SPIN MOTION PERTURBATION EFFECT ON THE EDM STATISTIC IN THE FREQUENCY DOMAIN METHOD

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Abstract

The spin precession axis of a particle involved in betatron motion precesses about the invariant spin axis defined on the closed orbit (CO). This precession can be observed in polarization data as a rapid, small-amplitude oscillation on top of the major effect oscillation caused by the precession of spin about the CO axis. The frequency of this latter oscillation is used in the Frequency Domain methodology as the EDM observable. It is estimated by fitting polarimetry data by a sine function; the rapid oscillations, therefore, constitute a model specification error. This model error will introduce a bias into the frequency estimate. In the present work we investigate how this bias changes depending on the beam revolution direction, its stability over time, and the EDM estimate error introduced by it.

FREQUENCY DOMAIN METHODOLOGY

Frequency Domain (FD) [1] is a Storage Ring method of search for the Electric Dipole Moment (EDM) of a fundamental particle. [2] It belongs to the Frozen Spin [3] category of such methods, i.e., the Magnetic Dipole Moment (MDM) component of spin precession is minimized. However, the original Frozen Spin method proposed in [3] is a Space Domain method [4, p. 4]: inferences about the EDM are drawn from the change of orientation of the polarization vector, as measured by the angle between its initial and final orientations. This approach has the following problems: *a*) it puts very stringent constraints on the precision of the accelerator optical element alignment, and *b*) it poses a challenging task for polarimetry. [5, p. 6]

The former is to minimize the magnitude of the vertical plane MDM precession frequency: [3, p. 11]

$$\omega_{syst} \approx \frac{\mu\langle E_v\rangle}{\beta c\gamma^2}, \tag{1}$$

induced by field imperfections. The latter is due to the requirement of detecting a change of about $5\cdot10^{-6}$ to the cross section ε_{LR} in order to get to the EDM sensitivity level of 10^{-29} $e\cdot cm$. [3, p. 18]

EDM search methods in the Frequency Domain circumvent the above problems: EDM inferences are based on the rate of change of the aforementioned polarization orientation angle. The polarization vector is made to roll about a nearly-constant, definite direction vector $\bar{n}$, with an angular velocity that is large enough for the beam polarization to be easily measureable at all times. This "Spin Wheel" may be externally applied [6], or otherwise the machine imperfection fields may be utilized for the same purpose (1) [1]. The latter is made possible by the fact that ω_{syst} changes sign when the beam revolution direction is reversed. [3, p. 11]

The frequency of oscillation of the vertical polarization component P_y is estimated via a fit of the polarimetry data to the model

$$f(t) = a\cdot\sin(\omega\cdot t+\delta). \tag{2}$$

PROBLEM STATEMENT

Consider the case of a single particle beam. The solution of the T-BMT equation for the vertical spin-vector component has the general form

$$s_y(t) = \sqrt{\left(\frac{\omega_y\omega_z}{\omega^2}\right)^2+\left(\frac{\omega_x}{\omega}\right)^2}\cdot\sin\left(\omega\cdot t+\delta\right), \tag{3}$$

where $\boldsymbol{\omega} = (\omega_x,\omega_y,\omega_z)$ is a function of time as a result of betatron motion.

Using $\boldsymbol{\omega} = 2\pi f_{rev}\nu_s\bar{n}$ [7, p. 4], equation (3) can be reformulated in terms of spin tune ν_s and invariant spin axis $\bar{n}$:

$$s_y(n_{turn}) = \sqrt{\left(\bar{n}_y\bar{n}_z\right)^2+\bar{n}_x^2}\cdot\sin\left(2\pi\nu_s\cdot n_{turn}+\delta\right), \tag{4}$$

where $\bar{n}=\bar{n}(n_{turn})$ and $\nu_s=\nu_s(n_{turn})$ are functions of the turn number n_{turn}.

Sufficiently large variation of $\bar{n}$ and/or ν_s can lead to model specification systematic error. Variation in ν_s is especialy problematic in this regard, as it directly affects the phase of the signal; however, this problem can be solved by the introduction of sextupole fields into the system, as described in [8]. In this paper we will, therefore, be concerned only with the $\bar{n}$ variation.

