Frequency Domain Methodto Search for the Deuteron Electric Dipole Moment in a Storage Ring Environment

Alexander Aksentev[e,f,g,*], Yury Senichev[f], Eremey Valetov[h]

[a]*Institut für Kernphysik (IKP-2), Forschungszentrum Jülich, Jülich, Germany*
[b]*Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia*
[c]*National Research Nuclear University "MEPhI," Moscow, Russia*
[d]*Department of Physics and Astronomy, Michigan State University, MI 48824, USA*

Abstract

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*Corresponding author

Email addresses: `alexaksentyev@gmail.com` (Alexander Aksentev), `y.senichev@inr.ru` (Yury Senichev), `eremey@valetov.com` (Eremey Valetov)

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[e] *Institut für Kernphysik (IKP-2), Forschungszentrum Jülich, Jülich, Germany*
[f] *Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia*
[g] *National Research Nuclear University "MEPhI," Moscow, Russia*
[h] *Department of Physics and Astronomy, Michigan State University, MI 48824, USA*

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Contents

1. Introduction

Spin rotations belong to the Spin(3) group, which is isomorphic to SU(2).

[*] Corresponding author
Email addresses: alexaksentyev@gmail.com (Alexander Aksentev), y.senichev@inr.ru (Yury Senichev), eremey@valetov.com (Eremey Valetov)

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Rotations in SU(2). Rotation by angle ψ about direction $\bar{n}$

$$R_{\bar{n}}(\psi) = \exp\left[-i\frac{\psi}{2}(\bar{n}\cdot\vec{\sigma})\right],$$

where $\vec{\sigma}$ is the Pauli matrix vector.

1.1. General spin rotation matrices

Denote

- $(\Theta^{mi}, \bar{n}_{mi})$ from machine imperfections;
- $(\Theta^{+}, \bar{n}_{sol})$ for the $+\Delta$ solenoidal field;
- $(\Theta^{-}, -\bar{n}_{sol})$ for the $-\Delta$ solenoidal field.

$$\begin{aligned} R^{+\Delta} &= \exp\left[-i\left(\frac{\Theta^{mi}}{2}(\bar{n}_{mi}\cdot\vec{\sigma}) + \frac{\Theta^{+}}{2}(\bar{n}_{sol}\cdot\vec{\sigma})\right)\right] \\ &= \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} + \Theta^{+}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \qquad (1) \\ R^{-\Delta} &= \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} - \Theta^{-}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \qquad (2) \end{aligned}$$

2. Preliminary analytic of the Spin Wheel method

In SW we posit

$$\left(\vec{\Omega}^{+\Delta}_{MDM}\cdot\hat{x}\right) = -\left(\vec{\Omega}^{-\Delta}_{MDM}\cdot\hat{x}\right). \qquad (3)$$

The spin precession angular velocity vector can be expressed via spin tune and invariant spin axis as

$$\vec{\Omega}_{spin} = \frac{2\pi}{\tau_{ring}}\cdot\nu\cdot\bar{n},$$

hence

$$\nu^{+\Delta}(\bar{n}_{+\Delta}\cdot\hat{x}) + \nu^{-\Delta}(\bar{n}_{-\Delta}\cdot\hat{x}) = 0 \qquad (4)$$

From $\Delta\Theta = \tau\Delta\Omega$ and $\Delta\Omega_x^{MDM} = \frac{q}{m}GB_x$, and **assuming**

$$B_{sol}^{\pm}\tau_{sol} = \langle B_{sol}^{\pm}\rangle\tau_{ring} : \tag{5}$$

$$\begin{cases} \Theta^+ & = \tau_{sol}\frac{q}{m}GB_{sol}^+ \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B_{sol}^+\rangle, \\ \Theta^- & = \tau_{sol}\frac{q}{m}GB_{sol}^- \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B_{sol}^-\rangle. \end{cases} \tag{6}$$

Remark 1. Assumption (5) is **required** if we want to obtain $B_{sol}^{\pm}$ from equations of group (11).

