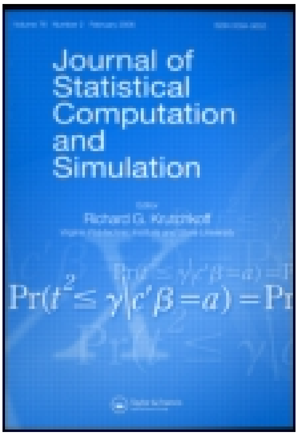

Journal of Statistical Computation and Simulation
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gscs20

An empirical power comparison of univariate goodness-of-fit tests for normality

Xavier Romão [a] , Raimundo Delgado [a] & Aníbal Costa [b]
[a] Civil Engineering Department, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal
[b] Autonomous Civil Engineering Section, Universidade de Aveiro, Campus Universitário de Santiago, 3810-193, Aveiro, Portugal
Version of record first published: 17 Mar 2009.

Journal of Statistical Computation and Simulation
Vol. 80, No. 5, May 2010, 545–591

An empirical power comparison of univariate goodness-of-fit tests for normality

Xavier Romão[a]*, Raimundo Delgado[a] and Aníbal Costa[b]

[a] *Civil Engineering Department, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal;* [b] *Autonomous Civil Engineering Section, Universidade de Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal*

(*Received 15 July 2008; final version received 12 January 2009*)

A comprehensive power comparison study of existing tests for normality is proposed. Given the importance of this subject and the widespread development of normality tests, comprehensive descriptions and power comparisons of such tests are of considerable interest. Since recent comparison studies do not include several interesting and more recently developed tests, a further comparison of normality tests is considered to be of foremost interest. The study addresses the performance of 33 normality tests, for various sample sizes, considering several significance levels and for a number of symmetric, asymmetric and modified normal distributions. General recommendations for normality testing resulting from the study are defined according to the nature of the non-normality.

Keywords: normality test; power comparison; normal distribution; goodness-of-fit test; univariate

1. Introduction

There is a multitude of statistical models and procedures that rely on the validity of a given data hypothesis, being the normality of the data assumption one of the most commonly found in statistical studies. As observed in many econometric models and in research on applied economics, following the normal distribution assumption blindly may affect the accuracy of inference and estimation procedures, in both cross-sectional and time series data sets [1]. The evaluation of this distributional assumption has been addressed, for example, in [2] where the conditional normality assumption in the sample selection model applied to housing demand is examined, or in [3,4] where the normality assumption has been addressed in the context of stock market data, a type of data that has been found to be typically heavy-tailed [5,6]. The analysis of the normality hypothesis can also be found in the characterization of error terms in the context of regression analysis models applied to economic time-series [7–9], to probit models [10] or to other types of time series [11,12]. In medical research the assumption of normality is also very common [13,14], but the suitability of this assumption must also be verified with adequate statistical tests as, for example, in the case

*Corresponding author. Email: xnr@fe.up.pt

ISSN 0094-9655 print/ISSN 1563-5163 online

DOI: 10.1080/00949650902740824
http://www.informaworld.com

of the variability of gene expression data [15] or in the case of assessing the effectiveness of new treatments using clinical trials [16]. Similarly, the normality hypothesis considered in the field of quality control [17–19] also needs verification, namely when using techniques based on Shewhart control charts that are based on the normality assumption [20]. In addition, it can be seen that the lognormality assumption, which is frequent in many science fields [21], can also be examined by testing the normality hypothesis after the logarithmic transformation of the data, e.g. [22–24].

