№2

$$f(y) = \frac{1}{y-2} \quad \text{при} \quad y \to 2 \quad - \quad \text{беск. большая}$$

$$\lim_{y\to 2} f(y) = \infty \iff \forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 : |y - y_0| < \delta \Rightarrow |f(y)| > \frac{1}{\varepsilon}$$

$$|y-2| < \delta \iff -\delta < y-2 < \delta$$

$$\left|\frac{1}{y-2}\right| < \frac{1}{\varepsilon} \iff -\frac{1}{\varepsilon} > \frac{1}{y-2} > \frac{1}{\varepsilon}$$

$$-\varepsilon < y-2 < \varepsilon$$

Пусть $\delta = \varepsilon$

№3

$r \to 0 \quad \lim\limits_{r\to 0} \frac{S}{V} = ?$

$S = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

$$\lim_{r\to 0} \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \lim_{r\to 0} \frac{3}{r} = \infty$$

№4

$\beta > 0 \quad \lim\limits_{x\to 0} \frac{f(x)}{x^\beta} = C \neq 0$

- $f(x) = \frac{3x}{1-x}$

$$\lim_{x\to 0} \frac{3x}{(1-x)x^\beta} = \overbrace{\left(\lim_{x\to 0} \frac{3}{1-x}\right)}^{=3} \cdot \left(\lim_{x\to 0} x^{1-\beta}\right)$$

$$\beta = 1 \Rightarrow \lim_{x\to 0} 1 = 1$$

- $f(x)=\sqrt{x+\sqrt{x}}$

$$\lim_{x\to 0}\frac{\sqrt{x+\sqrt{x}}}{x^{\beta}} = \lim_{x\to 0}\sqrt{\frac{x}{x^{2\beta}}+\sqrt{\frac{x}{x^{4\beta}}}} = \lim_{x\to 0}\sqrt{x^{1-2\beta}+x^{\frac{1}{2}-2\beta}}$$

$$\frac{\sqrt{x+\sqrt{x}}}{x^{\beta}} = \sqrt{\frac{x}{x^{2\beta}}+\sqrt{\frac{x}{x^{4\beta}}}} = \sqrt{x^{1-2\beta}+x^{\frac{1-4\beta}{2}}} = \text{“}\beta=\tfrac{1}{2}\text{”} = \lim_{x\to 0}\sqrt{\sqrt{x}+1} = 1$$

$$= \sqrt{x^{1-2\beta}+x^{\frac{1}{2}-2\beta}}$$

Εαν $\beta=\frac{1}{4}$

Το πηλικο $\frac{f(x)}{x^{\beta}} = \sqrt{x^{1-\frac{1}{2}}+x^{0}} = \sqrt{\sqrt{x}+1}$

- $f(x)=\sqrt[3]{x^{2}}-\sqrt{x^{3}}$

$\frac{1}{2}-\frac{2}{3}=\frac{9-4}{6}=\frac{5}{6}$

$$\frac{f(x)}{x^{\beta}} = \frac{x^{2/3}-x^{1/2}}{x^{\beta}} = x^{\frac{2}{3}-\beta}-x^{\frac{1}{2}-\beta} = \text{“}\beta=\tfrac{2}{3}\text{”} = x^{0}-x^{\frac{5}{6}} = 1-x^{\frac{5}{6}}$$

$$\lim_{x\to 0}\frac{f(x)}{x^{\beta}} = \lim_{x\to 0}\left(1-x^{\frac{5}{6}}\right) = 1$$

- $f(x)=\tan x-\sin x$

$$\frac{f(x)}{x^{\beta}} = \frac{\tan x}{x^{\beta}}-\frac{\sin x}{x^{\beta}} = \frac{\sin x}{x^{\beta}\cos x}-\frac{\sin x}{x^{\beta}}$$

$$\lim_{x\to 0}\frac{f(x)}{x^{\beta}} = \left(\lim_{x\to 0}\frac{1}{\cos x}\right)\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)-\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$$

№ 1ес.

$y = |x|$ непр. в т. $x=0$ по опр.

По опр. непр.

$\forall \varepsilon > 0 \quad \exists \delta(\varepsilon) > 0 : |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$

Тогда, должно быть выполнено ($x_0 = 0$, $f(x_0) = 0$)

$\forall \varepsilon > 0 \quad \exists \delta > 0 : |x| < \delta \Rightarrow |y| < \varepsilon$

$|y| = |x| \quad \Rightarrow \quad |x| < \delta \Rightarrow |x| < \varepsilon$

Пусть $\delta = \varepsilon$

№ 5

$f: \mathbb{R} \to \mathbb{R}$ непрер. и инъективна, имеет непод. точку x_0 и

$f(2x - f(x)) = x \quad \forall x$. Док-ть: $f(x) = x \quad \forall x$

- т.к. инъективна, то $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- т.к. непрерывна, то $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$

$f(2x - f(x)) = x$

$f(0 - f(0)) = 0$

$f(-f(0)) = 0$

$f(2 - f(1)) = 1$

Пусть $y = 2x - f(x)$

$f(y) = x$

$f(2x_0 - f(x_0)) = f(x_0) = x_0$

$y(x_0) = x_0$

$\Rightarrow f(2f(x) - f(f(x))) = f(x)$

$$\left.\begin{aligned} 2f(x) - f(f(x)) &= x \\ f(2x - f(x)) &= x \end{aligned}\right\} \Rightarrow$$

$\Rightarrow \quad x = f(2f(x) - 2f(f(x)) - f(x)) = x$

$x = f(3f(x) - 2f(f(x)))$

$x = f(2x - f(x))$

$f(2x$