10 MATH

Quarter 1

PIVOT 4A LEARNER'S MATERIAL

This module is a resource of information and guide in understanding the Most Essential Learning Competencies (MELCs). Understanding the target contents and skills can be further enriched thru the K to 12 Learning Materials and other supplementary materials such as worksheets/activity sheets provided by schools and/ or Schools Division Offices and thru other learning delivery modalities including radio-based and TV-based instruction (RB/TVI).

CLMD CALABARZON

Mathematics

Grade 10

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MATH Grade 10
PIVOT IV-A Learner's Material
Quarter 1
First Edition, 2020

Published by: Department of Education Region IV-A CALABARZON
Regional Director: Wilfredo E. Cabral
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Guide in Using PIVOT Learner's Material

For the Parents/Guardian

This module aims to assist you, dear parents, guardians, or siblings of the learners, to understand how materials and activities are used in the new normal. It is designed to provide the information, activities, and new learning that learners need to work on.

Activities presented in this module are based on the Most Essential Learning Competencies (MELCs) for English as prescribed by the Department of Education.

Further, this learning resource hopes to engage the learners in guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

You are expected to assist the child in the tasks and ensure the learner's mastery of the subject matter. Be reminded that **learners have to answer all the activities in their own notebook**.

For the Learners

The module is designed to suit your needs and interests using the IDEA instructional process. This will help you attain the prescribed grade-level knowledge, skills, attitude, and values at your own pace outside the normal classroom setting.

The module is composed of different types of activities that are arranged according to graduated levels of difficulty—from simple to complex. You are expected to **answer all activities on separate sheets of paper** and submit the outputs to your respective teachers on the time and date agreed upon.

PARTS OF PIVOT LEARNER'S MATERIAL

	Parts of the LM	Description
Introduction	What I need to know	The teacher utilizes appropriate strategies in presenting the MELC and desired learning outcomes for the day or week, purpose of the lesson, core content and relevant samples. This allows teachers to maximize learners awareness of their own knowledge as regards content and skills required for the lesson
	What is new	
Development	What I know	The teacher presents activities, tasks , contents of value and interest to the learners. This shall expose the learners on what he/she knew, what he /she does not know and what she/he wanted to know and learn. Most of the activities and tasks must simply and directly revolved around the concepts to develop and master the skills or the MELC.
	What is in	
	What is it	
Engagement	What is more	The teacher allows the learners to be engaged in various tasks and opportunities in building their KSA's to meaningfully connect their learnings after doing the tasks in the D. This part exposes the learner to real life situations /tasks that shall ignite his/ her interests to meet the expectation, make their performance satisfactory or produce a product or performance which lead him/ her to understand fully the skills and concepts .
	What I can do	
	What else I can do	
Assimilation	What I have learned	The teacher brings the learners to a process where they shall demonstrate ideas, interpretation , mindset or values and create pieces of information that will form part of their knowledge in reflecting, relating or using it effectively in any situation or context. This part encourages learners in creating conceptual structures giving them the avenue to integrate new and old learnings.
	What I can achieve	

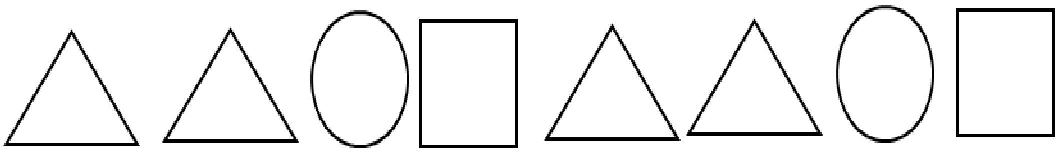
WEEK 1

Sequence

Lesson

I

After going through this lesson, you are expected to generate patterns and sequences. Probably without even knowing it, you have been observing and creating patterns ever since you were a very small child. You probably made repeating patterns with shapes, such the one below with triangles, circles, and squares.



A sequence is a set of numbers written in a specific order: a_1, a_2, a_3, a_4, a_5, a_6,…, a_n. The number a_1 is called the 1st term, a_2 is the 2nd term, and in general, a_n is the nth term.

Illustrative Example 1

Finite Sequence is a sequence with last term while infinite sequence is a sequence without last term.

Finite Sequence	Infinite Sequence
12, 15, 17, 19, 21,…27 1, 1, 2, 3, 5, 8, 13 5, 5, 5, 5, 5	3, 5, 7, 9, 11, … 1, 1, 2, 3, 5, … 1, -1, 1, -1, …

Illustrative Example 2

Find the next term of the sequence 19, 24, 29, 34, …

Solution:

Observe the pattern of the sequence. Observe the interval of each term. To get the next term of the pattern you add 5 to the previous number.

Illustrative Example 3

What is the next term of the sequence 6, 12, 24, 48, …?

Solution:

For this example, 6 is multiplied by 2 to get 12, 12 is multiplied by 2 to get 24 and 24 is also multiplied by 2 to get 48. So the next term is 96.

Illustrative Example 4

Find the first 4 terms of the given rule an=3n-2.

$a_n = 3n-2$	$a_n = 3n-2$	$a_n = 3n-2$	$a_n = 3n-2$
$a_1 = 3(1)-2 = 1$	$a_2 = 3(2)-2 = 4$	$a_3 = 3(3)-2 = 7$	$a_4 = 3(4)-2 = 10$

D

Learning Task 1

An old woman is overweight. Her doctor told her to decrease 35 kilos. If she loss 11 kilos in the 1st week, 9 kilos in the 2nd week and 7 kilos on the 3rd week. If she continues losing at this rate, how long will it take her to lose 35 kilos?

Fill out the table below to get the answer.

Week	1	2	3	4	5
Kilos					

E

Learning Task 2

A. Write F if the sequence is finite or I if the sequence is infinite before the number.

__1. 5, 15, 25, 35

__2. 2, 4, 8, 16, …

__3. 1, 9, 17, 25

__4. -9, -4, 1, 6, …

__5. 2, 9, 16, 23

__6. 2, 6, 18, 54,…

__7. 3, 6, 9, 12, … 30

__8. 7, 7, 7, 7, 7, 7

__9. 16, 21, 26, 31

__10. 24, 19, 14, 9,…

B. Find the next three terms of the given sequences.

1. 2, 5, 8, 11, __, __, __
2. 1, -3, -7, -11, __, __, __
3. 1, 4, 16, 64, __, __, __
4. 60, 48, 36, 24, __, __, __
5. 1, 4, 9, 16, 25, __, __, __
6. 2, 4, 8, 16, __, __, __
7. 2, 6, 18, 54, __, __, __
8. 3.2, 4.3, 5.4, __, __, __
9. -7,-9, -11, -13, __, __, __
10. , , , , __, __, __

C. Write the first four terms of the sequence whose n^{th} term is given by the rule.

1. $a_n = 2n-1$
2. $a_n = 12-3n$
3. $a_n = 3n$
4. $a_n = 5n + 5$
5. $a_n = 8 - 2n$

A

Learning Task 3. Determine your family's expenses in the last 4 months. Analyze it and find a pattern. Create a sequence equation for it and determine the next 3 months' expenses.

