FREQUENCY DOMAIN METHOD OF SEARCH FOR THE DEUTERON ELECTRIC DIPOLE MOMENT

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CONTENTS

MOTIVATION

Storage ring-based methods of search for the electric dipole moments (EDMs) of fundamental particles can be classified into two major categories, which we will call *a*) Space Domain, and *b*) Frequency Domain methods.

In the Space Domain paradigm, one measures a *change in the spatial orientation* of the beam polarization vector *caused by the EDM*.

The original storage ring, frozen spin-type method, proposed in [1], is a canonical example of a methodology in the space domain: an initially longitudinally-polarized beam is injected into the storage ring; the vertical component of its polarization vector is observed. Under ideal conditions, any tilting of the beam polarization vector from the horizontal plane is attributed to the action of the EDM.

Two technical difficulties are readily apparent with this approach:

1. it poses a challenging task for polarimetry [2];
2. it puts very stringent constraints on the precision of the accelerator optical element alignment.

The former is due to the requirement of detecting a change of about $5 \cdot 10^{-6}$ to the cross section asymmetry ε_{LR} in order to get to the EDM sensitivity level of 10^{-29} $e \cdot cm$. [1, p. 18]

The latter is to minimize the magnitude of the vertical plane MDM precession frequency: [1, p. 11]

$$\omega_{syst} \approx \frac{\mu\langle E_v\rangle}{\beta c\gamma^2}, \tag{1}$$

induced by machine imperfection fields. According to estimates done by Y. Senichev, if it is to be fulfilled, the geodetic installation precision of accelerator elements must reach 10^{-14} m. Today's technology allows only for about 10^{-4} m.

At the practically-achievable level of element alignment uncertainty, $\omega_{syst} \gg \omega_{edm}$, and changes in the orientation of the polarization vector are no longer EDM-driven.

Another crucial problem one faces in the space domain is geometric phase error. [3, p. 6] The problem here lies in the fact that, even if one can somehow make field imperfections (either due to optical element misalignment or spurious electro-magnetic fields) zero *on average*, since spin rotations are non-commutative, the polarization rotation angle due to them will not be zero.

By contrast, the Frequency Domain methodology is founded on measuring the EDM *contribution* to the total (MDM and EDM together) spin precession *angular velocity*.

The polarization vector is made to roll about a nearly-constant, definite direction vector $\bar{n}$, with an angular velocity that is high enough for its magnitude to be easily measureable at all times. Apart from easier polarimetry, the definiteness of the angular velocity vector is a safeguard against geometric phase error.

This "Spin Wheel" may be externally applied [4], or otherwise the machine imperfection fields may be utilized for the same purpose (wheel roll rate determined by equation (1)). The latter is made possible by the fact that ω_{syst} changes sign when the beam revolution direction is reversed. [1, p. 11]

BASIC PRINCIPLES

The method we propose has four fundamental features:

1. It is a frequency domain method;
2. The fields induced by machine imperfections, instead of being suppressed, are used as a Koop Wheel;
    - The Koop Wheel roll direction is reversed by flipping the direction of the guide field;
    - its roll rate is controlled through observation of the horizontal plane polarization precession frequency.

We have already mentioned the advantages of the frequency domain, such as *a*) ease of polarimetry, and *b*) immunity to geometric phase error. Let us expand on the latter point.

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Frequency domain protection against geometric phase error

The magnitude of the spin precession angular velocity vector has a general form $\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$.

In the space domain, one tries to reduce ω to just ω_{edm}; by reducing machine imperfections, for example, or otherwise.

$$
\begin{aligned}
\omega &= \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} \\
&= \sqrt{(\omega_{edm} + \omega_{\langle E_v \rangle})^2 + \omega_y^2 + \omega_z^2} \\
&\approx (\omega_{edm} + \omega_{\langle E_v \rangle}) \left[1 + \frac{\omega_y^2 + \omega_z^2}{\omega_{\langle E_v \rangle}^2} \right]^{1/2}
\end{aligned}
$$

Machine imperfection fields as a Koop Wheel

The feasibility of using machine imperfection fields as a Koop Wheel comes from two properties of ω_{syst}: *a*) its magnitude; *b*) it changes sign when the accelerator guide field polarity is reversed.

EDM ESTIMATOR STATISTIC

The EDM estimator statistic requires two cycles to compose: one in which the Koop Wheel rolls forward, the other backward.

The change in the Koop Wheel roll direction is affected by flipping the direction of the guide field. When this is done: $\vec{B} \mapsto -\vec{B}$, the beam circulation direction changes from clockwise (CW) to counter-clockwise (CCW): $\vec{\beta} \mapsto -\vec{\beta}$, while the electrostatic field remains constant: $\vec{E} \mapsto \vec{E}$. According to the T-BMT equation, spin precession frequency components change like:

$$
\begin{aligned}
\omega_x^{CW} &= \omega_x^{MDM,CW} + \omega_x^{EDM}, \\
\omega_x^{CCW} &= \omega_x^{MDM,CCW} + \omega_x^{EDM}, \\
\omega_x^{MDM,CW} &= -\omega_x^{MDM,CCW},
\end{aligned} \tag{2a}
$$

and the EDM estimator

$$
\hat{\omega}_x^{EDM} := \frac{1}{2}\left(\omega_x^{CW} + \omega_x^{CCW}\right) \tag{2b}
$$

$$
= \omega_x^{EDM} + \underbrace{\frac{1}{2}\left(\omega_x^{MDM,CW} + \omega_x^{MDM,CCW}\right)}_{\varepsilon \to 0}. \tag{2c}
$$

To keep the systematic error term ε below required precision, i.e. that equation (2a) holds with sufficient accuracy, we devised a guide field flipping procedure based on observation of the horizontal plane spin precession frequency.

