Contents

1 Thomas-BMT equation **2**

2 Spin tune **2**

3 The Frozen Spin concept **2**
3.1 Integer resonance 3
3.2 Realization of the FS condition in a storage ring 3

4 Non-resonance methods **3**
4.1 COSY Spin Tune Mapping + RF Wien Filter Method 3

5 Resonance methods **4**
5.1 BNL Frozen Spin Method 4
5.2 I. Koop's Spin Wheel 5
5.3 Frequency Domain Method 5

1 Thomas-BMT equation

The Thomas-BMT equation describes the dynamics of spin vector $\boldsymbol{s}$ in magnetic field $\boldsymbol{B}$ and electrostatic field $\boldsymbol{E}$. Generalized to account for the EDM effects, it can be written (in the rest frame) as follows: [1, p. 6]

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = \boldsymbol{s} \times \left(\boldsymbol{\Omega}_{MDM} + \boldsymbol{\Omega}_{EDM}\right), \tag{1a}$$

where the MDM and EDM angular frequencies $\boldsymbol{\Omega}_{MDM}$ and $\boldsymbol{\Omega}_{EDM}$ are

$$\boldsymbol{\Omega}_{MDM} = \frac{q}{m}\left[G\boldsymbol{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\boldsymbol{E}\times\boldsymbol{\beta}}{c}\right], \tag{1b}$$

$$\boldsymbol{\Omega}_{EDM} = \frac{q}{m}\frac{\eta}{2}\left[\frac{\boldsymbol{E}}{c} + \boldsymbol{\beta}\times\boldsymbol{B}\right]. \tag{1c}$$

In the equations above, m, q, G are the particle mass, electric charge, and anomalous MDM respectively; $\beta = {}^{v_0}/_{c}$, is the ratio of the particle velocity to the speed of light; γ is the Lorentz factor. The EDM factor η is defined by $d = \eta\frac{q}{2mc}s$, where d is the particle EDM and s is its spin.

2 Spin tune

In the standard spinor formalism, the spin transfer matrix per turn in a ring R equals [p. 4][2]

$$\boldsymbol{t}_R = \exp\left(-i\pi\nu_s\boldsymbol{\sigma}\cdot\boldsymbol{c}\right) = \cos\pi\nu_s - i(\boldsymbol{\sigma}\cdot\boldsymbol{c})\sin\pi\nu_s,$$

where $\boldsymbol{\sigma}$ is the Pauli matrix vector, $\boldsymbol{c}$ is a unit vector, pointing along the local spin precession axis. The spin precession angular velocity can be written as

$$\boldsymbol{\Omega}_s = 2\pi f_s\boldsymbol{c} = 2\pi f_R\nu_s\boldsymbol{c},$$

where f_R is the beam revolution frequency, and ν_s is the *spin tune*, i.e. the number of spin revolutions per turn.

3 The Frozen Spin concept

It can be observed in eq (1b) that, in the absence of an EDM, the spin of a beam particle can be frozen along its momentum direction: $\boldsymbol{\Omega}_{MDM} = \boldsymbol{0}$, i.e., the so-called Frozen Spin (FS) condition can be realized. The advantage of imposing the FS condition on the beam is as follows: according to equations equations (1a–1c), in a storage ring, the MDM angular velocity points mainly along the vertical axis, while the EDM — along the radial one. This means that for the total precession frequency, these components add in squares, and hence the small EDM component is completely drowned by the stronger MDM one:

$$\Omega_{net} = \Omega_{MDM}\sqrt{1 + \left(\frac{\Omega_{EDM}}{\Omega_{MDM}}\right)^2}.$$

3.1 Integer resonance

A particle in a Frozen Spin state has a spin tune $\nu_s = 0 \in \mathbb{N}$ in the rest frame (and $\nu_s = 1$ in the laboratory frame), i.e., it is susceptible to *integer resonance*. [4] As a result of integer resonance, any magnetic field imperfection causing the appearance of a radial component will inevitably tilt the polarizatrion of the particle bunch into the horizontal plane. [4, p. 8] Once there, due to the energy-dependence of spin tune, the beam particles' spin vectors will decohere, depolarizing the beam.

We can classify EDM experiment methodologies into 1) Resonance, and 2) Non-resonance, depending on whether or not they put the particles into an integer resonance.

