Inm: The Set P= Eprimes 3 was in finitely Idea: If P= & Pi,-, Pm3, Produce a PEP not equal to any of the p LOOK OF N=P, Pm+1 has the Property that P: \N. (60 if it did

2.3+1=7 , 23+7. Foind any prime divisor PIN

Why are Primes important Thm (Fundamental theorem of arishmotiz): Every possitive integer can be factored uniquely as a product of primes.

Definition; Prime factor: Zation X = P, e, --- Pm unique if X= 7, --- (n f) and P1 < P2 < .. < Pm and 2, < --- < 2 h m=n 9 nd --- j Pm= 2n e/ =+, em fr.

(2,3,0,0,0 - 23,335,79 11913 finitely many

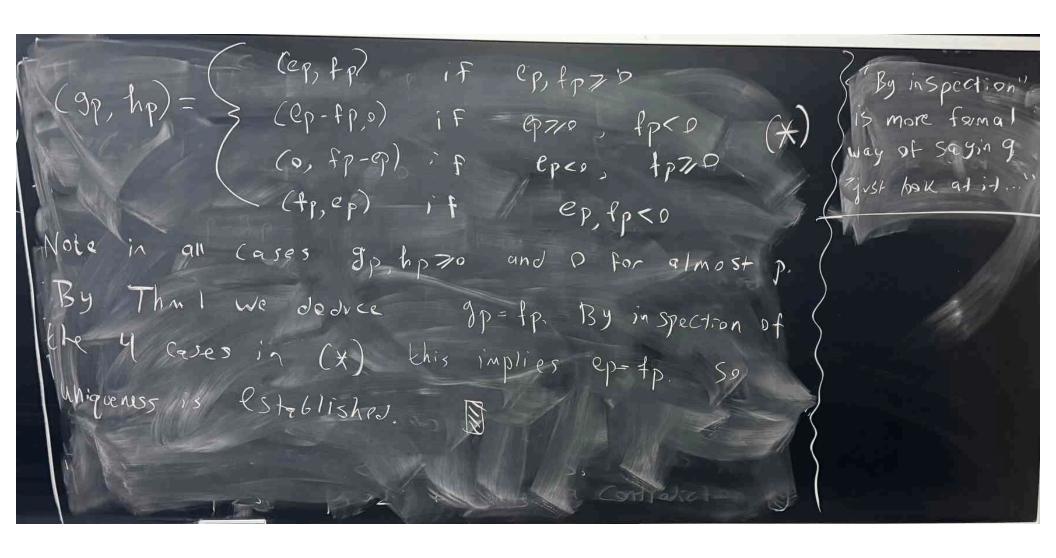
PEP ep=fp PER P Intuition: Primes are building blocks of integens Pf (existence por (ish): Let $S = \{x \in \mathbb{N}: x \text{ does not factor into pirks}\}$ Suppose S is non-empty. Then, by L.I.D., there

is a minimal element x_0 of S.

Note, Xo Can't be a prine, or
V T C Q S F $P \neq XQ$
no= Hep, where ep=3
Tell 1
is a prime factor: Zation.
Since Xo is not prime we can write Xo= 40.20 w)
(Yo Zarv integrand) C U a
Minimal Clauses of C
1, Zo € S. So
to- PEP and Zo- Dep P
Thus The cold
Pc D
Economic prime features to contradiction.
Xo = TT ep, where ep = \(\) if p \(\text{xo} \) is a prime factor: \(\text{Zation.} \) Since Xo is not prime we can write \(\text{Xo = 4.20 w} \) \(\text{Yo, Zo < Xo} \) integers. As \(\text{Yo, Zo < Xo} \) and \(\text{Xo is} \) integers. As \(\text{Yo, Zo < Xo} \) and \(\text{Xo is} \) is \(\text{Minimal clement of S}, \(\text{Yo, Zo } \text{ES. So} \) Thus, \(\text{Yo = 7.20 } \text{Trept}, \) Thus, \(\text{Yo = 7.20 } \text{Trept}, \) So we have prime factorized \(\text{Xo. Contradiction.} \)

