

# Frozen spin method of search for the deuteron EDM in a storage ring

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# Research goal and objectives

**Goal:** to evaluate the proposed method's capability to detect the deuteron EDM at the sensitivity level  $10^{-29} \text{e}\cdot\text{cm}$

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## **Objectives:**

- effect of betatron oscillations on the validity of the EDM statistic
- spin-decoherence near zero resonance
- properties of the MDM faking signal due to machine imperfections
- the calibration and exclusion of the faking signal from the EDM static
- evaluation of statistical sensitivity

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- The machine imperfections faking signal's properties
  - ▶ necessitate the use of a frequency domain method of search for the EDM
  - ▶ make it possible to exclude this signal from the target statistic

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- The effective Lorentz factor is susceptible to calibration
- It is statistically possible to reach a standard error of the mean EDM estimate at the level  $10^{-29}$  e·cm in year's worth of beam time

# Space vs frequency domain methods

$$P_y = A \cdot \sin \left( \underbrace{\sqrt{(\omega^{edm} + \omega^{imp})^2 + \omega_y^2 + \omega_z^2}}_{\Omega} \cdot t + \delta \right)$$

- **Space domain:**

- ▶ (Must!) Stop MDM precession in the **vertical**, as well as **horizontal**, plane
- ▶ Track the change in spatial orientation of the polarization vector

- **Frequency domain:**

- ▶ Stop **only** the horizontal plane precession
- ▶ Track the change in the vertical plane precession angular velocity

# What are the advantages of frequency domain?

- 1 Element alignment specifications aren't as severe
- 2 Stable spin wheel state solves the geometric phase error problem
- 3 Easier polarimetry

# Betatron motion effect

## EDM statistic

$$\omega^{\hat{edm}} = \frac{1}{2}(\hat{\omega}_x^+ + \hat{\omega}_x^-), \text{ where } \omega_x^\pm = \omega^{edm} \pm \omega^{mdm}$$

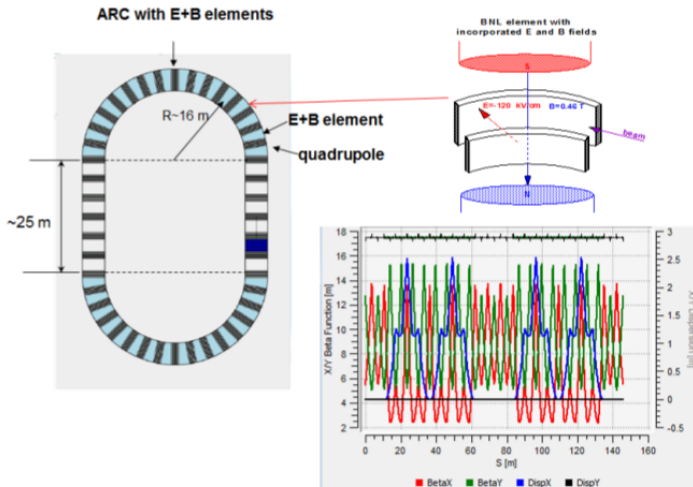
## Frequency estimated via fit

$$f(t) = a \cdot \sin(\omega_x \cdot t + \delta) \mapsto \hat{\omega}_x, \text{ where } (a, \omega_x, \delta) = \text{const}$$

## While the solution of T-BMT

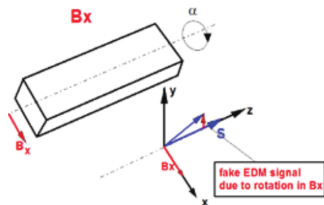
$$a = \sqrt{\bar{n}_x^2 + (\bar{n}_y \cdot \bar{n}_z)^2}, \text{ where } \bar{n} = g(\mathbf{E}, \mathbf{B})$$

# Simulation



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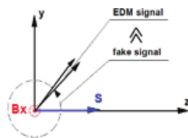
## Machine imperfections



- $\alpha \sim N(\mu_i, 3 \cdot 10^{-4})^\circ$
- $\mu_i$  simulates the application of a spin wheel driver

## Particles

- betatron oscillating in the vertical plane
  - $E_{FS} \neq E_{kin} \rightarrow E_{FS}$
- $\Rightarrow \bar{n}_x \ll 1 \Rightarrow$  high sensitivity to perturbations



# Analysis

## Data

- TRK** data from the COSY  
Infinity tracker
- GEN** computed from the fit  
function, with  $\bar{n}$ ,  $\nu_s$   
from tracking
- IDL** as in GEN, but  
 $\bar{n} = \langle \bar{n} \rangle$ ,  $\nu_s = \langle \nu_s \rangle$

