SIMULATION OF THE GUIDE FIELD FLIPPING PROCEDURE FOR THE FREQUENCY DOMAIN METHOD

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Abstract

The spin vector of a particle injected into a perfectly aligned storage ring precesses about the vertically-orientated guide field. In the presence of an Electric Dipole Moment (EDM), the spin precession axis acquires a proportional radial component. However, in an imperfect ring, rotational magnet misalignments induce a radial component to the spin precession axis, related to the Magnetic Dipole Moment (MDM). In the Frequency Domain Method, this additional precession is dealt with by consecutively injecting the beam in opposite directions, and constructing the EDM estimator as the sum of the clockwise and counter-clockwise vertical plane precession frequencies. Since the radial MDM component changes sign when the magnetic field direction is reversed, it cancels in the sum, leaving only the EDM effect. In order to reproduce the guide field magnitude with a precision sufficient for the cancellation of the MDM effect, we propose to calibrate the guide field via the horizontal plane precession frequency. In the present work we describe the algorithm of the field flipping procedure, and do a numerical simulation.

SPIN DYNAMICS IN A STORAGE RING

The dynamics of a spin-vector s in a magnetic field B and an electrostatic field E is described by the Thomas-BMT equation. Its generalized version, accounting for the effect of the particle's electric dipole moment, can be written in the rest frame as:

$$\frac{ds}{dt} = s \times (\mathbf{\Omega}_{MDM} + \mathbf{\Omega}_{EDM}), \qquad (1a)$$

where the magnetic (MDM) and electric (EDM) dipole moment angular velocities $\mathbf{\Omega}_{MDM}$ and $\mathbf{\Omega}_{EDM}$

$$\mathbf{\Omega}_{MDM} = \frac{q}{m}\left[GB - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{E \times \beta}{c}\right], \qquad (1b)$$

$$\mathbf{\Omega}_{EDM} = \frac{q}{m}\frac{\eta}{2}\left[\frac{E}{c} + \beta \times B\right]. \qquad (1c)$$

In the above equations, m, q, $G = (g-2)/2$ are, respectively, the mass, charge, and anomalous magnetic moment of the particle; $\beta = v_0/c$ is its normalized speed; γ its Lorentz-factor. The EDM factor η is defined by the equation $d = \eta\frac{q}{2mc}$, in which d is the particle EDM, s its spin.

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BNL FROZEN SPIN METHOD

The original method for the measurement of the electric dipole moment of an elementary particle was first proposed by the Storage Ring EDM Collaboration [1] of Brookhaven National Laboratory. In the proposed method, [2] a longitudinally-polarized beam is injected into a storage ring designed on the basis of the Frozen Spin (FS) concept: by applying a radial electric field $E_r = \frac{GB_y c\beta\gamma^2}{1 - G\beta^2\gamma^2}$, [2, p. 10] the MDM component in (1a) is set to zero: $\mathbf{\Omega}_{MDM} = \mathbf{0}$. Then, any tilting of the beam polarization vector out of the horizontal plane is attributed to the presence of an EDM; specifically, the vertical component P_y grows as

$$P_y = P\frac{\Omega_{edm}}{\Omega}\sin(\Omega t + \Theta_0) \approx P\Omega_{EDM} \cdot t,$$

where $\Omega = \sqrt{\Omega_{EDM}^2 + \Omega_{MDM}^2}$. [2, p. 8]

This method has two inherent weaknesses, due to the smallness of the hypothesized EDM value: *a*) the expected polarization tilt angle after 1,000 seconds is on the order of microradians, [2, p. 18] which makes for difficult polarimetry [3, p. 6] and *b*) the main systematic effect, $\Omega_{syst} \approx \frac{\mu\langle E_y\rangle}{c\beta\gamma^2}$ (μ being the MDM of the particle) [2, p. 10] must be reduced to less than Ω_{EDM} if one is to measure the polarization tilt angle. The systematic error is caused by accelerator element alignment error. For a practical value of $100\mu m$ of element installation uncertainty, this means a Ω_{syst} on the order of 50 rad/sec. [4]

Both these problems can be mitigated if the net spin precession frequency is used as the EDM observable.

FREQUENCY DOMAIN METHOD

The Frequency Domain Methodology (FDM) [4] was designed specifically to address the problem of element alignment uncertainty. The FS condition condition is fulfilled as in the BNL method; however, instead of the polarization tilt angle, the combined MDM+EDM precession frequency is measured in two cases: once when the beam is injected clockwise, and once counter-clockwise. The EDM-effect is extracted by comparing the measured frequencies. When the guide field polarity is reversed $B \mapsto -B$, $\beta \mapsto -\beta$, and $E \mapsto E$, precession frequency components change thus:

$$\Omega_x^{CW/CCW} = \Omega_x^{MDM,CW/CCW} + \Omega_x^{EDM,CW/CCW},$$
$$\Omega_x^{MDM,CW} = -\Omega_x^{MDM,CCW} \equiv \Omega_x^{MDM},$$
$$\Omega_x^{EDM,CW} = \Omega_x^{EDM,CCW} \equiv \Omega_x^{EDM}, \qquad (2a)$$

and the EDM estimator

$$\hat{\Omega}_x^{EDM} := \frac{1}{2}\left(\Omega_x^{CW} + \Omega_x^{CCW}\right) \tag{2b}$$

$$= \Omega_x^{EDM} + \underbrace{\frac{1}{2}\left(\Omega_x^{MDM,CW} + \Omega_x^{MDM,CCW}\right)}_{\varepsilon \to 0}. \tag{2c}$$

In order to guarantee that ε is less than the required EDM measurement precision, i.e. that equation (2a) holds with sufficient accuracy, a guide field flipping algorithm has been devised, that uses the horizontal plane precession frequency as a means to calibrating the guide field.