From eqs (1) and (2):

$$\begin{cases} \Theta^{mi}\bar{n}_{mi} + \Theta^+\bar{n}_{sol} = \nu^{+\Delta}\bar{n}_{+\Delta}, \\ \Theta^{mi}\bar{n}_{mi} - \Theta^-\bar{n}_{sol} = \nu^{-\Delta}\bar{n}_{-\Delta}. \end{cases} \tag{7}$$

Substituting eq (7) into (4), and assuming $\bar{n}_{sol} = \hat{x}$:

$$2\Theta^{mi}(\bar{n}_{mi}\cdot\hat{x}) + (\Theta^+ - \Theta^-) = 0. \tag{8}$$

Assuming[1]

$$\Theta^{mi} = \tau_{ring}\cdot\frac{q}{m}G\cdot\langle B_x\rangle^{mi}, \tag{9}$$

from (8) and (5) obtain:

$$2\langle B_x\rangle^{mi} + \left(\langle B_{sol}^+\rangle - \langle B_{sol}^-\rangle\right) = 0. \tag{10}$$

From eq (9) in Koop2015, assuming in the $+\Delta$ case the machine imperfections and solenoid fields are co-aligned, in the $-\Delta$ anti-aligned:

$$\begin{cases} \Delta^+ & = \frac{\beta_1-\beta_2}{\langle G_z\rangle}\langle B_x\rangle = \frac{\beta_1-\beta_2}{\langle G_z\rangle}\left(\langle B_x\rangle^{mi} + \langle B_{sol}^+\rangle\right), \\ & \Rightarrow \langle B_{sol}^+\rangle = \frac{\langle G_z\rangle}{\beta_1-\beta_2}\Delta^+ - \langle B_x\rangle^{mi}; \\ \Delta^- & = \frac{\beta_1-\beta_2}{\langle G_z\rangle}\langle B_x\rangle = \frac{\beta_1-\beta_2}{\langle G_z\rangle}\left(\langle B_x\rangle^{mi} - \langle B_{sol}^-\rangle\right), \\ & \Rightarrow -\langle B_{sol}^-\rangle = \frac{\langle G_z\rangle}{\beta_1-\beta_2}\Delta^- - \langle B_x\rangle^{mi}. \end{cases} \tag{11}$$

[1] This is a generous assumption implying that $\bar{n}_{mi} = \hat{x}$; i.e., this is **not** a non-commutativity-based argument; we assume all spin rotations commute.

Substituting this into (10):

$$2\langle B_x\rangle^{mi} + \left(\frac{\langle G_z\rangle}{\beta_1 - \beta_2} \left[\Delta^+ - \Delta^-\right] - 2\langle B_x\rangle^{mi} \right) = 0.$$

In the original method, we are to make

$$\Delta^- = -\Delta^+, \tag{12}$$

so the term in the square brackets is zero, and we are left with

$$(1-1)\,\langle B_x\rangle^{mi} = 0. \tag{13}$$

So, seems that SW works, but we did two important assumptions here: (a) commutativity (in order to get eq (9)), and (b) "averaging" of B_{sol} over the ring (in order to get eq (5) and remove the τ_{sol}/τ_{ring} from (10)).

Remark 2. If we don't use (9) (but still use (5) in order to obtain $B^{\pm}_{sol}$ from group (11)), then eq (13) becomes

$$\Theta^{mi}\,(\bar{n}_{mi}\cdot\hat{x}) - \frac{q}{m}G\cdot\tau_{ring}\langle B_x\rangle^{mi} = 0, \tag{14}$$

which is not very informative.

Remark 3. To check that eq (14) is correct, assume (9). Then

$$\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}\,(\bar{n}_{mi}\cdot\hat{x}) - \frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi} = 0,$$

and hence

$$\bar{n}_{mi}\cdot\hat{x} = 1,$$

which is implied by machine imperfection spin rotations adding up commutatively.