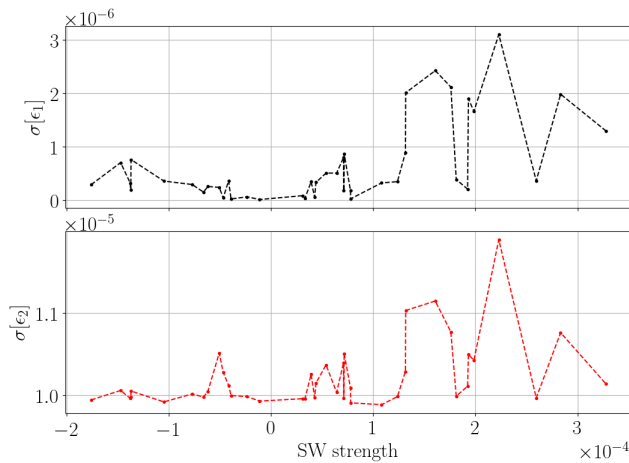
## Comparator stats

$$\epsilon_1(t) = s_y^{gen}(t) - s_y^{idl}(t)$$

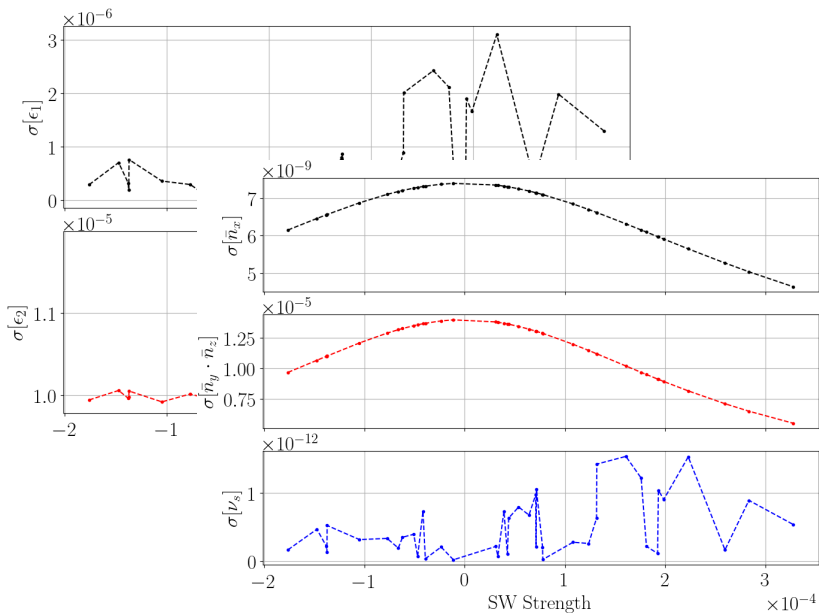
$$\epsilon_2(t) = s_y^{trk}(t) - s_y^{idl}(t)$$



# Results



# Results



# Conclusions

- 1 The signal amplitude oscillations (as estimated by  $\epsilon_2$ ) are small. They occur at the level at least two orders of magnitude smaller than the expected polarization measurement error. This means the superposition of this systematic error with the random measurement error will exhibit no statistically-significant systematicity.

# Conclusions

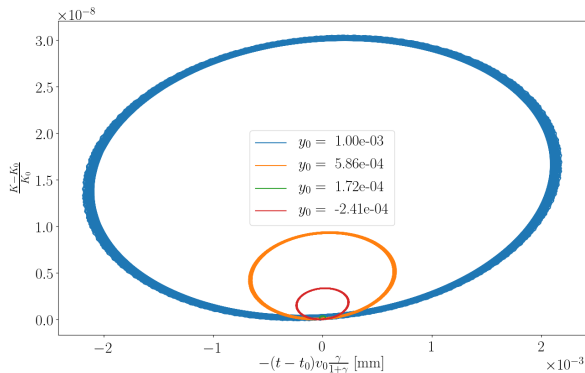
- 2 The correlation coefficient between the amplitude and frequency estimates is not significant. The amplitude oscillations affect the  $\hat{a}$ -estimate foremost; their effect on the  $\hat{\omega}$ -estimate is secondary, and is described by the correlation coefficient. Since it is less than 10%, even if the oscillations happen to be strong enough to affect the amplitude estimate, their effect on the frequency estimate will be reduced by at least a factor of 10.

# Conclusions

- 3 This systematic effect is controllable. And this point is the major advantage of the FD methodology. By applying an external Spin Wheel, the  $\bar{n}$  oscillations can be continuously minimized as much as necessary, without changing the pattern of the experiment.

# Spin decoherence

## Cause



- $\nu_s = \gamma G$
- because of the orbit length difference, beam particles have different  $\gamma_{eq}$

# Supression via sextupole field

## Equilibrium level momentum shift

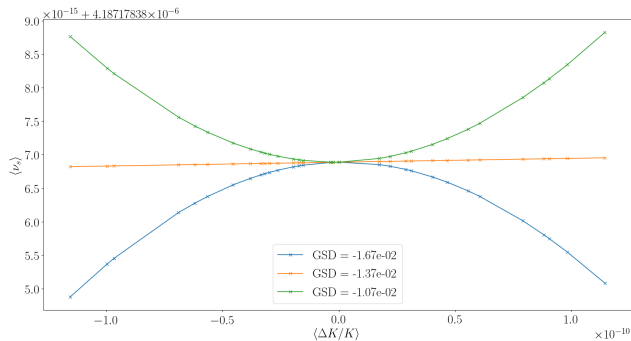
$$\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0-1} \left[ \frac{\delta_m^2}{2} (\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}) + \left(\frac{\Delta L}{L}\right)_\beta \right]$$

## Sextupole field effects

$$S_{sext} = \frac{1}{B\rho} \frac{\partial^2 B_y}{\partial x^2} \begin{cases} \rightarrow \Delta\alpha_{1,sext} = -\frac{S_{sext}D_0^3}{L} \\ \rightarrow \left(\frac{\Delta L}{L}\right)_{sext} = \mp \frac{S_{sext}D_0\beta_{x,y}\varepsilon_{x,y}}{L} \end{cases}$$

