

# **LING2136 / 4136**

## **Advanced Statistical Methods for Language Students**



**Session-02: MLMs**

Lecturer: Timo Roettger

# Multilevel linear models

aka hierarchical linear model

aka linear mixed effects models



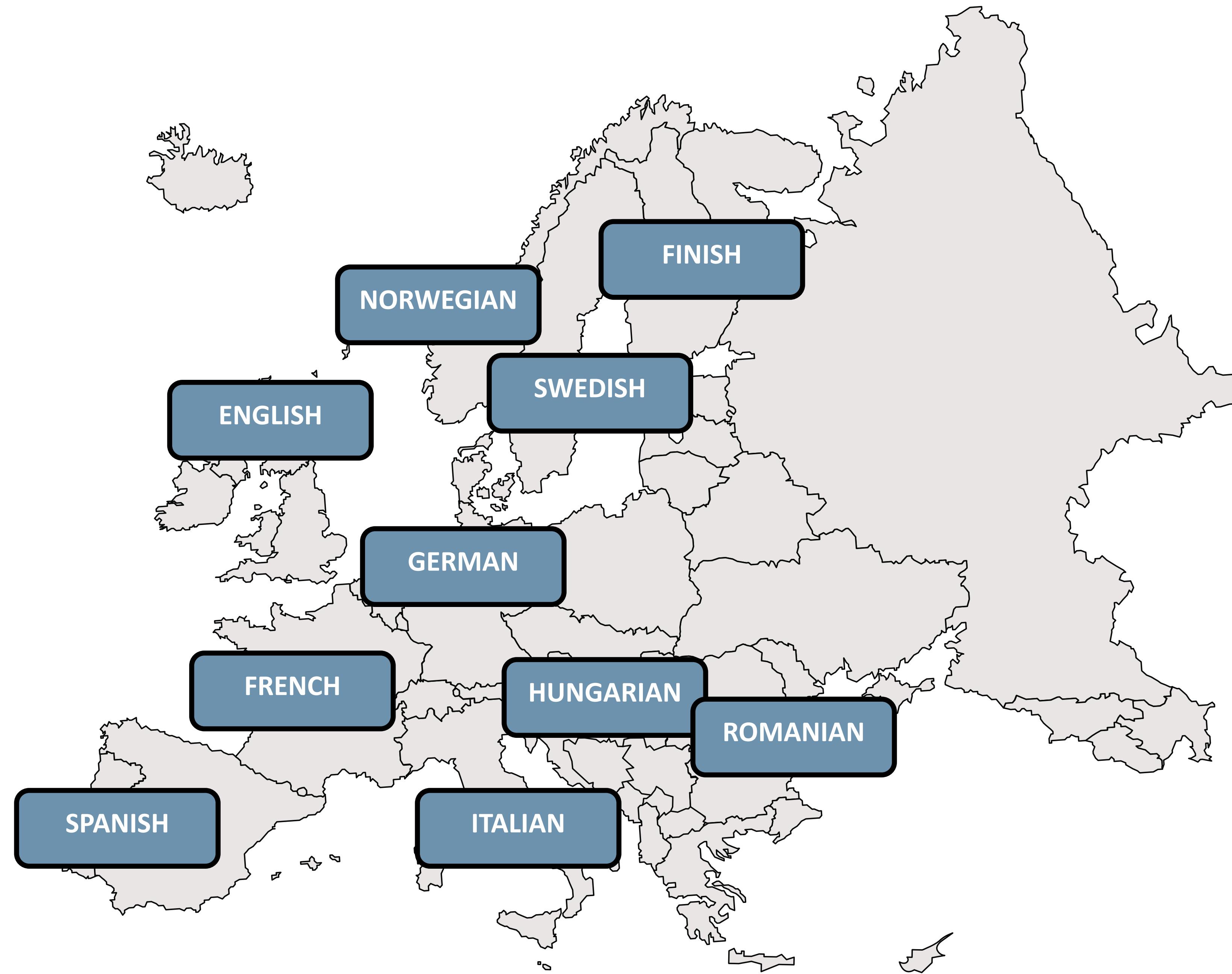
# EXAMPLE: TENSE AND SAVINGS BEHAVIOR

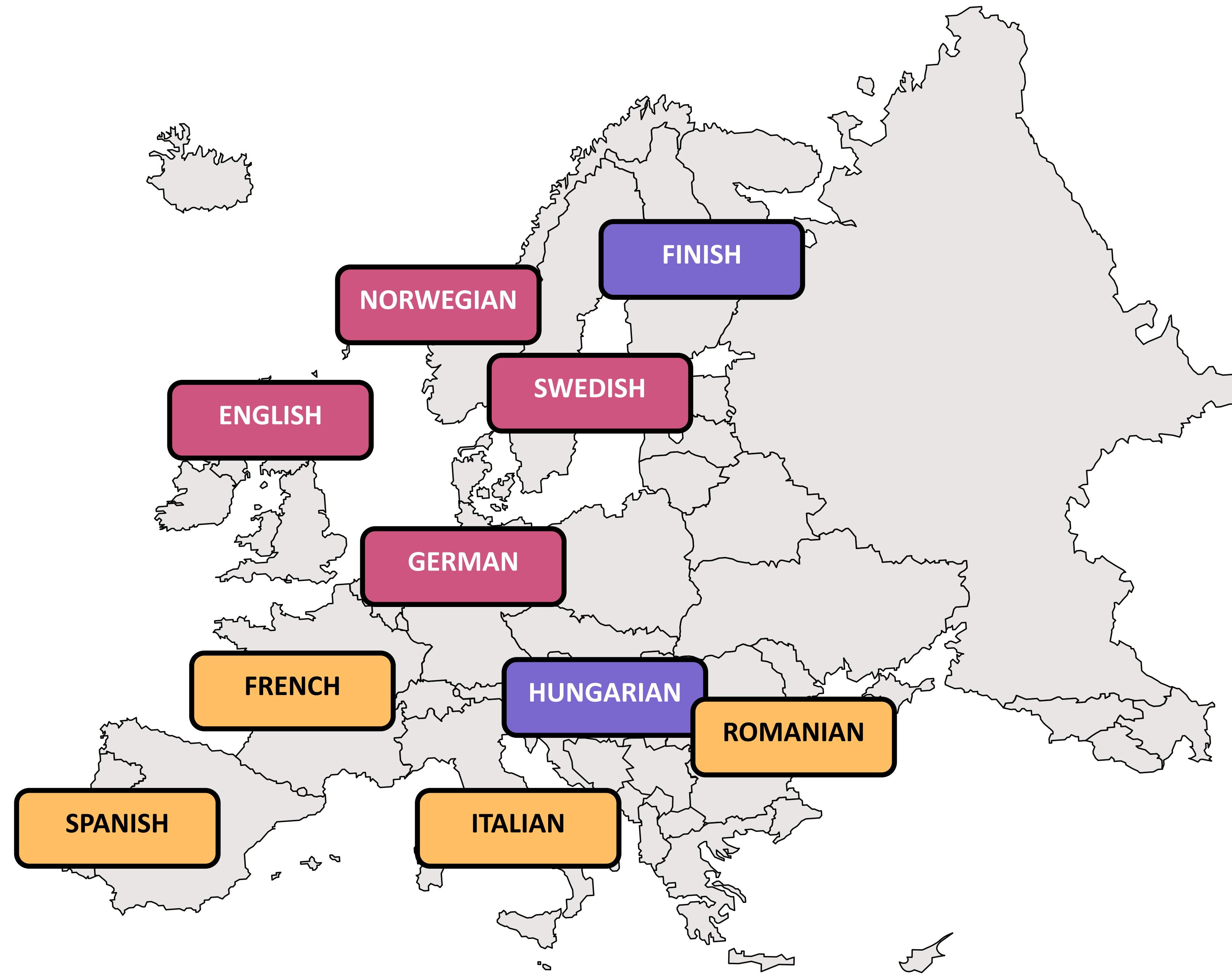
Chen (2013) "The Effect of Language on Economic Behavior: Evidence from Savings Rates, Health Behaviors, and Retirement Assets."

