Ejercicio 3

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 2 \end{pmatrix} = A$$

$$A' = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 5 \\ 5 & 6 \end{pmatrix} = (A'A)_{2\times 2}$$

$$|A'A| = (5)(6) - (5)(5) = 30 - 25 = 5$$

$$tr(A'A) = 5 + 6 = 11$$

$$(A'A)^{-1} = \frac{1}{30-25} \begin{pmatrix} 6 & -5 \\ -5 & 5 \end{pmatrix} = \begin{pmatrix} 6/5 & -1 \\ -1 & 1 \end{pmatrix} = (A'A)^{-1}$$

$$\begin{pmatrix} 5 & 5 \\ 5 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 6/5 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

$$A_{3\times 2} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix} = A'_{2\times 3}$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 1 & 1 & 2 \\ 4 & 2 & 8 \end{pmatrix} = (AA')_{3\times 3}$$

$$|AA'| = \cancel{16} + 8 + \cancel{8} - \cancel{16} - \cancel{8} - 8 = 0 \Rightarrow \nexists (AA')^{-1}$$

$$tr(AA') = 2 + 1 + 8 = 11$$

Ejercicio 4

a $(A+BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1} \Rightarrow$

$\Rightarrow (A+BCD)[A^{-1} - A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1}] = I \Rightarrow$

$\Rightarrow (A+BCD)A^{-1} - (A+BCD)A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1} =$

$= I + BCDA^{-1} - (\cancel{AA^{-1}}B + BCDA^{-1}B)(C^{-1}+DA^{-1}B)^{-1}DA^{-1} =$

$= I + BCDA^{-1} - (B + BCDA^{-1}B)(C^{-1}+DA^{-1}B)^{-1}DA^{-1} =$

$= I + BCDA^{-1} - BC\cancel{(C^{-1}+DA^{-1}B)}\cancel{(C^{-1}+DA^{-1}B)^{-1}}DA^{-1} =$

$= I + \cancel{BCDA^{-1}} - \cancel{BCDA^{-1}} = I$ lqqd

Ejercicio 6

$(*)\ (A-\lambda I) = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$

$\boxed{a}$ $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$(2-\lambda)^2 - 1 = 0 \Rightarrow$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda_1 = 3, \ \lambda_2 = 1$

$\underline{\lambda = 3}$ $(A-\lambda I) = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$\Rightarrow x_1 = x_2 \underset{\boxed{x_1=1}}{\Rightarrow} v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \boxed{v_1' = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

$\underline{\lambda = 1}$ $(A-\lambda I) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$

$\Rightarrow -x_1 = x_2 \underset{\boxed{x_1=1}}{\Rightarrow} v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow \boxed{v_2' = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

$(*)\ A^{-1} = \frac{1}{4-1}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$

$|A^{-1} - \lambda I| = \begin{vmatrix} \frac{2}{3}-\lambda & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3}-\lambda \end{vmatrix} = \left(\frac{2}{3}-\lambda\right)^2 - \frac{1}{9} = 0 \Rightarrow$

$\Rightarrow \lambda^2 - \frac{4}{3}\lambda + \frac{1}{3} = 0 \Rightarrow \lambda_1 = 1 ; \lambda_2 = 1/3$

$\underline{\lambda = 1}$ $(A^{-1} - \lambda I) = \begin{pmatrix} 2/3 - 1 & -1/3 \\ -1/3 & 2/3 - 1 \end{pmatrix} = \begin{pmatrix} -1/3 & -1/3 \\ -1/3 & -1/3 \end{pmatrix} \Rightarrow$

$\Rightarrow \left(\begin{array}{cc|c} -1/3 & -1/3 & 0 \\ -1/3 & -1/3 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} -1/3 & -1/3 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow -x_1 = x_2$

$\underset{\boxed{x_2=1}}{\Rightarrow} v_1 = \begin{pmatrix} -1 \\ +1 \end{pmatrix} \Rightarrow \boxed{v_1' = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ +1 \end{pmatrix}}$