# Sextupole field effects

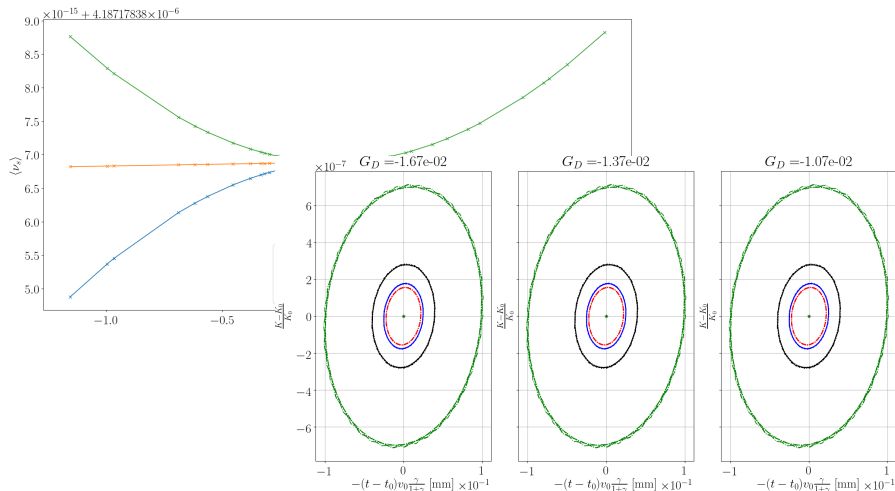
Momentum compaction factor





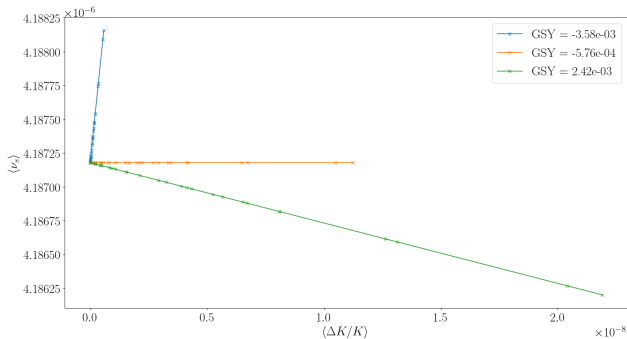
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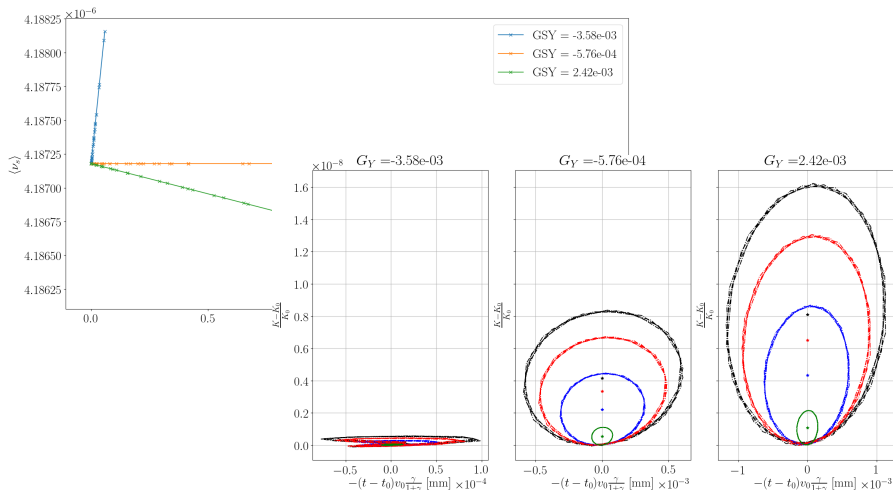
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## Orbit length



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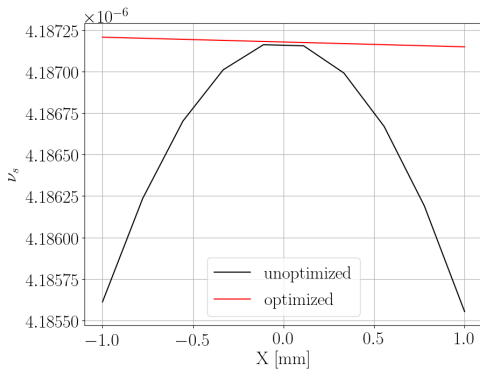
# Conclusions

- 1 The signature of the sextupole field's momentum compaction effect is the change in the functional form of  $\langle \nu_s \rangle (\langle \Delta K / K \rangle)$

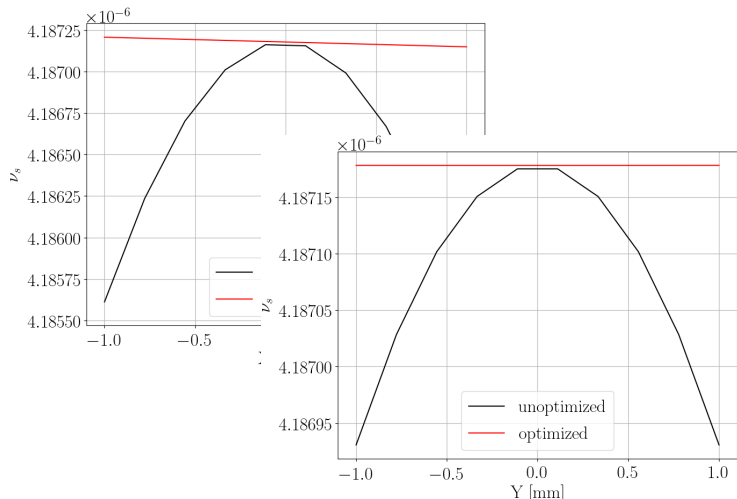
# Conclusions

- 1 The signature of the sextupole field's momentum compaction effect is the change in the functional form of  $\langle \nu_s \rangle (\langle \Delta K / K \rangle)$
- 2 ... orbit length effect — reduction in the dispersion of  $\langle \Delta K / K \rangle$

