Joni	Hello, I am Joana Kicheva and these are my teammates Margarita Stefanova and Alex Tsvetanov. We are from Bulgaria, Sofia high school of mathematics and our supervisior is Vanya Danova. This project is about Catalan numbers. Hope you like it.
Alex	What have these objects in common? Our presentation Is dedicated to this question.
Magi	The mathematician Catalan studied the number of ways to place n pairs of brackets in a row.
Joni	Later it became clear that the same numbers apply for other objects, outwardly different to each other. We will call each group of this type Catalan's.
Magi	For each group we will find uniquely conformity (bijections) between it and other sets.
Joni	Consider a problem similar to considered by Catalan. We have an expression of n operators and n+1 numbers. IN how many ways it can be divided to subexpressions.
Magi	Each division of the expression bijectively matches to binary tree, showing the hierarchy. In suffix notation of a tree replace each oprator with 1 and each number with -1. We will obtain new expression compound of n+1 ones and n minus ones and will call it correct expression.
Joni	The number of all possible expressions is 2n+1 choose n. The number of correct expressions is Cn where Cn is equal to 1 over n+1 times 2n choose n. C0=1
Magi	From this it follows that the set of necklace which is composed of n white and n+1 black beads is Catalan's. 1,44
Joni	We make the conformity between correct expression of ones and minus ones and balanced parenthesis. Therefor the group of brackets is Catalan's.
Magi	We find that the balanced parenthesis expression corresponds to mountain range where the peculiarity of such set is that it never intersects the horizon.
Joni	If inflict the horizon of a mountain range on the diagonal of a grid n by n, we will obtain a new set called diagonal avoiding path, which is Catalan's. Each rout outlines a poliomino hence it is Catalan's too.
Magi	How many different piles of logs can be traced on the basis of n closely spaced logs. Build a mountain range as the shown bellow which means the set of pile of logs is Catalan's.
Joni	Let's permute the sequence a1,a2,…,an by using stack. For example from 1,2,3,4,5,6,7 we receive 2,3,1,6,5,4,7 where the workflow is aababbaaabbbab. When we make a conformity between this set and the balanced parenthesis obtain the stack permutation is also Catalan's.
Magi	A rooted tree is a non-empty tree. A forest is a set of rooted trees. When the set of forest is ordered the corresponding trees are ordered too. Each forest with n>=0 vertices uniquely corresponds to rooted tree whit n+1 vertices.
Joni	Here we have shown bijectively correspondence between the balanced balanced parenthesis expression and forest. Therefor the sets of forest and ordered trees are Catalan's. 1,44
Magi	Binary tree is a tree compound of root ;;;;lwo children which are reffered to left and right children which are binary trees too. We make a conformity

	between forest and binary tree by the following rule. Jence the binary tree is Catalan's. This figure shows all the possible forests and the corresponding binary trees for n=0
Joni Magi	Full binary is a binary tree where each vertex has 0 or 2 children. Each binary tree has one more leaves then the number of non-leaves vertices. Easily can obtain a binary tree from full binary tree so this set is Catalan's.
Joni	We find parity between the set of tillings in a staircase so it is Catalan's.
Magi	The set of chords is equivalent to the group of binary trees, so it is Catalan's too.
Joni	There is a accordance between the triangulation and binary trees so the triangulation is Catalan's.
Magi	In the Catalan triangle each row starts with the second element of the one above and the other numbers are equal to the sum of the one above and the left one. The first element of each rout is Ci. 1,2,5
Alex	In the first of next lines have implemented in programming language Haskell sequences of sequences – i.e. the triangle. We have shown some standart tasks which illustrate the information. To explore Catalan numbers we have built some programs. One of them generates all the possible balanced parenthesis expressions. The current expression differs from the previous exchange of only two bracket's locations. Another program translates the balanced parenthesis expression to a binary tree. Further there are programs for translations from balanced parenthesis to stack permutation. - From balanced parenthesis to the pile of logs - From binary tree to tillings in the staircase - From binary tree to triangualation - and ect. Program illustrates a graphical view of trees, paths, necklaces, triangulations and other objects. Catalan numbers are very interesting because they apply for a lot of different objects. Bringing one object to another often has an algorithmic nature. Eugene Catalan was born in Belguim. He received his education in Paris. He has worked on repeating fractions, descriptive geometry, theory of numbers and combinatorics. Before Catalan Leonard Euler recived the triangulation problem and invent the sequence wich was called to Catalan. The literature that we used is this and for programs we used this. Special thanks to Vanya Danova, Vasil Tinchev and Boyko Banchev. That was our project. Thanks for the attention. Hope you liked it.