$\underline{\lambda = 1/3}$ $(A^{-1} - \lambda I) = \begin{pmatrix} 2/3 - 1/3 & -1/3 \\ -1/3 & 2/3 - 1/3 \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 \\ -1/3 & 1/3 \end{pmatrix}$

$\Rightarrow \left(\begin{array}{cc|c} 1/3 & -1/3 & 0 \\ -1/3 & 1/3 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1/3 & -1/3 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow x_1 = x_2 \Rightarrow$

$\underset{\boxed{x_1 = 1}}{\Rightarrow} v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \boxed{\frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

$\boxed{b}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$B' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$(B'B - \lambda I) = \begin{pmatrix} 2-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix} \Rightarrow \lambda_1 = 2, \lambda_2 = 1$

$\underline{\lambda_1 = 2}$ $\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow x_2 = 0 \Rightarrow v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\underline{\lambda_2 = 1}$ $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 = 0 \Rightarrow v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow V' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$D = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$B' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = BB'$$

$$(BB' - \lambda I) = \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix} \Rightarrow$$

$$\Rightarrow (1-\lambda)^3 - (1-\lambda) = 0 \Rightarrow (1-\lambda)\left[(1-\lambda)^2 - 1\right] = 0$$

$$\Rightarrow (1-\lambda)\left[\lambda^2 - 2\lambda\right] = \lambda(1-\lambda)(\lambda-2) = 0 \Rightarrow$$

$$\Rightarrow \lambda_1 = 1, \ \lambda_2 = 2 ; \ \lambda_3 = 0$$

$\underline{\lambda_1 = 1}$ $$(BB' - \lambda I) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\underline{\lambda_2 = 2}$ $$(BB' - \lambda I) = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} x_2 = 0 \\ x_1 = x_3 \end{array} \Rightarrow$$

$\boxed{x_1 = 1}$

$$\Rightarrow \quad v_2' = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad v_2 = \frac{v_2'}{\|v_2'\|} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = v_2}$$

$$U = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix}$$

$$SVD(B): \quad B = UDV' =$$

$$= \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = B$$

$$\underset{2\times 3}{C} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix} \qquad C' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 1 & 1 & 2 \\ 4 & 2 & 8 \end{bmatrix} = C'C$$

$$\left|(C'C - \lambda I)\right| = \left|\begin{bmatrix} 2-\lambda & 1 & 4 \\ 1 & 1-\lambda & 2 \\ 4 & 2 & 8-\lambda \end{bmatrix}\right| =$$

$$= (2-\lambda)(1-\lambda)(8-\lambda) + 8 + 8 - 16(1-\lambda) - 4(2-\lambda) -$$

$$-(8-\lambda) =$$

$$= (2 - 3\lambda + \lambda^2)(8-\lambda) + 16 - 16(1-\lambda) - 4(2-\lambda) - (8-\lambda) =$$

$$= \cancel{16} - 2\lambda - 24\lambda + 3\lambda^2 + 8\lambda^2 - \lambda^3 + \cancel{16} - \cancel{16} + 16\lambda -$$

$$- \cancel{8} + 4\lambda - \cancel{8} + \lambda =$$

$$= -\lambda^3 + 11\lambda^2 - 5\lambda = \lambda(-\lambda^2 + 11\lambda - 5) = 0 \Rightarrow$$

$$\Rightarrow \lambda_1 = \frac{-11 + \sqrt{101}}{-2} = \frac{11 - \sqrt{101}}{2} \simeq 0{,}4751$$

$$\Rightarrow \lambda_2 = \frac{-11 - \sqrt{101}}{-2} = \frac{11 + \sqrt{101}}{2} \simeq 10{,}5249$$

$$\Rightarrow \lambda_3 = 0$$

$$\circledast \quad \lambda_1 = \frac{11+\sqrt{101}}{2}$$

$$\begin{bmatrix} 2-\frac{11-\sqrt{101}}{2} & 1 & 4 \\ 1 & 1-\frac{11-\sqrt{101}}{2} & 2 \\ 4 & 2 & 8-\frac{11-\sqrt{101}}{2} \end{bmatrix} =$$

