2 VQ:

z_1, z_2, z_3 : independent, uniform distribution on $[0,1]$,

$f: [0,1]^3 \to R$. $f(z) = z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2$.

What is the Sobol representation of $f(z)$?

~~Only consider second order term.~~

$$f_0 = \int_\Gamma f(z)dz = \int_0^1\int_0^1\int_0^1 (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2) dz_1 dz_2 dz_3$$

$$= \iiint z_1 dz_1 dz_2 dz_3 + \iiint z_2^2 dz_1 dz_2 dz_3 + \iiint z_1 z_2 dz_1 dz_2 dz_3 + \iiint z_2 z_3^2 dz_1 dz_2 dz_3$$

$$= 1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{7}{4}$$

$$f_1(z_1) = \int_{\Gamma^2} f(z) dz_{\sim i} - f_0 = \int_0^1\int_0^1 (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2) dz_2 dz_3 - f_0$$

$$= z_1 + \frac{1}{3} + \frac{1}{2} z_1 + \frac{1}{6} - \frac{7}{4} = \frac{3}{2} z_1 + \frac{1}{2} - \frac{7}{4} = \frac{3}{2} z_1 - \frac{5}{4}.$$

$$f_2(z_2) = \iint (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2) dz_1 dz_3 - f_0$$

$$= \frac{1}{2} + z_2^2 + \frac{1}{2} z_2 + \frac{1}{3} z_2 - \frac{7}{4} = z_2^2 + \frac{5}{6} z_2 - \frac{5}{4}.$$

$$f_1(z_3) = \iint (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2) dz_1 dz_2 - f_0$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2} z_3^2 - \frac{7}{4} = \frac{1}{2} z_3^2 - \frac{2}{3}.$$

$$f_{12}(z_1, z_2) = \int (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2) dz_3 - f_1(z_1) - f_2(z_2) - f_0$$

$$= z_1 + z_2^2 + z_1 z_2 + \frac{1}{3} z_2 - \left(\frac{3}{2} z_1 - \frac{5}{4}\right) - \left(z_2^2 + \frac{5}{6} z_2 - \frac{5}{4}\right) - \frac{7}{4}$$

$$= z_1 z_2 - \frac{1}{2}(z_1 + z_2) + \frac{3}{4}$$

$$f_{13}(z_1, z_3) = \int (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2)\, dz_2 - f_1(z_1) - f_1(z_3) - f_0$$

$$= z_1 + \frac{1}{3} + \frac{1}{2} z_1 + \frac{1}{2} z_3^2 - \left(\frac{3}{2} z_1 - \frac{5}{4}\right) - \left(\frac{1}{2} z_3^2 - \frac{2}{3}\right) - \frac{7}{4}$$

$$= \frac{1}{2}.$$

$$f_{23}(z_2, z_3) = \int (z_1 + z_2^2 + z_1 z_2 + z_2 z_3^2)\, dz_1 - f_1(z_2) - f_1(z_3) - f_0$$

$$= \frac{1}{2} + z_2^2 + \frac{1}{2} z_2 + z_2 z_3^2 - \left(z_2^2 + \frac{5}{6} z_2 - \frac{5}{4}\right) - \left(\frac{1}{2} z_3^2 - \frac{2}{3}\right) - \frac{7}{4}$$

$$= \left(z_2 - \frac{1}{2}\right) z_3^2 - \frac{1}{3} z_2 + \frac{2}{3}.$$

$$f_{123}(z_1, z_2, z_3) = 0$$

Thus, $f(z) = f_0 + f_1(z_1) + f_1(z_2) + f_1(z_3) + f_{12}(z_1, z_2) + f_{13}(z_1, z_3) + f_{23}(z_2, z_3)$

(Exact expansion terms).