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JÜLICH FORSCHUNGSZENTRUM

Model of statistical errors in the search for the deuteron EDM in the storage ring

03/20/17

Methodology

When put into an electromagnetic field, the particle spin begins to precess according to the T-BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

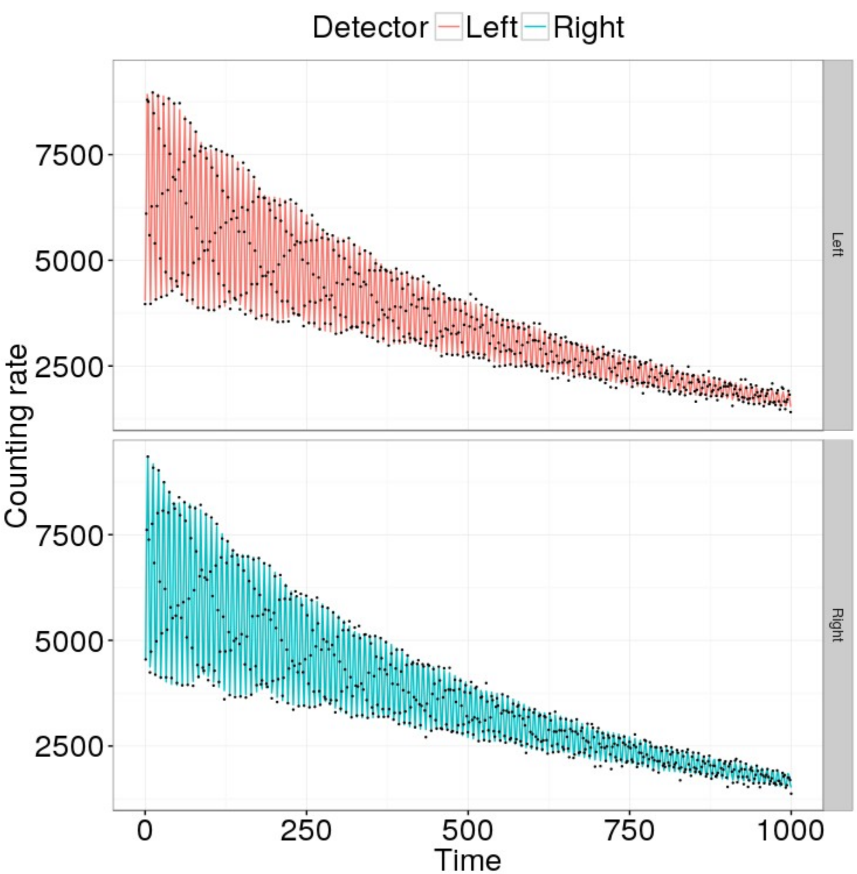
$$\vec{\Omega} = -\frac{e}{m}\left\{ \underbrace{G\vec{B} + \left(\frac{1}{\gamma^2 - 1} - G\right)\left(\vec{\beta} \times \vec{E}\right)}_{\text{MDM}} + \underbrace{\frac{\eta}{2}\left(\vec{E} + \vec{\beta} \times \vec{B}\right)}_{\text{EDM}} \right\}$$

By measuring the beam's polarization, we can determine the frequency

$$\vec{\Omega}^{\pm} = \vec{\Omega}_{MDM} \pm \vec{\Omega}_{EDM}$$

Comparing the CW vs CCW frequencies, determine Ω_{EDM}

Detector counting rate



$$\tilde{N}(t) = N_0(t)\left[1 + P\cdot e^{-t/\tau_d}\cdot\sin(\omega t+\phi)\right] + \varepsilon_t$$

