

Ohio State University ICPC Team Notebook

Contents

Sublinear Substring Seekers

1 Essentials

1.1	C++ header	1
1.2	C++ flags	1
1.3	C++ input/output	1

2 Data structures

2.1	Unordered Set/Map	2
2.2	Ordered Set/Map	2
2.3	Suffix array	3
2.4	Union-find set	3
2.5	KD-tree	4
2.6	Segment tree	5
2.7	Lazy segment tree	5
2.8	Lowest common ancestor	6
2.9	Li Chao Tree	6

3 Combinatorial optimization

3.1	Sparse max-flow	7
3.2	Min-cost max-flow	7
3.3	Min-cost matching	8
3.4	Max bipartite matching	9
3.5	Global min-cut	10

4 Geometry

4.1	Python geometry	10
4.2	3D geometry	12
4.3	Slow Delaunay triangulation	12

5 Numerical algorithms

5.1	Number theory (modular, Chinese remainder, linear Diophantine)	13
5.2	Modular Arithmetic	13
5.3	Systems of linear equations, matrix inverse, determinant	14
5.4	Reduced row echelon form, matrix rank	14
5.5	Fast Fourier transform	15
5.6	Euler's Totient Function	15
5.7	Partitions	16

6 Graph algorithms

6.1	Bellman-Ford shortest paths with negative edge weights (C++)	17
6.2	Topological sort (C++)	17
6.3	Fast Dijkstra's algorithm	17
6.4	Strongly connected components	18
6.5	Eulerian path	18
6.6	Minimum spanning trees	18

7 Strings

7.1	AhoCorasick	19
7.2	Longest increasing subsequence	19
7.3	Longest common subsequence	20
7.4	Knuth-Morris-Pratt	20
7.5	Longest Common Prefix	21
7.6	Palindromes	21

8 Miscellaneous

8.1	Prime numbers	22
8.2	Binary Search	22
8.3	Latitude/longitude	22
8.4	Hilbert curve for Mo's Algorithm	23

1 Essentials

1.1 C++ header

```
#include <bits/stdc++.h>
using namespace std;
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
```

1.2 C++ flags

```
# Add this to the CMakeLists in CLion to crash with bad memory accesses and give
better warnings.
# Don't include this comment, comments don't work in CMakeLists.
set(CMAKE_CXX_STANDARD 17)
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -Wall -Wextra -Wno-sign-compare -D
_GLIBCXX_DEBUG -D _GLIBCXX_DEBUG_PEDANTIC ")
```

1.3 C++ input/output

```
#include <iostream>
#include <iomanip>
#include <bitset>
using namespace std;

int main() {

    // Output a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed);
    cout << setprecision(5);
    cout << 100.0 / 7.0 << " " << 10.0 << endl; // 14.28571 10.00000
    cout.unsetf(ios::fixed);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl; // +100 -100
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal. Also works for oct
    cout << hex << 500 << dec << endl; // 1f4 (1*256 + 15*16 + 4*1)
    // Output numerical values in binary
    std::bitset<10> bs(500);
    cout << bs << endl; // 0111110100

    // Read until end of file.
    string line;
    getline(cin, line);
    while (!line.empty()) { // Input in CP problems always ends with an empty
        line.
        int intV; string stringV;
        stringstream line_stream(line);
        line_stream >> stringV >> intV; // Just read like usual from the stream
        getline(cin, line);
    }
}
```

2 Data structures

2.1 Unordered Set/Map

```
// An example of policy hashtable with a custom object in cpp. It is
// it is better than the built in unordered_map in that
// it is ~5 times faster. (https://codeforces.com/blog/entry/60737)
// No real downsides (normal map is just as annoying with custom objects),
// but be careful with the hash function, the number of buckets is a power of 2.
#include <bits/stdc++.h>
using namespace std;

struct Coordinate {
    int x;
    int y;
    bool operator==(const Coordinate &other) const {
        return x == other.x && y == other.y;
    }
};

ostream &operator<<(ostream &stream, const Coordinate &l) {
    return stream << "{" << l.x << " " << l.y << "}";
}

#include <ext/pb_ds/assoc_container.hpp>

struct chash {
    static auto const c = uint64_t(7e18) + 13; // Big prime
    uint64_t operator()(const Coordinate &l) const {
        return __builtin_bswap64((l.x + l.y) * c);
    }
};

template<class k, class v>
using hash_map = __gnu_pbds::gp_hash_table<k, v, chash>;
template<class k>
using hash_set = __gnu_pbds::gp_hash_table<k, __gnu_pbds::null_type, chash>;
template<typename k, typename v>
bool contains(hash_map<k, v> map, k val) {
    return map.find(val) != map.end();
}

int main() {
    // After importing, writing the template code, overloading ==
    // and << (print) operator like above, you can use the map
    hash_map<Coordinate, int> my_map;
    my_map[{1, 2}] = 17;
    cout << my_map[{1, 2}] << endl; // Prints 17
    assert(contains(my_map, {1, 2}));
    assert(!contains(my_map, {3, 4}));
    cout << my_map[{3, 4}] << endl; // Prints 0
    assert(my_map.size() == 2); // We just set {3, 4} to 0 by accessing it.
    for (auto pair : my_map) {
        cout << pair.first << " = " << pair.second << " "; // {3 4}=0 {1 2}=17
    }

    hash_set<Coordinate> my_set;
    assert(my_set.empty());
    my_set.insert({1, 2});
    assert(contains(my_set, {1, 2}));
    my_set.insert({4, 5});
    // hash_set does the correct thing, and when you iterate over it you get
    // keys,
    // not key-value pairs with a null value.
    for (auto it = my_set.begin(); it != my_set.end(); it++) {
        cout << *it << " "; // print {4, 5} {1, 2}.
    }
}
```

```
}
// Standard C Library Equivalent Declarations:
// unordered_map<Coordinate, int, chash> my_map;
// unordered_set<Coordinate, chash> my_set;
}
```

2.2 Ordered Set/Map

```
// An example of using an ordered map with a custom object.
// Also include code for the gnu policy tree, which gives
// a easy (~2x slower) segment tree by implementing
// find_by_order and order_of_key
#include <bits/stdc++.h>

using namespace std;

struct Coordinate {
    int x;
    int y;
    // Overloaded for ordered map. If !(c1<c2), !(c2<c1), then
    // c1 will be considered equal to c2.
    bool operator<(const Coordinate &o) const {
        return x == o.x ? y < o.y : x < o.x;
    }
};

ostream &operator<<(ostream &stream, const Coordinate &l) {
    return stream << "{" << l.x << " " << l.y << "}";
}

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;
template<class k, class v>
using ordered_map = tree<k, v, less<k>,
    rb_tree_tag, // Red black tree. Can use splay_tree_tag for a splay tree,
    // but split operation for splay is linear time so it may be terrible.
    tree_order_statistics_node_update // To get find_by_order and
    order_of_key methods
>;
template<class k> // Same as ordered map almost
using ordered_set = tree<k, null_type, less<k>,
    rb_tree_tag, tree_order_statistics_node_update>;

int main() {
    map<Coordinate, int> c_map; // Standard C Library Ordered Map
    set<Coordinate> c_set; // Standard C Library Ordered Set
    ordered_map<Coordinate, int> gnu_map; // Gnu map declaration
    ordered_set<Coordinate> gnu_set; // Gnu set declaration
    for (int i = 0; i < 10; i++) {
        gnu_set.insert({0, i * 10});
    }
    cout << *gnu_set.find({0, 30}) << endl; // {0, 30}
    cout << *gnu_set.lower_bound({0, 53}) << endl; // {0, 60}
    cout << *gnu_set.upper_bound({0, 53}) << endl; // {0, 60}
    cout << *gnu_set.lower_bound({0, 50}) << endl; // {0, 50}
    cout << *gnu_set.upper_bound({0, 50}) << endl; // {0, 60}
    // Example of the operations only supported by gnu_set
    cout << *gnu_set.find_by_order(2) << endl; // {0 20}
    cout << *gnu_set.find_by_order(4) << endl; // {0 40}
    assert(end(gnu_set) == gnu_set.find_by_order(10));
    cout << gnu_set.order_of_key({0, -99}) << endl; // 0
    cout << gnu_set.order_of_key({0, 0}) << endl; // 0
    cout << gnu_set.order_of_key({0, 11}) << endl; // 2
    cout << gnu_set.order_of_key({0, 999}) << endl; // 10
}
```

