Frequency Domain Methodto Search for the Deuteron Electric Dipole Moment in a Storage Ring Environment

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Abstract

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Contents

1. Introduction

Spin rotations belong to the Spin(3) group, which is isomorphic to SU(2).

Rotations in SU(2). Rotation by angle ψ about direction $\bar{n}$

$$R_{\bar{n}}(\psi) = \exp\left[-i\frac{\psi}{2}(\bar{n}\cdot\vec{\sigma})\right],$$

where $\vec{\sigma}$ is the Pauli matrix vector.

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1.1. General spin rotation matrices

Denote

- $(\Theta^{mi}, \bar{n}_{mi})$ from machine imperfections;
- $(\Theta^{+}, \bar{n}_{sol})$ for the $+\Delta$ solenoidal field;
- $(\Theta^{-}, -\bar{n}_{sol})$ for the $-\Delta$ solenoidal field.

$$
\begin{aligned}
R^{+\Delta} &= \exp\left[-i\left(\frac{\Theta^{mi}}{2}(\bar{n}_{mi}\cdot\vec{\sigma}) + \frac{\Theta^{+}}{2}(\bar{n}_{sol}\cdot\vec{\sigma})\right)\right] \\
&= \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} + \Theta^{+}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \qquad (1) \\
R^{-\Delta} &= \exp\left[-\frac{i}{2}\left(\Theta^{mi}\bar{n}_{mi} - \Theta^{-}\bar{n}_{sol}\right)\cdot\vec{\sigma}\right], \qquad (2)
\end{aligned}
$$

2. Preliminary analytic of the Spin Wheel method

In SW we posit

$$\left(\vec{\Omega}^{+\Delta}_{MDM}\cdot\hat{x}\right) = -\left(\vec{\Omega}^{-\Delta}_{MDM}\cdot\hat{x}\right). \qquad (3)$$

The spin precession angular velocity vector can be expressed via spin tune and invariant spin axis as

$$\vec{\Omega}_{spin} = \frac{2\pi}{\tau_{ring}}\cdot\nu\cdot\bar{n},$$

hence

$$\nu^{+\Delta}(\bar{n}_{+\Delta}\cdot\hat{x}) + \nu^{-\Delta}(\bar{n}_{-\Delta}\cdot\hat{x}) = 0 \qquad (4)$$

From $\Delta\Theta = \tau\Delta\Omega$ and $\Delta\Omega_x^{MDM} = \frac{q}{m}GB_x$, and **assuming**

$$B^{\pm}_{sol}\tau_{sol} = \langle B^{\pm}_{sol}\rangle\tau_{ring} : \qquad (5)$$

$$
\begin{cases}
\Theta^{+} &= \tau_{sol}\frac{q}{m}GB^{+}_{sol} \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B^{+}_{sol}\rangle, \\
\Theta^{-} &= \tau_{sol}\frac{q}{m}GB^{-}_{sol} \overset{(5)}{=} \tau_{ring}\frac{q}{m}G\langle B^{-}_{sol}\rangle.
\end{cases} \qquad (6)
$$

Remark 1. Assumption (5) is **required** if we want to obtain $B^{\pm}_{sol}$ from equations of group (11).

From eqs (1) and (2):

$$\begin{cases} \Theta^{mi}\bar{n}_{mi} + \Theta^{+}\bar{n}_{sol} = \nu^{+\Delta}\bar{n}_{+\Delta}, \\ \Theta^{mi}\bar{n}_{mi} - \Theta^{-}\bar{n}_{sol} = \nu^{-\Delta}\bar{n}_{-\Delta}. \end{cases} \tag{7}$$

Substituting eq (7) into (4), and assuming $\bar{n}_{sol} = \hat{x}$:

$$2\Theta^{mi}(\bar{n}_{mi} \cdot \hat{x}) + (\Theta^{+} - \Theta^{-}) = 0. \tag{8}$$

Assuming[1]

$$\Theta^{mi} = \tau_{ring} \cdot \frac{q}{m} G \cdot \langle B_x \rangle^{mi}, \tag{9}$$

from (8) and (5) obtain:

$$2\langle B_x \rangle^{mi} + \left(\langle B^{+}_{sol} \rangle - \langle B^{-}_{sol} \rangle\right) = 0. \tag{10}$$

