Comparison of Frozen Spin-type EDM search methods

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Considered methods

- BNL Frozen Spin
- I.Koop's Spin Wheel
- Frequency Domain Method

BNL FS

- Observation of the vertical polarization component[1] $\Delta P_V \approx P \cdot \omega_{EDM} \cdot t$ (making it a Space Domain method)
- Cross section asymmetry $\varepsilon_{LR} \approx 5 \cdot 10^{-6}$ for smallest practical values of (horizontal plane) ω_{MDM}[2]

* Challenging task for polarimetry[3]

[1] **BNL:Deuteron2008**.
[2] **BNL:Deuteron2008**.
[3] **Mane:SpinWheel**.

BNL FS

Systematics

- Only known first-order systematic effect pertaining to the spin dynamics is the existence of $\langle E_V \rangle \neq 0$[4]
- Error frequency $\omega_{syst} \approx \frac{\mu \langle E_V \rangle}{\beta c \gamma^2}$ changes sign when reversing the beam circulation direction (CW/CCW)[5]
- However, at practical values of element alignment error, $\omega_{syst} \gg \omega_{EDM}$, hence $P_V = P\frac{\omega_{EDM}}{\omega}\sin(\omega t + \Theta_0) \not\approx P\omega_{EDM} t$; a Space Domain method is inapplicable under such conditions

* At $\langle E_V \rangle \to 0$, Space Domain methods are vulnerable to the geometric phase error[6]

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[4] **BNL:Deuteron2008**.
[5] **BNL:Deuteron2008**.
[6] **BNL:Proton**.

Geometric phase error

- Caused by the non-commutativity of rotations
- Formulated in the angular momentum language, it means the *absence of a definite orientation* of the spin precession axis (SPA): $\bar{n} \to 0$

* Call that the *3D Frozen Spin* state

- 3D FS is unstable: any stray magnetic field can tilt the precession plane

FS-type methodology

Conditions of success

- One must always have a definite direction of the SPA
- Measurements must be done in the frequency domain

These conditions are satisfied by two methods:

- I.Koop's "Spin Wheel"
- Y.Senichev's "Frequency Domain"

(Both of which belong to the Frequency Domain category.)

Spin Wheel

The Spin Wheel is great; it satisfies both success conditions.

- Apply a radial magnetic field of strength B_x sufficient to turn the spin vector about the $\hat{x}$-axis with a frequency of 1 Hz
- $\omega_{B_X} \parallel \omega_{EDM}$ hence $\omega_{net} \propto \omega_{EDM} + \omega_{B_X}$[7]
- EDM effect $\hat{\omega}_{EDM} = \frac{1}{2}\left[\omega_{net}(+B_X) + \omega_{net}(-B_X)\right]$
- Value of B_X is calibrated by measuring the vertical orbit splitting

[7] **Mane:SpinWheel**.

Spin Wheel

The good, the bad, the ugly

- Higher polarization growth rate greatly simplifies the task for polarimetry
- Magnetic field calibration by means of orbit split measurements seems unfeasible
- Element misalignment-induced error is not accounted for:

$$\begin{aligned}\hat{\omega}_{EDM} &= \frac{1}{2}\left(\omega_{EDM} + \cancel{\omega_{B_X}} + \omega_{mis} + \omega_{EDM} - \cancel{\omega_{B_X}} + \omega_{mis}\right) \\ &= \omega_{EDM} + \omega_{mis}\end{aligned}$$

Frequency Domain Method

This methodology has been developed specifically to deal with misalignment error.

- No reason to apply an external B-field; misalignment B_X-field provides a sufficiently fast wheel
- The FS condition ensures that $\omega_{net} \propto \omega_{EDM} + \omega_{mis}$
- The same EDM estimator $\hat{\omega}_{EDM} = \frac{\omega_{net}(+B_X)+\omega_{net}(-B_X)}{2}$
- To flip the sign of B_X one must reverse the guide field polarity (CW/CCW comeback)
- The value of B_X is calibrated via horizontal plane precession frequency

Thank you!

