

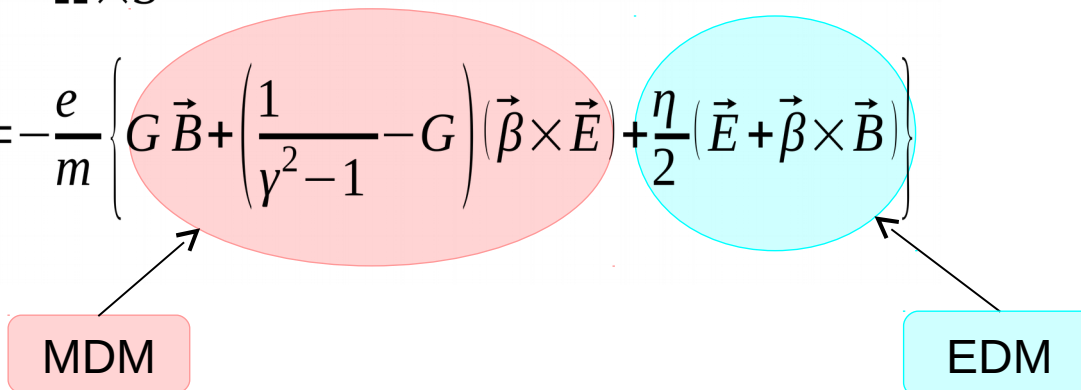
# Model of statistical errors in the search for the deuteron EDM in the storage ring

03/21/17

# Methodology

When put into an electromagnetic field, the particle spin begins to precess according to the T-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

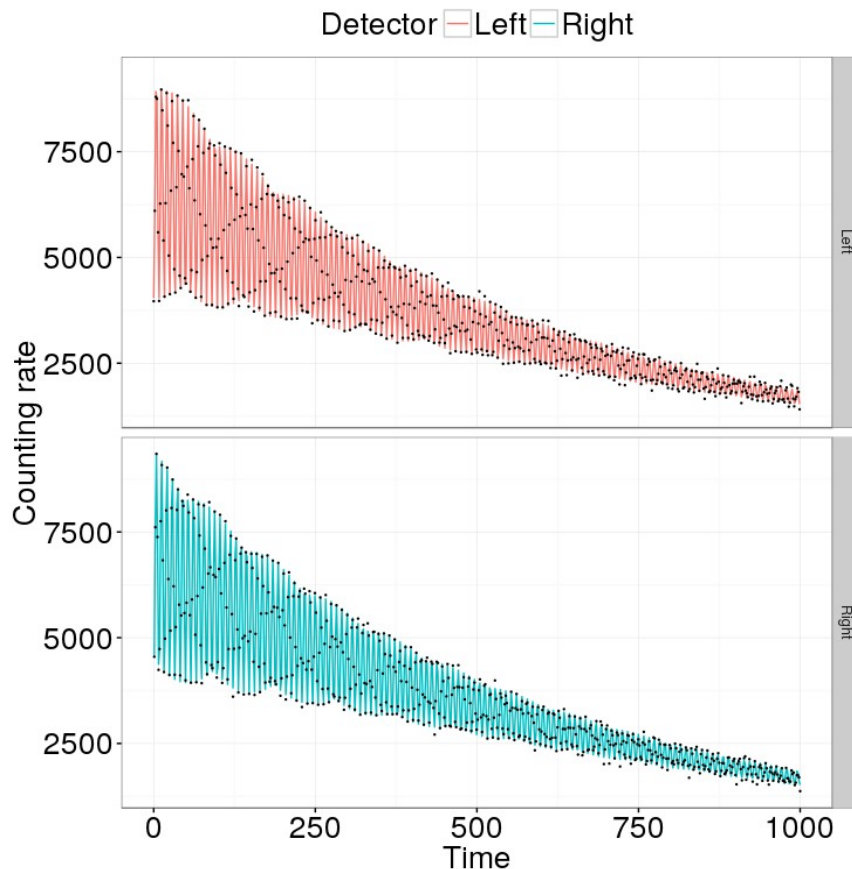
$$\vec{\Omega} = -\frac{e}{m} \left\{ G \vec{B} + \left( \frac{1}{\gamma^2 - 1} - G \right) (\vec{\beta} \times \vec{E}) + \frac{\eta}{2} (\vec{E} + \vec{\beta} \times \vec{B}) \right\}$$


By measuring the beam's polarization, we can determine the frequency

$$\vec{\Omega}^{\pm} = \vec{\Omega}_{MDM} \pm \vec{\Omega}_{EDM}$$

Comparing the CW vs CCW frequencies, determine  $\Omega_{EDM}$

# Detector counting rate



$$\tilde{N}(t) = N_0(t) \left[ 1 + P \cdot e^{-t/\tau_d} \cdot \sin(\omega t + \phi) \right] + \varepsilon_t$$

