The probability of observing the value $y_i \equiv y(t_i)$ when the expectation value is $\mu(t_i)$ and the error is gaussian is

$$
\begin{aligned}
f(y_i|\boldsymbol{\theta}) &= \frac{1}{\sqrt{2\pi\nu}} \exp\left(-\frac{1}{2}\frac{(y_i-\mu(t_i))^2}{\nu}\right), \\
\boldsymbol{\theta} &= (\nu, \omega, \phi), \\
\mu(t_i) &= N_0\left(1 + P\sin(\omega t_i + \phi)\right).
\end{aligned}
$$

The likelihood of observing a set of observations $\mathbf{y} = (y_1, \ldots, y_K)$, under the i.i.d. assumption, is the product of propabilities taken as a function of the parameters:

$$
\mathcal{L}(\boldsymbol{\theta}|\mathbf{y}) = \prod_i f(y_i|\boldsymbol{\theta}),
$$

and the log-likelihood

$$
\ell(\boldsymbol{\theta}|\mathbf{y}) = -\frac{K}{2}\log 2\pi - \frac{K}{2}\log\nu - \frac{1}{2\nu}\sum_i \epsilon_i^2, \;\; \epsilon_i = y_i - \mu(t_i).
$$

The usual assumptions for the error term are zero expectation and strict exogeneity

$$
\mathrm{E}\left[\epsilon_i |\, \boldsymbol{\theta}_0\right] = \mathrm{E}\left[t_i\epsilon_i |\, \boldsymbol{\theta}_0\right] = 0,
$$

and the relations between the mean's derivatives are

$$
\begin{aligned}
\mu'_\phi &= N_0 P\cos(\omega t + \phi), \\
\mu'_\omega &= t \cdot \mu'_\phi, \epsilon'_\xi = -\mu'_\xi.
\end{aligned}
$$

The log-likelihood derivatives:

$$
\begin{aligned}
\ell'_\nu &= -\frac{K}{2\nu} + \frac{1}{2\nu^2}\sum_i \epsilon_i^2; \\
\ell'_\omega &= \frac{1}{\nu}\sum_i \mu'_\phi(t_i) t_i \epsilon_i; \\
\ell'_\phi &= \frac{1}{\nu}\sum_i \mu'_\phi(t_i)\epsilon_i; \\
\ell''_{\nu^2} &= \frac{K}{2\nu^2} - \frac{1}{\nu^3}\sum_i \epsilon_i^2, & -\mathrm{E}\left[\ell''_{\nu^2} |\, \boldsymbol{\theta}_0\right] &= \frac{K}{2\nu^2} - \frac{1}{\nu^3}\sum_i \nu = \frac{K}{2\nu^2}; \\
\ell''_{\nu\omega} &= -\frac{1}{\nu^2}\sum_i \mu'_\phi(t_i) t_i\epsilon_i, & -\mathrm{E}\left[\ell''_{\nu\omega} |\, \boldsymbol{\theta}_0\right] &= \frac{1}{\nu^2}\sum_i \mu'_\phi(t_i)\mathrm{E}\left[t_i\epsilon_i |\, \boldsymbol{\theta}_0\right] = 0; \\
\ell''_{\nu\phi} &= -\frac{1}{\nu^2}\sum_i \mu'_\phi(t_i)\epsilon_i, & -\mathrm{E}\left[\ell''_{\nu\phi} |\, \boldsymbol{\theta}_0\right] &= \frac{1}{\nu^2}\sum_i \mu'_\phi(t_i)\mathrm{E}\left[\epsilon_i |\, \boldsymbol{\theta}_0\right] = 0; \\
\ell''_{\phi^2} &= \frac{1}{\nu}\sum_i \left(\mu''_{\phi^2}(t_i)\epsilon_i - \left(\mu'_\phi(t_i)\right)^2\right), & -\mathrm{E}\left[\ell''_{\phi^2} |\, \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i \left(\left(\mu'_\phi(t_i)\right)^2 - \mu''_{\phi^2}(t_i)\mathrm{E}\left[\epsilon_i |\, \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i \left(\mu'_\phi(t_i)\right)^2; \\
\ell''_{\phi\omega} &= \frac{1}{\nu}\sum_i \left(\mu''_{\phi^2}(t_i)t_i\epsilon_i - \left(\mu'_\phi(t_i)\right)^2 t_i\right), & -\mathrm{E}\left[\ell''_{\phi\omega} |\, \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i \left(t_i\left(\mu'_\phi(t_i)\right)^2 - \mu''_{\phi^2}(t_i)\mathrm{E}\left[t_i\epsilon_i |\, \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i t_i\left(\mu'_\phi(t_i)\right)^2; \\
\ell''_{\omega^2} &= \frac{1}{\nu}\sum_i \left(\mu''_{\phi^2}(t_i)t_i^2\epsilon_i - \left(\mu'_\phi(t_i)t_i\right)^2\right), & -\mathrm{E}\left[\ell''_{\omega^2} |\, \boldsymbol{\theta}_0\right] &= \frac{1}{\nu}\sum_i \left(\left(t_i\mu'_\phi(t_i)\right)^2 - \mu''_{\phi^2}(t_i)\mathrm{E}\left[t_i^2\epsilon_i |\, \boldsymbol{\theta}_0\right]\right) = \frac{1}{\nu}\sum_i \left(t_i\mu'_\phi(t_i)\right)^2.
\end{aligned}
$$