Doubly-magic ring

Fundamental assumptions

1. Both beams are at Frozen Spin: $\omega = \omega_X = \omega_{EDM} + \omega_{\langle B_r \rangle}$
2. EDM of the secondary beam $\ll$ EDM of the primary beam: $\omega^{PRI}_{EDM} \gg \omega^{SEC}_{EDM} \to 0 \Rightarrow \omega^{SEC}_X \approx \omega^{SEC}_{\langle B_r \rangle}$;
3. Beams on the same design orbit $\Leftrightarrow$ experience same fields: $\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC}$

* MDM's of both beams are known to high precision (what for?)

** Assumption 1 is formulated in the simplest form (we'll address that).

D-M Ring

Addressing initial objections

- Precession frequency difference (given 2): $\omega_X^{PRI} - \omega_X^{SEC} \approx \omega_{EDM}^{PRI} + \omega_{\langle B_r \rangle}^{PRI} - \omega_{\langle B_r \rangle}^{SEC}$

Objection (to assumption 3): The beams have different mass $\Leftrightarrow$ $\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC} \not\Rightarrow \omega_{\langle B_r \rangle}^{PRI} = \omega_{\langle B_r \rangle}^{SEC}$

- Using the Koop Wheel, $\omega_X^{SEC} = 0 = \omega_{\langle B_r \rangle}^{SEC} \Rightarrow \langle B_r \rangle^{SEC} = 0$ (again require 2)
- Given the design orbit is shared by both beams, $\omega_{\langle B_r \rangle}^{PRI}$ is also 0, b/c $\forall m, \gamma, G \left[\omega_{\langle B_r \rangle} = \frac{q}{m} G \langle B_r \rangle = 0 \Leftrightarrow \langle B_r \rangle = 0\right]$
- Sameness of the design orbits is guaranteed by the equation: $p^4 - 2\mathcal{B}p^3 + (\mathcal{B}^2 - \mathcal{E}^2)p^2 - \mathcal{E}^2 m^2 = 0$, where $\mathcal{B} = qcB_0 r_0$, $\mathcal{E} = qE_0 r_0$, (E_0, B_0, r_0) are defined by the primary beam FS condition

D-M Ring

Fundamental flaw

- But by nulling $\omega_{\langle B_r \rangle}^{PRI/SEC}$ we go to the unstable 3D FS state
- Which also forces us back to the Space Domain, since $\omega_X^{PRI} \approx \omega_{EDM}^{PRI} \ll 1$
- Thus, both the FS success conditions are violated

Concl'n D-MR solves the machine imperfection fields problem, but, other than that, inherits all of the original BNL FS weaknesses

D-M Ring

Let's go back to Assumption 1

- Our formulation of Assumption 1 as $\omega_X = \omega_{EDM} + \omega_{\langle B_r \rangle}$ is unrealistic: the existence of $\langle B_r \rangle$ must cause $\langle E_v \rangle$, since we have a closed orbit
- So really, it should be

$$\omega_X = \omega_{EDM} + \omega_{MDM}(\langle B_r \rangle + \langle E_v \rangle),$$
$$\omega_{MDM} = \frac{q}{m}\left[G\langle B_r \rangle + a(\gamma, G)\beta\langle E_v \rangle\right]$$

- Still, we have the system

$$\begin{cases} c\beta\langle B_r \rangle + \langle E_v \rangle & = 0, \\ G\langle B_r \rangle + a\beta\langle E_v \rangle & = 0 \end{cases}$$

  w/solution $(\langle B_r \rangle, \langle E_v \rangle) = (0, 0)$, and the defense argument holds up

Universal SR EDM measurement problems

And their canonical solutions

Solved by Spin Wheel

- Stray fields
- Betatron motion

* Both cause variation of $\bar{n}$

Solved elsewise

- Spin decoherence

Sol'n : Sextupole fields

- Machine imperfections

Sol'n : CW/CCW injection