1) $f(x) = \frac{x^2+3}{x-1}$

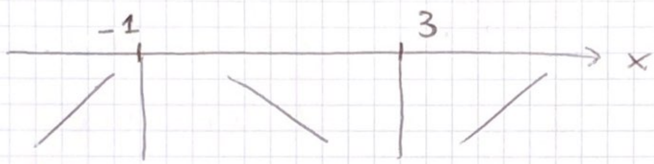
$$f'(x) = (2x) \cdot \frac{1}{x-1} + (x^2+3) \cdot \frac{1}{(x-1)^2}(-1) =$$

$$= \frac{2x(x-1) - x^2 - 3}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 3}{(x-1)^2} = \frac{x^2 - 2x - 3}{(x-1)^2}$$

$f'(x) \geq 0 \qquad \frac{x^2 - 2x - 3}{(x-1)^2} \geq 0 \qquad x^2 - 2x - 3 \geq 0$

$$x_{1,2} = 1 \pm \sqrt{1+3} = \begin{cases} 3 \\ -1 \end{cases}$$

$\Rightarrow \quad x \leq -1 \quad \vee \quad x \geq 3$



$x = -1$ MAX locale

$x = 3$ MIN locale

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2) $f(x) = \sqrt{x^2 - x} - x$

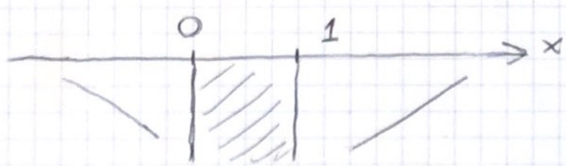
$$f'(x) = \frac{1}{2\sqrt{x^2-x}}(2x-1) - 1 = \frac{2x - 1 - 2\sqrt{x^2-x}}{2\sqrt{x^2-x}}$$

$f'(x) \geq 0 \qquad 2x - 1 - 2\sqrt{x^2-x} \geq 0 \qquad 2\sqrt{x^2-x} \leq 2x - 1$

per $x < \frac{1}{2} \quad f'(x) < 0$

per $x > \frac{1}{2} \qquad \cancel{4x^2} - \cancel{4x} \leq \cancel{4x^2} + 1 - \cancel{4x} \qquad 0 \leq 1$

$\Rightarrow \quad x > \frac{1}{2} \rightarrow x \geq 1$ per le C.E



$x = 0, 1$ MIN locali

$\lim_{x \to 0} f'(x) = -\infty \qquad \lim_{x \to 1} f'(x) = +\infty$

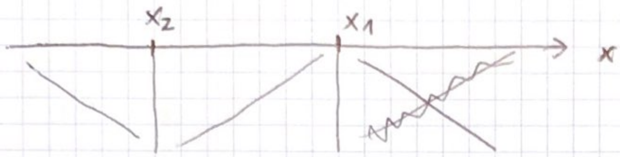
3) $f(x) = \dfrac{x}{4x+1}\, e^{-x}$

$$f'(x) = \frac{e^{-x}}{4x+1} - \frac{x e^{-x}}{4x+1} + x e^{-x} \cdot \frac{-4}{(4x+1)^2} =$$

$$= \frac{(4x+1)e^{-x} - x(4x+1)e^{-x} - 4x e^{-x}}{(4x+1)^2} = \frac{4x+1-4x^2-x-4x}{(4x+1)^2}\, e^{-x} =$$

$$= \frac{-4x^2-x+1}{(4x+1)^2}\, e^{-x}$$

$f'(x) \geq 0 \qquad -4x^2-x+1 \geq 0 \qquad x_{1,2} = \dfrac{+1 \pm \sqrt{1+16}}{-8} = \dfrac{-1 \pm \sqrt{17}}{8}$



