Estimation of the total cross section and cross section asymmetry in a transmission experiment

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Abstract The optical theorem guarantees the existence of a genuine null observable for T-symmetry in double-polarized proton-deuteron scattering: the total cross section asymmetry $A_{y,xz}$. An experiment exploiting this possibility (TRIC) is being planned at the cooler synchrotron COSY-Jülich, Germany. TRIC aims at achieving an accuracy in the order of 10^{-6} in the asymmetry estimate. The major source of systematic error in the experiment is the scattering of the beam by the target's vector polarization. In order to correct for this, a dedicated experiment to measure the cross section asymmetry $A_{y,y}$, must be performed. In this contribution we outline the procedure to estimate it, as well as the unpolarized *pp* and *pd* cross sections, and provide some preliminary estimates.

I. TRIC EXPERIMENT

TRIC (test of Time-Reversal Invariance at COSY) is a transmission experiment planned at the Cooler SYnchrotron COSY-Jülich for the purpose of testing Time-Reversal Invariance. Its physical foundation is the use of a genuine null-observale for T-symmetry, — the total cross section asymmetry in double-polarized proton-deuteron scattering, — whose existence is guaranteed by the optical theorem [1]. TRIC is aimed at achieving the accuracy of 10^{-6} in the cross section asymmetry estimate $\hat{A}_{y,xz}$.

The total cross section in a double-polarized scattering involves a number of polarization-dependent terms:

$$\sigma_{tot} = \sigma_0 \cdot \left(1 + \sum_{i,j} A_{i,j} P_i P_j^t + \sum_{k,mn} A_{k,mn} P_k P_{mn}^t\right), \tag{1}$$

where P_j^t and P_i are respectively the j-projection of target and the i-projection of beam polarizations, P_{mn}^t is the mn-tensor component of target polarization, σ_0 is the unpolarized cross section component, and $A_{i,j}/A_{k,mn}$ is the appropriate asymmetry.

The asymmetry that serves as the null-observable of T-symmetry is $A_{y,xz}$, all others being faking observables [1]. TRIC's experimental design limits the influence of all faking observables to below the experimental accuracy [2], except for that of $A_{y,y}$, caused by the misalignment of the target and beam polarizations. Thus arises the problem of knowing the extent to which vector target polarization must be controlled, for which the knowledge of the value of $A_{y,y}$ is required.

Unpolarized cross section σ_0 is a parameter in both estimators' distributions, and hence it must be known as well.

In this proceeding we summarize the results of two experiments preliminary to TRIC, in order to assess the viability of the transmission-experiment-based method of accessing the $A_{y,xz}$ observable.

II. THEORETICAL BACKGROUND

A. Physics

The intensity of a particle beam revolving inside an accelerator decreases according to the Beer-Lambert law:

$$\begin{aligned} I_{n+1} &= I_n \cdot \exp\left(-\sum_{i=i}^{N} \sigma_i \cdot \int_0^L n_i(z)\mathrm{d}z\right) \\ &= I_n \cdot \exp\left(-\sum_{i=i}^{N} \sigma_i \cdot \Theta_i\right) \\ &= I_n \cdot \exp\left(-\sum_i \frac{1}{\tau\,i}\right), \end{aligned}$$

where L is the beam path length, N is the number of attenuating species, σ_i is the attenuation cross section, n is the number of passed revolutions, $\Theta_i = \int_L n_i(z)\mathrm{d}z$ is the thickness of the corresponding attenuating element.

For the average beam current, integration of the above yields

$$I_t = I_0 \cdot \exp(\beta \cdot t), \tag{2}$$

with $\beta = \sum_i \beta_i = -\nu \cdot \sum_i {}^1/_{\tau_i}$, ν — the beam revolution frequency.

In the case of unpolarized scattering, an unpolarized beam interacts with an unpolarized gas target with cross section σ_0; to that add all the losses in the accelerator ring ($\sigma_x\Theta_x$), to produce the following expression for beam loss:

$$\beta = -\nu\left(\sigma_0\Theta + \sigma_x\Theta_x\right). \tag{3}$$

Since $\sigma_x\Theta_x$ is independent from the presence of the target, an estimate of the cross section is obtained from

$$\hat{\sigma}_0 = \frac{\hat{\beta}_{off} - \hat{\beta}_{on}}{\nu\Theta_{on}}, \tag{4}$$

where $\hat{\beta}_{on/off}$ is the slope estimate in an on-/off-cycle.

