Frequency Domain Method

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Abstract

A new method for searching for the electric dipole moment (EDM) of the deuteron and other nuclei is presented. When trying to measure the EDM in a storage ring environment, magnetic dipole moment (MDM) spin precession due to machine imperfections becomes the primary source of systematic error. The proposed method aims at providing a solution to the machine imperfection problem. The method is based on estimating the combined MDM + EDM spin precession angular velocity, in which the MDM contribution is due only to field imperfections. The MDM term is canceled in the final statistic by adding angular velocity estimates from cycles with counter-circulating beams. Spin precession rate depends on the particle's effective Lorentz factor; the proposed method's core feature is a procedure for equalizing the effective Lorentz factors of the clockwise and counter-clockwise circulating beams, thus enabling the cancelation.

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Tilting of the accelerator optical elements about the beam axis induces a non-zero average radial magnetic field, which causes an EDM-faking MDM precession.

We have simulated the machine imperfection precession rate Ω_{MDM} for the frozen spin lattice depicted in Figure 1. The lattice utilizes cylindrical E+B field spin rotators in the arc sections in order to effect the frozen spin condition. Imperfections were simulated via rotations of the E+B elements about the optical axis by normally-distributed angles $\Theta_{tilt} \sim N(0, 10^{-4})$ rad. The standard deviation of 10^{-4} rad was chosen as an estimate of a practically-achievable element alignment error level. Analytical estimates [1] show, that at this level, the machine imperfection Ω_{MDM} should be expected in the range of 50 to 100 rad/sec, assuming an $n = 100$ element lattice.

Simulation results are presented in Figure 2. One can observe that at $\langle\Theta_{tilt}\rangle = 10^{-4}$ rad the radial component of Ω_{MDM} is approximately 500 rad/sec. Since $\sigma[\langle\Theta_{tilt}\rangle] = \sigma[\Theta_{tilt}]/\sqrt{n} = 10^{-4}/\sqrt{100} = 10^{-5}$ rad. The dependence in Figure 2 is linear, hence the probability of observing $\Omega_{MDM} \leq 50$ rad/sec is 68%, $\Omega_{MDM} \leq 100$ rad/sec is 95%, and $50 \leq \Omega_{MDM} \leq 100$ rad/sec with a 27% probability.

3 Frequency Domain Method

3.1 EDM estimator statistic

Since the measured angular velocity $\Omega = \Omega_{MDM} + \Omega_{EDM}$ includes a contribution due to the MDM, one has to find a way to eliminate the Ω_{MDM} term from the final $\hat{\Omega}_{EDM}$ estimator.

In the proposed methodology, non-spurious Ω_{MDM} is generated only by the radial magnetic fields induced by accelerator element tilts about the optical axis. Therefore, by reversing the polarity of the guide field one also reverses the sign of Ω_{MDM}. The EDM estimator is constructed as a sum of positive (beam circulates clockwise) and negative (counter-clockwise) polar-

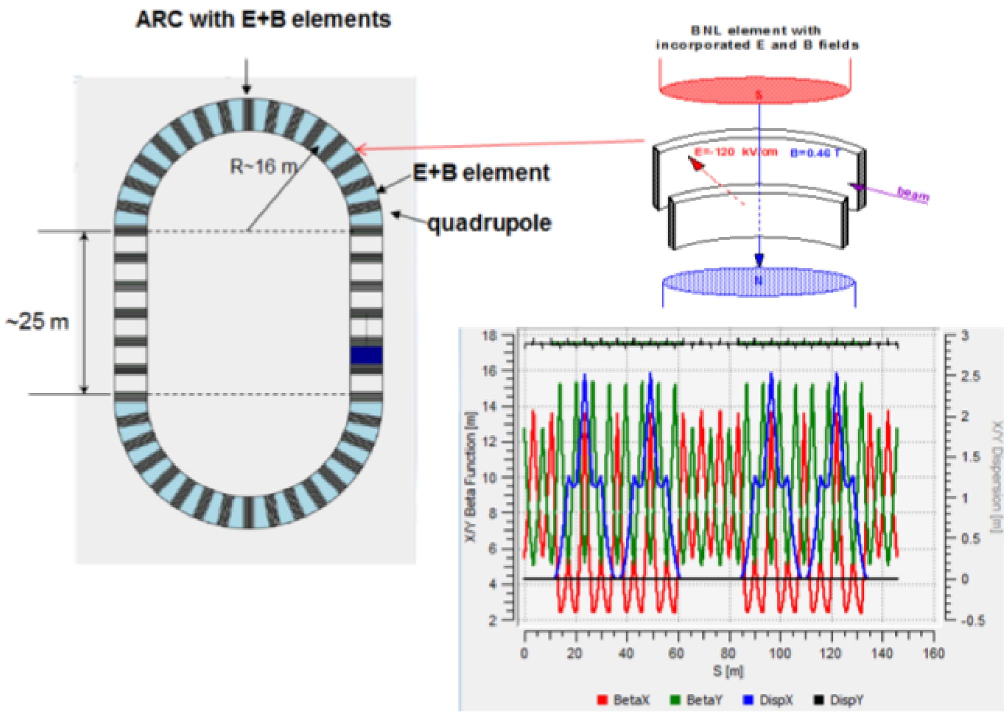

Figure 1: Frozen spin lattice with cylindrical E+B field spin rotators inserted into the arc sections.

ity cycles' angular momentum estimates:

$$\Omega^{\pm} = \pm\Omega_{MDM} + \Omega_{EDM}, \tag{1}$$

$$\begin{aligned} \hat{\Omega}_{EDM} &= \frac{1}{2}\left[\hat{\Omega}^{+} + \hat{\Omega}^{-}\right] \\ &= \Omega_{EDM} + \frac{1}{\sqrt{2}} \cdot \sigma_{MDM} + \epsilon, \end{aligned} \tag{2}$$

where σ_{MDM} is the statistical (model parameter estimate) error, and the difference between the two cycles' MDM spin precession rates $\epsilon = \Omega^{+}_{MDM} - \Omega^{-}_{MDM}$ is the systematic error term.