$x = x_2$ MIN locale

$x = x_1$ MAX locale

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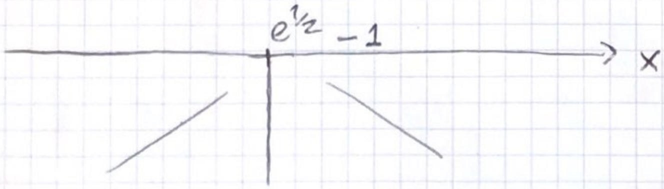
4) $f(x) = \dfrac{1}{(x+1)^2} \log(x+1)$

$$f'(x) = \frac{-2}{(x+1)^3}\log(x+1) + \frac{1}{(x+1)^3} = \frac{1-2\log(x+1)}{(x+1)^3}$$

$f'(x) \geq 0 \qquad \dfrac{1-2\log(x+1)}{(x+1)^3} \geq 0 \quad \rightarrow \quad 1-2\log(x+1) \geq 0$ per C.E.

$\log(x+1) \leq \dfrac{1}{2} \qquad x+1 \leq e^{1/2} \qquad x \leq e^{1/2} - 1$



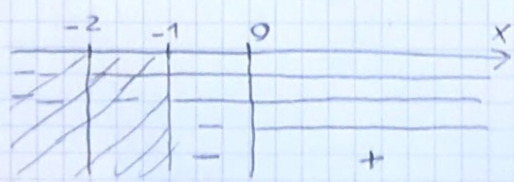
$x = e^{1/2} - 1$ MAX assoluto

---

5) $f(x) = \log\left(\dfrac{x^2}{x+1}\right)$

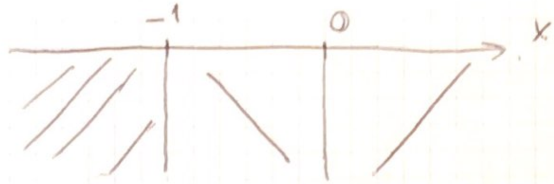
$$f'(x) = \frac{1}{\frac{x^2}{x+1}} \cdot \left[\frac{2x}{x+1} - \frac{x^2}{(x+1)^2}\right] = \frac{x+1}{x^2}\left[\frac{2x(x+1)-x^2}{(x+1)^2}\right] =$$

$$= \frac{2(x+1)-x}{x(x+1)} = \frac{x+2}{x(x+1)}$$



$\frac{x+2}{x(x+1)} \geq 0$

$x > 0$



$f(x)$ non definita in $x = 0$

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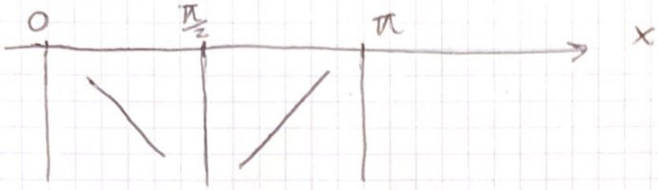
6) $f(x) = -\log(\operatorname{sen} x)$

$f'(x) = -\frac{1}{\operatorname{sen} x} \cdot \cos x = -\operatorname{cotg}(x)$

$-\operatorname{cotg}(x) \geq 0 \qquad \frac{\pi}{2} \leq x < \pi$



$x = \frac{\pi}{2}$ MIN assoluto

---

7) $f(x) = \sqrt{\frac{x^3}{x+3}}$

$$f'(x) = \frac{1}{2\sqrt{\frac{x^3}{x+3}}} \cdot \left[\frac{3x^2}{x+3} - \frac{x^3}{(x+3)^2} \right] =$$

$$= \frac{\sqrt{x+3}}{2\sqrt{x^3}} \cdot \frac{3x^2(x+3) - x^3}{(x+3)^2} =$$

$$= \frac{\sqrt{x+3} \cdot (\cancel{3x^3}\ 2x^3 + 9x^2)}{2\sqrt{x^3}\ (x+3)^2} = \frac{2x^3 + 9x^2}{2\sqrt{x^3(x+3)^3}}$$

