# PRÁCTICO 3

## Ejercicio 1

$$S = \begin{pmatrix} 1+d & 1 & 1 \\ 1 & 1+d & 1 \\ 1 & 1 & 1+d \end{pmatrix}$$

$$(S-\lambda I)a_1 = 0 \Rightarrow \det(S-\lambda I) = 0$$

$$\begin{vmatrix} 1+d-\lambda & 1 & 1 \\ 1 & 1+d-\lambda & 1 \\ 1 & 1 & 1+d-\lambda \end{vmatrix} \sim \begin{vmatrix} 1+d-\lambda & 1 & 1 \\ 1 & 1+d-\lambda & 1 \\ 0 & \lambda-d & d-\lambda \end{vmatrix} \sim$$

$$\sim \begin{vmatrix} 1+d-\lambda & 1 & 1 \\ 1 & 1+d-\lambda & 1 \\ 0 & \lambda-d & -(\lambda-d) \end{vmatrix} \sim$$

$$\sim (\lambda-d) \begin{vmatrix} 1+d-\lambda & 1 & 1 \\ 1 & 1+d-\lambda & 1 \\ 0 & 1 & -1 \end{vmatrix} \sim$$

$$\sim (\lambda-d)^2 \begin{vmatrix} -1 & 1 & 0 \\ 1 & 1+d-\lambda & 1 \\ 0 & 1 & -1 \end{vmatrix} =$$

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$$= (\lambda-d)^2 \left[ 0 - 1 \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 1 \\ 1 & 1+d-\lambda \end{vmatrix} \right] =$$

$$= (\lambda-d)^2 \left[ 1+2+d-\lambda \right] = (\lambda-d)^2 (3+d-\lambda) = 0$$

⇒ <u>opción 1</u> $\lambda - d = 0 \Rightarrow \lambda = d$ (doble)

⇒ <u>opción 2</u> $3 + d - \lambda = 0 \Rightarrow \lambda = 3 + d$

④ <u>$\lambda = 3 + d$</u>

$$(S - \lambda I) = \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -1{,}5 & 1{,}5 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 0 & -1{,}5 & 1{,}5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$\Rightarrow -1{,}5y + 1{,}5z = 0 \Rightarrow \boxed{y = z} \Rightarrow -2x + 2y = 0 \Rightarrow$

$\Rightarrow \boxed{x = y}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = y \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow$ unitario : $\boxed{a_1 = \frac{\sqrt{3}}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$

<u>Por lo tanto</u> : $\boxed{z_1 = x \begin{pmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix}}$

* $\underline{\lambda = d}$

$$(S - \lambda d) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$$\Rightarrow x = -y - z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y-z \\ y \\ z \end{pmatrix} = y \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \boxed{\begin{matrix} \text{tomo} \\ y = 1 \\ z = -1 \end{matrix}} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \Rightarrow \begin{matrix} \text{norma} \\ \text{uno} \end{matrix} \Rightarrow \boxed{a_2 = \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ -\sqrt{2}/2 \end{pmatrix}}$$

Por lo tanto: $\boxed{z_2 = z_3 = x \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ -\sqrt{2}/2 \end{pmatrix}}$

* Porcentaje de variabilidad de i:

$$\frac{V(z_i)}{\sum_{i=1}^{3} V(z_i)} = \frac{\lambda_i}{\sum_{i=1}^{3} \lambda_i} = \frac{\lambda_i}{2d + 3 + d} = \frac{\lambda_i}{3(1+d)}$$

$$\rightarrow z_1 = \frac{3+d}{3(1+d)} \quad \rightarrow z_2 = z_3 = \frac{d}{3(1+d)}$$

## ⊛ Correlaciones

$$COR(z_i; x_j) = a_{ij} \left( \frac{\sqrt{\lambda i}}{s_j} \right)$$

$\rightarrow COR(z_1; x_1) = \frac{\sqrt{3}}{3} \quad \frac{\sqrt{3+d}}{s_1}$

$\rightarrow COR(z_1; x_2) = \frac{\sqrt{3}}{3} \quad \frac{\sqrt{3+d}}{s_2}$

$\rightarrow COR(z_1; x_3) = \frac{\sqrt{3}}{3} \quad \frac{\sqrt{3+d}}{s_3}$

$\rightarrow COR(z_2; x_1) = COR(z_3; x_1) = 0$

$\rightarrow COR(z_2; x_2) = COR(z_3; x_2) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{d}}{s_2}$

$\rightarrow COR(z_2; x_3) = COR(z_3; x_3) = -\frac{\sqrt{2}}{2} \quad \frac{\sqrt{d}}{s_3}$