Frequency Domain Methodto Search for the Deuteron Electric Dipole Moment in a Storage Ring Environment

Alexander Aksentev[e,f,g,*], Yury Senichev[f], Eremey Valetov[h]

[a] *Institut für Kernphysik (IKP-2), Forschungszentrum Jülich, Jülich, Germany*
[b] *Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia*
[c] *National Research Nuclear University "MEPhI," Moscow, Russia*
[d] *Department of Physics and Astronomy, Michigan State University, MI 48824, USA*

Abstract

*Corresponding author

Email addresses: alexaksentyev@gmail.com (Alexander Aksentev), y.senichev@inr.ru (Yury Senichev), eremey@valetov.com (Eremey Valetov)

Frequency Domain Methodto Search for the Deuteron Electric Dipole Moment in a Storage Ring Environment

Alexander Aksentev[e,f,g,*], Yury Senichev[f], Eremey Valetov[h]

[e]*Institut für Kernphysik (IKP-2), Forschungszentrum Jülich, Jülich, Germany*
[f]*Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia*
[g]*National Research Nuclear University "MEPhI," Moscow, Russia*
[h]*Department of Physics and Astronomy, Michigan State University, MI 48824, USA*

Keywords: electric dipole moment; storage ring; frozen spin; Frequency Domain Method.

Contents

1 Introduction **2**
1.1 General spin rotation matrices 3

2 Preliminary analytic of the Spin Wheel method **3**

3 Assumptions of the Spin Wheel method **6**

4 Argumument against the SW method **6**

1. Introduction

Spin rotations belong to the Spin(3) group, which is isomorphic to SU(2).

Rotations in SU(2). Rotation by angle ψ about direction $\bar{n}$

$$R_{\bar{n}}(\psi) = \exp\left[-i\frac{\psi}{2}(\bar{n}\cdot\vec{\sigma})\right],$$

where $\vec{\sigma}$ is the Pauli matrix vector.

*Corresponding author
Email addresses: alexaksentyev@gmail.com (Alexander Aksentev), y.senichev@inr.ru (Yury Senichev), eremey@valetov.com (Eremey Valetov)

1.1. General spin rotation matrices

Denote

- $(\Theta^{mi}, \bar{n}_{mi})$ from machine imperfections;
- $(\Theta^{+}, \bar{n}_{sol})$ for the $+\Delta$ solenoidal field;
- $(\Theta^{-}, -\bar{n}_{sol})$ for the $-\Delta$ solenoidal field.

$$R^{+\Delta} = \exp\left[-i\left(\frac{\Theta^{mi}}{2}(\bar{n}_{mi}\cdot\vec{\sigma}) + \frac{\Theta^{+}}{2}(\bar{n}_{sol}\cdot\vec{\sigma})\right)\right]$$

$$= \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} + \Theta^{+}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \tag{1}$$

$$R^{-\Delta} = \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} - \Theta^{-}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \tag{2}$$

2. Preliminary analytic of the Spin Wheel method

In SW we posit

$$\left(\vec{\Omega}^{+\Delta}_{MDM}\cdot\hat{x}\right) = -\left(\vec{\Omega}^{-\Delta}_{MDM}\cdot\hat{x}\right). \tag{3}$$

The spin precession angular velocity vector can be expressed via spin tune and invariant spin axis as

$$\vec{\Omega}_{spin} = \frac{2\pi}{\tau_{ring}}\cdot\nu\cdot\bar{n},$$

hence

$$\nu^{+\Delta}(\bar{n}_{+\Delta}\cdot\hat{x}) + \nu^{-\Delta}(\bar{n}_{-\Delta}\cdot\hat{x}) = 0 \tag{4}$$

From $\Delta\Theta = \tau\Delta\Omega$ and $\Delta\Omega^{MDM}_{x} = \frac{q}{m}GB_x$, and **assuming**

$$B^{\pm}_{sol}\tau_{sol} = \langle B^{\pm}_{sol}\rangle\tau_{ring}: \tag{5}$$

$$\begin{cases} \Theta^{+} & = \tau_{sol}\frac{q}{m}GB^{+}_{sol} \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B^{+}_{sol}\rangle, \\ \Theta^{-} & = \tau_{sol}\frac{q}{m}GB^{-}_{sol} \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B^{-}_{sol}\rangle. \end{cases} \tag{6}$$

Remark 1. Assumption (5) is **required** if we want to obtain $B^{\pm}_{sol}$ from equations of group (11).

From eqs (1) and (2):

$$\begin{cases} \Theta^{mi}\bar{n}_{mi} + \Theta^{+}\bar{n}_{sol} = \nu^{+\Delta}\bar{n}_{+\Delta}, \\ \Theta^{mi}\bar{n}_{mi} - \Theta^{-}\bar{n}_{sol} = \nu^{-\Delta}\bar{n}_{-\Delta}. \end{cases} \tag{7}$$

Substituting eq (7) into (4), and assuming $\bar{n}_{sol} = \hat{x}$:

$$2\Theta^{mi}(\bar{n}_{mi} \cdot \hat{x}) + (\Theta^{+} - \Theta^{-}) = 0. \tag{8}$$

Assuming[1]

$$\Theta^{mi} = \tau_{ring} \cdot \frac{q}{m} G \cdot \langle B_x \rangle^{mi}, \tag{9}$$

from (8) and (5) obtain:

$$2\langle B_x \rangle^{mi} + \left(\langle B^{+}_{sol} \rangle - \langle B^{-}_{sol} \rangle\right) = 0. \tag{10}$$

