Frozen spin method of searching for the deuteron EDM in a storage ring

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18 July 2019

Research goal and objectives

Goal: to evaluate the proposed method's capability to detect the deuteron EDM at the sensitivity level $10^{-29} e{\cdot}\text{cm}$

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Objectives:

- effect of betatron oscillations on the validity of the EDM statistic
- spin-decoherence near zero resonance
- properties of the MDM faking signal due to machine imperfections
- the calibration and exclusion of the faking signal from the EDM static
- evaluation of statistical sensitivity

Defended propositions

- The frequency domain method's EDM statistic is robust with regard to perturbations from the particles' betatron motion

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- The machine imperfections faking signal's properties
  - necessitate the use of a frequency domain method of searching for the EDM
  - make it possible to exclude this signal from the target statistic

Defended propositions

- Spin tune can be expressed as a function of a **single** parameter (effective Lorentz factor), which is a measure of the particle's longitudinal emittance

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- The effective Lorentz factor is susceptible to calibration
- It is statistically possible to reach a standard error of the mean EDM estimate at the level $10^{-29} e{\cdot}\mathrm{cm}$ in year's worth of beam time

Space vs frequency domain methods

$$P_y = A \cdot \sin\left(\underbrace{\sqrt{(\omega^{edm} + \omega^{imp})^2 + \omega_y^2 + \omega_z^2}}_{\Omega} \cdot t + \delta\right)$$

- **Space domain:**
  - (Must!) Stop MDM precession in the **vertical**, as well as **horizontal**, plane
  - Track the change in spatial orientation of the polarization vector
- **Frequency domain:**
  - Stop **only** the horizontal plane precession
  - Track the change in the vertical plane precession angular velocity

What are the advantages of frequency domain?

1. Element alignment specifications aren't as severe
2. Stable spin wheel state solves the geometric phase error problem
3. Easier polarimetry

Betatron motion effect

EDM statistic

$\widehat{\omega^{edm}} = \frac{1}{2}(\hat{\omega}_x^+ + \hat{\omega}_x^-)$, where $\omega_x^\pm = \omega^{edm} \pm \omega^{mdm}$

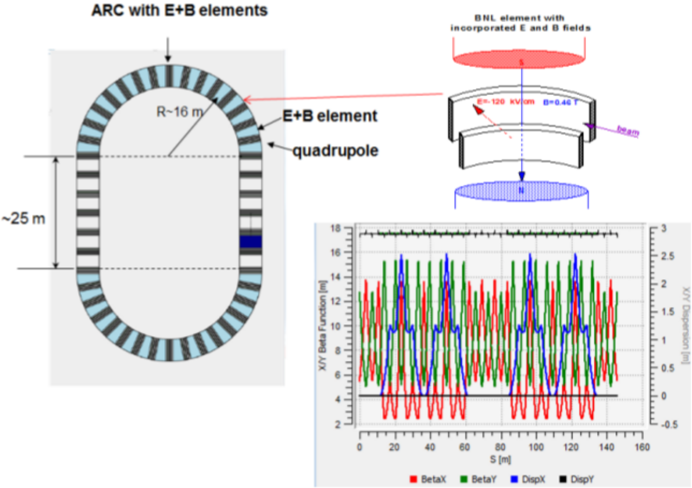
Frequency estimated via fit

$f(t) = a \cdot \sin(\omega_x \cdot t + \delta) \mapsto \hat{\omega}_x$, where $(a, \omega_x, \delta) = \mathrm{const}$

While the solution of T-BMT

$a = \sqrt{\bar{n}_x^2 + (\bar{n}_y \cdot \bar{n}_z)^2}$, where $\bar{n} = g(\boldsymbol{E}, \boldsymbol{B})$

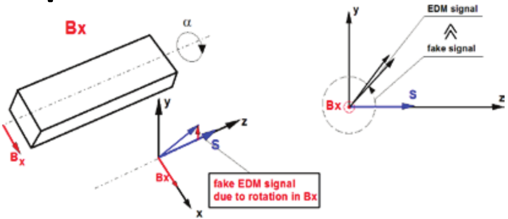
Simulation



ARC with E+B elements
R~16 m
E+B element
quadrupole
~25 m
BNL element with
incorporated E and B fields
E=120 kV/cm
B=0.46 T
beam
X/Y Beta Function [m]
X/Y Dispersion [m]
S [m]
BetaX
BetaY
DispX
DispY

Simulation

Machine imperfections



- $\alpha \sim N(\mu_i, 3 \cdot 10^{-4})^\circ$
- μ_i simulates the application of a spin wheel driver

Particles

- betatron oscillating in the vertical plane
- $E_{FS} \neq E_{kin} \to E_{FS}$

$\Rightarrow$ $\bar{n}_x \ll 1 \Rightarrow$ high sensitivity to perturbations

Analysis

Data

TRK data from the COSY Infinity tracker

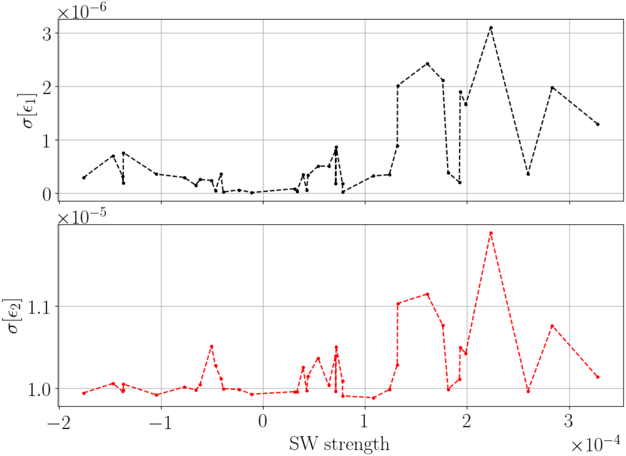
GEN generated from the fit function, with $\bar{n}$, ν_s from tracking