RESEARCH ARTICLE

## Future Tense and Economic Decisions: Controlling for Cultural Evolution

Seán G. Roberts<sup>1\*</sup>, James Winters<sup>2</sup>, Keith Chen<sup>3</sup>





<b>Subject</b>	<b>Item</b>	<b>Condition</b>
S1	1	Test
	1	Control
S1	2	Test
	2	Control
S2	1	Test
S2	1	Control
S2	2	Test
S2	2	Control

# Multilevel perspective & lme4 syntax

intercept

slope

$$y_i = b_0 + b_1 * x_i$$

```
lm(y ~ 1 + x)
```

# Multilevel perspective & lme4 syntax

intercept

slope

$$y_{ij} = b_{0j} + b_1 * x_{ij}$$

```
lmer(y ~ 1 + x + (1 | group))
```

random  
intercepts

# Multilevel perspective & lme4 syntax

intercept

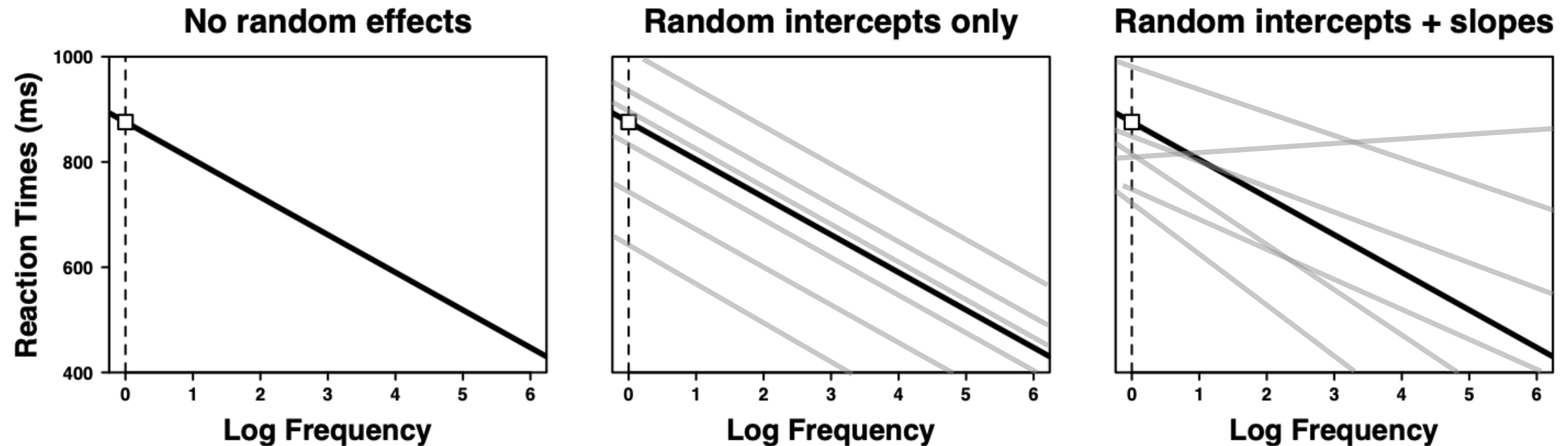
slope

$$y_{ij} = b_{0j} + b_{1j} * x_{ij}$$

```
lmer(y ~ 1 + x + (1 + x | group))
```

random  
slope

random  
intercepts



# Multilevel perspective & lme4 syntax

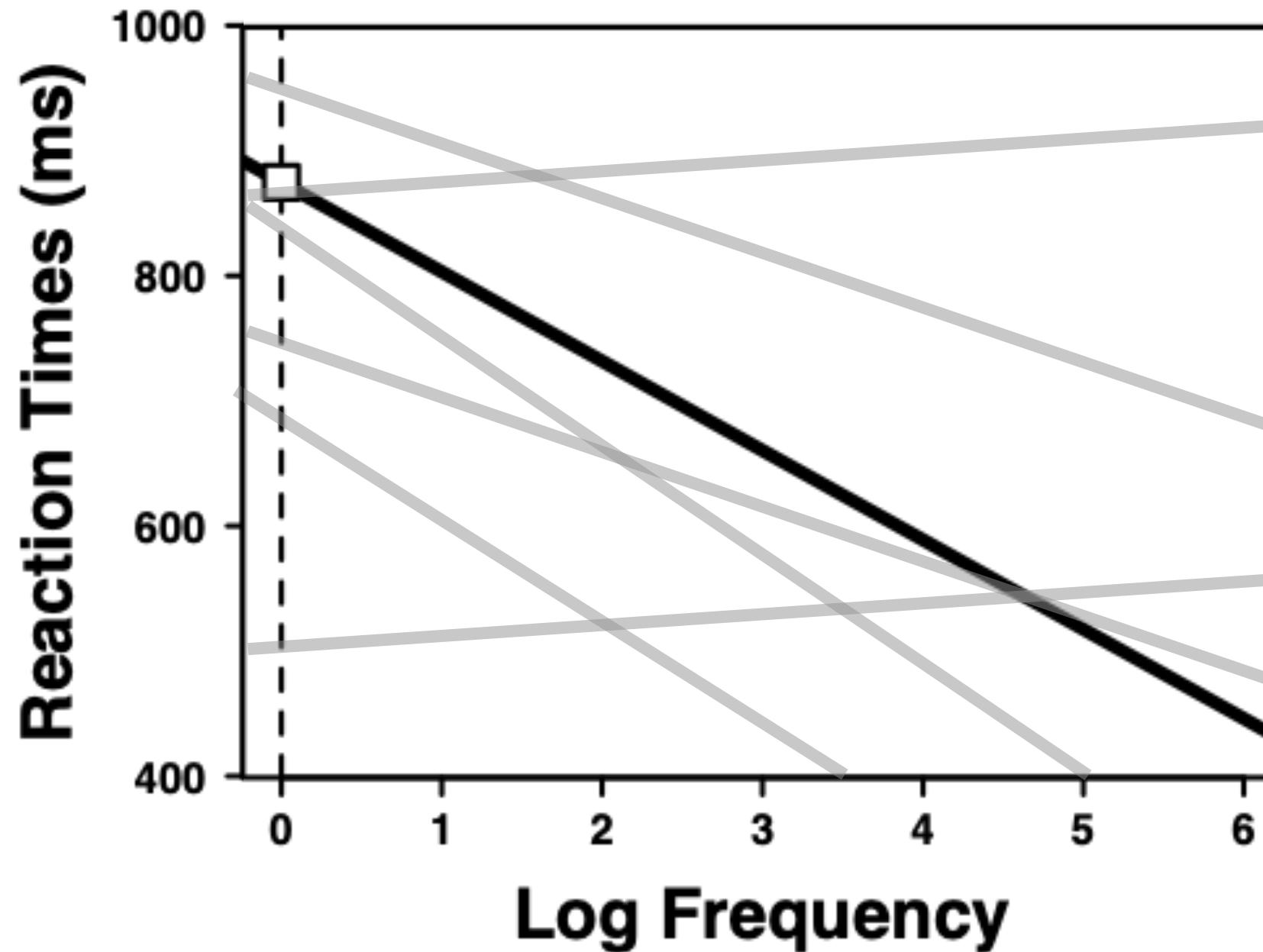
```
lmer(y ~ 1 + x + (1 + x | group))
```