Direction: Answer the following questions.

1. Is analyzing a sequence challenging for you?
2. Were you able to find patterns and get the unknown?
3. What mathematical concept/s did you use to find the unknown?
4. What realization did you have with this lesson?

WEEK 2

Arithmetic Sequence

I Lesson

After going through this lesson, you are expected to illustrate an arithmetic sequence, determine arithmetic means and nth term of an arithmetic sequence.

On the previous lesson, you work with sequences or patterns. Recognizing and extending patterns are important skills needed for learning concepts about arithmetic sequence. Knowledge on patterns will be able to help you determine the nth term and terms between any two consecutive terms of an arithmetic sequence.

An **arithmetic sequence** is a sequence where each succeeding term is obtained by adding a fixed number. The fixed number is called the common difference which is denoted as d. To find the next terms in an arithmetic sequence, we use the formula:

$$\mathbf{a_n = a_1 + (n - 1)d}$$

where;

a_n – the last nth term

a_1 – the first term

n – the number of terms in the sequence

Study the given examples below and then identify if it is arithmetic or not.

1. 10, 13, 16, 19,...
2. 2, 6, 18, 54...
3. 57, 49, 41

Study the illustrative examples below.

Illustrative Example 1

Determine the 10^{th} term in the sequence 4, 6, 8, 10, ...

$$\mathbf{a_n = a_1 + (n - 1)d}$$

The first element: $a_1 = 4$. The common difference: $d = 2$. The term: $n = 10$.

$a_{10} = 4 + (10 - 1)2$

$a_{10} = 4 + (9)2$

$a_{10} = 4 + 18$

$a_{10} = 22$

Therefore, 22 is the 10^{th} term of the arithmetic sequence.

The terms between any two nonconsecutive terms of an arithmetic sequence are known as **arithmetic means**.

Illustrative Example 2

Find two arithmetic means between 2 and 8.

Using d = 2, generate the next terms by adding "d" to the previous term. So $a_2 = a_1 + d$ and $a_3 = a_2 + d$ which means $a_2 = 2 + 2 = 4$ and $a_3 = 4 + 2 = 6$.

$$\mathbf{2 ,\ a_2,\ a_3,\ 8}$$

$$a_n = a_1 + (n - 1)d$$

$$8 = 2 + (4 - 1)d$$

$$8 = 2 + 3d$$

$$8 - 2 = 3d$$

$$6 = 3d$$

$$d = 2$$

The numbers 4 and 6 are the two arithmetic means between 2 and 8.

D

Learning Task 1

Each item below shows a pattern. Determine the next term in the sequence.

1. What is the next shape?

♥,□,♥,♥,□,♥,♥,♥,□,♥,♥,♥,♥,___

2. What is the next shape? _______

3. What is the next number?

5, 15, 25, 35, ___

4. What is the next number?

2, 4, 8, 16, ___

5. What is the next number?

-9, -4, 1, 6, ___

E

Learning Task 2

A. Determine whether the sequence is arithmetic or not. If it is, find the common difference and the next three terms.

1. 2, 5, 8, 11,...
2. 2, -4, 6, -8, 10,...
3. -6, -10, -14, -18,...
4. 40, 42, 44, 46,...
5. 1.2, 1.8, 2.4,...
6. 1, 5, 9, 13,...

B. After familiarizing yourself on finding the nth term of an arithmetic sequence, let us try to answer the exercises below.

Find the term indicated in each of the following arithmetic sequences.

1. 2, 4, 6, ... 15th term _______
2. 13, 16, 19, 22, ... 25th term _______
3. 99, 88, 77, 66, ... 18th term_______
4. $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \ldots$ 20th term _______
5. 99, 87, 75, 63, ... 12th term _______

C. Solve what is asked:

1. Insert four arithmetic means between -1 and 14.
2. Insert five arithmetic means between 14 and 86.
3. Insert three arithmetic means between -16 and 4.

A

Learning Task 3

1. How do you find the nth term of the arithmetic sequence? Discuss the mathematics concepts and the formula used.
2. 2. What new realizations do you have about finding the nth term of the arithmetic sequence and finding the arithmetic means? How would you use this in making decisions?

Geometric Sequence

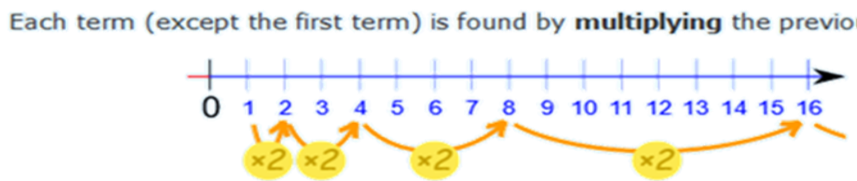
Lesson

I

A **Geometric Sequence,** also known as Geometric Progression, is a set of terms in which each term after the first is obtained by multiplying the preceding term by the same fixed number called the Common Ratio which is commonly represented by r.

Illustrative Example 1

The number pattern 1, 2, 4, 8, 16, … is a geometric sequence. Two is multiplied to any term to get the next term. Therefore, we can say that 2 is the common ratio. The common ratio may be an integer or fraction, negative or positive. It can be found by dividing any term by the term that precedes it. The number line below illustrates the sequence:



Using the picture above, we illustrated and proved that the common ratio is equal to 2.

Now, let us determine the next two terms in the given sequence 1, 2, 4, 8, 16,…

Since the common ratio is 2 and the first term is 1,

$$
\begin{array}{ll}
1(2) = 2 & ---\ \textit{this is the 2nd term} \\
2(2) = 4 & ---\ \textit{this is the 3rd term} \\
4(2) = 8 & ---\ \textit{this is the 4th term} \\
8(2) = 16 & ---\ \textit{this is the 5th term} \\
16(2) = 32 & ---\ \textit{this is the 6th term} \\
32(2) = 64 & ---\ \textit{this is the 7th term}
\end{array}
$$

Thus, the next two terms of the geometric sequence 1, 2, 4, 8, 16, are 32 and 64.

What do you think are the 8th and 9th terms of this sequence? _______________

Illustrative Example 2.

In the sequence 3, 8, 13, 18, 23 …there is no common ratio among the terms. However, there is a common difference of 5. To prove, let us have the following solution:

$d = 23 - 18 = 5$ $d = 18 - 13 = 5$ $d = 13 - 8 = 5$ $d = 8 - 3 = 5$

Thus, this is not a geometric sequence but an arithmetic sequence.

Illustrative Example 3.

In the sequence -20, 10, -5, , the *Common Ratio* is To prove, let us have the following solution:

$$r = \frac{5}{2} \div (-5) = \frac{5}{2}\left(-\frac{1}{5}\right) = -\frac{1}{2}$$

$$r = \frac{-5}{10} = -\frac{1}{2}$$

$$r = \frac{10}{-20} = -\frac{1}{2}$$

Thus, we proved that the common ratio is and this sequence is a **Geometric Sequence**

Illustrative Example 4

Determine if there is a common ratio or common difference in 6, 12, 14, 28, … To prove, let us have the following solutions:

Let us solve for the common Arithmetic:	Let us solve for the common Geometric:
$d = 28 - 14 = 14$	$r = \frac{28}{14} = 2$
$d = 14 - 12 = 2$	$r = \frac{14}{12} = \frac{7}{6}$
$d = 12 - 6 = 6$	$r = \frac{12}{6} = 2$

D

After going through this lesson, you are expected to illustrate a geometric sequence, differentiate a geometric sequence from an arithmetic sequence; and determine geometric means and nth term of a geometric sequence.

