Spacecraft Mission

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This report will cover all parts of assignment one in the Aerospace 720 course

1 Orbital Propagation

1.1 Solving Kepler's Equation

I) We need to solve Kepler's Equation using numerical methods. Using the Newton-Rasphon Method we can take the eccentricity and mean anamoly as inputs and numerically solve for the eccentric anamoly.

```
def Kepler(e, M, tol = 1e-12, max_i = 1000):
E = M                              # Guess solution
for i in range(max_i):
    f_E = E - e * np.sin(E) - M   # Define the function in terms of f(E) = 0
     f_prime = 1 - e * np.cos(E) # Derive the function in terms of E
    del_E = f_E / f_prime
    E_new = E - del_E             # Calculate the new eccentric anamoly
    if np.abs(del_E) < tol:       # If the value is within the set tolerance
        theta = 2*np.arctan(np.tan(E_new/2) * ((1+e)/(1-e))**(0.5))
        return theta              # Return true anamoly
    E = E_new
```

II) If we set the tolerance to $1e-12$, we can compute the true anamoly of the asteroid at t_0 and $t_0 + 100$ days. A_ae0 is the OBJ data of the asteroid, it is an array.

```
trueAnamoly_asteroidt_0 = Kepler(A_ae0[2], A_ae0[6])
meanAnamolyt_100 = get_mean_anamoly(100*(3600*24), A_ae0[6], A_ae0[1])
trueAnamoly_asteroidt_100 = Kepler(A_ae0[2], meanAnamolyt_100)
```

Printing these values gives the following

2 Mathematics

To derive the necessary state function we have:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} \tag{1}$$

From here we know that $\frac{dv}{dt} = \dot{r}$ giving:

$$\frac{d\mathbf{v}}{dt} = -\frac{\mu}{r^3}\mathbf{r} \tag{2}$$

Expanding each vector as three-dimensional components in x, y, z:

$$\frac{dx}{dt} = v_x, \frac{dy}{dt} = v_y, \frac{dz}{dt} = v_z \tag{3}$$

$$\frac{dv_x}{dt} = -\frac{\mu}{r^3}x, \frac{dv_y}{dt} = -\frac{\mu}{r^3}y, \frac{dv_z}{dt} = -\frac{\mu}{r^3}z \tag{4}$$

Where $r = \sqrt{x^2 + y^2 + z^2}$
We can now define a state vector $\bar{\mathbf{X}}$

$$\bar{\mathbf{X}} = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} \tag{5}$$

Finally, deriving this state vector gives the following:

$$\dot{\bar{\mathbf{X}}} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ -\frac{\mu}{r^3}x \\ -\frac{\mu}{r^3}y \\ -\frac{\mu}{r^3}z \end{bmatrix} \tag{6}$$

3 Figures

Figure example:

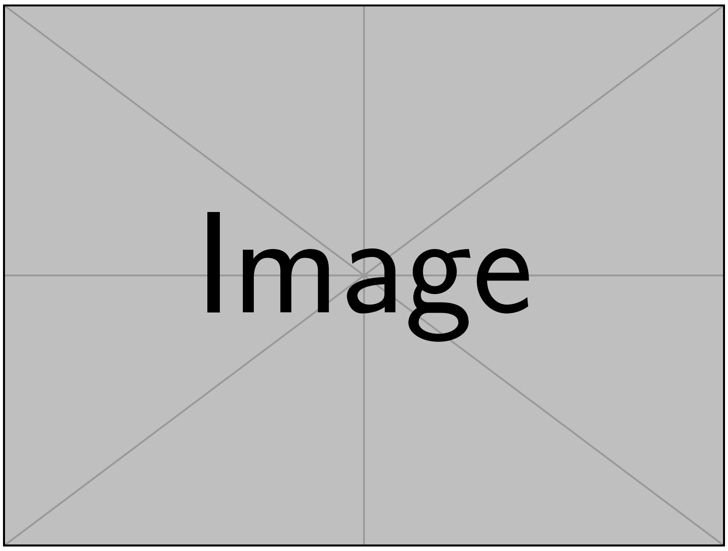

Figure 1: Example figure caption.

4 Tables

Left	Center	Right
A	B	C
1	2	3

Table 1: Example table.