To explain how it works, we need to introduce the concept of the effective Lorentz factor.

EFFECTIVE LORENTZ FACTOR

Spin dynamics is described by the concepts of *spin tune* ν_s and *invariant spin axis* $\bar{n}$. Spin tune depends on the the particle's equilibrium-level energy, expressed by the Lorentz factor:

$$
\begin{cases}
\nu_s^B &= \gamma G, \\
\nu_s^E &= \beta^2\gamma\left(\frac{1}{\gamma^2-1} - G\right) \\
&= \frac{G+1}{\gamma} - G\gamma.
\end{cases} \tag{3}
$$

Unfortunately, not all beam particles share the same Lorentz factor. A particle involved in betatron motion will have a longer orbit, and as a direct consequence of the phase stability principle, in an accelerating structure utilizing an RF cavity, its equilibrium energy level must increase. Otherwise it cannot remain the bunch. In this section we analyze how the particle Lorentz factor should be modified when betatron motion, as well as non-linearities in the momentum compaction factor are accounted for.

The longitudinal dynamics of a particle on the reference orbit of a storage ring is described by the system of equations:

$$
\begin{cases}
\frac{\mathrm{d}}{\mathrm{d}t}\Delta\varphi &= -\omega_{RF}\eta\delta, \\
\frac{\mathrm{d}}{\mathrm{d}t}\delta &= \frac{qV_{RF}\omega_{RF}}{2\pi h\beta^2 E}\left(\sin\varphi - \sin\varphi_0\right).
\end{cases} \tag{4}
$$

In the equations above, $\Delta\varphi = \varphi - \varphi_0$ and $\delta = (p - p_0)/p_0$ are the deviations of the particle's phase and normalized momentum from those of the reference particle; all other symbols have their usual meanings.

The solutions of this system form a family of ellipses in the (φ, δ)-plane, all centered at the point (φ_0, δ_0). However, if one considers a particle involved in betatron oscillations, and uses a higher-order Taylor expansion of the momentum compaction factor $\alpha = \alpha_0 + \alpha_1\delta$, the first equation of the system transforms into: [5, p. 2579]

$$
\begin{aligned}
\frac{\mathrm{d}\Delta\varphi}{\mathrm{d}t} = -\omega_{RF}\Bigg[&\left(\frac{\Delta L}{L}\right)_\beta + \left(\alpha_0 + \gamma^{-2}\right)\delta \\
&+ \left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma^{-4}\right)\delta^2\Bigg],
\end{aligned}
$$

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where $\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x Q_x + \varepsilon_y Q_y\right]$, is the betatron motion-related orbit lengthening; ε_x and ε_y are the horizontal and vertical beam emittances, and Q_x, Q_y are the horizontal and vertical tunes.

The solutions of the transformed system are no longer centered at the same single point. Orbit lengthening and momentum deviation cause an equilibrium-level momentum shift [5, p. 2581]

$$
\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right], \tag{5}
$$

where δ_m is the amplitude of synchrotron oscillations.

We call the equilibrium energy level associated with the momentum shift (5), the *effective Lorentz factor*:

$$\gamma_{eff} = \gamma_0 + \beta_0^2\gamma_0 \cdot \Delta\delta_{eq}, \tag{6}$$

where γ_0, β_0 are the Lorentz factor and relative velocity factor of the reference particle.

Observe, that the effective Lorentz factor enables us to account for variation in the value of spin tune due to variation in the particle orbit length. It is crucial in the analysis of spin decoherence [6] and its suppression by means of sextupole fields.

It plays a big role, as well, in the successfull reproduction of the MDM component to the total spin precession angular velocity.

GUIDE FIELD FLIPPING

Two aspects of the problem need to be paid attention to:

1. What needs to be kept constant from one measurement cycle to the next;
2. How it can be observed.

The goal of flipping the direction of the guide field is to accurately reproduce the radial component of the MDM spin precession frequency induced by machine imperfection fields. This point should not be overlooked: a mere reproduction of the *magnetic field strength* would not suffice, since the injection point of the beam's centroid, and hence its orbit length — and, via equations (6) and (3), spin tune, — is subject to variation. (Apart from that, the accelerating structure might not be symmetrical, in terms of spin dynamics, with regard to reversal of the beam circulation direction.)

What needs to be reproduced, therefore, is not the field strength, but the effective Lorentz factor of the centroid.

