SPIN DECOHERENCE IN THE FREQUENCY DOMAIN METHOD FOR THE SEARCH OF A PARTICLE EDM

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Abstract

Spin coherence refers to a measure of preservation of polarization in an initially polarized beam. The spin vector of a particle injected into a storage ring starts to precess about the vertical magnetic field vector in accordance with the Thomas-BMT equation. The precession frequency is dependent on the equilibrium-level energy, which differs across the beam particles. This does not pose a problem when the initial polarization is vertical; however, the Frozen Spin Storage Ring EDM search method requires beam polarization along the momentum vector, i.e., in the horizontal plane.

In the present work we analyze the source of decoherence, and investigate the way it can be suppressed in the horizontal plane in a perfectly aligned ring by means of sextupole fields. We also consider the case of an imperfect ring, the vertical plane decoherence introduced by field imperfections, and its effect on the EDM estimator used in the Frequency Domain method.

SPIN DYNAMICS IN A STORAGE RING

The dynamics of a spin-vector $\boldsymbol{s}$ in a magnetic field $\boldsymbol{B}$ and an electrostatic field $\boldsymbol{E}$ is described by the Thomas-BMT equation. Its generalized version, accounting for the effect of the particle's electric dipole moment, can be written in the rest frame as:

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = \boldsymbol{s} \times (\boldsymbol{\Omega}_{MDM} + \boldsymbol{\Omega}_{EDM}), \tag{1a}$$

where the magnetic (MDM) and electric (EDM) dipole moment angular velocities $\boldsymbol{\Omega}_{MDM}$ and $\boldsymbol{\Omega}_{EDM}$

$$\boldsymbol{\Omega}_{MDM} = \frac{q}{m}\left[G\boldsymbol{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\boldsymbol{E}\times\boldsymbol{\beta}}{c}\right], \tag{1b}$$

$$\boldsymbol{\Omega}_{EDM} = \frac{q}{m}\frac{\eta}{2}\left[\frac{\boldsymbol{E}}{c} + \boldsymbol{\beta}\times\boldsymbol{B}\right]. \tag{1c}$$

In the above equations, m, q, $G = (g-2)/2$ are, respectively, the mass, charge, and magnetic anomaly of the particle; $\beta = \dfrac{v_0}{c}$ is its normalized speed; γ its Lorentz-factor. The EDM factor η is defined by the equation $d = \eta\frac{q}{2mc}$, in which d is the particle EDM, s its spin.

In the standard formalizm one operates with the spin transfer matrix [1, p. 4]

$$\boldsymbol{t}_R = \exp\left(-i\pi\nu_s\boldsymbol{\sigma}\cdot\bar{n}\right) = \cos\pi\nu_s - i(\boldsymbol{\sigma}\cdot\bar{n})\sin\pi\nu_s,$$

where $\nu_s = {}^{\Omega_s}/_{\Omega_{cyc}}$, the ratio of the particle's spin precession frequency to its cyclotron frequency, is termed *spin tune*, and $\bar{n}$, termed the *invariant spin axis*, defines the spin precession axis. They relate to the spin precession angular velocity as in $\boldsymbol{\Omega}_s = \omega_{cyc}\cdot\nu_s\bar{n}$.

ORIGIN OF DECOHERENCE

Spin decoherence in a particle beam is a result of the difference of the particles' spin precession angular velocities ($\boldsymbol{\Omega}_s$), which, in turn, is caused by the difference of their orbit lengths.

The longitudinal dynamics of a particle on the reference orbit in a storage ring is described by the system of equations:

$$\begin{cases} \frac{\mathrm{d}\varphi}{\mathrm{d}t} &= -\omega_{RF}\eta\delta, \\ \frac{\mathrm{d}\delta}{\mathrm{d}t} &= \frac{qV_{RF}\omega_{RF}}{2\pi h\beta^2 E}\sin\varphi. \end{cases} \tag{2}$$

In the equations above, φ and $\delta = {}^{\Delta p}/_{p_0}$ are the particle's phase and normalized momentum deviations from those of the reference particle; V_{RF}, ω_{RF} are the amplitude and frequency of the RF field; $\eta = \alpha_0 - \gamma^{-2}$ is the slip-factor, where α_0 is the momentum compaction factor defined by ${}^{\Delta L}/_L = \alpha_0\delta$, L being the orbit length; h is the harmonic number; E the total energy of the particle.

