Statistical Methods - MATH2715 - University of Leeds

WORKSHOP 1

PRACTICE ON:

- Exercise n.56 Chapter 1 *Math. Stat. and Data Anal.*, by J.A. Rice
- Homework Problem Assignment
- Monty Hall Problem
- Birthday Problem
- R Homework assignment: R implementation of the birthday problem

Facts: A couple has two children.

Questions

- (a) What is the probability that both are girls given that the oldest is a girl?
- (b) What is the probability that both are girls given that one of them is a girl?

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Solution

Observe that:

- the sample space is $\Omega = \{(f,f),(f,m),(m,f),(m,m)\}$ where in each couple the gender of in the first position corresponds to the older.
- def. $A = \{(f,f)\}$ (both are girls); $B = \{(f,f),(f,m)\}$ (oldest is a girl); $C = \{(f,f),(f,m),(m,f)\}$ (at least one is a girl).

At this point it comes that:

- (a) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{2/4} = 1/2$; (b) $P(A|C) = \frac{1/4}{3/4} = 1/3$.

Homework Problem (Will be marked)

Questions

Show that if two events A and B are independent, then A and B^c are independent and so are A^c and B^c.

Homework Problem Solution

Solution

Given that: $P(A \cap B) = P(A)P(B)$ and $A = (A \cap B) \cup (A \cap B^c)$. Compute:

$$P(A \cap B^c) = P(A) - P(A \cap B) = P(A) - P(A)P(B) = \\ = P(A)(1 - P(B)) = P(A)P(B^c) \quad (1)$$

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) \\ = 1 - P(A) - P(B) + P(A)P(B) = (1 - P(A))(1 - P(B)) = P(A^c)P(B^c) \quad (2)$$

Birthday Problem

In a classroom of n students, what is the probability p_n that two (or more) students share the same birthday?

Birthday Problem Solution

In a classroom of n students, what is the probability p_n that two (or more) students share the same birthday?

It's easier to compute the probability that no two students share a birthday. Let's look at the students one at a time. The first student can have any birthday he/she likes. The second student can not share the first student's birthday: 364 choices. The third student cannot share either of the first two birthdays: 363 choices. ... Etc. ... The $n-th$ student cannot share any of the previous $n-1$ birthdays: $365-n+1$ choices.
Therefore, probability of no shared birthdays is

$$\frac{365 \times 364 \times \ldots (365-n+1)}{365^n} \tag{3}$$

and the probability of a shared birthday is

$$1-\frac{365 \times 364 \times \ldots (365-n+1)}{365^n} \tag{4}$$

For $n = 23$, the answer is $p_{23} = 0.507$

R Homework: Birthday Problem R implementation

Instructions

Please develop a R script implementing the discussed solution of the Birthday Problem.
You can write a function that implements the solution for every $n \leq 365$) and a function call that prints out the solution corresponding to n=23.
Upload your script as a **SURNAMEstudentid.txt** in MINERVA (you will find an assignment).

Birthday Problem R implementation

```
birthday <- function(n){
  p = 1;
  for(a in seq(365,365-n+1,-1)){
    p = p * a/365;
    }
  return(1-p)
  }

birthday(23)
```

Monty Hall Problem

Suppose you are on a game show, and you are given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what?s behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

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Short Answer

If you switch doors you?ll win 2/3 of the time!

Monty Hall Problem

Long answer

If I pick a door and hold, I have a $1/3$ chance of winning.
My first guess is 1 in 3. there are 3 random options, right?
If I rigidly stick with my first choice no matter what, I can not improve my chances. The best I can do with my original choice is 1 in 3. The other doors must have the rest of the chances, i.e. $2/3$. Given that one of the remaining is now known to have a goat, the $2/3$ "concentrates" on the left door! Hence switching increase the odds!

Long answer on larger scale

Imagine this variant:
There are 100 doors to pick from in the beginning. You pick one door. Monty looks at the 99 others, finds 98 goats, and opens all of the corresponding doors. Do you stick with your original door (1/100), or the other door, which was filtered from 99? Monty is taking a set of 99 choices and improving them by removing 98 goats. He has the top door out of 99 for you to pick.