intercepts & slopes are  
correlated

```
lmer(y ~ 1 + x + (0 + x | group))
```

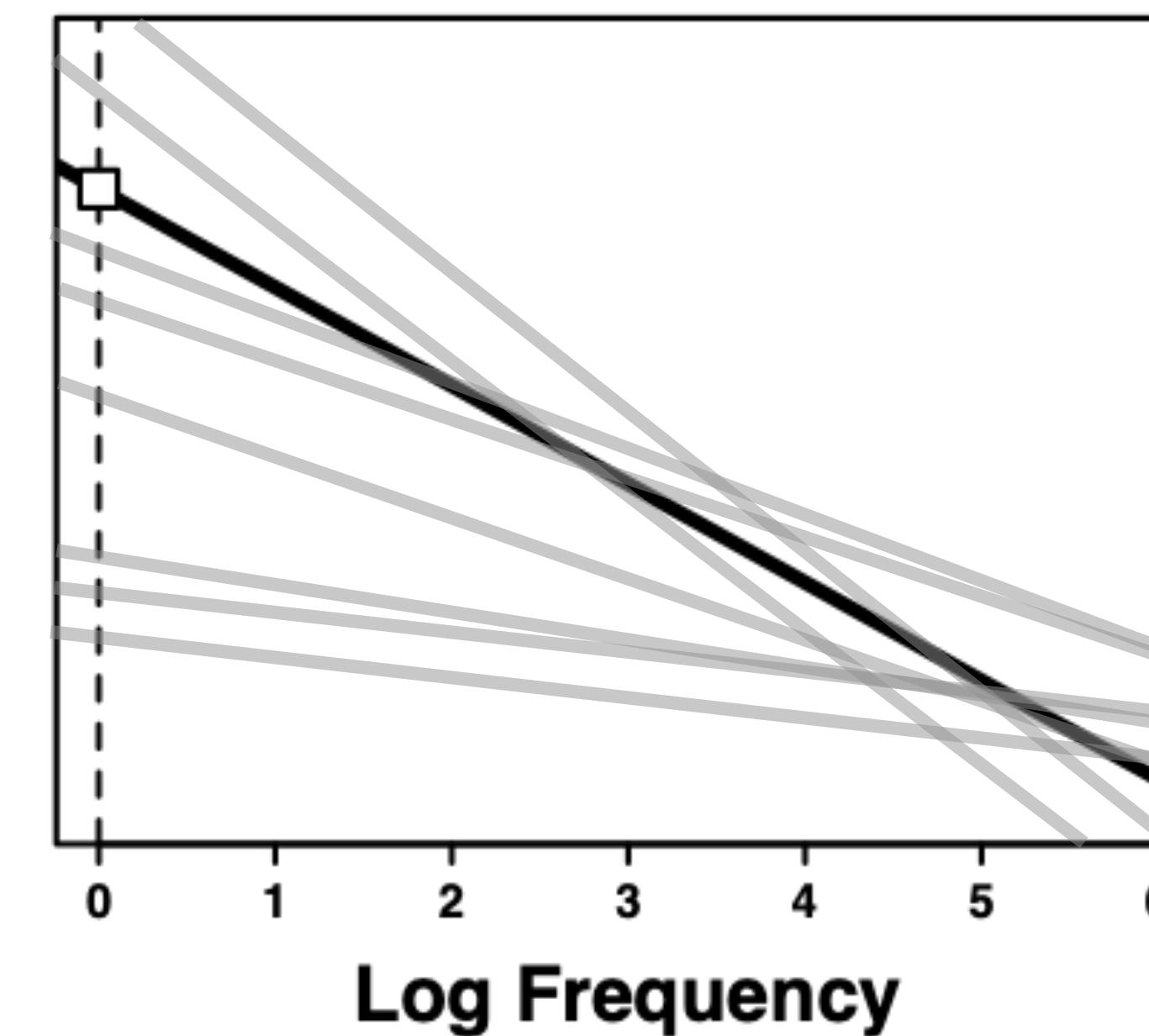
intercepts & slopes are NOT  
correlated

**intercepts & slopes are NOT correlated**

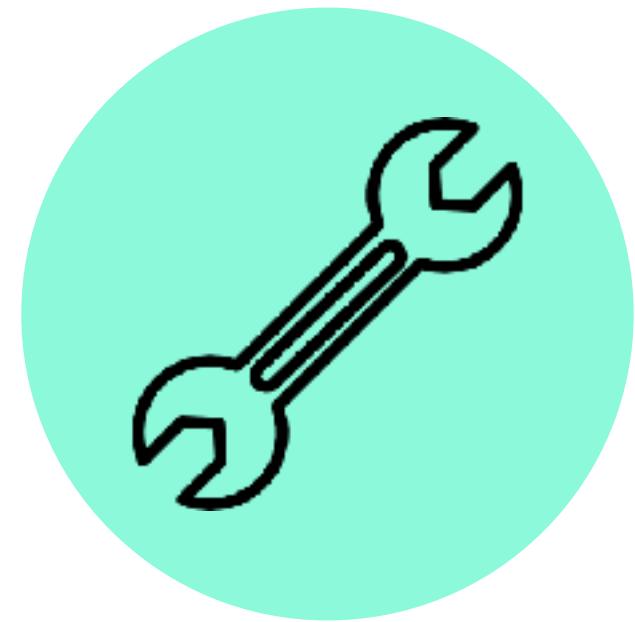


Slope variation between groups  
does not depend on their intercepts

**intercepts & slopes are correlated**



Slope variation between groups  
depends on their intercepts, e.g. higher  
intercepts have steeper slopes