2.3 Suffix array

```
// Suffix array construction in  $O(L \log^2 L)$  time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in  $O(\log L)$  time.
//
// INPUT:  string s
//
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
//         of substring s[i...L-1] in the list of sorted suffixes.
//         That is, if we take the inverse of the permutation suffix[],
//         we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int, int>, int> > M;

    SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)),
        M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[level - 1][i], i + skip < L ? P[
                    level - 1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first == M[i - 1].first)
                    ? P[level][M[i - 1].second] : i;
        }

        vector<int> GetSuffixArray() { return P.back(); }

        // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
        int LongestCommonPrefix(int i, int j) {
            int len = 0;
            if (i == j) return L - i;
            for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
                if (P[k][i] == P[k][j]) {
                    i += 1 << k;
                    j += 1 << k;
                    len += 1 << k;
                }
            }
            return len;
        }
    };

    // BEGIN CUT
    // The following code solves UVA problem 11512: GATTACA.
    #define TESTING
    #ifndef TESTING
    int main() {
        int T;
        cin >> T;
        for (int caseno = 0; caseno < T; caseno++) {
            string s;
```

```
        cin >> s;
        SuffixArray array(s);
        vector<int> v = array.GetSuffixArray();
        int bestlen = -1, bestpos = -1, bestcount = 0;
        for (int i = 0; i < s.length(); i++) {
            int len = 0, count = 0;
            for (int j = i + 1; j < s.length(); j++) {
                int l = array.LongestCommonPrefix(i, j);
                if (l >= len) {
                    if (l > len) count = 2; else count++;
                    len = l;
                }
            }
            if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) >
                s.substr(i, len)) {
                bestlen = len;
                bestcount = count;
                bestpos = i;
            }
        }
        if (bestlen == 0) {
            cout << "No repetitions found!" << endl;
        } else {
            cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
        }
    }
}

#else
// END CUT
int main() {

    // bobocel is the 0'th suffix
    // obocel is the 5'th suffix
    // bocel is the 1'st suffix
    // ocel is the 6'th suffix
    // cel is the 2'nd suffix
    // el is the 3'rd suffix
    // l is the 4'th suffix
    SuffixArray suffix("bobocel");
    vector<int> v = suffix.GetSuffixArray();

    // Expected output: 0 5 1 6 2 3 4
    //
    //
    for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
    cout << endl;
    cout << suffix.LongestCommonPrefix(0, 2) << endl;
}

// BEGIN CUT
#endif
// END CUT
```

2.4 Union-find set

```
/**
 * Description: Disjoint-set data structure.
 * Time:  $O(\alpha(N))$ 
 */
struct UF {
    // E is parent set number if positive, and the size if negative.
    // If negative, it's the root of a set.
    vi e;
    UF(int n) : e(n, -1) {}
    bool sameSet(int a, int b) { return find(a) == find(b); }
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
    int find2(int x) { // Dennis's faster union find
```

```

    while (e[x] >= 0) {e[x] = e[e[x]]; x = e[x]}
    return x;
}
bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;
    return true;
}
};

```

2.5 KD-tree

```

// -----
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
// - constructs from n points in O(n lg^2 n) time
// - handles nearest-neighbor query in O(lg n) if points are well
//   distributed
// - worst case for nearest-neighbor may be linear in pathological
//   case
//
// Sonny Chan, Stanford University, April 2009
// -----

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b) {
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b) {
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b) {
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b) {
    ntype dx = a.x - b.x, dy = a.y - b.y;
    return dx * dx + dy * dy;
}

// bounding box for a set of points
struct bbox {
    ntype x0, x1, y0, y1;
    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}
    // computes bounding box from a bunch of points

```

```

void compute(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) {
        x0 = min(x0, v[i].x);
        x1 = max(x1, v[i].x);
        y0 = min(y0, v[i].y);
        y1 = max(y1, v[i].y);
    }
}

// squared distance between a point and this bbox, 0 if inside
ntype distance(const point &p) {
    if (p.x < x0) {
        if (p.y < y0) return pdist2(point(x0, y0), p);
        else if (p.y > y1) return pdist2(point(x0, y1), p);
        else return pdist2(point(x0, p.y), p);
    } else if (p.x > x1) {
        if (p.y < y0) return pdist2(point(x1, y0), p);
        else if (p.y > y1) return pdist2(point(x1, y1), p);
        else return pdist2(point(x1, p.y), p);
    } else {
        if (p.y < y0) return pdist2(point(p.x, y0), p);
        else if (p.y > y1) return pdist2(point(p.x, y1), p);
        else return 0;
    }
}

// stores a single node of the kd-tree, either internal or leaf
struct kndnode {
    bool leaf; // true if this is a leaf node (has one point)
    point pt; // the single point of this is a leaf
    bbox bound; // bounding box for set of points in children

    kndnode *first, *second; // two children of this kd-node

    kndnode() : leaf(false), first(0), second(0) {}
    ~kndnode() {
        if (first) delete first;
        if (second) delete second;
    }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp) {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        } else {
            // split on x if the bbox is wider than high (not best heuristic...)
            if (bound.x1 - bound.x0 >= bound.y1 - bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size() / 2;
            vector<point> vl(vp.begin(), vp.begin() + half);
            vector<point> vr(vp.begin() + half, vp.end());
            first = new kndnode();
            first->construct(vl);
            second = new kndnode();
            second->construct(vr);

```

```

    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree {
    kdnnode *root;
    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }
    // recursive search method returns squared distance to nearest point
    ntype search(kdnnode *node, const point &p) {
        if (node->leaf) {
            // commented special case tells a point not to find itself
            if (p == node->pt) return sentry;
            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        } else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }
    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

// -----
// some basic test code here

int main() {
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand() % 100000, rand() % 100000));
    }
    kdtree tree(vp);

    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand() % 100000, rand() % 100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y << ")"
              << " is " << tree.nearest(q) << endl;
    }

    return 0;
}

// -----

```

2.6 Segment tree

```

#include <bits/stdc++.h>
using namespace std;
typedef vector<int> vi;
struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
    T query(int b, int e) { // query [b, e)
        T ra = unit, rb = unit;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb);
        }
        return f(ra, rb);
    }
};

```

2.7 Lazy segment tree

```

#include <bits/stdc++.h>
using namespace std;
typedef vector<int> vi;
const int inf = 1e9;
// A lazy segment tree supporting range add, range set, and range get max
struct Node {
    Node *l = 0, *r = 0;
    int lo, hi, mset = inf, madd = 0, val = -inf;
    Node(int lo, int hi) : lo(lo), hi(hi) {} // Large interval of -inf
    // Initialize based on the values in the vector v.
    // main will call this with Node(v, 0, v.size())
    Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {
        if (lo + 1 < hi) {
            int mid = lo + (hi - lo) / 2;
            l = new Node(v, lo, mid); r = new Node(v, mid, hi);
            val = max(l->val, r->val);
        }
        else val = v[lo];
    }
    // query [L, R)
    int query(int L, int R) {
        if (R <= lo || hi <= L) return -inf;
        if (L <= lo && hi <= R) return val;
        push();
        return max(l->query(L, R), r->query(L, R));
    }
    // set all elements in [L, R) to x
    void set(int L, int R, int x) {
        if (R <= lo || hi <= L) return;
        if (L <= lo && hi <= R) {
            // Update the range [lo, hi) to x
            mset = val = x, madd = 0;
        }
        else {
            push(), l->set(L, R, x), r->set(L, R, x);
            val = max(l->val, r->val);
        }
    }
};

```

```

}
// add x to all elements in [L, R)
void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
        // Add x to all elements in the range [lo, hi)
        if (mset != inf) mset += x;
        else madd += x;
        val += x;
    }
    else {
        push(), l->add(L, R, x), r->add(L, R, x);
        val = max(l->val, r->val);
    }
}
// Push the lazily stored values.
void push() {
    if (!l) {
        int mid = lo + (hi - lo) / 2;
        l = new Node(lo, mid); r = new Node(mid, hi);
    }
    if (mset != inf)
        l->set(lo, hi, mset), r->set(lo, hi, mset), mset = inf;
    else if (madd)
        l->add(lo, hi, madd), r->add(lo, hi, madd), madd = 0;
}
};

```

2.8 Lowest common ancestor

```

/**
 * Description: Calculate power of two jumps in a tree,
 * to support fast upward jumps and LCAs.
 * Assumes the root node points to itself.
 * Time: construction  $O(N \log N)$ , queries  $O(\log N)$ 
 * Status: Tested at Petrozavodsk, also stress-tested via LCA.cpp
 */
// Takes an array of parent pointers and returns
// the LCA table.
vector<vi> treeJump(vi& P) {
    int on = 1, d = 1;
    while (on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]];
    return jmp;
}
// helper function to jump a certain number of parents
// up from a node in  $O(\log(\text{steps}))$  time.
int jmp(vector<vi>& tbl, int nod, int steps) {
    rep(i, 0, sz(tbl))
        if (steps & (1 << i)) nod = tbl[i][nod];
    return nod;
}
// Find least common ancestor of two nodes.
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a;
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    }
    return tbl[0][a];
}