The definition of adequate normality tests can, therefore, be seen to be of much importance since the acceptance or rejection of the normality assumption of a given data set plays a central role in numerous research fields. As such, the problem of testing normality has gained considerable importance in both theoretical and empirical research and has led to the development of a large number of goodness-of-fit tests to detect departures from normality. Given the importance of this subject and the widespread development of normality tests over the years, comprehensive descriptions and power comparisons of such tests have also been the focus of attention, thus helping the analyst in the choice of suitable tests for his particular needs. Examples of such comprehensive reviews on the effectiveness of many normality tests towards a wide range of non-normality alternatives may be found, for example, in [25–37] and in the references cited therein. Since the tests that have been developed are based on different characteristics of the normal distribution, it can be seen from these comparison studies that their power to detect departures from normality can be significantly different depending on the nature of the non-normality.

Furthermore, although the referred comparison studies have been appearing over the years, it is worth mentioning that some of the more recent ones, e.g. [35,37], do not include several interesting and more recently developed tests. Moreover, power results presented in [37] appear to contradict those resulting from previous studies. A further comparison of normality tests, such as the one proposed herein, can therefore be considered to be of foremost interest.

An extensive simulation study is presented herein to estimate the power of 33 tests aiming to assess the validity of the univariate normality assumption of a data set. The selected tests include a group of well-established normality tests as well as more recently developed ones. Section 2 presents a general description of the normality tests selected for the study, while Section 3 discuses, for some of the considered tests, the adequacy of the asymptotical critical values when compared with the empirical ones. The effects on the power of the tests due to the sample size, the selected significance level and the type of alternative distribution are also considered in the proposed study. The study is carried out for various sample sizes n and considering several significance levels α. With respect to the considered alternative distributions, the study considers a number of statistical distributions that are categorized into three sets. The first set includes several types of symmetric non-normal distributions, the second set includes several types of asymmetric distributions and the third set comprises a number of modified normal distributions with various shapes. Section 4 presents a more detailed description of the distributions included in these three sets. Section 5 presents the simulation approach considered in the study and the power results of the normality tests for the different alternative distribution sets, which are then discussed in Section 6. Finally, conclusions and recommendations resulting from the study are provided in Section 7.

2. Goodness-of-fit tests for normality

The selected normality tests are considered for testing the composite null hypothesis for the case where both location and scale parameters, μ and σ, respectively, are unknown. Normality test formulations differ according to the different characteristics of the normal distribution they focus. The goodness-of-fit tests considered in the proposed study are grouped into four general categories and a brief review of each test is presented herein.

In the following review, it is considered that $x_1, x_2, \ldots, x_n$ represent a random sample of size n; $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ represent the order statistics of that sample; $\bar{x}$, s^2, $\sqrt{b_1}$ and b_2 are the sample mean, variance, skewness and kurtosis, respectively, given by

$$\bar{x} = n^{-1}\sum_{i=1}^{n} x_i; \quad s^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$
$$\sqrt{b_1} = \frac{m_3}{(m_2)^{3/2}}; \quad b_2 = \frac{m_4}{(m_2)^2} \tag{1}$$

where the jth central moment m_j is defined by

$$m_j = n^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^j.$$

2.1. *Tests based on the empirical distribution function*

2.1.1. *The Kolmogorov-Smirnov test modified by Lilliefors*

Lilliefors [38] proposed a modification of the Kolmogorov–Smirnov test for normality when the mean and the variance are unknown, and must be estimated from the data. The test statistic K–S is defined as

$$\text{K–S} = \max_{1\le i\le n}\left[\Phi(x_i;\bar{x};s^2) - \frac{(i-1)}{n}; \frac{i}{n} - \Phi(x_i;\bar{x};s^2)\right]$$

where $\Phi(x_i;\bar{x};s^2)$ is the cumulative distribution function of the normal distribution with parameters estimated from the data. The normality hypothesis of the data is then rejected for large values of K–S. Although the competitiveness of this test has been contested in several comparison studies (see e.g. [28,30]), it is considered in the proposed study due to its large availability in commercial software and also due to the recent performance results presented in [37] which contradict the aforementioned about K–S.

