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1 Spin decoherence in a perfectly aligned ring

Spin coherence refers to a measure or quality of preservation of polarization in an initially fully polarized beam. [1, p. 205]

When a polarized beam is injected into a storage ring, the spins of the beam particles start precessing around the vertical (guiding) magnetic field. The precession frequency is dependent upon the equilibrium-level energy of the particle, which differs for the particles in the beam.

This does not pose a problem when the initial polarization is vertical; however, the FS storage ring method requires spin polarization along the momentum vector, i.e., in the horizontal plane. Therefore, spin decoherence is an inherent weakness of the FS method.

1.1 Spin coherence time requirements

The spin coherence time (SCT) for an FS method performed in a perfectly aligned storage ring is determined by the smallest detectable angle by which the beam polarization vector is tilted from the horizontal plane by EDM alone. For the sensitivity level of $10^{-29}\ e \cdot cm$ it is about $5 \cdot 10^{-6}$. [2]

According to the T-BMT equation,

$$\Omega_{EDM,x} = \eta \frac{qE_x}{2mc},$$

where η is the proportionality factor between the EDM and spin, equaling 10^{-15} for the deuteron, for the given EDM sensitivity limit. [1, p. 206]

For the deuteron BNL FS storage ring, $E_x = 12$ MV/m, [2, p. 19] and so $\Omega_{EDM,x} \approx 10^{-9}$ rad/sec. This gives an SCT of approximately 1000 seconds in order that the vertical polarization reaches a detectable level of 1μrad. [1, p. 207]

1.2 Origin of decoherence

The longitudinal dynamics of a charged particle on the reference orbit in a storage ring is described by the system of equations:

$$\begin{cases} \frac{\mathrm{d}\varphi}{\mathrm{d}t} &= -\omega_{RF}\eta\delta, \\ \frac{\mathrm{d}\delta}{\mathrm{d}t} &= \frac{qV_{RF}\omega_{RF}}{2\pi h \beta^2 E}\sin\varphi. \end{cases}$$

In the equations above: φ is the phase deviation from the reference $\varphi_0 = 0$; $\delta = \frac{\Delta p}{p_0}$ is the relative momentum deviation from the momentum p_0 of the reference particle; V_{RF}, ω_{RF} are the voltage and oscilation frequency of the RF field; $\eta = \alpha_0 - \gamma^{-2}$ is the slip factor, with α_0 being the compaction factor defined by ${}^{\Delta L}/{}_{L} = \alpha_0\delta$, and L being the orbit length; h is the harmonic number; E is the total energy of the accelerated particle. $\omega_{RF} = 2\pi h f_{rev}$, where $f_{rev} = T_{rev}^{-1}$ is the beam revolution frequency.

The solutions of this system form a family of ellipses in the (φ, δ) space, centered at $(0, 0)$. However, if we consider a particle involved in betatron oscillations, and use a higher-order Taylor expansion of the compaction factor $\alpha = \alpha_0 + \alpha_1\delta$, the first equation of the system transforms into: [3, p. 2579]

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\omega_{RF}\left[\left(\frac{\Delta L}{L}\right)_\beta + \left(\alpha_0 + \gamma^{-2}\right)\delta + \left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma^{-4}\right)\delta^2\right],$$

where $\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x Q_x + \varepsilon_y Q_y\right]$, is the betatron motion-related orbit lengthening; ε_x and ε_y are the horizontal and vertical beam emittances, and Q_x and Q_y are the horizontal and vertical tunes. [3, p. 2580]

The solutions of the modified system are no longer centered at the same point. Orbit-lengthening and momentum deviation cause an equilibrium-level momentum momentum shift [3, p. 2581]

$$\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right],$$

where δ_m is the amplitude of synchrotron oscillations.

The equilibrium energy spread, associated with this momentum shift, of the beam particles results in a spin tune spread [3, p. 2581]

1.3 Sextupoles for the reduction of decoherence

To minimize the spin decoherence dur to betatron motion and momentum deviation, sextupoles (or octupoles) may be used [1, p. 212]

A sextupole of strength

$$S_{sext} = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2},$$

where $B\rho$ is the magnetic rigidity, affects the first-order compaction factor as [3, p. 2581]

$$\Delta\alpha_{1,sext} = -\frac{S_{sext}D_0^3}{L}, \tag{1}$$

and simultaneously the orbit length as

$$\left(\frac{\Delta L}{L}\right)_{sext} = \mp\frac{S_{sext}D_0\beta_{x,y}W_{x,y}}{L}, \tag{2}$$

where $D(s,\delta) = D_0(s) + D_1(s)\delta$ is the dispersion.

In the following sections, we will call the decoherence accociated with horizontal/vertical betatron, and synchrotron oscillations, respectively X-/Y-, and D-decoherence.

It can be observed from eqs equations (1, 2), that three sextupole families are required for the reduction of decoherence, placed in the maxima of: β_x, $beta_y$ for the reduction of X-,Y-decoherence, and D_0 for D-decoherence.

1.3.1 Simulation

In order to check the effectiveness of the sextupole method for the suppression of decoherence, a simulation was carried out using the COSY INFINITY code.

We took a perfectly aligned FS lattice, with three families of sextupoles (SX, SY, SD) placed as was explained at the end of section 1.3. Then we varied the strengths of each sextupole family (GSX, GSY, GSD) individually, and computed the spin tunes of particles offset at injection from the reference particle in: *a*) the x-coordinate for the SX-family, *b*) y-coordinate for the SY-family, and *c*) $d = \Delta K/K_0$ for the SD-family.