# Simulation

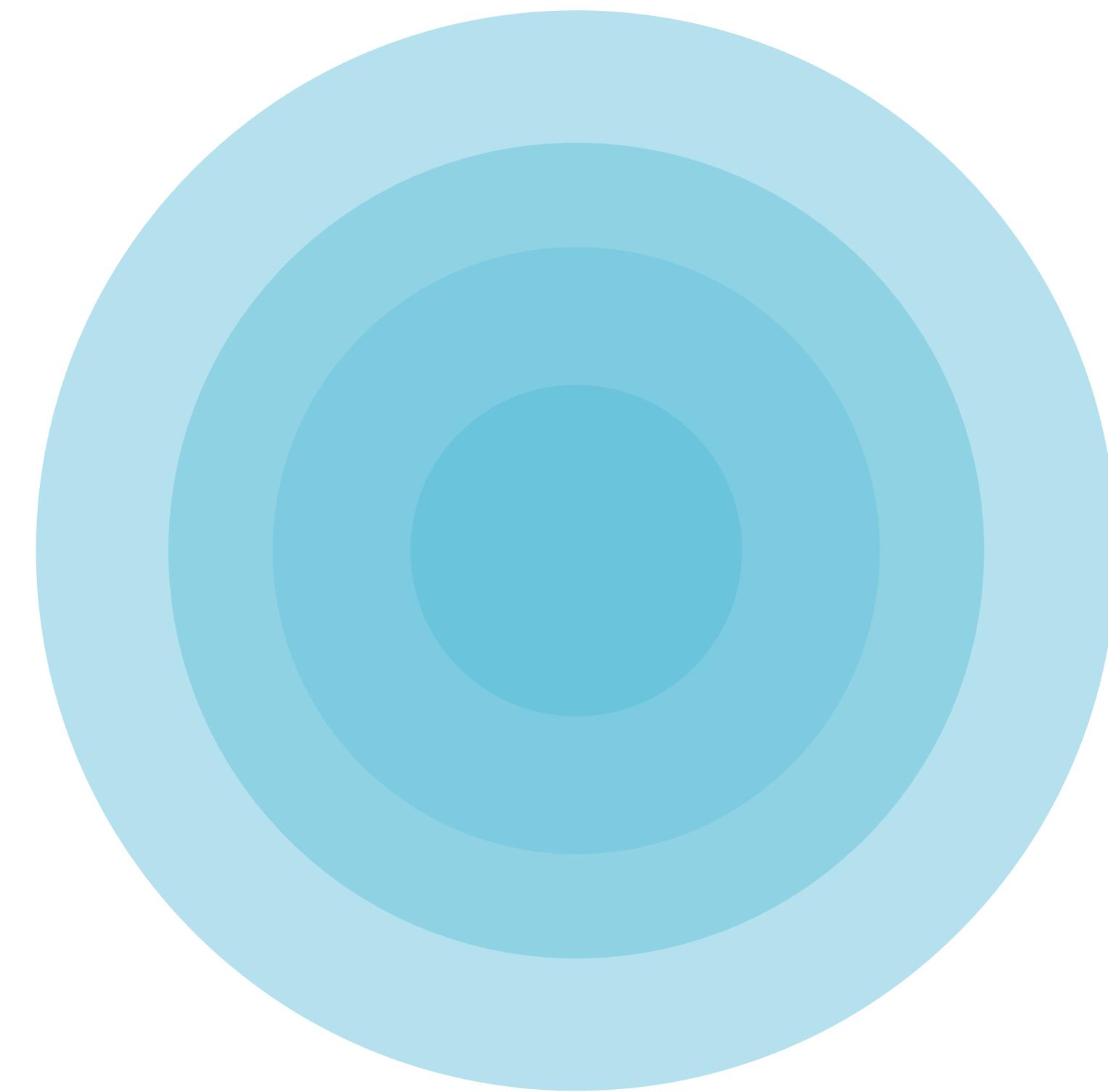
# “Fixed” vs. “Random” effects

(1) Effects are fixed if they are **interesting in themselves** or random if there is interest in the underlying population. Searle et al. (1992)

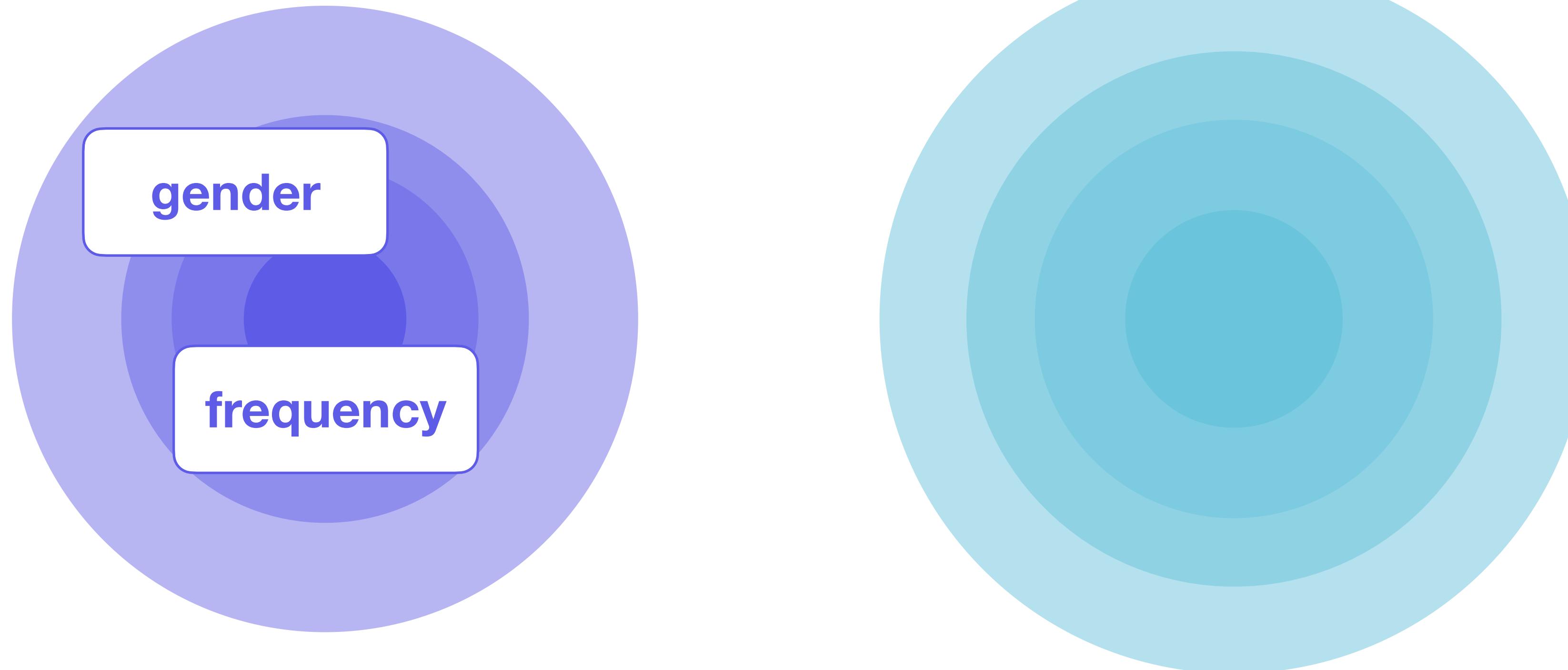
(2) “When a sample **exhausts the population**, the corresponding variable is *fixed*; when the sample is **a small (i.e., negligible) part of the population** the corresponding variable is *random*.” Green & Tukey (1960)