IDL as in GEN, but $\bar{n} = \langle\bar{n}\rangle$, $\nu_s = \langle\nu_s\rangle$

Comparator stats

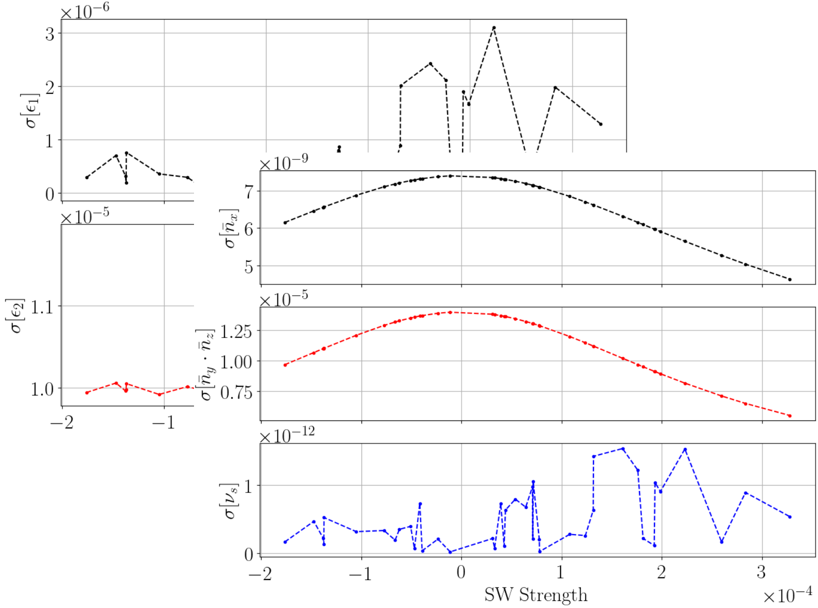
$$\epsilon_1(t) = s_y^{gen}(t) - s_y^{idl}(t)$$
$$\epsilon_2(t) = s_y^{trk}(t) - s_y^{idl}(t)$$

Results



$\times 10^{-6}$
3
2
1
0
$\sigma[\epsilon_1]$
$\times 10^{-5}$
1.1
1.0
$\sigma[\epsilon_2]$
−2
−1
0
1
2
3
SW strength
$\times 10^{-4}$

Results



$\times 10^{-6}$
$\sigma[\epsilon_1]$
$\times 10^{-5}$
$\sigma[\epsilon_2]$
$\times 10^{-9}$
$\sigma[\bar{n}_x]$
$\times 10^{-5}$
$\sigma[\bar{n}_y \cdot \bar{n}_z]$
$\times 10^{-12}$
$\sigma[\nu_s]$
SW Strength
$\times 10^{-4}$

Conclusions

1. The signal amplitude oscillations (as estimated by ϵ_2) are small. They occur at the level at least two orders of magnitude smaller than the expected polarization measurement error. This means the superposition of this systematic error with the random measurement error will exhibit no statistically-significant systematicity.

Conclusions

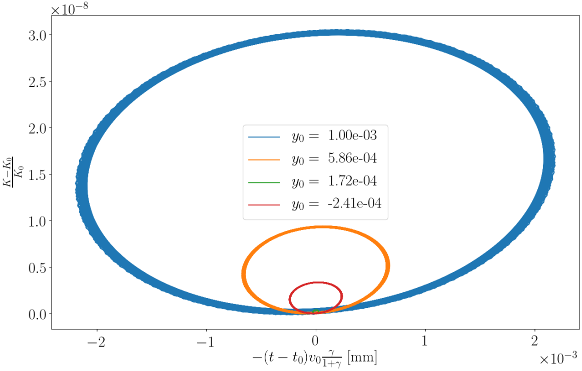
2. The correllation coefficient between the amplitude and frequency estimates is not significant. The amplitude oscillations affect the $\hat{a}$-estimate foremost; their effect on the $\hat{\omega}$-estimate is secondary, and is described by the correlation coefficient. Since it is less than 10%, even if the oscillations happen to be strong enough to affect the amplitude estimate, their effect on the frequency estimate will be reduced by at least a factor of 10.

Conclusions

3. This systematic effect is controllable. And this point is the major advantage of the FD methodology. By applying an external Spin Wheel, the $\bar{n}$ oscillations can be continuously minimized as much as necessary, without changing the pattern of the experiment.

Spin decoherence

Cause



- $\nu_s = \gamma G$
- because of the orbit length difference, beam particles have different γ_{eq}

Supression via sextupole field

Equilibrium level momentum shift

$$\Delta\delta_{eq} = \frac{\gamma_0^2}{\gamma_0^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \alpha_0\gamma^{-2} + \gamma_0^{-4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right]$$

Sextupole field effects

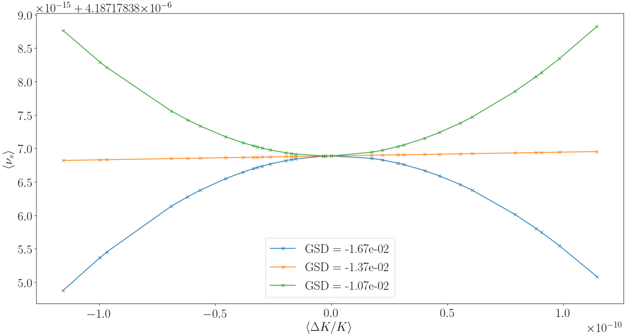
$$S_{sext} = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2}$$

$$\longrightarrow \Delta\alpha_{1,sext} = -\frac{S_{sext} D_0^3}{L}$$

$$\longrightarrow \left(\frac{\Delta L}{L}\right)_{sext} = \mp\frac{S_{sext} D_0 \beta_{x,y}\varepsilon_{x,y}}{L}$$