Learning Task 1

Determine if the pattern illustrates Geometric Sequence or not.

1. 5, 10, 20, 40, 80, …
2. 3, 6, 9, 12, 15, …
3. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$, 1, 2, …
4. 1, 3, 9, 27, 81, …
5. 10, 9, 8, 7, 6, …

Thus, we proved that this sequence is neither an *Arithmetic* nor *Geometric Sequence.*

To solve for the geometric means of a given geometric sequence, the formula for the nth term of a geometric sequence is also used.

$$a_n = a_1 \cdot r^{n-1}$$

where n is the number of the term (term number) and a_1 is the first term.

Let us use the geometric sequence in Activity, (4, 8, 16, 32, 64). For instance, the only given in the problem are the first term and the last term, as in 4, __, __, __, 64. How can we solve the geometric means?

Solution:

Step 1: Find the common ratio

$a_n = a_1r^{n-1}$

$64 = 4r^{5-1}$ substitute the first and last terms

$64 = 4r^4$ simplify the exponent

$\frac{64}{4} = \frac{4}{4}r^4$ apply MPE

$16 = r^4$ coefficient of r now is 1

$2^4 = r^4$ exponential equation

$\pm 2 = r$ common ratio

Step 2: Multiply the first term by the common ratio $r = 2$ to get the second term. Repeat the process until you solve the three geometric means. Use r = -2 to find the other geometric sequence

$a_2 = 4 \times 2$; $a_2 = 8$ $a_3 = 8 \times 2$; $a_3 = 16$ $a_4 = 16 \times 2$; $a_4 = 32$

Note: There are two common ratios, therefore there are also two sets of geometric sequences: 4, 8, 16, 32, 64 and 4, -8, 16, -32, 64

Answer: The three geometric means are 8, 16, 32 and -8, 16, -32

E

Learning Task 2

A. Determine the common ratio and the next 3 terms of the following geometric sequence. Write your answer on the space provided. Item number 1 is done for you...

Given	**Common Ratio**	***Next 3 terms***
4, 8, 16, ...	2	*32, 64, 128*
972, 324, 108, ...		
-3, 12, -48, ...		
0.1, 0.5, 2.5, ...		
10 000, 1 000, 100, ...		

B. Translate the word "I love you very much!" in French by determining and matching the common ratio or common difference of the following and then placing each letter on the spaces below the decoder:

JE' 1, 4, 7, 10, …	***EA*** 1, 4, 16, 64, …	***P*** -7, -56, -392, …	***IM*** 2, -5, -12, -19, …
EB **88, 83, 78, 73, …**	***OU*** 250, 50, 10, 2, …	***TA*** $\frac{1}{2}, 1, 1\frac{1}{2}, 2$ …	***UC*** -3, 6, -12, 24, …

Write your answer on the decoder below:

d = 3	**d =** $\frac{1}{2}$	**d = -7**	**d = -5**	**r = 4**	**r = -2**	**r =** $\frac{1}{5}$	**r = 8**

And the French word for "I love you very much!" is ….

___ ___ ___ ___ ___ ___ ___   ___ ___ ___ ___   ___ ___ ___ ___

C. Find the 10th term of the geometric progression.

1. 4, 20 , 100,…
2. 4. -6, 9, ,…
3. -2, 6, -18,…
4. 5, -2, 4/5 ,…
5. 6, 3, 6/3 …

A

Learning Task 3. Tell something about what you have learned by answering the following questions.

1. Explain how to determine the next term of the geometric sequence 3, 6, 12, 24, ____,…

2. In your opinion, where can you apply the concepts and skills that you learned about geometric sequences? Explain.

3. What are the differences between an arithmetic sequence and a geometric sequence?

Application of Sequences

Lesson

I

After going through this lesson, you are expected to solve problems involving sequences.

GREET THEM IN KHMER

In Cambodia, Khmer is the languageof the Cambodians. Today, you will learn the basic Khmer greeting by simply answering the following questions below.

Learning Task 1. Write the letter before the number. The letters will spell out the Khmer greeting.

_____1. What is the 5^{th} term of the arithmetic sequence $a_n = 5n + 1$?

G. 23 H. 24 I. 25 J. 26

_____2. What is the next 3 terms of the Fibonacci sequence 0, 1, 1, 2, 3, 5,...?

T. 8, 10, 18 U. 8, 13, 21 V. 8, 12, 16 W. 8, 11, 14

_____3. What are the missing numbers in the sequence 4, --__, __, __, 64 ?

M. 8, 16, 32 N. 8, 14, 28 O. 8, 18, 28 P. 8, 12, 16

_____4. What is the common ratio of the given sequence 3, -6, 12,-24, 48 ?

Q. -2 R. 2 S. 3 T. -3

_____5. What is the 10^{th} term of the arithmetic sequence $a_n = 3n - 5$?

I. 25 J. 30 K. 35 L. 40

_____6. If $a_1 = 2$, $d = -2$, then what is a_{12}?

T. -20 U. -22 V. 20 W. 22

_____7. What is the common ratio of the given sequence 1000, 500, 250, 125?

P. 2 Q. -2 R. S. $\frac{1}{2}$ $-\frac{1}{2}$

_____8. What is the 8^{th} term of the sequence -10, -8, -6, -4, ...?

S. 2 T. 4 U. 6 V. 8

What is the Khmer greetings? ______________________________.

Now, try to consider the situation below.

Illustrative Example 1

Suppose the auditorium of the Tagaytay International Convention Center (TICC) has 20 seats in the first row and that each row has 2 more seats than the previous row. If there are 30 rows in the auditorium, how many seats are in the last row?

To solve real-life problems involving sequences, remember the words "SEE, PLAN, DO and LOOK BACK".

Solution:
SEE - What kind of sequence is involve in the problem?

$$20 + 22 + 24 + 26 + . . ., + a_{30}$$

PLAN – What is the appropriate formula to be used and the needed values?

$$a_n = a_1+(n - 1)d$$

Where $a_1 = 20$: $d = 2$: $n = 30$

DO – Perform the indicated operation and simplify.

$$a_n = a_1+(n - 1)d$$
$$= 20+(30 – 1)2$$
$$= 20+(29)2$$
$$= 20+58$$
$$= \mathbf{78}$$

LOOK BACK – The answer should satisfy all the given information in the problem

20 + 22 + 24 + 26 + 28 + 30 + 32 + 34 + 36 + 38 + 40 + 42 + 44 + 46 + 48 + 50 + 52 + 54 + 56 + 58 + 60 + 62 + 64 + 66 + 68 + 70 + 72 + 74 + 76 + **78**

Finding the total number of seats is solving for arithmetic series or sum given the formula:

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2}(2a_1 + (n-1)d$$

n is the number of terms in the sequence

$\boldsymbol{a_1}$ is the first term of the sequence

$\boldsymbol{a_n}$ is the last or the nth term of the sequence

d is the common difference

The sum of a geometric series id obtained using the formula:

$$S_n = \frac{a(1-r^n}{(1-r)}$$

a is the first tem

n number of terms

r common ratio

Illustrative Example 2.

