Substitution
$\overline{HE} = 16$	Subtraction Property
$\overline{DE} = \frac{(18)}{2}$	Substitution
$\overline{DE} = 9$	Division Property

3. Given: $\overline{DL}//\overline{KM}$. Complete the following table below using Triangle Proportionality Theorem.



Set	$\overline{AD}$	$\overline{AK}$	$\overline{AL}$	$\overline{AM}$
1	4	10	x	$x+9$
2	12	30	x	$x+30$

a. Solving for set 1 value of $\overline{AL}$ and $\overline{AM}$.

$\frac{\overline{AD}}{\overline{AK}} = \frac{\overline{AL}}{\overline{AM}}$	Triangle Proportionality Theorem
$\frac{4}{10} = \frac{x}{x+9}$	Substitution
$4(x+9) = 10x$	Multiplication Property of Equality
$4x + 36 = 10x$	Distributive Property
$4x - 4x + 36 = 10x - 4x$	Addition Property of Equality
$6x = 36$	Subtraction Property
$x = 6$	Dividing both sides by 6
$\overline{AL} = 6$	Substitution
$\overline{AM} = (6) + 9$	Substitution
$\overline{AM} = 15$	Addition Property

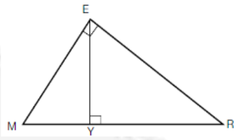
b. Solving for set 2 values of $\overline{AL}$ and $\overline{AM}$.

$\frac{\overline{AD}}{\overline{AK}} = \frac{\overline{AL}}{\overline{AM}}$	Triangle Proportionality Theorem
$\frac{12}{30} = \frac{x}{x+30}$	Substitution
$12(x+30) = 30x$	Multiplication Property of Equality
$12x + 360 = 30x$	Distributive Property
$12x - 12x + 360 = 30x - 12x$	Addition Property of Equality
$18x = 360$	Subtraction Property
$x = 20$	Dividing both sides by 18
$\overline{AL} = 20$	Substitution
$\overline{AM} = (20) + 30$	Substitution
$\overline{AM} = 50$	Addition Property

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

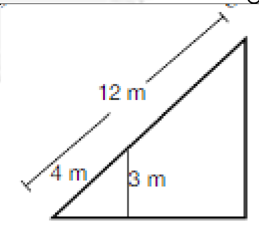
Proof of Pythagorean Theorem

- The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

Given: Right ΔMER with altitude $\overline{EY}$. Prove: $(MR)^2 = (ME)^2 + (ER)^2$ 	Statements	Reasons
	1. Right ΔMER with altitude $\overline{EY}$.	Given
	2. $\Delta MER \sim \Delta EYR \sim \Delta MYE$	Right Triangle Similarity Theorem
	3. $\frac{MY}{ME} = \frac{ME}{MR}$ and $\frac{YR}{ER} = \frac{ER}{MR}$	Special Properties of Right Triangle
	4. $(ME)^2 = (MY)(MR)$ and $(ER)^2 = (YR)(MR)$	Cross Multiplication
	5. $(ME)^2 + (ER)^2 = (MY)(MR) + (ER)^2$	Addition Property of Equality
	6. $(ME)^2 + (ER)^2 = (MY)(MR) + (YR)(MR)$	Substitution
	7. $(ME)^2 + (ER)^2 = (MR)((MY) + (YR))$	Factoring
	8. $(ME)^2 + (ER)^2 = (MR)(MR)$	Segment Addition Postulate
	9. $(MR)^2 = (ME)^2 + (ER)^2$	Product Law of Exponents

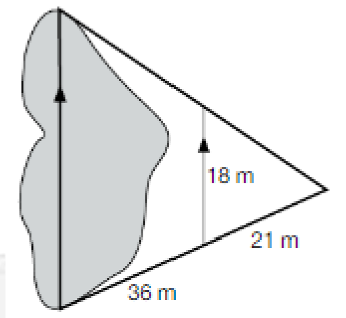
Problem Solving Involving Similar Triangles and Right Triangles

1. A 12-meter fire truck ladder leaning on a vertical fence also leans on the vertical wall of a burning three-storey building. How high does the ladder reach?

