Effect of spin motion perturbation on the EDM statistic

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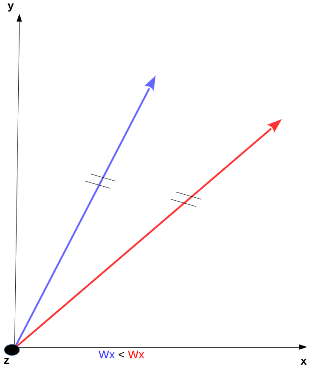
Problem statement

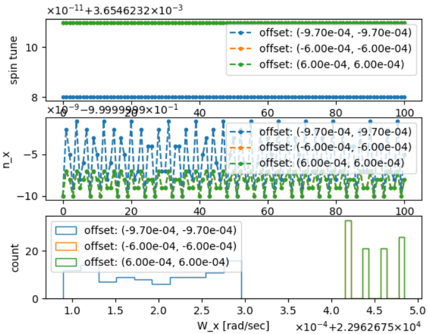
- The spin precession axis (SPA) of a particle involved in betatron motion moves about the invariant spin axis defined on the CO: $\mathbf{\Omega} = \mathbf{\Omega}_0(\Theta) + \boldsymbol{\omega}(\Theta, \Delta\mathbf{r})$.
- Simultaneously, it was claimed that: for two beams, $\gamma^1_{eff}(\frac{\Delta L}{L}, \frac{\Delta p}{p}) = \gamma^2_{eff}(\frac{\Delta L}{L}, \frac{\Delta p}{p}) \to (\Omega^1_x, \Omega^1_y, \Omega^1_z) = (\Omega^2_x, \Omega^2_y, \Omega^2_z)$, **regardless** of the particulars of their orbital motion.

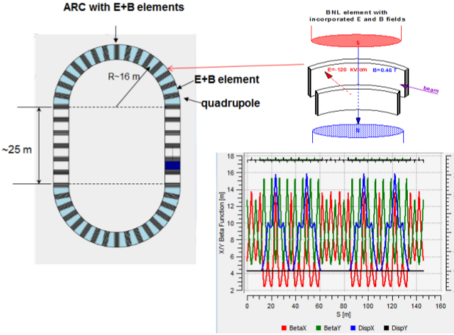


The last statement makes sense so long as by "frequency" we mean $|\mathbf{\Omega}|$. Couldn't see how γ_{eff} alone can guarantee the equality of the $\bar{n}$ orientations. Must be an implicit assumption.

Single particle ν_s, $\bar{n}$, and Ω_x

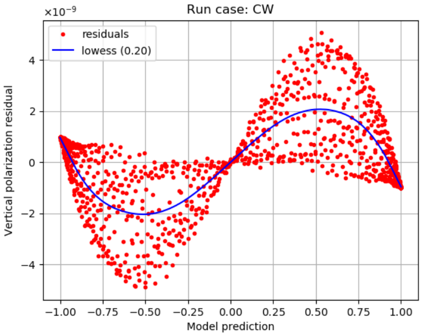
Implicit assumption (sic!): All spin vectors in the beam precess about the same $\bar{n}_{CO}$. (More carefuly: $\bar{n}_i - \bar{n}_{CO} \ll 1$.)
Below: 270 MeV (FS@270.0092 MeV), FS lattice w/E+B elements tilted about the optic axis by $\theta \sim N(4 \cdot 10^{-3}, 5 \cdot 10^{-4})$ rad. Observe a significant $\sigma[\Omega_x]$.
Question: How does this affect the net beam polarization?





Simulation: Uniform beam

- Same lattice; beam represented by 4,000 rays;
  $x_0,\ y_0 \in [-1mm, +1mm]$,
  $d_0 := \Delta K / K_{ref} \in [-1 \cdot 10^{-4}, +1 \cdot 10^{-4}]$.
- $\mathbf{P} = \frac{\sum_{i \in E} \mathbf{s}_i}{|| \sum_{i \in E} \mathbf{s}_i ||}$.
- Fit P_y by model $g(t) = \sin(2\pi f \cdot t)$.