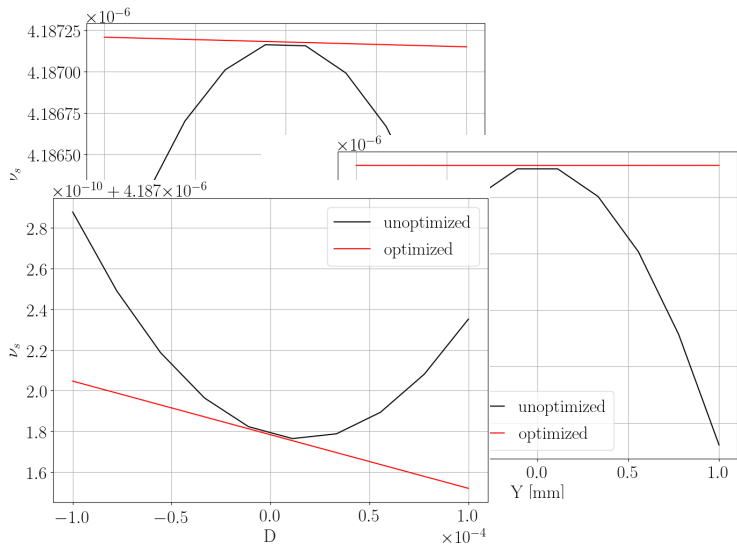
# Decoherence suppression in an ideal lattice



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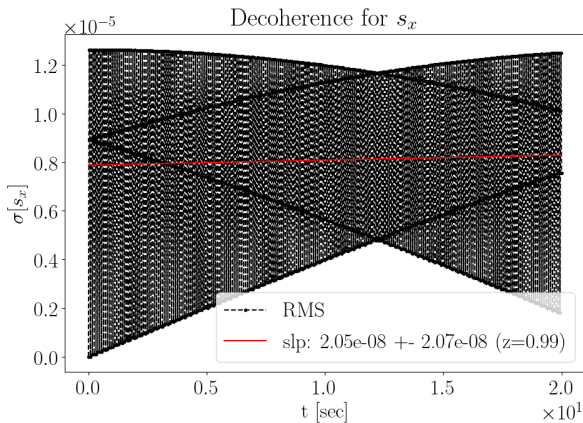


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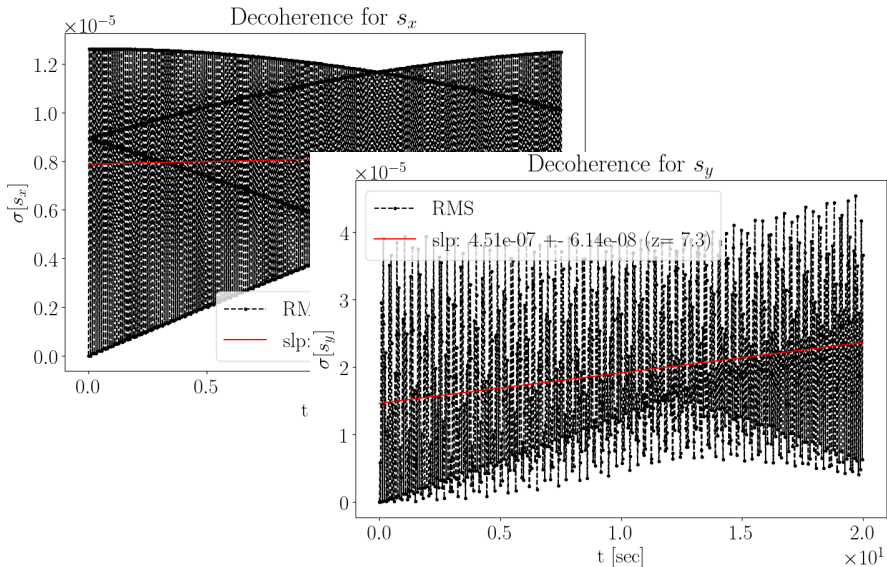




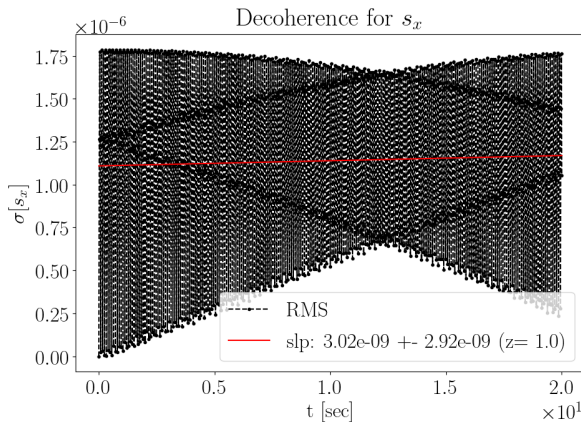
# Decoherence in an imperfect lattice



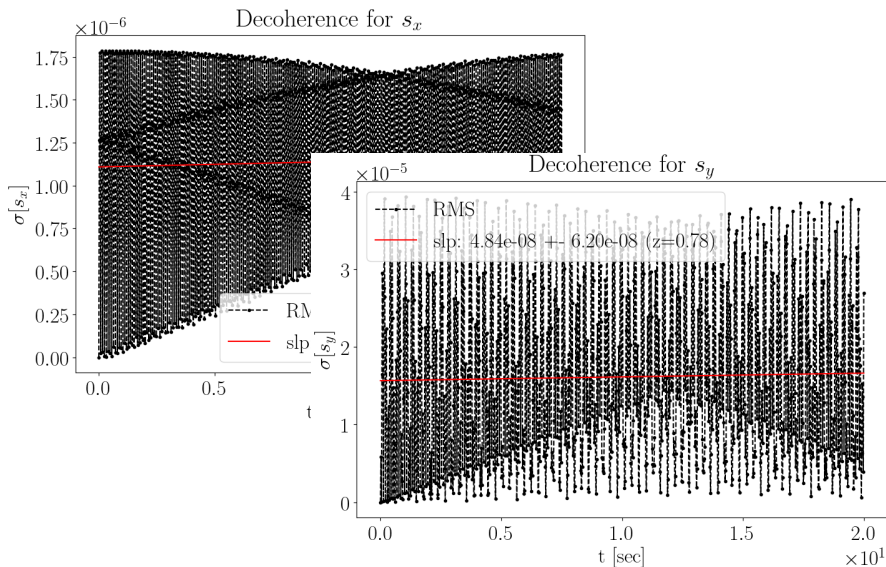
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# Turning on the sextupoles