The spin tune depends parabolically for each direction of offset:

$$\mu(x, y, d) = \mu_0 + \mu_{xx}x^2 + \mu_{yy}y^2 + \mu_{dd}d^2 + O(x^3) + O(y^3) + O(d^3).$$

The search for the optimal sextupole strengths was done in two iterations:

1. first, we manually varied the strengths in a wide range, and observed the flattening of the spin tune curve;

2. when the global minima were found approximately, we continued the optimization automatically (for each family independently), using the second-order Taylor expansion coefficient before the corresponding coordinate.

The simulation results at 300 MeV (30 MeV above the FS energy) are presented in Figure 1. After the optimization of the sextupole strengths, spin tune shows almost no dependence on the variable offsets.[1]

2 Fake signal simulation

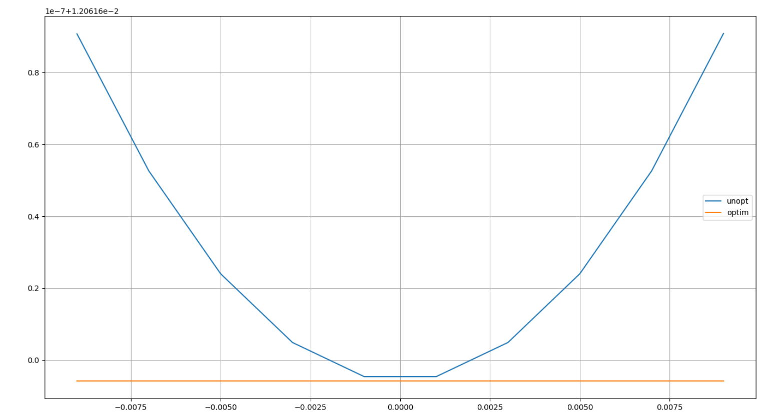
Analytical estimates of the MDM precession frequency about the radial axis. Description of how element misalignments were introduced and why so (to preserve the closed orbit). Plots: precession frequency vs the mean tilt angle.

References

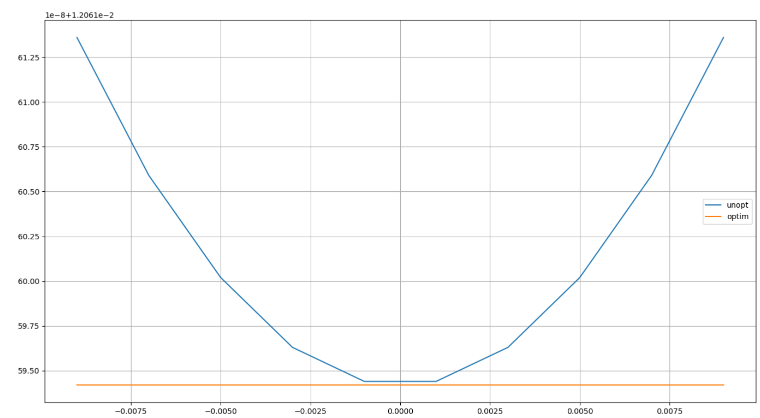
[1] Eremey Valetov. FIELD MODELING, SYMPLECTIC TRACKING, AND SPIN DECOHERENCE FOR EDM AND MUON G-2 LATTICES. Michigan State University. Michigan, USA;. Available from: http://collaborations.fz-juelich.de/ikp/jedi/public_files/theses/valetovphd.pdf.

[2] D Anastassopoulos, V Anastassopoulos, D Babusci. AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the 10 29 e cm level. BNL; 2008. Available from: https://www.bnl.gov/edm/files/pdf/deuteron_proposal_080423_final.pdf.

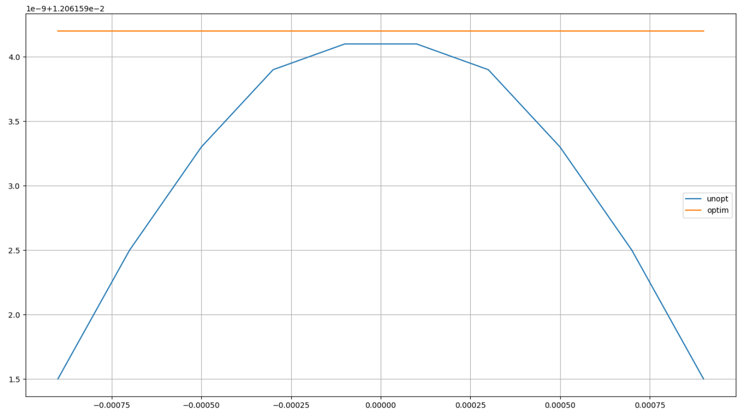
[1] We were unable to suppress all three types simultaneously in this lattice. Specifically, when the SD-type sextupoles are turned on, the D-decoherence is suppressed as before, but the X- and Y-types degrade even further.



(a) GSX optimized for the X bunch



(b) GSY optimized for the Y bunch



(c) GSD optimized for the D bunch

Figure 1: Spin tunes of offset particles vs the offsets, at 300 MeV, with/out the corresponding sextupole.

[3] Senichev Y, Zyuzin D. SPIN TUNE DECOHERENCE EFFECTS IN ELECTRO- AND MAGNETOSTATIC STRUCTURES. In: Beam Dynamics and Electromagnetic Fields. vol. 5. Shanghai, China: JACoW; 2013. p. 2579–2581. OCLC: 868251790. Available from: `https://accelconf.web.cern.ch/accelconf/IPAC2013/papers/wepea036.pdf`.