$$\left[\begin{array}{ccc|c} \frac{-7+\sqrt{101}}{2} & 1 & 4 & 0 \\ 1 & \frac{-9+\sqrt{101}}{2} & 2 & 0 \\ 4 & 2 & \frac{5+\sqrt{101}}{2} & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} \frac{-7+\sqrt{101}}{2} & 1 & 4 & 0 \\ 0 & \frac{-62+6\sqrt{101}}{13} & \frac{12-2\sqrt{101}}{13} & 0 \\ 0 & \frac{12-2\sqrt{101}}{13} & \frac{-47-3\sqrt{101}}{26} & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -\frac{7+\sqrt{101}}{2} & 1 & 4 & 0 \\ 0 & \frac{-62+6\sqrt{101}}{13} & \frac{12-2\sqrt{101}}{13} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] =$$

$$\left(\frac{-62+6\sqrt{101}}{13}\right)x_2 + \left(\frac{12-2\sqrt{101}}{13}\right)x_3 = 0 \Rightarrow$$

$$\Rightarrow (-62+6\sqrt{101})x_2 = -(12-2\sqrt{101})x_3 \Rightarrow$$

$$\Rightarrow x_2 = \left(\frac{-(12-2\sqrt{101})}{-62+6\sqrt{101}}\right)x_3 \simeq -4{,}762\, x_3$$

$$\left(\frac{-7+\sqrt{101}}{2}\right)x_1 + x_2 + 4x_3 = 0 \Rightarrow$$

$$\Rightarrow \left(\frac{-7+\sqrt{101}}{2}\right)x_1 - 4{,}762\, x_3 + 4x_3 = 0 \Rightarrow$$

$$\Rightarrow \left(\frac{-7+\sqrt{101}}{2}\right)x_1 = 0{,}762\, x_3 \Rightarrow$$

$$\Rightarrow x_1 = \left(\frac{2(0{,}762)}{-7+\sqrt{101}}\right)x_3 \Rightarrow x_1 = \frac{1}{2}x_3$$

$$\therefore\ v_1 = \begin{pmatrix} x_3/2 \\ -4{,}762\, x_3 \\ x_3 \end{pmatrix} \underset{\boxed{x_3=1}}{\Rightarrow} v_1 = \begin{pmatrix} 1/2 \\ -4{,}762 \\ 1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \frac{v_1}{\|v_1\|} \cong \begin{pmatrix} 0,1022 \\ -0,9735 \\ 0,2044 \end{pmatrix}$$

$$\circledast\ \lambda_2 = \frac{11+\sqrt{101}}{2}$$

$$\begin{bmatrix} 2-\frac{11+\sqrt{101}}{2} & 1 & 4 \\ 1 & 1-\frac{11+\sqrt{101}}{2} & 2 \\ 4 & 2 & 8-\frac{11+\sqrt{101}}{2} \end{bmatrix} \to$$

$$\left[\begin{array}{ccc|c} \frac{-7-\sqrt{101}}{2} & 1 & 4 & 0 \\ 1 & \frac{-9-\sqrt{101}}{2} & 2 & 0 \\ 4 & 2 & \frac{5-\sqrt{101}}{2} & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} \frac{-7-\sqrt{101}}{2} & 1 & 4 & 0 \\ 0 & \frac{-62-6\sqrt{101}}{13} & \frac{12+2\sqrt{101}}{13} & 0 \\ 0 & \frac{12+2\sqrt{101}}{13} & \frac{-47+3\sqrt{101}}{26} & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} \frac{-7-\sqrt{101}}{2} & 1 & 4 & 0 \\ 0 & \frac{-62-6\sqrt{101}}{13} & \frac{12+2\sqrt{101}}{13} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left(\frac{-62-6\sqrt{101}}{13}\right) x_2 + \left(\frac{12+2\sqrt{101}}{13}\right) x_3 = 0 \Rightarrow$$

$$\Rightarrow \left(-62-6\sqrt{101}\right) x_2 = -\left(12+2\sqrt{101}\right) x_3 \Rightarrow$$

