Spin decoherence in a Frozen Spin lattice, its suppression and effect on the Frequency Domain EDM statistic

Alexander Aksentev

January 15, 2019

Basic info

T-BMT equation

A spin vector placed into a magnetic field is subject to precession described by the T-BMT equation (rest frame):

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \mathbf{s} \times (\mathbf{\Omega}_{MDM} + \mathbf{\Omega}_{EDM})\,, \tag{1a}$$

where MDM and EDM angular velocities $\mathbf{\Omega}_{MDM}$ and $\mathbf{\Omega}_{EDM}$

$$\mathbf{\Omega}_{MDM} = \frac{q}{m}\left[G\mathbf{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\mathbf{E} \times \boldsymbol{\beta}}{c}\right], \tag{1b}$$

$$\mathbf{\Omega}_{EDM} = \frac{q}{m}\frac{\eta}{2}\left[\frac{\mathbf{E}}{c} + \boldsymbol{\beta} \times \mathbf{B}\right]. \tag{1c}$$

Basic info

Frozen Spin condition

From eq. (1b) one can observe that the direction of the spin vector can be fixed relative to the momentum vector, i.e., $\boldsymbol{\Omega}_{MDM} = \mathbf{0}$.
This is called the Frozen Spin condition.
Why is it important?

- In a storage ring, of necessity, $\boldsymbol{\Omega}_{MDM} \perp \boldsymbol{\Omega}_{EDM}$
- Hence, the measured net angular velocity $\omega \propto \sqrt{\Omega^2_{MDM} + \Omega^2_{EDM}} \approx \Omega_{MDM} + \frac{\Omega^2_{EDM}}{2\Omega_{MDM}}$ (EDM is a second-order effect)
- When the FS condition is fulfilled, the only remaining (if any) $\boldsymbol{\Omega}_{MDM} \parallel \boldsymbol{\Omega}_{EDM}$
- Meaning that the EDM-related spin precession frequency shift becomes a first-order effect

Basic info

Spin tune and Invariant spin axis

The standard formalism operates with spin tune ν_s and invariant spin axis $\bar{n}$, determining the one-turn spin transfer matrix

$$\mathbf{t}_R = \exp\left(-i\pi\nu_s\boldsymbol{\sigma}\cdot\bar{n}\right) = \cos\pi\nu_s - i(\boldsymbol{\sigma}\cdot\bar{n})\sin\pi\nu_s.$$

They relate to the spin precession angular velocity as in

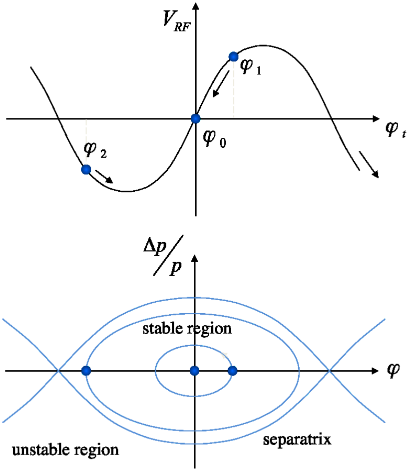
$$\boldsymbol{\Omega}_s = 2\pi f_s\bar{n} = 2\pi f_R\nu_s\bar{n}.$$

The invariant spin axis (a.k.a. the spin precession axis) is called such because this is the only direction in which the polarization of a beam survives.

Decoherence origins

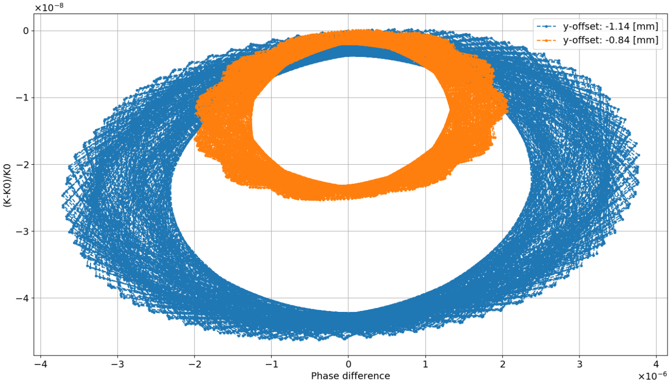
Phase stability principle

- In a lattice utilizing an RF cavity, particles travel in bunches
- Therefore, the Phase Stability Principle demands that a particle with a **longer orbit** should have a **higher equilibrium-level energy**, so that it doesn't fall from the bunch



