

MACM day9 - Conditional Probability and Independence

Summary

Probabilities can be dependent on each other. For example what's the chances of something happening if something else happens. We have a couple fun laws for this :)

Four consequences of $\Pr(B|A) = \Pr(B \cap A) / \Pr(A)$

1. switching A and B
 $\Pr(A|B) = \Pr(A \cap B) / \Pr(B)$
2. Multiplicative rule
 $\Pr(A \cap B) = \Pr(B) * \Pr(A|B)$
3. Law of total probability
 $B = [A \cap B] \cup [B \cap A(\text{bar})]$
 $\Pr(B) = \Pr([A \cap B] \cup [B \cap A(\text{bar})])$
 $\Pr(B) = \Pr(A \cap B) + \Pr(B \cap A(\text{bar}))$
 $\Pr(B) = \Pr(A) * \Pr(B|A) + \Pr(A(\text{bar})) * \Pr(B|A)$
4. Bayes' Theorem
 $\Pr(B|A) = \Pr(A \cap B) / \Pr(A) = \Pr(B) * \Pr(A|B) / \Pr(A)$
 $\Pr(B|A) = \Pr(A|B) * \Pr(B) / \Pr(A)$.

Independent

Two events A and B are independent if either one of them has probability 0 or both have positive probability and

$$\Pr(B|A) = \Pr(B) \text{ and } \Pr(A|B) = \Pr(A)$$

Two events A and B are independent if and only if

$$\Pr(A \cap B) = \Pr(A)\Pr(B)$$