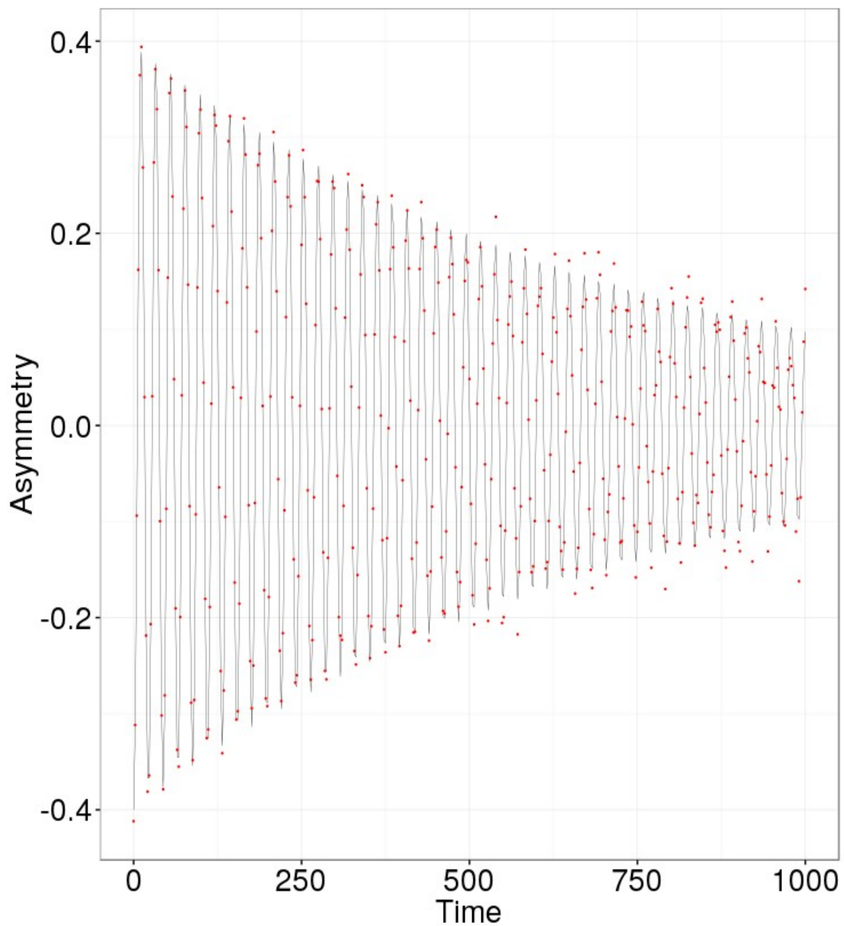
Number of counts is Poisson distributed, hence

$$\sigma^2_{\tilde{N}_0} = N_0(t)$$

$$\sigma_{N_0}(t) = \sigma_{\tilde{N}_0}(t)/\sqrt{n_{c/\epsilon}}$$

$$\frac{\sigma_{N_0}(t)}{N_0(t)} \propto \frac{1}{\sqrt{\Delta t_\epsilon}}\cdot\exp\left(\frac{t}{2\tau_b}\right)$$

Cross section asymmetry



A measure of polarization

Definition: $$A = \frac{N_L - N_R}{N_L + N_R}$$

Model: $$A(t) = A(0) \cdot e^{\lambda t} \cdot \sin(\omega \cdot t + \phi)$$

$$\sigma_A^2(t) \approx \frac{1}{2 N_0(t)}$$

Error: $$\sigma^2[\hat{\omega}] = \frac{\sigma^2[\varepsilon]}{\sum_i f(t_i) \cdot \sigma_w^2[t]}$$

Standard Error

- No decoherence
- Uniform sampling

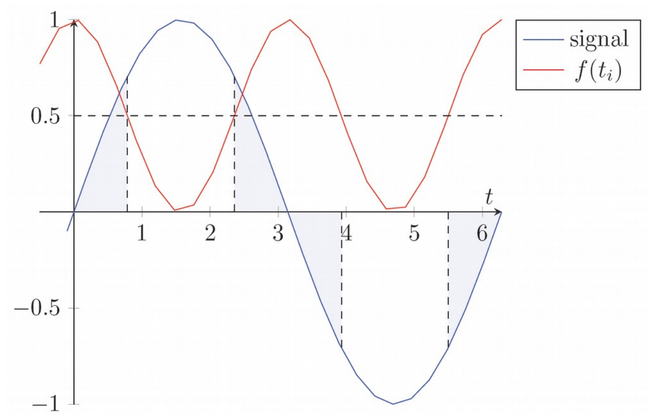
$$\sum f(t_i) = N \cdot \tilde{x}$$

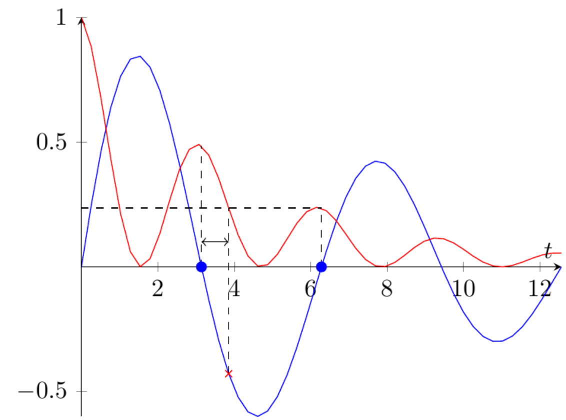
$$\tilde{x} = \frac{1}{2}(N_0 P)^2$$

$$\sigma_w^2 = \frac{T^2}{12}$$

$$\sigma_{\hat{\omega}}^2 = \frac{24}{N(PT)^2} \cdot \left(\frac{\sigma[\varepsilon]}{N_0} \right)^2$$