The padic valuation MB: We are assuming That in this section $Q = \{\frac{a}{b}, \sigma, b \in \mathbb{Z}\}$ = 7 rational numbes Proposition: E very 07 XEQ admits 9 unique factor: zation x = c.15 | ep w/
epez and ep=0 for almost all p, and SEE0,13. Write X = (-1)3 a w/ a, beIN. By Thal a: The paper and 6= The paper where fragely and one Zero for almost all p. The X-(-1) P P-50

and fp-gp CZ and is a for almost p. So existence	
For uniqueness suppose X = (-1) The per per per per per per per per per pe	
There Per P	

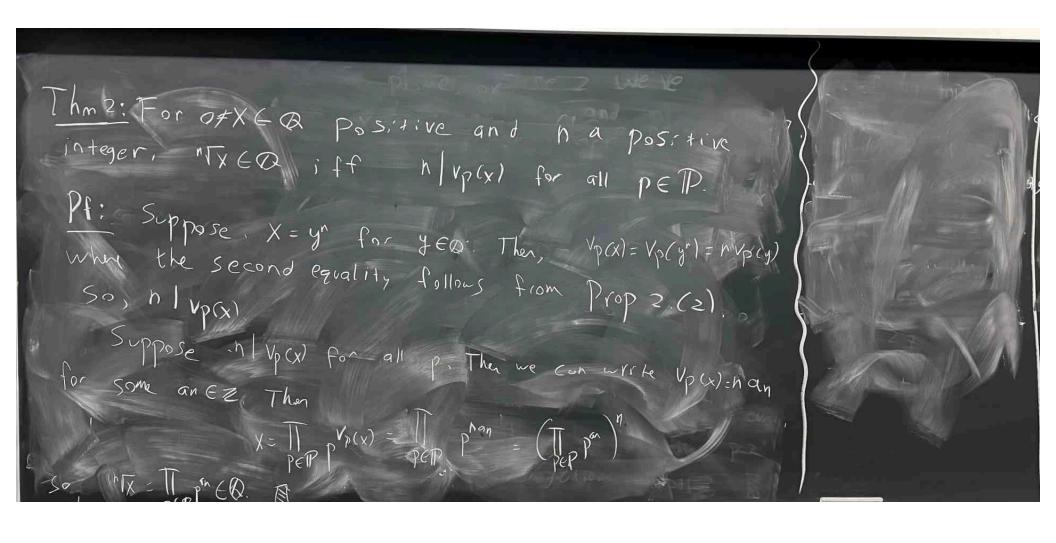


Definition: The P-adic valuation (for a prime p) A function is the function Up on Q defined by on the input $V_{p}(x) = \begin{cases} e_{p} & \text{if } 0 \neq x = \prod_{p \in p} p^{e_{p}} \\ \infty & \text{if } x = 0 \end{cases}$ Remark: This is only well-defined lecause of Prop1

IP Prop: (1) 72 = {x \in \mathbb{Q}; \mathbb{P}(x) > 0} For Statements (Pand Q, the (2) $\forall p(xy) = \forall p(x) + \forall p(y) \text{ for } X, y \in \mathbb{Q}$ and only of Q" (also writte Pitte" Or =) () Many For X, y EZ, one has x | y if and only P is the then Qis true (ie, P=0) AM IFR IS true the Pistive (i.e. R=D)

If $y \in \mathbb{Z}$, then by That $y = \pm ||$ PeP epto and ep= o for almost p, so vp(y) -epto So y & EXEQ: VP(x) >0 for all p3. which is clearly in Z. But, xy= TT property So, by proper, Vpray = Vp

(3) $0 = Vp(1) = Vp(x \cdot x^{-1}) = Vp(x) + Vp(x^{-1})$ or statements 69 (21. 50, Vp(x-1)=-Vp(x). Prop (4) Note xly iff y (1) happens iff /p(x) 70 for all p, but live 7000 by (1)+(2), $V_p(x) = V_p(y \cdot x^{-1}) = V_p(y) + V_p(x^{-1}) = V_p(y) - V_p(y)$ Pf: 10



Corollary: $\sqrt{2}$ is irrational.

Pf: If $\sqrt{2} \in D$, then by $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$ even for all p But $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$