Mitglied der Helmholtz-Gemeinschaft

Statistical Error

A.E. AKSENTEV

IKP, Forschungszentrum Jülich, Germany

National Research Nuclear University «MEPhI», Russia

03/06/17

Methodology

When put into an electromagnetic field, the particle spin begins to precess according to the T-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m}\left\{ \underbrace{G\vec{B} + \left(\frac{1}{\gamma^2 - 1} - G\right)\left(\vec{\beta} \times \vec{E}\right)}_{\text{MDM}} + \underbrace{\frac{\eta}{2}\left(\vec{E} + \vec{\beta} \times \vec{B}\right)}_{\text{EDM}} \right\}$$

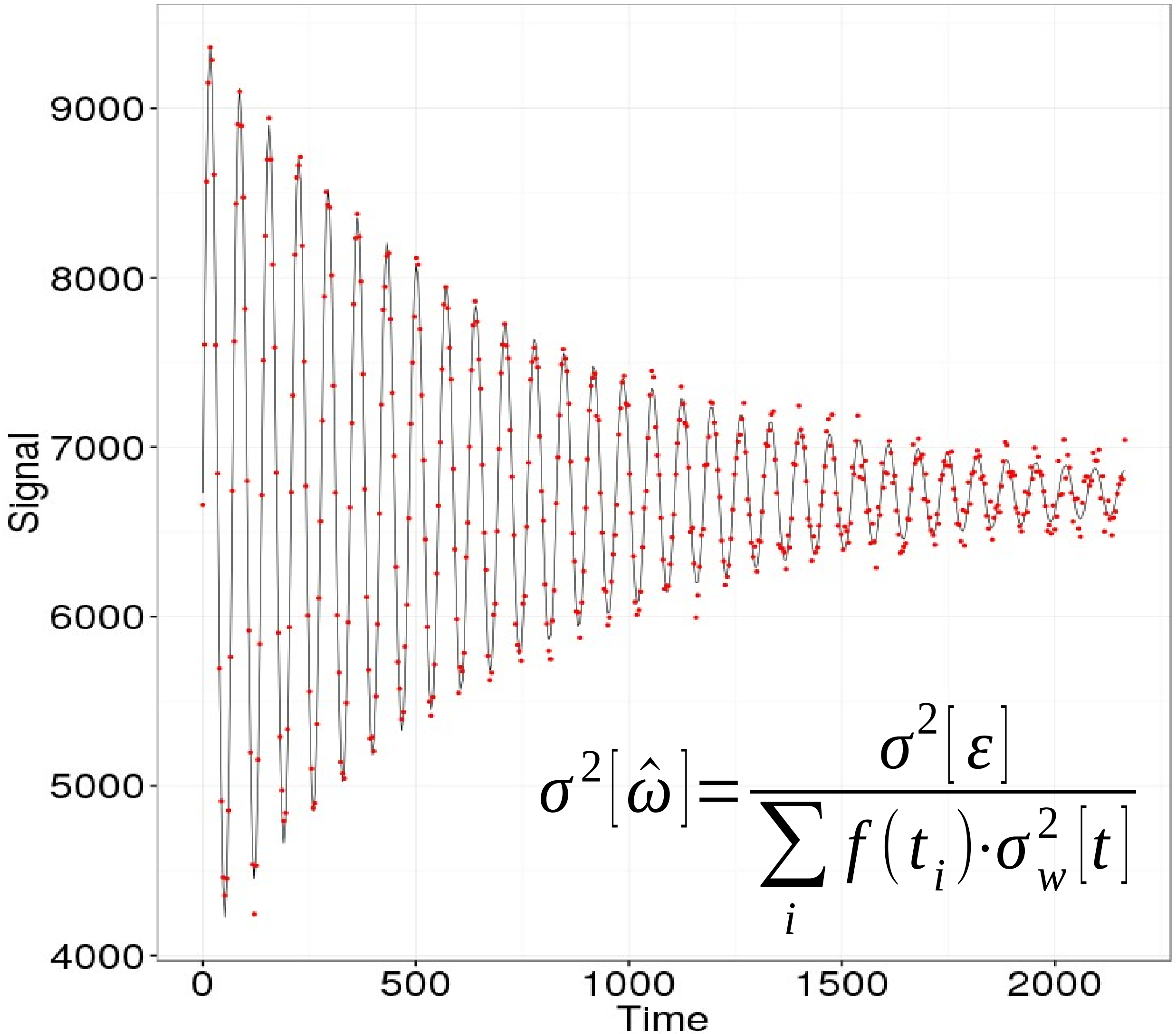
By measuring the beam's polarization, we can determine the frequency

