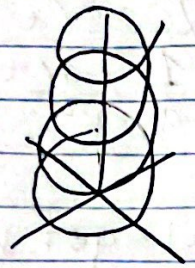
APSC-172 January 10th 2023

example 1.4 $x = 2\cos(t)$ (a) sketch

$y = 2\sin(t)$

$z = 3t$



(b) find velocity & speed

$$\text{vel} = \frac{d}{dt}\begin{bmatrix} 2\cos t \\ 2\sin t \\ 3t \end{bmatrix} = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 3 \end{bmatrix}$$

$$\text{Speed} = \|\text{vel}\| = \sqrt{(-2\sin(t))^2 + (2\cos(t))^2 + 3^2}$$

$$= \sqrt{4\sin^2 t + 4\cos^2 t + 9}$$

$$= \sqrt{4(1) + 9} = \sqrt{13}$$

(c) take the z axis as vertical and calculate m(slope)

Slope of curve = Slope of tangent vector

= Slope of velocity vector

$$\text{Slope} = \frac{c}{\sqrt{a^2 + b^2}}$$

$$= \frac{3}{\sqrt{(-2\sin t)^2 + (2\cos t)^2}}$$

$$= \frac{3}{2}$$ this is unitless

every 2 steps in xy means 3 steps up in z

aside
"dot notation" means time derivative

example 1.5

$z = 9 - x^2 - y^2$ → Represents hill

path: $x^2 + 4y^2 = 4$

When is the path the steepest

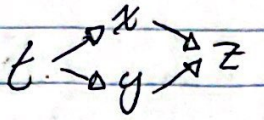
need slope → need tangent line

↓

parameterize path/add time → Velocity Vector is tangent

Hilroy

Paramdterization



$x = 2\cos(t)$
$y = 1\sin(t)$
$z = 9 - x^2 - y^2$

$\frac{d}{dt} \rightarrow$ $\dot{x} = -2\sin t$
$\dot{y} = \cos t$
$\dot{z} = -2x(\dot{x}) - 2y(\dot{y})$

now get velocity

$\frac{dx}{dt}$ $\frac{dy}{dt}$

$= -2(2\cos(t))(-2\sin(t)) - 2(\sin(t))(\cos(t))$
$= 6(\cos(t))(\sin(t))$

$$m = \frac{6(\cos t)(\sin t)}{\sqrt{(-2\sin t)^2 + (\cos(t))^2}}$$

Now we optimize
we can minimize m^2 → take on faith

$$= 36\,\frac{\sin^2 t \cdot \cos^2 t}{4\sin^2 t + \cos^2 t}$$

let $s = \sin^2 t$

$\cos^2 t = 1 - \sin^2 t$
$= 1 - s$

$$= 36 \cdot \left(\frac{s \cdot \cos^2 t}{4s + \cos^2 t}\right)$$

$$= 36 \cdot \left(\frac{s \cdot (1-s)}{4s + 1 - s}\right) = 36\left(\frac{s - s^2}{3s+1}\right)$$

find crit points of $m^2(s)$

$$= \frac{d}{ds}(m^2(s)) = 0$$

$(1-2s)(3s+1) - (s-s^2)3 = 0$
$-6s^2 + s + 1 - 3s + 3s^2 = 0$
$-3s^2 - 2s + 1 = 0$
$s+1$ is a factor $s = -1$
$-(s+1)(3s-1) = 0$ $3s = 1$
$s = 1/3$

$\sin^2 t = -1$
all real → no solns

$\sin^2 t = 1/3$

$\sin t = \pm\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

we know t, now we can find all values