FREQUENCY DOMAIN METHOD OF SEARCH FOR THE DEUTERON ELECTRIC DIPOLE MOMENT

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CONTENTS

MOTIVATION

Storage ring-based methods of search for the electric dipole moments (EDMs) of fundamental particles can be classified into two major categories, which we will call *a*) Space Domain, and *b*) Frequency Domain methods.

In the Space Domain paradigm, one measures a *change in the spatial orientation* of the beam polarization vector *caused by the EDM*.

The original storage ring, frozen spin-type method, proposed in [1], is a canonical example of a methodology in the space domain: an initially longitudinally-polarized beam is injected into the storage ring; the vertical component of its polarization vector is observed. Under ideal conditions, any tilting of the beam polarization vector from the horizontal plane is attributed to the action of the EDM.

Two technical difficulties are readily apparent with this approach:

1. it poses a challenging task for polarimetry [2];
2. it puts very stringent constraints on the precision of the accelerator optical element alignment.

The former is due to the requirement of detecting a change of about $5 \cdot 10^{-6}$ to the cross section asymmetry ε_{LR} in order to get to the EDM sensitivity level of 10^{-29} $e \cdot cm$. [1, p. 18]

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The latter is to minimize the magnitude of the vertical plane magnetic dipole moment (MDM) precession frequency: [1, p. 11]

$$\omega_{syst} \approx \frac{\mu\langle E_v\rangle}{\beta c\gamma^2}, \tag{1}$$

induced by machine imperfection fields. According to estimates done by Y. Senichev, if it is to be fulfilled, the geodetic installation precision of accelerator elements must reach 10^{-14} m. Today's technology allows only for about 10^{-4} m.

At the practically-achievable level of element alignment uncertainty, $\omega_{syst} \gg \omega_{edm}$, and changes in the orientation of the polarization vector are no longer EDM-driven.

Another crucial problem one faces in the space domain is geometric phase error. [3, p. 6] The problem here lies in the fact that, even if one can somehow make field imperfections (either due to optical element misalignment or spurious electro-magnetic fields) zero *on average*, since spin rotations are non-commutative, the polarization rotation angle due to them will not be zero.

By contrast, the Frequency Domain methodology is founded on measuring the EDM *contribution* to the total (MDM and EDM together) spin precession *angular velocity*.

The polarization vector is made to roll about a nearly-constant, definite direction vector $\bar{n}$, with an angular velocity that is high enough for its magnitude to be easily measureable at all times. Apart from easier polarimetry, the definiteness of the angular velocity vector is a safeguard against geometric phase error.

This "Spin Wheel" may be externally applied [4], or otherwise the machine imperfection fields may be utilized for the same purpose (wheel roll rate determined by equation (1)). The latter is made possible by the fact that ω_{syst} changes sign when the beam revolution direction is reversed. [1, p. 11]

UNIVERSAL SR EDM MEASUREMENT PROBLEMS

By way of introduction to the proposed measurement methodology, let us briefly summarize some measurement problems encountered by any EDM experiment performed in a storage ring; they can be grouped into two big categories:

- Problems solved by a Spin Wheel:
  - spurious electro-magnetic fields;
  - betatron motion.

- Problems having specific solutions:
  - spin decoherence;
  - machine imperfections.

Spin motion perturbation

Problems from the first category are ones introducing geometric phase error. Indeed, both the spurious and the focusing fields, when acting on a betatron-oscillating particle, perturb the direction and magnitude of its spin precession angular velocity vector. The effect is a spin kick in the direction defined by the perturbation.

Assume that the EDM provides a spin kick about the radial ($\hat{x}$-) axis. The magnitude of the angular velocity vector has a general form

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2},$$

where ω_y is minimized by fulfilling the frozen spin condition; ω_z (the constant part of which is due to machine imperfections) can be minimized via the installation of a longitudinal solenoid on the optic axis.[1] In the space domain, one also tries to minimize the $\omega_{\langle E_v \rangle}$ contribution to $\omega_x = \omega_{edm} + \omega_{\langle E_v \rangle}$. Consequently, spin kicks must be minimized to (significantly) less than ω_{edm}, so as to reduce geometric phase to less than the accumulated EDM phase.