The solutions of this system form a family of ellipses in the (φ,δ) plane, all centered at point $(0,0)$. However, if one considers a particle involved in betatron oscillations, and uses a higher-order Taylor expansion of the momentum compaction factor $\alpha = \alpha_0 + \alpha_1\delta$, the first equation of the system transforms into: [2, p. 2579]

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\omega_{RF}\left[\left(\frac{\Delta L}{L}\right)_\beta + \left(\alpha_0 + \gamma^{-2}\right)\delta + \left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma^{-4}\right)\delta^2\right],$$

where $\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x Q_x + \varepsilon_y Q_y\right]$, is the betatron motion-related orbit lengthening; ε_x and ε_y are the horizontal and vertical beam emittances, and Q_x, Q_y are the horizontal and vertical tunes.

The solutions of the transformed system are no longer centered at the same single point. Orbit lengthening and momentum deviation cause an equilibrium-level momentum shift [2, p. 2581]

$$\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right], \tag{3}$$

where δ_m is the amplitude of synchrotron oscillations.

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The equilibrium energy level, associated with the momentum shift (3), termed the *effective Lorentz-factor*, is [3]

$$\gamma_{eff} = \gamma_0 + \beta_0^2 \gamma_0 \cdot \Delta\delta_{eq}, \tag{4}$$

where γ_0, β_0 are the Lorentz-factor and normalized speed of the reference particle.

From the T-BMT equation (1b), and the equation of cyclotron frequency in a magnetic field $\omega_{cyc} = \frac{q}{m}\frac{B}{\gamma}$, spin tune $\nu_s = \gamma_{eff} G$; therefore, the variation of orbit lengths over the beam particles casues a spin tune dispersion, and consequently — spin decoherence.

SEXTUPOLE FIELD SUPPRESSION OF DECOHERENCE

A sextupole field with gradient $S_{sext} = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2}$, has a two-fold effect on decoherence:

- it directly affects the particles' orbit lengths: $\left(\frac{\Delta L}{L}\right)_{sext} = \mp \frac{S_{sext} D_0 \beta_{x,y} \varepsilon_{x,y}}{L}$, and
- it modifies the lattice's momentum compaction factor: $\Delta\alpha_{1,sext} = -\frac{S_{sext} D_0^3}{L}$.

Above, $\beta_{x,y}$ are the horizontal, vertical beta-functions; $D(s) = D_0(s) + D_1(s)\delta$ is the dispersion function; $B\rho$ is magnetic rigidigy.

Consequently, in order to reduce decoherence associated with horizontal, and vertical betatron oscillations, and with synchrotron oscillations, correcting sextupoles must be placed, respectively, in the maxima of the β_x-, β_y- and D-functions.

SIMULATION OF DECOHERENCE SUPPRESSION

In the following simulation we used a Frozen Spin-type lattice represented in Figure 1. All optical elements are perfectly aligned, i.e., there's no spin precession about the radial coordinate system axis. Particles were injected at 270 MeV kinetic energy, which is the frozen spin energy for the deuteron in this lattice. Computations were performed using the COSY Infinity code. [4] Spin tune and invariant spin axis Taylor expansions were computed up to 5th order. Simulation results are presented in Figure 2.

SIMULATION OF DECOHRENCE IN AN IMPERFECT LATTICE

In this simulation we rotated the E+B elements about the optic axis by angles generated from the normal distribution $\alpha \sim N(0, 5\cdot 10^{-4})$ rad. The value $\sigma_\alpha = 5\cdot 10^{-4}$ is a realistic [3] element alignment error standard deviation. A rotation of an E+B element does not cause a perturbation of the closed orbit, since the net Lorentz force is kept constant.