Regarding the second question, we mentioned earlier that the Koop Wheel roll rate is controlled through measurement of the horizontal plane spin precession frequency. This plane was chosen because the EDM angular velocity vector points (mainly) in the radial direction; its vertical component is due to machine imperfection fields, and is small compared to the measured EDM effect. Therefore, in first approximation, when we manipulate the vertical component of the combined spin precession angular velocity, we manipulate the vertical component of the MDM angular velocity vector.

Moving on to the effective Lorentz factor calibration procedure. Let $\mathcal{T}$ denote the set of all trajectories that a particle might follow in the accelerator. $\mathcal{T} = \mathcal{S} \bigcup \mathcal{F}$, where $\mathcal{S}$ is the set of all stable trajectories, $\mathcal{F}$ are all trajectories such that if a particle gets on one, it will be lost from the bunch.

Calibration is done in two phases:

1. In the first phase, the guide field value is set so that the beam particles are injected onto trajectories $t \in \mathcal{S}$.
2. In the second phase, it is fine-tuned further, so as to fulfill the FS condition in the horizontal plane. By doing this, we physically move the beam trajectories into the subset $\mathcal{S}|_{\omega_y=0} \subset \mathcal{S}$ of trajectories for which $\omega_y = 0$.

Spin tune (and hence precession frequency) is an injective function of the effective Lorentz-factor γ_{eff}, which means $\omega_y(\gamma^1_{eff}) = \omega_y(\gamma^2_{eff}) \to \gamma^1_{eff} = \gamma^2_{eff}$. The trajectory space $\mathcal{T}$ is partitioned into equivalence classes according to the value of γ_{eff}: trajectories characterized by the same γ_{eff} are equivalent in terms of their spin dynamics (possess the same spin tune and invariant spin axis direction), and hence belong to the same equivalence class. Since $\omega_y(\gamma_{eff})$ is injective, there exists a unique γ^0_{eff} at which $\omega_y(\gamma^0_{eff}) = 0$:

$$[\omega_y = 0] = [\gamma^0_{eff}] \equiv \mathcal{S}|_{\omega_y=0}.$$

If the lattice didn't use sextupole fields for the suppression of decoherence, $\mathcal{S}|_{\omega_y=0}$ would be a singleton set. We have shown in [6] that if sextupoles are utilized, then $\exists\mathcal{D} \subset \mathcal{S}$ such that $\forall t_1, t_2 \in \mathcal{D}$: $\nu_s(t_1) = \nu_s(t_2)$, $\bar{n}(t_1) = \bar{n}(t_2)$. By adjusting the guide field strength we equate $\mathcal{D} = \mathcal{S}|_{\omega_y=0}$, and hence $\mathcal{S}|_{\omega_y=0}$ contains multiple trajectories. [1]

Therefore, once we ensured that the beam polarization does not precess in the horizontal plane, all of the beam particles have γ^0_{eff}, equal for the CW and CCW beams.

Guide field flipping procedure simulation results can be found in **??**.

STATISTICAL PRECISION

First, we see in (**??**) that the accuracy of the frequency measurements of Ω_r^{CW} and Ω_r^{CCW} determines the precision of the EDM measurement. In [**?**], it is shown that the relative accuracy of the polarization precession frequency measurement, 10^{-10} to 10^{-11}, is achievable even when the frequency of polarization measurements (a detector rate) is much less than the polarization precession frequency. In our case, we have an inverse relationship between the polarimeter rate and the measured spin frequency, which extends the range of frequencies where statistical estimates are legitimate. As shown in [**?**], for an absolute statistical error of measuring a frequency of the spin oscillation, we can use $\sigma_\Omega = \delta\epsilon_A\sqrt{24/N}/T$, where N is the total number of recorded events, $\delta\epsilon_A$ is the relative error in measuring the asymmetry, and $T \approx 1000$ sec is the measurement duration. If we assume a beam of 10^{11} particles per fill and a polarimeter efficiency of one percent, this leads to an absolute error of frequency measurement of $\sigma_\Omega = 2 \cdot 10^{-7}$ rad/sec. With a nominal accelerator beam time of 6,000 hours per year, we can reach $\sigma_\Omega = 2 \cdot 10^{-9}$ rad/sec during one year. If we take into account that formula (**??**) with the EDM $d_d \approx 10^{-30} e \cdot cm$ gives a value of the spin precession frequency of $\Omega_{\text{edm}} \approx 10^{-8}$, we can state that the accuracy

[1] Strictly speaking, even if sextupoles are used there remains some negligible dependence of spin tune on the particle orbit length (linear decoherence effects, cf. [6]). Because of that, the equalities for ν_s and $\bar{n}$ are approximate, and the set $\mathcal{S}|_{\omega_y=0}$ should be viewed as fuzzy: we will consider trajectories for which $|\omega_y| < \delta$ for some small δ as belonging to $[\omega_y = 0]$.

for the frequency of $\sigma_\Omega = 1.4 \cdot 10^{-9}$ is satisfactory and sufficient for reaching a sensitivity of $d_d \approx 10^{-30} e \cdot cm$ (where $\eta \approx 2 \cdot 10^{-15}$).

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