2.1.2. *The Anderson-Darling test*

Anderson and Darling [39] proposed a test statistic AD of the form

$$\text{AD} = n\int_{-\infty}^{\infty}[F_n(x) - \Phi(x)]^2\psi(x)\,\mathrm{d}F(x)$$

where $F_n(x)$ is the empirical distribution function (EDF), $\Phi(x)$ is the cumulative distribution function of the standard normal distribution and $\psi(x)$ is a weight function given by $[\Phi(x)\cdot(1-\Phi(x))]^{-1}$. It can be seen [40] that AD can be written as

$$\text{AD} = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln(p_i) + \ln(1-p_{n+1-i})]$$

where the p_i values are given by $\Phi(z_{(i)})$, with $z_{(i)} = (x_{(i)} - \bar{x})/s$. In order to increase its power when μ and σ are estimated from the sample, a modification factor has been proposed for AD [26] resulting in the new statistic AD*:

$$\text{AD}^* = \text{AD}\left(1 + \frac{0.75}{n} + \frac{2.25}{n^2}\right).$$

The normality hypothesis of the data is then rejected for large values of the test statistic.

2.1.3. *The Zhang–Wu Z_C and Z_A tests*

Zhang and Wu [41] recently proposed test statistics Z_C and Z_A of the general form

$$Z = \int_{-\infty}^{\infty} 2n \left\{ F_n(x) \ln\left(\frac{F_n(x)}{F_0(x)}\right) + (1 - F_n(x)) \ln\left[\frac{(1 - F_n(x))}{(1 - F_0(x))}\right]\right\} \mathrm{d}w(x)$$

where $F_0(x)$ is a hypothetical distribution function completely specified and $w(x)$ is a weight function. In the case where $\mathrm{d}w(x)$ is considered to be $[1/F_0(x)] \cdot [1/(1 - F_0(x))]\mathrm{d}F_0(x)$ and $F_0(x)$ is $\Phi(x)$, the test statistic Z_C is obtained [41] by

$$Z_C = \sum_{i=1}^{n} \left[\ln \frac{(1/\Phi(z_{(i)}) - 1)}{(n - 0.5)/(i - 0.75) - 1}\right]^2.$$

In the case where $\mathrm{d}w(x)$ is considered to be $[1/F_n(x)] \cdot [1/(1 - F_n(x))]\mathrm{d}F_n(x)$, the test statistic Z_A is then obtained [41] by

$$Z_A = -\sum_{i=1}^{n} \left[\frac{\ln \Phi(z_{(i)})}{n - i + 0.5} + \frac{\ln[1 - \Phi(z_{(i)})]}{i - 0.5}\right].$$

For both tests, the normality hypothesis of the data is rejected for large values of the test statistic. Zhang and Wu [41] have also proposed another test statistic, Z_K, which is not included in the proposed study, as results presented in [41] indicate that Z_C and Z_A are generally more powerful than Z_K.

2.1.4. *The Glen–Leemis–Barr test*

Glen, Leemis and Barr [42] recently proposed a test statistic based on the quantiles of the order statistics. Given the relation between the order statistics and the EDF, this test was included in this category. The Glen–Leemis–Barr test statistic P_s is given by

$$P_s = -n - \frac{1}{n} \sum_{i=1}^{n} [(2n + 1 - 2i) \ln(p_{(i)}) + (2i - 1) \ln(1 - p_{(i)})]$$

where $p_{(i)}$ are the elements of the vector p containing the quantiles of the order statistics sorted in ascending order. Following the proposal in [42], the elements of p can be obtained by defining vector u, with elements sorted in ascending order and given by $u_{(i)} = \Phi(z_{(i)})$. Considering that $u_{(1)}, u_{(2)}, \ldots, u_{(n)}$ represent the order statistics of a sample taken from a uniform distribution $U(0;1)$, their quantiles, which correspond to the elements of p, can be determined knowing that $u_{(i)}$ follows a Beta distribution $B(i; n - i + 1)$ [28]. The normality hypothesis of the data is rejected for large values of the test statistic.