Sextupole field effects

Momentum compaction factor



Sextupole field effects

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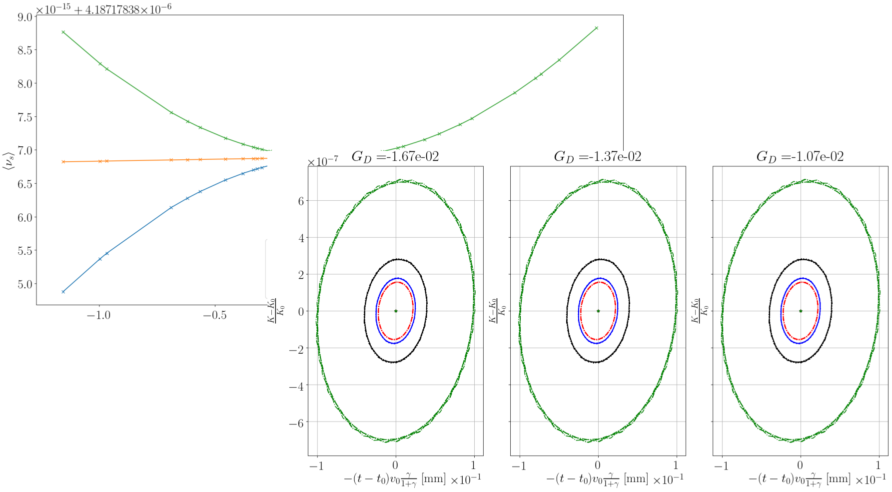
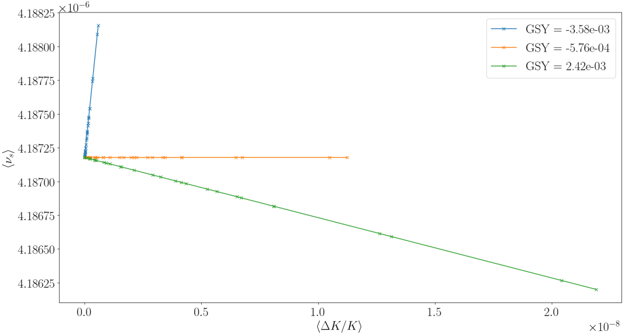


Sextupole field effects

Orbit length



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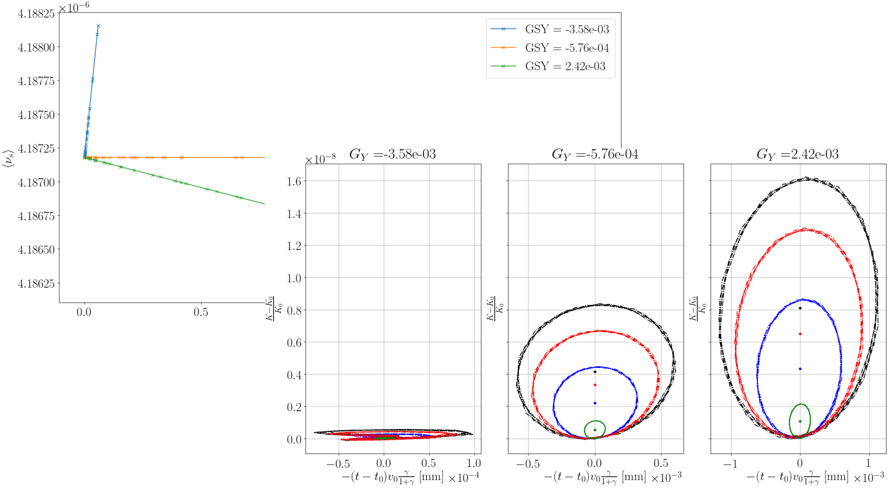

Conclusions

1. The signature of the sextupole field's momentum compaction effect is the change in the functional form of $\langle\nu_s\rangle(\langle\Delta K/K\rangle)$

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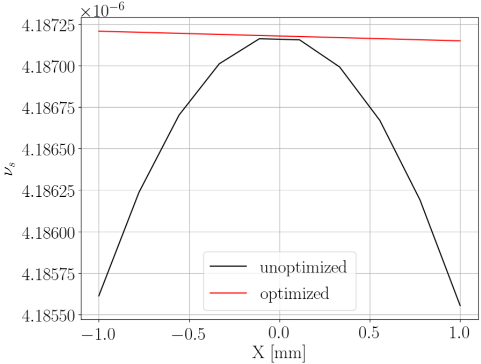
2. ... orbit length effect — reduction in the dispersion of $\langle \Delta K/K \rangle$

Decoherence supression in an ideal lattice



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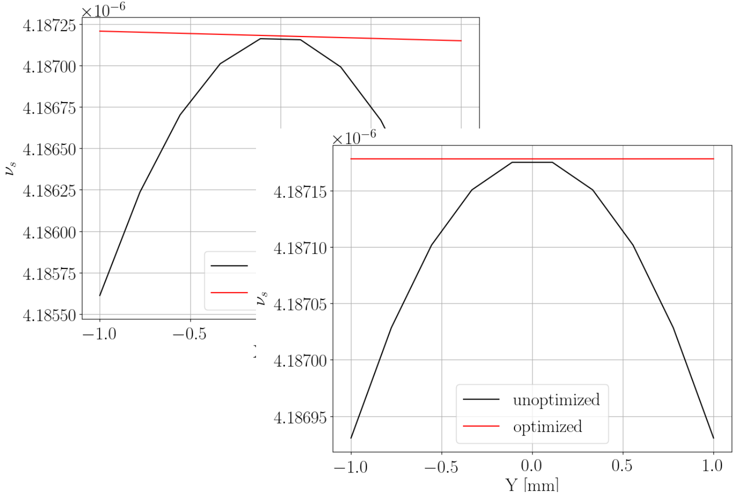