Number of counts is Poisson distributed, hence

$$\sigma_{\tilde{N}_0}^2 = N_0(t)$$

$$\sigma_{N_0}(t) = \sigma_{\tilde{N}_0}(t) / \sqrt{n_{c/\epsilon}}$$

$$\frac{\sigma_{N_0}(t)}{N_0(t)} \propto \frac{1}{\sqrt{\Delta t_\epsilon}} \cdot \exp\left(\frac{t}{2\tau_b}\right)$$

# Cross section asymmetry

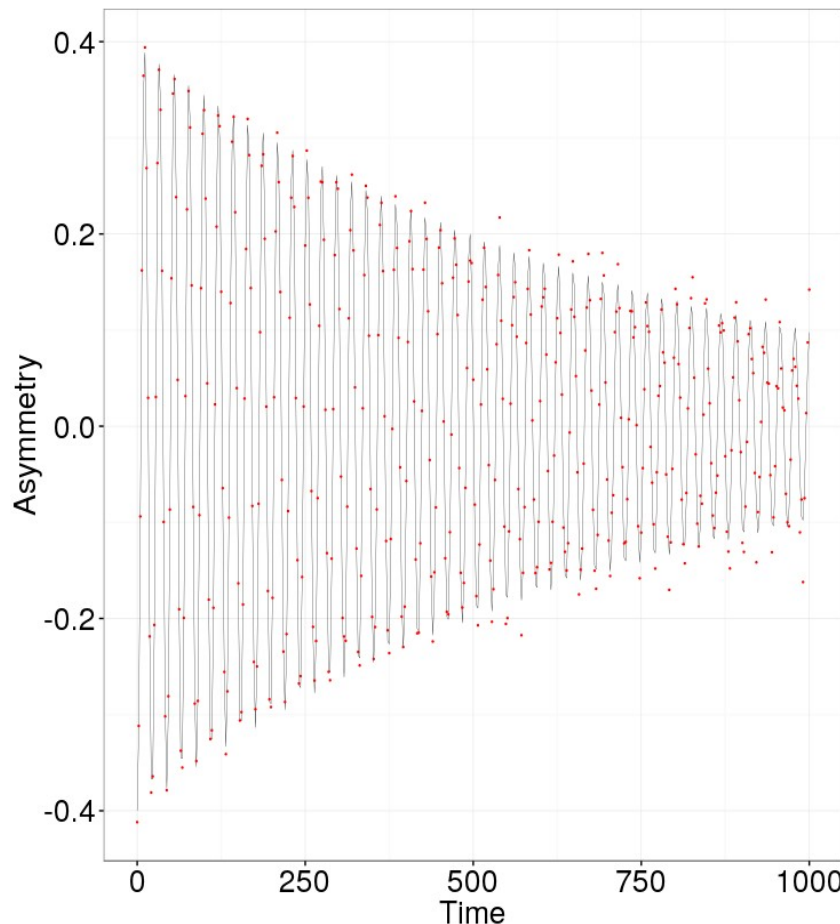
A measure of polarization

Definition: 
$$A = \frac{N_L - N_R}{N_L + N_R}$$

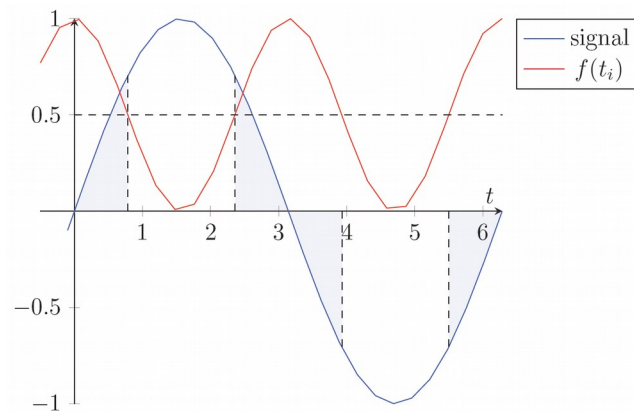
Model: 
$$A(t) = A(0) \cdot e^{\lambda t} \cdot \sin(\omega \cdot t + \phi)$$

$$\sigma_A^2(t) \approx \frac{1}{2N_0(t)}$$

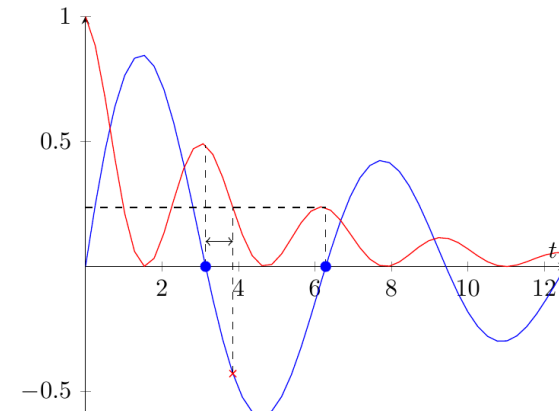
Error: 
$$\sigma^2[\hat{\omega}] = \frac{\sigma^2[\varepsilon]}{\sum_i f(t_i) \cdot \sigma_w^2[t]}$$