The benefit of having a Spin Wheel aligned with the EDM angular velocity is that orthogonal MDM contributions to the total angular velocity vector add up in squares, and hence their effect is greatly diminished:

$$\begin{aligned}
\omega &= \sqrt{(\omega_{edm} + \omega_{SW})^2 + \omega_y^2 + \omega_z^2} \\
&\approx (\omega_{edm} + \omega_{SW}) \cdot \left[1 + \frac{\omega_y^2 + \omega_z^2}{\omega_{SW}^2}\right]^{1/2} \\
&\approx (\omega_{edm} + \omega_{SW}) \cdot \left(1 + \frac{\omega_y^2 + \omega_z^2}{2\omega_{SW}^2}\right) \\
&\approx \omega_{SW} + \omega_{edm} + \underbrace{\frac{1}{2}\frac{\omega_y^2 + \omega_z^2}{\omega_{SW}}}_{\epsilon}.
\end{aligned}$$

Since our goal is to observe the EDM-related value shift in ω, we need to minimize random variable ϵ:

$$\frac{1}{2}\frac{\omega_y^2 + \omega_z^2}{\omega_{SW}} < \omega_{edm}.$$

Let's make some preliminary estimates. Suppose $\omega_{SW} \approx 50$ rad/sec (the reason for choosing this value will be explained shortly), $\omega_{edm} \approx 10^{-9}$ rad/sec (corresponding to the EDM value 10^{-29} $e\cdot$ cm). Then, $\omega_y^2 + \omega_z^2$ must be reduced to less than 10^{-7} rad/sec, or equivalently, either angular velocity to less than $3 \cdot 10^{-4}$ rad/sec. This is several orders of magnitude greater than the expected standard error on the angular velocity estimate, [5] and hence should not be a problem to achieve.

One case left to be considered is MDM spin kicks about the $\hat{x}$-axis. These are not attenuated, and cause the most trouble. They come in three varieties: *a*) permanent, not caused by optical element misalignments; *b*) semi-permanent, caused by element tilts about the optic axis; *c*) spurious.

Semi-permanent radial spin kicks (be they caused by magnetic or electric fields) change sign when the beam revolution direction is reversed from clockwise (CW) to counter-clockwise (CCW). Spurious kicks can be dealt with by statistical averaging. Permanent, insensitive to either the guide field or the beam circulation direction, cannot be controlled. On the bright side, their sources should not be present under normal circumstances.

For more details on spin motion perturbation effects on the measurement of the EDM in frequency domain, please refer to [6].

Expected machine imperfection SW roll rate

In the estimates above, we used a roll rate $\omega_{SW} \approx 50$ rad/sec for the spin wheel. This is our expected ω_{syst} caused by machine imperfections.

Denote the standard deviation of the imperfection radial magnetic field distribution $\sigma[B_x]$. For the whole ring, MDM precession will be distributed with a standard deviation [7]

$$\sigma[\omega_x^{MDM}] = \frac{e}{m\gamma}\frac{G+1}{\gamma}\frac{\sigma[B_x]}{\sqrt{n}},$$

where n is the number of misaligned elements, $G = (g-2)/2$ is the anomalous magnetic dipole moment.

For deuterons in lattices [8] of n on the order of 100 elements, rotated about the optic axis by angles $\Theta_{tilt} \sim N(0, 10^{-4})$ rad, Y. Senichev estimates [7] ω_x^{MDM} between 50 and 100 rad/sec.

Our simulations done in COSY INFINITY seem to confirm this result. In Figure 1 you see the results of the simulation in which we rotated the 32 E+B spin rotator elements used in the frozen spin (codename BNL) lattice [8] by angles randomly picked from the distribution $N(\mu_0 \cdot (i-5), \sigma_0)$, where $\mu_0 = 10 \cdot \sigma_0 = 10^{-4}$ rad, $i \in \{0, \dots, 10\}$.

At $\langle \Theta_{tilt} \rangle = 10^{-4}$ we observe a roll rate of 500 rad/sec. We should keep in mind, however, that Senichev assumes $\sigma_{\Theta_{tilt}} = 10^{-4}$ rad, which means, for a lattice with $n = 100$ tilted elements, a standard deviation of the mean $\sigma_{\langle \Theta_{tilt} \rangle} = \sigma_{\Theta_{tilt}}/\sqrt{100} = 10^{-5}$. The dependence of ω_x^{MDM} on $\langle \Theta_{tilt} \rangle$ is linear, which means in an actual lattice we would observe an $\omega_{syst} \leq 50$ rad/sec with 68% probability, and $\omega_{syst} \leq 100$ rad/sec with 95% probability, and with 27% probability $50 \leq \omega_{syst} \leq 100$.