Figure 3 shows the standard deviation of the radial spin-vector components in a particle bunch before and after turning on correcting sextupoles. Because an imperfect lattice

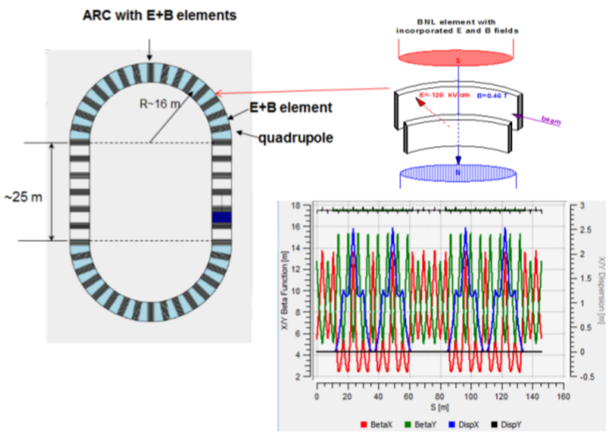

Figure 1: Frozen spin-type lattice used in simulations.

was used, the particles' spin-vectors precess in the vertical plane at a rapid pace, and hence the RMS-value of S_x is an oscillating function that does not show a long-term growth trend (slope estimate $(2\pm 2)\cdot 10^{-8}$ 1/sec); there's no decoherence in the horizontal plane. Use of correcting sextupoles further reduces the amplitude of the σ_{S_x}-oscillations.

Figure 4 shows the same statistic for the vertical components. One observes a presence of a long-term trend (slope estimate $(4.5\pm 0.6)\cdot 10^{-7}$ 1/sec) prior to turning on correcting sextupoles (Figure 4a). Sextupole correction does not reduce the amplitude of the σ_{S_y}-oscillations, but it does remove the long-term trend (Figure 4b, slope estimate $(5\pm 6)\cdot 10^{-8}$ 1/sec).

CONCLUSIONS

Spin decoherence is a major problem for any experiment aiming to measure the EDM of an elementary particle using the storage ring method, [5] as it drastically reduces the possible measurement cycle time. In the present work we have described the mechanism by which it arises, and a method by which it can be suppressed. We have shown that by means of this method the parabolic dependence of spin tune on the particle coordinate can be removed. The remaining linear decoherence effect observed in Figures 2a and 2c is suppressed by use of an RF-cavity.

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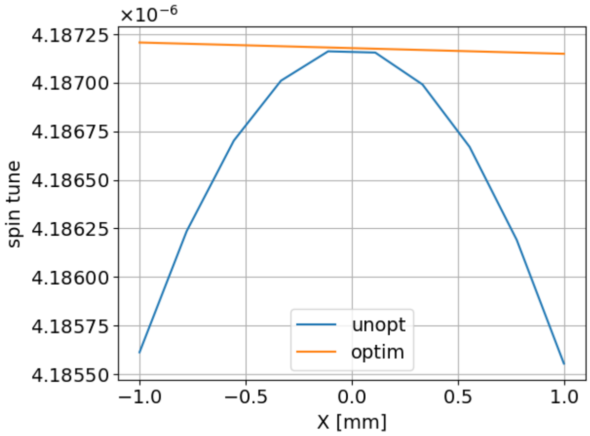
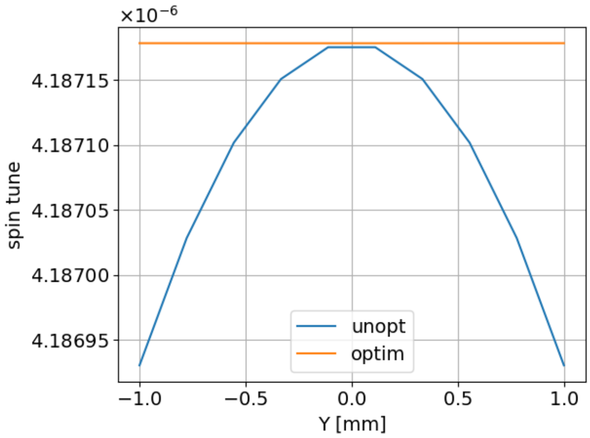
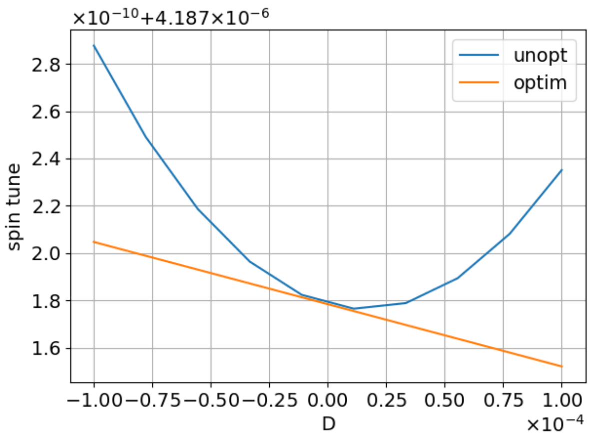