Remark 4. In general, since

$$\Theta^{mi} = \tau_{ring}\cdot\frac{q}{m}G\sqrt{\langle B^{mi}_x\rangle^2 + \langle B^{mi}_y\rangle^2 + \langle B^{mi}_z\rangle^2},$$

eq (14) implies that

$$\begin{aligned}(\bar{n}_{mi}\cdot\hat{x}) &= \frac{\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}}{\Theta^{mi}}\\ &= \frac{\langle B^{mi}_x\rangle}{\sqrt{\langle B^{mi}_x\rangle^2 + \langle B^{mi}_y\rangle^2 + \langle B^{mi}_z\rangle^2}}.\end{aligned} \tag{15}$$

Which is correct.

$$\begin{tikzcd}
(8) \arrow[r, "(9)+(5)"] \arrow[d, "\begin{matrix}(6)+\\(5)+\\(11)\end{matrix}"] & (10) \arrow[r, "(11)"] & (13) \\
(14) & &
\end{tikzcd}$$

Figure 1: Argument diagram.

Conclusion. In view of Remark 4, since eq (14) implies a valid statement, our conclusion is that the SW method resists the argument from non-commutativity.

3. Assumptions of the Spin Wheel method

Orbital dynamics. Koop2015 eq (7) (henceforth referred to as K(7)) and

$$\langle E_z \rangle = \langle E_z(0) \rangle + \langle G_z \rangle \cdot z \tag{K$\langle E_z \rangle$}$$

$$\rightarrow \langle z \rangle = \frac{\langle E_z(0) \rangle}{\langle G_z \rangle} - \frac{\beta}{\langle G_z \rangle} \cdot \langle B_x \rangle \tag{16}$$

$$\rightarrow \Delta = \frac{\beta_1 - \beta_2}{\langle G_z \rangle} \langle B_x \rangle. \tag{17}$$

This is as far as the argument from the non-linearity of the closed orbit shift dependence on the magnetic field is concerned. So long as we believe K(7) and K$\langle E_z \rangle$, we must believe K(9), and hence we cannot use that argument.

Spin dynamics. This is the argument from non-commutativity. For this argument cf. eq (14) and Remark 4, and the following conclusion.

4. Argumument against the SW method

The three-fold argument against the SW method is as follows (in the order of strength):

(1) The possibility of measuring the vertical orbit separation of two co-circulating beams at the sensitivity level of 10^{-12} m has not been shown by experiment. **Counter-argument**: there's reference [1] to commercially-available SQUIDs capable of detecting magnetic fields on the order of fT, which is equivalent to the beam separation of 10^{-12} m.

(2) Even if a SQUID-based BPM is capable of measuring orbit separation to such precision *locally*, the evaluation of the *mean* orbit separation requires multiple local measurements, and is not identical to the local measurement precision.

(3) Orbital and spin dynamics are idependent of each other, meaning that the observables $\vec{\Omega}$ and Δ are not directly related.

Regarding part (1): a counter to the counter-argument could be that the SQUID magnetic field measurements aren't linearly related to the beam orbit separation.

Regarding part (3) of the above argument (argument from statistics): we did a simulation, and confirmed that

$$\sigma[\langle\Delta\rangle] = \frac{a_y}{\sqrt{N_{BPM}}},$$

where a_y is the amplitude of betatron oscillaitons, N_{BPM} is the number of local BPM measurements.

5. Absence of the a_y^2 term in SW equations

Koop stats from the T-BMT equation K(3), which is a differential equation, defining

$$\Omega_x = \frac{q}{m} G B_x$$

locally. Then, in K(8) he transitions to the average

$$\langle\Omega_x\rangle = \frac{q}{m} G \langle B_x\rangle.$$

I think this is where he performes an invalid operation, by just formally including the LHS and RHS into the angle-brackets.

In our formalism,

$$\langle \Omega_x \rangle \propto G\gamma; \tag{18}$$

and since

$$\gamma \propto \frac{\langle B_x \rangle}{Q_y} + \kappa \cdot a_y^2,$$

so is $\langle \Omega_x \rangle$.

However, eq (18) is obtained from

$$\begin{aligned}
\Omega_x &= \frac{q}{m} G B_x, \\
\Omega_v &= \frac{qB}{m\gamma}, \\
\frac{\Omega_x}{\Omega_v} &= \frac{qGB}{m} \frac{m\gamma}{qB} = \gamma G.
\end{aligned}$$

References

[1] D. Kawal, "Relative Beam Position Monitors for the pEDM Experiment." https://apps.fz-juelich.de/pax/paxwiki/images/a/a9/DKawal_longapp_dmk_20110621.pdf