Suppose in illustrative example 1 you are asked to find the total number of seats, how will you do it?

Instead of adding the number of seats per row, you must use the formula.

Given in the problem : a_1 = 20 number of seats in the first row

n = 30 rows

d = 2 common difference

Solve:

$$S_n = \frac{n}{2}(2a_1 + (n-1)d \longrightarrow S_n = \frac{30}{2}(2(20) + (30-1)2)$$

$$S_n = 15\left(40 + 29(2)\right) = 1470 \ \ \textit{total number of seats}$$

Illustrative Example 3.

Find the sum of the sequence, 2, 6, 18, 54, …, a_5.

Given; a_1 = 2, r = 3 n = 5

Solve:

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{2(1-3^5)}{1-3} = \frac{2(-242)}{-2} = 242$$

Check: The 5 terms of the sequence are 2,6,18,54,162

The sum is 2 + 6 + 18 + 54 + 162 = 242

Using long addition and formula will give you the same answer.

D

The concepts of arithmetic and geometric sequence are very essential in real life situations. Do you know that there are real-life situations that can be modeled by arithmetic and geometric sequence? This lesson will help you find solutions to solve problems involving sequences.

E

Learning Task 2. Solve the following problems.

1. To replace the trees destroyed by typhoon Yolanda, the forestry department of Tagaytay has developed a ten-year plan. During the first year they will plant 100 trees. Each succeeding year, they will plant 50 more trees than they planted the year before.

   a. How many trees will they plant during the fifth year?

   b. How many trees will they have planted by the end of the tenth year?

Solution:

Complete the table.

No. Of Years	1	2	3	4	5	6	7	8	9	10
No. Of Trees	100	150	200							

SEE

- What kind of sequence is involved in the problem?

PLAN

- What is the appropriate formula to be used and the needed values?

  Formula: ___________________________

  Determine the values of:

  a_1 = ___; n = ___; d = ___ ; S_n = ___; r = ___

DO

- Perform the indicated operation and simplify.

  Write the formula. ____________________

  Substitute the values. ____________________

LOOK BACK

- The answer should satisfy all the given information in the problem.
- Now, back to the questions:

  A. How many trees will they plant during the fifth year? _______________

  B. How many trees will they have planted by the end of the tenth year?

2. Every December, Tagaytay City Science National High School is sponsoring a Gift-giving program for an orphanage. A newspaper fund drive to collect fund was launch. A student promised that he will bring 2 newspapers on the launching day of the drive, 6 on the second day and triple the number of newspapers each day until the last day of the fund drive. If the fund drive is set from December 1 to December 5.

   a. How many newspapers will the student bring on the last day?

   b. What is the total number of newspapers that he will contribute?

A

Learning Task 3,

1. How do you solve problems involving sequence? Discuss the mathematics concepts and principles applied when solving problems involving sequence.
2. What new realizations do you have about solving problems involving sequence? How would you connect this to real life? How would you use this in making decisions?

Solve

The logs are piled such that each row is 2 less than the one below. If the there are 30 logs at bottom and the top most is 2, how many logs are there in all?

Division of Polynomials

WEEK 6

I

Lesson

After going through this lesson, you are expected to perform division of polynomials using long division and synthetic division. These procedures can be used in dividing polynomial by a binomial which is the focus of this lesson.

Steps

1. Arrange the terms of the dividend and divisor in decreasing powers. If there are missing terms, write them with a coefficient of zero.
2. Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.
3. Multiply the result in step 2 by the divisor.
4. Subtract the result from step 3. Bring down the next term of the dividend.
5. Repeat the entire process using the result in step 4 as the new dividend.
6. Express the result as:

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

Illustrative example: $(3x^3 + 19x - 10 + 16x^2) \div (x + 3)$

Solution: Following the procedure given in the chart, we have,

$$\begin{array}{rl}
 & 3x^2 + 7x - 2 \\
x + 3 & 3x^3 + 16x^2 + 19x - 10 \\
 & (-)3x^3 + 9x^2 \\
 & \qquad 7x^2 + 19x \\
 & \qquad (-)\ 7x^2 + 21x \\
 & \qquad\qquad -2x - 10 \\
 & \qquad\qquad (-)\ -2x - 6 \\
 & \qquad\qquad\qquad -4
\end{array}$$

If the remainder is zero or its degree is less than that of the divisor, you will stop dividing. Since the difference is not zero, -4 is called the remainder. The

quotient or answer is $3x^2 + 7x - 2 \quad \frac{-4}{x+3} \quad +$

SYNTHETIC DIVISION OF POLYNOMIALS

Procedure:	Solution:
1. Write the numerical coefficients in one row. If there is a missing term, write 0 to represent that missing term. Write the test zero at the left. We are dividing by x + 3 so the test zero is x = -3	-3 3 16 19 -10
2. Carry down the first number, multiply by test zero and add to the second column. 3• -3 = -9	-3 3 16 19 -10 -9 3 7
3. Add down the column. Now, multiply the sum in the second column (that is, 7) by the test zero. Thus 7(-3) = -21	-3 3 16 19 -10 -9 3 7
4. Write the product 21 on the third column then add. Repeat the steps for the next column. Multiply 7 by the test zero, add the result to the next column and subtract.	-3 3 16 19 -10 -9 -21 3 7 -2
5. Repeat the steps for the next column. Multiply -3 by the test zero, add the result to the next column until you reach the last column	-3 3 16 19 -10 -9 -21 6 3 7 -2 -4 remainder
The third row that we obtained represents the numerical coefficients of the terms of the quotient. The degree to be used is one less than the degree of the dividend. The right most number is the remainder. Thus, in $(3x^3 + 16x^2 + 19x - 10) \div (x + 3)$, the quotient is $3x^2 + 7x - 2$, with -4 as remainder or $3x^2 + 7x - 2 +$ or $3x^3 + 16x^2 + 19x - 10 = (x + 3)(3x^2 + 7x - 2) - 4$	

If P(x) is the dividend , D(x) is the divisor, Q(x) is the quotient and R is the remainder, then

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad or \quad P(x) = D(x)[Q(x)] + R$$

D

Learning Task 1. On the chart below, the long division method is shown in Column A with missing expressions. You are to complete the division process by finding the correct expressions for each box in Column B.

