

# Chapter 1 Day 2

## Summary

The **normal curve** is a **density curve**. The normal curve has the **normal table** which can be used to look up what **proportion** is up to a point. Data can be converted to a **Z-value** where the normal curve and **table** can be used. Knowing the **standard deviation**, **mean**, and z-value we can work backwards to get the **x-value**.

## Terminology

Variance / Standard Deviation

- measure variability around the mean
- if number is small, then the spread of the numbers is small
- the unit of variance is squared and not very useful, so we often take the square root to get standard deviation

Density Curve

- mathematical model of distribution
- an approximation
- The total area under the curve is equal to 1 or 100%

Population

- **all** items collected
- use greek letters when analysing

Sample

- by taking a different sample, we will get different numbers
- use variable names

## Variance and Standard Deviation

A simple example:

Data set - 1,1,3,4,6

mean =  $(1+1+3+4+6)/5 = 3$

$x_i - \bar{x}$

```
1-3=-2
1-3=-2
3-3= 0 # middle
4-3= 1
6-3= 3
```

Now we square it to get rid of negative. Add the squares up to get the **sum of squares**.

$4+4+0+1+9=18$

Then we divide it by  $n-1$ . Sum of squares is always an underestimate, so we need to divide by  $n-1$  instead of  $n$ .

$18/(5-1) = 4.5$

## 5 number summary vs mean and std dev

Can use either.

5 number summary usually better if distribution skewed or with strong outliers.

Mean, std. dev. suitable if distribution is reasonably symmetric and free of outliers

## Describing distributions

Plot your data.

Look for patterns and outliers. Can they be explained?

Calculate appropriate numerical summary measures.

## The density curve

Sometimes the pattern of observations is so strong we can approximate it with a *density curve*.

Can interpret the area under the curve. That is the proportion of data between  $x_1$  and  $x_2$ .

The **median** of a density curve is the equal areas point.

The **mean** of a density curve is the balance point.

Can be the same number for a symmetric density curve.

Use English letters for samples, because the numbers will be different for different samples; they are variables. When using the population, however, we use Greek letters.

## Normal Curves

The most important class of density curves.

Symmetric with one peak; bell shape.

## 68, 95, 99.7 rule

Three symmetrical intervals containing proportions.

68 - 1 standard deviation

95 - 2 standard deviations

99.7 - 3 standard deviations

## Z-value / Z standard / standardized value of x

Get distance between mean and point, then divide by the standard deviation.

$$Z = (x - \mu) / \sigma$$

$$N(\mu, \sigma)$$

## Normal Tables

Table A (in textbook) gives proportion of observations that fall to the left of  $z$  standard deviations from the mean.

**P** for proportion

```
P(-2.15 <= Z <= 1.20) # Proportion between 2 areas
P(Z <= 1.20) - P(Z <= -2.15)
= 0.8849 - 0.0158
= 0.8691 or 86.91%
```

Standardize all terms!

$$P((240-\mu)/\sigma \leq (x-\mu)/\sigma \leq (275-\mu)/\sigma)$$
$$P((240-250)/15 \leq Z \leq (275-250)/15)$$
$$\dots$$

If you cannot find a Z-value in the table, it is either 0 or 100%!

### Working "backwards"

Given proportion, find value

eg.1) 4% of values for the standard normal distribution are bigger than what value.

$$1 - 0.04 = 0.9600$$

Check closest value at 0.9600. If equally close, take average  
1.75

eg.2) The "most central" 90% of standard normal distribution lies between what values?

A: Use symmetry,  $1 - 0.90 = 0.10$  for the tails.  $0.10/2 =$  size of each tail.

Between -1.645 and 1.645

Also can go the other way, if you have Z, mean, and SD, but need x

$$Z = (x - \mu) / \sigma$$
$$Z\sigma = x - \mu$$
$$x = \mu + Z\sigma$$