—> If you want to **generalize beyond a sample** of those levels, you treat it as random.

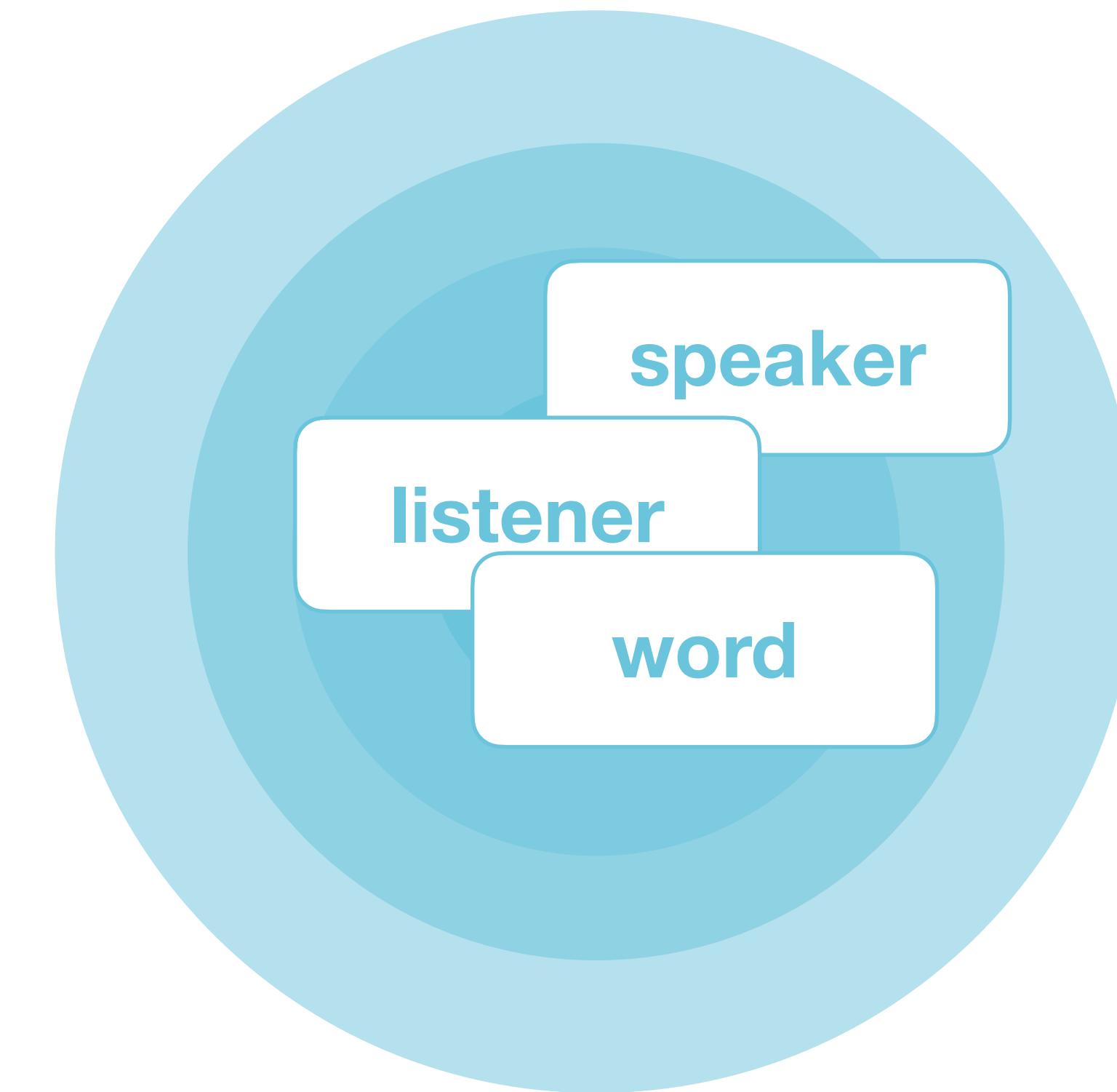
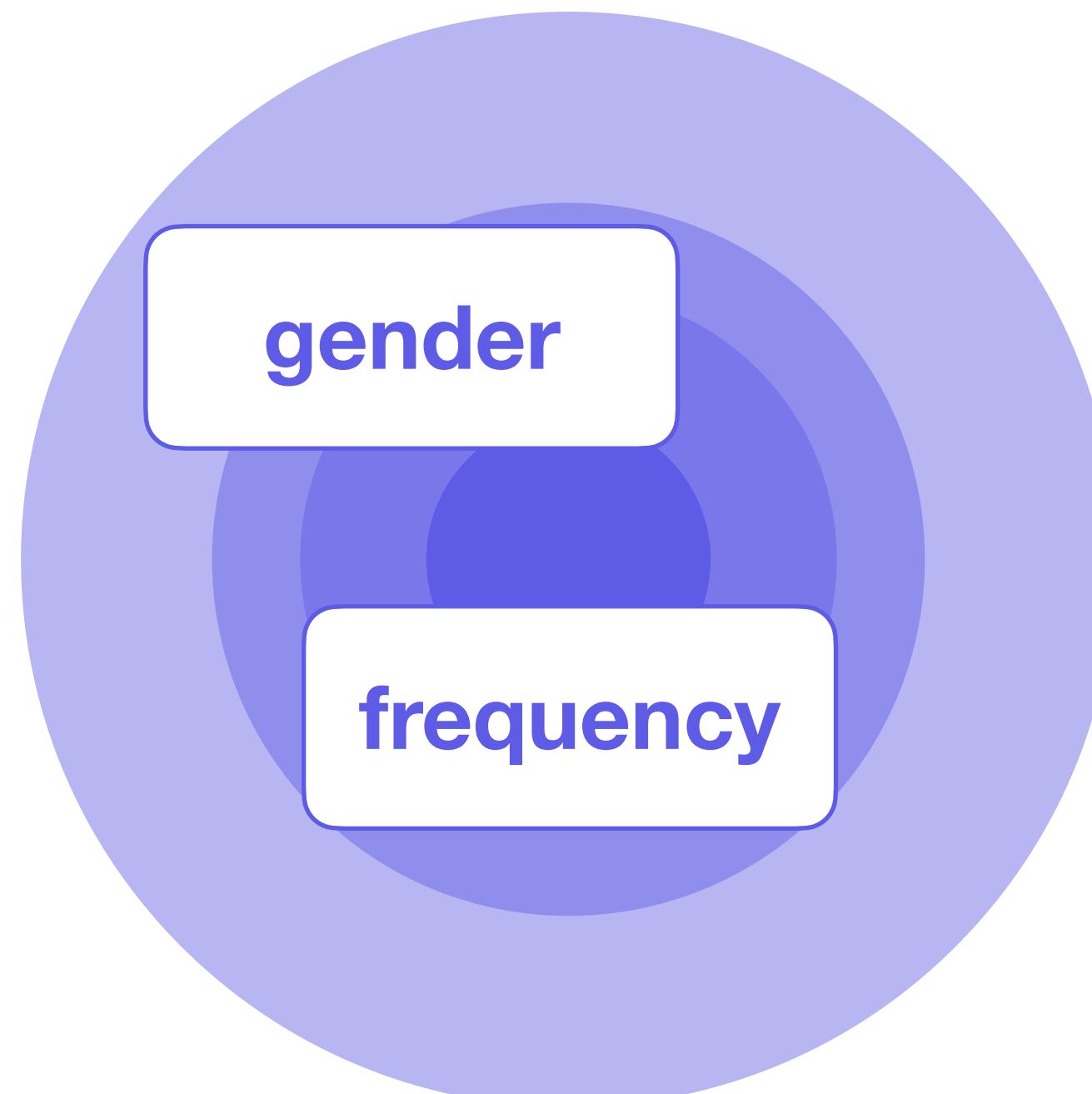
# **“Fixed” vs. “Random” effects**