```

2.9 Li Chao Tree

```

// Li-Chao Tree. Store a family of functions with domain a subset of R
// where no two functions intersect
// more than once. Support querying for the max of all functions in the tree
// in  $O(\log(n))$  with  $O(\log(n))$  insertion
struct LiChao {
    typedef int ftype;
    typedef pair<int, int> params;
    int maxn;

    vector<params> best_params;

    LiChao(int maxN) {
        maxn = maxN;
        best_params = vector<params>(maxn * 4);
    }

    // The function you add to the tree. It is a family of functions
    // parameterized by a
    // Any two functions  $f(a, -)$ ,  $f(b, -)$  must intersect at most once,
    // else the tree will not work.
    ftype f(params a, ftype x) {
        return a.first * x + a.second;
    }

    // Add the function parameterized by nw to the tree
    void add_fn(params nw, int v, int l, int r) {
        int m = (l + r) / 2;
        bool lef = f(nw, l) < f(best_params[v], l);
        bool mid = f(nw, m) < f(best_params[v], m);
        if (mid) {
            swap(best_params[v], nw);
        }
        if (r - l == 1) {
            return;
        }
        else if (lef != mid) {
            add_fn(nw, 2 * v, l, m);
        }
        else {
            add_fn(nw, 2 * v + 1, m, r);
        }
    }

    void add_fn(params nw) {
        return add_fn(nw, 1, 0, maxn);
    }

    // Compute the maximum valued function over all x
    int get(int x, int v, int l, int r) {
        int m = (l + r) / 2;
        if (r - l == 1) {
            return f(best_params[v], x);
        }
        else if (x < m) {
            return min(f(best_params[v], x), get(x, 2 * v, l, m));
        }
        else {
            return min(f(best_params[v], x), get(x, 2 * v + 1, m, r));
        }
    }

    int get(int x) {
        return get(x, 1, 0, maxn);
    }
};

```

3 Combinatorial optimization

3.1 Sparse max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//  $O(|V|^2 |E|)$ 
//
// INPUT:
// - graph, constructed using AddEdge()
// - source and sink
//
// OUTPUT:
// - maximum flow value
// - To obtain actual flow values, look at edges with capacity > 0
// (zero capacity edges are residual edges).

#include<cstdio>
#include<vector>
#include<queue>
using namespace std;
typedef long long LL;

struct Edge {
    int u, v;
    LL cap, flow;
    Edge() {}
    Edge(int u, int v, LL cap) : u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
    int N;
    vector<Edge> E;
    vector<vector<int>> g;
    vector<int> d, pt;
    Dinic(int N) : N(N), E(0), g(N), d(N), pt(N) {}
    void AddEdge(int u, int v, LL cap) {
        if (u != v) {
            E.emplace_back(u, v, cap);
            g[u].emplace_back(E.size() - 1);
            E.emplace_back(v, u, 0);
            g[v].emplace_back(E.size() - 1);
        }
    }
    bool BFS(int S, int T) {
        queue<int> q({S});
        fill(d.begin(), d.end(), N + 1);
        d[S] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            if (u == T) break;
            for (int k: g[u]) {
                Edge &e = E[k];
                if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
                    d[e.v] = d[e.u] + 1;
                    q.emplace(e.v);
                }
            }
        }
        return d[T] != N + 1;
    }
    LL DFS(int u, int T, LL flow = -1) {
```

```
        if (u == T || flow == 0) return flow;
        for (int &i = pt[u]; i < g[u].size(); ++i) {
            Edge &e = E[g[u][i]];
            Edge &oe = E[g[u][i] ^ 1];
            if (d[e.v] == d[e.u] + 1) {
                LL amt = e.cap - e.flow;
                if (flow != -1 && amt > flow) amt = flow;
                if (LL pushed = DFS(e.v, T, amt)) {
                    e.flow += pushed;
                    oe.flow -= pushed;
                    return pushed;
                }
            }
        }
        return 0;
    }
    LL MaxFlow(int S, int T) {
        LL total = 0;
        while (BFS(S, T)) {
            fill(pt.begin(), pt.end(), 0);
            while (LL flow = DFS(S, T))
                total += flow;
        }
        return total;
    }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

int main() {
    int N, E;
    scanf("%d%d", &N, &E);
    Dinic dinic(N);
    for (int i = 0; i < E; i++) {
        int u, v;
        LL cap;
        scanf("%d%d%lld", &u, &v, &cap);
        dinic.AddEdge(u - 1, v - 1, cap);
        dinic.AddEdge(v - 1, u - 1, cap);
    }
    printf("%lld\n", dinic.MaxFlow(0, N - 1));
    return 0;
}

// END CUT
```

3.2 Min-cost max-flow

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
//
// Running time,  $O(|V|^2)$  cost per augmentation
// max flow:  $O(|V|^3)$  augmentations
// min cost max flow:  $O(|V|^4 * \text{MAX\_EDGE\_COST})$  augmentations
//
// INPUT:
// - graph, constructed using AddEdge()
// - source
// - sink
//
// OUTPUT:
// - (maximum flow value, minimum cost value)
// - To obtain the actual flow, look at positive values only.
```

```

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;
const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;
    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}
    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }
    void Relax(int s, int k, L cap, L cost, int dir) {
        L val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }
    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
            s = best;
        }

        for (int k = 0; k < N; k++)
            pi[k] = min(pi[k] + dist[k], INF);
        return width[t];
    }
    pair<L, L> GetMaxFlow(int s, int t) {
        L totflow = 0, totcost = 0;
        while (L amt = Dijkstra(s, t)) {
            totflow += amt;
            for (int x = t; x != s; x = dad[x].first) {
                if (dad[x].second == 1) {
                    flow[dad[x].first][x] += amt;
                    totcost += amt * cost[dad[x].first][x];
                } else {
                    flow[x][dad[x].first] -= amt;

```

```

                    totcost -= amt * cost[x][dad[x].first];
                }
            }
            return make_pair(totflow, totcost);
        }
    };

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
    int N, M;

    while (scanf("%d%d", &N, &M) == 2) {
        VVL v(M, VL(3));
        for (int i = 0; i < M; i++)
            scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
        L D, K;
        scanf("%Ld%Ld", &D, &K);

        MinCostMaxFlow mcmf(N + 1);
        for (int i = 0; i < M; i++) {
            mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
            mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
        }
        mcmf.AddEdge(0, 1, D, 0);

        pair<L, L> res = mcmf.GetMaxFlow(0, N);

        if (res.first == D) {
            printf("%Ld\n", res.second);
        } else {
            printf("Impossible.\n");
        }
    }

    return 0;
}

// END CUT

```

3.3 Min-cost matching

```

////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[][] matrix.
////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

```



```

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
    int n = int(cost.size());

    // construct dual feasible solution
    VD u(n);
    VD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
                Lmate[i] = j;
                Rmate[j] = i;
                mated++;
                break;
            }
        }
    }

    VD dist(n);
    VI dad(n);
    VI seen(n);

    // repeat until primal solution is feasible
    while (mated < n) {

        // find an unmatched left node
        int s = 0;
        while (Lmate[s] != -1) s++;

        // initialize Dijkstra
        fill(dad.begin(), dad.end(), -1);
        fill(seen.begin(), seen.end(), 0);
        for (int k = 0; k < n; k++)
            dist[k] = cost[s][k] - u[s] - v[k];

        int j = 0;
        while (true) {

            // find closest
            j = -1;
            for (int k = 0; k < n; k++) {
                if (seen[k]) continue;
                if (j == -1 || dist[k] < dist[j]) j = k;
            }
            seen[j] = 1;

            // termination condition
            if (Rmate[j] == -1) break;

            // relax neighbors
            const int i = Rmate[j];
            for (int k = 0; k < n; k++) {
                if (seen[k]) continue;

```

```

                const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
                if (dist[k] > new_dist) {
                    dist[k] = new_dist;
                    dad[k] = j;
                }
            }

            // update dual variables
            for (int k = 0; k < n; k++) {
                if (k == j || !seen[k]) continue;
                const int i = Rmate[k];
                v[k] += dist[k] - dist[j];
                u[i] -= dist[k] - dist[j];
            }
            u[s] += dist[j];

            // augment along path
            while (dad[j] >= 0) {
                const int d = dad[j];
                Rmate[j] = Rmate[d];
                Lmate[Rmate[j]] = j;
                j = d;
            }
            Rmate[j] = s;
            Lmate[s] = j;

            mated++;

        }

        double value = 0;
        for (int i = 0; i < n; i++)
            value += cost[i][Lmate[i]];

        return value;
    }
}

```

3.4 Max bipartite matching

```

// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
// INPUT: w[i][j] = edge between row node i and column node j
// OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//         mc[j] = assignment for column node j, -1 if unassigned
// function returns number of matches made

#include <vector>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {
        if (w[i][j] && !seen[j]) {
            seen[j] = true;
            if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
                mr[i] = j;
                mc[j] = i;
                return true;
            }
        }
    }
    return false;
}

```

```
int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);

    int ct = 0;
    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}
```