An actual bunch

COSY Infinity 3rd order Taylor model

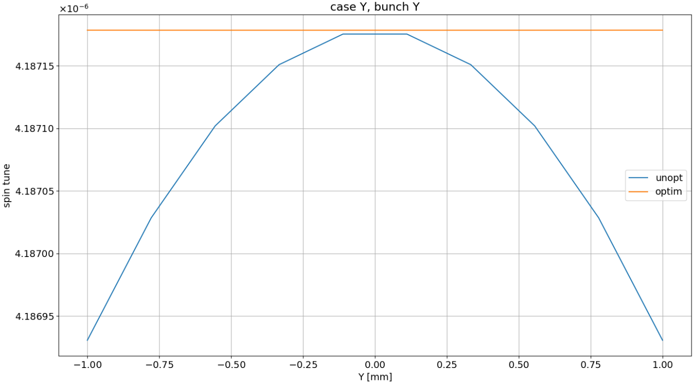


Spin tune decoherence

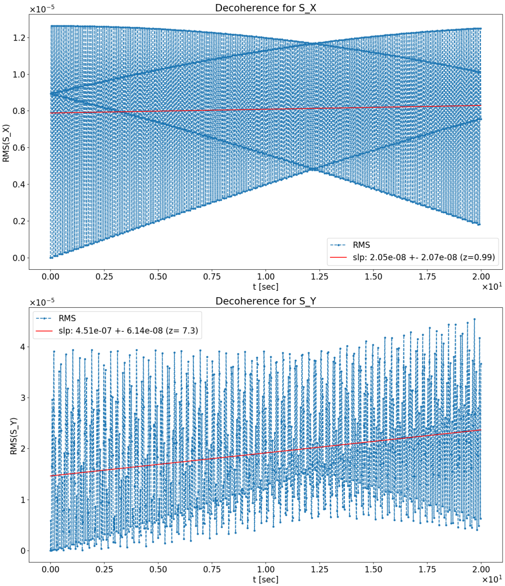
- Orbit lengthening occurs as a result of:
  - Betatron motion: $\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x Q_x + \varepsilon_y Q_y\right]$
  - Initial momentum deviation: $\left(\frac{\Delta L}{L}\right)_\alpha = \alpha_0\delta + \alpha_1\delta^2 + \ldots$
- Causes an equilibrium-level momentum shift $\Delta\delta_{eq}$
- The particle's Lorentz factor after accounting for the orbit lengthening effects: $\gamma_{eff} = \gamma_0 + \beta_0^2\gamma_0\Delta\delta_{eq}$
- Spin tune is proportional to the particle's Lorentz factor: $\nu_s = \gamma G$
- Spin tune dispersion in a particle bunch is one mechanism of depolarization (spin tune decoherence)

Spin tune decoherence

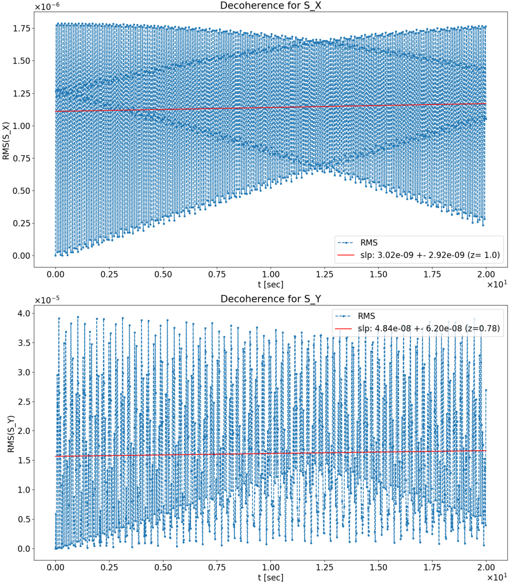
- Y-plane flat bunch
- Perfect FS lattice, only Y-family sextupoles optimized.