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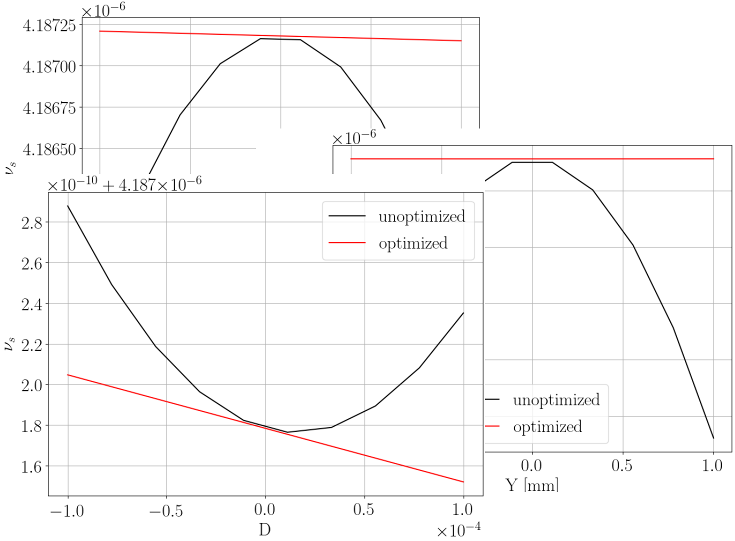
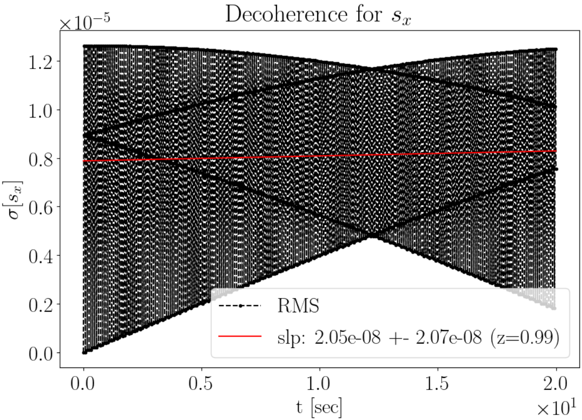


Decoherence in an imperfect lattice



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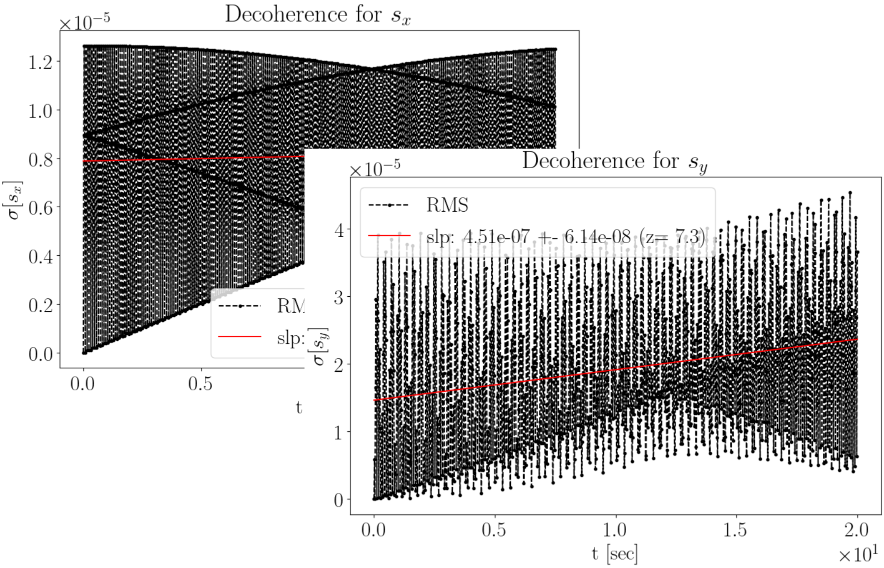
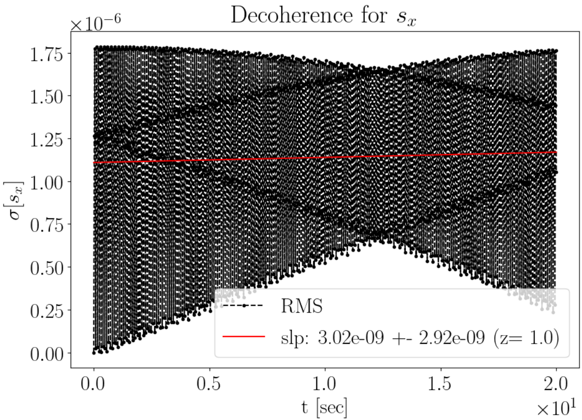


Turning on the sextupoles



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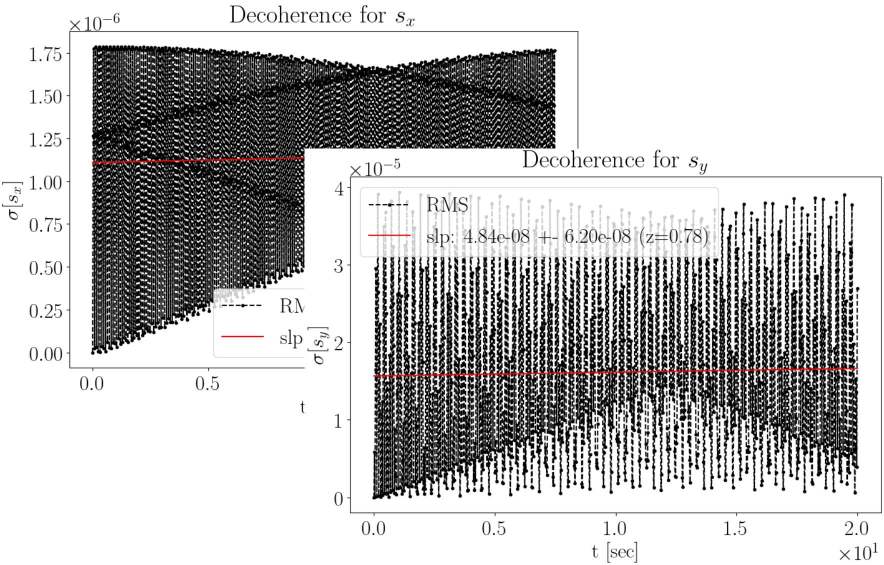