$$\vec{\Omega}^{\pm} = \vec{\Omega}_{MDM} \pm \vec{\Omega}_{EDM}$$

Comparing the CW vs CCW frequencies, determine Ω_{EDM}

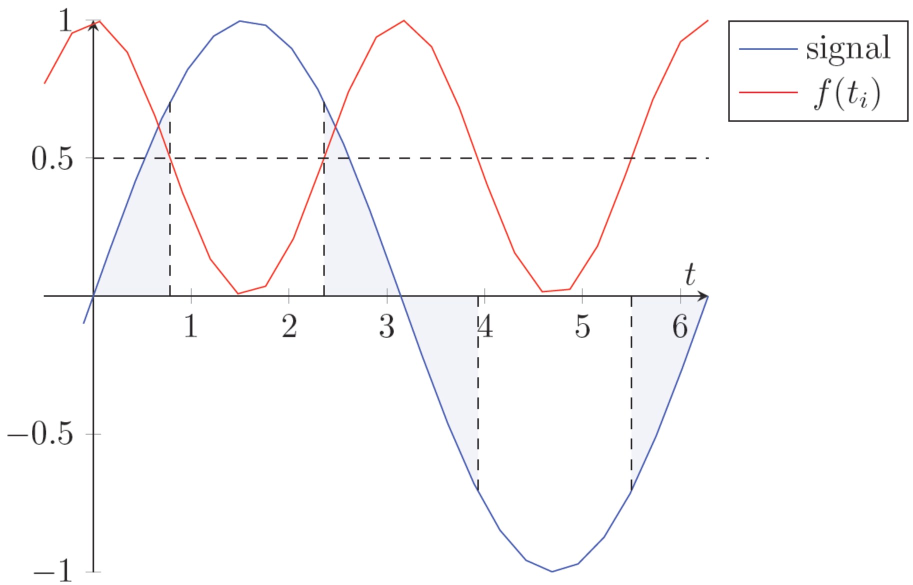
Problem statement

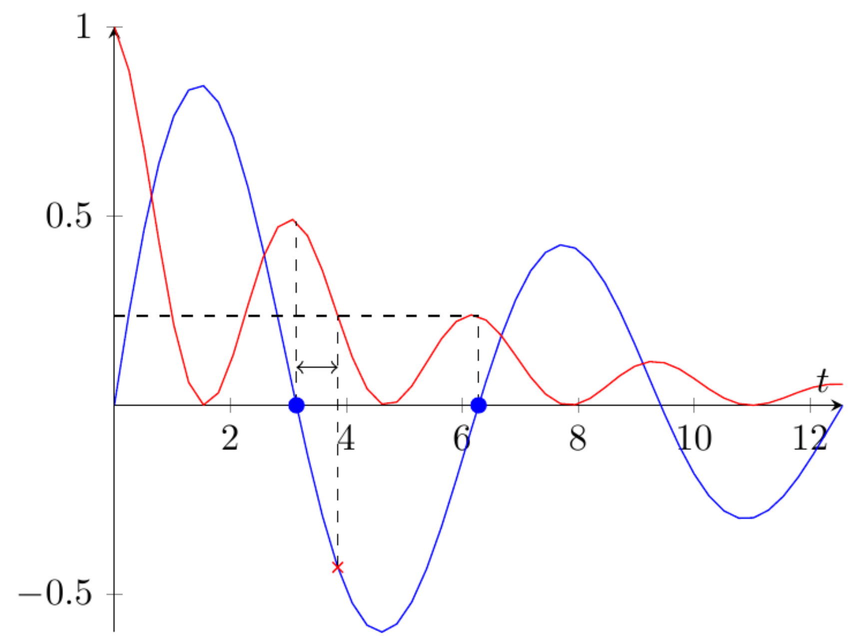
$$N(t) = N_0 \cdot \left(1 + P \cdot e^{-t/\tau_d} \cdot \sin(\omega \cdot t + \varphi)\right) + \varepsilon_t$$



$$\sigma^2[\hat{\omega}] = \frac{\sigma^2[\varepsilon]}{\sum_i f(t_i) \cdot \sigma_w^2[t]}$$