LONG DIVISION OF POLYNOMIALS

COLUMN A	COLUMN B
$\boxed{} + 7x + 24$ $x - 3 \overline{) x^3 + 4x^2 + 3x - 2}$ $x^3 - 3x^2$ $\boxed{} + 3x$ $7x^2 - \boxed{}$ $\boxed{}\ \boxed{}$ $24x - 72$ $\boxed{}$	-2 7x $7x^2$ 70 24x x^2 21x

E

Learning Task 2

A. Divide each of the following: using the two methods

1. $(x^2 - 7x + 10) \div (x - 5)$
2. $(x^2 + x - 20) \div (x - 4)$
3. $(x^3 + 3x^2 + 5x + 3) \div (x + 1)$
4. $(2x^3 - 18x - 45 + 5x^2) \div (x + 5)$
5. $(3x^2 + 7x^3 + 11x - 4) \div (x + 2)$

B. Write the dividend , P(x) in $P(x) = D(x)[Q(x)] + R$ of the above items.

A

Learning Task 3. This lesson was about performing division of polynomials using long method and synthetic division. Based on your understanding about the lesson, make your own reflection by answering the following questions:

1. How would you compare the two methods? What are their similarities and differences?
2. What new realizations do you have about performing division of polynomials using long method and synthetic division? How would you connect this to real life? How would you use this in making decisions?

Remainder Theorem and Factor Theorem

I

After going through this lesson, you are expected to prove the Remainder Theorem and the Factor Theorem.

If your goal is to find only the remainder when a polynomial in x is divided by a binomial in the form (x – c), using the synthetic division is quite long. Actually, the one that you did in the previous activity is another way and easy way of getting the remainder without using the synthetic division. That is what we call **The Remainder Theorem.**

REMAINDER THEOREM:

If the polynomial P(x) is divided by x - C, then the remainder is P (C).

Now, here is the proof of the remainder theorem

STATEMENT

$P(x) = (x - c) \bullet Q(x) + R$

$P(c) = (c - c) \bullet Q(x) + R$

$P(c) = 0 \bullet Q(x) + R$

$P(c) = R$

Note: P(x) is the given polynomial in x

(x – c) is the divisor

Q(x) is the quotient

R is the remainder

By substituting the value of (c) of the divisor x – c in the polynomial P(x), you can also test whether a certain polynomial is exactly divisible by another or is a factor by the Remainder Theorem.

The Remainder Theorem states that P(c) is the remainder when the polynomial P(x) is divided by (x – c). The divisor x – c is then restated as x=c.

$$R = P(c)$$

Illustrative Example 1

$P(x) = x^3+4x^2+3x-2 \div x-3$

$P(x) = x^3 + 4x^2 + 3x - 2;$ $x = 3$

$P(3) = (3)^3 + 4(3)^2 + 3(3) - 2$ Substitute 3 for x.

$= 27 + 36 + 9 - 2$

$P(3) = 70$ the remainder

To check if the remainder is correct use the synthetic division that was discussed in the previous lesson.

3⌋					
	1	4	3	-2	
		3	21	72	
	1	7	24	70	Remainder

Notice that the value obtained in two processes is the same.

Illustrative Example 2

$P(x) = 2x^3+4x^2+3x-2 \div x-3$

$P(x) = 2x^3+4x^2+3x-2$; $x = 3$

$P(3) = 2(3)^3 + 4(3)^2 + 3(3) - 2$ Substitute 3 for x.

$= 54 + 36 + 9 - 2$

$P(3) = 97$ the remainder

Check using synthetic division

3⌋					
	2	4	3	-2	
		6	30	99	
	2	10	33	97	Remainder

Now, the question is what happens when the remainder is 0? Is 9 a factor of 36? If your answer is Yes, what makes 9 a factor of 36? That is, 9 is a factor of 36 since 9 divides 36 without a remainder or the remainder is 0. This concept leads to another theorem which we call The Factor Theorem:

FACTOR THEOREM:

If the remainder comes out to be 0 (zero), then x - c is a factor of P (X).

A zero remainder obtained when applied using the Remainder Theorem will give rise to another theorem called the factor theorem. This is a test to find if a polynomial is a factor of another polynomial.

The Factor Theorem states that:

If P(x) is a polynomial and

1. $P(c) = 0$, then $x - c$ is a factor of P(x).
2. $x - c$ is a factor of P(x), then $P(c) = 0$.

Illustrative Example 1

$P(x) = x^3 - x^2 - 4x + 4$ divided by $(x - 2)$.

To determine whether $(x - 2)$ is a factor of $P(x) = x^3 - x^2 - 4x + 4$.

Use the remainder theorem to find the remainder.

$$\begin{aligned} P(2) &= (2)^3 - (2)^2 - 4(2) + 4 \\ &= 8 - 4 - 8 + 4 \\ &= 4 - 8 + 4 \\ &= -4 + 4 \\ &= 0 \end{aligned}$$

Since, the remainder is 0. Then we can say that $(x - 2)$ is a factor of $P(x) = x^3 - x^2 - 4x+4$ by factor theorem.

Illustrative Example 2

$P(x) = 2x^4 - x^3 - 18x^2 - 7$ divided by $(x - 3)$

$$\begin{aligned} P(3) &= 2x^4 - x^3 - 18x^2 - 7 \\ &= 2(3)^4 - (3)^3 - 18(3)^2 - 7 \\ &= 2(81) - 27 - 162 - 7 \\ &= 162 - 27 - 162 - 7 \\ &= -34 \end{aligned}$$

Since, the remainder is -34. Then we can say that $(x - 3)$ is NOT a factor of $P(x) = 2x^4 - x^3 - 18x^2 - 7$ by factor theorem.

D

The expression $ax^2 + bx + c = 0$, where a, b, and c are real numbers has no meaning unless you replace the variable x with a number. For instance, you are given this statement "the area of the lot is $5x^2 + 7x - 2$ m^2", can you tell us how big is the lot? However, if the value of x is 10, you can find the value of the area by substituting to x. $5(10)^2 + 7(10) - 2 = 568$. Therefore the area of the lot is 568 m^2.

Learning Task 1. Perform what is asked.

1. Evaluate $3x^3 + 2x^2 - 5x + 1$ when $x = 32$

2. What is the remainder if $x^3 + 2x^2 - 4x + 3$ is divided by $(x - 3)$?

3. Given $P(x) = 2x^3 + 5x^2 - 3$, find $P(-1)$.

4. If $f(x) = x^3 + 4x^2 + 3x - 2$, what will be the value of $f(x)$ at $x = 3$?

5. Use synthetic division to find the remainder when $(5x^2 - 2x + 1)$ is divided by $(x + 2)$.

E

Learning Task 2.

A. Use the Remainder Theorem to find the remainder **R** in each of the following. Then check using synthetic division.

1. $(x^4 – x^3 + 2) \div (x + 2)$
2. $(x^3 – 2x^2 + x + 6) \div (x – 3)$
3. $(x^4 – 3x^3 + 4x^2 – 6x + 4) \div (x – 2)$
4. $(x^4 – 16x^3 + 18x^2 – 128) \div (x + 2)$
5. $(3x^2 + 5x^3 – 8) \div (x – 4)$

B. Use the Factor Theorem to determine whether or not the first polynomial is a factor of the second.

1. $x – 1; x^2 + 2x + 5$
2. $x + 1; x^3 – x – 2$
3. $x – 4; 2x^3 – 9x^2 + 9x – 20$
4. $a – 1; a^3 – 2a^2 + a – 2$
5. $y + 3; 2y^3 + y^2 – 13y +6$

A

Learning Task 3

1. What is the indication of having a zero remainder? What happens if the remainder is zero?
2. Which process do you preferred in identifying if the binomial is a factor of the polynomial?
3. What is the relation between the remainder and the value of the polynomial at $x = r$ when the polynomial $P(x)$ is divided by a binomial of the form $x – r$?
4. How will you find the remainder when a polynomial in x is divided by a binomial of the form $x – r$?