$f'(x) \geq 0 \qquad 2x^3 + 9x^2 \geq 0 \qquad x^2(2x+9) \geq 0$

$x \geq -\frac{9}{2}$

$x = -\frac{9}{2}$ MIN locale

$x = 0$ MIN assoluto

8) $f(x) = \dfrac{e^x+1}{e^x-1}$

$$f'(x) = \frac{e^x}{e^x-1} - \frac{(e^x+1)(e^x)}{(e^x-1)^2} = \frac{e^x-1-e^x-1}{(e^x-1)^2}\, e^x =$$

$$= \frac{-2e^x}{(e^x-1)^2} < 0 \quad \forall x$$

---

9) $f(x) = \log\left|\dfrac{x+1}{x+3}\right| + x$

$$f'(x) = \frac{1}{\frac{x+1}{x+3}} \cdot \left[\frac{1}{x+3} - \frac{x+1}{(x+3)^2}\right] + 1 =$$

$$= \frac{x+3}{x+1} \cdot \frac{x+3-x-1}{(x+3)^2} + 1 = \frac{2+(x+1)(x+3)}{(x+1)(x+3)} =$$

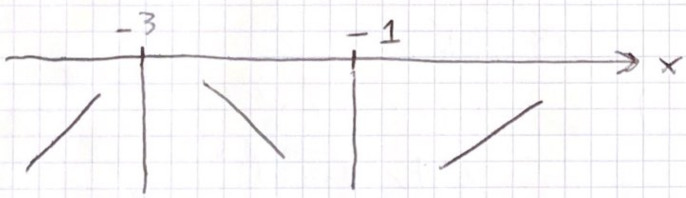
$$= \frac{x^2+4x+5}{(x+1)(x+3)}$$

$x^2+4x+5=0$

$x_{1,2} = -2 \pm \sqrt{4-5} \notin \mathbb{R}$

$\Rightarrow f'(x) \geq 0 \qquad \dfrac{x^2+4x+5}{(x+1)(x+3)} \geq 0 \rightarrow (x+1)(x+3) > 0$

$x > -1 \quad \vee \quad x < -3$



$x = -3, -1$ non appartengono al dominio

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10) $f(x) = \begin{cases} x^2 \operatorname{sen}\left(\frac{1}{x}\right) & x > 0 \\ x^2 + bx + c & x \leq 0 \end{cases} \qquad b, c \in \mathbb{R}$

f continua e derivabile

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$c = \lim_{x \to 0} x^2 \operatorname{sen}\left(\frac{1}{x}\right) = 0$$

$$f'(x) = \begin{cases} 2x+b & x \leq 0 \\ 2x\,\text{sen}\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x+b & x \leq 0 \\ -\cos\left(\frac{1}{x}\right) + 2x\,\text{sen}\left(\frac{1}{x}\right) & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$b = \lim_{x \to 0} \left[2x\,\text{sen}\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)\right] \quad \nexists \text{ limite}$$

$\Rightarrow$ rapporto incrementale

$$f'(0) = \lim_{x \to 0} \frac{f(\cancel{0}) - f(x)}{-x} = \cancel{\lim_{x\to 0}}$$

$$b = \lim_{x \to 0} \frac{x^2\,\text{sen}\left(\frac{1}{x}\right) - 0}{x} = \lim_{x \to 0} x\,\text{sen}\left(\frac{1}{x}\right) = 0$$

$$\Rightarrow f(x) = \begin{cases} x^2\,\text{sen}\left(\frac{1}{x}\right) & x \geq 0 \\ x^2 & x \leq 0 \end{cases}$$

---

(11) $f(x) = (x^3+1)^{\sqrt{x+2}} = e^{\sqrt{x+2}\,\log(x^3+1)}$

$$f'(x) = e^{\sqrt{x+2}\,\log(x^3+1)} \cdot \left[\frac{1}{2\sqrt{x+2}} \log(x^3+1) + \frac{\sqrt{x+2}}{x^3+1} \cdot 3x^2\right] =$$

$$= (x^3+1)^{\sqrt{x+2}} \left[\frac{(x^3+1)\log(x^3+1) + 2(x+2) \cdot 3x^2}{2\sqrt{x+2}\,(x^3+1)}\right] =$$

$$= (x^3+1)^{\sqrt{x+2}} \left[\frac{(x^3+1)\log(x^3+1) + 6x^3 + 12x^2}{2\sqrt{x+2}\,(x^3+1)}\right]$$