MDM faking signal

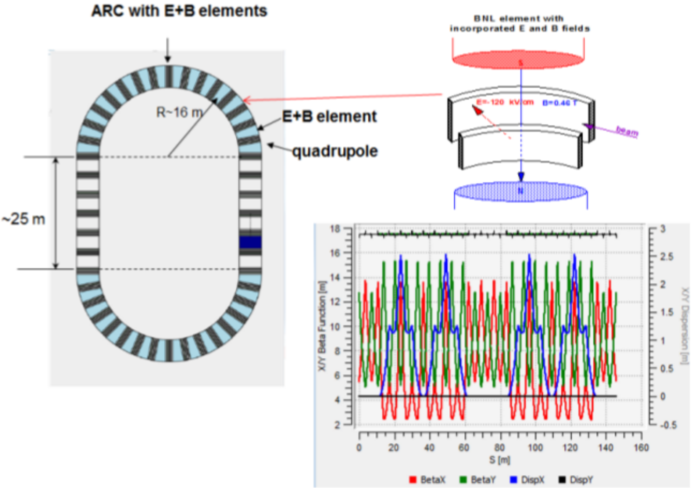
Magnitude

$$\sigma\left[\Omega_x^{MDM}\right] = \frac{q}{m\gamma}\frac{G+1}{\gamma}\frac{\sigma[B_x]}{\sqrt{n}}$$

Questions to consider

- Is the error linear, i.e. $\Omega_x^{MDM} = f(\langle\Theta_{tilt}\rangle)$?
- Is it symmetric with regard to the beam circulation direction change, i.e. $|\Omega_x^{CW}| = |\Omega_x^{CCW}|$?

Analyzed lattice



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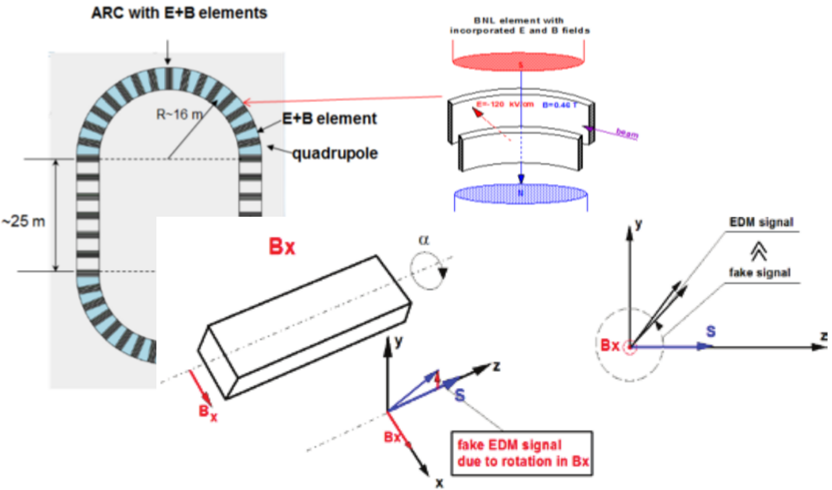
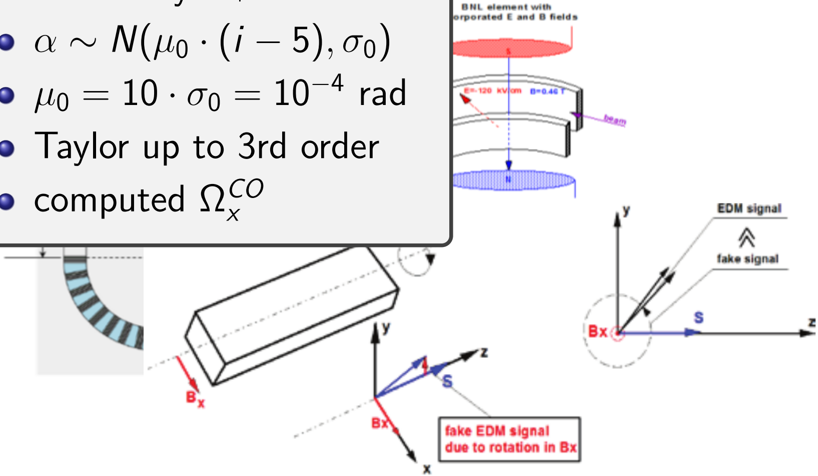
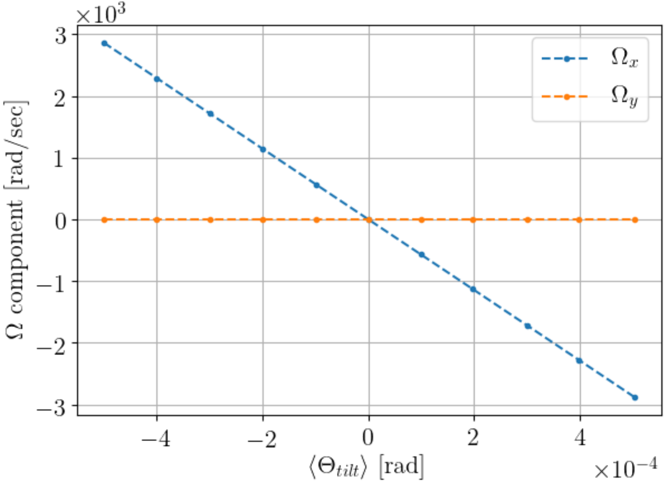