Two aspects





Goals

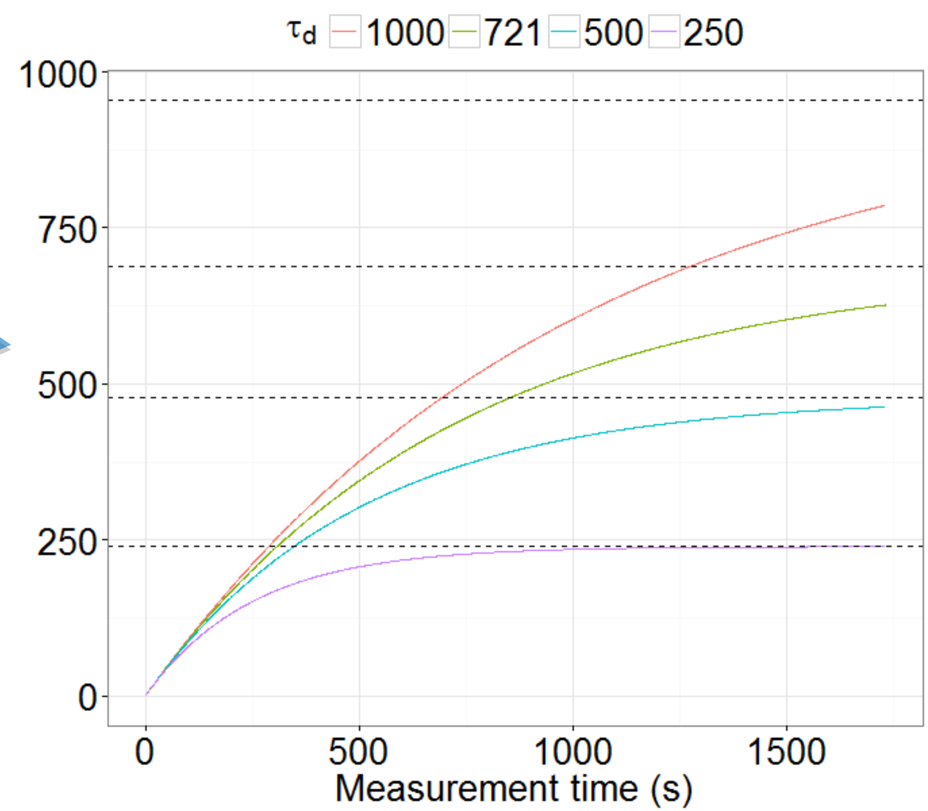
- Under what conditions sampling modulation is beneficial
- How much modulation is appropriate
- At what point measurement is no longer informative

