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Statistical Error

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Methodology

When put into an electromagnetic field, the particle spin begins to precess according to the T-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m}\left\{ \underbrace{G\vec{B} + \left(\frac{1}{\gamma^2 - 1} - G\right)\left(\vec{\beta} \times \vec{E}\right)}_{\text{MDM}} + \underbrace{\frac{\eta}{2}\left(\vec{E} + \vec{\beta} \times \vec{B}\right)}_{\text{EDM}} \right\}$$

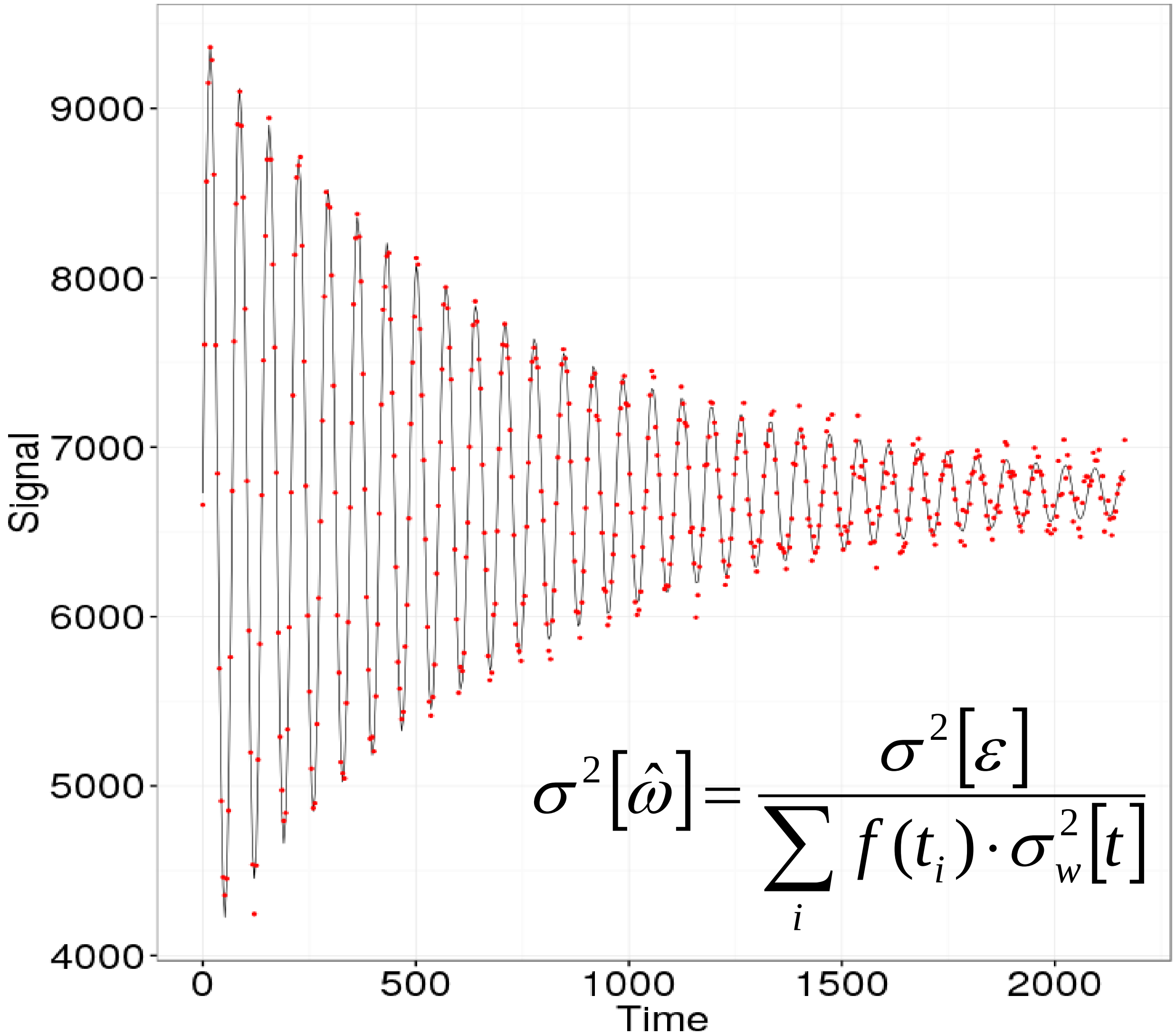
By measuring the beam's polarization, we can determine the frequency

