During the simulations, we computed the degree of curing and temperature distributions, which were then used to compute the stress-strain state in a composite [1]. To accelerate the computations, we considered a two-dimensional (2D) setting both for the problems of thermal conductivity and the degree of polymerization distribution [2], as well as for the problem of stress-strain state calculations [3]. All of the calculations presented here are conducted for a transversely isotropic composite material. The anisotropic axis of the material coincides with the pulling direction and with the orientation of unidirectional rovings. The axis of anisotropy coincides with the OZ axis of the Cartesian coordinate system, with the OX- and OY-axes lying in the cross-sectional plane of the profile.

*2D Thermal Model*

Here, we consider a stationary pulling process with a pulling speed . The heat conductivity equation for the profile during pultrusion, in the Lagrangian (material) frame of reference, can be expressed as follows, disregarding the longitudinal conduction [2]:

(1)

where and  are the cross-sectional coordinates;  represents the temperature; is the specific heat of composite material depending on the degree of polymerization and temperature;  is the density of a composite material; and is the thermal conductivity of a composite material in the cross-sectional plane. It should be noted that as the process equation is expressed in the Lagrangian frame of reference, the linear speed, , is absent from (1) and appears in boundary conditions [2]. Boundary conditions determine the interactions between the moving profile and the die block and the ambient air. The convective boundary conditions were used in computations:

inside the die: ; (2)

outside the die: ; (3)

where is the surface of the profile, is the coefficient of convective heat transfer between the die block and the profile, is the coefficient of convective heat transfer between the ambient air and the profile after exiting the die block, is the temperature of the metallic die block, which varies along the direction of pulling, , and is the ambient temperature.

The temperature of the material at the die entrance, , is set assuming its uniformity in a cross section:

(4)

The temperature of the material at the die entrance, , is set assuming its uniformity in a cross section:

(5)

where is the density of resin, is the total heat released during curing, is the volume fraction of reinforcement in a composite, is the curing rate of the resin, and is the degree of curing. The degree of curing is determined by the ratio of current heat released, , to the total heat released during curing, , i.e., . The curing degree rate can be expressed using the Arrhenius type equation as follows:

, (6)

where is the pre-exponential coefficient;  is the activation energy;  is the order of the reaction; and is the universal gas constant.

We assume that preheating the material before the die entrance will not result in significant polymerization of the resin; hence, the degree of material polymerization at the die entrance, , is taken to be zero:

. (7)

*2D Mechanical Model*

To describe the mechanical behavior of the matrix during curing, we use the Cure Hardening Instantaneous Linear Elastic (CHILE) model [4], which accounts for changes in the elastic modulus during phase transitions. The following form of the relationship for the resin elastic modulus is used:

(8)

where , , , , and represents the elastic modulus of polymerized matrix at temperatures of 20 , 40 , 60 , 80 , and , respectively. Further, term  represents the glass transition temperature depending on the degree of curing, and can be expressed as follows [5,6]:

(9)

where  is the glass transition temperature of the uncured matrix, and  represents the glass transition temperature of the fully cured () matrix; is the material parameter.

In order to describe changes in the Poisson’s ratio during phase transitions, we considered that during phase transitions, the bulk compression modulus does not change as significantly as does the elastic modulus. For example, according to some studies [7,8] the bulk compression modulus exhibits a reduction of only 2.5 times during the transition to the rubber-like state. Therefore, we used the following expression for changes in the bulk compression modulus:

(10)

where , and represents the bulk compression modulus of polymerized matrix at temperatures of 20 , and .

Then, we calculate the Poisson’s ratio using the classical relation as follows:

. (11)

In this work, we describe the formation of cracks at the surface of the pultruded rod. The crack forms immediately after the die exit (see Figure 1а) and propagates along the profile (Figure 1b). Opening of the crack after initiation, as shown in Figure 1c, indicates the existence of transverse tension stresses in the crack formation region. As this takes place, the temperature at the surface of the profile exceeds the glass transition temperature of the material. Considering the symmetry of the profile cross section as well as the presumed position of the crack at the surface of the product, we assume that the strength of the composite can be deemed sufficient if transverse stresses (e.g., stresses at the axis (see Figure 2)) do not exceed the tensile strength of the matrix, i.e.,

(12)

where , , , , and represent the strength of the polymerized matrix at temperatures of 20 , 40 , 60 , 80 , and , respectively. Here, the values , , , and are determined experimentally, and the value of is taken such that it correlates with experimental observations where cracks are formed at the pulling speed of 5 cm/min.

In order to describe the mechanical behavior of the composite, we used the self-consistent field micromechanics (SCFM) approach [9–11].

|  |  |
| --- | --- |
|  | (13) |

Here, the index *f* refers to the characteristics of the fiber, the index *m* - to the matrix. Direction 1 corresponds to the direction of laying fibers of transversally isotropic composite material.

The effective thermal expansion coefficients of the composite material are calculated by the following formulas

|  |  |
| --- | --- |
|  | (14) |

In order to calculate displacements and stresses, we used the incremental linear elastic approach [1] implemented in the ABAQUS environment [12].

The stress increment at the integration step is related to the mechanical strain vector increment by the linear relation

|  |  |
| --- | --- |
|  | (15) |

To calculate the stiffness tensor, the effective modules of the composite are first determined in accordance with the micromechanical model. The compliance matrix is determined by

|  |  |
| --- | --- |
|  | (16) |

Then we [inverse the matrix](https://www.multitran.com/m.exe?s=inverse+of+a+matrix&l1=1&l2=2).

The increments of temperature deformation are defined as

|  |  |
| --- | --- |
|  | (17) |

The increment of the volumetric chemical shrinkage of the matrix is determined by

|  |  |
| --- | --- |
|  | (18) |

where is the relative change in the resin volume at full polymerization.

The increment of linear chemical deformation is

|  |  |
| --- | --- |
|  | (19) |

The effective chemical deformations of composite material are determined by

|  |  |
| --- | --- |
|  | (20) |

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