		
	$\frac{4}{3} = \frac{12}{x}$	Given
	$x = \frac{(3)12}{4}$	Multiplication Property of Equality
	$x = 9$ meters	Multiplication and Division Property

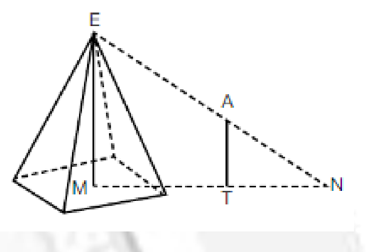
IV. LEARNING PHASES	Suggested Time Frame	Learning Activities

2. Solve for the distance across the lake.



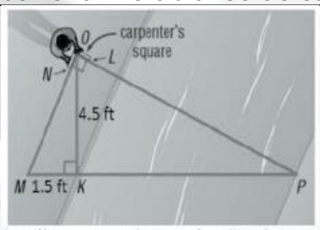
$\frac{21}{18} = \frac{21+36}{x}$	Given
$x = \frac{(18)57}{21}$	Multiplication Property of Equality
$x = 48.86$ meters	Multiplication and Division Property

3. Determine the height of a pyramid given that it cast a shadow 80 ft. wide while at the same time, a 6ft person cast an 8ft shadow.



$\frac{6}{8} = \frac{x}{80}$	Given
$x = \frac{(6)80}{8}$	Multiplication Property of Equality
$x = 60$ feet	Multiplication and Division Property

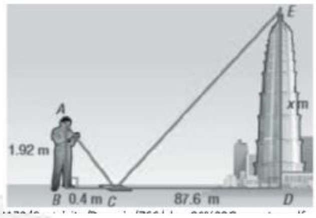
4. Solve for the distance across the river.



$\frac{1.5}{4.5} = \frac{4.5}{KP}$	Geometric Mean
$KP = \frac{(4.5)4.5}{1.5}$	Multiplication Property of Equality
$KP = 13.5$ feet	Multiplication and Division Property
$(MO)^2 = (MK)^2 + (OK)^2$	Pythagorean Theorem
$(MO)^2 = (1.5)^2 + (4.5)^2$	Substitution
$MO = 4.73$ feet	Square root of the sum of the squares of the legs
$(MP)^2 = (MO)^2 + (OP)^2$	Pythagorean Theorem
$(15)^2 = (4.73)^2 + (OP)^2$	Substitution
$MN = 14.23$ feet	Square root of the difference of the squares of the hypotenuse and a leg

http://www.agusta.k12.va.us/cms/lib01/VA01000173/Centricity/Domain/766/chap06%20Geometry.pdf

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
		5. Solve for the height of the skyscraper whose top is reflected on the mirror.
C. Engagement	40 minutes	**Learning Task 1** Directions: Solve the following.



$\frac{x}{87.6} = \frac{1.92}{0.4}$	Given
$x = \frac{(1.92)87.6}{0.4}$	Multiplication Property of Equality
$x = 420.48$ meters	Multiplication and Division Property

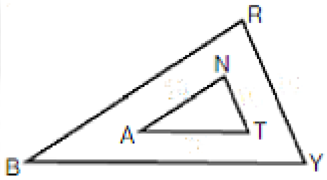
http://www.agusta.k12.va.us/cms/lib01/VA01000173/Centricity/Domain/766/chap06%20Geometry.pdf

Learning Task 1

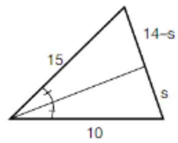
Directions: Solve the following.

