Chapter 1: Linear Equations in linear algebra

Summary

Every matrix can be reduced to Echelon form. Echelon form is useful for determining how many solutions we have.

Terminology

non-zero

- a row is non-zero contains at least one non-zero entry
- can also have a column that's non-zero
- **all zeros**

leading entry

- ... of a non-zero row is the leftmost non-zero entry
- [0 0 -1 1] *// -1*

echelon form (EF)

1. zero-rows on bottom
2. each leading entry in a row is strictly to the right of the leading entry of the row above it (staircase down)
3. all entries below a leading entry are zero

reduced echelon form (REF)

4. the **leading** entry in each non-zero row is 1
5. each leading 1 is the only nonzero entry in it's column

Pivot position

- leading entry **in echelon form**

Pivot column

- the column that contains a *pivot position*

Basic variables

- variables corresponding to pivot columns

Free variables

- does not correspond to a pivot column
- MUST state a free variable
- can be any value

Details

zero column remains invariant under all elementary matrix operations.

Section 1.2

Row reduction and echelon forms

Transform a matrix into it's echelon form

> Every matrix can be row reduced to an echelon form, which is NOT unique

1. Omit zero columns if necessary
2. make each entry underneath the top left zero
3. repeat steps 1 and 2 for the submatrix until we get an EF

Pivots

> if the last column is a pivot column, then it's inconsistent

... because **0 0 0 n** is inconsistent

> if last column isn't a pivot column

... you have at **least** one solution

> ignore the last column, if all remaining columns are pivot columns

... you have exactly one solution *consistent* and *unique*.
Otherwise infinitely many solutions.

Reduced Echelon Form

> every matrix can be transformed into exactly one reduced echelon matrix

Row Reduction Algorithm

```
let A be a matrix

Forward Phase:
  1. if  all entries of A are zero, then done
     else  the leftmost non-zero column is a pivot.
           the corresponding pivot position is at the top.
  2. Select  a non-zero entry in the pivot column
     if  this entry is not in the pivot position.
     interchange  rows to move it to the pivot position
  3. Use replacement operations to make all positions below the pivot position
be zeros.
  3. Apply steps 1-3 to the submatrix.

Backward Phase (step 5).
  5.1 Select the rightmost pivot position.
  5.2 Scale so the entry in pivot position is 1.
  5.3 Use replacement operations to make all entries above be zeros.
  5.4 Apply steps 5.1-5.3 to the next rightmost pivot position
```

Matrix --forward-phase--> Echelon Form --backward-phase--> Reduced echelon form

A consistent system is unique (exactly one solution) if it has no free variable and is not unique (infinitely many solutions) otherwise.

No free variables = unique. Free variables = infinitely many