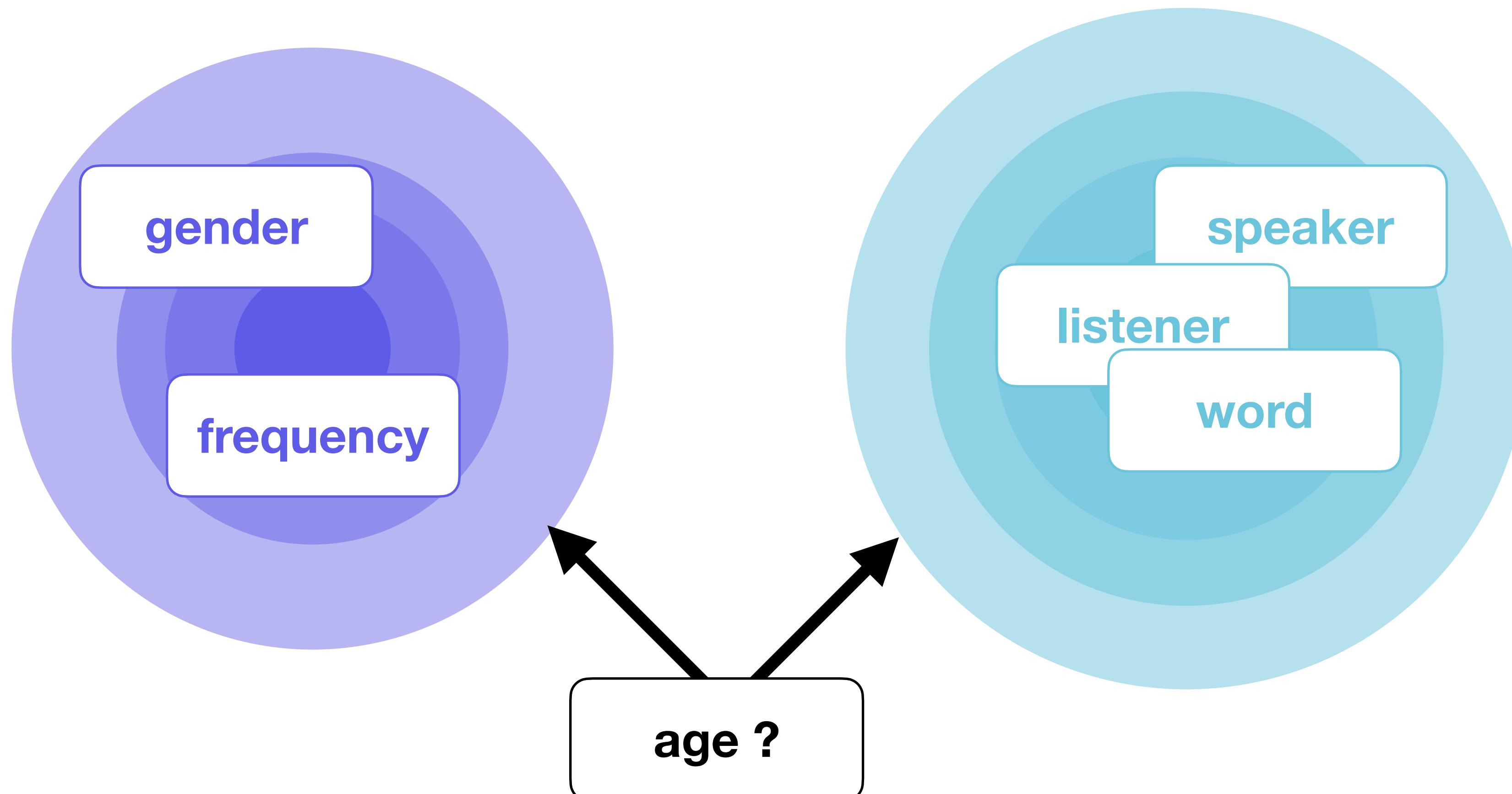
# “Fixed” vs. “Random” effects



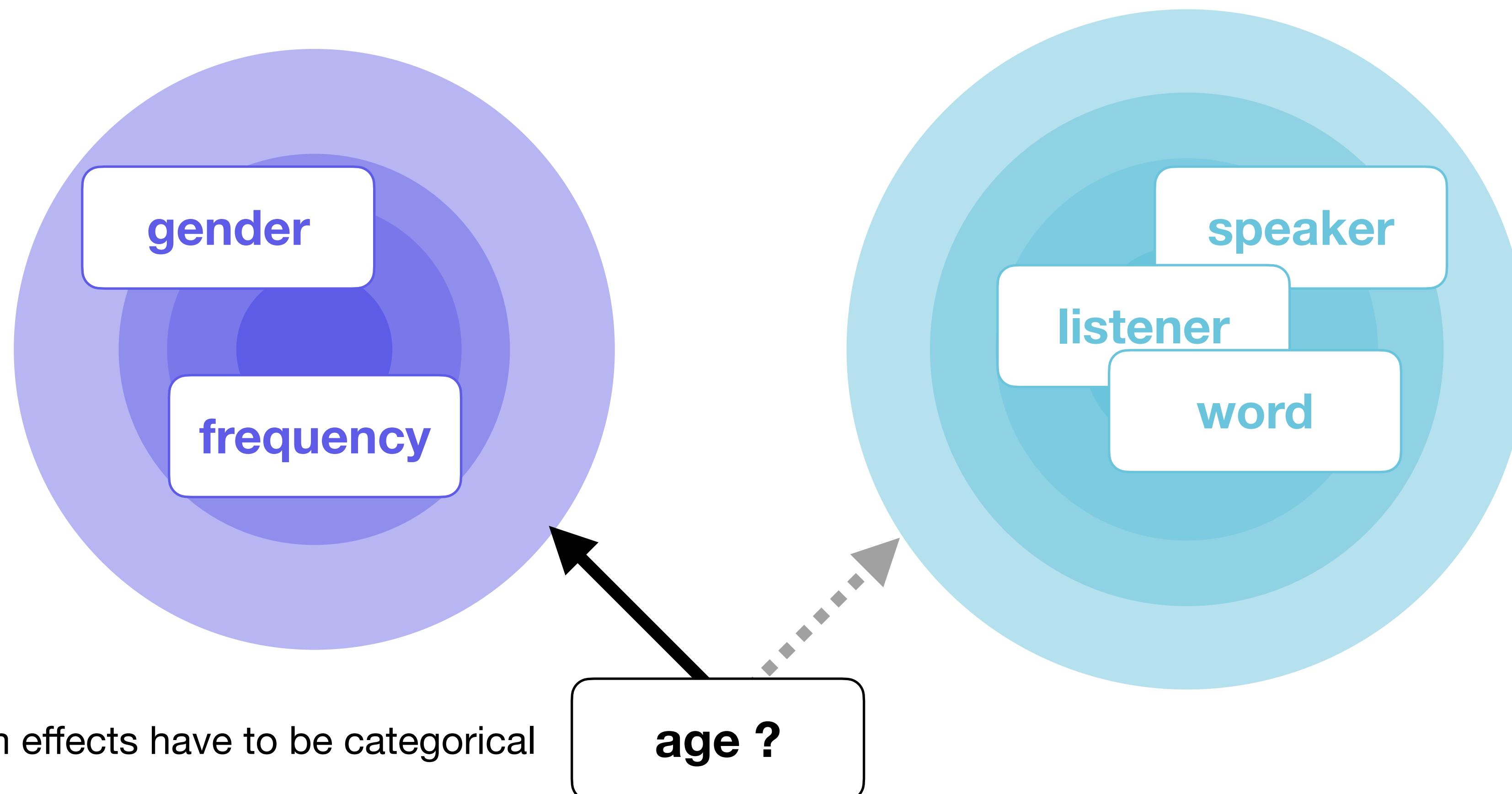
# “Fixed” vs. “Random” effects



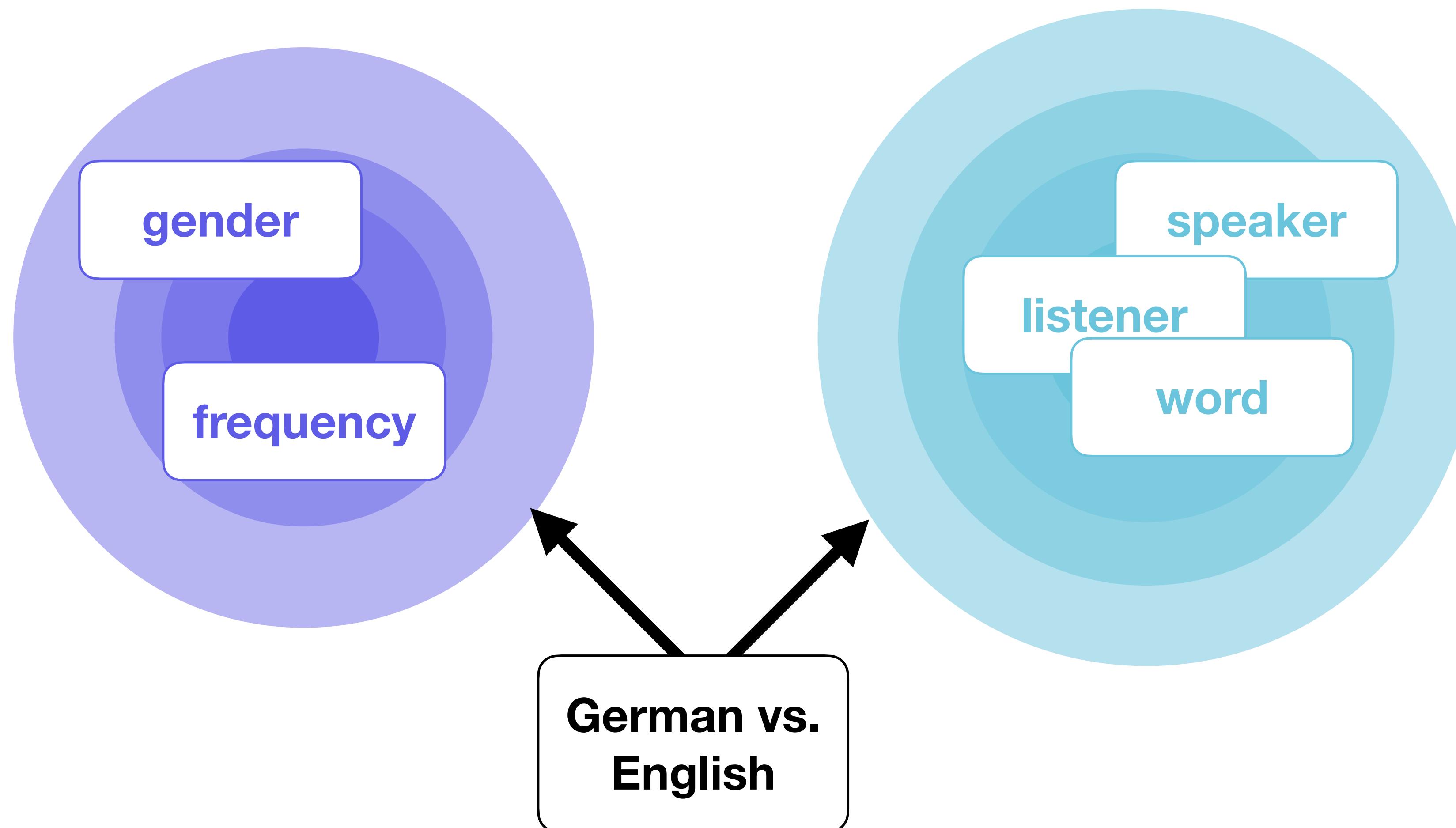
# “Fixed” vs. “Random” effects



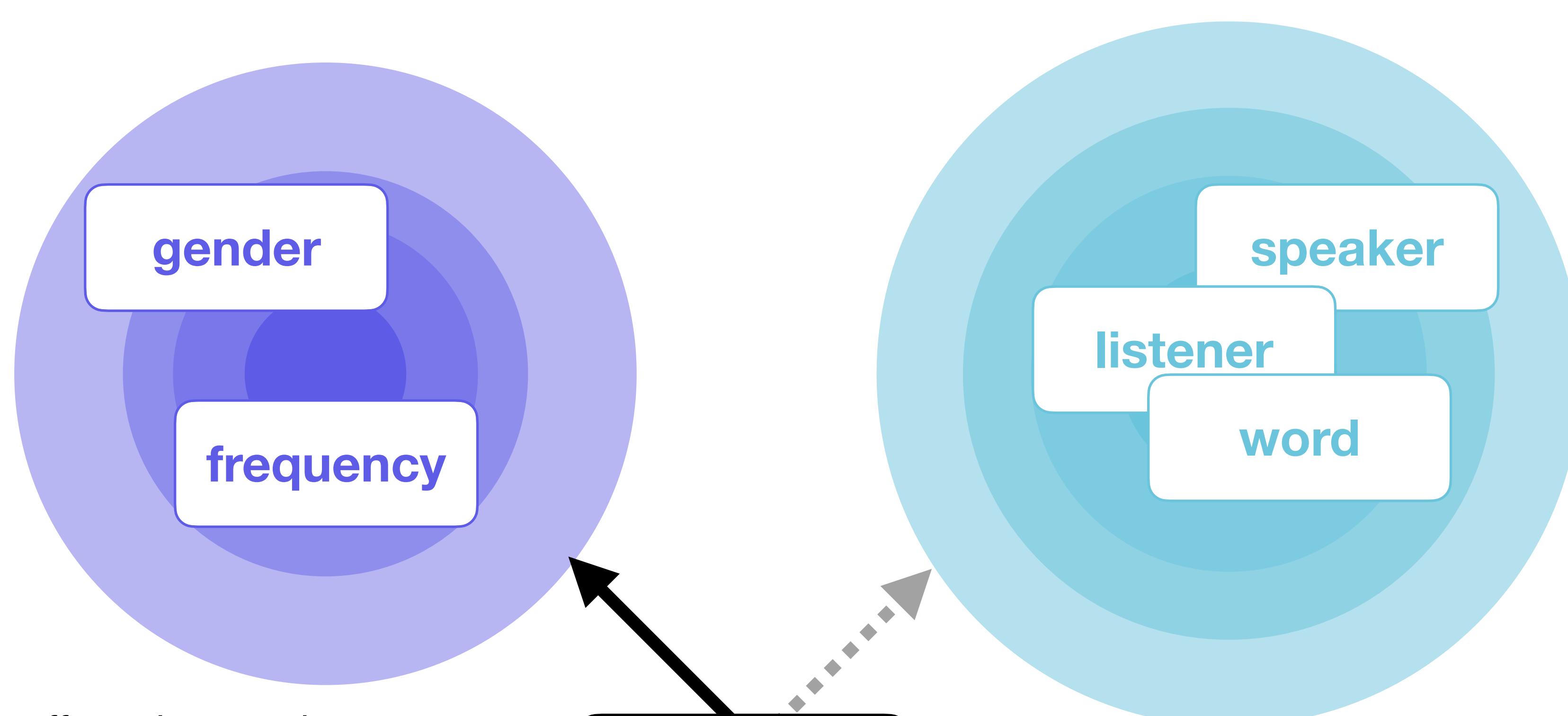
# “Fixed” vs. “Random” effects



# “Fixed” vs. “Random” effects



# “Fixed” vs. “Random” effects



Random effects have to have at least 5-6 levels for estimation to be robust,  
cf. Gelman & Hill (2007)

**German vs.  
English**

<b>Subject</b>	<b>Item</b>	<b>Condition</b>
S1	1	Test
S1	2	Control
S1	3	Test
S1	4	Control
S2	5	Test
S2	6	Control
S2	7	Test
S2	8	Control

subjects-n = 20  
item-n = 20

```
lmer(y ~ 1 + Condition +
```

...

...

<b>Subject</b>	<b>Item</b>	<b>Condition</b>	
S1	1	Test	subjects-n = 20
S1	2	Control	item-n = 20
S1	3	Test	
S1	4	Control	
S2	5	Test	
S2	6	Control	
S2	7	Test	
S2	8	Control	

```
lmer(y ~ 1 + Condition +
      (1 + Condition | subject))
```

...

<b>Subject</b>	<b>Item</b>	<b>Condition</b>	