Analyzed lattice

- 11 simulations
- tilted only E+Bs
- $\alpha \sim N(\mu_0 \cdot (i-5), \sigma_0)$
- $\mu_0 = 10 \cdot \sigma_0 = 10^{-4}$ rad
- Taylor up to 3rd order
- computed Ω_x^{CO}

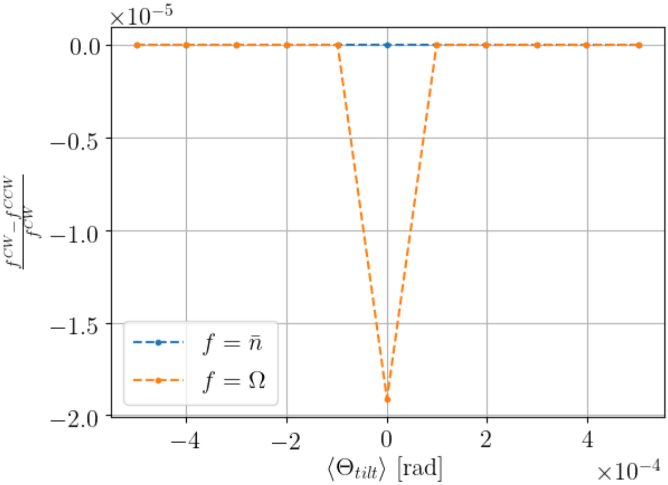


Linearity

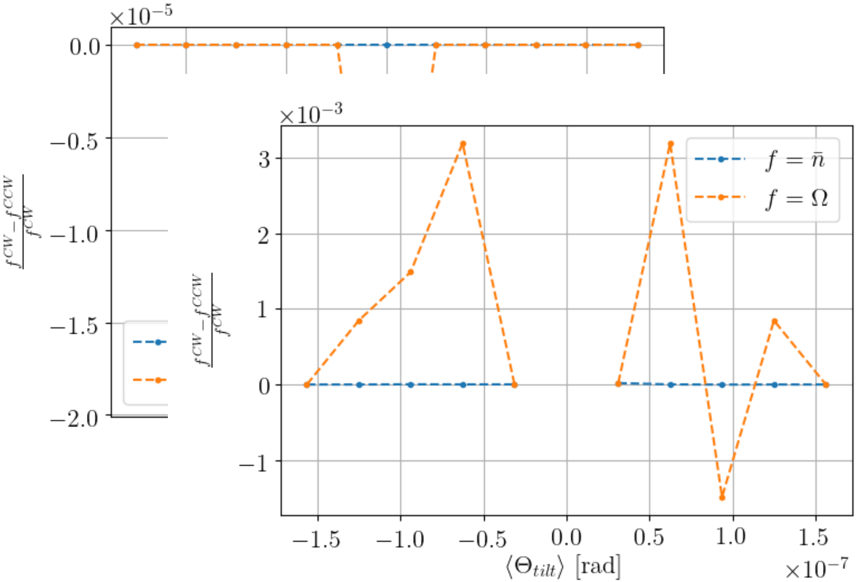


$\times 10^3$
3
2
1
0
−1
−2
−3
Ω component [rad/sec]
−4
−2
0
2
4
$\langle\Theta_{tilt}\rangle$ [rad]
$\times 10^{-4}$
Ω_x
Ω_y

Symmetricity



Symmetricity



Conclusions

$\sigma_\theta = 10^{-4}$ rad, $n = 100$ elements

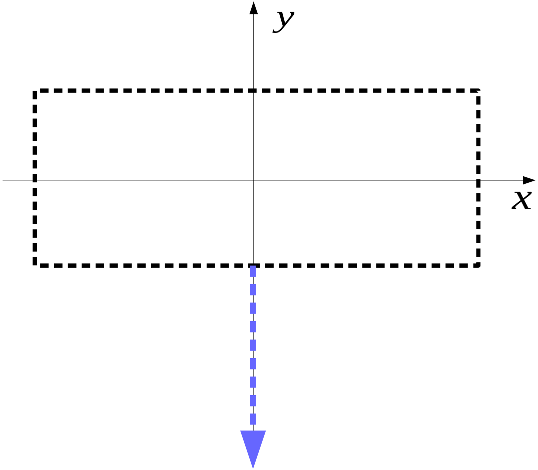
ω^{max} [rad/sec]	$P(\Omega_x^{MDM} < \omega^{max})$
50	67%
100	95%

Properties

1. Linearity
2. Asymmetricity, likely due to the difference between the CW and CCW beams' closed orbits
3. Asymmetricity is less pronounced at higher SW roll rates

Guide field polarity reversal

Why?



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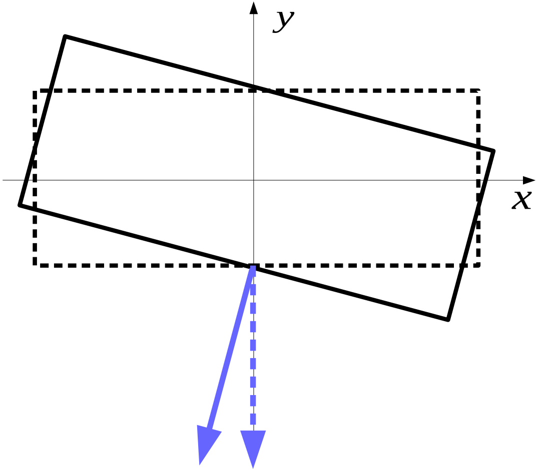