(a) For particles involved in horizontal betatron oscillation only

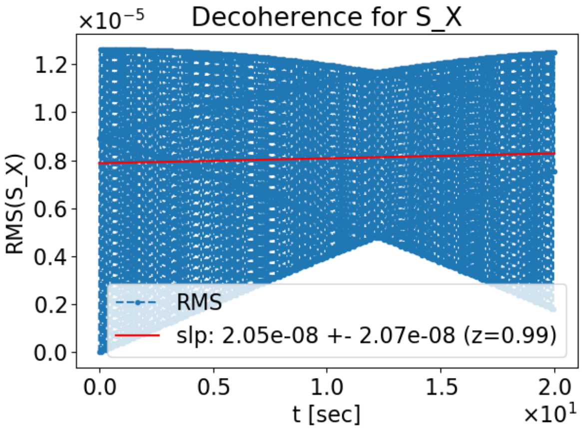


(b) For particles involved in vertical betatron oscillation only

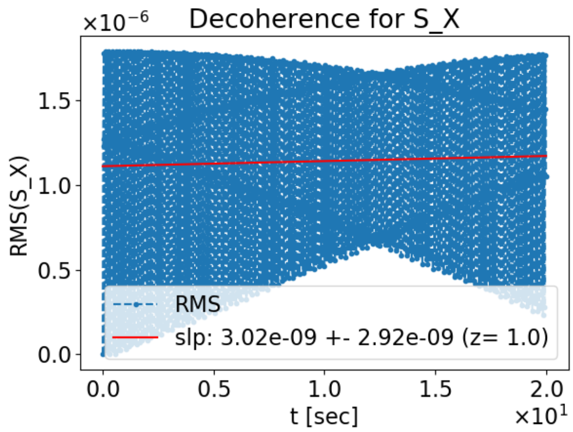


(c) For particles involved in synchrotron oscillation only

Figure 2: Spin tune a as function of initial horizontal, vertical, or momentum offset of the particle, with and without correcting sextupoles

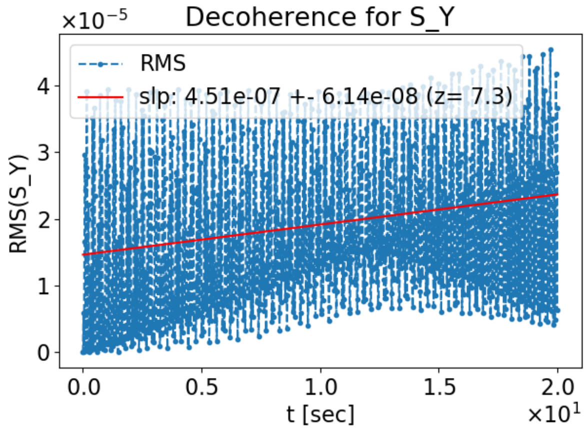


(a) Sextupoles turned off

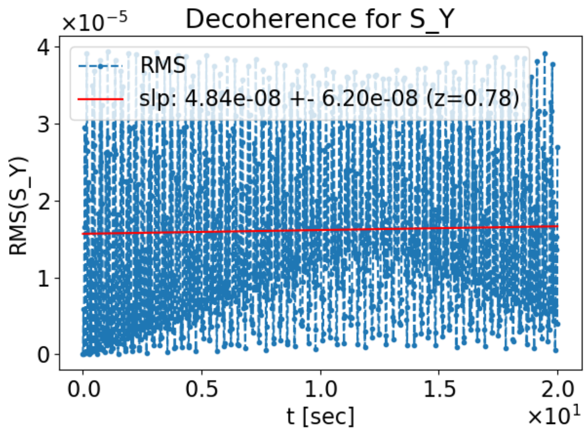


(b) Sextupoles turned on

Figure 3: Standard deviation of the radial spin vector components in a particle bunch prior and after turning on correcting sextupoles



(a) Sextupoles turned off



(b) Sextupoles turned on

Figure 4: Standard deviation of the vertical spin vector components in a particle bunch prior and after turning on correcting sextupoles