

Systematic Errors

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Introduction

The Test of Time-Reversal Invariance experiment (TRIC) is a transmission experiment aimed at achieving an accuracy in the order of 10^{-6} in the estimate of a cross section asymmetry. For that purpose, a polarized charged particle beam is scattered on a polarized target, and the rate of decay of the beam current is measured, which is proportional to the total scattering cross section. The difference in the cross sections for two opposite polarized states is proportional to the sought-after asymmetry. If the asymmetry is zero, Time-Reversal Invariance holds.

Here we will describe some possible problems with the experimental design that have to be overcome if one is to measure the asymmetry with a required accuracy.

1 The Design

In the following we describe the general idea of the experimental design of TRIC.

A particle beam circulates in the accelerator ring. At every turn it is scattered by the internal target (accumulated in a specially made cell) at the rate $\sigma_T \Theta$, but there is also background loss happening at the rate $\sigma_X \Theta_X$. (Notation: σ is the scattering cross section, $\Theta = \int_L n(z) dz$ is the thickness of the scatterer, L is the accelerator circumference.) At every turn the beam excites an active current transformer, the excitation signal is integrated, and the value of the average beam current is being recorded.

All of this boils down to the following formula for the measured signal:

$$I_t = I_0 \cdot e^{-\nu(\sigma_T \Theta + \sigma_X \Theta_X) \cdot t} \equiv I_0 \cdot e^{\beta \cdot t},$$

where ν is the circulation frequency.

In TRIC, we use the deuterium target, and hence it has two polarization components: vector and tensor. The vector component P_y^t aligns with the guiding magnetic field, as does the beam polarization (purely vector) P_y , and the tensor component P_{xz}^t with the specially generated holding field. The target cross section has the form

$$\sigma_T^\pm = \sigma_0 \cdot (1 + A_{y,y} P_y^t P^\pm + A_{y,xz} P_{xz}^t P^\pm). \quad (1)$$

The data are log-transformed, and the linear model $\ln I_t = \ln I_0 + \beta^\pm \cdot t + \epsilon_t$ is fitted to it; the difference $\beta^- - \beta^+ = \nu \Theta \sigma_0 \cdot (P^+ - P^-) \cdot (A_{y,y} P_y^t + A_{y,xz} P_{xz}^t)$. Ideally, the vector polarization component is $P_y^t = 0$, and we estimate the asymmetry as

$$\begin{cases} \hat{A}_{y,xz} &= (\nu \Theta \sigma_0 \Delta P P_{xz}^t)^{-1} \cdot (\hat{\beta}^- - \hat{\beta}^+), \\ \sigma [\hat{A}_{y,xz}] &= (\nu \Theta \sigma_0 \Delta P P_{xz}^t)^{-1} \cdot \sqrt{2} \cdot \sigma [\hat{\beta}]. \end{cases} \quad (2)$$

2 Systematic errors

We will distinguish between two sources of systematic error when estimating a quantity: statistical methodology, and physical interpretation.

The methodology source is the more easily accessible one; the errors of this kind are due to the contradiction between the assumptions made versus the properties of the data, and the latter can be easily checked. In essence, these errors arise from a lack of vigilance on the analysts' part.

By an interpretation error, on the other hand, we mean a fundamental experimental design flaw, one that prohibits distinguishing between multiple channels the outcome could've come about.¹

An example of a methodological systematic error would be using an ordinary least squares approach to fitting a time series with the autocorrelated disturbance term (generalized least squares are more appropriate here); or disregarding the fact that measurement error deterministically depends on time.

For an example of an interpretation error, consider the following: the ideal TRIC experiment requires that at least one of the two conditions hold: either 1) the injected ABS beam consists only of particles with the appropriate spin states, or 2) the target chamber magnetic field does not have a vertical component.