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# MDM faking signal

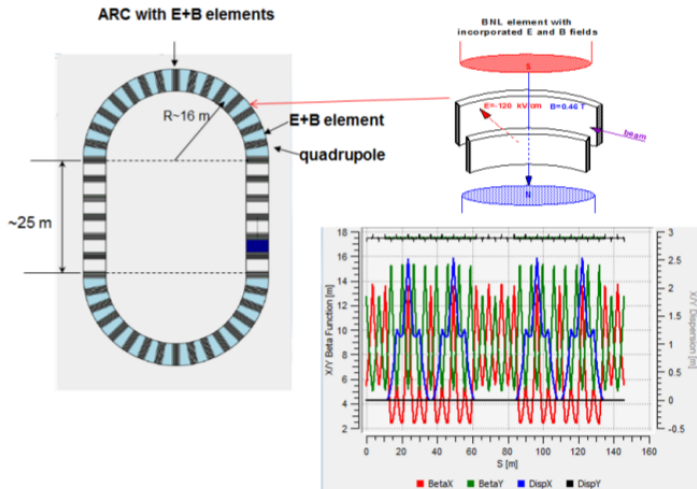
## Magnitude

$$\sigma [\Omega_x^{MDM}] = \frac{q}{m\gamma} \frac{G+1}{\gamma} \frac{\sigma[B_x]}{\sqrt{n}}$$

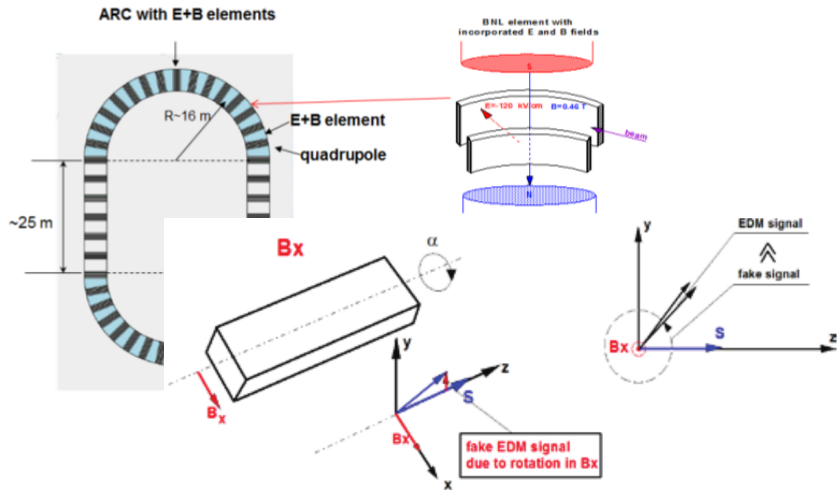
## Questions to consider

- Is the error linear, i.e.  $\Omega_x^{MDM} = f(\langle \Theta_{tilt} \rangle)$ ?
- Is it symmetric with regard to the beam circulation direction change, i.e.  $|\Omega_x^{CW}| = |\Omega_x^{CCW}|$ ?

# Analyzed lattice

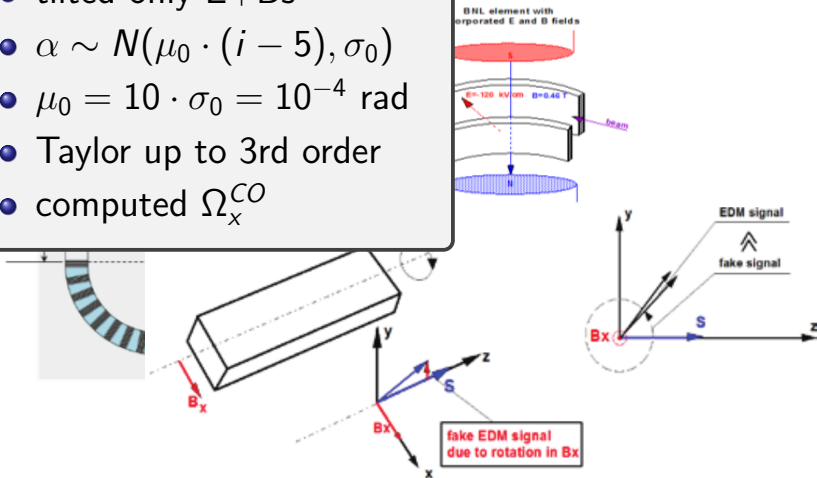


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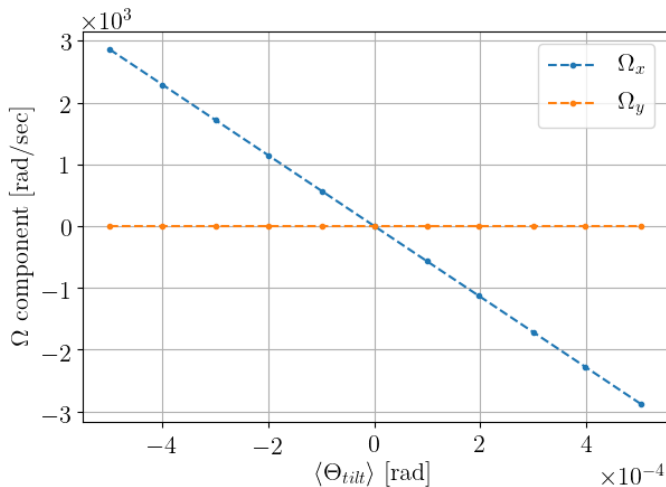
# Analyzed lattice