WEEK 7

Factoring Polynomial

I

Lesson

After going through this lesson, you are expected to factor polynomials (using the Remainder Theorem and the Factor Theorem).

To determine the factor of a polynomial you can use Factor Theorem, Synthetic Division, or Remainder Theorem. Let us consider the given polynomial on your previous activity.

Illustrative Example 1

What are the factors of $x^2 + 6x + 8$?

Using the remainder theorem and x + 2 (one of the choices in column B of the previous activity) as your divisor, x = -2

$P(x) = x^2 + 6x + 8$
$P(-2) = (-2)^2 + 6(-2) + 8$
$P(-2) = 4 - 12 + 8$
$P(-2) = 0$

So, x + 2 is indeed a factor of $x^2 + 6x + 8$.

The value of c should be taken from the constant 8. The factors of 8 are 1, 2, 4, 8 either both positive or negative. Since the polynomial is of degree to then there are two factors.

Now, use long division or synthetic division to get the other factor.
Since -2 was used as the value of , the the other factor that wen multiplied by –2 gives the product of 8 is –4.

Using Synthetic Division:

- 4	1	6	8
		-4	-8
	1	2	0

This means the other factor is x + 4. So, the factors of $x^2 + 6x + 8$ are x+2 and x + 4.

Illustrative Example 2

$x^3 - 7x + 6$ what is your answer in no. 2 in the activity?

Try x – 1 as one of the factors.

Using synthetic division:

1	1	0	-7	6
		1	1	-6
	1	1	-6	0

Since the remainder is zero, then x - 1 is one of the factors of $x^3 - 7x + 6$.

What are the other factors of the polynomial?

The resulting expression or quotient in the synthetic division is $x^2 + x - 6$, you can now find the factors of this expression by using your factoring skills or by trial and error. Think of two numbers that have a product of -6 and the sum of 1.

Those numbers are 3 and -2. So,

$x^2 - + x - 6$

$(x + 3)(x - 2)$.

Therefore, the factors of the polynomial $x^3 - 7x + 6$ are $(x - 1)(x + 3)(x - 2)$.

A third degree polynomial has at most 3 factors.

Illustrative Example 3

What are the factors of $x^3 - 2x^2 - 5x + 6$?

Choosing x – 3 as one of the factors, we do the synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -5 & 6 \\ & & 3 & 3 & -6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

since the remainder is zero, then x - 3 is one of the factors of $x^3 - 2x^2 - 5x + 6$.

What are the other factors of the polynomial?

The resulting expression or quotient in the synthetic division is $x^2 + x - 2$, you can now find the factors of this expression by using your factoring skills or by trial and error. Think of two numbers that have a product of -2 and the sum of 1.

Those numbers are 2 and -1.

So, $x^2 + x - 2 = (x + 2)(x - 1)$.

Therefore, the factors of the polynomial $x^3 - 2x^2 - 5x+6$ are $(x - 3)(x + 2)(x - 1)$.

D

Learning Task 1. On the chart below, find a factor in Column B of each of the given polynomials in Column A using the Factor Theorem.

Column A	
1. $x^2 + 6x + 8$	$x + 3$ $x - 3$
2. $x^3 - 7x + 6$	$x - 1$
3. $x^3 - 2x^2 - 5x + 6$	$x + 2$ $x + 1$

E

Learning Task 2

A. Determine the factors of the given polynomial.

1. $x^3 + 2x^2 - 5x - 6$
2. $x^3 + x^2 - x - 1$
3. $x^3 - x^2 - 10x - 8$

B. Solve the following problems using factoring polynomials.

1. A rectangular garden in a backyard has an area of $(3x^2 + 5x - 2)$ square meters. Its width is $(x + 2)$ meters.

    a. Find the length of the garden.

    b. You decided to partition the garden into two or more smaller congruent gardens. Design a possible model and include mathematical concepts in your design.

2. If one ream of bond paper costs $(x - 4)$ pesos, how many reams can you buy for $(6x^4 - 17x^3 + 24x^2 - 34x + 24)$ pesos?

A

Learning Task 3

A. Answer the following:

1. What is the remainder when $2x^{10} + 7x^5 - 5$ is divided by $x - 1$?
2. What is the remainder $x^2 + 8x + 12$ when divided by $x - 2$?
3. What are the factors of $x^2 + 8x + 12$?
4. If the remainder of the $x^3 - kx^2 + 3x - 2$ when divided by $x - 1$ is 4, find the value of k.

B. Answering the following questions.

1. How do you factor polynomials? Discuss the mathematics concepts and principles applied when factoring polynomials.
2. What new realizations do you have about factoring polynomials? How would you connect this to real life? How would you use this in making decisions?

WEEK 8

Polynomial Equations

I Lesson

After going through this module, you are expected to illustrate polynomial equations and determine the degree, leading coefficient and constant of the polynomial.

A **polynomial equation** is a special kind of algebraic equation where each term is a constant, a variable, or a product of constants and variables raised to whole number exponents.

It is defined by $a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_1x+a_0 = 0$, where n is a positive integer. The largest exponent n denotes the degree of the polynomials.

From the previous activity you identified which equations are polynomial and which are not. Hereunder are the answers and explanations:

Polynomial Equation

$3x^4 + 5x - 3 = 0$

$2x^4 + 2x^3 - 6x + 12 = 0$

4. $x^5 - 32 = 0$

5. $x^2 - \frac{1}{4} = 0$

7. $5x^3 + 4x^9 - x^3 + \sqrt{3} = 0$

9. $(x - 3)(x^2 - 4x) = 0$

10. $x(x - 2)(3x + 5) = 0$

Not Polynomial Equation

3. $x^3 - 6x^{-2} - x - 1 = 0$

-because of negative exponent.

6. $x^2 + \sqrt{x} - 2 = 0$

-because of there is a variable in side the radical sign.

8. $x^3 - \frac{2}{x} + 3 = 0$

-because of there is a variable in the denominator.

D

Remember that polynomial $P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0$, where a is real number, a_n is not 0 and n is a positive integer.

Learning Task 1. Identify if the given equation is a polynomial equation or not. Write P if polynomial equation and NP if not.

1. $3x^4 + 5x - 3 = 0$ _____

2. $2x^4 + 2x^3 - 6x + 12 = 0$ _____

3. $x^3 - 6x^{-2} - x - 1 = 0$ _____

4. $x^5 - 32 = 0$ _____

5. $x^2 - \frac{1}{4} = 0$ _____

6. $x^2 + \sqrt{x} - 2 = 0$ _____

7. $5x^3 + 4x^9 - x^3 + \sqrt{3} = 0$ _____

8. $x^3 - \frac{2}{x} + 3 = 0$ _____

9. $(x - 3)(x^2 - 4x) = 0$ _____

10. $x(x - 2)(3x + 5) = 0$ _____

1. How you will know if the equation is a polynomial equation?
2. What are the restrictions in determining if the given equation is a polynomial?