SIMULATION

The simulation setup was as follows: a particle, offset from the design orbit in the vertical direction by 0.3 mm, is injected multiple times into an imperfect Frozen Spin lattice [9] utilizing sextupoles for the reduction of spin decoherence caused by vertical plane betatron oscillations [8]. Lattice imperfections are simulated by rotations of the E+B spin rotator elements. Imperfections introduced this way do not perturb the design orbit.

Each injection, the rotation angles are randomly generated from the normal distribution $\alpha\sim N(\mu_i, 5\cdot10^{-6})$ rad, $i\in\{1,\dots,11\}$, where μ_i varies in the range $[-5\cdot10^{-5},+5\cdot10^{-5}]$. The non-zero expectation values μ_i simulate the application of a Koop Spin Wheel. [6] The magnitudes of μ_i and σ_α are chosen for effect detalization purposes.

Spin tracking is done in COSY Infinity [10], for $1.2\cdot10^6$ turns; each 800 turns ν_s and $\bar{n}$ are computed (by means

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of procedure TSS [11, p. 41]) at the phase space point occupied by the particle at the time, giving us the series $(\nu_s(n), \bar{n}(n))$. The corresponding spin vector components $(s_x^{trk}(n), s_y^{trk}(n), s_z^{trk}(n))$, computed by the tracker (procedure TR [11, p. 41]), constitute the second series used in the analysis.

ANALYSIS

Using the first series data, we generated the expected $s_y^{gen}(t)$ "generator" series according to equation (4), as well as the "ideal" series s_y^{idl}, in which we assumed constant values of $\nu_s = \langle \nu_s(t) \rangle$ and $\bar{n} = \langle \bar{n}(t) \rangle$.

Our hypothesis is that the particle's betatron motion should introduce a mismatch between the sinusoidal model (2) and tracker data, by varying the direction of the spin precession axis $\bar{n}$, and hence the amplitude of the fitted signal. The "ideal" series serves as the baseline of our analysis, as it's a perfect match to the model; the "generator" series incorporates $\bar{n}$ variation, still remaining within the confines of the model. The "tracker" series is the closest approximation to real measurement data.

To compare these series with one another, we *a)* computed and analyzed the residuals $\epsilon_1(t) = s_y^{gen}(t) - s_y^{idl}(t)$ and $\epsilon_2(t) = s_y^{trk}(t) - s_y^{idl}(t)$; *b)* fitted model (2) to the three time series and compared its goodness-of-fit; *c)* computed the standard deviations of $\bar{n}$ components at each spin wheel strength.

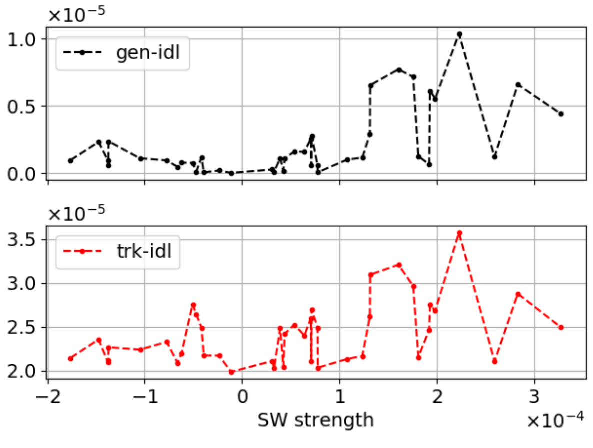
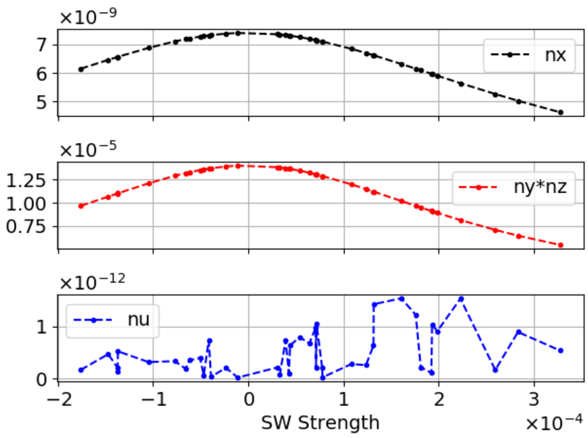