$$\vec{\Omega}^{\pm} = \vec{\Omega}_{MDM} \pm \vec{\Omega}_{EDM}$$

Comparing the CW vs CCW frequencies, determine Ω_{EDM}

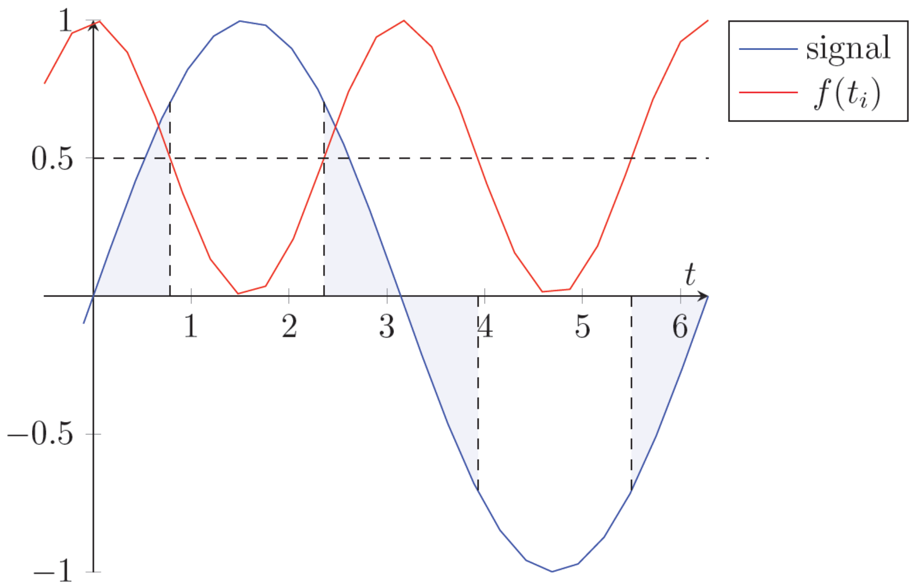
Problem statement

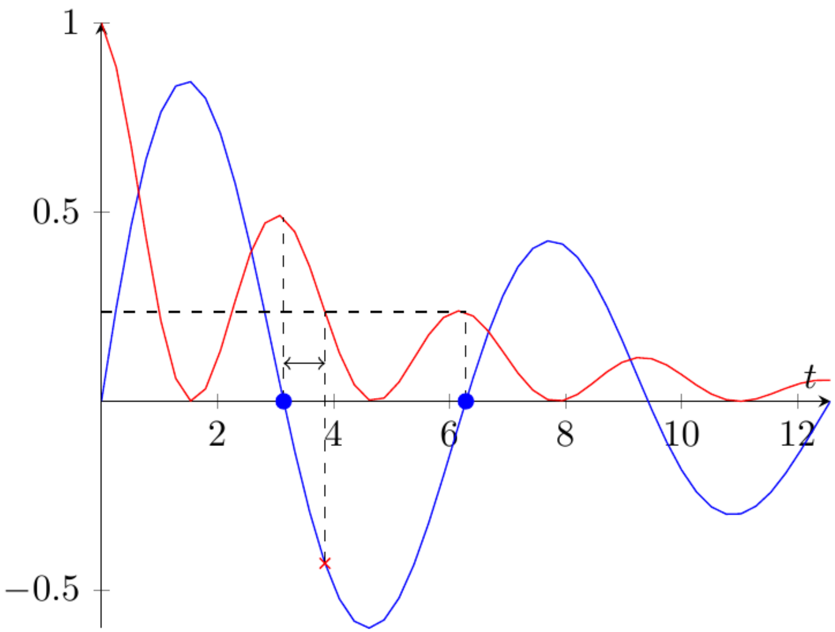
$$N(t) = N_0 \cdot \left(1 + P \cdot e^{-t/\tau_d} \cdot \sin(\omega \cdot t + \phi)\right) + \varepsilon_t$$



$$\sigma^2[\hat{\omega}] = \frac{\sigma^2[\varepsilon]}{\sum_i f(t_i) \cdot \sigma_w^2[t]}$$

