21/04/2022

Limiti

1) $\lim_{n \to \infty} \frac{2n-1}{3n+2} = \frac{2}{3}$ con la definizione

$\forall \varepsilon > 0 \quad \exists N$ tale che per $n \geq N \quad \left| \frac{2n-1}{3n+2} - \frac{2}{3} \right| < \varepsilon$

$$\left| \frac{2n-1}{3n+2} - \frac{2}{3} \right| = \left| \frac{3(2n-1)-2(3n+2)}{9n+6} \right| = \left| \frac{6n-3-6n-4}{9n+6} \right| =$$

$$= \left| \frac{-7}{9n+6} \right| = \frac{7}{9n+6}$$

- $\frac{7}{9n+6} < \varepsilon \qquad \frac{7}{\varepsilon} < 9n+6 \qquad n > \frac{1}{9}\left[\frac{7}{\varepsilon} - 6 \right]$

- $\frac{7}{9n+6} < \frac{7}{9n} < \frac{9}{9n} = \frac{1}{n}$

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2) $\lim_{n \to \infty} \sqrt[n]{n \log n}$

$$n \leq n \log n \leq n^2 \quad \text{per} \quad n > e \text{, cioè} \quad n \geq 3$$

$$\Rightarrow \quad \sqrt[n]{n} \leq \sqrt[n]{n \log n} \leq \sqrt[n]{n^2}$$

$$\lim_{n \to \infty} \sqrt[n]{n^\alpha} = \lim_{n \to \infty} e^{\log n^{\alpha/n}} = \lim_{n \to \infty} e^{\frac{\alpha}{n} \log n} = e^0 = 1$$

$\Rightarrow$ teorema dei 2 carabinieri

$$\lim_{n \to \infty} \sqrt[n]{n \log n} = 1$$

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3) $\lim_{n \to \infty} \sqrt[n]{2^n + 3^n} = \lim_{n \to \infty} 3 \sqrt[n]{1 + \frac{2^n}{3^n}}$

$$= \lim_{n \to \infty} 3 \sqrt[n]{1 + \left(\frac{2}{3}\right)^n} \sim \lim_{n \to \infty} 3 \left[1 + \frac{1}{n} \left(\frac{2}{3}\right)^n \right] = 3$$

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4) $\lim_{n \to \infty} \frac{n^2 + n \operatorname{sen} n}{1 + n^2 + n} = \lim_{n \to \infty} \frac{n^2 \left(1 + \frac{\operatorname{sen} n}{n}\right)}{n^2 \left(1 + \frac{1}{n} + \frac{1}{n^2}\right)}$

$$\sim \lim_{n \to \infty} \frac{n^2 (1+0)}{n^2 (1+0+0)} = 1$$

5) $\lim_{n \to \infty} \left[\frac{n^3}{n^2+1} - \frac{n^3-1}{n^2} \right] =$

$$\lim_{n \to \infty} \frac{n^5 - (n^3-1)(n^2+1)}{n^2(n^2+1)} = \lim_{n \to \infty} \frac{n^5 - n^5 - n^3 + n^2 + 1}{n^4\left(1 + \frac{1}{n^2}\right)} =$$

$$= \lim_{n \to \infty} \frac{-n^3\left(1 - \frac{1}{n} - \frac{1}{n^3}\right)}{n^4\left(1 + \frac{1}{n^2}\right)} = 0$$

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6) $\lim_{n \to \infty} \left(\sqrt{n} - \sqrt[3]{n^3 - n + \operatorname{sen} n} \right) =$

$$= \lim_{n \to \infty} \sqrt{n} \left(1 - \sqrt{n} \sqrt[3]{1 - \frac{1}{n^2} + \frac{\operatorname{sen} n}{n^3}} \right) =$$

$$\sim \lim_{n \to \infty} \sqrt{n} \left[1 - \sqrt{n}\left(1 + \frac{\operatorname{sen} n}{3n^3} - \frac{1}{3n^2}\right) \right] =$$

$$\cancel{= \lim_{n \to \infty} \sqrt{n} \left[1 - \sqrt{n}\, \frac{\operatorname{sen} n}{3n^3} - \frac{1}{3n^2} \right] =}$$

$$= \lim_{n \to \infty} \sqrt{n} \left[1 - \sqrt{n} + \frac{\operatorname{sen} n}{3n^{5/2}} - \frac{1}{3n^{3/2}} \right] = -\infty$$

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7) $\lim_{x \to 0} \frac{\ln(1+x)^3}{\operatorname{sen} 5x + \sqrt[3]{x^4} \operatorname{sen} x} =$

$$\sim \lim_{x \to 0} \frac{\log(1+3x)}{x \left(\frac{\operatorname{sen} 5x}{x} + \sqrt[3]{x} \operatorname{sen} x \right)} =$$

$$\sim \lim_{x \to 0} \frac{3x}{x\left(5 + x^{4/3}\right)} = \frac{3}{5}$$

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8) $\lim_{x \to +\infty} \frac{\operatorname{sen}\left(\frac{x-1}{x^3+2}\right) \log_2\left(\frac{x-2}{x+1}\right)}{1 - \cos\left(\frac{1}{\sqrt{x^3}}\right)} =$