E

Learning Task 2. Write the terms of expression in the descending order. Determine the degree, the leading coefficient and the constant term.

Polynomial Equation	Descending Order	Degree	Leading coeffi-cient	Constant Term
1. $-4x^3-15x + 6 + 7x^5 = 0$				
2. $-11 + x^4 - 3x^2 = 0$				
3. $8x^3 + x^6 - 6x^4 + 1 = 0$				
4. $x(5x^3 + 7) = 0$				
5. $(x - 3)(x - 2)^2 = 0$				

A

Learning Task 3

1. How you to transform a polynomial equation in standard form? State some steps based on your understanding.

D

In solving polynomial equations, we apply the Factor Theorem. But when the equation is expressed as linear factors, the Zero Product Property will be directly applied to solve its solution.

Illustrative Example 1.

Consider the equation, (x –2)(2x + 3)(x+1) = 0, since the equation is presented as linear factors, then we can say that **, (x—2) =0, (2x + 3) = 0** and (x + 1) = 0 by Zero Product Property. Solving each linear equations would yield to

$\boldsymbol{x = 2},\ \boldsymbol{x = -\frac{3}{2}}$, and $\boldsymbol{x = -1}$ which is the solution to the given polynomial equation.

Illustrative Example 2

Now what if the polynomial is not in factored formed? How are we going to solve its solution? Below are the steps in solving polynomial equations in standard form.

Step 1. List all the possible roots of the polynomial equation using Rational Root Theorem.

$$2x^3 - x^2 - 4x + 3 = 0$$

From the equation $p = 2$ and $q = 3$ where the factors of p and q written in the form of $\frac{p}{q}$ are the possible solution to the equation. Therefore the possible roots are $\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}$

Step 2. Apply the Factor Theorem and use the synthetic division to check if one of the listed roots is a factor. If it's a factor then its also one of the solutions to the polynomial equation.

$$2x^2 + x - 3 = 0$$

. Let's try 1

1	2	-1	-4	3
		2	1	-3
	2	1	-3	0

Since the last entry on the last row is 0, then **1** is a solution to the polynomial equation where its depressed equation would be .

$$2x^2 + x - 3 = 0$$

$2x^2 + x - 3 = 0$ is derived by using the entry of the third row as numerical coefficients of the depressed equation of degree lower than 1 from the polynomial equation $2x^3 - x^2 - 4x + 3 = 0$

WEEK 8

I

Problem Solving Involving Polynomial Equations

Lesson

After going through this lesson, you are expected to solve problems involving polynomials and polynomial equations.

The fundamental theorem of algebra taught you that the number of zeros (including repeated zeros) of polynomial function of degree n are equal. This means that a cubic equation has 3 roots and the quartic equation has 4 roots because cubic equation is a 3^{rd} degree equation and quartic equation is a 4^{th} degree equation.

Learning Task 1. Complete the table below by identifying the degree and real roots of polynomial equations (if a root occurs twice then use multiplicity 2 or if it occurs thrice use multiplicity 3) Three problems are done for you as example.

Polynomial Equations	Degree	Roots	Number of Real Roots
1. $(x-1)(x+2)=0$	2	1, 2	2
2. $2x(x-3)(x+2)=0$	3	-2, 0, 3	3
3. $(x+2)^3(x-2)=0$	4	-2 multiplicity 3, 2	4
4. $(x-2)(x+2)(x-4=0$			
5. $x(x-1)^2(x+2)=0$			
6. $(x+4)^2(x-1)^3=0$			
7. $(x+5)(x+1)=0$			
8. $x^2(x-1)(x+1)=0$			

Step 3. Solved the depressed equation to find the other solution.

Example: The depressed equation is $2x^2 + x - 3 = 0$

It is a quadratic equation so we can solve it either using factoring or quadratic formula, which is

$2x^2 + x - 3 = 0$ $\quad$ $(2x+3)(x-1) = 0$

By Zero Product Property then we can say that, $(2x+3) = 0$ and $(x-1) = 0$

and solving each linear equations would give us $x = -\frac{3}{2}$ and $x = 1$

So, the solutions of the polynomial equation $2x^3 - x^2 - 4x + 3 = 0$ are **1 multiplicity 2** and . $-\frac{3}{2}$

Illustrative Example 3

Find the roots / solutions of the polynomial equation . $2x^4 + 7x^3 + 4x^2 - 7x - 6 = 0$

The possible roots are $\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

Setting up the synthetic division and choosing 1 as a solution,

1	2	7	4	-7	-6
		2	9	13	6
	2	9	13	6	0

The last entry of the third row is 0, therefore 1 is a solution with $2x^3 + 9x^2 + 13x + 6 = 0$ as its depressed equation. Repeating the process but this time using the coefficients of depressed equation. Let's try -1,

-1	2	9	13	6
		-2	-7	-6
	2	7	6	0

Again, the last entry of the third row is 0, therefore -1 is a solution reducing the polynomial equation to $2x^2 + 7x + 6 = 0$ as its depressed equation. Since the reduced equation is a quadratic, we can now solve it using either factoring or quadratic formula. By factoring,

$$2x^2 + 7x + 6 = 0$$

$$(2x+3)(x+2) = 0$$

Equating and solving each linear factor to zero,

$(2x+3) = 0$ $\quad$ $(x+2) = 0$

$x = -\frac{3}{2}$ $\quad$ $x = -2$

Therefore, the solutions / roots of the polynomial equation

$2x^4 + 7x^3 + 4x^2 - 7x - 6 = 0$ are . $-1, -\frac{3}{2}, -2, and\ 1$

Illustrative Example 4

Find a polynomial equation in standard form whose roots are 1, 2, and -3.

Solution

Since a polynomial equation have roots 1, 2, and -3, we can find the polynomial equation by writing each root as a factor of polynomial. That is, if 1, 2, and -3 are solutions then $(x-1), (x-2)\ , and\ (x+3)$ are factors of the polynomial equation.

Therefore the polynomial equation should be the product of all the factors, which is, $(x-1)(x-2)(x+3) = 0$ the factored form of polynomial equation, and $\mathbf{x^3 - 7x + 6 = 0}$ is the standard form.

Let us now summarize what we have learned on this lesson on solving problems involving polynomials and polynomial equation

To solve polynomial equations, consider the following steps:

1. List all the possible roots of the polynomial equation using Rational Root Theorem.
2. Apply the Factor Theorem and use the synthetic division to check if one of the listed roots is a factor.
3. Solved the depressed equation to find the other solution.

E

Learning Task 2. Find the roots of the following polynomial equations given one of its roots on the right side.