- 11 simulations
- tilted only E+B<sub>s</sub>
- $\alpha \sim N(\mu_0 \cdot (i - 5), \sigma_0)$
- $\mu_0 = 10 \cdot \sigma_0 = 10^{-4}$  rad
- Taylor up to 3rd order
- computed  $\Omega_x^{CO}$

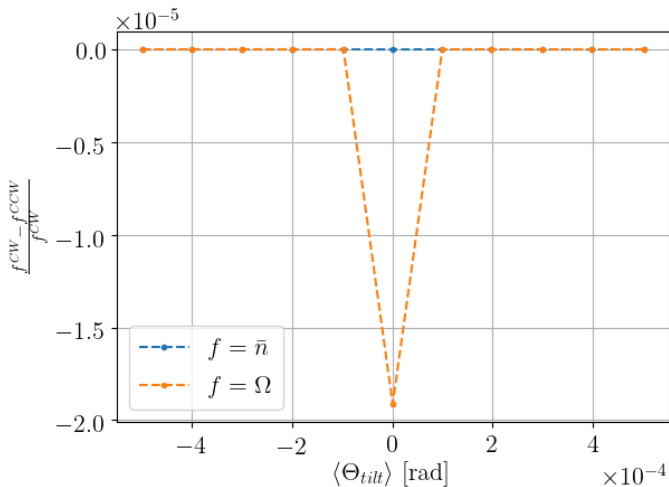




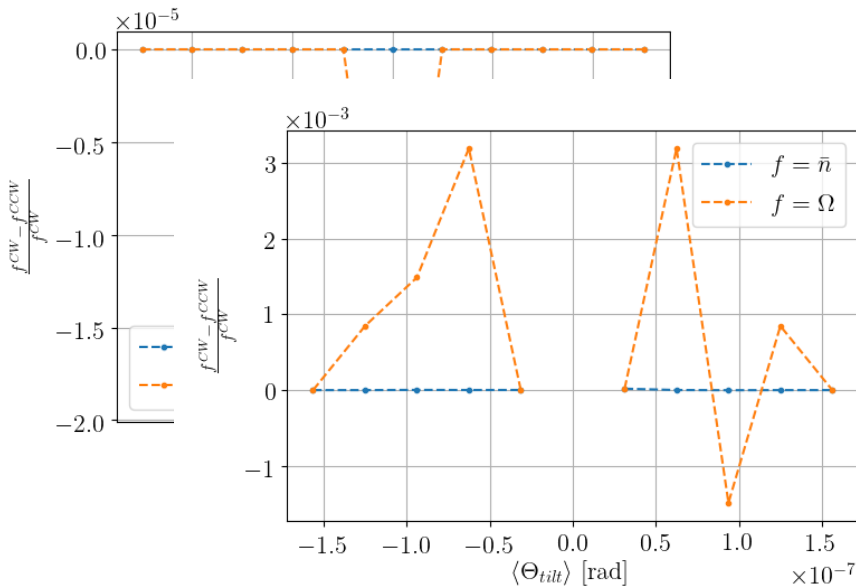
# Linearity



# Symmetry



# Symmetricity



# Conclusions

$$\sigma_{\theta} = 10^{-4} \text{ rad}, n = 100 \text{ elements}$$

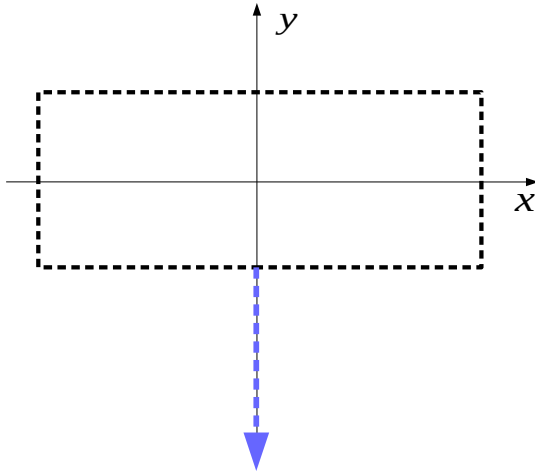
$\omega^{max}$ [rad/sec]	$P(\Omega_x^{MDM} < \omega^{max})$
50	67%
100	95%

## Properties

- 1 Linearity
- 2 Asymmetry, likely due to the difference between the CW and CCW beams' closed orbits
- 3 Asymmetry is less pronounced at higher SW roll rates

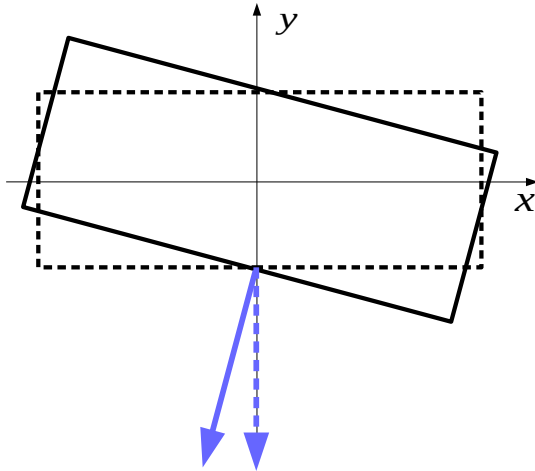
# Guide field polarity reversal

Why?



