Comparison of Frozen Spin-type EDM search methods

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Considered methods

- BNL Frozen Spin
- I.Koop's Spin Wheel
- Frequency Domain Method

BNL FS

- Observation of the vertical polarization component[1] $\Delta P_V \approx P \cdot \omega_{EDM} \cdot t$ (making it a Space Domain method)
- Cross section asymmetry $\varepsilon_{LR} \approx 5 \cdot 10^{-6}$ for smallest practical values of (horizontal plane) ω_{MDM}[2]

* Challenging task for polarimetry[3]

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[1] D. Anastassopoulos et al., *AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the* 10^{-29} $e \cdot cm$ *level.* p. 9.

[2] Ibid., p. 18.

[3] Mane, "Spin Wheel", p. 6.

BNL FS

Systematics

- Only known first-order systematic effect pertaining to the spin dynamics is the existence of $\langle E_V \rangle \neq 0$[4]
- Error frequency $\omega_{syst} \approx \frac{\mu \langle E_V \rangle}{\beta c \gamma^2}$ changes sign when reversing the beam circulation direction (CW/CCW)[5]
- However, at practical values of element alignment error, $\omega_{syst} \gg \omega_{EDM}$, hence $P_V = P\frac{\omega_{EDM}}{\omega}\sin(\omega t + \Theta_0) \not\approx P\omega_{EDM} t$; a Space Domain method is inapplicable under such conditions

* At $\langle E_V \rangle \to 0$, Space Domain methods are vulnerable to the geometric phase error[6]

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[4] D. Anastassopoulos et al., *AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the* 10^{-29} *e · cm level.* p. 10.

[5] Ibid., p. 11.

[6] V. Anastassopoulos et al., *A Storage Ring Experiment to Detect a Proton Electric Dipole Moment*, p. 6.

Geometric phase error

- Caused by the non-commutativity of rotations
- Formulated in the angular momentum language, it means the *absence of a definite orientation* of the spin precession axis (SPA): $\bar{n} \to 0$

  * Call that the *3D Frozen Spin* state
- 3D FS is unstable: any stray magnetic field can tilt the precession plane

FS-type methodology

Conditions of success

- One must always have a definite direction of the SPA
- Measurements must be done in the frequency domain

These conditions are satisfied by two methods:

- I.Koop's "Spin Wheel"
- Y.Senichev's "Frequency Domain"

(Both of which belong to the Frequency Domain category.)

Spin Wheel

The Spin Wheel is great; it satisfies both success conditions.

- Apply a radial magnetic field of strength B_x sufficient to turn the spin vector about the $\hat{x}$-axis with a frequency of 1 Hz
- $\omega_{B_X} \parallel \omega_{EDM}$ hence $\omega_{net} \propto \omega_{EDM} + \omega_{B_X}$[7]
- EDM effect $\hat{\omega}_{EDM} = \frac{1}{2}\left[\omega_{net}(+B_X) + \omega_{net}(-B_X)\right]$
- Value of B_X is calibrated by measuring the vertical orbit splitting

[7] Mane, "Spin Wheel", p. 6.

Spin Wheel

The good, the bad, the ugly

- Higher polarization growth rate greatly simplifies the task for polarimetry
- Magnetic field calibration by means of orbit split measurements seems unfeasible
- Element misalignment-induced error is not accounted for:

$$\begin{aligned}\hat{\omega}_{EDM} &= \frac{1}{2}\left(\omega_{EDM} + \cancel{\omega_{B_X}} + \omega_{mis} + \omega_{EDM} - \cancel{\omega_{B_X}} + \omega_{mis}\right) \\ &= \omega_{EDM} + \omega_{mis}\end{aligned}$$

Frequency Domain Method

This methodology has been developed specifically to deal with misalignment error.

- No reason to apply an external B-field; misalignment B_X-field provides a sufficiently fast wheel
- The FS condition ensures that $\omega_{net} \propto \omega_{EDM} + \omega_{mis}$
- The same EDM estimator $\hat{\omega}_{EDM} = \frac{\omega_{net}(+B_X)+\omega_{net}(-B_X)}{2}$
- To flip the sign of B_X one must reverse the guide field polarity (CW/CCW comeback)
- The value of B_X is calibrated via horizontal plane precession frequency

Thank you!

Doubly-magic ring

Fundamental assumptions

1. Both beams are at Frozen Spin: $\omega = \omega_X = \omega_{EDM} + \omega_{\langle B_r \rangle}$
2. EDM of the secondary beam $\ll$ EDM of the primary beam: $\omega_{EDM}^{PRI} \gg \omega_{EDM}^{SEC} \to 0 \Rightarrow \omega_X^{SEC} \approx \omega_{\langle B_r \rangle}^{SEC}$;
3. Beams on the same design orbit $\Leftrightarrow$ experience same fields: $\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC}$

* MDM's of both beams are known to high precision (what for?)

D-M Ring

Addressing initial objections

- Precession frequency difference (given 2):
  $\omega_X^{PRI} - \omega_X^{SEC} \approx \omega_{EDM}^{PRI} + \omega_{\langle B_r \rangle}^{PRI} - \omega_{\langle B_r \rangle}^{SEC}$

Objection (to assumption 3): The beams have different mass $\Leftrightarrow$
$\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC} \not\Rightarrow \omega_{\langle B_r \rangle}^{PRI} = \omega_{\langle B_r \rangle}^{SEC}$

- Using the Koop Wheel, $\omega_X^{SEC} = 0 = \omega_{\langle B_r \rangle}^{SEC} \Rightarrow \langle B_r \rangle^{SEC} = 0$ (again require 2)
- Given the design orbit is shared by both beams, $\omega_{\langle B_r \rangle}^{PRI}$ is also 0, b/c $\forall m, \gamma, G \left[\omega_{\langle B_r \rangle} = \frac{q}{m} \gamma G \langle B_r \rangle = 0 \Leftrightarrow \langle B_r \rangle = 0 \right]$
- Sameness of the design orbits is guaranteed by the equation:
  $p^4 - 2\mathcal{B} p^3 + (\mathcal{B}^2 - \mathcal{E}^2) p^2 - \mathcal{E}^2 m^2 = 0$,
  where $\mathcal{B} = qcB_0 r_0$, $\mathcal{E} = qE_0 r_0$, (E_0, B_0, r_0) are defined by the primary beam FS condition

D-M Ring

Main objection

- But by nulling $\omega_{\langle B_r \rangle}^{PRI/SEC}$ we go to the unstable 3D FS state
- Which also forces us back to the Space Domain, since $\omega_X^{PRI} \approx \omega_{EDM}^{PRI} \ll 1$
- Thus, both the FS success conditions are violated

Concl'n D-MR solves the machine imperfection fields problem, but, other than that, inherits all of the original BNL FS weaknesses

Universal SR EDM measurement problems

And their canonical solutions

Solved by Spin Wheel

- Stray fields
- Betatron motion

* Both cause variation of $\bar{n}$

Solved elsewise

- Spin decoherence

Sol'n : Sextupole fields

- Machine imperfections

Sol'n : CW/CCW injection