Spin decoherence

Spin coherence is a measure or quality of preservation of polarization in an initially fully-polarized beam. [9] Spin

[1] 1 m long, magnetic field approximately 10^{-6} T.

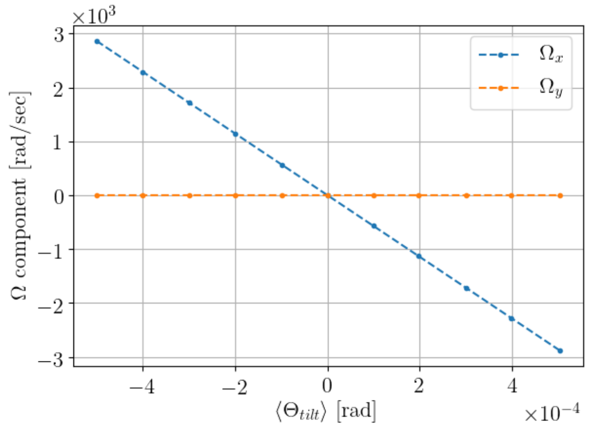

Figure 1: Spin precession frequency (radial and vertical components) versus the mean E+B element tilt angle

decoherence refers to the depolarization caused by the difference in the beam particles' spin precession frequencies.

The difference in spin tunes is due to the difference of the particles' orbit lengths, and hence their equilibrium energy levels, on which spin tune depends. One way spin decoherence can be suppressed is by utilization of sextupole fields. We consider how this can be accomplished in [10].

Machine imperfections

As we have seen, the problem with machine imperfections is twofold: *a*) they are practically impossible to remove at the present level of technology; but what's even worse, *b*) their removal leaves one in the space domain, and opens the measurement up to geometric phase error.

Fortunately for us, the imperfection spin kicks they induce change sign when the beam circulation direction is reversed. Their magnitude is also sufficient for use as a Koop Wheel. The one remaining difficulty is the accuracy of the Koop wheel roll direction flipping. Hopefully, we can make a persuasive enough argument as to how this can accomplished.

METHODOLOGY MAIN FEATURES

The method we propose is characterized by two main features:

1. It is a frequency domain method;
2. The fields induced by machine imperfections, instead of being suppressed, are used as a Koop Wheel.
    - The Koop Wheel roll direction is reversed by flipping the direction of the guide field;
    - its roll rate is controlled through observation of spin precession in the horizontal plane.

The advantages of the frequency domain, such as *a*) ease of polarimetry, and *b*) immunity to geometric phase error, have been discussed in prevous sections. Now we will turn to the description of how machine imperfection fields can be used as a Koop Wheel.

EDM ESTIMATOR STATISTIC

Since the angular velocity measured in the frequency domain methodology includes contributions due to both the magnetic and electric dipole moments, the EDM estimator statistic requires two cycles to compose: one in which the Koop Wheel rolls forward, the other backward.

The change in the Koop Wheel roll direction is affected by flipping the direction of the guide field. When this is done: $\vec{B} \mapsto -\vec{B}$, the beam circulation direction changes from clockwise (CW) to counter-clockwise (CCW): $\vec{\beta} \mapsto -\vec{\beta}$, while the electrostatic field remains constant: $\vec{E} \mapsto \vec{E}$. According to the T-BMT equation, spin precession frequency components change like:

$$\begin{aligned} \omega_x^{CW} &= \omega_x^{MDM,CW} + \omega_x^{EDM}, \\ \omega_x^{CCW} &= \omega_x^{MDM,CCW} + \omega_x^{EDM}, \\ \omega_x^{MDM,CW} &= -\omega_x^{MDM,CCW}, \end{aligned} \tag{2a}$$

and the EDM estimator

$$\hat{\omega}_x^{EDM} := \frac{1}{2}\left(\omega_x^{CW} + \omega_x^{CCW}\right) \tag{2b}$$

$$= \omega_x^{EDM} + \underbrace{\frac{1}{2}\left(\omega_x^{MDM,CW} + \omega_x^{MDM,CCW}\right)}_{\varepsilon\to 0}. \tag{2c}$$

To keep the systematic error term ε below required precision, i.e. ensure that equation (2a) holds with sufficient accuracy, Y. Senichev devised [7] a guide field flipping procedure based on observation of the beam polarization precession frequency in the horizontal plane.