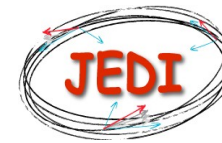
# Limiting factors



- Sample Fisher information can be increased by sampling during rapid change
- Limited by polarimetry sampling rate



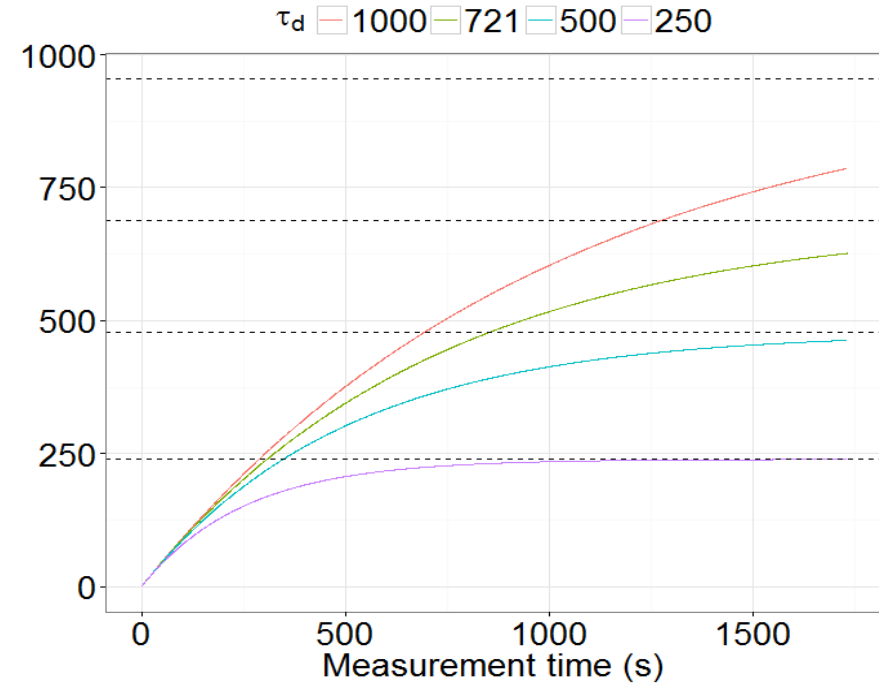
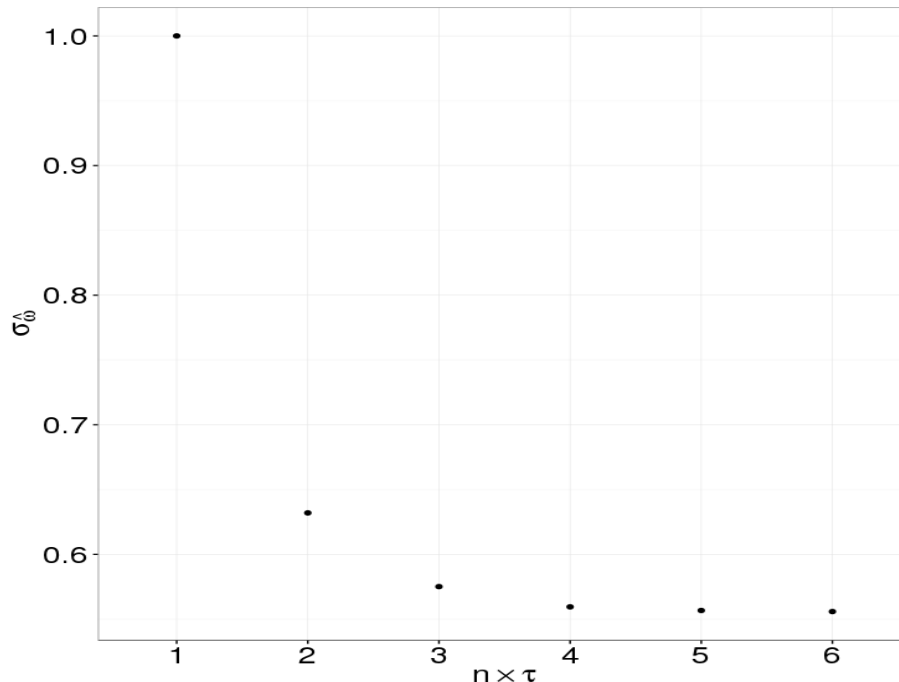
- Point Fisher information falls exponentially due to decoherence
- Can't economize the beam too much