Spread

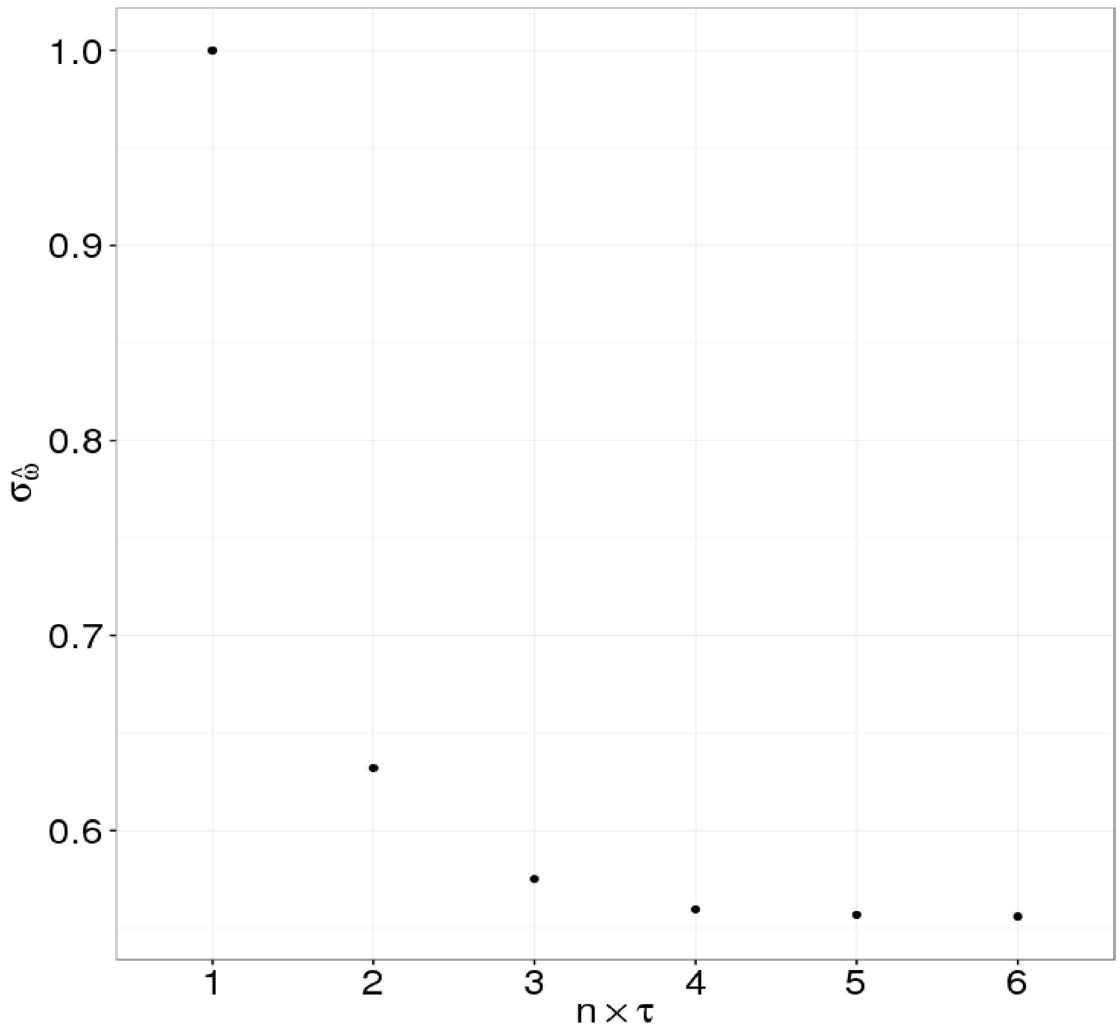
$$\sum f(t_i) = n_{\varepsilon/zc} \cdot x_{01} \cdot \frac{\exp\left(-\frac{\pi}{\omega\tau_d} n_{zc}\right) - 1}{\exp\left(-\frac{\pi}{\omega\tau_d}\right) - 1}$$

