

Quiz #1'

Date: September 19, 2025

Question 1. 5 points

Identify the error in the following proof, and explain why it is incorrect.

Proposition: For all $n \geq 2$, the number n is even.

Proof: We proceed by strong induction.

Base case: The base case $n = 2$ is true, as 2 is even.

Strong induction hypothesis: Assume that every number less than n is even. Then, as $n = (n - 2) + 2$, and $n - 2$ is less than n , so even by the strong induction hypothesis, we deduce that n is even, as desired. ■

Question 2. 10 points

1. **(8 points)** Let $r \neq 1$ be a real number. Prove that for all $n \geq 1$ the equality

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}, \quad (1)$$

holds.

2. **(2 points)** Formulate a version of Equation (1) which works even when $r = 1$, and justify this one missing case.