SPIN MOTION PERTURBATION EFFECT ON THE EDM STATISTIC IN THE FREQUENCY DOMAIN METHOD

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Abstract

The spin precession axis of a particle involved in betatron motion precesses about the invariant spin axis defined on the closed orbit (CO). This precession can be observed in polarization data as a rapid, small-amplitude oscillation on top of the major effect oscillation caused by the precession of spin about the CO axis. The frequency of this latter oscillation is used in the Frequency Domain methodology as the EDM observable. It is estimated by fitting polarimetry data by a sine function; the rapid oscillations, therefore, constitute a model specification error. This model error will introduce a bias into the frequency estimate. In the present work we investigate how this bias changes depending on the beam revolution direction, its stability over time, and the EDM estimate error introduced by it.

FREQUENCY DOMAIN METHODOLOGY

Frequency Domain (FD) [1] is a Storage Ring method of search for the Electric Dipole Moment (EDM) of a fundamental particle. [2] It belongs to the Frozen Spin [3] category of such methods, i.e., the Magnetic Dipole Moment (MDM) component of spin precession is minimized. However, the original Frozen Spin method proposed in [3] is a Space Domain method [4, p. 4]: the EDM is estimated from the value P_y of the vertical component of the beam polarization vector at the end of the measurement cycle. This approach has two obvious weaknesses: *a*) it puts very stringent constraints on the precision of the accelerator optical elements alignment, and *b*) it poses a challenging task for polarimetry. [5, p. 6]

The former is to minimize the magnitude of the vertical plane MDM precession frequency: [3, p. 11]

$$\omega_{syst} \approx \frac{\mu\langle E_v\rangle}{\beta c\gamma^2}, \tag{1}$$

induced by field imperfections. The latter is due to the requirement of detecting a change of about $5\cdot10^{-6}$ to the cross section ε_{LR} in order to get to the EDM sensitivity level of 10^{-29} $e\cdot cm$. [3, p. 18]

EDM search methods in the Frequency Domain circumvent the above problems: the polarization vector is made to rotate about a nearly-constant, definite direction vector $\bar{n}$, with a frequency that is large enough so that the beam polarization is easily measureable at all times. This "Spin Wheel" may be externally applied [6], or it may use the field imperfection frequency (1) [1]. The latter is made possible by the fact that ω_{syst} changes sign when the beam revolution direction is reversed. [3, p. 11]

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The frequency of oscillation of P_y is estimated via a fit of the polarimetry data to the model

$$f(t) = a\cdot\sin(\omega\cdot t+\delta). \tag{2}$$

PROBLEM STATEMENT

Consider the case of a single particle beam. The solution of the T-BMT equation for the vertical spin-vector component has the general form

$$s_y(t) = \sqrt{\left(\frac{\omega_y\omega_z}{\omega^2}\right)^2+\left(\frac{\omega_x}{\omega}\right)^2}\cdot\sin\left(\omega\cdot t+\delta\right), \tag{3}$$

where $\boldsymbol{\omega} = (\omega_x,\omega_y,\omega_z)$ is a function of time as a result of betatron motion.

Using $\boldsymbol{\omega} = 2\pi f_{rev}\nu_s\bar{n}$ [7, p. 4], equation (3) can be reformulated in terms of spin tune ν_s and invariant spin axis $\bar{n}$:

$$s_y(n_{turn}) = \sqrt{\left(\bar{n}_y\bar{n}_z\right)^2+\bar{n}_x^2}\cdot\sin\left(2\pi\nu_s\cdot n_{turn}+\delta\right), \tag{4}$$

where $\bar{n} = \bar{n}(n_{turn})$ and $\nu_s = \nu_s(n_{turn})$ are functions of the turn number n_{turn}.

Sufficiently large variation of $\bar{n}$ and/or ν_s can lead to model specification systematic error. Variation in ν_s is especialy problematic in this regard, as it directly affects the phase of the signal; however, this problem can be solved by the introduction of sextupole fields into the system, as described in [8]. In this paper we will, therefore, be concerned only with the $\bar{n}$ variation.

SIMULATION

The simulation setup was as follows: an array of particles, offset from the design orbit in the vertical direction, is injected into an imperfect Frozen Spin lattice [9] utilizing sextupoles for the reduction of spin decoherence caused by vertical plane betatron oscillations [8]. Lattice imperfections are simulated by rotations of the E+B spin rotator elements, randomly generated from the normal distribution $\alpha\sim N(0,5\cdot10^{-4})$ rad. The value of the standard deviation approximates the practically-achievable element installation error. [1] Imperfections introduced this way do not perturb the design orbit.

Tracking was done in COSY Infinity [10], for $1.2\cdot10^6$ turns; each 800 turns ν_s and $\bar{n}$ are computed (by means of procedure TSS [11, p. 41]) at the phase space point occupied by the particle at the time, giving us a series $(\nu_s(n),\bar{n}(n))$. The corresponding spin vector components $(S_X(n),S_Y(n),S_Z(n))$, computed by the COSY Infinity tracker (procedure TR [11, p. 41]), constitute the second series used in the analysis.

Using the first series data, we generated the expected $s_y^{gen}(t)$ "generator" series according to equation (4), as well as the "ideal" series s_y^{idl}; in the latter case, ν_s and $\bar{n}$ assumed constant values of $\nu_s = \langle \nu_s \rangle$ and $\bar{n} = \langle \bar{n} \rangle$.