$$t(z) = \tau_d \cdot \ln\left(\frac{1}{1-z}\right)$$

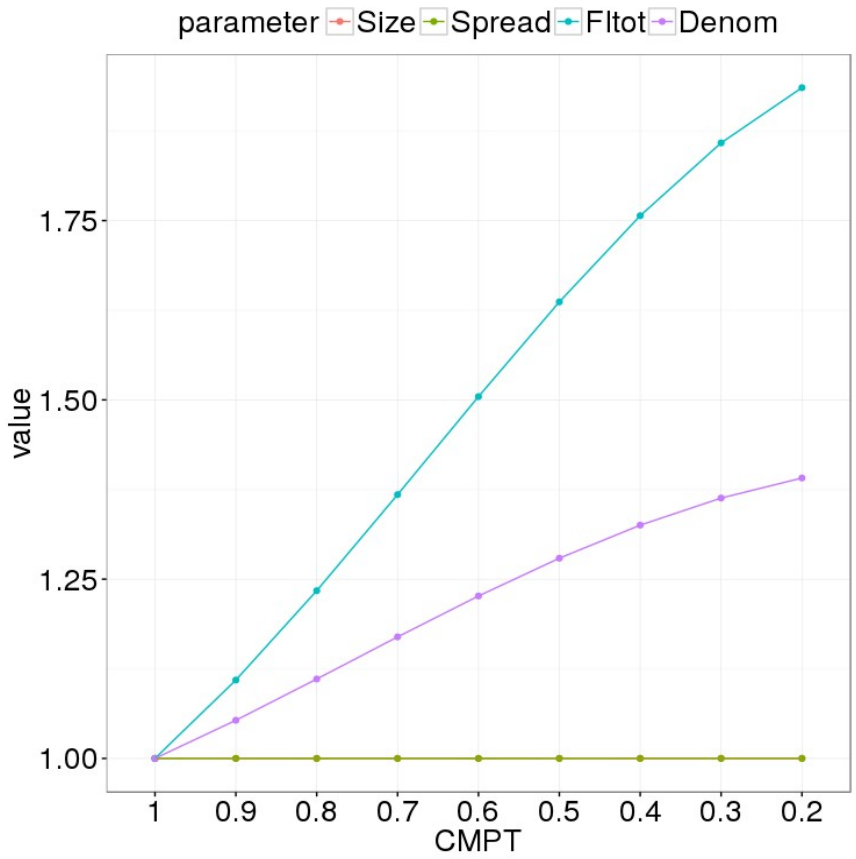
FI limit (%)	Reached (×τ_d)	SNR@3% error
95	3.0	1.7
90	2.3	3.3
70	1.2	10.0
50	0.7	16.5



