1 $A_{y,y}$ estimators

Assuming the model for the slope of $\ln I_t = \ln I_0 + \beta t + \epsilon_t$ is

$$\beta = -\nu\sigma_X\Theta_X - \nu\sigma_0\Theta\left(1 + PP^t A_{y,y}\right),$$

two ways to construct the estimator for $A_{y,y}$ are via the slope difference $D = \beta^- - \beta^+ = \nu\sigma_0\Theta P^t(P^+ - P^-)A_{y,y}$, with the corresponding *difference* estimator

$$\hat{A}^D_{y,y} = (\nu\sigma_0\Theta)^{-1} \cdot \frac{\hat{D}}{P^t\Delta P},$$

or the slope ratio $\delta = \beta^+/\beta^-$, and the R-statistic

$$R = \frac{\delta - 1}{\delta + 1} = \frac{(P^+ - P^-)P^t A_{y,y}}{2\cdot(1+x) + (P^+ + P^-)P^t A_{y,y}},\ \ x = \frac{\sigma_X\Theta_X}{\sigma_0\Theta},$$

with

$$\hat{A}^R_{y,y} = \frac{2\hat{R}}{P^t\left(\Delta P - \hat{R}\cdot\Sigma P\right)} \cdot (1 + x)\,.$$

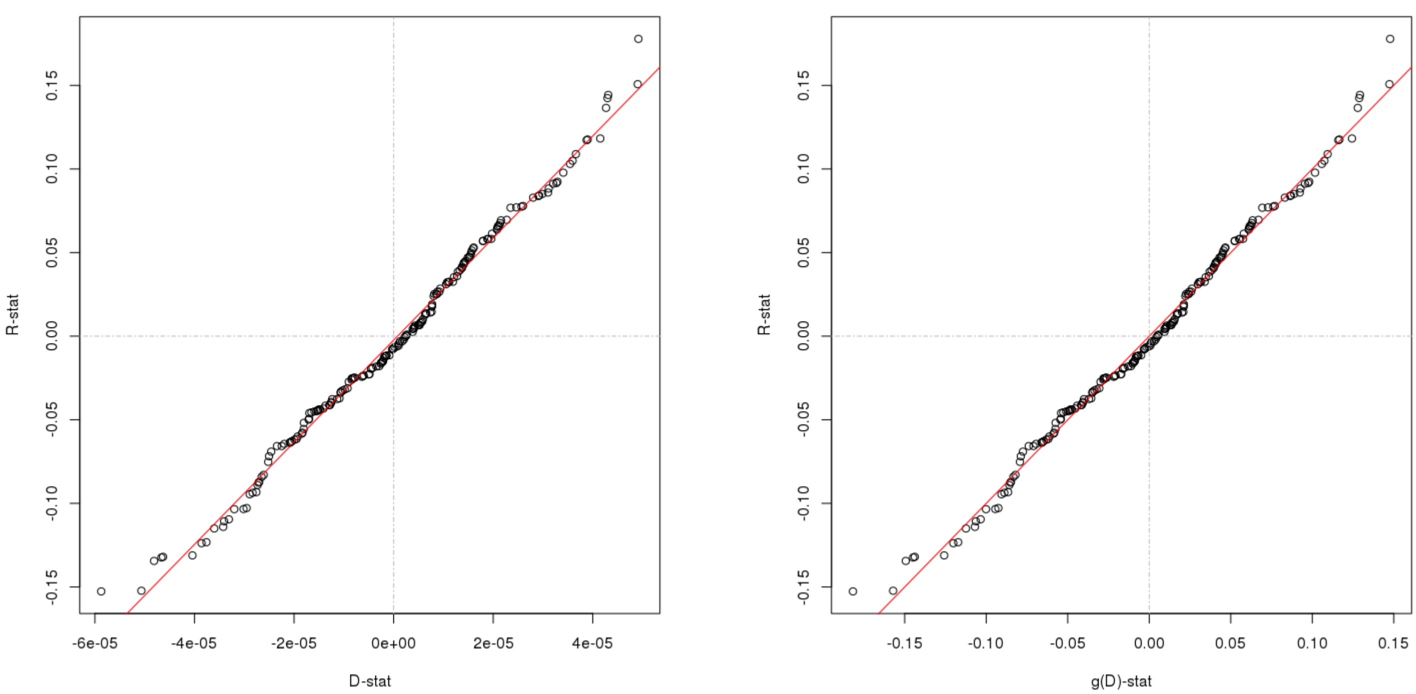
2 Comparison of the estimators

Def. 1 (Congruent in distribution). *Congruent in distribution* means

$$X \overset{d}{\cong} Y \Leftrightarrow f(X|\theta) = f(g(Y)|\theta).$$

The transformation g is called the d-congruency transformation.

We can compare whether $\hat{A}^D_{y,y}$ and $\hat{A}^R_{y,y}$ are congruent in distribution by using the QQ-plot. Here it is:



(a) Red line $y = a + b \cdot x$.

(b) Red line $y = x$.

Figure 1: QQ-Plot for the sample quantiles of the R vs D statistics.

If the distributions of the D- and R-stats are linearly related, the points of the QQ-plot will lie on a line. The transformation $g(\hat{D}) = \alpha + \beta\hat{D} = \frac{2\hat{R}}{P^t\left(\Delta P - \hat{R}\Sigma P\right)}$.

This means that

$$\begin{cases} \hat{A}^R_{y,y} & = g(\hat{D}) \cdot (1+x), \\ \hat{D} & = \hat{A}^D_{y,y} \cdot (P^t \Delta P \nu \sigma_0 \Theta)\, ; \end{cases}$$

hence

$$\hat{A}^R_{y,y} = (1+x) \cdot \Big[\alpha + \beta \cdot \big(P^t \Delta P \nu \sigma_0 \Theta \big) \cdot \hat{A}^D_{y,y} \Big]$$