1. $x^3 - x^2 - x + 1 = 0$	$x = 1$
2. $3x^4 + 2x^3 - 13x^2 - 8x + 4 = 0$	$x = -2$
3. $x^4 - 2x^3 - 3x^2 + 4x + 4 = 0$	$x = 2$
4. $x^3 - x^2 - 20x = 0$	$x = 5$
5. $2x^4 - 7x^3 + 6x^2 = 0$	$x = 2$

B. Find a polynomial equation with integer coefficients that has the following roots.

1. $1, \ -2, \ -5$

2. $\pm 1, \ 2$

3. $\pm 2, \ \pm 3$

4. $-1, \ 2, \ -\frac{1}{2}$

5. $\frac{2}{5}, \ -\frac{1}{3}, \ \pm 3$

A

Learning Task 3.

A. Perform as instructed

1. Find the roots of $(2x + 1)(x + 1)(x - 1) = 0$

2. Find the zeros of the polynomial equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

3. Find the polynomial equation whose zeros are 2 of multiplicity 2 , 1 and –2

4. Find the equation if the zeros are 2, -1, 2, -3.

B. Answer the following questions.

1. What do you observe about the relationship between the number of roots and the degree of a polynomial?

2. What generalization can you make when polynomial equation is expressed as a product of linear factors?

3. How does the “Rational Root Theorem” and “Factor Theorem” helps you in solving polynomial equation?

Answer Key

WEEK 1

Learning Task 1

Week	1	2	3	4	5
Kilos	11	9	7	5	3

5 weeks

Learning Task 2

1. F 4. I 7. F 10. I
2. I 5. F 8. F
3. F 6. I 9. F

B

1. 14,17,20 6. 32,64,128
2. -15, -19, -23 7. 162, 486, 1458
3. 256, 1024, 4096 8. 6.5, 7.6, 8.7
4. 12, 0, -12 9. –15, -17, -19
5. 36, 49, 54 10. ., ., .

C.

1. 1,3,5,7 4. 10,15,20,25
2. 9, 6, 3, 0 5. 6, 4, 2, 0
3. 3, 6,9,12

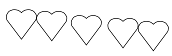
Learning Task 3

Answer may vary

WEEK 2

Learning Task 1

1. 

2. . 

3. 5 4. 11

Learning Task 2

A.

1. d = 3 14, 17, 20
2. Not
3. D = -4 -22, -26, , -30
4. D = 2 48, 50, 52
5. D = 0.6 3.0, 3.6, 4.2
6. D = 4 17, 21, 25

B, 1. 30 2. 85 3. –88 4. 5/2 5. –43

C. 1. 2,5,8,11 3. -11, -6, -1

2. 26,38,50,62,74

Learning Task 3

Answer may vary

WEEKS 3-4

Learning Task 1

1. Yes 2. No 3. Yes 4. Yes 5. No

Learning Task 2

1/3	36, 12, 4
-4	192, -768, 3072
5	12.5, 62.5,312.5
1/10	10, 1,1/10

B. JE-TAIMEBEAUCOUP

C.

1. 7,812,500 5. 3/256
2. 153 99/125 Learning Task 3
3. -39,366 1. Multiply the

WEEK 5

Learning Task 1

1. J 3. M 5. I 7. S
2. U 4. Q 6. T 8. T

Learning Task 2

A. 300 B. 600

Learning Task 3

240 loga

WEEK 6 Lesson 2

Learning Task 1

1. 34, 657
2. 26
3. 0
4. 52
5. 25

Learning Task 2

1. 26
2. 18
3. 0
4. 8
5. 365

B. 1. Not 2. Not 3. Not 4. Not 5. Yes

Learning Task 3

1. x –c is a gactor
2. Synthetic Division
3. R = P(c) 4. P(r) = R

WEEK 6 Lesson 1

Learning Task 1

1 x^2 , $7x^2$, -21x, 24x, -2, 70

Learning Task 2

1. x—2
2. X + 5
3. $X^2 + 4$ R. –1
4. $2x^2 -5x + 7$ R. 10
5. $7x^2 -11x +33$ R. –70

B,

1. (x—5)(x—2)
2. (x—4)(x +5)
3. $(x + 1)(x^2 + 4) - 1$
4. $(x +5)(2x^2 –5x + 7) + 10$
5. $(x +2)(7x^2 - 11x +33) -1$

Learning Task 3

Answer may vary

WEEK 7

Learning Task 1

1. x + 2
2. X—1
3. X—3

Learning Task 2

1. (x +1)(x + 3)(x—2)
2. (x = 10(x = 1)(x– 1)
3. (x + 2)(x + 1)(x—4)

B. 1. 3x—1

2. 720

Learning Task 3

1. 4
2. 24
3. (x + 2)(x + 6)
4. K = -2

WEEK 8

Lesson1

Learning Task 1

1. P	6. NP
2. P	7. P
3. NP	8. P
4. P	9. P
5. P	10. P

It is a polynomial is all the exponents are positive integers.

Learning Task 2

Week 8 Lesson 1 Learning Task 2

Polynomial Equation	Descending Order	Degree	Leading coefficient	Constant Term
1. $-4x^3-15x + 6 + 7x^5 = 0$	$6 -15x -4x^3 + 7x5 = 0$	5	7	6
2. $-11 + x^4 - 3x^2 = 0$	$-11—3x^2 + x^4 = 0$	4	1	-11
3. $8x^3 + x^6 - 6x^4 + 1 = 0$	$1 + 8x^3—6x^4 + x^5 = 0$	5	1	1
4. $x(5x^3 + 7) = 0$	$7x + 5x^4 = 0$	4	5	0
5. $(x - 3)(x - 2)^2 = 0$	$-12 + 16x -7x^2 + x^3 = 0$	3	1	-12

Learning Task 3

1. apply properties of equality by equating the polynomial to zero. The polynomial must be arranged in ascending order.

WEEK 8

Lesson 2

Learning Task 1

Polynomial Equations	Degree	Roots	Number of Real Roots
1. $(x-1)(x+2)=0$	2	1, 2	2
2. $2x(x-3)(x+2)=0$	3	-2, 0, 3	3
3. $(x+2)^3(x-2)=0$	4	-2 multiplicity 3, 2	4
4. $(x-2)(x+2)(x-4=0$	3	2, -2, 4	3
5. $x(x-1)^2(x+2)=0$	4	0, -2, 1 of multi-plicity 2	4
6. $(x+4)^2(x-1)^3=0$	5	-4 of multiplicity, 2 of multiplicity 3	5
7. $(x+5)(x+1)=0$	2	-5, -1	2
8. $x^2(x-1)(x+1)=0$	4	0,1, -1	3

Learning Task 2

A,

1. 1, -1
2. 2, -1, 1/3
3. 2, -1, -1
4. 0, -4
5. 3, 0

B.

1. $x^3 + 6x^2 + 3x - 10 = 0$
2. $x^3 - 3x^2 + 1 = 0$
3. $X^4 + 5x^2 + 36 = 0$
4. $2x^3 - x^2 - 5x - 2 = o$
5. $15x^4 - x^3 - 137x^2 + 9x$ 18

Learning Task 3

1. 1/2, -1, 1
2. 1, -4, ,-2, 3
3. $X^4 - 3x^3 + 2x^2 + 12x - 8 = 0$
4. $X^4 - 9x^2 + 4x + 12 = 0$

B.

The degree tess us the number of zeros or the number of zeros is at most the number of de-gree.

For inquiries or feedback, please write or call:

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