CALIBRATION ALGORITHM

The goal of flipping the direction of the guide field is to accurately reproduce the radial component of the MDM spin precession frequency due to element misalignment.

Let $\mathcal{T}$ denote the set of all trajectories that a particle might follow in the accelerator. $\mathcal{T} = \mathcal{S} \bigcup \mathcal{F}$, where $\mathcal{S}$ is the set of all stable trajectories, $\mathcal{F}$ are all trajectories such that if a particle gets on one, it will be lost from the bunch.

Calibration is done in two phases:

1. In the first phase, the guide field value is set so that the beam particles are injected onto trajectories $t \in \mathcal{S}$.

2. In the second phase, it is fine-tuned further, so as to fulfill the FS condition in the horizontal plane: by doing this, we select the subset $\mathcal{S}|_{\Omega_y=0} \subset \mathcal{S}$ of trajectories for which $\Omega_y = 0$.

Spin tune (and hence precession frequency) is an injective function of the effective Lorentz-factor γ_{eff} [5], which means $\Omega_y(\gamma_{eff}^1) = \Omega_y(\gamma_{eff}^2) \to \gamma_{eff}^1 = \gamma_{eff}^2$. The trajectory space $\mathcal{T}$ is partitioned into equivalence classes according to the value of γ_{eff}: trajectories characterized by the same γ_{eff} are equivalent in terms of their spin dynamics (possess the same spin tune and invariant spin axis direction), and hence belong to the same equivalence class. Since $\Omega_y(\gamma_{eff})$ is injective, there exists a unique γ_{eff}^0 at which $\Omega_y(\gamma_{eff}^0) = 0$:

$$[\Omega_y = 0] = [\gamma_{eff}^0] \equiv \mathcal{S}|_{\Omega_y=0}.$$

If the lattice didn't use sextupole fields for the suppression of decoherence, $\mathcal{S}|_{\Omega_y=0}$ would be a singleton set. We have shown in [5] that if sextupoles are utilized, then $\exists \mathcal{D} \subset \mathcal{S}$ such that $\forall t_1, t_2 \in \mathcal{D}: \nu_s(t_1) = \nu_s(t_2), \bar{n}(t_1) = \bar{n}(t_2)$. By adjusting the guide field strength we equate $\mathcal{D} = \mathcal{S}|_{\Omega_y=0}$, and hence $\mathcal{S}|_{\Omega_y=0}$ contains multiple trajectories. [1]

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[1] Strictly speaking, even if sextupoles are used there remains some negligible dependence of spin tune on the particle orbit length (linear decoherence effects, cf. [5]). Because of that, the equalities for spin tune and $\bar{n}$ are approximate, and the set $\mathcal{S}|_{\Omega_y=0}$ should be viewed as fuzzy: we will consider trajectories for which $|\dot{\Omega}_y| < \delta$ for some small δ as belonging to $[\Omega_y = 0]$.

Therefore, once we ensured that the beam polarization does not precess in the horizontal plane, all of the beam particles have γ_{eff}^0, equal for the CW and CCW beams.

SIMULATION

In order to confirm that the proposed calibration procedure works, we need to show that:

1. $\mathcal{S}|_{\Omega_y=0}^{CW} = \mathcal{S}|_{\Omega_y=0}^{CCW}$, that is $\Omega_y = 0$ for the same set of trajectories (equivalently, the same γ_{eff}) in the CW and CCW cases.

2. $\forall t_1, t_2 \in \mathcal{S}|_{\Omega_y=0}^{CCW}: \nu_s(t_1) = \nu_s(t_2), \bar{n}(t_1) = \bar{n}(t_2)$, i.e., the same sextupole fields reduce decoherence in the CW and CCW beams.

We do this by first computing the function $\nu_s(z)$ (where z is the particle's horizontal, vertical, or momentum offset from the reference particle) for the CW and CCW beams; and then compute their discrepancy $\epsilon(z) = \nu_s^{CW}(z) - \nu_s^{CCW}(z)$. If the discrepancy is small in a wide range of z, then

1. sextupole decoherence suppression works for both beams without gradient value change;

2. spin tune (respectively γ_{eff}) is equal for both beams, and hence their Spin Wheels roll at the same rate.

In the simulation, we use an imperfect FS lattice [6], in which the E+B spin rotator elements are tilted about the optic axis by angles $\alpha \sim N(0, 5 \cdot 10^{-4})$ radians. The simulation is repeated three times; each time only one sextupole family is turned on.

