

Comparison of Frozen Spin-type EDM search methods

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Considered methods

- ▶ BNL Frozen Spin
- ▶ I.Koop's Spin Wheel
- ▶ Y.Senichev's Frequency Domain Method

- ▶ Observation of the vertical polarization component¹
 $\Delta P_V \approx P \cdot \omega_{EDM} \cdot t$ (making it a Space Domain method)
- ▶ Cross section asymmetry $\varepsilon_{LR} \approx 5 \cdot 10^{-6}$ for smallest practical values of (horizontal plane) ω_{MDM} ²
- * Challenging task for polarimetry³

¹D. Anastassopoulos et al., *AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the 10^{-29} e · cm level.* p. 9.

²Ibid., p. 18.

³Mane, “Spin Wheel”, p. 6.

- ▶ Only known first-order systematic effect pertaining to the spin dynamics is the existence of $\langle E_V \rangle \neq 0$ ⁴
- ▶ Error frequency $\omega_{syst} \approx \frac{\mu \langle E_V \rangle}{\beta c \gamma^2}$ changes sign when reversing the beam circulation direction (CW/CCW)⁵
- ▶ However, at practical values of element alignment error, $\omega_{syst} \gg \omega_{EDM}$, hence $P_V = P \frac{\omega_{EDM}}{\omega} \sin(\omega t + \Theta_0) \not\approx P \omega_{EDM} t$; a Space Domain method is inapplicable under such conditions
- * At $\langle E_V \rangle \rightarrow 0$, Space Domain methods are vulnerable to the geometric phase error⁶

⁴D. Anastassopoulos et al., *AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the 10^{-29} e · cm level*. p. 10.

⁵Ibid., p. 11.

⁶V. Anastassopoulos et al., *A Storage Ring Experiment to Detect a Proton Electric Dipole Moment*, p. 6.

Geometric phase error

- ▶ Caused by the non-commutativity of rotations
- ▶ Formulated in the angular momentum language, it means the *absence of a definite orientation* of the spin precession axis (SPA): $\bar{n} \rightarrow 0$
 - * Call that the *3D Frozen Spin* state
- ▶ 3D FS is unstable: any stray magnetic field can tilt the precession plane

FS-type methodology

Conditions of success

- ▶ One must always have a definite direction of the SPA
- ▶ Measurements must be done in the frequency domain

These conditions are satisfied by two methods:

- ▶ I.Koop's "Spin Wheel"
- ▶ Y.Senichev's "Frequency Domain"

(Both of which belong to the Frequency Domain category.)

Spin Wheel

The Spin Wheel is great; it satisfies both success conditions.

- ▶ Apply a radial magnetic field of strength B_X sufficient to turn the spin vector about the \hat{x} -axis with a frequency of 1 Hz
- ▶ $\omega_{B_X} \parallel \omega_{EDM}$ hence $\omega_{net} \propto \omega_{EDM} + \omega_{B_X}$ ⁷
- ▶ EDM effect $\hat{\omega}_{EDM} = \frac{1}{2} [\omega_{net}(+B_X) + \omega_{net}(-B_X)]$
- ▶ Value of B_X is calibrated by measuring the vertical orbit splitting

⁷Mane, “Spin Wheel”, p. 6.

Spin Wheel

The good, the bad, the ugly

- ▶ Higher polarization growth rate greatly simplifies the task for polarimetry
- ▶ Magnetic field calibration by means of orbit split measurements seems unfeasible
- ▶ Element misalignment-induced error is not accounted for:

$$\begin{aligned}\hat{\omega}_{EDM} &= \frac{1}{2} (\omega_{EDM} + \cancel{\omega_{B_X}} + \omega_{mis} + \omega_{EDM} - \cancel{\omega_{B_X}} + \omega_{mis}) \\ &= \omega_{EDM} + \omega_{mis}\end{aligned}$$

Frequency Domain Method

This methodology has been developed specifically to deal with misalignment error.

- ▶ No reason to apply an external B-field; misalignment B_X -field provides a sufficiently fast wheel
- ▶ The FS condition ensures that $\omega_{net} \propto \omega_{EDM} + \omega_{mis}$
- ▶ The same EDM estimator $\hat{\omega}_{EDM} = \frac{\omega_{net}(+B_X) + \omega_{net}(-B_X)}{2}$
- ▶ To flip the sign of B_X one must reverse the guide field polarity (CW/CCW comeback)
- ▶ The value of B_X is calibrated via horizontal plane precession frequency

Thank you!