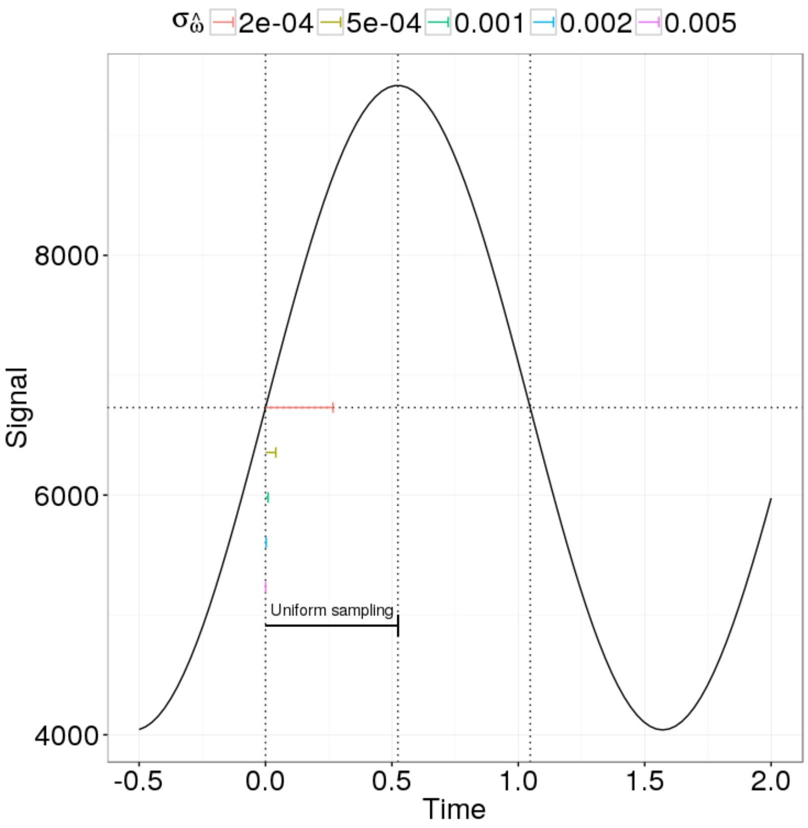
Simulation



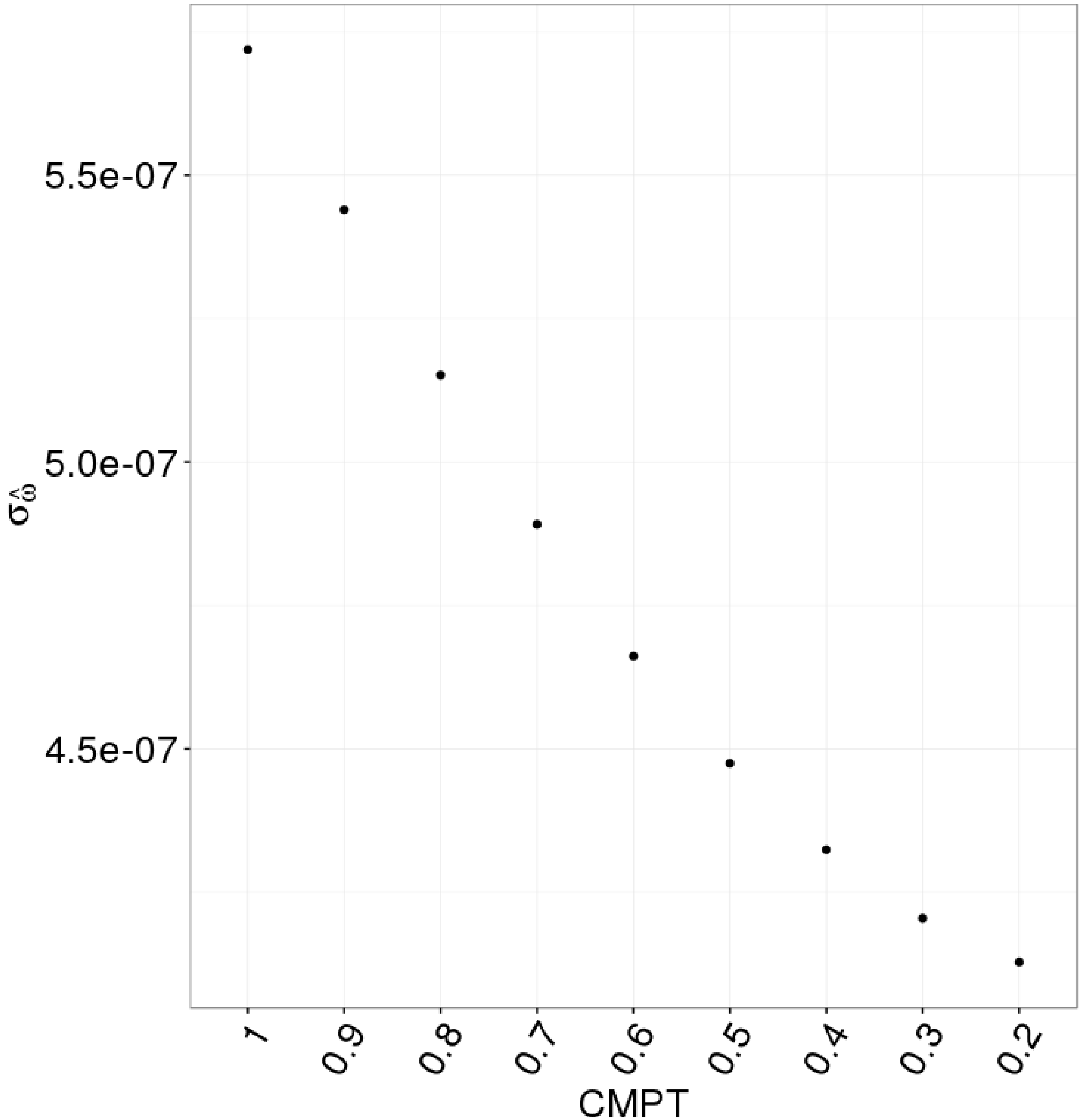
Modulation



parameter Size Spread Fltot Denom
value
1.75
1.50
1.25
1.00
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2
CMPT