3.5 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
// O(|V|^3)
//
// INPUT:
// - graph, constructed using AddEdge()
//
// OUTPUT:
// - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
const int INF = 1000000000;
pair<int, VI> GetMinCut(VVI &weights) {
    int N = weights.size();
    VI used(N), cut, best_cut;
    int best_weight = -1;

    for (int phase = N - 1; phase >= 0; phase--) {
        VI w = weights[0];
        VI added = used;
        int prev, last = 0;
        for (int i = 0; i < phase; i++) {
            prev = last;
            last = -1;
            for (int j = 1; j < N; j++)
                if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
            if (i == phase - 1) {
                for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
                for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
                used[last] = true;
                cut.push_back(last);
                if (best_weight == -1 || w[last] < best_weight) {
                    best_cut = cut;
                    best_weight = w[last];
                }
            } else {
                for (int j = 0; j < N; j++)
                    w[j] += weights[last][j];
                added[last] = true;
            }
        }
    }
    return make_pair(best_weight, best_cut);
}
```

```
// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
    int N;
    cin >> N;
    for (int i = 0; i < N; i++) {
        int n, m;
        cin >> n >> m;
        VVI weights(n, VI(n));
        for (int j = 0; j < m; j++) {
            int a, b, c;
            cin >> a >> b >> c;
            weights[a - 1][b - 1] = weights[b - 1][a - 1] = c;
        }
        pair<int, VI> res = GetMinCut(weights);
        cout << "Case #" << i + 1 << ": " << res.first << endl;
    }
    // END CUT
```

4 Geometry

4.1 Python geometry

```
from collections import namedtuple
import math

class Point(namedtuple("_Point", "x y")):
    def __add__(self, other):
        return Point(self.x + other.x, self.y + other.y)
    def __sub__(self, other):
        return Point(self.x - other.x, self.y - other.y)
    def __mul__(self, scalar):
        return Point(scalar * self.x, scalar * self.y)
    def __truediv__(self, scalar):
        return Point(self.x / scalar, self.y / scalar)
    __rmul__ = __mul__
    def dist2(self):
        return self.x**2 + self.y**2
    def dist(self):
        return math.sqrt(self.dist2())
    __abs__ = dist
    def theta(self):
        return math.atan2(self.y, self.x)
    def dot(self, other):
        return self.x*other.x + self.y*other.y
    def cross(self, other):
        return self.x*other.y - self.y * other.x
    def unit(self):
        return (1/abs(self)) * self
    def rotate(self, theta):
        cos_t = math.cos(theta)
        sin_t = math.sin(theta)
        return Point(self.x*cos_t - self.y*sin_t,
                    self.x*sin_t + self.y*cos_t)
    def perp(self):
        return Point(-self.y, self.x)

class Circle(namedtuple("_Circle", "center radius")):
    pass

def circle_circle_intersect(circle1, circle2):
    a, r1 = circle1
    b, r2 = circle2
    if (a == b):
```

```

        if r1 == r2:
            raise ValueError
        return []
    vec = b - a
    d2 = vec.dist2()
    sum = r1 + r2
    dif = r1 - r2
    p = (d2 + r1 * r1 - r2 * r2) / (d2 * 2)
    h2 = r1 * r1 - p * p * d2
    if sum * sum < d2 or dif * dif > d2:
        return []
    mid = a + vec * p
    per = vec.perp() * math.sqrt(max(0, h2) / d2)
    return [mid + per, mid - per]

def circle_tangents(circle1, circle2):
    """
    Get 0, 1, or 2 outer tangents as a list of pairs of points.
    Negate r2 to get the inner tangents.
    """
    c1, r1 = circle1
    c2, r2 = circle2
    d = c2 - c1
    dr = r1 - r2
    d2 = d.dist2()
    h2 = d2 - dr * dr
    if d2 == 0 or h2 < 0:
        return []
    out = []
    for sign in (-1, 1):
        v = (d * dr + d.perp() * math.sqrt(h2) * sign) / d2
        pair = (c1 + v * r1,
                c2 + v * r2)
        out.append(pair)
    if h2 == 0:
        out.pop()
    return out

def circle_line_intersect(circle, a, b):
    """a and b are endpoints"""
    assert isinstance(a, Point) and isinstance(b, Point)
    c, r = circle
    ab = b - a
    p = a + ab * (c - a).dot(ab) / ab.dist2()
    s = Point.cross(b-a, c-a)
    h2 = r*r - s * s / ab.dist2()
    if h2 < 0:
        return ()
    if h2 == 0:
        return (p,)
    h = ab.unit() * math.sqrt(h2)
    return (p - h, p + h)

def segment_intersection(seg1, seg2):
    a, b = seg1
    c, d = seg2
    oa = Point.cross(d-c, a-c)
    ob = Point.cross(d-c, b-c)
    oc = Point.cross(b-a, c-a)
    od = Point.cross(b-a, d-a)
    if on_segment(*seg1, c):
        return c
    if on_segment(*seg1, d):
        return d
    if on_segment(*seg2, a):
        return a
    if on_segment(*seg2, b):
        return b

    return b
    if oa * ob < 0 and oc * od < 0:
        return (a * ob - b * oa) / (ob - oa)
    return None
    # set < P > s;
    # if (onSegment(c, d, a)) s.insert(a);
    # if (onSegment(c, d, b)) s.insert(b);
    # if (onSegment(a, b, c)) s.insert(c);
    # if (onSegment(a, b, d)) s.insert(d);
    # return {all(s)};

def on_segment(start, end, x):
    u = start - x
    v = end - x
    return u.cross(v) == 0 and u.dot(v) <= 0

def distance_to_segment(start, end, x):
    if start == end:
        return (start-x).dist()
    d = (end-start).dist2()
    t = min(d, max(0, (x-start).dot(end-start)))
    return abs((x-start)*d - (end-start)*t) / d

def in_polygon(poly, point):
    """poly is a list of points"""
    wind = 0
    for i, p in enumerate(poly):
        q = poly[(i + 1) % len(poly)]
        # maybe handle on_segment here
        if p.y <= point.y:
            if q.y > point.y:
                # upward crossing
                if turnLeft(p, q, point):
                    wind += 1
            else:
                if q.y <= point.y:
                    if turnLeft(q, p, point):
                        wind -= 1
    return wind

def turnLeft(p0, p1, p2):
    # Are the points counterclockwise-oriented?
    u = p1 - p0
    v = p2 - p1
    return u.cross(v) >= 0

def hull_one_side(points):
    hull = []
    for point in points:
        while len(hull) >= 2 and turnLeft(hull[-2], hull[-1], point):
            hull.pop()
        hull.append(point)
    return hull

def convex_hull(points):
    if len(points) <= 1:
        return points
    points = sorted(points)
    h1 = hull_one_side(points)
    points.reverse()
    h1.pop()
    h2 = hull_one_side(points)
    h2.pop()
    h1.extend(h2)
    return h1

```

```
def shoelace(points):
    return sum(points[i-1].cross(points[i])
               for i in range(len(points))) / 2
```

4.2 3D geometry

```
public class Geom3D {
    // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
    public static double ptPlaneDist(double x, double y, double z,
        double a, double b, double c, double d) {
        return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance between parallel planes aX + bY + cZ + d1 = 0 and
    // aX + bY + cZ + d2 = 0
    public static double planePlaneDist(double a, double b, double c,
        double d1, double d2) {
        return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
    // (or ray, or segment; in the case of the ray, the endpoint is the
    // first point)
    public static final int LINE = 0;
    public static final int SEGMENT = 1;
    public static final int RAY = 2;
    public static double ptLineDistSq(double x1, double y1, double z1,
        double x2, double y2, double z2, double px, double py, double pz,
        int type) {
        double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);

        double x, y, z;
        if (pd2 == 0) {
            x = x1;
            y = y1;
            z = z1;
        } else {
            double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) / pd2;
            x = x1 + u * (x2 - x1);
            y = y1 + u * (y2 - y1);
            z = z1 + u * (z2 - z1);
            if (type != LINE && u < 0) {
                x = x1;
                y = y1;
                z = z1;
            }
            if (type == SEGMENT && u > 1.0) {
                x = x2;
                y = y2;
                z = z2;
            }
        }

        return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
    }

    public static double ptLineDist(double x1, double y1, double z1,
        double x2, double y2, double z2, double px, double py, double pz,
        int type) {
        return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz, type));
    }
}
```

4.3 Slow Delaunay triangulation

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:    x[] = x-coordinates
//           y[] = y-coordinates
//
// OUTPUT:    triples = a vector containing m triples of indices
//               corresponding to triangle vertices

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T> &x, vector<T> &y) {
    int n = x.size();
    vector<T> z(n);
    vector<triple> ret;

    for (int i = 0; i < n; i++)
        z[i] = x[i] * x[i] + y[i] * y[i];

    for (int i = 0; i < n - 2; i++) {
        for (int j = i + 1; j < n; j++) {
            for (int k = i + 1; k < n; k++) {
                if (j == k) continue;
                double xn = (y[j] - y[i]) * (z[k] - z[i]) - (y[k] - y[i]) * (z[j]
                    ] - z[i]);
                double yn = (x[k] - x[i]) * (z[j] - z[i]) - (x[j] - x[i]) * (z[k]
                    ] - z[i]);
                double zn = (x[j] - x[i]) * (y[k] - y[i]) - (x[k] - x[i]) * (y[j]
                    ] - y[i]);
                bool flag = zn < 0;
                for (int m = 0; flag && m < n; m++)
                    flag = flag && ((x[m] - x[i]) * xn +
                        (y[m] - y[i]) * yn +
                        (z[m] - z[i]) * zn <= 0);
                if (flag) ret.push_back(triple(i, j, k));
            }
        }
    }
    return ret;
}

int main() {
    T xs[] = {0, 0, 1, 0.9};
    T ys[] = {0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //           0 3 2

    int i;
    for (i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

5 Numerical algorithms

5.1 Number theory (modular, Chinese remainder, linear Diophantine)