Unoptimized imperfect lattice



Optimized imperfect lattice



Sextupole decoherence suppression theory

A sextupole of strength

$$S_{sext} = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2},$$

has a two-fold effect on decoherence:

- Momentum compaction factor effect: $\Delta\alpha_{1,sext} = -\frac{S_{sext} D_0^3}{L}$
- Orbit length effect: $\left(\frac{\Delta L}{L}\right)_{sext} = \mp\frac{S_{sext} D_0 \beta_{x,y}\varepsilon_{x,y}}{L}$

Where

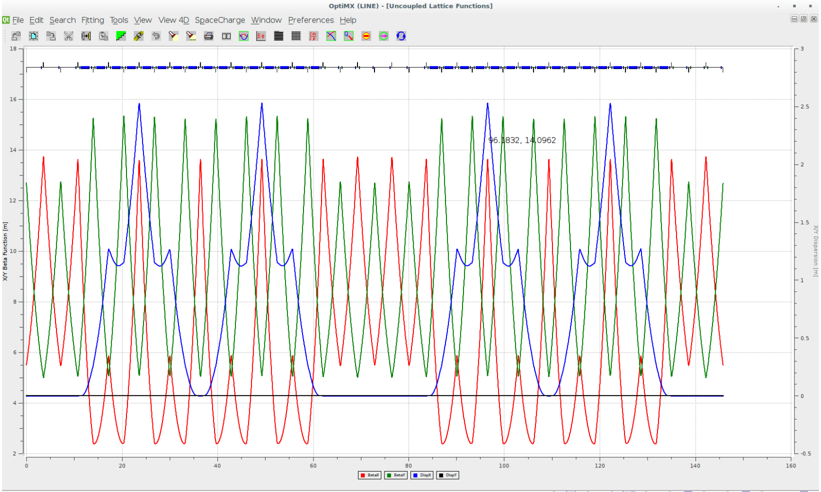
$$D(s,\delta) = D_0(s) + D_1(s)\delta$$

is the dispersion function

Decoherence suppression theory

Sextupole placement

Sextupoles are placed in the maxima of the $\beta_x(s)$, $\beta_y(s)$, or $D_0(s)$ functions, depending on the plane in which we want to suppress decoherence (X-, Y-, and D-family sextupoles)

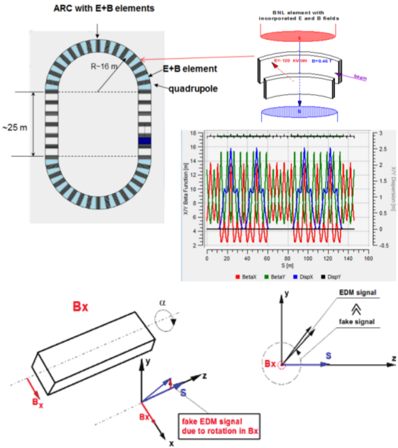


Simulation setup: the beam

- Y-plane flat, normally-distributed beam of 30 particles: $y \sim N(y_0, 0.1)$ mm
- $y_0 \in [-1, +1]$ mm (10 beams)
- Initial spin vector $\mathbf{S}(t = 0) = (0, 0, 1)$
- Kinetic energy 270.0092 MeV (strict FS)

The lattice

- Imperfect Frozen Spin lattice
- E+B elements tilted about the optic axis by $\theta \sim N(0, 5 \cdot 10^{-4})$ rad
- Y-family sextupole gradient $\mathrm{GSY} \in \mathrm{GSY0} \pm 5 \cdot 10^{-3}$, $\mathrm{GSY0} = -2.5 \cdot 10^{-3}$ is the optimal setting for the perfect lattice