3.2 Realization of the FS condition in a storage ring

Storage rings can be classified into three groups:

1. purely magnetic (like COSY, NICA, etc),
2. purely electrostatic (Brookhaven AGS Analog Ring),
3. combined rings.

In view of eq (1b), the FS condition cannot be realized in a purely magnetic ring.

For a number of particles, such as the proton, whose $G > 0$, a purely electrostatic ring can be used in a resonance-type EDM experiment methodology with a beam at a so-called "magic" energy, defined by $\gamma_{mag} = \sqrt{(1+G)/G}$.

For particles with $G < 0$ (such as the deuteron), this is not an option, and a combined ring must be used. In order to realize the FS condition in a combined ring, a radial E-field of magnitude

$$E_r = \frac{GBc\beta\gamma^2}{1 - G\beta^2\gamma^2}, \tag{2}$$

is introduced. [3]

4 Non-resonance methods

4.1 COSY Spin Tune Mapping + RF Wien Filter Method

This method has been devised by the Jülich Electric Dipole Moments Investigations (JEDI) collaboration, based in Forschungszentum Jülich GmbH, Jülich, Germany. It proposes to conduct a precusror experiment for finding the deuteron EDM at the Cooler Synchrotron COSY. COSY is a purely magnetic storage ring approximately 184 m in circumferance. It provides un-/polarized proton and deuteron beams in the momentum range of 300 MeV/c to 3.7 GeV/c. [6]

5 Resonance methods

5.1 BNL Frozen Spin Method

The BNL FS method, proposed by BNL's Storage Ring EDM collaboration in 2008, [3] is a resonance method in a combined ring. A beam of longitudinally-polarized deuterons is injected into a storage ring; the spin precession in both the vertical and horizontal planes is probed by polarimetry; the EDM signal is the change in the vertical polarization over time, given by: [3, p. 8]

$$\Delta P_V = P\frac{\omega_{edm}}{\Omega}\sin\left(\Omega t + \Theta_0\right), \tag{3}$$

where $\Omega = \sqrt{\omega_{edm}^2 + \omega_a^2}$, ω_a, ω_{edm} are the angular velocities arising, respectively, from the magnetic and electric dipole moments.

By applying a radial electric field E_r (2), the ω_a component is expected to be reduced by at least a factor of 10^9; in view of the small value of the hypothesized ω_{edm}, $\Delta P_V \approx P\omega_{edm}t$, and the maximum value of ΔP_V is amplified by 10^9.

The expected one-sigma measurement sensitivity is 10^{-29} $e \cdot cm$ per 10^7 seconds (6 months) of total run time. At this sensitivity level, the left-right cross-section asymmetry $\varepsilon_{LR} \approx 5 \cdot 10^{-6}$ for the smallest practical values of ω_a. [3, p. 18] This is a challenging task for polarimetry. [5] One way to deal with this challenge is to apply an external radial magnetic field and measure the combined MDM + EDM spin precession frequency. This is the basis of the so-called Spin Wheel method, which will be outlined below.

The only known first-order spin dynamics systematic effect is the presence of a non-zero average vertical component of the electric field $\langle E_V \rangle$. In that case, the spin will precess about the radial direction with a frequency [3, p. 11]

$$\omega_{syst} \approx \frac{\mu\langle E_V \rangle}{\beta c \gamma^2}.$$

There are two points to consider with this effect:

- the presence of $\langle E_V \rangle \neq 0$ is due to accelerator element misalignment;
- this systematic effect changes sign when the beam is injected in the opposite direction.

The latter is the reason for the clockwise/counter-clockwise (CW/CCW) beam injection structure used in this method. Though it is true that ω_{syst} flips sign when the beam direction is reversed, what this methodology doesn't account for is its *magnitude*. Our simulations[1] show, that at the realistic value of 100 μm deflector installation error, the MDM precession frequency about the radial axis is in the order of 50–100 rad/sec. [7] For this reason, it is impossible to use this methodology as is.

[1] REFERENCE TO SIMULATION

5.2 I. Koop's Spin Wheel

The above problems with polarimetry and high spin precession rate are solved in the Spin Wheel method, proposed by Ivan Koop of BINP, Novosibirsk, Russia. The outline of the idea is as follows: first, the FS condition is imposed; then, a radial magnetic field

5.3 Frequency Domain Method

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