1 $A_{y,y}$ estimators

Assuming the model for the slope of $\ln I_t = \ln I_0 + \beta t + \epsilon_t + \sigma_0$ is

$$\begin{aligned}
\beta &= -\nu\sigma_X\Theta_X - \nu\sigma_0\Theta\left(1 + PP^t A_{y,y}\right) \\
&= -\nu\sigma_0\Theta\left(1 + x + PP^t A_{y,y}\right), \\
x &= \frac{\sigma_X\Theta_X}{\sigma_0\Theta},
\end{aligned}$$

two ways to construct the estimator for $A_{y,y}$ are via the slope difference $D = \beta^- - \beta^+ = \nu\sigma_0\Theta P^t(P^+ - P^-)A_{y,y}$, with the corresponding *difference* estimator

$$\hat{A}^D_{y,y} = (\nu\sigma_0\Theta)^{-1} \cdot \frac{\hat{D}}{P^t\Delta P},$$

or the slope ratio $\delta = \beta^+/\beta^-$, and the R-statistic

$$R = \frac{\delta - 1}{\delta + 1} = \frac{(P^+ - P^-)P^t A_{y,y}}{2\cdot(1+x) + (P^+ + P^-)P^t A_{y,y}},$$

with the *ratio* estimator

$$\hat{A}^R_{y,y} = \frac{2\hat{R}}{P^t\left(\Delta P - \hat{R}\cdot\Sigma P\right)} \cdot (1+x).$$

2 Comparison of the estimators

2.1 Estimation of the asymmetry

The compution of the asymmetry from the difference statistic requires knowledge of the product $\sigma_0\Theta$. The non-target scattering term x could take any (unknown) value, however, so long as it is fixed and its variance satisfies statistical precision requirements. This cannot be said of the ratio estimator, in which x remains as a nuisance parameter.

The trade-off between the two estimators, therefore, is: if the value of $\sigma_0\Theta$ is known to a reasonable precision, the difference estimator trupms the ratio one; if, however, there's no way to know $\sigma_0\Theta$ itself reasonably well, it might be easier to estimate the ratio ${}^{\sigma_X\Theta_X}/_{\sigma_0\Theta}$.

It must be said, though, that systematic varying of x, causing grief in to both estimator, causes *more* to the R one. Compare:

$$\hat{A}^D_{y,y} = \frac{\hat{D}}{\nu\sigma_0\Theta P^t\Delta P} - \frac{\Delta x}{P^t\Delta P},$$

versus

$$\hat{A}^R_{y,y} = \frac{(2+\Sigma x)\,\hat{R}}{P^t\left(\Delta P - \hat{R}\Sigma P\right)} - \frac{\Delta x}{P^t\left(\Delta P - R\Sigma P\right)}.$$

2.2 Systematic effects

We could try to learn *something* about the structure of the experiment by comparing the R- and D-statistics.

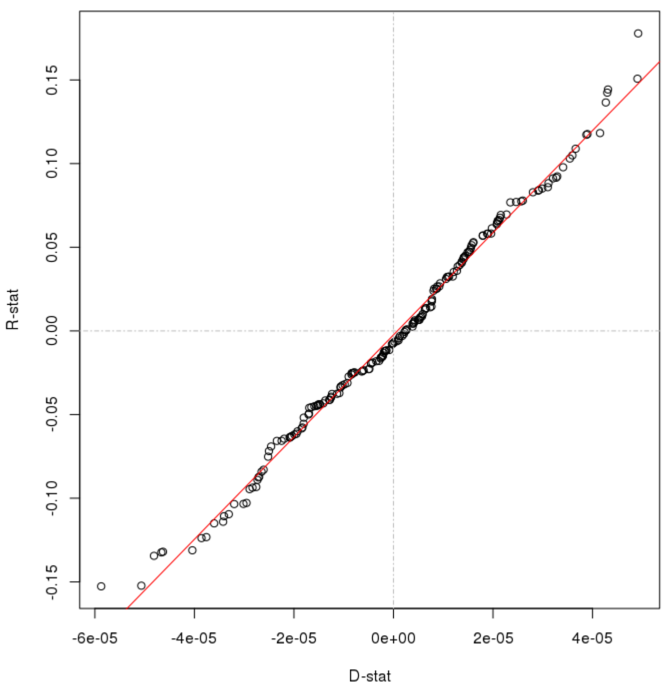
If we plot the QQ-plot of $\hat{R}' \equiv {}^{2\hat{R}}/_{P^t}\left(\Delta P - \hat{R}\Sigma P\right)$ against $\hat{D}$, Fig. 1, we see that the statistics' distributions are linearly related (the points of the QQ-plot lie on a line).

By fitting this plot, we can find the linear transformation *for the given sample* of slopes $\hat{g}(\hat{D}) = \hat{\alpha} + \hat{\beta}\hat{D} = {}^{2\hat{R}}/_{P^t}\left(\Delta P - \hat{R}\Sigma P\right)$ This means that

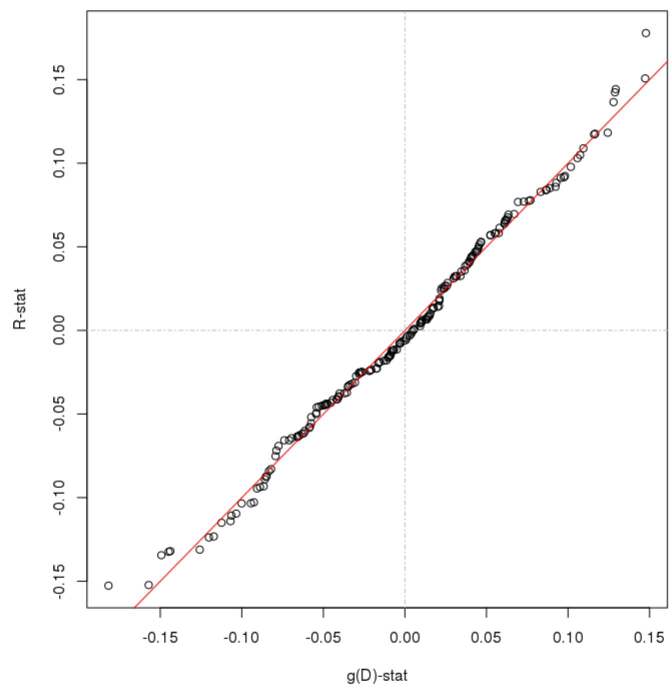
$$\begin{cases} \hat{A}^R_{y,y} &= \hat{g}(\hat{D})\cdot(1+x), \\ \hat{D} &= \hat{A}^D_{y,y}\cdot\left(P^t\Delta P\nu\sigma_0\Theta\right); \end{cases}$$

hence

$$\hat{A}^R_{y,y} = (1+x)\cdot\left[\hat{\alpha} + \hat{\beta}\cdot\left(P^t\Delta P\nu\sigma_0\Theta\right)\cdot\hat{A}^D_{y,y}\right]$$



(a) Red line $y = a + b \cdot x$.



(b) Red line $y = x$.

Figure 1: QQ-Plot for the sample quantiles of the R vs D statistics.