```
// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;
// return a % b (positive value)
int mod(int a, int b) {
    return ((a % b) + b) % b;
}
// computes gcd(a,b)
int gcd(int a, int b) {
    while (b) {
        int t = a % b;
        a = b;
        b = t;
    }
    return a;
}
// computes lcm(a,b)
int lcm(int a, int b) {
    return a / gcd(a, b) * b;
}
// (a^b) mod m via successive squaring
int powermod(int a, int b, int m) {
    int ret = 1;
    while (b) {
        if (b & 1) ret = mod(ret * a, m);
        a = mod(a * a, m);
        b >>= 1;
    }
    return ret;
}
// Finds two integers $x$ and $y$, such that $ax+by=\gcd(a,b)$. If
// If $a$ and $b$ are coprime, then $x$ is the inverse of $a \pmod{b}$.
// Returns gcd(a, b)
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    VI ret;
    int g = extended_euclid(a, n, x, y);
    if (!(b % g)) {
        x = mod(x * (b / g), n);
        for (int i = 0; i < g; i++)
            ret.push_back(mod(x + i * (n / g), n));
    }
    return ret;
}
```

```
// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int g = extended_euclid(a, n, x, y);
    if (g > 1) return -1;
    return mod(x, n);
}
// compute mod inverse of all numbers up to n
vector<ll> precompute_inv_mod(int n, ll mod) {
    vector<ll> inv(n + 1);
    inv[1] = 1;
    for (int i = 2; i <= n; ++i) {
        inv[i] = mod - (mod / i) * inv[mod % i] % mod;
    }
    return inv;
}
// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
    int s, t;
    int g = extended_euclid(m1, m2, s, t);
    if (r1 % g != r2 % g) return make_pair(0, -1);
    return make_pair(mod(s * r2 * m1 + t * r1 * m2, m1 * m2) / g, m1 * m2 / g);
}
// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i. Note that the solution is
// unique modulo M = lcm_i (m[i]). Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
    PII ret = make_pair(r[0], m[0]);
    for (int i = 1; i < m.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
        if (ret.second == -1) break;
    }
    return ret;
}
// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
    if (!a && !b) {
        if (c) return false;
        x = 0; y = 0;
        return true;
    }
    if (!a) {
        if (c % b) return false;
        x = 0; y = c / b;
        return true;
    }
    if (!b) {
        if (c % a) return false;
        x = c / a; y = 0;
        return true;
    }
    int g = gcd(a, b);
    if (c % g) return false;
    x = c / g * mod_inverse(a / g, b / g);
    y = (c - a * x) / b;
    return true;
}
int main() {
    int x, y;
    int g = extended_euclid(14, 30, x, y);
    cout << g << " " << x << " " << y << endl; //2 -2 1
    VI sols = modular_linear_equation_solver(14, 30, 100);
    for (int i = 0; i < sols.size(); i++) cout << sols[i] << " "; // 95 451
    cout << endl;
}
```

```

cout << mod_inverse(8, 9) << endl; // 8
PII ret = chinese_remainder_theorem(VI({3, 5, 7}), VI({2, 3, 2}));
cout << ret.first << " " << ret.second << endl; // 23 105
ret = chinese_remainder_theorem(VI({4, 6}), VI({3, 5}));
cout << ret.first << " " << ret.second << endl; // 11 12
if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
cout << x << " " << y << endl; // 5 -15
return 0;
}

```

5.2 Modular Arithmetic

```

template <int MOD=998'244'353>
struct Modular {
    int value;
    static const int MOD_value = MOD;

    Modular(long long v = 0) { value = v % MOD; if (value < 0) value += MOD; }
    Modular(long long a, long long b) : value(0) { *this += a; *this /= b; }

    Modular& operator+=(Modular const& b) {value += b.value; if (value >= MOD)
        value -= MOD; return *this;}
    Modular& operator-=(Modular const& b) {value -= b.value; if (value < 0)
        value += MOD; return *this;}
    Modular& operator*=(Modular const& b) {value = (long long)value * b.value %
        MOD; return *this;}

    friend Modular mexp(Modular a, long long e) {
        Modular res = 1; while (e) { if (e&1) res *= a; a *= a; e >>= 1; }
        return res;
    }
    friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

    Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
    friend Modular operator+(Modular a, Modular const b) { return a += b; }
    friend Modular operator-(Modular a, Modular const b) { return a -= b; }
    friend Modular operator-(Modular const a) { return 0 - a; }
    friend Modular operator*(Modular a, Modular const b) { return a *= b; }
    friend Modular operator/(Modular a, Modular const b) { return a /= b; }
    friend std::ostream& operator<<(std::ostream& os, Modular const& a) {return
        os << a.value;}
    friend bool operator==(Modular const& a, Modular const& b) {return a.value
        == b.value;}
    friend bool operator!=(Modular const& a, Modular const& b) {return a.value
        != b.value;}
};

// Chained Multiplication or Successive Simple Multiplication
Modular<998244353> a=1, m=123456789;
a *= m * m * m; // a = 519994069
// Inverse
a=inverse(m) // a=25170271
// fractions
Modular<> frac=(1,2); // frac=1*2^(-1) % 998244353 = 499122177
// Modular exponentiation
Modular<> power(2);
power=mexp(power,500); // power = 616118644

```

5.3 Systems of linear equations, matrix inverse, determinant

```

// Gauss-Jordan elimination with full pivoting.
//
// Uses:

```

```

// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxn matrix
//          b[][] = an nxm matrix
//
// OUTPUT:  X      = an nxm matrix (stored in b[][])
//          A^{-1} = an nxn matrix (stored in a[][])
//          returns determinant of a[][]

```

```

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++)
            if (!ipiv[j])
                for (int k = 0; k < n; k++)
                    if (!ipiv[k])
                        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) {
                            pj = j;
                            pk = k;
                        }
        if (fabs(a[pj][pk]) < EPS) {
            cerr << "Matrix is singular." << endl;
            exit(0);
        }
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;

        T c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++)
            if (p != pk) {
                c = a[p][pk];
                a[p][pk] = 0;
                for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
                for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
            }
    }

    for (int p = n - 1; p >= 0; p--)
        if (irow[p] != icol[p]) {
            for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
        }
}

```

```

    return det;
}
int main() {
    const int n = 4;
    const int m = 2;
    double A[n][n] = {{1, 2, 3, 4},
                      {1, 0, 1, 0},
                      {5, 3, 2, 4},
                      {6, 1, 4, 6}};

    double B[n][m] = {{1, 2},
                      {4, 3},
                      {5, 6},
                      {8, 7}};

    VVT a(n), b(m);
    for (int i = 0; i < n; i++) {
        a[i] = VT(A[i], A[i] + n);
        b[i] = VT(B[i], B[i] + m);
    }

    double det = GaussJordan(a, b);

    // expected: 60
    cout << "Determinant: " << det << endl;

    // expected: -0.233333 0.166667 0.133333 0.0666667
    //              0.166667 0.166667 0.333333 -0.333333
    //              0.233333 0.833333 -0.133333 -0.0666667
    //              0.05 -0.75 -0.1 0.2
    cout << "Inverse: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++)
            cout << a[i][j] << ' ';
        cout << endl;
    }

    // expected: 1.63333 1.3
    //              -0.166667 0.5
    //              2.36667 1.7
    //              -1.85 -1.35
    cout << "Solution: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++)
            cout << b[i][j] << ' ';
        cout << endl;
    }
}

```

5.4 Reduced row echelon form, matrix rank

```

// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

```

```

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
    int n = a.size();
    int m = a[0].size();
    int r = 0;
    for (int c = 0; c < m && r < n; c++) {
        int j = r;
        for (int i = r + 1; i < n; i++)
            if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
        if (fabs(a[j][c]) < EPSILON) continue;
        swap(a[j], a[r]);

        T s = 1.0 / a[r][c];
        for (int j = 0; j < m; j++) a[r][j] *= s;
        for (int i = 0; i < n; i++)
            if (i != r) {
                T t = a[i][c];
                for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
            }
        r++;
    }
    return r;
}

int main() {
    const int n = 5, m = 4;
    double A[n][m] = {
        {16, 2, 3, 13},
        {5, 11, 10, 8},
        {9, 7, 6, 12},
        {4, 14, 15, 1},
        {13, 21, 21, 13}};