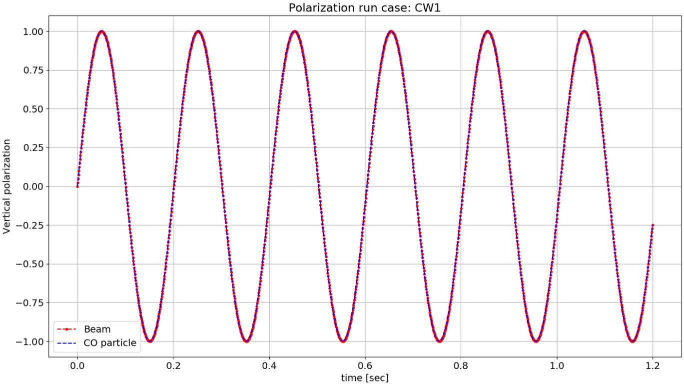


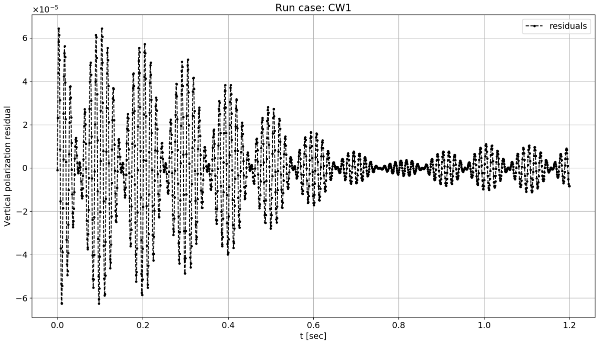
Tracking parameters and written data

- Tracking for $1.2 \cdot 10^6$ turns (approx. 1.2 sec); data written every 800 turns
- Normal Form-computed (COSY Infinity procedure TSS) spin tune and invariant spin axis: $(\nu_s, \bar{n}_x, \bar{n}_y, \bar{n}_z)$
- Spin components (S_X, S_Y, S_Z)
- Phase space components (X, A, Y, B, T, D)

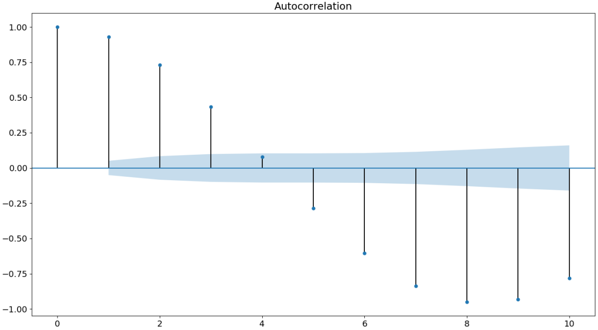
Computed data

- Polarization $\mathbf{P} = \frac{\sum_i \mathbf{S}_i}{|\sum_i \mathbf{S}_i|}$
- Model $f(t; a, f, \phi) = a \cdot \sin(2\pi \cdot f \cdot t + \phi)$
- Fitted parameters: $(\hat{a}, \hat{f}, \hat{\phi})$





Run case: CW1
×10⁻⁵
residuals
Vertical polarization residual
t [sec]



Autocorrelation

Spin precession axis effect

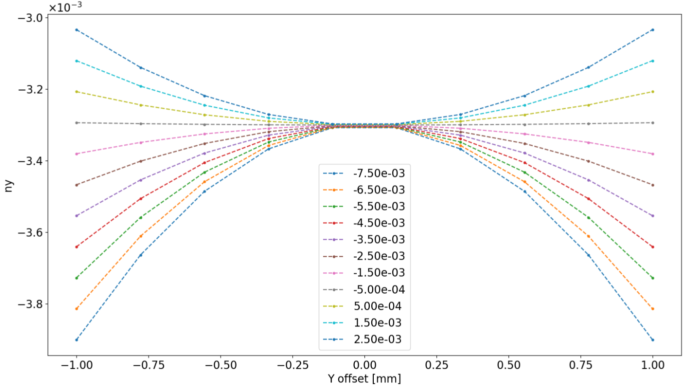


Figure 3: SPA component $\bar{n}_y$ as a function of the vertical particle offset, sextupole gradient value.

SPA: zoom

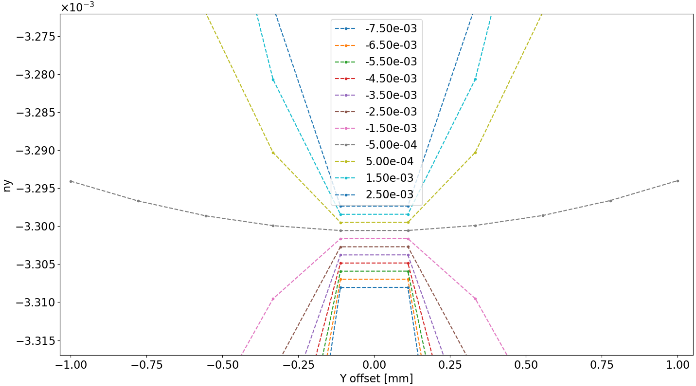


Figure 4: Zoom of Figure 3. SPA component $\bar{n}_y$ (as well as $\bar{n}_x$) is a parabola in the neighborhood of the reference orbit at the optimal GSY value, unlike ν_s, which is **linear**.