$$= \lim_{x \to +\infty} \frac{1}{\log 2} \frac{\operatorname{sen}\left(\frac{x-1}{x^3+2}\right) \log\left(\frac{x-2}{x+1}\right)}{1 - \cos\left(\frac{1}{\sqrt{x^3}}\right)}$$

$$\operatorname{sen}\left(\frac{x-1}{x^3+2}\right) \sim \operatorname{sen}\left(\frac{1}{x^2}\right) \sim \frac{1}{x^2}$$

$$\log\left(\frac{x-2}{x+1}\right) = \log\left(1 - \frac{3}{x+1}\right) \sim -\frac{3}{x+1}$$

$$1 - \cos\left(\frac{1}{\sqrt{x^3}}\right) \sim \frac{1}{2x^3}$$

$$\lim_{x \to \infty} \frac{\operatorname{sen}\left(\frac{x-1}{x^3+2}\right) \log\left(\frac{x-2}{x+1}\right)}{1 - \cos\left(\frac{1}{\sqrt{x^3}}\right)} \cdot \frac{1}{\log 2} =$$

$$= - \frac{\frac{1}{x^2} \cdot \frac{3}{x+1}}{\frac{1}{2x^3} \log 2} = - \frac{6}{\log 2}$$

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9) $$\lim_{x \to 1^-} \frac{\operatorname{sen}\sqrt{1-x} - x + 1}{e^{1-x} + 3\sqrt{1-x} - 1}$$

$$\lim_{x \to 1^-} \operatorname{sen}\sqrt{1-x} \sim \lim_{x \to 1^-} \cancel{\operatorname{sen}}\sqrt{1-x}$$

$$\lim_{x \to 1^-} e^{1-x} - 1 \sim \lim_{x \to 1^-} (1-x)$$

$$\lim_{x \to 1^-} \frac{\sqrt{1-x} + (1-x)}{1 - x + 3\sqrt{1-x}} = \lim_{x \to 1^-} \frac{\cancel{\sqrt{1-x}}}{\cancel{\sqrt{1-x}}} \frac{1 + \sqrt{1-x}}{3 + \sqrt{1-x}} = \frac{1}{3}$$

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10) ordine di infinito

$$\lim_{x \to \infty} \frac{3x^2 - \operatorname{sen} x}{3\sqrt{x} - 2} = \lim_{x \to \infty} \frac{3x^2\left(1 - \frac{\operatorname{sen} x}{3x^2}\right)}{3\sqrt{x}\left(1 - \frac{2}{3}\frac{1}{\sqrt{x}}\right)} =$$

$$= \lim_{x \to \infty} x^{3/2} = \infty$$

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11) $x^3 + x^5 \operatorname{sen}\left(\frac{1}{x}\right)$ per $x \to \infty$

$$x^5 \operatorname{sen}\frac{1}{x} \sim x^4$$

$$\lim_{x \to \infty} \left[x^3 + x^5 \operatorname{sen}\left(\frac{1}{x}\right)\right] \sim \lim_{x \to \infty} \left[x^3 + x^4\right] = \lim_{x \to \infty} x^4 \left[1 + \frac{1}{x}\right]$$

$$\sim \lim_{x \to \infty} x^4 = +\infty$$

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12) $$\frac{\sqrt[3]{n+3}}{\log(1+e^n) + \sqrt{n^2+n}}$$ per $n \to \infty$

$$\sim \frac{n^{1/3}}{\log(e^n) + n\sqrt{1+\frac{1}{n}}} \sim \frac{n^{1/3}}{n+n} \sim \frac{1}{2n^{2/3}}$$

13) $\lim_{n \to \infty} \frac{2^{\sqrt{(\log n)^2 + \log n^2}}}{n^2+1}$

$\sqrt{(\log n)^2 + \lg n^2} = \sqrt{(\log n)^2 + 2\log n} =$

$= \log n \sqrt{1 + \frac{2}{\log n}} \sim \log n \left(1 + \frac{1}{\log n}\right) =$

$= \log n + 1$

$\lim_{n \to \infty} \frac{2^{\sqrt{(\log n)^2 + \log(n^2)}}}{n^2+1} \sim \lim_{n \to \infty} \frac{2 \cdot 2^{\log n}}{n^2+1} =$

$= \lim_{n \to \infty} \frac{2 \cdot \left(2^{\log_2 n}\right)^{1/\log_2 e}}{n^2+1}$

$\sim \lim_{n \to \infty} 2 \cdot n^{1/\log_2 e - 2} = 0$

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14) $\lim_{x \to \frac{\pi}{2}} \tan(x) \left(e^{\cos x} - 1\right) =$

$\sim \lim_{x \to \frac{\pi}{2}} \frac{\operatorname{sen}(x)}{\cos x} \cdot \cos x = \lim_{x \to \frac{\pi}{2}} \operatorname{sen} x = 1$

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15) ordine di infinitesimo per $x \to 0$

$\frac{x^2 \log(1+x) + \tan x}{\operatorname{sen} x + \sqrt[3]{x}} \sim \frac{x}{x^{1/3}} \sim x^{2/3} \to 0$

$x^2 \log(1+x) + \tan x \sim x^2 \cdot x + x \sim x$

$\operatorname{sen} x + \sqrt[3]{x} \sim x + x^{1/3} \sim x^{1/3}$