1. Given $\Delta BRY \sim \Delta ANT$ with AN = 4, NT = 3, and RY = 15. What is the measure of BR?

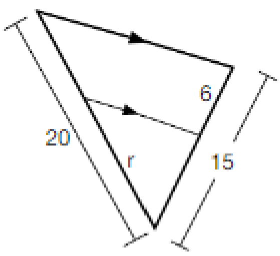
2. Given $\Delta BRY \sim \Delta ANT$ with BY = 10x + 5, BR = 10x + 2, AT= 4x – 3 and AN= 3x + 3. What is the value of x?



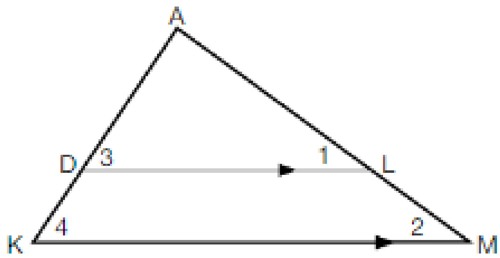
3. Solve for s.



4. Solve for the value of r.

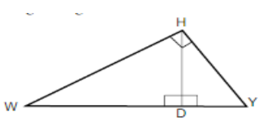


5. Given $\Delta ADL \sim \Delta AKM$ with AD = 14, DK = 21, and AL= 15. What is the measure of LM?

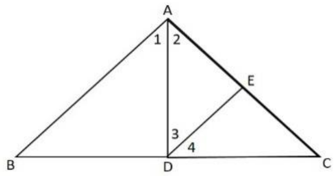
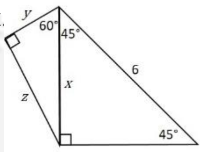
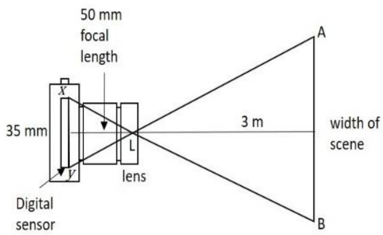


Learning Task 2

Directions: Solve the following using Pythagorean Theorem given the figure below.



1. If DY is 12, DW is 27, what is DH?
2. If HW is 41, DW is 40, what is DH?
3. If DY is 12, DH is 35, what is HY?
4. If HW is 8, HY is 15, what is WY?
5. If HW is 20, HY is 21, what is WY?

IV. LEARNING PHASES	Suggested Time Frame	Learning Activities
D. Assimilation	30 minutes	**Learning Task 3** Directions: Solve the following problems. 1. If the hypotenuse of a right triangle measures 25 meters while a leg is 24 meters, what is the measure of the other leg? 2. A tower casts a shadow 7m long. A vertical stick casts a shadow 0.6m long. If the stick is 1.2m high, how high is the tower? 3. The length of the shadow of your 1.6-meter height is 2.8 meters at a certain time in the afternoon. How high is an electrical post in your back yard if the length of its shadow is 20 meters? 4. The size of a TV screen is given by the length of its diagonal. If the dimension of a TV screen is eight inches by 15 inches, what is the size of the TV screen? 5. A 13-meter ladder is leaning against a vertical wall. If the foot of the ladder is five meters away from the wall, how high does the ladder reach?
V. ASSESSMENT (Learning Activity Sheets for Enrichment, Remediation or Assessment to be given on Weeks 3 and 6)	30 minutes	Directions: Answer each of the following accordingly. 1. Complete each statement. a. $\frac{AE}{EC}$ = ___ b. $\frac{AB}{AC}$ = ____ c. If BD = 3, DC = 4, and AB = 6 , then AC = __  2. Find the length x, y, and z.  3. The figure describes a camera with digital sensor width xy that is 35mm and with focal length 50mm. What is the width of the scene AB? 
VI. REFLECTION	10 minutes	• The learners communicate the explanation of their personal assessment as indicated in the Learner's Assessment Card. • The learners will write their personal insights about the lesson in their notebook using the prompts below: I understand that ____________________. I realize that ________________________. I need to learn more about ___________.

Prepared by:	Wilson Anzures and Edgar V. Tuico	**Checked by:**	MA. FILIPINA M. DRIO

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	