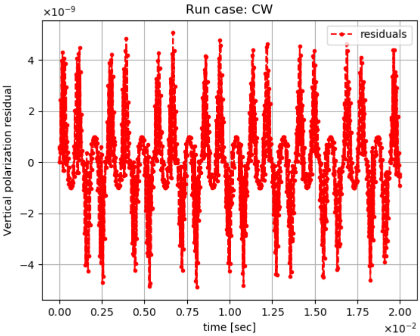


Table 1: Frequency estimates for the Uniform CW & CCW beams, reference ray and full beam, in Hz

Data		Polarization		Reference ray	
Frequency	Offset	CW	CCW	CW	CCW
Estimate	360.90365	$1.58 \cdot 10^{-7}$	$1.57 \cdot 10^{-7}$	$3.42902 \cdot 10^{-6}$	$3.42902 \cdot 10^{-6}$
SE	–	$1 \cdot 10^{-9}$	$2 \cdot 10^{-9}$	$5 \cdot 10^{-10}$	$5 \cdot 10^{-10}$

- Residuals exhibit a systematic pattern (model error); also $\hat{f}_{P_y} < \hat{f}_{s_y}^{CO}$.
- However, $\hat{f}_{P_y}^{CW} - \hat{f}_{P_y}^{CCW}$ is below statistical precision.
- But what if the CW & CCW beams are **not** identical?

Simulation: Gaussian beams

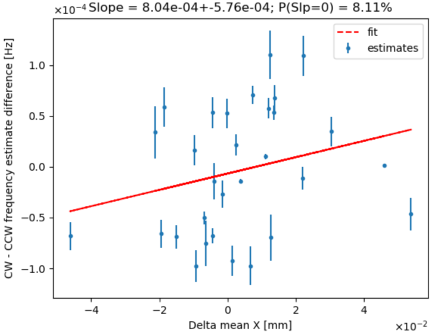
Table 2: Frequency estimates for the Gaussian CW & CCW beams, reference ray and full beam, in Hz

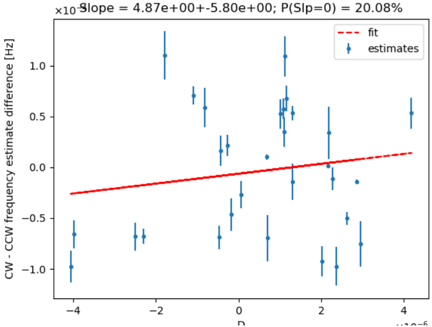
Data		Polarization		Reference ray	
Frequency	Offset	CW	CCW	CW	CCW
Estimate	352.99403	$9.0405 \cdot 10^{-6}$	$7.792 \cdot 10^{-6}$	$4.149017 \cdot 10^{-5}$	$4.149017 \cdot 10^{-5}$
SE	–	$7 \cdot 10^{-10}$	$9 \cdot 10^{-9}$	$2 \cdot 10^{-11}$	$2 \cdot 10^{-11}$

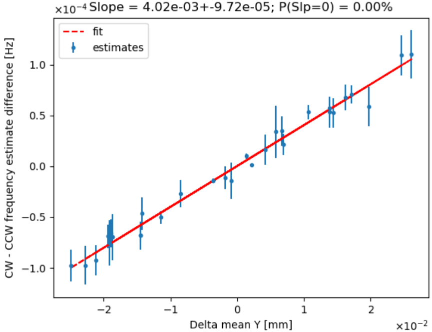
Error: $\varepsilon := \hat{f}^{CW} - \hat{f}^{CCW} = 1.249 \cdot 10^{-6}$ (model) $\pm 9 \cdot 10^{-9}$ (fit) Hz.

Multiple runs

Hypothesis: the systematic error component likely depends on the beam centroid difference. Define the centroid by $\mathbf{c} = (\langle x_0 \rangle, \langle y_0 \rangle, \langle d_0 \rangle)$. Then do linear regression of $\hat{f}^{CW} - \hat{f}^{CCW}$ on $\mathbf{c}^{CW} - \mathbf{c}^{CCW}$.




×10⁻³ Slope = 4.87e+00+-5.80e+00; P(Slp=0) = 20.08%
fit
estimates
CW - CCW frequency estimate difference [Hz]
1.0
0.5
0.0
−0.5
−1.0
−4
−2
0
2
4


×10⁻⁴ Slope = 4.02e-03+-9.72e-05; P(Slp=0) = 0.00%
fit
estimates
CW - CCW frequency estimate difference [Hz]
1.0
0.5
0.0
−0.5
−1.0
−2
−1
0
1
2
Delta mean Y [mm]
×10⁻²

Effect size dependence on the beam size

- $\varepsilon = a_0 + a_1 \Delta \mathbf{c}_y$;
- all beams in the simulation had the same CO; $\mathbf{c}_y$ deviated from 0 only b/c of a finite sample size;
- $\sigma_{\langle y \rangle} \equiv \sigma_{4k} = \sigma / \sqrt{n}$, hence if $n = 4 \cdot 10^3 \to n = 4 \cdot 10^9$, then $\sigma_{4b} = \sigma_{4k} \cdot 10^{-3}$;
- then the model part of ε would also drop 3 orders of magnitude, and would be comparable with fit error.

Extra