

Quantum variability of polarization

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In this note I will be concerned with the effect of the quantum nature of spin on the EDM measurement. Spin is a quantum quantity, subject to probabilistic fluctuation. However, the expectation value of spin

$$\mathbb{E}[\vec{S}] = \langle \phi | (\sigma_x, \sigma_y, \sigma_z) | \phi \rangle,$$

behaves like a classical vector. (Ehrenfest's theorem. [1])

The polarization vector $\vec{P} = \sum_i \vec{s}_i / \|\sum_i \vec{s}_i\| = \langle \vec{S} \rangle$ is a finite-sample (beam number of particles) approximation of $\mathbb{E}[\vec{S}]$; it is an *average* statistic. As such, it naturally has an uncertainty $\sigma[\vec{P}]$ (standard deviation of the mean).

This uncertainty is due to the statistical/quantum nature of spin. It is not an epistemic uncertainty of our not knowing the value of \vec{P} ; if we could measure \vec{P} with an infinitely small error, we'd still observe that the value of \vec{P} varies. And it's not due to the variation of the experimental conditions. But it is due to the finite beam size; if we had an infinitely large number of beam particles, then $\vec{P} \equiv \langle \vec{S} \rangle \rightarrow \mathbb{E}[\vec{S}]$.

1 What this means for our EDM measurement

In the experiment, we measure the signal

$$P_y(t) = a \cdot \sin(\omega \cdot t + \phi_0).$$

Quantum statistical variation occurs at the level of a single $P_y(t_i)$ measurement. We can formulate it as follows:

$$P_y(\theta) = a \cdot \sin \theta,$$

and for all $\theta_i, \theta_j = \theta_i + 2\pi n$, due to quantum variability,

$$P_y(\theta_i) \neq P_y(\theta_j). \quad (1)$$

Suppose we have exactly non-destructive polarimetry. Not a single beam particle is lost. Suppose also that the spin coherence time is infinite; that all particles are on the closed orbit, there are no stray fields, experimental conditions are as perfect as we want, and unchanging.

We measure the vertical component of polarization with negligible uncertainty for a year. Then we split all that data into frames of 10,000 seconds, fit $f(t) = a \cdot \sin(\omega \cdot t + \phi_0)$, and get an $\hat{\omega}$ -estimate.

By hypothesis, the fit standard error $\sigma[\hat{\omega}]$ is negligibly small (but **finite**, b/c polarization measurements are based on a finite number of detector counts), say 10^{-20} [rad/sec].

If we plot a histogram of $\hat{\omega}$, its standard deviation will (very likely) exceed 10^{-20} by orders of magnitude (if not, then assume an even smaller $\sigma[\hat{\omega}]$). This is b/c every time a cycle is started, P_y follows a slightly different sinusoid, all b/c of equation (1).

2 How to deal with that

So long as the quantum standard deviation does not change the shape of the $\hat{\omega}$ distribution, the law of large numbers holds, and you deal with it just as you would with any other statistical uncertainty.

If it does change the shape of the distribution, then you have a fundamental problem of interpretation. But that aside, you may have to change the EDM-estimator statistic.

3 A big BUT

Apparently, this is not exactly true; according to this [2, p. 193], there's a fundamental certainty limit $\sigma[\hat{A}] = \sqrt{\mathbb{E}[\hat{A}^2] - \mathbb{E}[\hat{A}]^2} = \text{const}$ for observable \hat{A} .

Therefore, when we take a difference between $\hat{\omega}_1$ and $\hat{\omega}_2$, and that difference is greater than statistical uncertainty, classically, we'd say the difference is proportional to the EDM (assuming systematic uncertainty is negligibly small). But since $\hat{\omega}$ is a quantum-mechanical operator, it varies naturally due to the Heisenberg uncertainty principle. Then, the difference could be greater than statistical uncertainty even if the EDM is 0.

4 In summary

An estimate of the EDM $\hat{d} = \langle d \rangle \pm \sigma_{stat} \pm \sigma_{syst} \pm \sigma_{quant}$, where σ_{stat} is the statistical uncertainty (epistemic), σ_{syst} is the systematic (due to variation in experimental conditions, say) uncertainty (ontic), and σ_{quant} is the quantum-effect uncertainty (also ontic), due to the ‘‘Heisenberg principle.’’ To the best of my knowledge, it is interpreted as ontic, i.e. relating to the uncertainty in the actual physical state of the system under consideration.

The only situation in which that last uncertainty might be of concern, is when we managed to reduce (and prove that it is so)

$$\sqrt{\sigma_{stat}^2 + \sigma_{syst}^2} \ll \sigma_{quant}. \quad (2)$$

4.1 What does that mean?

If we managed to fulfill condition (2), then we would observe a normal (maybe not normal) distribution $d \sim N(E[d], \sigma_{quant})$. And we could not in principle give an estimate of d more precise than σ_{quant} . If $\sigma_{quant} < 10^{-29} \text{ e} \cdot \text{cm}$, we don't care; if it is greater, say 10^{-28} , then it's impossible to achieve the desired accuracy.

But really, what does it mean?

It just means that at the achieved level of experimental precision the EDM no longer belongs to the set of real numbers. It belongs to the set of probability distributions on real numbers:

$$d \notin \mathbb{R}, \quad d \in \mathcal{P}(\mathbb{R}).$$

Take natural numbers for example: in set theory, a natural number is a set. $\emptyset = 0$, $\{0\} = 1$, $\{0, 1\} = 2$, etc. However, when doing arithmetics, we don't think of numbers as sets. Up until the point where (2) is true, we can think of the electric dipole moment as a real number, but once there, this mathematical description is no longer valid, and its quantum mechanical nature manifests itself.

I don't think it is a problem. If our goal as experimentalists is to describe natural phenomena as accurately as possible, what could be more accurate than exposing the probabilistic nature of the quantity?

References

- [1] http://www.niser.ac.in/~sbasak/p303_2010/17_18.08.pdf
- [2] <http://physics.mq.edu.au/~jcresser/Phys301/Chapters/Chapter14.pdf>