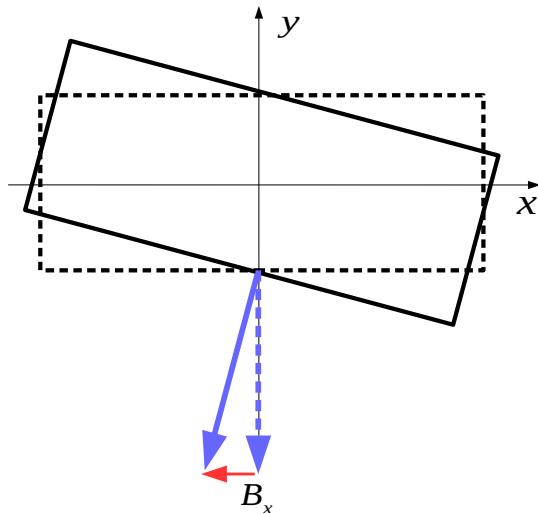
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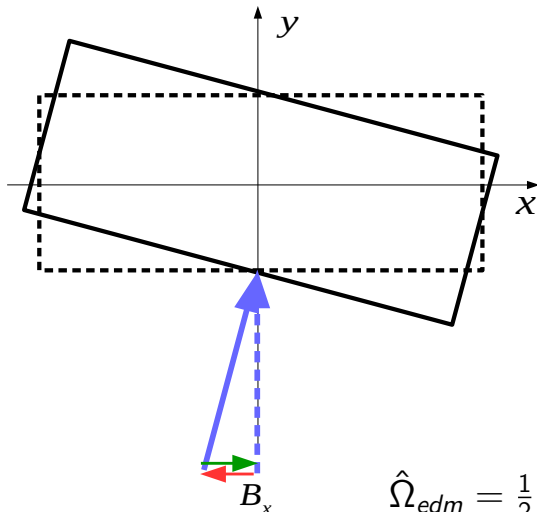
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$$\hat{\Omega}_{edm} = \frac{1}{2} (\Omega_x^{CW} + \Omega_x^{CCW})$$



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  - Besides, one has the lattice's spin dynamics asymmetry to consider (above)
- ⇒ Must reproduce the beam's effective Lorentz factor

# Calibration of the effective L-factor

- $\nu_S$  is an injective function of  $\gamma_{eff}$ , meaning
$$\Omega_y(\gamma_{eff}^1) = \Omega_y(\gamma_{eff}^2) \rightarrow \gamma_{eff}^1 = \gamma_{eff}^2$$

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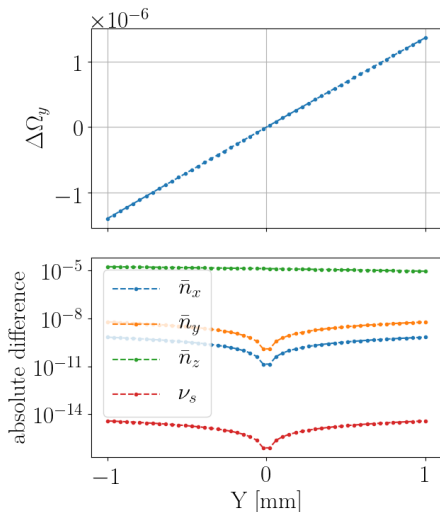
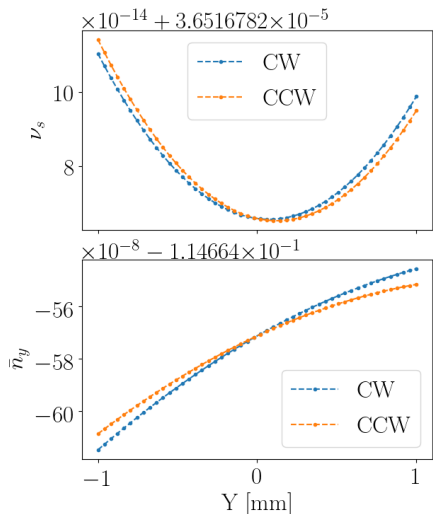
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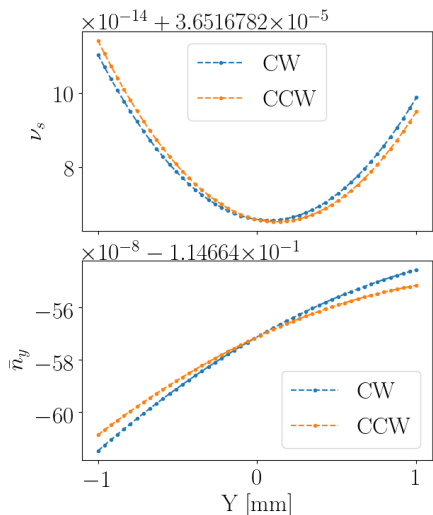
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- $\Rightarrow \exists! \gamma_{eff}^0: [\gamma_{eff}^0] = [\Omega_y = 0]$
- $\Rightarrow$  if both CW, CCW beams are “frozen” in the horizontal plane, their  $\gamma_{eff}$  are equal

# Simulation

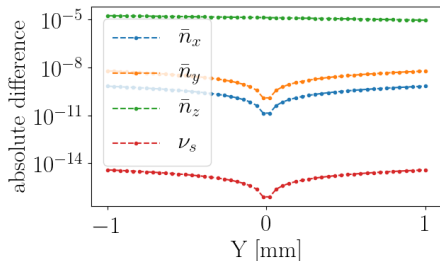


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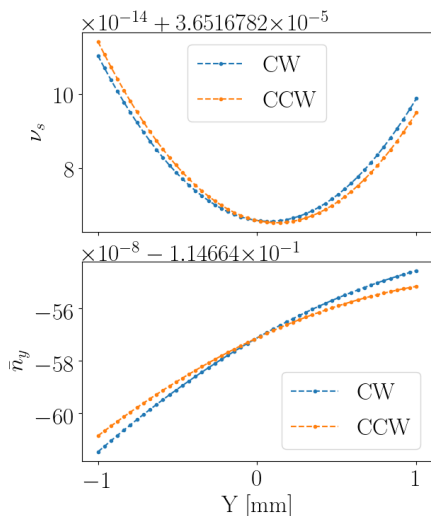