# Time spread

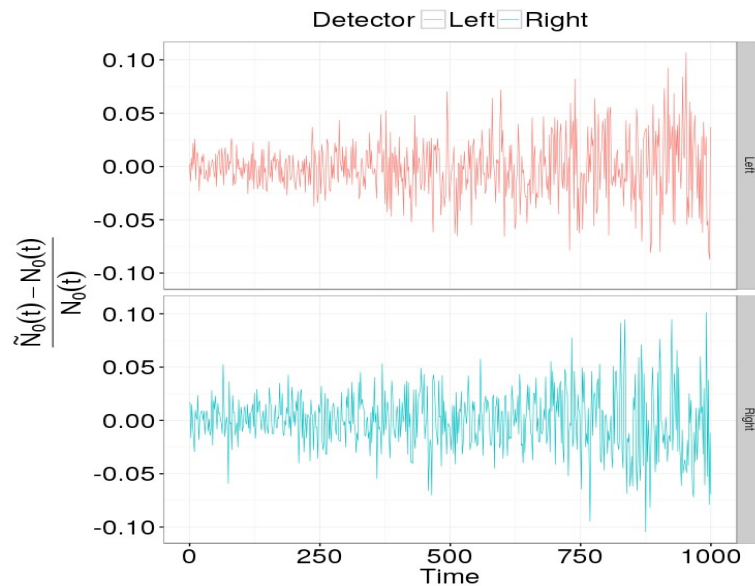
$$\sum f(t_i) = n_{\varepsilon/zc} \cdot x_{01} \cdot \frac{\exp\left(-\frac{\pi}{\omega\tau_d} n_{zc}\right) - 1}{\exp\left(-\frac{\pi}{\omega\tau_d}\right) - 1}$$

$$t(z) = \tau_d \cdot \ln\left(\frac{1}{1-z}\right)$$

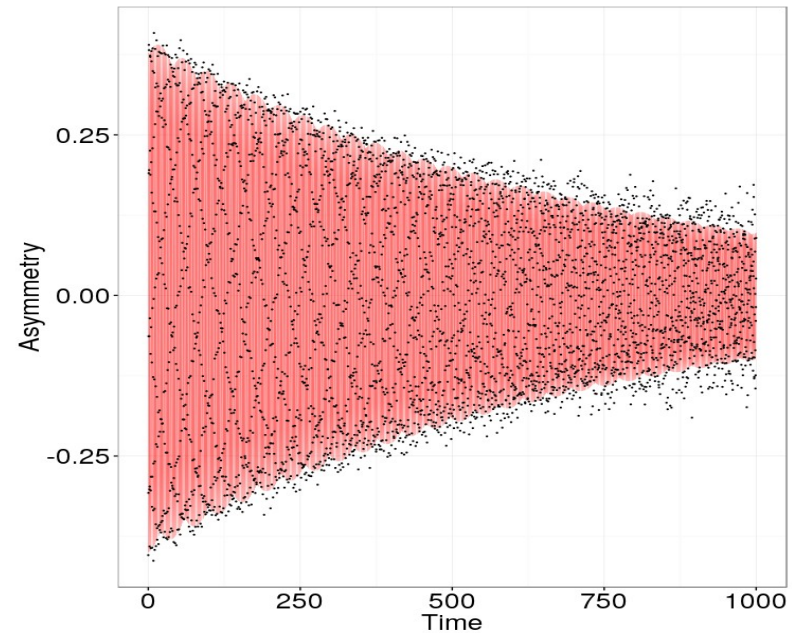


FI limit (%)	By ( $\times \tau_d$ )	SNR@3% error
95	3.0	1.7
90	2.3	3.3
70	1.2	10.0

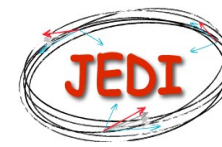
# Simulation



- Uniform sampling
- 75% of the beam ( $7.5 \cdot 10^8$  useful scatterings)
- 3% initial counting rate error



- Standard error  $7.55 \cdot 10^{-7}$  rad/sec
- If  $\omega$  is known down to  $10^{-6}$ , can improve the result by 30%



# Thank You



# Standard Error

- No decoherence
- Uniform sampling

$$\sum f(t_i) = N \cdot \tilde{x}$$

$$\tilde{x} = \frac{1}{2} (N_0 P)^2$$

$$\sigma_w^2 = \frac{T^2}{12}$$

$$\sigma_{\hat{\omega}}^2 = \frac{24}{N (PT)^2} \cdot \left( \frac{\sigma[\varepsilon]}{N_0} \right)^2$$