Limiting factors





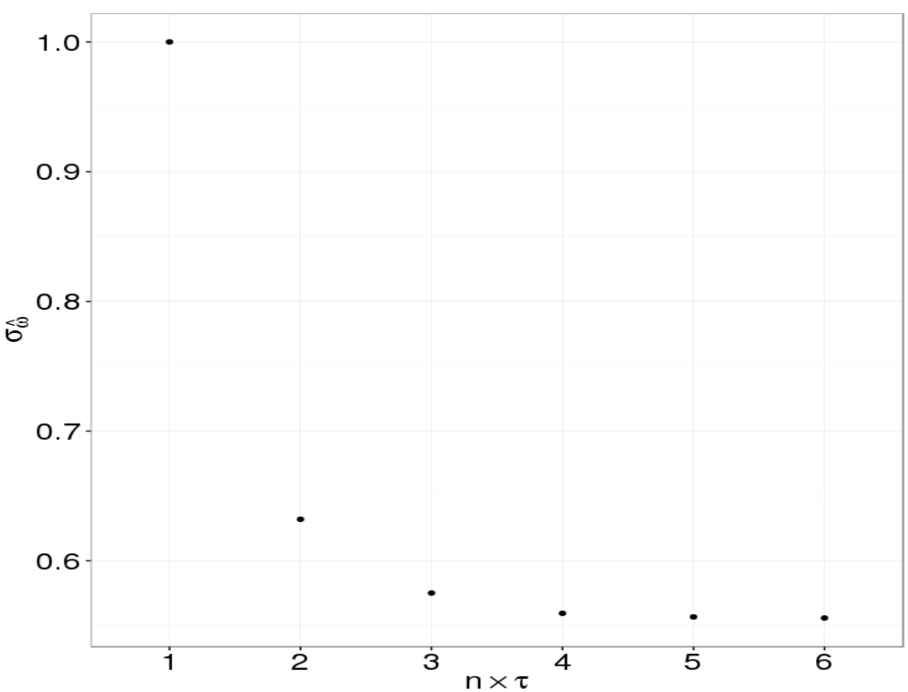
- Sample Fisher information can be increased by sampling during rapid change
- Limited by polarimetry sampling rate

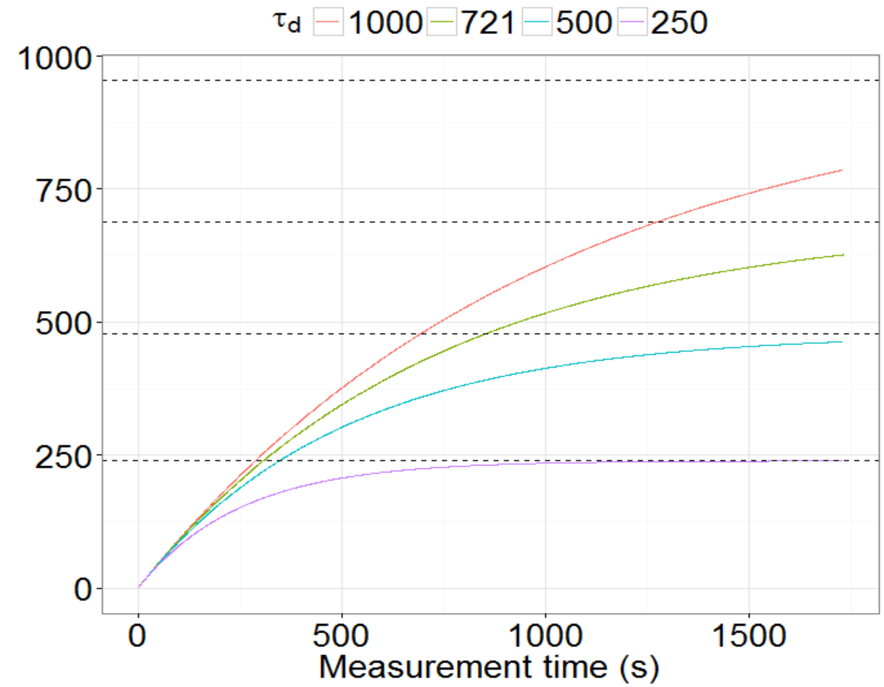
- Point Fisher information falls exponentially due to decoherence
- Can't economize the beam too much