The Fisher matrix

$$
I(\boldsymbol{\theta}_0) = \begin{bmatrix} K/2\nu & 0 & 0 \\ 0 & 1/\nu\sum\left(t_i\mu'_\phi(t_i)\right)^2 & 1/\nu\sum t_i\left(\mu'_\phi(t_i)\right)^2 \\ 0 & 1/\nu\sum t_i\left(\mu'_\phi(t_i)\right)^2 & 1/\nu\sum\left(\mu'_\phi(t_i)\right)^2 \end{bmatrix}.
$$

The determinant

$$
|I(\boldsymbol{\theta}_0)| = \frac{K}{2\nu^4}\underbrace{\left(\sum\left(t_i\mu'_\phi(t_i)\right)^2\sum\left(\mu'_\phi(t_i)\right)^2 - \left(\sum t_i\left(\mu'_\phi(t_i)\right)^2\right)^2\right)}_{\Omega}.
$$

The variance-covariance matrix

$$vcov = \begin{bmatrix} 2\nu^2/K & 0 & 0 \\ 0 & \nu\frac{\sum\left(\mu'_\phi(t_i)\right)^2}{\Omega} & \nu\frac{\sum t_i\left(\mu'_\phi(t_i)\right)^2}{\Omega} \\ 0 & \nu\frac{\sum t_i\left(\mu'_\phi(t_i)\right)^2}{\Omega} & \nu\frac{\sum\left(t_i\mu'_\phi(t_i)\right)^2}{\Omega} \end{bmatrix}.$$

Variance of the frequency estimate

$$var(\hat{\omega}) = \nu\frac{\sum\left(\mu'_\phi(t_i)\right)^2}{\sum\left(t_i\mu'_\phi(t_i)\right)^2\sum\left(\mu'_\phi(t_i)\right)^2 - \left(\sum t_i\left(\mu'_\phi(t_i)\right)^2\right)^2}.$$

Cross-check. Let $\mu(t_i) = \phi + \omega t_i$. In that case $\mu'_\phi(t_i) = 1$, $\mu'_\omega(t_i) = t_i = t_i \cdot \mu'_\phi(t_i)$, the determinant of the Fisher matrix simplifies to

$$\begin{aligned} |I(\boldsymbol{\theta}_0)| &= \frac{K}{2\nu^4}\left(K\sum_i t_i^2 - \left(\sum t_i\right)^2\right) \\ &= \frac{K^3}{2\nu^4}\left(\frac{1}{K}\sum t_i^2 - \langle t\rangle^2\right) \\ &= \frac{K}{2\nu^4}\cdot\underbrace{K\sum\left(t_i - \langle t\rangle\right)^2}_{\Omega} \end{aligned}$$

and the variance-covariance matrix becomes

$$vcov = \begin{bmatrix} 2\nu^2/K & 0 & 0 \\ 0 & \frac{\nu}{\sum(t_i-\langle t\rangle)^2} & \nu\frac{\sum t_i}{K\sum(t_i-\langle t\rangle)^2} \\ 0 & \nu\frac{\sum t_i}{K\sum(t_i-\langle t\rangle)^2} & \nu\frac{\sum t_i^2}{K\sum(t_i-\langle t\rangle)^2} \end{bmatrix},$$

with the well-known expression for the slope variance

$$var(\hat{\omega}) = \frac{\nu}{\sum\left(t_i - \langle t\rangle\right)^2}.$$