Canonical Research Designs II: Difference-in-Differences II: Event Studies, Synthetic Control, and Synthetic DinD

Paul Goldsmith-Pinkham

March 25, 2021

Today’s Topics

- Today, touching on two (related) topics
- First, finishing up conversation related to last class and diff-in-diff, focusing again on discussion of *event studies*
  - Focusing on how event studies generate a counteractual control unit
- Second, discuss synthetic control (and dind) methods
  - Not completely new methods, but big upswing in research

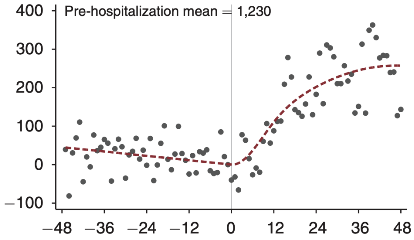
Event study

- Distinction betwen "event study" and "diff-in-diff" is not formal
  - My view: event study are panel (or time series) settings where all units are eventually treated
- What does this imply? No group is a "pure control"
- Without a true control group, can't have both time fe, unit fe and the full non-parametric spec.

$$Y_{it} = \alpha_i + \gamma_t + \sum_{s=L_0, s\neq -1}^{L_1} 1(t - T_i = s)\mu_s$$

- Need to exclude both the baseline period AND at least some periods outside the treatment window

Panel B. Collection balances



- Dobkin et al. (2018)
- Comparison is between those not yet hospitalized and those hospitalized

Event study continued

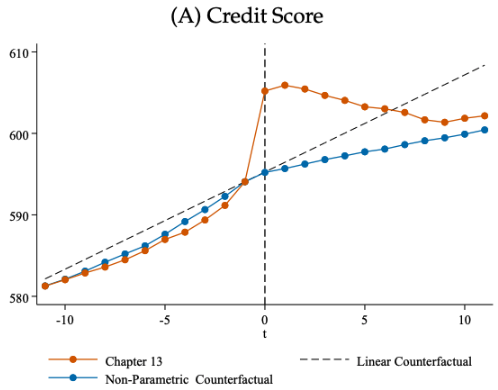
- The necessary assumption is the same (or similar) what we discussed last class
  - Parallel trends – but amongst who?
  - Turns out, all of the groups need to be parallel.
  - That might be an awful assumption (e.g. very far apart from one another)
- Sun and Abraham (2020) point out that heterogeneity in treatment effects can have big impacts as well
  - In fact, it can violate the pre-trend test
  - The use of “excluded” periods potentially contaminates pre-periods
  - Solution: use “late” adopters as control group.
- Additional untestable assumptions are required as we allow for more types of heterogeneity

Aside in event studies

- A key factor in how you construct your counterfactual (and what assumptions you find plausible) are a function of how far into the future you want to estimate outcomes

- An extremely short-run counterfactual could potentially just be a linear extrapolation
  - This assumes that the underlying model is locally linear, rather than globally
  - Construct a counterfactual from just a single time series, but highly non-robust

- Example from a robustness check in my own work (Dobbie et al. 2020)



Constructing a counterfactual is the key goal

- Issue in event study was the attempt to get a "free lunch" – we always need a control group

- Think back to cross-sectional setting with ATT
  - We always knew $Y_i(1)$. Key issue is an estimator for $Y_i(0)$.
  - Event study approaches had issues by ignoring this point and hoping regression would solve problem
  - Notably, this problem disappears if we have full homogeneity + no anticipation and only exclude pre-periods

- Point of emphasis – we need parallel trends to hold to construct a counterfactual in these settings. Why? $Y_{jt}(0) - Y_{j,t-1}(0)$ needs to be a good approximator of $Y_{i,t}(0) - Y_{i,t-1}(0)$.
  - Since we imposed $Y_{it} = \alpha_i + \gamma_t + D_{it}\tau$, the first differencing makes them good approximations

Generalizing the Dind approach

- Pivoting slightly: instead of imposing the parallel trends assumption directly through the linear model, we could construct a combination of units to approximate $Y_{it}(0)$
  - This is what one does in the cross-sectional setting with a pscore method! E.g. consider the ATT:

$$\tau_{ATT} = \underbrace{Y(1)}_{\text{Fully observed}} - \underbrace{\hat{Y}(0)}_{\text{Constructed}}$$

- How would one pick? Recall that with p-score methods or regression, weights effectively reweight based on comparability to treated group
  - With panel data, can use pre-treatment data to construct these weights
  - This method is known as synthetic control (and its various descendents)

Synthetic Control example - Abadie

- Consider following problem: California bans smoking in 1989. What does that do to smoking?
    - Define estimand: $\tau_{ban,CA} = Y_{california,post}(1) - Y_{california,post}(0)$
    - This is the effect of the *California* smoking ban
    - How can we get at it?