The beam's kinetic energy is 270.00 MeV. We compute third-order Taylor expansions of the spin and orbital transfer maps.

The main body of the simulation consists in the following: using the TSS [7, p. 41] procedure of COSY Infinity we compute the ν_s and $\bar{n}$ third-order Taylor expansions for the lattice traversed in the forward direction. Then, using the combinations of procedures MR and SMR [8, p. 233], we reverse the lattice's orbital and spin transfer maps, and compute ν_s and $\bar{n}$ for the reversed lattice (as it is seen by the counter-circulating beam).

RESULTS

In Fig. 1 are shown the ν_s and $\bar{n}_y$ dependencies on the particle's vertical offset from the reference orbit, when the corresponding sextupoles are operational. Specifically, in Fig. 1a one can observe that there's a difference between the CW and CCW beams in the values of spin tune and invariant spin axis vertical component for trajectories deviating from the reference orbit. Figure 1b indicates that if one can make the difference between the CW and CCW particle's horizontal plane precession frequencies smaller than 10^{-7} rad/sec, the difference between their vertical plane precession frequencies will be smaller than 10^{-8} rad/sec. This confirms that the equalization of the vertical plane
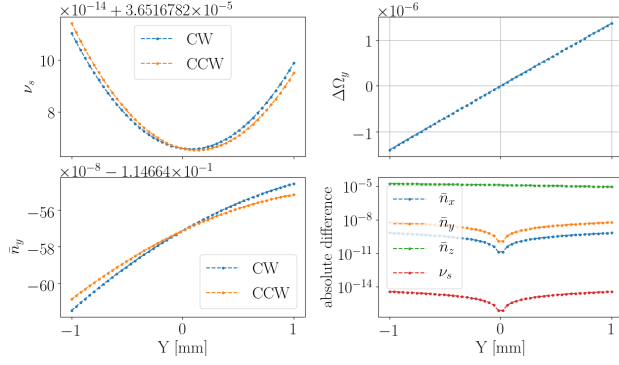
(a) Spin tune and invariant spin axis dependencies on the particle vertical offset from the reference orbit.
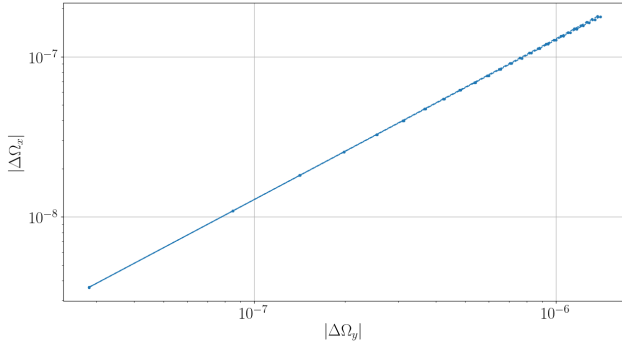
(b) Difference between the CW and CCW radial plane precession frequencies vs the corresponding horizontal plane frequencies (calibration plot).
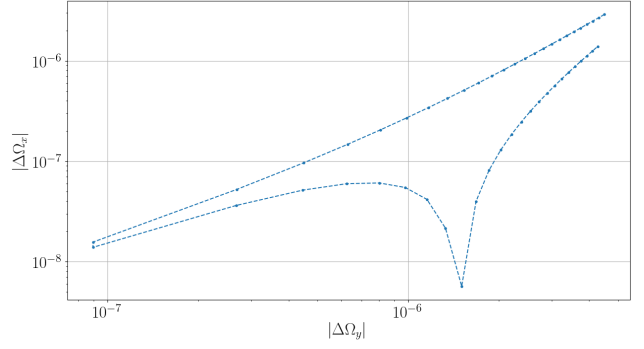
Figure 1: Simulation results for the decoherence caused by vertical plane betatron motion.

(a) Horizontal betatron motion case.

(b) Synchrotron motion case.

Figure 2: Calibration plots in the cases of decoherence caused by horizontal betatron motion and synchtorton oscillations.

MDM precession frequencies of counter-circulating beams by means of equalizing their horizontal plane precession frequencies is a viable technique.
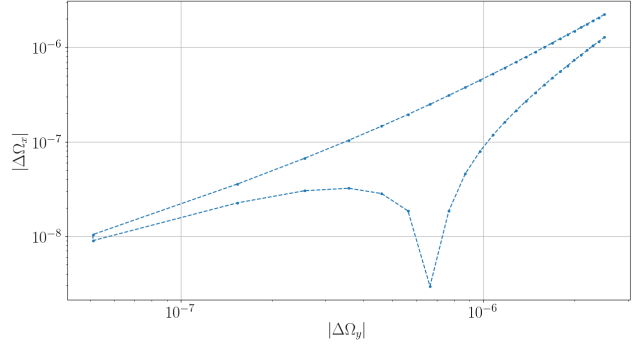
In Fig. 2 you see the calibration plots when we varied the horizontal offset, and the momentum offset.

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