# Workshop2- Ex. 1

Let $X$ and $Y$ be independent exponential random variables with the same parameter $\lambda$.

Find the distribution of their sum: $Z = X + Y$.

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The distribution of the sum is given by the convolution:

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\begin{aligned}
f_Z(z) &= \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) dx \\
&= \int_0^z \lambda e^{-\lambda x}\, \lambda e^{-\lambda(z-x)}\, dx \\
&= \lambda^2 \int_0^z e^{-\lambda z} dx \\
&= \lambda^2 z e^{-\lambda z} \qquad \Longleftarrow \text{Gamma}
\end{aligned}
$$

NOTE: The sum of $n$ independent exponential random variables with parameter $\lambda$ is a random variable with a Gamma distribution with parameters $n$ and $\lambda$.

# Workshop2- Ex. 2

Let $X$ and $Y$ be independent standard normal random variables. That is, $N(0,1)$.

Find the distribution of the ratio: $Z = Y/X$.

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$$\begin{aligned}
F_Z(z) &= P(Y/X \leq z) = P(Y \leq zX,\ X \geq 0) + P(Y \geq zX,\ X < 0) \\
&= P(Y \leq zX,\ X \geq 0) + P(-Y \leq z(-X),\ -X \geq 0) \\
&= 2P(Y \leq zX,\ X \geq 0) \\
&= 2\int_0^\infty \int_{-\infty}^{xz} f_X(x) f_Y(y)\, dy\, dx = \frac{1}{\pi}\int_0^\infty \int_{-\infty}^{xz} e^{-x^2/2} e^{-y^2/2}\, dy\, dx
\end{aligned}$$

Differentiating, we get

$$\begin{aligned}
f_Z(z) &= \frac{1}{\pi}\int_0^\infty e^{-x^2/2}\, e^{-(xz)^2/2} x\, dx = \frac{1}{\pi}\int_0^\infty e^{-x^2(1+z^2)/2}\, x\, dx \\
&= \frac{1}{\pi}\int_0^\infty e^{-u(1+z^2)}\, du = \frac{1}{\pi(1+z^2)} \qquad \Longleftarrow \text{Cauchy}
\end{aligned}$$