- We need a "synthetic California" as our control

- In an ideal world, the average of the other states would work – however, not clear empirically that they are a good counterfactual

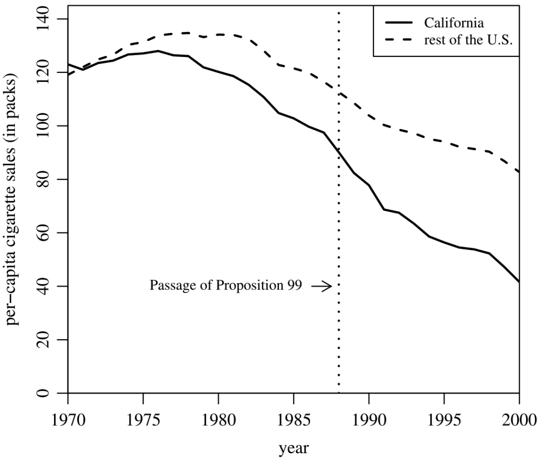


Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

Generalized setup (Doudchenko and Imbens (2018))

- Consider the following general problem

- We have a panel with T time periods and $N+1$ units. Intervention D_{it} at time T_0 for one unit (unit $i=0$)

- Potential outcomes $Y_{it}(D_{it})$, and we only observe one of the potential outcomes (as per usual)
  - Fundamental problem of causal inference
  - We can also have fixed characteristics X_{it}

- Let $\mathbf{Y}_{a,b}$ denote the vector (or matrix in control case) for $a \in \{\text{treatment, control}\}$ and $b \in \{\text{pre, post}\}$ for the treated and control groups in the pre or post period.

- Then, we have observations (analogous setup for the covariates):

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

Generalized panel setup

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

- To estimate $\tau_i = Y_{t,post}(1) - Y_{t,post}(0)$, we need an estimate for $Y_{t,post}(0)$
- What if we just had the cross-section?
    - Note that if D_{it} were randomly assigned, we can derive an estimate using our p-score or regression methods
    - Even without random assignment, one could use covariates to match
    - Our main concern with p-score matching is bias
- Diff-in-diff exploited the panel structure by asserting a particular functional form

$$Y_{it} = \alpha_i + \gamma_t + D_{it}\tau + \epsilon_{it}$$

- Is there something particularly special about this linear additive factor structure?

Generalized panel setup

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

- Recall that our problem boils down to the estimate of an untreated "synthetic" unit

- Following Doudchenko and Imbens (2018), note estimators of the following form:

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

  - A constant μ allows for very different averages (common in diff-in-diff)
  - Weights are allowed to vary across i – a simple average would be diff-in-diff

- We can now consider deviations from diff-in-diff

The synthetic control method (Abadie et al. (2010)

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

- In ADH, they impose
  1. $\mu = 0$
  2. $\sum_i \omega_i = 1$
  3. $\omega_i \geq 0 \ \forall i$

- These three restrictions create a counterfactual California whose outcomes are within the support of the other states, and is a weighted sum of a subset of states

The synthetic control method (Abadie et al. (2010)

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

- Formally, the ω_i need to be estimated, and are constructed by minimizing the distance between covariates in the pre-period:

$$||\boldsymbol{X}_{\text{treat}} - \boldsymbol{X}_{\text{control}} \boldsymbol{W}||$$

- The crucial piece tying this together: $\boldsymbol{X}$ can include both lagged outcomes, and covariates.

- Note we can now re-envision our panel data:
  - Observed outcomes: $\mathbf{Y}_{t,post}(1)$, $\mathbf{Y}_{c,post}(0)$
  - Observed covariates / predictors: $\mathbf{Y}_{t,pre}(0)$, $\mathbf{Y}_{c,pre}(0)$, $\mathbf{X}_t$, $\mathbf{X}_c$

- In many ways, this is just a matching problem using many characteristics!

The synthetic control method (Abadie et al. (2010)

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

- Formally, the ω_i need to be estimated, and are constructed by minimizing the distance between covariates in the pre-period:

$$\{\hat{\omega}\}_i = \arg\min_{\boldsymbol{W}} ||\boldsymbol{X}_{\text{treat}} - \boldsymbol{X}_{\text{control}}\boldsymbol{W}||$$

- The crucial piece tying this together: $\boldsymbol{X}$ can include both lagged outcomes, and covariates.

- Note we can now re-envision our panel data:
  - Observed outcomes: $\mathbf{Y}_{t,post}(1)$, $\mathbf{Y}_{c,post}(0)$
  - Observed covariates / predictors: $\mathbf{Y}_{t,pre}(0)$, $\mathbf{Y}_{c,pre}(0)$, $\mathbf{X}_t$, $\mathbf{X}_c$

- In many ways, this is just a matching problem using many characteristics!