- Same  $S_{\text{sext}}$  works for both beams

- $\nu_s^{\text{CW}} \approx \nu_s^{\text{CCW}} \Rightarrow |\Omega_x^{\text{CW}}| \approx |\Omega_x^{\text{CCW}}|$

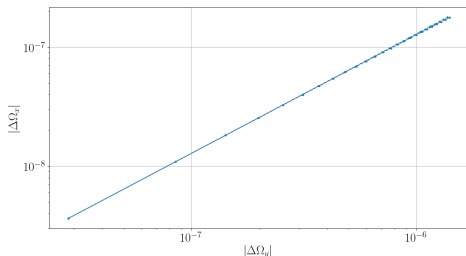


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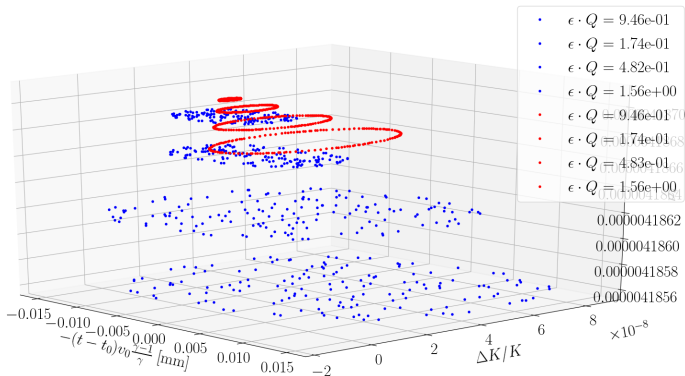


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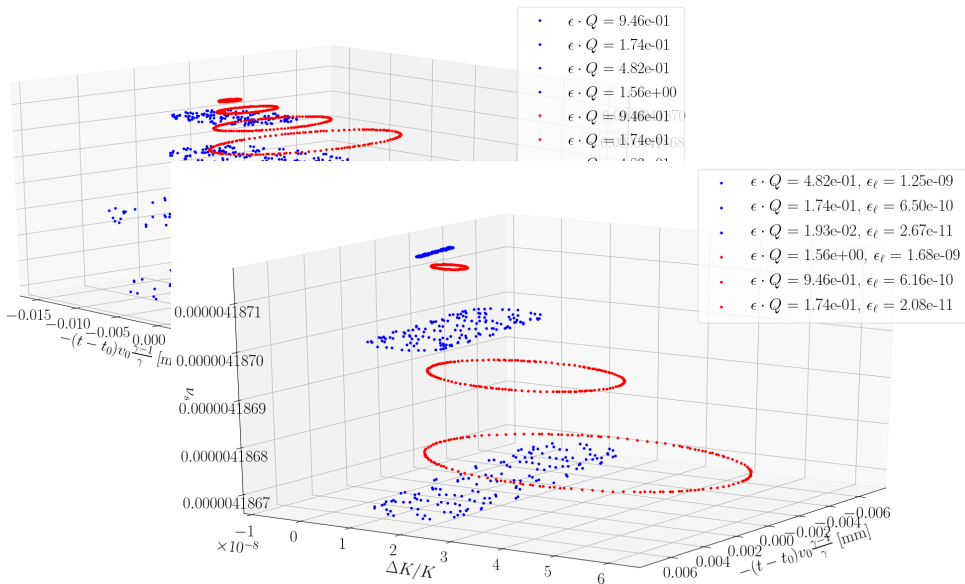
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# Spin tune a univariate function



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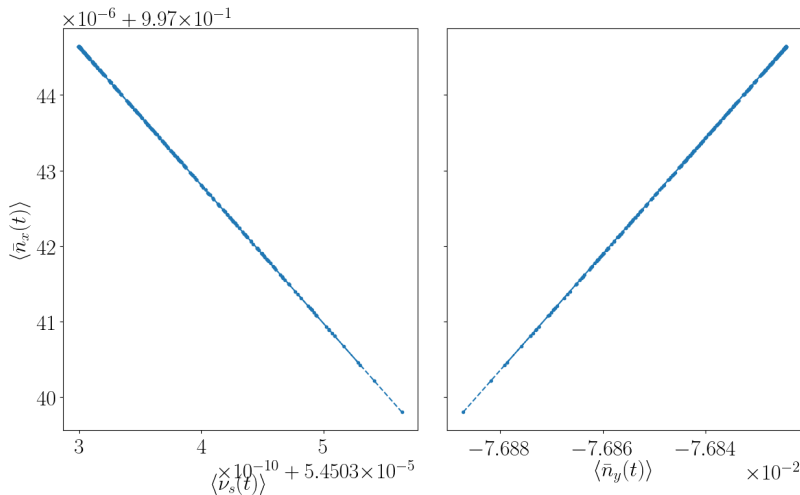


# Conclusions

## Conclusion 1

- Spin tune is reducible to a univariate function
- Effective Lorentz-factor is a measure of the particle's longitudinal emittance

# Conclusions





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## Conclusion 2

The spin dynamics of particles with the same value of  $\gamma_{eff}$  are equivalent in the general sense  $(\nu_s, \vec{n})$

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## Disclaimer

At least for the Frozen Spin lattice that I used in simulations