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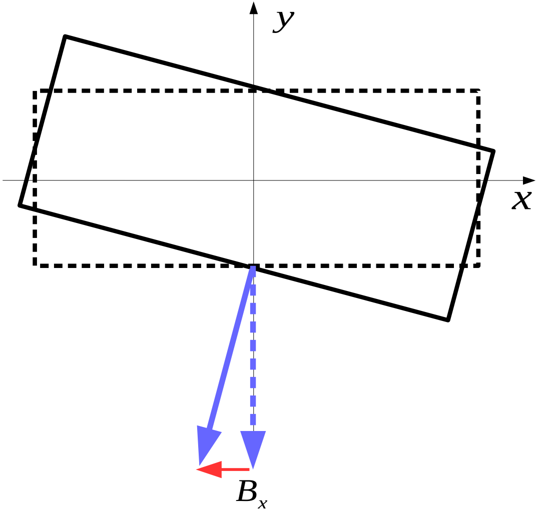

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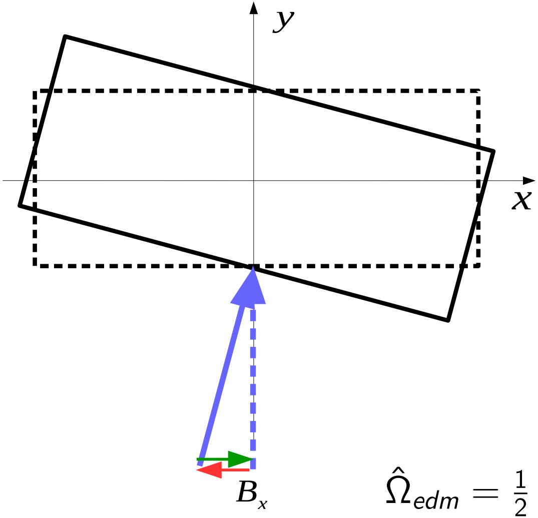

$$\hat{\Omega}_{edm} = \frac{1}{2}\left(\Omega_x^{CW} + \Omega_x^{CCW}\right)$$

Wherein lies the problem?

- $\Omega_x^{MDM} = \frac{q}{m} G B_x$

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- Besides, one has the lattice's spin dynamics asymmetry to consider (above)

$\Rightarrow$ Must reproduce the beam's effective Lorentz factor

Calibration of the effective L-factor

- ν_s, $\bar{n}$ are injective functions of γ_{eff}, meaning $\Omega_y(\gamma_{eff}^1) = \Omega_y(\gamma_{eff}^2) \to \gamma_{eff}^1 = \gamma_{eff}^2$

Calibration of the effective L-factor

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$\Rightarrow$ $\exists! \gamma^0_{eff}$: $\Omega_y = 0$

Calibration of the effective L-factor

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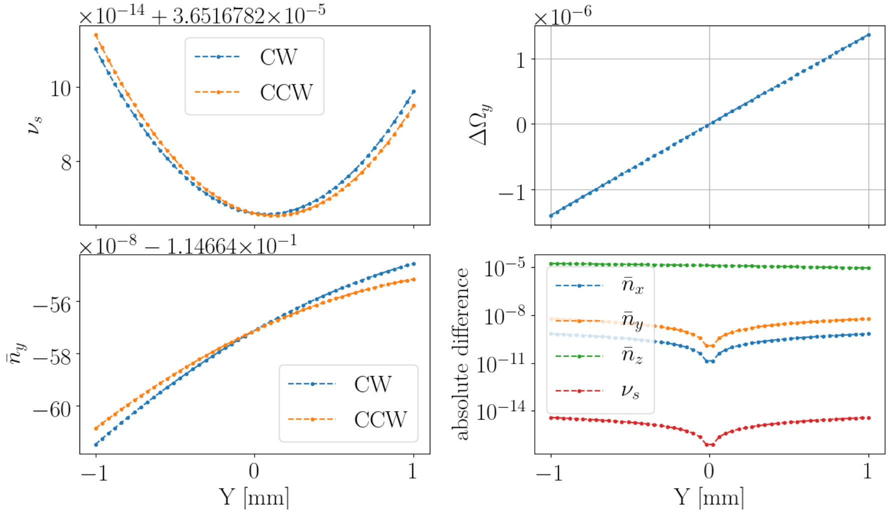
$\Rightarrow$ $\exists! \gamma^0_{eff}$: $\Omega_y = 0$

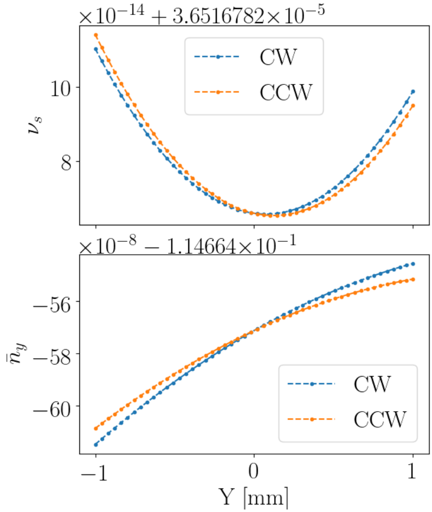
$\Rightarrow$ if both CW, CCW particles are “frozen” in the horizontal plane, their γ_{eff} are equal

Simulation

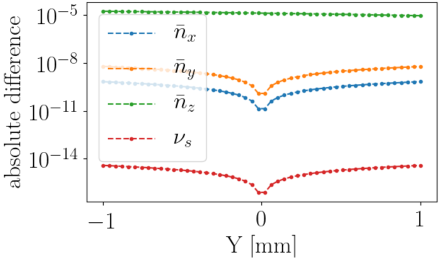


$\times 10^{-14} + 3.6516782\times 10^{-5}$
ν_s
10
8
CW
CCW
$\times 10^{-6}$
$\Delta\Omega_y$
1
0
−1
$\times 10^{-8} - 1.14664\times 10^{-1}$
$\bar{n}_y$
−56
−58
−60
CW
CCW
−1
0
1
Y [mm]
absolute difference
10^{-5}
10^{-8}
10^{-11}
10^{-14}
$\bar{n}_x$
$\bar{n}_y$
$\bar{n}_z$
ν_s
−1
0
1
Y [mm]