In the *pd* scattering in which both the beam and the target have vector polarization, from equation (1), beam loss is

$$\beta = -\nu \left(\sigma_0 (1 + A_{y,y} P_y^t P_y) \Theta_{on} + \sigma_x \Theta_x \right) ,$$

from which an asymmetry estimate can be computed as the difference between the slopes with spin states *up* and *down*:

$$\hat{A}_{y,y} = \frac{\hat{\beta}_{on}^{-} - \hat{\beta}_{on}^{+}}{\nu P_y^t \Delta P_y \cdot \sigma_0 \Theta_{on}}. \quad (5)$$

B. Statistics

We estimate β by fitting a linear model to logarithmized beam current data, $\ln I_t = \ln I_0 + \beta \cdot t + \epsilon_t$, using the least squares method. In order that the estimate be minimum-variance mean-unbiased, the data must satisfy the Gauss-Markov conditions:

1. Linearity and additivity of the relationship;
2. Independence of the time and error variables (strict exogeneity);
3. No serial correlation of the error;
4. Constant variance of the error (homoskedasticity).

Linearity is necessary for the validity of using linear regression; homoskedasticity and absence of serial correlation are required for the efficiency, and exogeneity for the consistency of the estimator.

This means the following series of questions has to be answered in order to verify the validity of our results:

1. Is the logarithm of beam current a linear function of time?
2. Are the errors uncorrelated with time?
   - Is measurement time measured with negligible error?
   - Are there predictors other than time?
   - Is there among the omitted variables a predictor dependent on current?
3. What is the interpretation of the slope?

Below, we will be concerned with the former two.

III. OVERVIEW OF DATA

We have analyzed two data sets: one was measured in 2012, the other in 2016.

In the 2012 experiment, the proton beam was scattered on the hydrogen target, both unpolarized; the beam was cooled using electron cooling and bunched by the barrier bucket; the cycles lasted for one hour each.

In the 2016 experiment the proton beam was scattered on the deuteron target, both polarized; the beam had undergone RF-bunching and electron-cooling; the cycles lasted for 12 minutes, the first half of which the target was turned on, in the second off; the beam spin state alternated from spin up to spin down through no spin, while the target spin state remained constant (spin up).

In both experiments the target was produced by an Atomic Beam Source (ABS) capable of producing jets of polarized hydrogen and deuterium, and was concentrated within a storage cell located inside the PAX target chamber, described in more detail in [3]. The cycles for both experiments are presented in FIG. 1. The measurements were made with a Bergoz Beam Current Transformer, and read out by a standard Siemens APC. In the 2012 experiment, the BCT offset was around 2300 a.u., in 2016, the offset it was about 4300 a.u. The zeroing of current before each cycle, that can be observed in the 2012 panel, is due to the overflow of the measurement system, since the beam current at injection lead to a voltage that is out of range for this ADC.

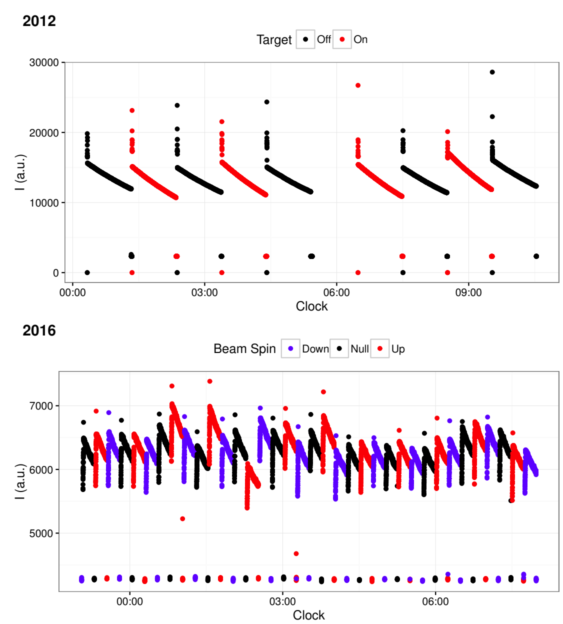

FIG. 1: Average beam current as a function of time. Upper panel: experiment in 2012; the cycles with the target present are drawn in red, those without in black. Lower panel: experiment in 2016. The cycles are colored according to the beam spin state (red for spin up, blue for down, and black for cycles with unpolarized beam). The first half of a cycle is collected with the target turned on, the second half off.

TABLE I: Characteristics of a typical cycle.

Charactetistic	Test	P-value
Linearity	Harvey-Collier	0%
-	Rainbow	0%
Constant slope	Chow[a]	100%
-	Moving estimates	1%
Homoskedasticity	Breusch-Pagan	0%
Autocorrelation	Durbin-Watson	0%

[a] The Chow test was performed at every point in the fitting range. The average of F-statistics is used as the test statistic.