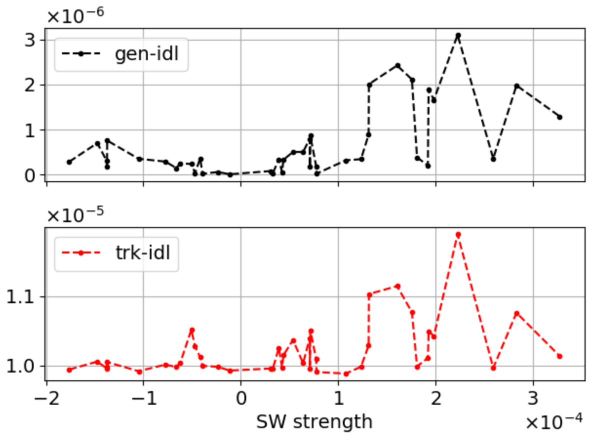
Figure 1: Amplitudes of the time series' comparator residuals as a function of the relative Spin Wheel strength. Top panel: residual ϵ_1; bottom panel: residual ϵ_2

What we observe in Figure 1 is that the "generator" series is nearly identical to the "ideal" series, with $\epsilon_1 \leq 6 \cdot 10^{-6}$ during run time, while the "tracker" series deviates from it at the level $\epsilon_2 \leq 3.5 \cdot 10^{-5}$. This discrepancy, which can get as high as four order of magnitude at higher SW strengths (not shown here), between ϵ_1 and ϵ_2 has no explanation as of yet.

In Figure 2b we see that both residual standard deviations exhibit the same pattern as the standard deiviation of ν_s



(a) Of the $\bar{n}$ components



(b) Of the comparator residuals. Top panel: residual ϵ_1; bottom panel: residual ϵ_2

Figure 2: Standard deviations versus relative Spin Wheel strength

Table 1: Model parameter estimates

Series	Par.	Value	St.Error	AIC
s_y^{idl}	$\hat{f}$	74.452466549766214	$6 \cdot 10^{-15}$	-86246
	$\hat{a}$	0.99841729771960	$1 \cdot 10^{-14}$	
	$\hat{\delta}$	3.14159265358978	$2 \cdot 10^{-14}$	
s_y^{gen}	$\hat{f}$	74.452466548	$1 \cdot 10^{-9}$	-49917
	$\hat{a}$	0.998417300	$2 \cdot 10^{-9}$	
	$\hat{\delta}$	3.1415926564	$4 \cdot 10^{-9}$	
s_y^{trk}	$\hat{f}$	4.3276781	$6 \cdot 10^{-7}$	-30665
	$\hat{a}$	0.998418	$1 \cdot 10^{-6}$	
	$\hat{\delta}$	3.141589	$3 \cdot 10^{-6}$	

(Figure 2a, bottom panel), but not the $\bar{n}$ components. This is an indication that frequency variation is a much more significant factor in the mismatch between model (2) and tracker data, than is the presumed amplitude variation due to the change of orientation of $\bar{n}$.

Table 1 characterizes the fit model's goodness-of-fit with respect to the time series.

CONCLUSIONS

The question of the influence of betatron motion on the EDM statistic in the FD method should be considered in view of three circumstances:

1. The signal amplitude oscillations are small. They occur at the 10^{-4} level (when $\sigma_\alpha = 5 \cdot 10^{-4}$), whereas the expected polarization measurement error is on the order of percents. This means the superposition of this systematic error with the random measurement error will exhibit no statistically-significant systematicity.

2. The correllation coefficient between the amplitude and frequency estimates is not significant. The amplitude oscillations affect the $\hat{a}$-estimate foremost; their effect on the $\hat{\omega}$-estimate is secondary, and is described by the correlation coefficient. Since it is less than 10%, even if the oscillations happen to be strong enough to affect the amplitude estimate, their effect on the frequency estimate will be reduced by at least a factor of 10.

3. This systematic effect is controllable. And this point is the major advantage of the FD methodology. By applying an external Spin Wheel, the $\bar{n}$ oscillations can be continuously minimized as much as necessary, without changing the experiment pattern.

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