Doubly-magic ring

Fundamental assumptions

1. Both beams are at Frozen Spin: $\omega = \omega_X = \omega_{EDM} + \omega_{\langle B_r \rangle}$
2. EDM of the secondary beam \ll EDM of the primary beam:
 $\omega_{EDM}^{PRI} \gg \omega_{EDM}^{SEC} \rightarrow 0 \Rightarrow \omega_X^{SEC} \approx \omega_{\langle B_r \rangle}^{SEC};$
3. Beams on the same design orbit \Leftrightarrow experience same fields:
 $\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC}$

Comments

- * MDM's of both beams are known to high precision (what for?)
- ** Assumption 1 is formulated in the simplest form (we'll address that later).

D-M Ring

Addressing the mass objection

Precession frequency difference (given 2)

$$\omega_X^{PRI} - \omega_X^{SEC} \approx \omega_{EDM}^{PRI} + \omega_{\langle B_r \rangle}^{PRI} - \omega_{\langle B_r \rangle}^{SEC}$$

The mass objection (to assumption 3)

The particles have different mass \Leftrightarrow

$$\langle B_r \rangle^{PRI} = \langle B_r \rangle^{SEC} \not\Rightarrow \omega_{\langle B_r \rangle}^{PRI} = \omega_{\langle B_r \rangle}^{SEC}$$

- ▶ Using the Koop Wheel, $\omega_X^{SEC} = 0 = \omega_{\langle B_r \rangle}^{SEC} \Rightarrow \langle B_r \rangle^{SEC} = 0$
(again require 2)
- ▶ Given the design orbit is shared by both beams, $\omega_{\langle B_r \rangle}^{PRI}$ is also 0, b/c $\forall m, \gamma, G \left[\omega_{\langle B_r \rangle} = \frac{q}{m} G \langle B_r \rangle = 0 \Leftrightarrow \langle B_r \rangle = 0 \right]$
- ▶ Sameness of the design orbits is guaranteed by equation $p^4 - 2\mathcal{B}p^3 + (\mathcal{B}^2 - \mathcal{E}^2)p^2 - \mathcal{E}^2m^2 = 0$,
where $\mathcal{B} = qcB_0r_0$, $\mathcal{E} = qE_0r_0$, (E_0, B_0, r_0) are defined by the primary beam FS condition

D-M Ring

Fundamental flaw

- ▶ But by nulling $\omega_{\langle B_r \rangle}^{PRI/SEC}$ we go to the unstable 3D FS state
- ▶ Which also forces us back to the Space Domain, since $\omega_X^{PRI} \approx \omega_{EDM}^{PRI} \ll 1$
- ▶ Thus, both the FS success conditions are violated

Conclusion

D-MR solves the machine imperfection fields problem, but, other than that, inherits all of the original BNL FS weaknesses

But does it really solve the imperfection fields problem?

D-M Ring

Let's go back to Assumption 1

- ▶ Our formulation of Assumption 1 as $\omega_X = \omega_{EDM} + \omega_{\langle B_r \rangle}$ is unrealistic: the existence of $\langle B_r \rangle$ must cause $\langle E_v \rangle$, since we have a closed orbit
- ▶ So really, it should be

$$\begin{aligned}\omega_X &= \omega_{EDM} + \omega_{MDM}(\langle B_r \rangle + \langle E_v \rangle), \\ \omega_{MDM} &= \frac{q}{m} [G\langle B_r \rangle + a(\gamma, G)\beta\langle E_v \rangle]\end{aligned}$$

- ▶ Still, we have the system

$$\begin{cases} c\beta\langle B_r \rangle + \langle E_v \rangle &= 0, \\ G\langle B_r \rangle + a\beta\langle E_v \rangle &= 0 \end{cases}$$

w/solution $(\langle B_r \rangle, \langle E_v \rangle) = (0, 0)$, and the argument against the mass objection holds up, with $\langle B_r \rangle \mapsto (\langle B_r \rangle, \langle E_v \rangle)$

Universal SR EDM measurement problems

And their canonical solutions

Solved by Spin Wheel

- ▶ Stray fields
- ▶ Betatron motion
- * Both cause variation of \bar{n}

Solved otherwise

- ▶ Spin decoherence

Sol'n : Sextupole fields

- ▶ Machine imperfections

Sol'n : CW/CCW injection

Method classification