IV. SLOPE

In order to correctly estimate a cycle's slope, we subtract the BCT offset Δ from the data. This is done because slope

$$\tilde{\beta} = \frac{\mathrm{d}\ln \tilde{I}_t}{\mathrm{d}t} = \frac{1}{\tilde{I}_t}\frac{\mathrm{d}\tilde{I}_t}{\mathrm{d}t},$$

where, if the measured current

$$\tilde{I}_t = I_t + \Delta_t = I_0 \exp\left(\beta \cdot t\right) + \Delta_t,$$

then

$$\tilde{\beta} = \frac{1}{1+\lambda_t}\left(\beta + \frac{1}{I_t}\frac{\mathrm{d}\Delta_t}{\mathrm{d}t}\right),$$
$$\lambda_t = {I_0}^{-1} \cdot \Delta_t \cdot \exp\left(-\beta \cdot t\right).$$

Even a constant offset must be removed in order to have a constant slope to estimate. The presence of an offset (let alone a time-dependent one) violates the exogeneity assumption, and hence biases the estimate. At this stage, estimation was done assuming offset was constant within a cycle, and its value was estimated as the median of the post-cycle current.

After subtracting the offset, the linear model $\ln I_t = \ln I_0 + \beta t + \epsilon_t$ is fitted via the ordinary least squares method. The reduced chi-squares computed from model residuals deviate from one starting from the fourth decimal place; however, one should note that the data are likely to have structural slope changes, and do not pass linearity tests as well (see TABLE I). Because the model residuals exhibit serial correlation (FIG. 2), the slope estimates' standard errors are estimated with robust estimators. This procedure is applied to both data sets.

The fit results for all the slopes are summarized in TABLE II. The results contradict our expectations, because, for target-on slopes, the cross section is expected to increase when the beam spin is up, and decrease when it's down, i.e. $\beta^+ < \beta^0 < \beta^-$; the estimates, however, satisfy $\hat{\beta}^0 < \hat{\beta}^- < \hat{\beta}^+$. This might have suggested that the labeling of the spin states was done incorrectly.

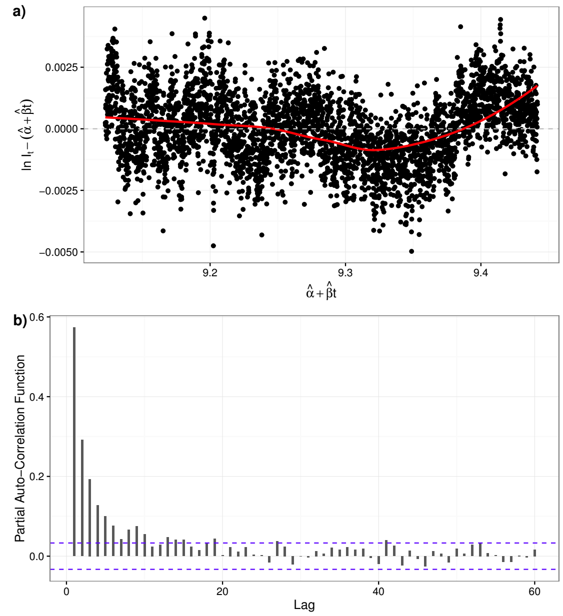

FIG. 2: Some diagnostic plots for a typical 2016 cycle. a) The plot of the model residuals against the fitted values. The red line indicates that the linear model systematically underestimates the beam current at the cycle start and end, and overestimates it in mid-cycle. b) The plot of the partial correlation coefficients between the model residuals and lags of themselves. The dashed horizontal lines mark the 95% confidence interval that the correlation is statistically insignificant.

Using the pull distribution $p(a,b) = \frac{\langle a\rangle - \langle b\rangle}{\sqrt{\sigma[a]^2+\sigma[b]^2}}$, we conducted a hypothesis test for the equality of the sample means, and concluded that the spin states aren't statistically different with respect to the slope means (the T-test P-values are: 72% (Up-Down), 73% (Up-Null), 47% (Null-Down)). For this reason, we assume that the spin states are labeled correctly.

TABLE II: Slope summary statistics.

Year	Target	Spin	#[a]	Mean [a.u.]	SE [a.u.]
2012	Off	Null	5	$-9.04 \cdot 10^{-5}$	$6 \cdot 10^{-7}$
	On	Null	4	$-1.20 \cdot 10^{-4}$	$1 \cdot 10^{-6}$
2016	Off	Up	12	$-2.26 \cdot 10^{-4}$	$7 \cdot 10^{-6}$
		Down	12	$-2.16 \cdot 10^{-4}$	$8 \cdot 10^{-6}$
		Null	12	$-2.12 \cdot 10^{-4}$	$9 \cdot 10^{-6}$
	On	Up	12	$-2.51 \cdot 10^{-4}$	$4 \cdot 10^{-6}$
		Down	12	$-2.53 \cdot 10^{-4}$	$5 \cdot 10^{-6}$
		Null	12	$-2.54 \cdot 10^{-4}$	$6 \cdot 10^{-6}$

[a] Sample size.