$$\Rightarrow x_2 = \left[\frac{-(12+2\sqrt{101})}{-62-6\sqrt{101}}\right] x_3 \Rightarrow \boxed{x_2 \cong 0{,}2625\, x_3}$$

$$\left[\frac{-7-\sqrt{101}}{2}\right] x_1 + x_2 + 4x_3 = 0 \Rightarrow$$

$$\Rightarrow \left[\frac{-7-\sqrt{101}}{2}\right] x_1 + 0{,}2625 x_3 + 4x_3 = 0 \Rightarrow$$

$$\Rightarrow \left(\frac{-7-\sqrt{101}}{2}\right) x_1 = -4{,}2625\, x_3 \Rightarrow$$

$$\Rightarrow x_1 = \left[\frac{(-4{,}2625)\,2}{-7-\sqrt{101}}\right] x_3 \Rightarrow \boxed{x_1 = \frac{1}{2} x_3}$$

$$v_2 = \begin{pmatrix} x_3/2 \\ 0{,}2625x_3 \\ x_3 \end{pmatrix} \underset{\boxed{x_3=1}}{\Rightarrow} v_2 = \begin{pmatrix} 1/2 \\ 0{,}2625 \\ 1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \frac{v_2}{\|v_2\|} = \begin{pmatrix} 0{,}4354 \\ 0{,}2285 \\ 0{,}8708 \end{pmatrix}$$

$$V = \begin{pmatrix} 0{,}4354 & 0{,}1022 \\ 0{,}2285 & -0{,}9735 \\ 0{,}8708 & 0{,}2044 \end{pmatrix} \Rightarrow$$

$$\Rightarrow V' = \begin{pmatrix} 0{,}4354 & 0{,}2285 & 0{,}8708 \\ 0{,}1022 & -0{,}9735 & 0{,}2044 \end{pmatrix}$$

$$D = \begin{bmatrix} \sqrt{\frac{11+\sqrt{101}}{2}} & 0 \\ 0 & \sqrt{\frac{11-\sqrt{101}}{2}} \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 5 \\ 5 & 6 \end{bmatrix} = CC'$$

$$|CC' - \lambda I| = \begin{vmatrix} 5-\lambda & 5 \\ 5 & 6-\lambda \end{vmatrix} \Rightarrow$$

$$\Rightarrow (5-\lambda)(6-\lambda) - 25 =$$

$$= 30 - 11\lambda + \lambda^2 - 25 = 0 \Rightarrow$$

$$\Rightarrow \lambda^2 - 11\lambda + 5 = 0 \Rightarrow \lambda_1 = \frac{11+\sqrt{101}}{2} \simeq 10{,}5$$

$$\Rightarrow \lambda_2 = \frac{11-\sqrt{101}}{2} \simeq 0{,}5$$

$$\lambda_1 = \frac{11+\sqrt{101}}{2}$$

$$(CC' - \lambda_1 I) = \begin{pmatrix} 5 - \frac{11+\sqrt{101}}{2} & 5 \\ 5 & 6 - \frac{11+\sqrt{101}}{2} \end{pmatrix} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cc|c} \frac{-1-\sqrt{101}}{2} & 5 & 0 \\ 5 & \frac{1-\sqrt{101}}{2} & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} \frac{-1-\sqrt{101}}{2} & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \left(\frac{-1-\sqrt{101}}{2}\right) x_1 + 5x_2 = 0 \Rightarrow x_1 = \left(\frac{-10}{-1-\sqrt{101}}\right) x_2 \Rightarrow$$

$$\Rightarrow x_1 = \left(\frac{-1+\sqrt{101}}{10}\right) x_2 \Rightarrow x_1 \simeq 0{,}9050\, x_2 \Rightarrow$$

$$\Rightarrow u_1 = \begin{pmatrix} 0{,}9050 \\ 1 \end{pmatrix} \Rightarrow \frac{u_1}{\|u_1\|} = \begin{pmatrix} 0{,}6710 \\ 0{,}7415 \end{pmatrix}$$