Time-spread

$$\sum f(t_i) = n_{\varepsilon/zc} \cdot x_{01} \cdot \frac{\exp\left(-\frac{\pi}{\omega\tau_d} n_{zc}\right) - 1}{\exp\left(-\frac{\pi}{\omega\tau_d}\right) - 1}$$

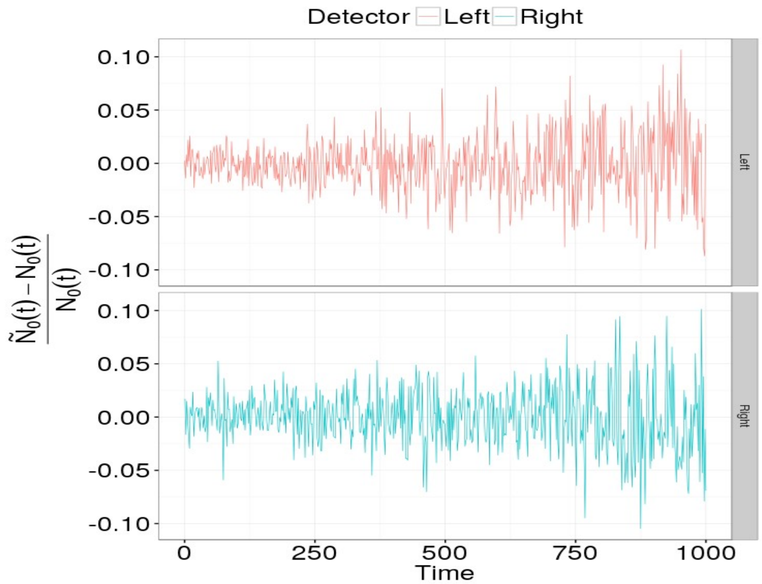
$$t(z) = \tau_d \cdot \ln\left(\frac{1}{1-z}\right)$$

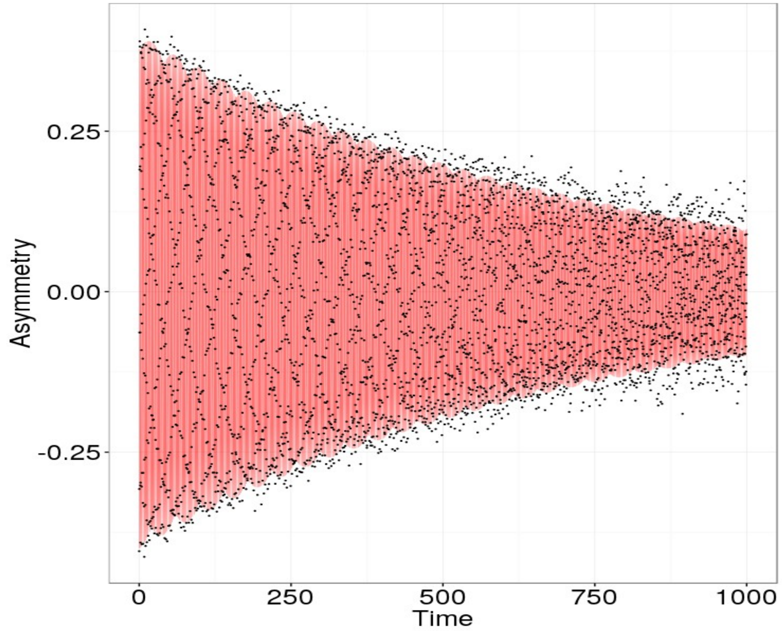




FI limit (%)	Reached ($\times\tau_d$)	SNR@3% error
95	3.0	1.7
90	2.3	3.3
70	1.2	10.0

Simulation





- Uniform sampling
- 75% of the beam ($7.5 \cdot 10^8$ useful scatterings)
- 3% initial counting rate error

- Standard error $7.55 \cdot 10^{-7}$ rad/sec
- If ω is known down to 10^{-6}, can improve the result by 30%

Thank You