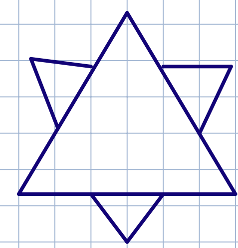
№1

$$\lim_{n\to\infty} \frac{n+1}{n} \overset{?}{=} 1$$

$$\left|\frac{n+1}{n} - 1\right| = \left|\frac{n+1-n}{n}\right| = \left|\frac{1}{n}\right| = \frac{1}{n} \quad // n>0 //$$

$$\frac{1}{n} < \varepsilon \Rightarrow \frac{1}{\varepsilon} < n \Rightarrow N = \left[\frac{1}{\varepsilon}\right] + 1$$



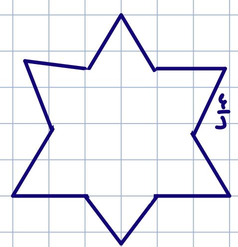
№2

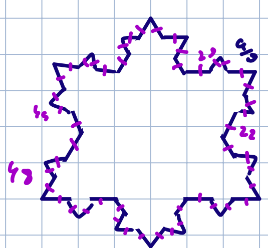
Снежинка Коха

N	1	2	3
K кол-во	3	12	48
l	a	$\frac{a}{3}$	$\frac{a}{9}$

каждая сторона → 4 новых
$a_n \to \frac{a_n}{3} = a_{n+1}$

a





$$K_1 = 3\cdot 4^{1-1} \qquad l_1 = a\cdot\left(\frac{1}{3}\right)^{1-1}$$

$$K_2 = 3\cdot 4^{2-1} \qquad l_2 = a\cdot\left(\frac{1}{3}\right)^{2-1}$$

$$K_3 = 3\cdot 4^{3-1} \qquad l_3 = a\cdot\left(\frac{1}{3}\right)^{3-1}$$

$$K_i = 3\cdot 4^{i-1} \qquad l_i = a\cdot\left(\frac{1}{3}\right)^{i-1}$$

$$\Rightarrow \quad P_n = 3\cdot 4^{n-1}\cdot a\cdot\left(\frac{1}{3}\right)^{n-1}$$

$$\underline{P_n = 3a\cdot\left(\frac{4}{3}\right)^{n-1}} \qquad P_n = \frac{9}{4}a\left(\frac{4}{3}\right)^n$$

$$// \lim_{n\to\infty} P_n = +\infty$$

Д-во:

$$/// \lim_{n\to\infty} P_n = +\infty \iff \forall\varepsilon>0\ \exists N\in\mathbb{N}\ \forall n\in\mathbb{N}: \left(n>N \Rightarrow P_n > \frac{1}{\varepsilon}\right)$$

Зафикс. ε. найдём N, для которого верно: $\forall n\in\mathbb{N}: \left(n>N \Rightarrow P_n > \frac{1}{\varepsilon}\right)$

$$\frac{9}{4}a\cdot\left(\frac{4}{3}\right)^n > \frac{1}{\varepsilon} \iff \left(\frac{4}{3}\right)^n > \frac{4}{9a\varepsilon} \qquad // \varepsilon>0,\ a>0 //$$

$$\left(\frac{4}{3}\right)^n > \left(\sqrt[n]{\frac{4}{9a\varepsilon}}\right)^n$$

$$\frac{4}{3} > \sqrt[n]{\frac{4}{9a\varepsilon}}$$

$$\xi = const$$

$$P_n = \underbrace{\frac{9}{4}a}_{\xi}\left(\frac{4}{3}\right)^n$$

найдём dim K

$$// \dim K = -\lim_{\varepsilon \to 0} \frac{\log(N(\varepsilon))}{\log(\varepsilon)} //$$

$N(\varepsilon) = K_n = \frac{3}{4} \cdot 4^n$ // по маршу на каждую сторону //

$\varepsilon = l_n = 3a \cdot \left(\frac{1}{3}\right)^n$

$$\dim K = -\lim_{\varepsilon \to 0} \frac{\log\left(\frac{3}{4}\cdot 4^n\right)}{\log\left(3a\cdot\left(\frac{1}{3}\right)^n\right)} = -\lim_{\varepsilon \to 0} \frac{n\log(4) + \log\left(\frac{3}{4}\right)}{n\log\left(\frac{1}{3}\right) + \log(3a)}$$

// $\varepsilon \to 0 \Rightarrow n \to \infty$ //

$$\dim K = \lim_{n\to\infty} \frac{n\log(4) + \log\left(\frac{3}{4}\right)}{n\log(3) - \log(3a)} = \lim_{n\to\infty} \frac{\log 4 + \frac{\log\frac{3}{4}}{n}}{\log 3 - \frac{\log 3a}{n}} = \frac{\log 4}{\log 3} = \log_3 4$$

№3

$$\lim_{n\to\infty} x^n \overset{?}{=} +\infty$$

$$x^n = (1+p)^n \geq 1 + np$$

$$1 + np > \frac{1}{\varepsilon} \iff n > \frac{1-\varepsilon}{p} \iff \forall \varepsilon > 0 \ \forall n \in \mathbb{N} \ \exists N \in \mathbb{N} : \left(n > N \Rightarrow a_n > \frac{1}{\varepsilon}\right)$$

№8

$$\lim_{n\to\infty} (b_{n+1} - b_n) = 0$$

$$\forall \varepsilon > 0 \ \exists N \in \mathbb{N} \ \forall n \in \mathbb{N} : \left(\left|(b_{n+1} - b_n) - 0\right| < \varepsilon\right)$$

$|b_{n+1} - b_n| < \varepsilon$ "$|b_n - b| < \varepsilon$"

$b_n - \varepsilon < b_{n+1} < b_n + \varepsilon$

№ 9 $\{b_n\} \longrightarrow L \Rightarrow \{\sqrt{b_n}\} \to \sqrt{L}$ "$\forall n: b_n > 0$"

Д-во:

т.к. $\{b_n\}$ - имеет предел $\Rightarrow$ ограниченная

$\Rightarrow \exists C > 0 \; \forall n \in \mathbb{N} : \underset{\text{"} b_n > 0 \text{"}}{|b_n| > C} \Rightarrow \frac{1}{\sqrt{b_n} + \sqrt{b}} < \frac{1}{\sqrt{c} + \sqrt{b}}$

$\exists N \in \mathbb{N} \; \forall n \in \mathbb{N} : |b_n - b| < \varepsilon \cdot (\sqrt{c} + \sqrt{b})$

$|\sqrt{b_n} - \sqrt{b}| = \frac{|b_n - b|}{\sqrt{b_n} + \sqrt{b}} < \frac{\sqrt{c} + \sqrt{b}}{\sqrt{c} + \sqrt{b}} \varepsilon = \varepsilon \quad \Leftrightarrow \quad |\sqrt{b_n} - \sqrt{b}| < \varepsilon$

№ 10

$\sqrt{2}, \; \sqrt{2\sqrt{2}}, \; \sqrt{2\sqrt{2\sqrt{2}}}$

$2^{\frac{1}{2}}, \; 2^{\frac{3}{4}}, \; 2^{\frac{7}{8}}$ $\qquad a_n = 2^{\frac{2^n - 1}{2^n}}$

$\lim\limits_{n \to \infty} 2^{1 - \frac{1}{2^n}} = 2$

Д-во:

$\left| 2^{1 - \frac{1}{2^n}} - 2 \right| < \varepsilon$

$2 \left| 2^{-\frac{1}{2^n}} - 1 \right| < \varepsilon$