Simulation



- Same S_{sext} works for both beams
- $\nu_s^{CW} \approx \nu_s^{CCW} \Rightarrow |\Omega_x^{CW}| \approx |\Omega_x^{CCW}|$



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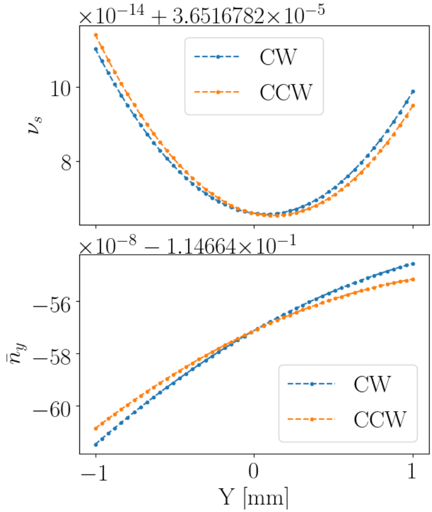

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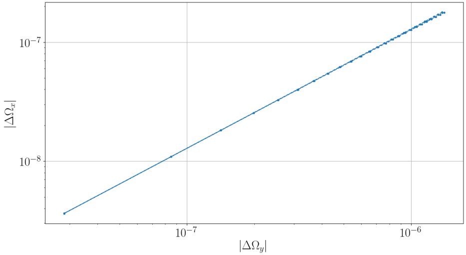
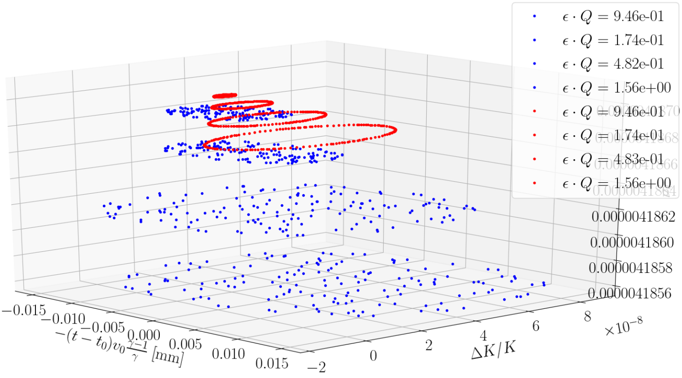


Spin tune a univariate function



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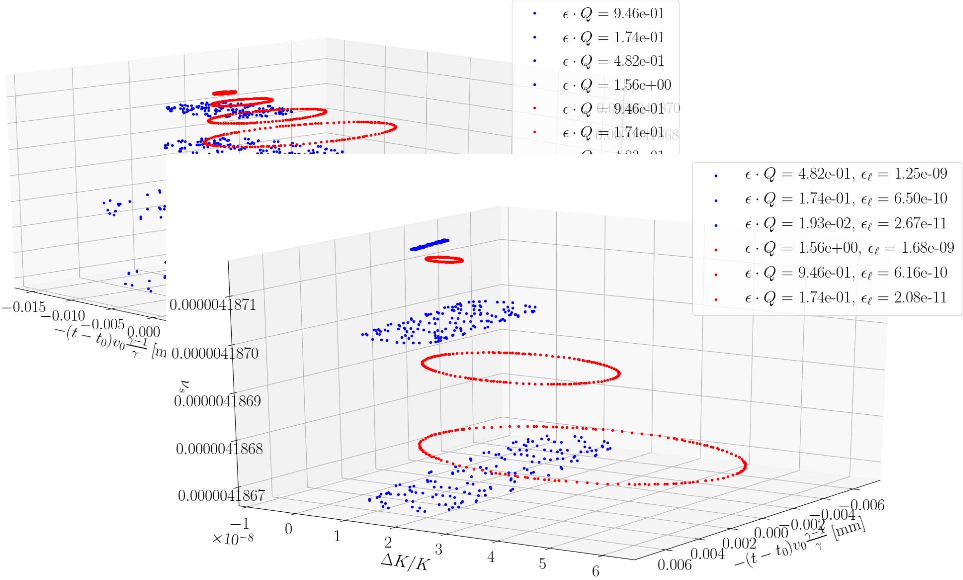
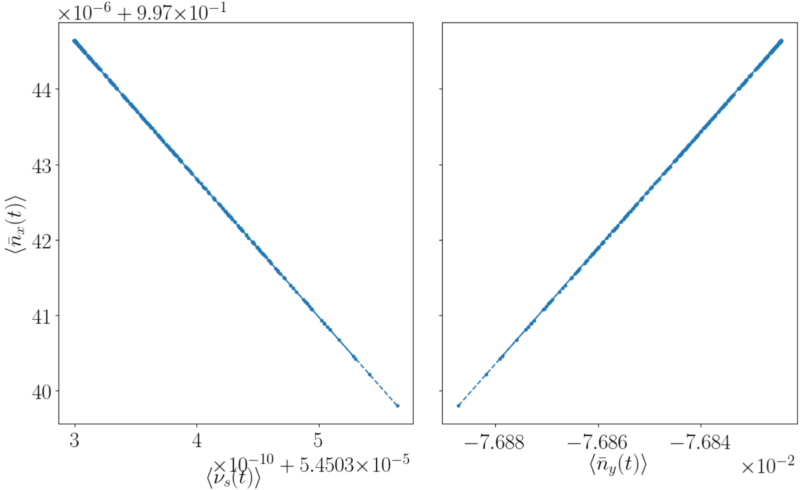


Conclusions

Conclusion 1

- Spin tune is reducible to a univariate function
- Effective Lorentz-factor is a measure of the particle's longitudinal emittance

Conclusions



Conclusions

Conclusion 2

The spin dynamics of particles with the same value of γ_{eff} are equivalent in the gerenal sense (ν_s, $\bar{n}$)

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Disclaimer

At least for the Frozen Spin lattice that I used in simulations

Thank you!