Neither of the conditions are practically realizable, meaning that the vertical component of the target vector polarization is not zero. This means a beam particle can be scattered via either of *two* channels which we have no way of distinguishing in our transmission experiment design. That means the estimator in (2) must be corrected by subtracting a scaled estimate of $A_{y,y}$:

$$\hat{A}_{y,xz} = C\Delta\hat{\beta} - P_y^t/P_{xz}^t \cdot \hat{A}_{y,y}. \quad (3)$$

It also means the value of $A_{y,y}$ has to be estimated first, making TRIC a two-stage experiment.

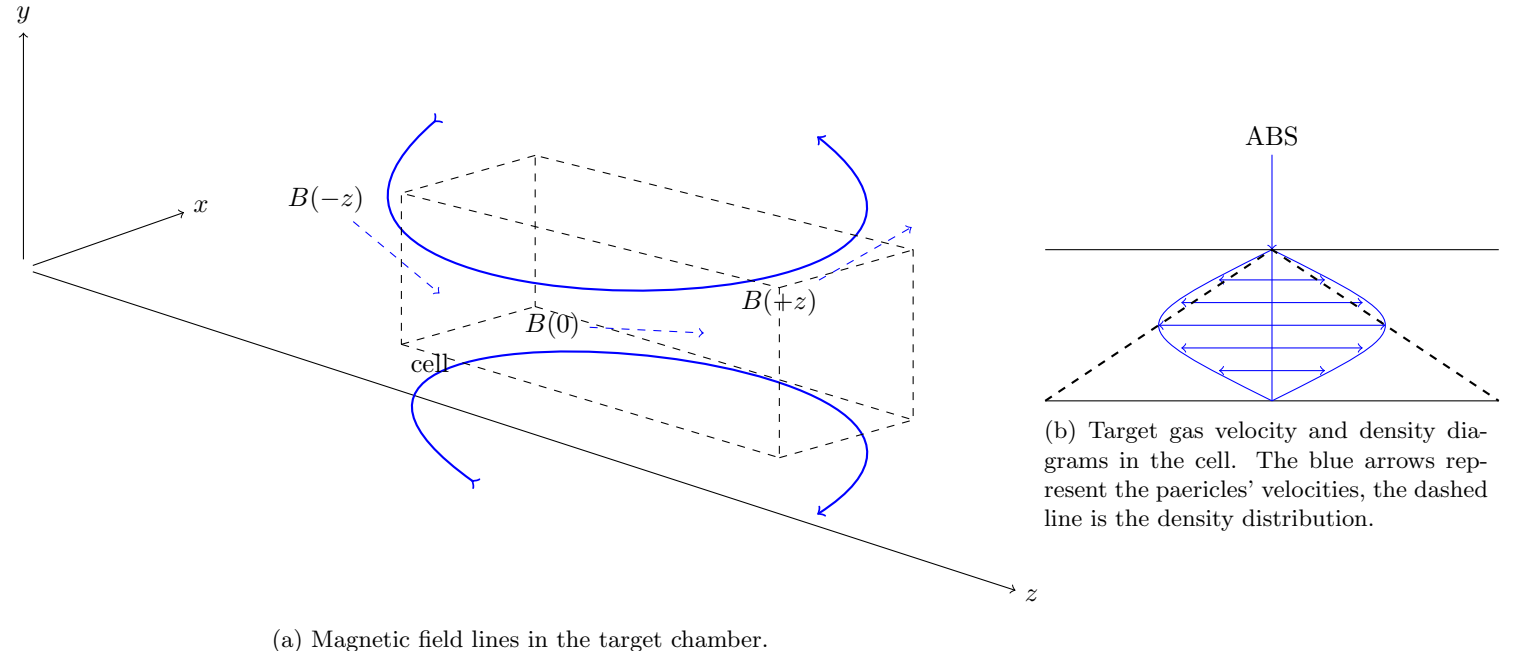
For an analysis of the statistical properties of the TRIC data obtained at COSY in 2016, please refer to [2]. Below, we will concern ourselves with the interpretation problem outlined above.