To explain how it works, we need to introduce the concept of the effective Lorentz factor.

EFFECTIVE LORENTZ FACTOR

Spin dynamics is described by the concepts of *spin tune* ν_s and *invariant spin axis* $\bar{n}$. Spin tune depends on the the particle's equilibrium-level energy, expressed by the Lorentz factor:

$$\begin{cases} \nu_s^B &= \gamma G, \\ \nu_s^E &= \beta^2\gamma\left(\frac{1}{\gamma^2-1} - G\right) \\ &= \frac{G+1}{\gamma} - G\gamma. \end{cases} \tag{3}$$

Unfortunately, not all beam particles share the same Lorentz factor. A particle involved in betatron motion will have a longer orbit, and as a direct consequence of the phase stability principle, in an accelerating structure utilizing an RF cavity, its equilibrium energy level must increase. Otherwise it cannot remain the bunch. In this section we analyze how the particle Lorentz factor should be modified when betatron motion, as well as non-linearities in the momentum compaction factor are accounted for.

The longitudinal dynamics of a particle on the reference orbit of a storage ring is described by the system of equations:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}\Delta\varphi &= -\omega_{RF}\eta\delta, \\ \frac{\mathrm{d}}{\mathrm{d}t}\delta &= \frac{qV_{RF}\omega_{RF}}{2\pi h\beta^2 E}\left(\sin\varphi - \sin\varphi_0\right). \end{cases} \tag{4}$$

In the equations above, $\Delta\varphi = \varphi - \varphi_0$ and $\delta = (p - p_0)/p_0$ are the deviations of the particle's phase and normalized momentum from those of the reference particle; all other symbols have their usual meanings.

The solutions of this system form a family of ellipses in the (φ, δ)-plane, all centered at the point (φ_0, δ_0). However, if one considers a particle involved in betatron oscillations, and uses a higher-order Taylor expansion of the momentum compaction factor $\alpha = \alpha_0 + \alpha_1\delta$, the first equation of the system transforms into: [11, p. 2579]

$$\frac{\mathrm{d}\Delta\varphi}{\mathrm{d}t} = -\omega_{RF}\left[\left(\frac{\Delta L}{L}\right)_\beta + \left(\alpha_0 + \gamma^{-2}\right)\delta + \left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma^{-4}\right)\delta^2\right],$$

where $\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x Q_x + \varepsilon_y Q_y\right]$, is the betatron motion-related orbit lengthening; ε_x and ε_y are the horizontal and vertical beam emittances, and Q_x, Q_y are the horizontal and vertical tunes.

The solutions of the transformed system are no longer centered at the same single point. Orbit lengthening and momentum deviation cause an equilibrium-level momentum shift [11, p. 2581]

$$\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right], \tag{5}$$

where δ_m is the amplitude of synchrotron oscillations.

We call the equilibrium energy level associated with the momentum shift (5), the *effective Lorentz factor*:

$$\gamma_{eff} = \gamma_0 + \beta_0^2\gamma_0 \cdot \Delta\delta_{eq}, \tag{6}$$

where γ_0, β_0 are the Lorentz factor and relative velocity factor of the reference particle.

Observe, that the effective Lorentz factor enables us to account for variation in the value of spin tune due to variation in the particle orbit length. It is crucial in the analysis of spin decoherence [10] and its suppression by means of sextupole fields.

It plays a big role, as well, in the successfull reproduction of the MDM component to the total spin precession angular velocity.

GUIDE FIELD FLIPPING

Two aspects of the problem need to be paid attention to:

1. What needs to be kept constant from one measurement cycle to the next;
2. How it can be observed.

The goal of flipping the direction of the guide field is to accurately reproduce the radial component of the MDM spin precession frequency induced by machine imperfection fields. This point should not be overlooked: a mere reproduction of the *magnetic field strength* would not suffice, since the injection point of the beam's centroid, and hence its orbit length — and, via equations (6) and (3), spin tune, — is subject to variation. (Apart from that, the accelerating structure might not be symmetrical, in terms of spin dynamics, with regard to reversal of the beam circulation direction.)

What needs to be reproduced, therefore, is not the field strength, but the effective Lorentz factor of the centroid.