    VVT a(n);
    for (int i = 0; i < n; i++)
        a[i] = VT(A[i], A[i] + m);

    int rank = rref(a);

    cout << "Rank: " << rank << endl; // 3

    // expected: 1 0 0 1
    //              0 1 0 3
    //              0 0 1 -3
    //              0 0 0 3.10862e-15
    //              0 0 0 2.22045e-15
    cout << "rref: " << endl;
    for (int i = 0; i < 5; i++) {
        for (int j = 0; j < 4; j++)
            cout << a[i][j] << ' ';
        cout << endl;
    }
}

```

5.5 Fast Fourier transform

```

#include <cstdio>
#include <cmath>

struct cpx {
    cpx() {}
    cpx(double aa) : a(aa), b(0) {}
    cpx(double aa, double bb) : a(aa), b(bb) {}

```

```

double a, b;
double modsq(void) const {
    return a * a + b * b;
}
cpx bar(void) const {
    return cpx(a, -b);
}
};

cpx operator+(cpx a, cpx b) {
    return cpx(a.a + b.a, a.b + b.b);
}
cpx operator*(cpx a, cpx b) {
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}
cpx operator/(cpx a, cpx b) {
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}
cpx EXP(double theta) {
    return cpx(cos(theta), sin(theta));
}
}

const double two_pi = 4 * acos(0);
// in:    input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT, 1 is first)
// RESULT: out[k] = \sum_{j=0}^{size-1} in[j] * exp(dir * 2pi * i * j * k /
//          size)
void FFT(cpx *in, cpx *out, int step, int size, int dir) {
    if (size < 1) return;
    if (size == 1) {
        out[0] = in[0];
        return;
    }
    FFT(in, out, step * 2, size / 2, dir);
    FFT(in + step, out + size / 2, step * 2, size / 2, dir);
    for (int i = 0; i < size / 2; i++) {
        cpx even = out[i];
        cpx odd = out[i + size / 2];
        out[i] = even + EXP(dir * two_pi * i / size) * odd;
        out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) *
            odd;
    }
}

// Usage:
// f[0...N-1] and g[0...N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
// 1. Compute F and G (pass dir = 1 as the argument).
// 2. Get H by element-wise multiplying F and G.
// 3. Get h by taking the inverse FFT (use dir = -1 as the argument)
//    and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void) {
    printf("If rows come in identical pairs, then everything works.\n");

    cpx a[8] = {0, 1, cpx(1, 3), cpx(0, 5), 1, 0, 2, 0};
    cpx b[8] = {1, cpx(0, -2), cpx(0, 1), 3, -1, -3, 1, -2};
    cpx A[8];
    cpx B[8];
    FFT(a, A, 1, 8, 1);
    FFT(b, B, 1, 8, 1);

```

```

    for (int i = 0; i < 8; i++) {
        printf("%7.2lf%7.2lf", A[i].a, A[i].b);
    }
    printf("\n");
    for (int i = 0; i < 8; i++) {
        cpx Ai(0, 0);
        for (int j = 0; j < 8; j++) {
            Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
        }
        printf("%7.2lf%7.2lf", Ai.a, Ai.b);
    }
    printf("\n");

    cpx AB[8];
    for (int i = 0; i < 8; i++)
        AB[i] = A[i] * B[i];
    cpx aconvb[8];
    FFT(AB, aconvb, 1, 8, -1);
    for (int i = 0; i < 8; i++)
        aconvb[i] = aconvb[i] / 8;
    for (int i = 0; i < 8; i++) {
        printf("%7.2lf%7.2lf", aconvb[i].a, aconvb[i].b);
    }
    printf("\n");
    for (int i = 0; i < 8; i++) {
        cpx aconvbi(0, 0);
        for (int j = 0; j < 8; j++) {
            aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
        }
        printf("%7.2lf%7.2lf", aconvbi.a, aconvbi.b);
    }
    printf("\n");

    return 0;
}

```

5.6 Euler's Totient Function

```

/**
 * Author: Hakan Terelius
 * Date: 2009-09-25
 * License: CC0
 * Description: Precompute the number of positive integers coprime to N up to a
 *              given limit.
 * - The sum phi(d) for all divisors d of n is equal to n.
 * - The sum of all positive numbers less than n that are coprime to n is n phi(
 *   n) / 2 (n > 1)
 * - For any a, n coprime, a^(phi(n)) = 1 mod n
 * - Specifically, for any prime p, any number a, a^{p-1} = 1 mod p
 * Status: Tested
 */
#pragma once

const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    rep(i, 0, LIM) phi[i] = i & 1 ? i : i / 2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}

```

5.7 Partitions


```
#include <bits/stdc++.h>
using namespace std;
typedef vector<int> vi;
// Ways to write n as a sum of positive numbers.
// partition(4)=5 because 4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1
int partition(int n) {
    if(n==0) return 1;
    assert(n > 0);
    vi dp = vi(n + 1);
    dp[0] = 1;
    for (int i = 1; i <= n; i++) {
        for (int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; j++, r *= -1) {
            dp[i] += dp[i - (3 * j * j - j) / 2] * r;
            if (i - (3 * j * j + j) / 2 >= 0) {
                dp[i] += dp[i - (3 * j * j + j) / 2] * r;
            }
        }
    }
    return dp[n];
}
int main() {
    // 0 1, 1 1, 2 2, 3 3, 4 5, 5 7, 6 11, 7 15, 8 22, 9 30, 10 42
    // 11 56, 12 77, 13 101, 14 135, 15 176, 16 231, 17 297
    for (int i = 0; i <= 17; ++i) {
        cout << i << " " << partition(i) << ", ";
    }
    return 0;
}
```

6 Graph algorithms

6.1 Bellman-Ford shortest paths with negative edge weights (C++)

```
// This function runs the Bellman-Ford algorithm for single source
// shortest paths with negative edge weights. The function returns
// false if a negative weight cycle is detected. Otherwise, the
// function returns true and dist[i] is the length of the shortest
// path from start to i.
//
// Running time:  $O(|V|^3)$ 
//
// INPUT: start, w[i][j] = cost of edge from i to j
// OUTPUT: dist[i] = min weight path from start to i
//         prev[i] = previous node on the best path from the
//         start node

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool BellmanFord(const VVT &w, VT &dist, VI &prev, int start) {
    int n = w.size();
    prev = VI(n, -1);
    dist = VT(n, 1000000000);
    dist[start] = 0;

    for (int k = 0; k < n; k++) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                if (dist[j] > dist[i] + w[i][j]) {
                    if (k == n - 1) return false;
                    dist[j] = dist[i] + w[i][j];
                    prev[j] = i;
                }
            }
        }
    }

    return true;
}
```

6.2 Topological sort (C++)

```
// This function uses performs a non-recursive topological sort.
//
// Running time:  $O(|V|^2)$ . If you use adjacency lists (vector<map<int> >),
// the running time is reduced to  $O(|E|)$ .
//
// INPUT: w[i][j] = 1 if i should come before j, 0 otherwise
// OUTPUT: a permutation of 0,...,n-1 (stored in a vector)
//         which represents an ordering of the nodes which
//         is consistent with w
//
// If no ordering is possible, false is returned.

#include <iostream>
#include <queue>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool TopologicalSort(const VVI &w, VI &order) {
    int n = w.size();
    VI parents(n);
    queue<int> q;
    order.clear();

    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++)
            if (w[j][i]) parents[i]++;
        if (parents[i] == 0) q.push(i);
    }

    while (q.size() > 0) {
        int i = q.front();
        q.pop();
        order.push_back(i);
        for (int j = 0; j < n; j++)
            if (w[i][j]) {
                parents[j]--;
                if (parents[j] == 0) q.push(j);
            }
    }

    return true;
}
```

```

    return (order.size() == n);
}

```

6.3 Fast Dijkstra's algorithm

```

// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time:  $O(|E| \log |V|)$ 

#include <queue>
#include <cstdio>

using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;
int main() {
    int N, s, t;
    scanf("%d%d%d", &N, &s, &t);
    vector<vector<PII> > edges(N);
    for (int i = 0; i < N; i++) {
        int M;
        scanf("%d", &M);
        for (int j = 0; j < M; j++) {
            int vertex, dist;
            scanf("%d%d", &vertex, &dist);
            edges[i].push_back(make_pair(dist, vertex)); // note order of
                arguments here
        }
    }

    // use priority queue in which top element has the "smallest" priority
    priority_queue<PII, vector<PII>, greater<PII> > Q;
    vector<int> dist(N, INF), dad(N, -1);
    Q.push(make_pair(0, s));
    dist[s] = 0;
    while (!Q.empty()) {
        PII p = Q.top();
        Q.pop();
        int here = p.second;
        if (here == t) break;
        if (dist[here] != p.first) continue;

        for (vector<PII>::iterator it = edges[here].begin(); it != edges[here].
            end(); it++) {
            if (dist[here] + it->first < dist[it->second]) {
                dist[it->second] = dist[here] + it->first;
                dad[it->second] = here;
                Q.push(make_pair(dist[it->second], it->second));
            }
        }
    }

    printf("%d\n", dist[t]);
    if (dist[t] < INF)
        for (int i = t; i != -1; i = dad[i])
            printf("%d%c", i, (i == s ? '\n' : ' '));
    return 0;
}

