APSC-174 January 11th

The language of sets

↳ A set is a collection of objects

-Objects in a set are called elements

↳ $\mathbb{R}$ ↳ $\mathbb{Z}$

↳ $\mathbb{C}$ ↳ all letters in all letters

Notation: $a \in S$

"a is an element of set S"

e.g. $\frac{1}{2} \in \mathbb{R}$, $i \notin \mathbb{R}$

Defining sets

$A = \{a, b, c, \dots, z\}$ $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$B = \{3\}$ ← singleton (set with 1 element)

Remarks:

- the order of elements in a set does not matter
- listing elements repeatedly does not matter

$$S = \{a, b\} = \{b, a\} = \{a, b, b, b, a\}$$

More general method

$S = \{x : P(x)\}$ "such that"

example: the set of all integers ≥ 3: $A = \{3, 4, 5, 6, \dots\}$

$= \{x : x \in \mathbb{Z} \text{ and } x \geq 3\}$

$= \{x \in \mathbb{Z} : x \geq 3\}$

• $B = \{x \in \mathbb{R} : 0 \leq x \leq 1\} = [0, 1]$

The empty set: $\{\ \}$ → $\emptyset$

given sets A & B, where A is a subset of B

$A \subset B$ → if $x \in A$, then $x \in B$

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{R} \subset \mathbb{C}$ $\mathbb{Z} \not\subset \mathbb{N}$

for any set S, $\emptyset \subseteq S$

Set equality
two sets are equal if they have the same elements

$$A = B \iff A \subset B \text{ and } B \subset A$$

"if and only if"

given 2 sets, the union is the combined elements: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.x.

$A = \{a, b, c\}$ $B = \{d e\}$ $C = \{c, d, e\}$

$A \cup B = \{a, b, c, d, e\}$ $B \cup C = \{d, e, c\}$

$A \cup C = \{a, b, c, d, e\}$

properties:
- $A \cup \emptyset = A$
- $A \cup A = A$
- $A \cup B = B \cup A$
- $A \cup (B \cup C) = (A \cup B) \cup C$

Intersection: shared elements

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

ex

$A = \{a, b, c\}$ $B = \{c, d, e, f\}$ $C = \{e, f, g\}$

$A \cap B = \{c\}$ singleton $A \cap C = \{\}$ empty set

Properties:
- $A \cap \emptyset = \emptyset$
- $A \cap A = A$
- $A \cap B = B \cap A$
- $A \cap (B \cap C) = (A \cap B) \cap C$

Cartesian Product of sets

(a, b) is an ordered pair, order is now important

-given A & B

$$A \times B = \{(a, b);\ a \in A, b \in B\}$$

$A = \{1, 2\}$ $B = \{a, b, c\}$

- $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$
- $B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$
- $A \times A = A^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$
- $B \times B = B^2$

Remark: if $A \neq B$, $A \times B \neq B \times A$