From eq (9) in Koop2015, assuming in the $+\Delta$ case the machine imperfections and solenoid fields are co-aligned, in the $-\Delta$ anti-aligned:

$$\begin{cases} \Delta^{+} & = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\langle B_x \rangle = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\left(\langle B_x \rangle^{mi} + \langle B^{+}_{sol} \rangle\right), \\ & \Rightarrow \langle B^{+}_{sol} \rangle = \frac{\langle G_z \rangle}{\beta_1 - \beta_2}\Delta^{+} - \langle B_x \rangle^{mi}; \\ \Delta^{-} & = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\langle B_x \rangle = \frac{\beta_1 - \beta_2}{\langle G_z \rangle}\left(\langle B_x \rangle^{mi} - \langle B^{-}_{sol} \rangle\right), \\ & \Rightarrow -\langle B^{-}_{sol} \rangle = \frac{\langle G_z \rangle}{\beta_1 - \beta_2}\Delta^{-} - \langle B_x \rangle^{mi}. \end{cases} \tag{11}$$

Substituting this into (10):

$$2\langle B_x \rangle^{mi} + \left(\frac{\langle G_z \rangle}{\beta_1 - \beta_2}\left[\Delta^{+} - \Delta^{-}\right] - 2\langle B_x \rangle^{mi}\right) = 0.$$

In the original method, we are to make

$$\Delta^{-} = -\Delta^{+}, \tag{12}$$

[1] This is a generous assumption implying that $\bar{n}_{mi} = \hat{x}$; i.e., this is **not** a non-commutativity-based argument; we assume all spin rotations commute.

so the term in the square brackets is zero, and we are left with

$$(1-1)\langle B_x\rangle^{mi} = 0. \tag{13}$$

So, seems that SW works, but we did two important assumptions here: *a)* commutativity (in order to get eq (9)), and *b)* "averaging" of B_{sol} over the ring (in order to get eq (5) and remove the τ_{sol}/τ_{ring} from (10)).

Remark 2. If we don't use (9) (but still use (5) in order to obtain $B_{sol}^{\pm}$ from group (11)), then eq (13) becomes

$$\Theta^{mi}\left(\bar{n}_{mi}\cdot\hat{x}\right) - \frac{q}{m}G\cdot\tau_{ring}\langle B_x\rangle^{mi} = 0, \tag{14}$$

which is not very informative.

Remark 3. To check that eq (14) is correct, assume (9). Then

$$\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}\left(\bar{n}_{mi}\cdot\hat{x}\right) - \frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi} = 0,$$

and hence

$$\bar{n}_{mi}\cdot\hat{x} = 1,$$

which is implied by machine imperfection spin rotations adding up commutatively.

Remark 4. In general, since

$$\Theta^{mi} = \tau_{ring}\cdot\frac{q}{m}G\sqrt{\langle B_x^{mi}\rangle^2 + \langle B_y^{mi}\rangle^2 + \langle B_z^{mi}\rangle^2},$$

eq (14) implies that

$$\begin{aligned}\left(\bar{n}_{mi}\cdot\hat{x}\right) &= \frac{\frac{q}{m}G\tau_{ring}\langle B_x\rangle^{mi}}{\Theta^{mi}}\\ &= \frac{\langle B_x^{mi}\rangle}{\sqrt{\langle B_x^{mi}\rangle^2 + \langle B_y^{mi}\rangle^2 + \langle B_z^{mi}\rangle^2}}.\end{aligned} \tag{15}$$

Which is correct.

Conclusion. In view of Remark 4, since eq (14) implies a valid statement, our conclusion is that the SW method resists the argument from non-commutativity.

$$\begin{tikzcd}
(8) \arrow[r, "(9)+(5)"] \arrow[d, "\begin{array}{l}(6)+\\(5)+\\(11)\end{array}"] & (10) \arrow[r, "(11)"] & (13) \\
(14) & &
\end{tikzcd}$$

Figure 1: Argument diagram.

3. Assumptions of the Spin Wheel method

Orbital dynamics. Koop2015 eq (7) (henceforth referred to as K(7)) and

$$\langle E_z \rangle = \langle E_z(0) \rangle + \langle G_z \rangle \cdot z \tag{K$\langle E_z\rangle$}$$

$$\to \langle z \rangle = \frac{\langle E_z(0) \rangle}{\langle G_z \rangle} - \frac{\beta}{\langle G_z \rangle} \cdot \langle B_x \rangle \tag{16}$$

$$\to \Delta = \frac{\beta_1 - \beta_2}{\langle G_z \rangle} \langle B_x \rangle. \tag{17}$$

This is as far as the argument from the non-linearity of the closed orbit shift dependence on the magnetic field is concerned. So long as we believe K(7) and K$\langle E_z \rangle$, we must believe K(9), and hence we cannot use that argument.

Spin dynamics. This is the argument from non-commutativity. For this argument cf. eq (14) and Remark 4, and the following conclusion.