$\boxed{x_2 = 1}$

$$\circledast\ \lambda_2 = \frac{11-\sqrt{101}}{2}$$

$$(CC^t - \lambda I) = \begin{pmatrix} 5 - \frac{11-\sqrt{101}}{2} & 5 \\ 5 & 6 - \frac{11-\sqrt{101}}{2} \end{pmatrix} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cc|c} \frac{-1+\sqrt{101}}{2} & 5 & 0 \\ 5 & \frac{1+\sqrt{101}}{2} & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cc|c} \frac{-1+\sqrt{101}}{2} & 5 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left(\frac{-1+\sqrt{101}}{2}\right) x_1 + 5x_2 = 0 \Rightarrow x_1 = \left(\frac{-10}{-1+\sqrt{101}}\right) x_2 \Rightarrow$$

$$\Rightarrow x_1 = \left(\frac{-1-\sqrt{101}}{10}\right) x_2 \simeq -1{,}1050\, x_2 \Rightarrow$$

$$\Rightarrow \quad \mu_2 = \begin{pmatrix} -1{,}1050 \\ 1 \end{pmatrix} \Rightarrow \frac{\mu_2}{\|\mu_2\|} = \begin{pmatrix} -0{,}7415 \\ 0{,}6710 \end{pmatrix}$$

$\boxed{x_2 = 1}$

$$U = \begin{pmatrix} 0{,}6710 & -0{,}7415 \\ 0{,}7415 & 0{,}6710 \end{pmatrix}$$

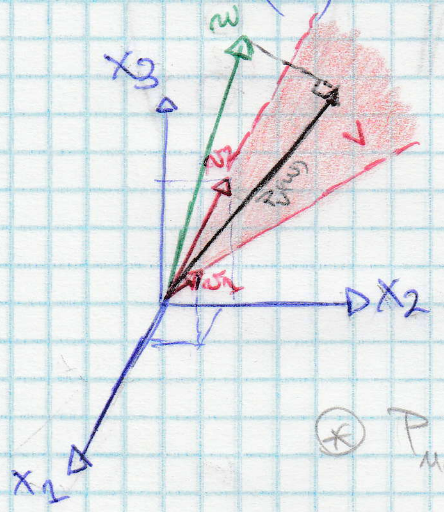
Ejercicio 8

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix} \Rightarrow A^{100} = B\,D^{100}\,B^{-1}$$

$$A^{100} = B \begin{bmatrix} 8^{100} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B^{-1}$$

Ejercicio 9

a) $w = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ $\quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ $\quad V = \{v_1 ; v_2\}$



⊛ Quiero hallar $P_V(w)$ ⇒

⇒ aplico Gram-Schmidt

⊛ $u_1 = v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

⊛ $P_{u_1}(v_2) = \dfrac{\langle v_2 ; u_1 \rangle}{\|u_1\|^2}\, u_1 = \dfrac{v_2' u_1}{u_1' u_1} \cdot u_1 = \dfrac{3}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = v_1$

⊛ $u_2 = v_2 - P_{u_1}(v_2) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

⊛ Por lo que $U = \{u_1 ; u_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$ tal que $\mathcal{L}(U) = \mathcal{L}(V)$

Verificación: $u_1 \perp u_2 \qquad u_1' u_2 = (1\ 1\ 1)\begin{pmatrix}-1\\0\\1\end{pmatrix} = 0$

(*) $P_U(w) = \dfrac{w'u_1}{\|u_1\|^2} \cdot u_1 + \dfrac{w'u_2}{\|u_2\|^2} \cdot u_2 = \dfrac{5}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} + \dfrac{2}{2}\begin{pmatrix}-1\\0\\1\end{pmatrix} =$

$\begin{pmatrix}1\\1\\1\end{pmatrix} \begin{pmatrix}-1\\0\\1\end{pmatrix} \qquad = \begin{pmatrix}5/3\\5/3\\5/3\end{pmatrix} + \begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}2/3\\5/3\\8/3\end{pmatrix} = \dfrac{1}{3}\begin{pmatrix}2\\5\\8\end{pmatrix} = h$