Two aspects





Goals

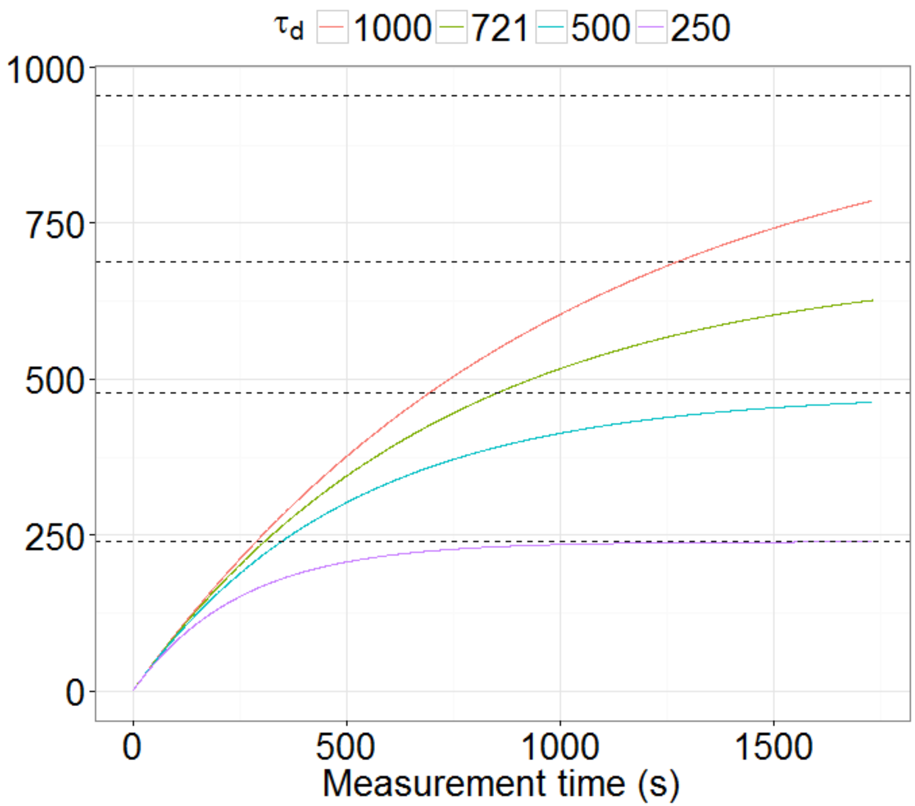
- Under what conditions sampling modulation is beneficial
- How much modulation is appropriate
- At what point measurement is no longer informative

