

# Chapter 1: Linear Equations in linear algebra

## Summary

Systems can have solutions. Namely, **exactly one solution**, **no solution**, or **infinitely many solutions**.  
Make simpler equivalent systems and solve those row equivalent matrixes.

## Terminology

- **solution**
  - sub this in the system to get a true statement
- **equivalent systems**
  - systems may look different but are equivalent if their solutions are the same
  - example: multiply whole system by a constant
- **consistent / does a solution exist**
  - at **least** one solution
- **inconsistent**
  - **no** solution exists
  - often when  $[0 \ 0 \ \dots \ 0 \ n]$  // *because  $0 \neq n$*
- **unique**
  - one solution
- **$m \times n$  matrix**
  - m is rows, n is columns
- **coefficient matrix**
  - does not include right-hand constants
- **augmented matrix**
  - does include right-hand constants
- **row equivalent**
  - elementary row operations changed  $B$  to  $A$ ; written  $B \sim A$
  - same solution set
  - elementary row operations are reversible

## Section 1.1

- a linear equation (what is it?)
  - $y = mx + b$  ||  $y - y_1 = m(x - x_1)$  ||  $x = 0$

- $4x + 3y + 2z = 1$  // *could be multiple dimensions*
- Don't need **x, y, z** instead use **x1, x2, ... xn** because there could be a wack ton of dimensions
- a linear equation (what is it not?)
  - $x^2 = y$  // *not parabola*
  - $x_1 * x_2 = 3$  // *variables multiplied*
  - **can't ever have anything raised to a power**

### In general...

A linear equation can be written like  **$a_1x_1 + a_2x_2 + \dots a_nx_n = b$** .

Constants can be "0".

- a system (what is it?)
  - you can use substitution
  - they are in the same universe
  - the value that satisfies it works for all

## Solving linear equations

*Big idea:* make an equivalent system that is simpler and solve that one.

### Three things you can do to make an equivalent system

1. Interchange two equations
  - move them around
2. Multiply an entire equation by a nonzero constant
  - scale it
3. Add / subtract two equations
  - $a = b, c = d \Rightarrow a + c = b + d$

## Format how we write linear equations

- linear terms on left side
- constants on right side
- variables in order
- welcome *matrix notation*

## Matrix notation

### Elementary Row Operations (*similar to equivalent system operations*)

1. *Exchange:* Interchange two rows
2. *Scale:* Multiply row by non-zero constant
3. *Replacement:* Replace one row by the sum of itself and a multiple of another row

Coming up...  
solve more systematically!