Regarding the second question, we mentioned earlier that the Koop Wheel roll rate is controlled through measurement of the horizontal plane spin precession frequency. This plane was chosen because the EDM angular velocity vector points (mainly) in the radial direction; its vertical component is due to machine imperfection fields, and is small compared to the measured EDM effect. Therefore, in first approximation, when we manipulate the vertical component of the combined spin precession angular velocity, we manipulate the vertical component of the MDM angular velocity vector.

Moving on to the effective Lorentz factor calibration procedure. Let $\mathcal{T}$ denote the set of all trajectories that a particle might follow in the accelerator. $\mathcal{T} = \mathcal{S} \bigcup \mathcal{F}$, where $\mathcal{S}$ is the set of all stable trajectories, $\mathcal{F}$ are all trajectories such that if a particle gets on one, it will be lost from the bunch.

Calibration is done in two phases:

1. In the first phase, the guide field value is set so that the beam particles are injected onto trajectories $t \in \mathcal{S}$.
2. In the second phase, it is fine-tuned further, so as to fulfill the FS condition in the horizontal plane. By doing this, we physically move the beam trajectories into the subset $\mathcal{S}|_{\omega_y=0} \subset \mathcal{S}$ of trajectories for which $\omega_y = 0$.

Spin tune (and hence precession frequency) is an injective function of the effective Lorentz-factor γ_{eff}, which means $\omega_y(\gamma_{eff}^1) = \omega_y(\gamma_{eff}^2) \to \gamma_{eff}^1 = \gamma_{eff}^2$. The trajectory space $\mathcal{T}$ is partitioned into equivalence classes according to the value of γ_{eff}: trajectories characterized by the same γ_{eff} are equivalent in terms of their spin dynamics (possess the same spin tune and invariant spin axis direction), and hence belong to the same equivalence class. Since $\omega_y(\gamma_{eff})$ is injective, there exists a unique γ_{eff}^0 at which $\omega_y(\gamma_{eff}^0) = 0$:

$$[\omega_y = 0] = [\gamma_{eff}^0] \equiv \mathcal{S}|_{\omega_y=0}.$$

If the lattice didn't use sextupole fields for the suppression of decoherence, $\mathcal{S}|_{\omega_y=0}$ would be a singleton set. We have shown in [10] that if sextupoles are utilized, then $\exists\mathcal{D} \subset \mathcal{S}$ such that $\forall t_1, t_2 \in \mathcal{D}$: $\nu_s(t_1) = \nu_s(t_2)$, $\bar{n}(t_1) = \bar{n}(t_2)$. By adjusting the guide field strength we equate $\mathcal{D} = \mathcal{S}|_{\omega_y=0}$, and hence $\mathcal{S}|_{\omega_y=0}$ contains multiple trajectories. [2]

Therefore, once we ensured that the beam polarization does not precess in the horizontal plane, all of the beam particles have γ^0_{eff}, equal for the CW and CCW beams.

Guide field flipping procedure simulation results can be found in [12].

STATISTICAL PRECISION

Members of the JEDI Collaboration have studied the statistical precision of spin precession angular velocity estimation from sparse (one detector event per 100 spin revolutions) [13] and dense [5] polarization data.

According to [13], the maximum likelihood estimator for the spin precession frequency estimate has a standard error

$$\sigma_{\hat{\omega}} = \frac{1}{PT}\sqrt{\frac{24}{N}},$$

where N is the total number of recorded detector events, P is the beam polarization, T is the measurement time.

Assuming $N = 7.5 \cdot 10^8$ events, polarization $P = 0.4$, and cycle duration $T = 1,000$ seconds (same parameters as in the simulation done in [5]), we have $\sigma_{\hat{\omega}} \approx 4.5 \cdot 10^{-7}$ rad/sec at the cycle level. Estimates made in [5] agree with this result.

This precision is sufficient to obtain a mean estimate with statistical uncertainty $\sigma_{\langle\hat{\omega}\rangle} \approx 3 \cdot 10^{-9}$ rad/sec in one year of measurement, with the accelerator operational 70% of the time. An EDM of 10^{-29} e·cm should induce an ω_{edm} on the level of 10^{-9} rad/sec in storage rings proposed in [8]. Thus, we expect to be able to measure the deuteron EDM at the 10^{-29} e·cm level in one year of measurement time.

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