Spread

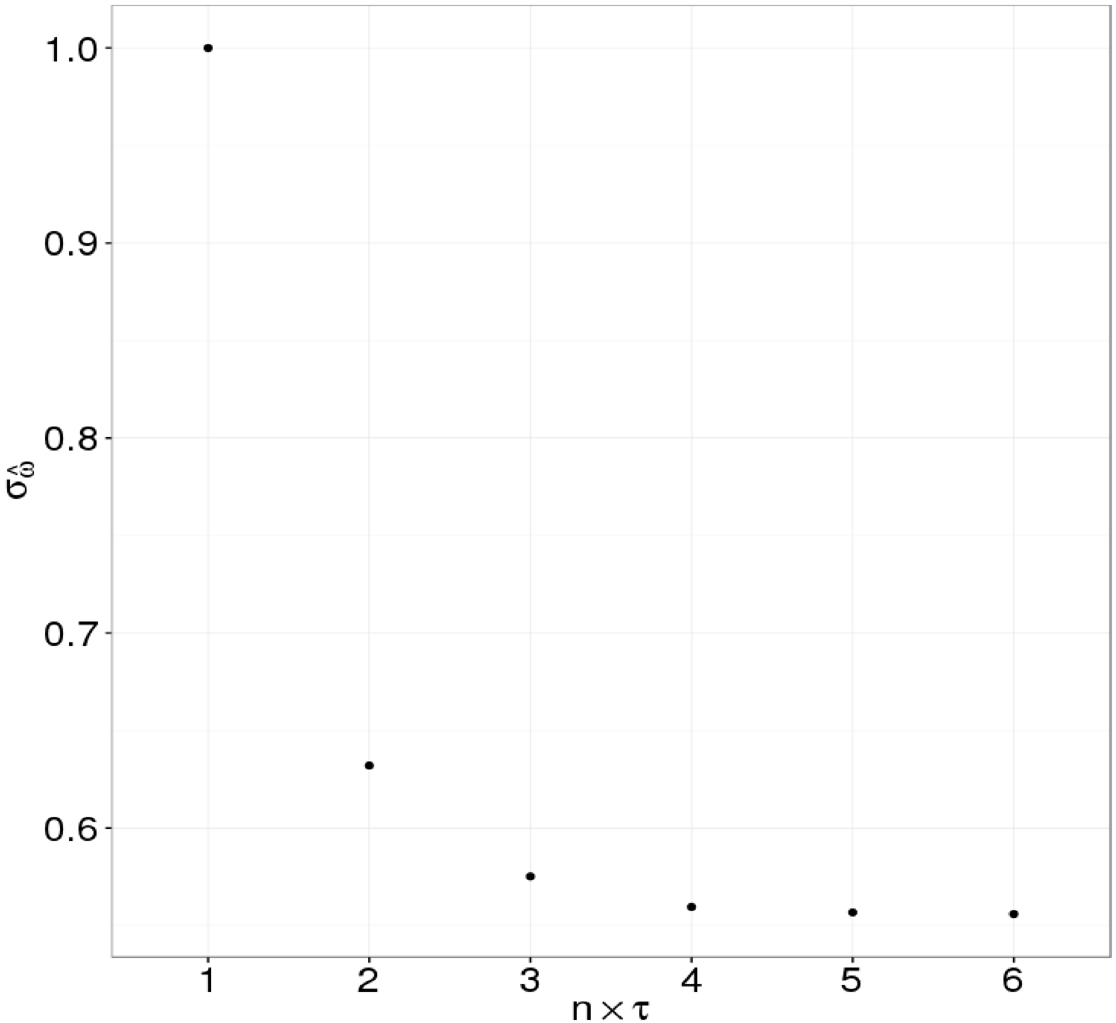
$$\sum f(t_i) = n_{\varepsilon/zc} \cdot x_{01} \cdot \frac{\exp\left(-\frac{\pi}{\omega\tau_d} n_{zc}\right)-1}{\exp\left(-\frac{\pi}{\omega\tau_d}\right)-1}$$

$$t(z) = \tau_d \cdot \ln\left(\frac{1}{1-z}\right)$$

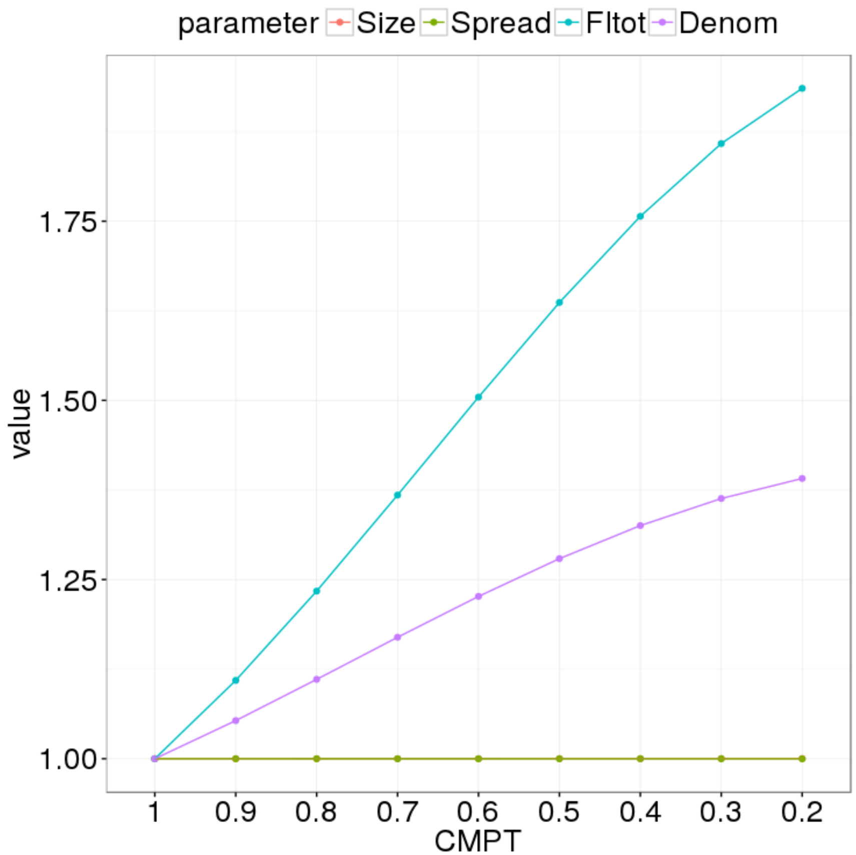
FI limit (%)	Reached (×τ_d)	SNR@3% error
95	3.0	1.7
90	2.3	3.3
70	1.2	10.0
50	0.7	16.5