Spin tune effect

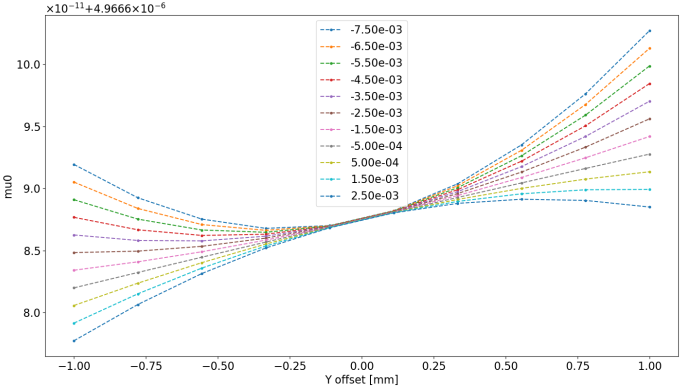


Figure 5: Spin tune ν_s Taylor expansion contains a linear term insensitive to sextupole optimization.

Frequency estimate effect

Fitted data: Polarization.

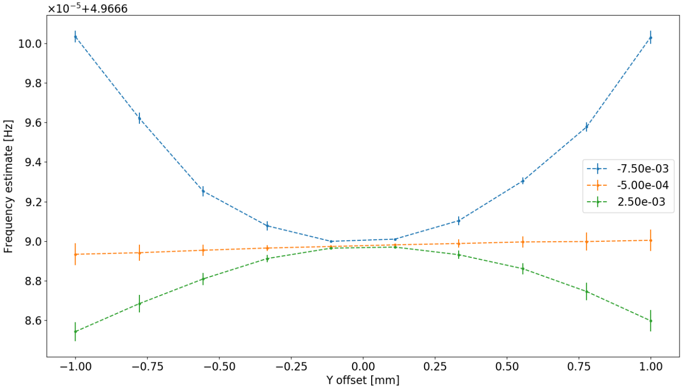


Figure 6: Frequency estimate for the optimal sextupole gradient (orange) and the values at the ends of the searched range.

Frequency estimate: zoom

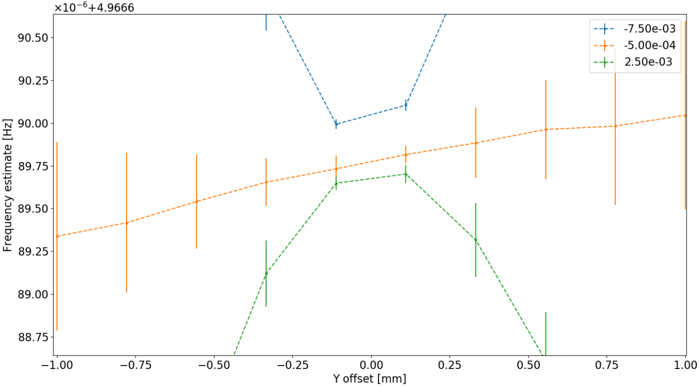


Figure 7: Zoom of Figure 6. Frequency estimate depends on the offset value linearly, like ν_s, and unlike $\bar{n}_y$.

Individual particle ST+SPA structure

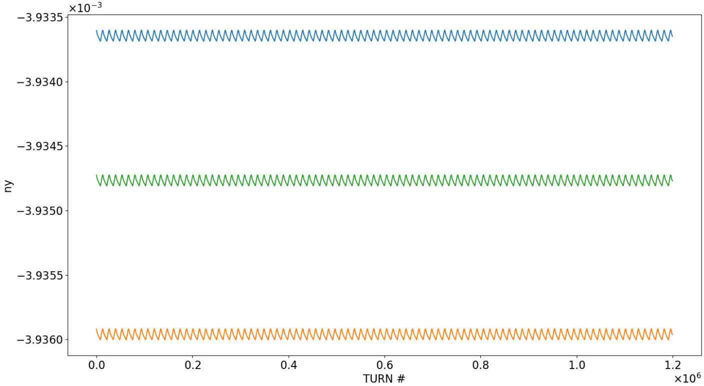
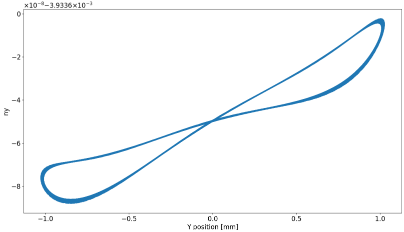
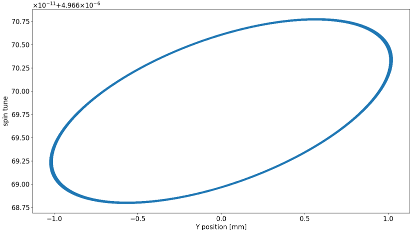


Figure 8: SPA component $\bar{n}_y$ for **particles** with offsets: [1.02749, 1.02937, 1.02840] mm. We observe small-amplitude rapid oscillations about an average level. This average level changes parabolically with the vertical offset (Figure 14 below). The rapid oscillations are due to betatron motion (Figures 10, and 12).