/*
Sample input:
5 0 4
2 1 2 3 1
2 2 4 4 5
3 1 4 3 3 4 1
2 0 1 2 3

```

```
2 1 5 2 1
```

```

Expected:
5
4 2 3 0
*/

```

6.4 Strongly connected components

```

vi val, comp, z, cont;
int Time, ncomps;
// A function that will be called with the indicies of all elements
// in each component as the parameter once per component after running scc.
void f(vi node_inds) {};
int dfs(int j, vector<vi>& g) {
    int low = val[j] = ++Time, x; z.push_back(j);
    for (auto e : g[j]) if (comp[e] < 0)
        low = min(low, val[e] ? : dfs(e, g));

    if (low == val[j]) {
        do {
            x = z.back(); z.pop_back();
            comp[x] = ncomps;
            cont.push_back(x);
        } while (x != j);
        f(cont); cont.clear();
        ncomps++;
    }
    return val[j] = low;
}

void scc(vector<vi>& g) {
    int n = g.size();
    val.assign(n, 0); comp.assign(n, -1);
    Time = ncomps = 0;
    rep(i, 0, n) if (comp[i] < 0) dfs(i, g);
}

```

6.5 Eulerian path

```

struct Edge;
typedef list<Edge>::iterator iter;

struct Edge {
    int next_vertex;
    iter reverse_edge;
    Edge(int next_vertex)
        : next_vertex(next_vertex) {}
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices]; // adjacency list

vector<int> path;
void find_path(int v) {
    while (adj[v].size() > 0) {
        int vn = adj[v].front().next_vertex;
        adj[vn].erase(adj[v].front().reverse_edge);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b) {

```

```

adj[a].push_front(Edge(b));
iter ita = adj[a].begin();
adj[b].push_front(Edge(a));
iter itb = adj[b].begin();
ita->reverse_edge = itb;
itb->reverse_edge = ita;
}

```

6.6 Minimum spanning trees

```

// This function runs Prim's algorithm for constructing minimum
// weight spanning trees.
//
// Running time:  $O(|V|^2)$ 
//
// INPUT:  w[i][j] = cost of edge from i to j
//
// NOTE: Make sure that w[i][j] is nonnegative and
// symmetric. Missing edges should be given -1
// weight.
//
// OUTPUT: edges = list of pair<int,int> in minimum spanning tree
// return total weight of tree

```

```

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

```

```
using namespace std;
```

```

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

```

```

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

```

```

T Prim(const VVT &w, VPII &edges) {
    int n = w.size();
    VI found(n);
    VI prev(n, -1);
    VT dist(n, 1000000000);
    int here = 0;
    dist[here] = 0;

    while (here != -1) {
        found[here] = true;
        int best = -1;
        for (int k = 0; k < n; k++)
            if (!found[k]) {
                if (w[here][k] != -1 && dist[k] > w[here][k]) {
                    dist[k] = w[here][k];
                    prev[k] = here;
                }
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
        here = best;
    }
}

```

```

T tot_weight = 0;
for (int i = 0; i < n; i++)
    if (prev[i] != -1) {
        edges.push_back(make_pair(prev[i], i));
        tot_weight += w[prev[i]][i];
    }

```

```

    }
    return tot_weight;
}

int main() {
    int ww[5][5] = {
        {0, 400, 400, 300, 600},
        {400, 0, 3, -1, 7},
        {400, 3, 0, 2, 0},
        {300, -1, 2, 0, 5},
        {600, 7, 0, 5, 0}
    };
    VVT w(5, VT(5));
    for (int i = 0; i < 5; i++)
        for (int j = 0; j < 5; j++)
            w[i][j] = ww[i][j];

    VPII edges;
    cout << Prim(w, edges) << endl; // 305
    for (int i = 0; i < edges.size(); i++)
        cout << edges[i].first << " " << edges[i].second << endl;
    //      2 1
    //      3 2
    //      0 3
    //      2 4
}

```

7 Strings

7.1 AhoCorasick

```

#define foreach(x, v) for (typeof (v).begin() x=(v).begin(); x!=(v).end(); ++x)
#define For(i, a, b) for (int i=(a); i<(b); ++i)
#define D(x) cout << #x " is " << x << endl
const int MAXS = 6 * 50 + 10; // Max number of states in the matching machine.
// Should be equal to the sum of the length of all keywords.
const int MAXC = 26; // Number of characters in the alphabet.
int out[MAXS]; // Output for each state, as a bitwise mask.
int f[MAXS]; // Failure function
int g[MAXS][MAXC]; // Goto function, or -1 if fail.
int buildMatchingMachine(const vector<string> &words, char lowestChar = 'a',
                        char highestChar = 'z') {
    memset(out, 0, sizeof out);
    memset(f, -1, sizeof f);
    memset(g, -1, sizeof g);
    int states = 1; // Initially, we just have the 0 state
    for (int i = 0; i < words.size(); ++i) {
        const string &keyword = words[i];
        int currentState = 0;
        for (int j = 0; j < keyword.size(); ++j) {
            int c = keyword[j] - lowestChar;
            if (g[currentState][c] == -1) { // Allocate a new node
                g[currentState][c] = states++;
            }
            currentState = g[currentState][c];
        }
        out[currentState] |= (1 << i); // There's a match of keywords[i] at node
                                   // currentState.
    }
    // State 0 should have an outgoing edge for all characters.
    for (int c = 0; c < MAXC; ++c) {
        if (g[0][c] == -1) {
            g[0][c] = 0;
        }
    }
}

```

```

}

// Now, let's build the failure function
queue<int> q;
for (int c = 0; c <= highestChar - lowestChar; ++c) { // Iterate over every
    possible input
    // All nodes s of depth 1 have f[s] = 0
    if (g[0][c] != -1 and g[0][c] != 0) {
        f[g[0][c]] = 0;
        q.push(g[0][c]);
    }
}
while (q.size()) {
    int state = q.front();
    q.pop();
    for (int c = 0; c <= highestChar - lowestChar; ++c) {
        if (g[state][c] != -1) {
            int failure = f[state];
            while (g[failure][c] == -1) {
                failure = f[failure];
            }
            failure = g[failure][c];
            f[g[state][c]] = failure;
            out[g[state][c]] |= out[failure]; // Merge out values
            q.push(g[state][c]);
        }
    }
}
return states;
}

int findNextState(int currentState, char nextInput, char lowestChar = 'a') {
    int answer = currentState;
    int c = nextInput - lowestChar;
    while (g[answer][c] == -1)
        answer = f[answer];
    return g[answer][c];
}

int main() {
    vector<string> keywords;
    keywords.push_back("he"); keywords.push_back("she");
    keywords.push_back("hers"); keywords.push_back("his");
    string text = "ahishers";
    buildMatchingMachine(keywords, 'a', 'z');
    int currentState = 0;
    for (int i = 0; i < text.size(); ++i) {
        currentState = findNextState(currentState, text[i], 'a');
        if (out[currentState] == 0)
            continue; // Nothing new, let's move on to the next character.
        for (int j = 0; j < keywords.size(); ++j) {
            if (out[currentState] & (1 << j)) { // Matched keywords[j]
                cout << "Keyword " << keywords[j] << " appears from "
                    << i - keywords[j].size() + 1 << " to " << i << endl;
            }
        }
    }
    return 0;
}

```

7.2 Longest increasing subsequence

```

// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers

```

```

// OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
    VPII best;
    VI dad(v.size(), -1);

    for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
        PII item = make_pair(v[i], 0);
        VPII::iterator it = lower_bound(best.begin(), best.end(), item);
        item.second = i;
#else
        PII item = make_pair(v[i], i);
        VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
        if (it == best.end()) {
            dad[i] = (best.size() == 0 ? -1 : best.back().second);
            best.push_back(item);
        } else {
            dad[i] = it == best.begin() ? -1 : prev(it)->second;
            *it = item;
        }
    }

    VI ret;
    for (int i = best.back().second; i >= 0; i = dad[i])
        ret.push_back(v[i]);
    reverse(ret.begin(), ret.end());
    return ret;
}

```

7.3 Longest common subsequence

```

/*
Calculates the length of the longest common subsequence of two vectors.
Backtracks to find a single subsequence or all subsequences. Runs in
O(m*n) time except for finding all longest common subsequences, which
may be slow depending on how many there are.
*/

#include <iostream>
#include <vector>
#include <set>
#include <algorithm>

using namespace std;

typedef int T;
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<int> VI;
typedef vector<VI> VVI;
void backtrack(VVI &dp, VT &res, VT &A, VT &B, int i, int j) {
    if (!i || !j) return;
    if (A[i - 1] == B[j - 1]) {

```