The synthetic control method (Abadie et al. (2010)

- This approach can be incredibly successful

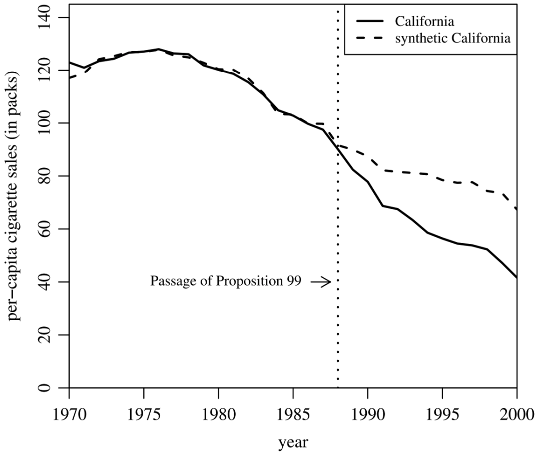


Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

The synthetic control method (Abadie et al. (2010)

- This approach can be incredibly successful
- By careful construction of a synthetic control, can calculate counterfactual impacts due to policy
- Still subject to same caveats from DinD – not invariant to some transformations (e.g. log and linear)

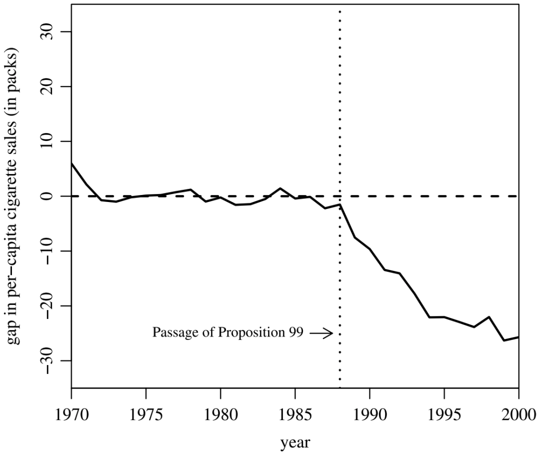


Figure 3. Per-capita cigarette sales gap between California and synthetic California.

Inference in the synthetic control method (Abadie et al. (2010))

- Inference for this method is slightly more complex, as there is only a single treated unit
  - Large sample asymptotics unlikely to work

- Placebo approach is standard: apply method to each potential control unit, and report effect in period

- Analogy here is to a randomization inference argument, comparing to a "null" effect

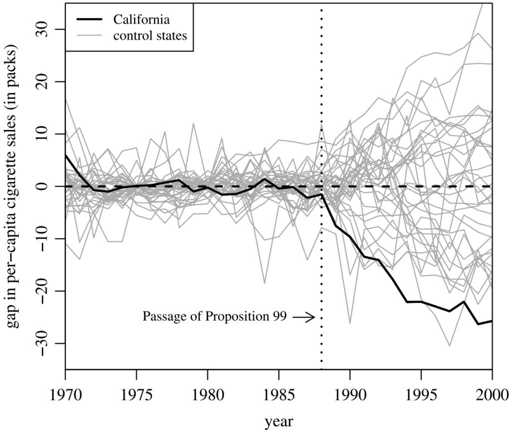


Figure 5. Per-capita cigarette sales gaps in California and placebo gaps in 34 control states (discards states with pre-Proposition 99 MSPE twenty times higher than California's).

Synthetic Diff-in-diff

- In Arkhangelsky et al. (2019), they show you can rewrite the synthetic control estimator as
$$(\hat{\mu}, \hat{\gamma}, \hat{\tau}) = \arg\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \gamma_t - D_{it}\tau)^2 \hat{\omega}_i,$$
subject to the $\hat{\omega}_i$ chosen via the SC approach

- Contrast that with DID:
$$(\hat{\mu}, \hat{\alpha}, \hat{\gamma}, \hat{\tau}) = \arg\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \alpha_i - \gamma_t - D_{it}\tau)^2$$

- They then propose a more robust approach, called Synthetic diff-in-diff, which estimates
$$(\hat{\mu}, \hat{\alpha}, \hat{\gamma}, \hat{\tau}) = \arg\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \alpha_i - \gamma_t - D_{it}\tau)^2 \hat{\omega}_i \hat{\lambda}_t$$

- This approach relaxes the parallel trends assumption by requiring parellel trends in an underlying approximate factor structure

Double Robustness

- In our approaches with DinD, I hightlighted that we were really leaning on the parametric assumption – namely that we could estimate the outcome given the α_i and γ_t.
  - This may not feel super robust
- In our analysis with pscore methods, we estimated models of the counterfactual that just used averages of the controls to get an estimate for the counterfactual. E.g. weighted sums
- This could be wrong if the treatment is not random (e.g. biased!). That is, in part, what the model is trying to account for.
- Key result from Arkhangelsky et al. (2019) is that these methods are robust if either assumption is right (weights are right, or model is right) – e.g. double robust

So what about synthetic methods?

- Many authors (mainly econometricians) argue that this is a burgeoning innovative field
  - It is a very cool method!
- But, I am not sure I see it breaking a huge amount of ground
- Researchers appear hesitant to use it. Why?
- My thoughts:
  - These are strong structural assumptions, and not clear we have good tests yet
  - Despite concerns re: pre-trends in dind, the assumptions felt testable
- Researcher degrees of freedom seem multifold. True in DinD too, but perhaps more transparent?
  - More worrisome: dind is equally problematic, but we aren’t aware of it

Highly recommend exploring

- Read Cunningham (2021) chapter
- Explore synth and synthdid packages
- Abadie (2020) JEL chapter