Vertical betatron motion dependence



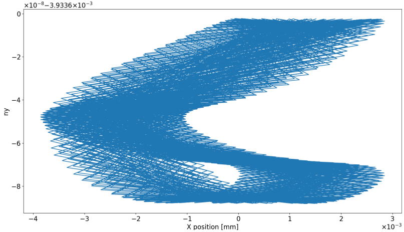
a: SPA component $\bar{n}_y$ as a function of the vertical particle position.



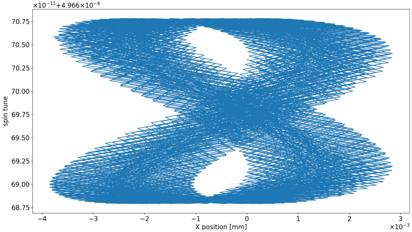
b: Spin tune ν_s as a function of the vertical particle position.

Figure 10: Particle spin precession frequency depending on its vertical position. The observed non-functionality of the parameters on the y-position os due to the dependence on the x-position as well, which also oscillates at a small amplitude (Figure 12).

Horizontal betatron motion dependence



a: SPA component $\bar{n}_y$ as a function of the horizontal particle position.



b: Spin tune ν_s as a function of the horizontal particle position.

Figure 12: Particle spin precession frequency as a function of its radial position.

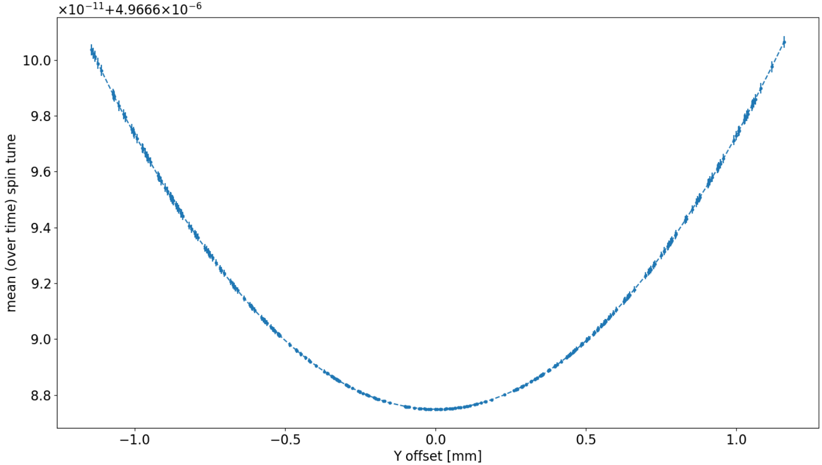


Figure 13: Mean level of spin tune as a function of beam offset

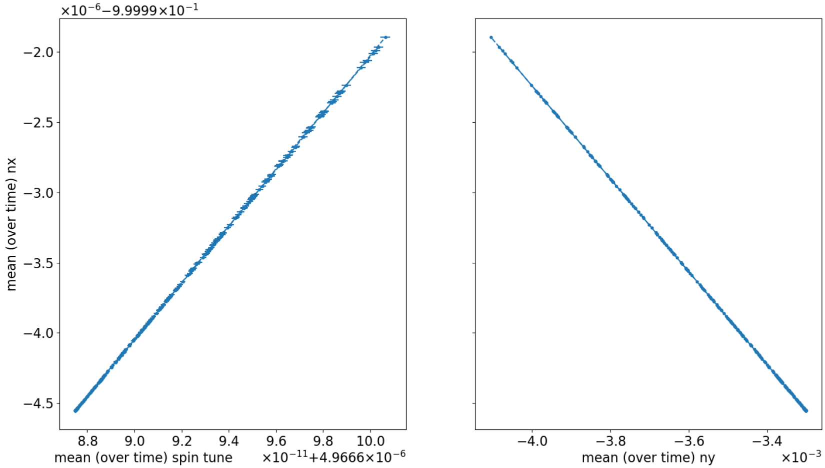


Figure 14: Mean SPA and ST levels versus each other. Observe a strong correlation.