SIMULATION RESULTS

The three spin component time series were analyzed via residual functions $\epsilon_1 = S_Y^{gen} - s_y^{idl}$ and $\epsilon_1 = S_Y^{trk} - s_y^{idl}$, as well as by fitting model (2) and comparing its goodness-of-fit.

The residuals ϵ_1 and ϵ_2 are plotted in Figure 1. From Figure 1a one can observe that the generator series nearly approximates the ideal, constant parameter sinusoid, whereas Figures 1b and **??** indicate that the tracker s_y^{trk}-component performs oscillations relative to s_y^{idl} with an amplitude $\approx 10^{-4}$, and frequency

Table 1: Parameter estimates table

Series	Par.	Value	St.Error	AIC
s_y^{idl}	$\hat{f}$	74.452466549766214	$6 \cdot 10^{-15}$	-86246
	$\hat{a}$	0.99841729771960	$1 \cdot 10^{-14}$	
	$\hat{\delta}$	3.14159265358978	$2 \cdot 10^{-14}$	
s_y^{gen}	$\hat{f}$	74.452466548	$1 \cdot 10^{-9}$	-49917
	$\hat{a}$	0.998417300	$2 \cdot 10^{-9}$	
	$\hat{\delta}$	3.1415926564	$4 \cdot 10^{-9}$	
s_y^{trk}	$\hat{f}$	74.4524675	$6 \cdot 10^{-7}$	-30665
	$\hat{a}$	0.998418	$1 \cdot 10^{-6}$	
	$\hat{\delta}$	3.141589	$3 \cdot 10^{-6}$	

CONCLUSIONS

The question of the influence of betatron motion on the EDM statistic in the FD method should be considered in view of three circumstances:

1. The signal amplitude oscillations are small. They occur at the 10^{-4} level, whereas the expected polarization measurement error is on the order of percents. This means the superposition of this systematic error with the random measurement error will exhibit no statistically-significant systematicity.

2. The correllation coefficient between the amplitude and frequency estimates is not significant. The amplitude oscillations affect the $\hat{a}$-estimate foremost; their effect on the $\hat{\omega}$-estimate is secondary, and is described by the correlation coefficient. Since it is less than 10‰, even if the oscillations happen to be strong enough to affect the amplitude estimate, their effect on the frequency estimate will be reduced by at least a factor of 10.

3. This systematic effect is controllable. And this point is the major advantage of the FD methodology. By applying an external Spin Wheel, the $\bar{n}$ oscillations can be continuously minimized as much as necessary, without changing the experiment pattern.

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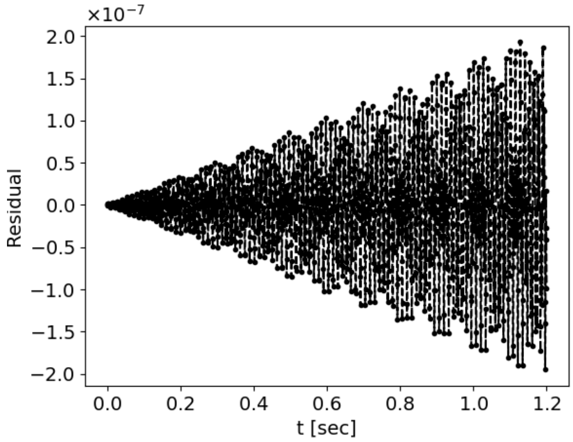
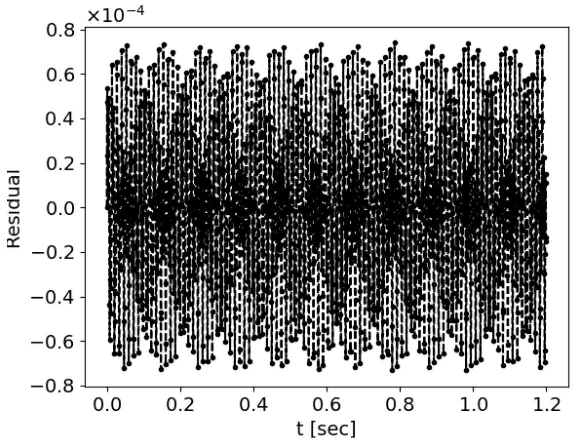
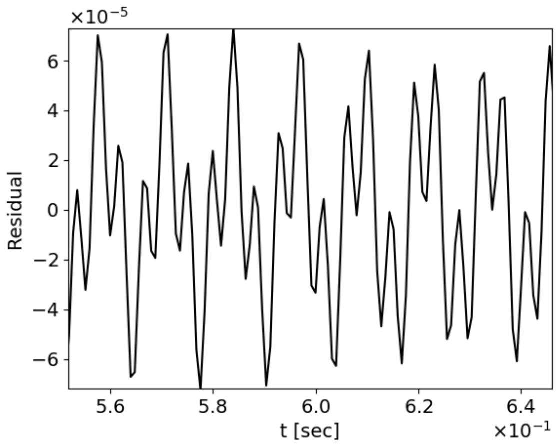


(a) $\epsilon_1 = s_y^{gen} - s_y^{idl}$ residual



(b) $\epsilon_2 = s_y^{trk} - s_y^{idl}$ residual



(c) Section of Figure 1b

Figure 1: Time series' comparator residuals