TABLE III: Cross section summary statistics.

Year	Soundness	Closeness	#	Mean[a][a.u.]	SE [a.u.]
2012	Sound	Close	4	507(507)	7
	Sound	Far	8	553(563)	14
	Unsound	Close	3	562(580)	36
	Unsound	Far	5	515(512)	20
	All		20	536(544)	10
2016	Sound	Close	40	409(411)	48
	Sound	Far	92	396(385)	34
	Unsound	Close	4	1400(1418)	170
	Unsound	Far	8	1453(1457)	69
	All		144	486(473)	35

[a] The value in parentheses is the variance-weighted mean.

V. CROSS SECTION

After the slopes are estimated from the raw measurement data, one can estimate the cross section. In this analysis, we used only the estimates from adjacent cycles, so as to minimize the effect of drifts of environmental variables such as target thickness (which is estimated to increase by 0.5 %/hour). The thickness by which the slope differences are divided, assumed constant, was provided by a Schottky measurement. Those estimates whose computation involved at least one slope which is deemed an outlier according to Tukey's range test are labeled unsound in TABLE III. There are two outlier slopes in the 2012 experiment, and one in the 2016 experiment; all three are from target-off cycles.

The cross section estimate's standard error (SE) is estimated by adding the squared standard errors of the paired slopes, not taking account of the covariance term. This is done so because depending on whether an on-slope is paired with the preceding or succeeding off-slope, the covariance term changes sign. Since there does not seem to be a reasonable criterion favoring either of the two mappings, the covariance term was omitted.

VI. ASYMMETRY

The asymmetry was estimated as in equation (5). In that expression, we used the target thickness as estimated using the Schottky measurements method [4] ($1.1 \cdot 10^{14}$ cm^{-2}, measured once), and our own best estimate for the unpolarized cross section (411 mb, see TABLE III).

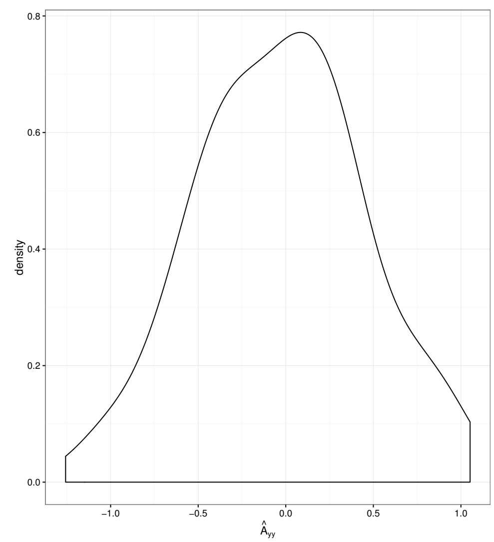

FIG. 3: Gaussian kernel density estimate of the double-polarized cross section asymmetry in proton-deuteron scattering.

VII. RESULTS

Our best estimate for the pp cross section is $\sigma_0^{pp} = 507{\pm}7$ a.u., that for the pd reaction is $\sigma_0^{pd} = 411{\pm}48$. The asymmetry $A_{y,y} = (2 \pm 4) \cdot 10^{-2}$. The Quantile-Quantile plot shows that the estimate is distributed normally (see FIG. 3); however the distribution's reduced chi-square $\chi^2_{red} = 7.8$. The upper limit of $A_{y,y}$ at the 95% confidence level is 10%.

There is a 31% chance that our estimator of the asymmetry is zero in expectation. Considering that, as was stated in section IV, the slope means' dependence on the beam spin state is not statistically significant, we find it likely that this is the case.

[1] Homer E. Conzett, in Proceedings of the 7th International Conference on Polarization Phenomena in Nuclear Physics, Paris, France (1990).

[2] P.D. Eversheim, B. Lorentz, and Yu. Valdau, Forschungszentrum Jülich, COSY, proposal #215, 2012.

[3] Weidemann, C., F. Rathmann, H.J. Stein, B. Lorentz, Z. Bagdasarian, L. Barion, S. Barsov, et al. Phys. Rev. ST Accel. Beams **18**, 2 (2015).

[4] H. J. Stein, M. Hartmann, I. Keshelashvili, et al., Phys. Rev. ST Accel. Beams **11**, 052801 (2008).