3 The holding field

TRIC is a double-polarized experiment. The target polarization is held oriented in the xz direction of the horizontal plane by the holding magnetic field. The field's structure influences two aspects of the experiment's success: in the first place, the presence of a vertical component increases the magnitude of the bias term in eq. (3); in the second, the deviation of the field's direction from 45 degrees in the horizontal plane decreases the magnitude of the useful signal.

3.1 The increase of the faking signal

The examination of the possible sources of systematic error [3] shows that the vector-polarized total cross section scattering asymmetry $A_{y,y}$ is the primary source of systematic error in the reaction zone. The interfering scattering channel appears for two reasons: the inevitable presence of vector-polarized quantum states in the target's population, and the presence of the vertical component in the target chamber's magnetic field.



¹More on the subject of the schematic description of experiments in [1].

In Figure 1a we drew a schematic representation of the structure of the holding field. According to it, the magnetic induction vector in the first half of the cell is related to that of the second half as in

$$\vec{B}(-z) = (-B_x, -B_y, B_z) \text{ and } \vec{B}(+z) = (B_x, B_y, B_z),$$

or

$$\vec{B}(-z) = R(\hat{z}, \pi) \vec{B}(+z). \quad (4)$$

Since the spin vector aligns itself with the magnetic field, $P_y(-z) = -P_y(+z)$ too, so the minimization of the faking signal reduces to symmetrizing the holding field radially and along the z -direction.

3.2 The diminution of the true signal

There's another aspect in which the magnetic field's structure influences eq. (3). In order to have the maximum tensor polarization in the xz -direction, we need the horizontal component of the field be turned at 45 degrees to the side of the beam's orbit. It can be shown [4] that the rate at which the beam scattering occurs is influenced by the field's inhomogeneity as in

$$\beta = \beta' \cdot \frac{\int_{\ell} \rho(s) \cos 2\Delta\Theta \, ds}{\int_{\ell} \rho(s) ds},$$

where ℓ is the cell length, $\rho(s)$ is the spatial distribution of the target density, and $\Delta\Theta$ is the angle deviation of the field's direction from 45 degrees.

4 Spin flipping

There are two possible ways to perform the time-reversed $\vec{p}\vec{d}$ scattering: we can flip the beam's polarization, or we can flip the target's. If we do the latter, this is accomplished by inverting the direction of the magnetic field. However, in this case we would have to ensure that the inverted field's deviation from the desired direction is not systematically different from the original. If it is (and it's likely to be), the scattering efficiencies of the two cases would be different, and a bias would be introduced.

Let us denote the angle deviation at a point z $\Delta\theta(z) = \mathfrak{B}_{\theta} + \delta_{\theta}$. Then the bias one can expect in the estimate $\hat{A}_{y,xz}$ is [4]

$$\mathfrak{B}_{\hat{A}_{y,xz}} = \frac{1}{2} \left[F(E[A(\chi^c)] \cos 2\mathfrak{B}_{\theta}^+) + F(E[A(\chi^c)] \cos 2\mathfrak{B}_{\theta}^-) \right] - 1, \quad (5)$$

where $F(f) = 1/\Theta_T \cdot \int_{\ell} \rho(z) f(z) \, dz$, χ^c a measure of the magnetic field strength, $A(\chi^c)$ is the proportionality coefficient between the magnetic field strength and the target polarization.

References

- [1] Simon Saunders. "Derivation of the Born Rule from Operational Assumptions." arXiv:quant-Ph/0211138, November 21, 2002. <http://arxiv.org/abs/quant-ph/0211138>.
- [2] Alexander Aksentyev. "Analysis of the 2016 data."
- [3] P.D. Eversheim, B. Lorentz, and Yu. Valdau, Forschungszentrum Jülich, COSY, proposal #215, 2012.
- [4] Alexander Aksentyev. Diploma thesis (2015).