$(1\ \ 1\ \ 3)\ (5) \quad (2)$

$(1\ \ 1\ \ 1)\ (3)$

$(-1\ \ 0\ \ 1) \quad (2)$

b

Quiero z tal que $\begin{cases} z'v_1 = 0 \\ z'v_2 = 0 \\ z'h = 0 \end{cases}$

① Hallo z

$\begin{cases} \alpha + \beta + \gamma = 0 \\ \beta + 2\gamma = 0 \\ \frac{2}{3}\alpha + \frac{5}{3}\beta + \frac{8}{3}\gamma = 0 \end{cases}$

$\beta = -2\gamma$

$\alpha = \gamma$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2/3 & 5/3 & 8/3 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \gamma \begin{pmatrix}1\\-2\\1\end{pmatrix} \Rightarrow \boxed{z = \begin{pmatrix}1\\-2\\1\end{pmatrix}}$

② Hallo la CL

$\begin{pmatrix}1\\1\\3\end{pmatrix} = \alpha\begin{pmatrix}1\\1\\1\end{pmatrix} + \beta\begin{pmatrix}0\\1\\2\end{pmatrix} + \gamma\begin{pmatrix}1\\-2\\1\end{pmatrix}$

$\begin{cases} \alpha + \gamma = 1 \\ \alpha + \beta - 2\gamma = 1 \\ \alpha + 2\beta + \gamma = 3 \end{cases}$

$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & -2 & 1 \\ 1 & 2 & 1 & 3 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) \Rightarrow \boxed{\beta = 1} \Rightarrow \boxed{\alpha = 2/3} \Rightarrow \boxed{\gamma = 1/3}$

Verificación

$$2/3\begin{pmatrix}1\\1\\1\end{pmatrix}+\begin{pmatrix}0\\1\\2\end{pmatrix}+1/3\begin{pmatrix}1\\-2\\1\end{pmatrix}=\begin{pmatrix}2/3\\2/3\\2/3\end{pmatrix}+\begin{pmatrix}0\\1\\2\end{pmatrix}+\begin{pmatrix}1/3\\-2/3\\1/3\end{pmatrix}=\begin{pmatrix}1\\1\\3\end{pmatrix}=w$$

$\boxed{c}$ $\mathbb{Y}=\begin{pmatrix}1\\1\\3\end{pmatrix}$ $\mathbb{X}=\begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix}$

$$\mathbb{Y}=\mathbb{X}\beta+\varepsilon \Rightarrow \beta=(\mathbb{X}'\mathbb{X})^{-1}\mathbb{X}'\mathbb{Y}$$

$$\begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix}=\mathbb{X}$$

$$\mathbb{X}'=\begin{pmatrix}1&1&1\\0&1&2\end{pmatrix}\quad\begin{pmatrix}3&3\\3&5\end{pmatrix}=\mathbb{X}\mathbb{X}$$

$$(\mathbb{X}'\mathbb{X})^{-1}=\frac{1}{15-9}\begin{pmatrix}5&-3\\-3&3\end{pmatrix}=\frac{1}{6}\begin{pmatrix}5&-3\\-3&3\end{pmatrix}$$

$$\mathbb{Y}=\begin{pmatrix}1\\1\\3\end{pmatrix}$$

$$\mathbb{X}'=\begin{pmatrix}1&1&1\\0&1&2\end{pmatrix}\quad\begin{pmatrix}5\\7\end{pmatrix}=\mathbb{X}'\mathbb{Y}$$

$$\begin{pmatrix}5\\7\end{pmatrix}=\mathbb{X}'\mathbb{Y}$$

$$(\mathbb{X}'\mathbb{X})^{-1}(\mathbb{X}'\mathbb{Y})=\frac{1}{6}\begin{pmatrix}5&-3\\-3&3\end{pmatrix}\begin{pmatrix}4\\6\end{pmatrix}=\begin{pmatrix}2/3\\1\end{pmatrix}=\begin{pmatrix}\beta_0\\\beta_1\end{pmatrix}=\beta$$

Por lo tanto: $w=\beta_0+\beta_1 v_1$ <u>lqqd</u>