The probability of observing the value $y_i \equiv y(t_i)$ when the expectation value is $\mu(t_i)$ and the error is gaussian:

$$
\begin{aligned}
f(y_i|\boldsymbol{\theta}) &= \frac{1}{\sqrt{2\pi\nu}} \exp\left(-\frac{1}{2}\frac{(y_i-\mu(t_i))^2}{\nu}\right),\\
\boldsymbol{\theta} &= (\nu,\omega,\phi),\\
\mu(t_i) &= N_0\left(1+P\sin(\omega t_i+\phi)\right).
\end{aligned}
$$

The likelihood of observing a set of observations $\mathbf{y}=(y_1,\ldots,y_K)$ is

$$
\mathcal{L}(\boldsymbol{\theta}|\mathbf{y}) = \prod_i f(y_i|\boldsymbol{\theta}),
$$

and the log-likelihood is

$$
\ell(\boldsymbol{\theta}|\mathbf{y}) = -\frac{K}{2}\log 2\pi - \frac{K}{2}\log\nu - \frac{1}{2\nu}\sum_i \epsilon_i^2,\ \ \epsilon_i = y_i-\mu(t_i)
$$

$$
\begin{aligned}
\mu'_\phi &= N_0 P\cos(\omega t+\phi),\\
\mu'_\omega &= t\cdot\mu'_\phi, \epsilon'_\xi = -\mu'_\xi\\
\mathrm{E}\left[\epsilon_i|\ \boldsymbol{\theta}_0\right] &= \mathrm{E}\left[t_i\epsilon_i|\ \boldsymbol{\theta}_0\right] = 0.
\end{aligned}
$$

The derivatives:

$$
\begin{aligned}
\ell'_\nu &= -\frac{K}{2\nu}+\frac{1}{2\nu^2}\sum_i \epsilon_i^2;\\
\ell'_\omega &= \frac{1}{\nu}\sum_i \mu'_\phi(t_i)t_i\epsilon_i;\\
\ell'_\phi &= \frac{1}{\nu}\sum_i \mu'_\phi(t_i)\epsilon_i;\\
\ell''_{\nu^2} &= \frac{K}{2\nu^2}-\frac{1}{\nu^3}\sum_i\epsilon_i^2, & -\mathrm{E}\left[\ell''_{\nu^2}|\ \boldsymbol{\theta}_0\right] &= \frac{K}{2\nu^2}-\frac{1}{\nu^3}\sum_i\nu = \frac{K}{2\nu^2};\\
\ell''_{\nu\omega} &= -\frac{1}{\nu^2}\sum_i\mu'_\phi(t_i)t_i\epsilon_i, & -\mathrm{E}\left[\ell''_{\nu\omega}|\ \boldsymbol{\theta}_0\right] &= \frac{1}{\nu^2}\sum_i\mu'_\phi(t_i)\mathrm{E}\left[t_i\epsilon_i|\ \boldsymbol{\theta}_0\right] = 0;\\
\ell''_{\nu\phi} &= -\frac{1}{\nu^2}\sum_i\mu'_\phi(t_i)\epsilon_i, & -\mathrm{E}\left[\ell''_{\nu\phi}|\ \boldsymbol{\theta}_0\right] &= \frac{1}{\nu^2}\sum_i\mu'_\phi(t_i)\mathrm{E}\left[\epsilon_i|\ \boldsymbol{\theta}_0\right] = 0;\\
\ell''_{\phi^2} &= \frac{1}{\nu}\sum_i\left(\mu''_{\phi^2}(t_i)\epsilon_i-\left(\mu'_\phi(t_i)\right)^2\right), & -\mathrm{E}\left[\ell''_{\phi^2}|\ \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i\left(\left(\mu'_\phi(t_i)\right)^2-\mu''_{\phi^2}(t_i)\mathrm{E}\left[\epsilon_i|\ \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i\left(\mu'_\phi(t_i)\right)^2;\\
\ell''_{\phi\omega} &= \frac{1}{\nu}\sum_i\left(\mu''_{\phi^2}(t_i)t_i\epsilon_i-\left(\mu'_\phi(t_i)\right)^2t_i\right), & -\mathrm{E}\left[\ell''_{\phi\omega}|\ \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i\left(t_i\left(\mu'_\phi(t_i)\right)^2-\mu''_{\phi^2}(t_i)\mathrm{E}\left[t_i\epsilon_i|\ \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i t_i\left(\mu'_\phi(t_i)\right)^2;\\
\ell''_{\omega^2} &= \frac{1}{\nu}\sum_i\left(\mu''_{\phi^2}(t_i)t_i^2\epsilon_i-\left(\mu'_\phi(t_i)t_i\right)^2\right), & -\mathrm{E}\left[\ell''_{\omega^2}|\ \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i\left(\left(t_i\mu'_\phi(t_i)\right)^2-\mu''_{\phi^2}(t_i)\mathrm{E}\left[t_i^2\epsilon_i|\ \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i\left(t_i\mu'_\phi(t_i)\right)^2.
\end{aligned}
$$

The Fisher matrix

$$
I(\boldsymbol{\theta}_0) = \begin{bmatrix} K/2\nu & 0 & 0\\ 0 & 1/\nu\sum\left(t_i\mu'_\phi(t_i)\right)^2 & 1/\nu\sum t_i\left(\mu'_\phi(t_i)\right)^2\\ 0 & 1/\nu\sum t_i\left(\mu'_\phi(t_i)\right)^2 & 1/\nu\sum\left(\mu'_\phi(t_i)\right)^2 \end{bmatrix}.
$$

The determinant

$$
|I(\boldsymbol{\theta}_0)| = \frac{K}{2\nu^4}\underbrace{\left(\sum\left(t_i\mu'_\phi(t_i)\right)^2\sum\left(\mu'_\phi(t_i)\right)^2-\left(\sum t_i\left(\mu'_\phi(t_i)\right)^2\right)^2\right)}_{\Omega}.
$$

The variance-covariance matrix

$$
vcov = \begin{bmatrix} 2\nu^2/K & 0 & 0\\ 0 & \nu\frac{\sum\left(\mu'_\phi(t_i)\right)^2}{\Omega} & \nu\frac{\sum t_i\left(\mu'_\phi(t_i)\right)^2}{\Omega}\\ 0 & \nu\frac{\sum t_i\left(\mu'_\phi(t_i)\right)^2}{\Omega} & \nu\frac{\sum\left(t_i\mu'_\phi(t_i)\right)^2}{\Omega} \end{bmatrix}.
$$

Variance of the frequency estimate

$$var(\hat{\omega}) = \nu \frac{\sum \left(\mu'_\phi(t_i)\right)^2}{\sum \left(t_i \mu'_\phi(t_i)\right)^2 \sum \left(\mu'_\phi(t_i)\right)^2 - \left(\sum t_i \left(\mu'_\phi(t_i)\right)^2\right)^2}.$$