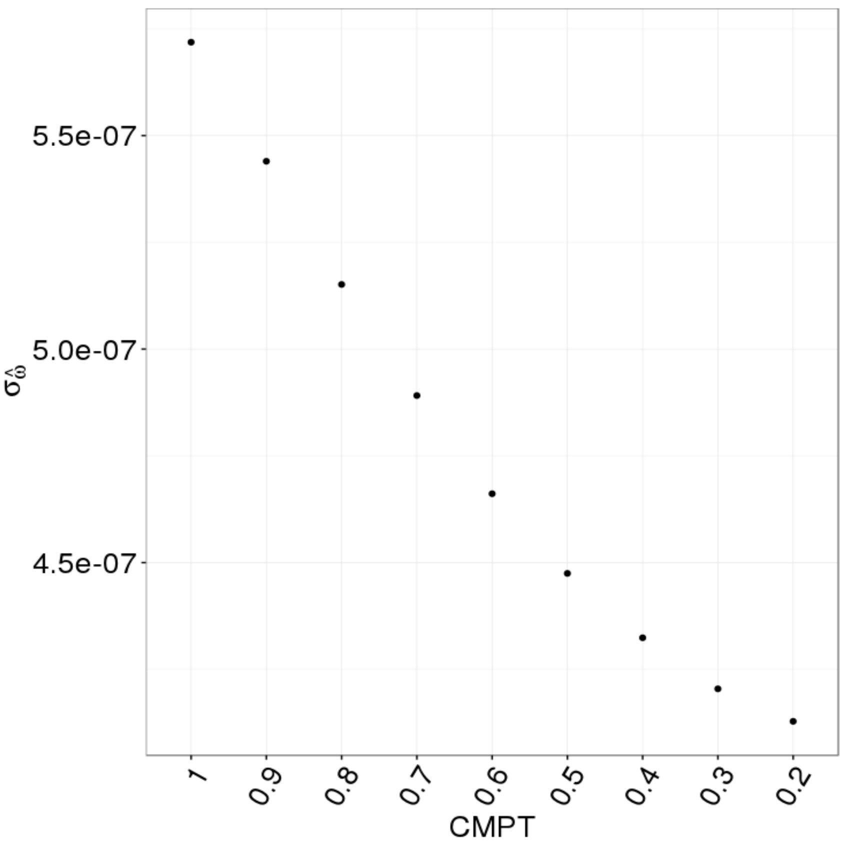


Simulation

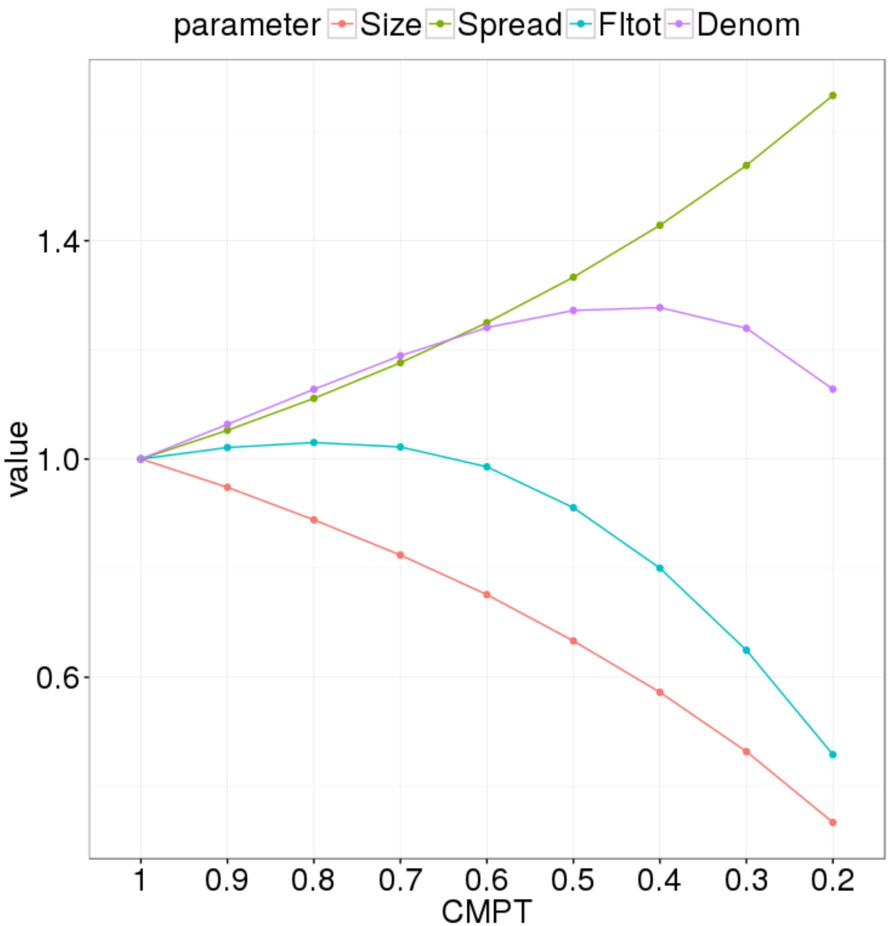


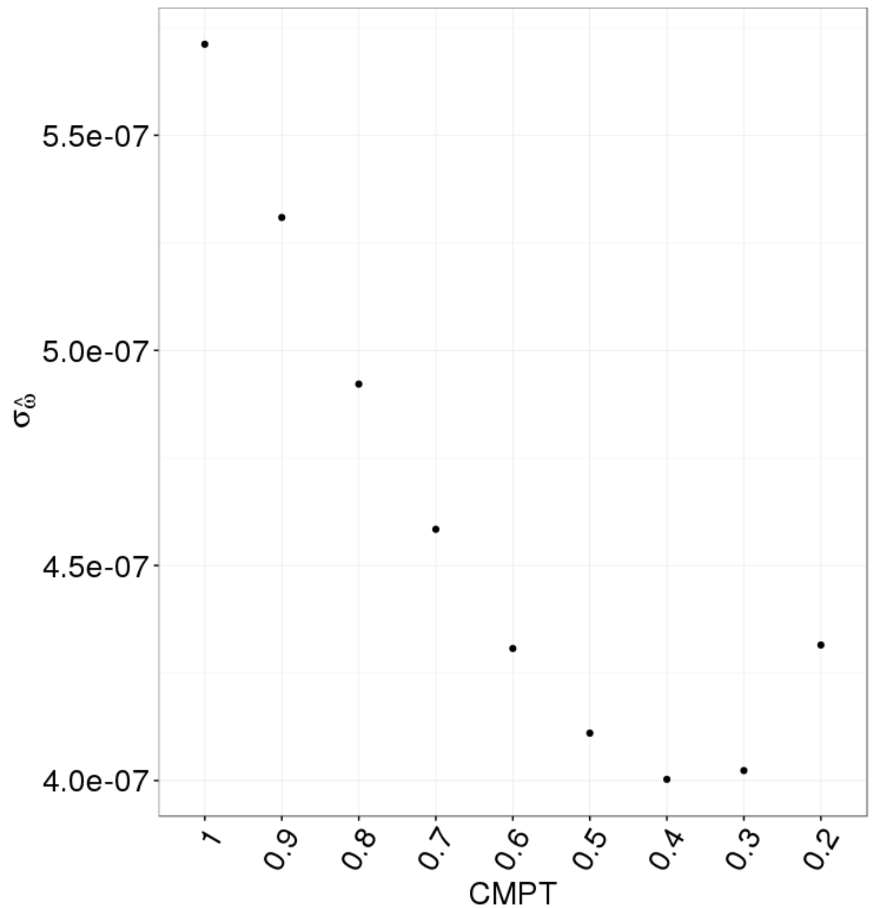
Modulation: Beam lifetime doesn't matter





Modulation: Beam lifetime does matter





Decoherence

$$S(t) = \sum_i \sin(\omega_i \cdot t + \phi_i)$$