S1	1	Test	subjects-n = 20
S1	2	Control	item-n = 20
S1	3	Test	
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S2	6	Control	
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S2	8	Control	

```
lmer(y ~ 1 + Condition +
      (1 + Condition | subject) +
      (1 + Condition | item))
```

<b>Subject</b>	<b>Item</b>	<b>Condition</b>	
S1	1	Test	subjects-n = 20
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```
lmer(y ~ 1 + Condition +
      (1 + Condition | subject) +
      (1 + Condition | item))
```



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# Random effects structure for confirmatory hypothesis testing: Keep it maximal

Dale J. Barr<sup>a</sup>  , Roger Levy<sup>b</sup>  , Christoph Scheepers<sup>a</sup>  , Harry J. Tily<sup>c</sup>  

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# But maximal models might not “converge”

Model did not arrive at stable estimates.

That means, we **cannot trust** the results.

Practically, we **simplify random effect** structure until convergence.

```
## Warning in checkConv(attr(opt, "derivs")), opt$par, ctrl =  
## control$checkConv, : Model failed to converge with max|grad| = 1.52673  
## (tol = 0.001, component 17)  
  
## boundary (singular) fit: see help('isSingular')
```

# Suggested workflow

1. Run **maximal** random effect structure, i.e. predictor slopes for all predictors for all grouping variables

2. if it does not converge, remove one slope at a time and rerun until convergence.

Usually random intercept models converge fine

3. Transparently report the final model, remain sceptical and advice reader to be sceptical

```
lmer(y ~ 1 + Condition +  
      (1 + Condition | subject) +  
      (1 + Condition | item))
```

```
## boundary (singular) fit: see help('isSingular')  
  
lmer(y ~ 1 + Condition +  
      (1 + Condition | subject) +  
      (1 | item))
```

```
## boundary (singular) fit: see help('isSingular')  
  
lmer(y ~ 1 + Condition +  
      (1 | subject) +  
      (1 | item))
```