From eq (9) in Koop2015, assuming in the $+\Delta$ case the machine imperfections and solenoid fields are co-aligned, in the $-\Delta$ anti-aligned:

$$\begin{cases} \Delta^{+} & = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\langle B_x \rangle = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\left(\langle B_x \rangle^{mi} + \langle B^{+}_{sol} \rangle\right), \\ & \Rightarrow \langle B^{+}_{sol} \rangle = \frac{\langle G_z \rangle}{\beta_1 - \beta_2}\Delta^{+} - \langle B_x \rangle^{mi}; \\ \Delta^{-} & = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\langle B_x \rangle = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\left(\langle B_x \rangle^{mi} - \langle B^{-}_{sol} \rangle\right), \\ & \Rightarrow -\langle B^{-}_{sol} \rangle = \frac{\langle G_z \rangle}{\beta_1 - \beta_2}\Delta^{-} - \langle B_x \rangle^{mi}. \end{cases} \tag{11}$$

Substituting this into (10):

$$2\langle B_x \rangle^{mi} + \left(\frac{\langle G_z \rangle}{\beta_1 - \beta_2}\left[\Delta^{+} - \Delta^{-}\right] - 2\langle B_x \rangle^{mi}\right) = 0.$$

In the original method, we are to make

$$\Delta^{-} = -\Delta^{+}, \tag{12}$$

[1] This is a generous assumption implying that $\bar{n}_{mi} = \hat{x}$; i.e., this is **not** a non-commutativity-based argument; we assume all spin rotations commute.

so the term in the square brackets is zero, and we are left with

$$(1-1)\,\langle B_x\rangle^{mi} = 0. \tag{13}$$

So, seems that SW works, but we did two important assumptions here: (a) commutativity (in order to get eq (9)), and (b) "averaging" of B_{sol} over the ring (in order to get eq (5) and remove the τ_{sol}/τ_{ring} from (10)).

Remark 2. If we don't use (9) (but still use (5) in order to obtain $B^{\pm}_{sol}$ from group (11)), then eq (13) becomes

$$\Theta^{mi}\,(\bar{n}_{mi}\cdot\hat{x}) - \frac{q}{m}G\cdot\tau_{ring}\langle B_x\rangle^{mi} = 0, \tag{14}$$

which is not very informative.

Remark 3. To check that eq (14) is correct, assume (9). Then

$$\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}\,(\bar{n}_{mi}\cdot\hat{x}) - \frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi} = 0,$$

and hence

$$\bar{n}_{mi}\cdot\hat{x} = 1,$$

which is implied by machine imperfection spin rotations adding up commutatively.

Remark 4. In general, since

$$\Theta^{mi} = \tau_{ring}\cdot\frac{q}{m}G\sqrt{\langle B^{mi}_x\rangle^2 + \langle B^{mi}_y\rangle^2 + \langle B^{mi}_z\rangle^2},$$

eq (14) implies that

$$\begin{aligned}(\bar{n}_{mi}\cdot\hat{x}) &= \frac{\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}}{\Theta^{mi}}\\ &= \frac{\langle B^{mi}_x\rangle}{\sqrt{\langle B^{mi}_x\rangle^2 + \langle B^{mi}_y\rangle^2 + \langle B^{mi}_z\rangle^2}}.\end{aligned} \tag{15}$$

Which is correct.

Conclusion. In view of Remark 4, since eq (14) implies a valid statement, our conclusion is that the SW method resists the argument from non-commutativity.

$$
\begin{tikzcd}
(8) \arrow[r, "(9)+(5)"] \arrow[d, "(6)+(5)+(11)"] & (10) \arrow[r, "(11)"] & (13) \\
(14) & &
\end{tikzcd}
$$

Figure 1: Argument diagram.

3. Assumptions of the Spin Wheel method

Orbital dynamics. Koop2015 eq (7) (henceforth referred to as K(7)) and

$$\langle E_z \rangle = \langle E_z(0) \rangle + \langle G_z \rangle \cdot z \tag{K$\langle E_z \rangle$}$$

$$\to \langle z \rangle = \frac{\langle E_z(0) \rangle}{\langle G_z \rangle} - \frac{\beta}{\langle G_z \rangle} \cdot \langle B_x \rangle \tag{16}$$

$$\to \Delta = \frac{\beta_1 - \beta_2}{\langle G_z \rangle} \langle B_x \rangle. \tag{17}$$

This is as far as the argument from the non-linearity of the closed orbit shift dependence on the magnetic field is concerned. So long as we believe K(7) and K$\langle E_z \rangle$, we must believe K(9), and hence we cannot use that argument.

Spin dynamics. This is the argument from non-commutativity. For this argument cf. eq (14) and Remark 4, and the following conclusion.

4. Argumument against the SW method

The three-fold argument against the SW method is as follows (in the order of strength):

(1) The possibility of measuring the vertical orbit separation of two co-circulating beams at the sensitivity level of 10^{-12} m has not been shown by experiment. **Counter-argument**: there's reference [1] to commercially-available SQUIDs capable of detecting magnetic fields on the order of fT, which is equivalent to the beam separation of 10^{-12} m.

(2) Even if a SQUID-based BPM is capable of measuring orbit separation to such precision *locally*, the evaluation of the *mean* orbit separation requires multiple local measurements, and is not identical to the local measurement precision.

(3) Orbital and spin dynamics are idependent of each other, meaning that the observables $\vec{\Omega}$ and Δ are not directly related.

Regarding part (3) of the above argument: suppose the amplitude a_y of betatron oscillations is on the order of a micro-meter;

References

[1] D. Kawal, "Relative Beam Position Monitors for the pEDM Experiment." https://apps.fz-juelich.de/pax/paxwiki/images/a/a9/DKawal_longapp_dmk_20110621.pdf