

# Chapter 1: Linear Equations in linear algebra - vector equations

## Summary

Vectors exist in  $\mathbb{R}^n$  space. You can add vectors in the same  $\mathbb{R}^n$ . A vector or a collection of vectors will have a span. The span is all the points in  $\mathbb{R}^n$  space the vector(s) can reach. You can tell if a vector exists in a span by making an augmented matrix and testing if the system is consistent.

## Terminology

### Vector

- list of ordered numbers
- matrix with **single** column
- arrow on top, could be bar on bottom

### Scalar multiple

- multiplying each entry of  $v$  by  $c$
- scale a vector

### $\mathbb{R}^n$

- n-dimensional real space
- the set of all column vectors with  $n$  real entries
- $\mathbb{R}^n = \{(x_1 \ x_2 \ \dots \ x_n) \mid x_1, x_2 \ \dots \ x_n \in \mathbb{R}\}$

### Zero vector

- all entries zero
- a zero vector of each size ( $\mathbb{R}^1, \mathbb{R}^2 \ \dots \ \mathbb{R}^n$ )
- but it should be clear from context what size is meant
- Graphically it's just the origin

### parallelogram rule

- adding vectors tip and tail

### weights

- constants in front of vectors

### linear combinations

- adding  $v_1, v_2 \ \dots \ v_n$  with weights  $c_1, c_2 \ \dots \ c_n$

### span

- the set of all linear combinations of  $v_1, \dots, v_n$

## Vectors

Two vectors in  $\mathbb{R}^n$  are equal if their corresponding entries are equal

$(1 \ -2) \neq (-2 \ 1)$   
order matters!

## $\mathbb{R}^n$

$\mathbb{R}^1$  = real line

$\{\mathbf{x} | \mathbf{x} \in \mathbb{R}\}$

$\mathbb{R}^2$  = real plane

$\{(\mathbf{x} \ y) | \mathbf{x}, \mathbf{y} \in \mathbb{R}\}$

Like cartesian plane but instead of  $(x, y)$  we use vector notation.  $(x \ y)$  *vertical*

## Arithmetic of vectors in $\mathbb{R}^n$

The sum of two vectors are obtained by adding the corresponding entries. (Slap em' together)

1.  $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2.  $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3.  $(\mathbf{u} + \mathbf{0}) = \mathbf{u}$
4.  $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5.  $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6.  $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7.  $c(d\mathbf{u}) = (cd)\mathbf{u}$
8.  $1\mathbf{u} = \mathbf{u}$

## Spans

The span of a single vector (multiply it by any Real number). The set of all multiples of that vector. Will get a line as long as it isn't a zero vector

in  $\mathbb{R}^2$ , if we take another vector  $\mathbf{w}$  not on the line, and not a zero vector, then  $\text{span}\{\mathbf{v}, \mathbf{w}\}$  is any Real number in  $\mathbb{R}^2$ .

in  $\mathbb{R}^3$ , if we take another vector  $\mathbf{w}$  not on the line, and not a zero vector, then  $\text{span}\{\mathbf{v}, \mathbf{w}\}$  is a plane through the origin in  $\mathbb{R}^3$  space containing  $\mathbf{w}$  and  $\mathbf{v}$ .

$$\text{span}\{\mathbf{0}\} = \mathbf{0}$$

$$\text{span}\{\mathbf{v}, 2\mathbf{v}\} = \text{span}\{\mathbf{v}\} \text{ // line through } \mathbf{v} \text{ and } \mathbf{0}$$

## Representing a linear system using vector equations

$$x_1 - 2x_2 + x_3 = 5$$

$$x_1 - x_2 - x_3 = 0$$

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$$x_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

solving a linear system is equivalent to finding a linear combination of the coefficient vectors which is equal to a particular other vector

From the point of view of spans, asking if  $b$  is in  $\text{span}\{v_1, \dots, v_p\}$  is the same as asking if  $x_1v_1 + x_2v_2 + \dots + x_pv_p = b$  has a solution. That is the same as asking if the system with augmented matrix is consistent.

Row Reduction Answers this!