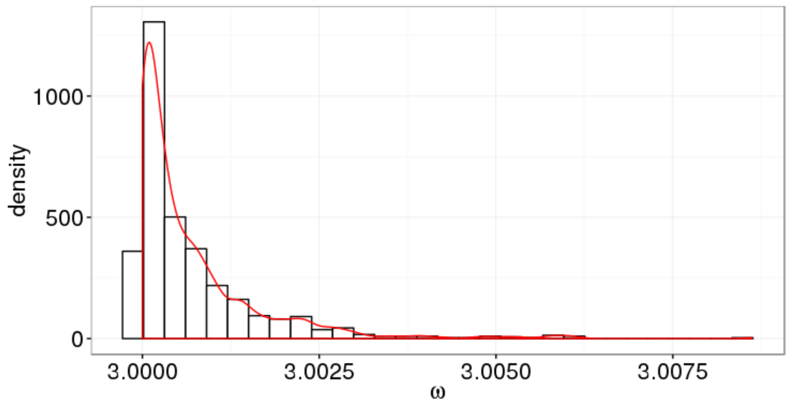


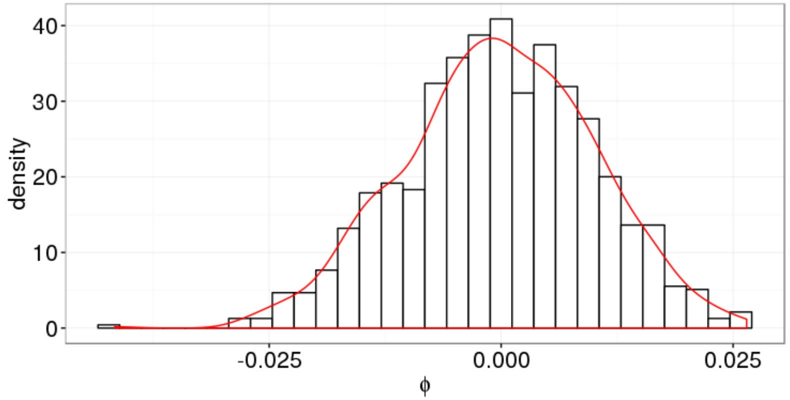
$\sigma_{\hat{\omega}}$ 2e-04 5e-04 0.001 0.002 0.005
Signal
8000
6000
4000
Uniform sampling
-0.5 0.0 0.5 1.0 1.5 2.0
Time

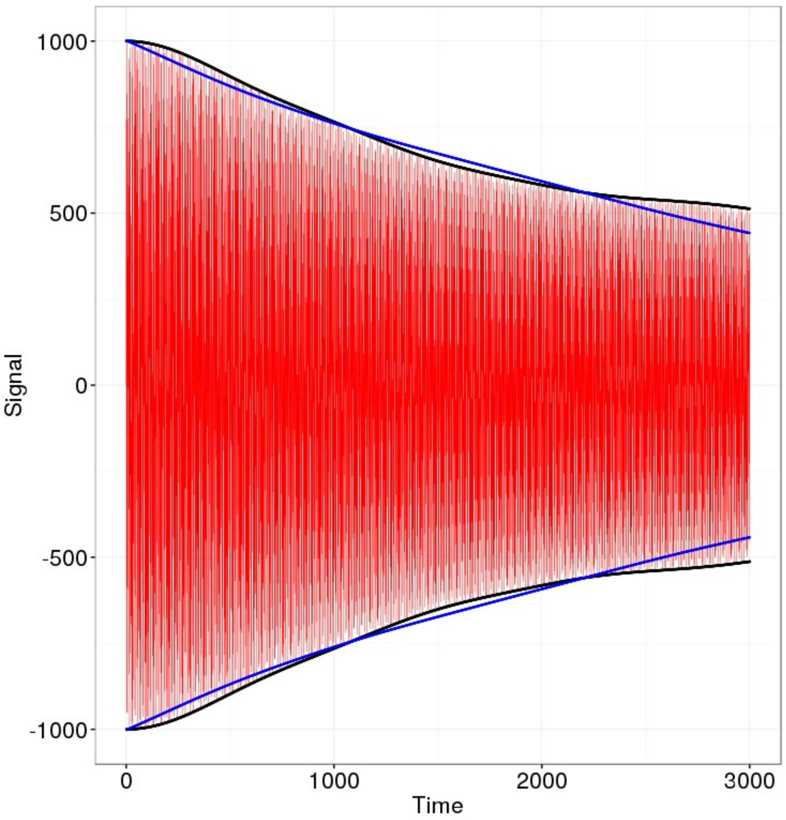

5.5e-07
5.0e-07
4.5e-07
$\sigma_{\hat{\omega}}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
CMPT

Decoherence

$$S(t) = \sum_i \sin(\omega_i \cdot t + \varphi_i)$$







Ways to proceed

1. Find the correct model specification

2. Make the frequency spectrum small enough so that the data fit sufficiently well to a simpler model

- Sextupoles to reduce X- & Y-emittance sizes
- RF for better momentum resolution