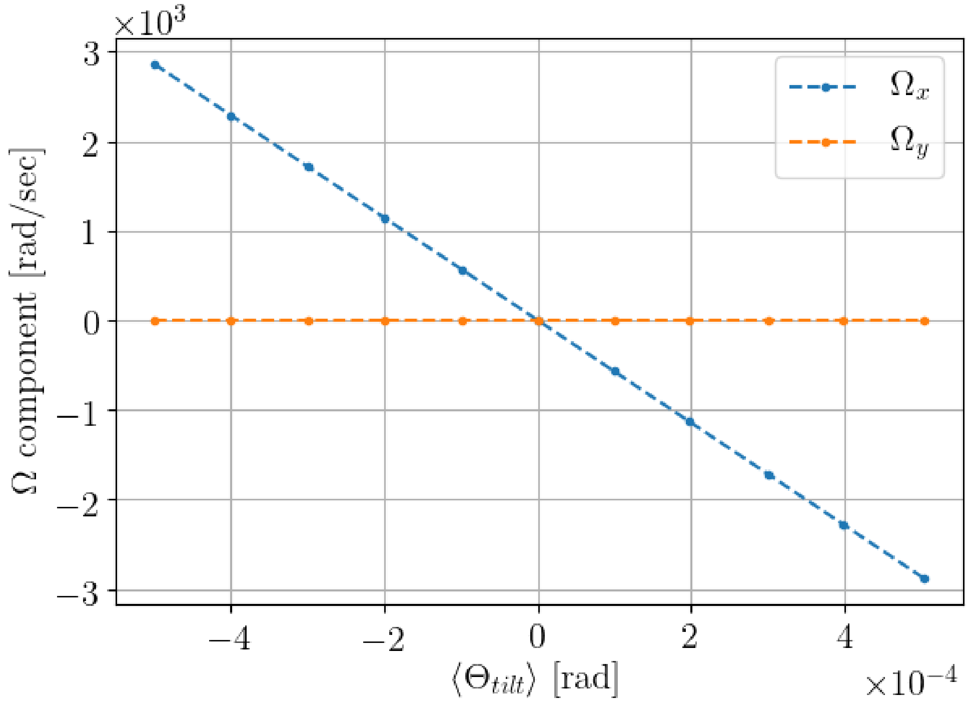

Figure 2: Spin precession angular velocity components vs mean E+B element tilt angle.

3.2 Effective Lorentz factor

In order to minimize systematic error ϵ, one needs a way to keep Ω_{MDM} constant across multiple runs.

The obvious way of trying to precisely reproduce the guiding field is inefficient for two major reasons: (1) standard magnetic field measurement methods do not yield sufficient precision; (2) the lattice might not be symmetric enough, in terms of spin dynamics, with respect to reversal of the beam circulation direction. Hence, we propose a different variable for calibration.

We note that the number of spin revolutions per turn (spin tune ν_s) depends on the particle's equilibrium-level energy, expressed by the Lorentz factor γ:

$$\nu_s^B = G\gamma, \qquad \text{(magnetic field)}$$
$$\nu_s^E = \frac{G+1}{\gamma} - G\gamma. \qquad \text{(electric field)}$$

Not all beam particles in a bunch are characterized by the same γ. A particle involved in betatron motion will have a longer orbit, and as a direct consequence of the phase stability principle, in an accelerating structure utilizing an RF cavity, its equilibrium energy level must increase.

The effective Lorentz factor is a generalization of the regular Lorentz factor accounting for betatron motion-related orbit lengthening and non-linearity of the momentum compaction factor.

It has been shown in [2, p. 56] that a particle's spin tune can be described by a univariate function; we associate the argument of that function with the effective Lorentz factor. Consequently, spin-vectors of two particles characterized by the same value of the effective Lorentz factor precess as the same rate.

Therefore, if the CW and CCW beam centroids' have equal γ_{eff}, we can expect the MDM components of the spin precession angular velocities to be equal as well.

3.3 Calibration of the ELF

Now the question becomes how the effective Lorentz factor can be calibrated.

4 Statistical precision

Spin precession angular velocity is estimated via non-linear fit of a constant-parameter harmonic function to polarization data. However, perturbations to the spin dynamics, caused by, for example, betatron motion, introduce a mismatch between the fit model and the data, and hence a model specification systematic error. This problem has been analyzed, [3] with the conclusion that this systematic error is negligible.

Effective duration of the measurement cycle cannot exceed three times the polarization lifetime τ_d, [4] where τ_d is the time during which beam polarization decreases by a factor of e.

Simulation shows [4] the possibility of reaching a statistical error $\sigma[\hat{\Omega}] = 8\cdot 10^{-7}$ rad/sec in one measurement cycle (at $\tau_d = 10^3$ sec), and $\sigma[\langle\hat{\Omega}\rangle] = 5\cdot 10^{-9}$ rad/sec in one year of measurement (at 70% accelerator time load). This should suffice to achieve an EDM estimate precision level of 10^{-29} $e\cdot$cm.

References

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