Frequency estimate offset dependence

Fitted data: spin.

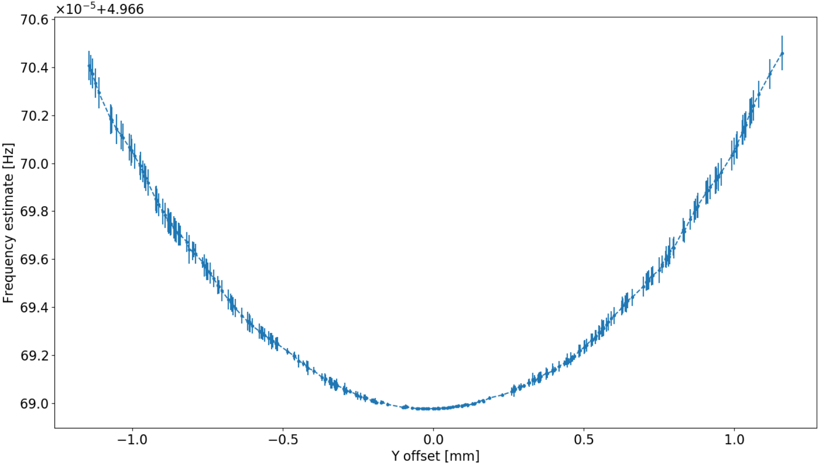


Figure 15: $\hat{f}$ as a function of the initial vertical offset.

Spin tune dependence

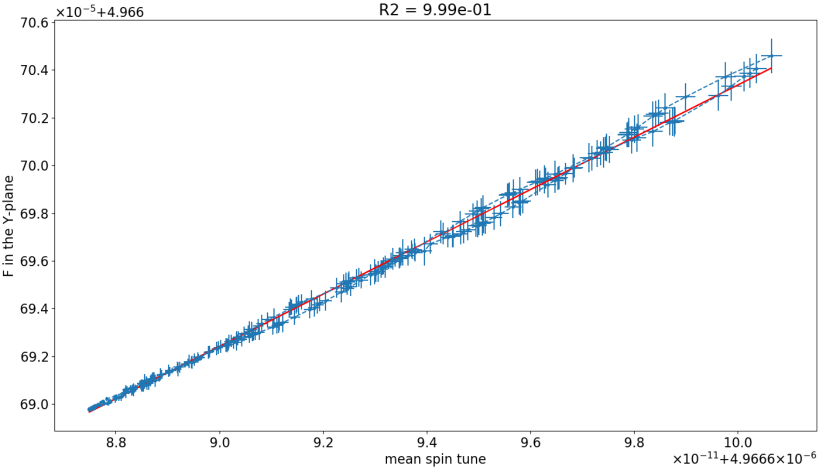


Figure 16: $\hat{f}$ as a function of the mean spin tune level.

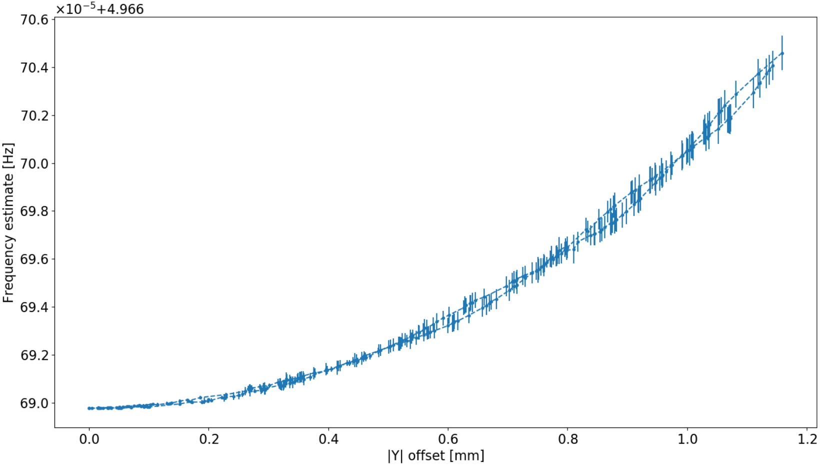


Figure 17: $\hat{f}$ as a function of the absolute value of the initial vertical offset.

Thank you!