```

        res.push_back(A[i - 1]);
        backtrack(dp, res, A, B, i - 1, j - 1);
    }
    else {
        if (dp[i][j - 1] >= dp[i - 1][j]) backtrack(dp, res, A, B, i, j - 1);
        else backtrack(dp, res, A, B, i - 1, j);
    }
}

void backtrackall(VVI &dp, set<VT> &res, VT &A, VT &B, int i, int j) {
    if (!i || !j) {
        res.insert(VI());
        return;
    }
    if (A[i - 1] == B[j - 1]) {
        set<VT> tempres;
        backtrackall(dp, tempres, A, B, i - 1, j - 1);
        for (set<VT>::iterator it = tempres.begin(); it != tempres.end(); it++)
            {
                VT temp = *it;
                temp.push_back(A[i - 1]);
                res.insert(temp);
            }
    }
    else {
        if (dp[i][j - 1] >= dp[i - 1][j]) backtrackall(dp, res, A, B, i, j - 1);
        if (dp[i][j - 1] <= dp[i - 1][j]) backtrackall(dp, res, A, B, i - 1, j);
    }
}

VT LCS(VT &A, VT &B) {
    VVI dp;
    int n = A.size(), m = B.size();
    dp.resize(n + 1);
    for (int i = 0; i <= n; i++) dp[i].resize(m + 1, 0);

    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= m; j++) {
            if (A[i - 1] == B[j - 1]) dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
        }

    VT res;
    backtrack(dp, res, A, B, n, m);
    reverse(res.begin(), res.end());
    return res;
}

set<VT> LCSall(VT &A, VT &B) {
    VVI dp;
    int n = A.size(), m = B.size();
    dp.resize(n + 1);
    for (int i = 0; i <= n; i++) dp[i].resize(m + 1, 0);
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= m; j++) {
            if (A[i - 1] == B[j - 1]) dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
        }

    set<VT> res;
    backtrackall(dp, res, A, B, n, m);
    return res;
}

int main() {
    int a[] = {0, 5, 5, 2, 1, 4, 2, 3}, b[] = {5, 2, 4, 3, 2, 1, 2, 1, 3};
    VI A = VI(a, a + 8), B = VI(b, b + 9);
    VI C = LCS(A, B);

    for (int i = 0; i < C.size(); i++) cout << C[i] << " ";
    cout << endl << endl;

    set<VI> D = LCSall(A, B);
    for (set<VI>::iterator it = D.begin(); it != D.end(); it++) {
        for (int i = 0; i < (*it).size(); i++) cout << (*it)[i] << " ";
    }
}

```

```

        cout << endl;
    }
}

```

7.4 Knuth-Morris-Pratt

```

/**
 * Finds all occurrences of the pattern string p within the
 * text string t. Running time is O(n + m), where n and m
 * are the lengths of p and t, respectively.
 */

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;
void buildPi(string &p, VI &pi) {
    pi = VI(p.length());
    int k = -2;
    for (int i = 0; i < p.length(); i++) {
        while (k >= -1 && p[k + 1] != p[i])
            k = (k == -1) ? -2 : pi[k];
        pi[i] = ++k;
    }
}

int KMP(string &t, string &p) {
    VI pi;
    buildPi(p, pi);
    int k = -1;
    for (int i = 0; i < t.length(); i++) {
        while (k >= -1 && p[k + 1] != t[i])
            k = (k == -1) ? -2 : pi[k];
        k++;
        if (k == p.length() - 1) {
            // p matches t[i-m+1, ..., i]
            cout << "matched at index " << i - k << ": ";
            cout << t.substr(i - k, p.length()) << endl;
            k = (k == -1) ? -2 : pi[k];
        }
    }
    return 0;
}

int main() {
    string a = "AABAACAADAABAABA", b = "AABA";
    KMP(a, b); // expected matches at: 0, 9, 12
    return 0;
}

```

7.5 Longest Common Prefix

```

/**
 * Author: chilli
 * License: CC0
 * Description: z[x] computes the length of the longest common prefix of s[i:]
 *             and s, except z[0] = 0. (abacaba -> 0010301)
 * Time: O(n)
 * Status: stress-tested
 */
#pragma once

vi Z(string S) {

```

```

vi z(sz(S));
int l = -1, r = -1;
rep(i, 1, sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - 1]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
        z[i]++;
    if (i + z[i] > r)
        l = i, r = i + z[i];
}
return z;
}

```

7.6 Palindromes

```

/**
 * Author: User adamant on CodeForces
 * Source: http://codeforces.com/blog/entry/12143
 * Description: For each position in a string, computes p[0][i] = half length of
 * longest even palindrome around pos i, p[1][i] = longest odd (half rounded
 * down).
 * Time: O(N)
 * Status: Stress-tested
 */
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi, 2> p = {vi(n+1), vi(n)};
    rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {
        int t = r-i+!z;
        if (i < r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L >= 1 && R+1 < n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R > r) l=L, r=R;
    }
    return p;
}

```

8 Miscellaneous

8.1 Prime numbers

```

#include <bits/stdc++.h>
using namespace std;
#define EPS 1e-7
typedef long long LL;

bool IsPrime(LL x) {
    if (x <= 1) return false;
    if (x <= 3) return true;
    if (!(x % 2) || !(x % 3)) return false;
    LL s = (LL) (sqrt((double) (x)) + EPS);
    for (LL i = 5; i <= s; i += 6) {
        if (!(x % i) || !(x % (i + 2))) return false;
    }
    return true;
}

// Factor every number up until n in O(n) time.
// minFact[i] = the minimum factor of i higher than 1. minFact[0] = minFact[1] =
// 0
// primes[i] = the ith prime.
vector<int> factorAll(int n) {
    vector<int> primes(0);

```

```

vector<int> minFact(n + 1);
for (int i = 2; i <= n; i++) {
    if (minFact[i] == 0) {
        primes.push_back(i);
        minFact[i] = i;
    }
    for (int j = 0; j < primes.size() && primes[j] <= minFact[i] && i *
        primes[j] <= n; ++j) {
        minFact[i * primes[j]] = primes[j];
    }
}
return primes;
}

// Primes close to 1e9: 999'999'937, 1'000'000'007, 1'000'000'009

```

8.2 Binary Search

```

// This code is guaranteed to work in the min number of ops
// for any MAX that fits in an ll.
ll MAX = 1LL << 62;
// Binary search integers in the range [0, MAX] (or higher)
// for the last element satisfying condition.
ll lo = 0
for (ll j = 1LL << (ll) (log2(MAX)); j != 0; j >= 1) {
    if (condition(lo + j)) {
        lo += j;
    }
}
// Binary search integers in the range (1, MAX] (or higher)
// for the first element satisfying condition.
ll hi = 1LL << (ll) (log2(MAX) + 1);
for (ll j = 1LL << (ll) (log2(MAX)); j != 0; j >= 1) {
    if (condition(hi - j)) {
        hi -= j;
    }
}
// Search list[lo:hi], targets occur at list[bisect_left:bisect_right]
while (lo < hi) { // bisect_right
    mid = lo + (hi - lo) / 2;
    if (target < list[mid]) { hi = mid; }
    else { lo = mid + 1; }
} return lo;
while (lo < hi) { // bisect_left
    mid = lo + (hi - lo) / 2;
    if (list[mid] < target) { lo = mid + 1; }
    else { hi = mid; }
} return lo;

```

8.3 Latitude/longitude

```

/*
 * Converts from rectangular coordinates to latitude/longitude and vice
 * versa. Uses degrees (not radians).
 */

#include <iostream>
#include <cmath>

using namespace std;

struct ll {
    double r, lat, lon;
};

struct rect {

```

```

    double x, y, z;
};
ll convert(rect &P) {
    ll Q;
    Q.r = sqrt(P.x * P.x + P.y * P.y + P.z * P.z);
    Q.lat = 180 / M_PI * asin(P.z / Q.r);
    Q.lon = 180 / M_PI * acos(P.x / sqrt(P.x * P.x + P.y * P.y));

    return Q;
}
rect convert(ll &Q) {
    rect P;
    P.x = Q.r * cos(Q.lon * M_PI / 180) * cos(Q.lat * M_PI / 180);
    P.y = Q.r * sin(Q.lon * M_PI / 180) * cos(Q.lat * M_PI / 180);
    P.z = Q.r * sin(Q.lat * M_PI / 180);

    return P;
}
int main() {
    rect A;
    ll B;

    A.x = -1.0;
    A.y = 2.0;
    A.z = -3.0;

    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;

    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}

```

8.4 Hilbert curve for Mo's Algorithm

```

struct Query {
    int l, r, idx;
    int64_t ord;
    inline void calcOrder() {
        ord = hilbertOrder(l, r, 21, 0);
    }
};

inline bool operator<(const Query &a, const Query &b) {
    return a.ord < b.ord;
}

// constant time optimization to Mo's algorithm (~3x faster lol)
// https://codeforces.com/blog/entry/61203
inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
    if (pow == 0) {
        return 0;
    }
    int hpow = 1 << (pow - 1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2 * pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow - 1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
    return ans;
}

```