2.2. ***Tests based on measures of the moments***

2.2.1. *The D'Agostino–Pearson K^2 test*

D'Agostino and Pearson [43] proposed the test statistic K^2 that combines normalizing transformations of skewness and kurtosis, $Z(\sqrt{b_1})$ and $Z(b_2)$, respectively. The test statistic K^2 is given by $[Z(\sqrt{b_1})]^2 + [Z(b_2)]^2$, in which the transformed skewness $Z(\sqrt{b_1})$ is obtained by [44]

$$Z(\sqrt{b_1}) = \frac{\ln(Y/c + \sqrt{(Y/c)^2 + 1})}{\sqrt{\ln(w)}} \tag{2}$$

with Y, c and w obtained by

$$Y = \sqrt{b_1} \cdot \sqrt{\frac{(n+1)(n+3)}{6(n-2)}}; \quad w^2 = -1 + \sqrt{2\beta_2 - 1};$$

$$\beta_2 = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}; \quad c = \sqrt{\frac{2}{(w^2 - 1)}}$$

and the transformed kurtosis $Z(b_2)$ is obtained by [44]

$$Z(b_2) = \left[\left(1 - \frac{2}{9A}\right) - \sqrt[3]{\frac{1 - 2/A}{1 + y\sqrt{2/(A-4)}}} \right] \sqrt{\frac{9A}{2}}$$

with A and y obtained by

$$A = 6 + \frac{8}{\sqrt{\beta_1}} \left(\frac{2}{\sqrt{\beta_1}} + \sqrt{1 + \frac{4}{\beta_1}} \right); \quad \sqrt{\beta_1} = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \sqrt{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}}$$

$$y = \frac{b_2 - 3(n-1)/(n+1)}{24n(n-2)(n-3)/[(n+1)^2(n+3)(n+5)]}.$$

The normality hypothesis of the data is rejected for large values of the test statistic. Furthermore, according to [44], the test statistic K^2 is approximately chi-squared distributed with two degrees of freedom.

2.2.2. *The Jarque–Bera test*

The Jarque–Bera test is a popular goodness-of-fit test in the field of economics. It has been first proposed by Bowman and Shenton [45] but is mostly known from the proposal of Jarque and Bera [46]. The test Statistic JB is defined by

$$\mathrm{JB} = \frac{n}{6} \left(b_1 + \frac{(b_2 - 3)^2}{4} \right).$$

The normality hypothesis of the data is rejected for large values of the test statistic. In addition, according to [45], it can be seen that JB is asymptotically chi-squared distributed with two degrees of freedom.

2.2.3. *The Doornik–Hansen test*

Various modifications of the Jarque–Bera test have been proposed over the years in order to increase its efficiency. For example, Urzua [47] introduced a modification consisting of a different standardization process for b_1 and b_2, though Thadewald and Büning [9] showed that such modification did not improve the power of the original formulation. A less known formulation is that of Doornik and Hansen [48], which suggests the use of the transformed skewness according to

Equation (2) and the use of a transformed kurtosis according to the proposal in [49]. The statistic of the Doornik–Hansen test DH is thus given by $[Z(\sqrt{b_1})]^2 + [z_2]^2$, in which the transformed kurtosis z_2 is obtained by [49]

$$z_2 = \left[\left(\frac{\xi}{2a} \right)^{1/3} - 1 + \frac{1}{9a} \right] (9a)^{1/2}$$

with ξ and a obtained by

$$\xi = (b_2 - 1 - b_1)2k; \quad k = \frac{(n+5)(n+7)(n^3 + 37n^2 + 11n - 313)}{12(n-3)(n+1)(n^2 + 15n - 4)}$$
$$a = \frac{(n+5)(n+7)[(n-2)(n^2 + 27n - 70) + b_1 \cdot (n-7)(n^2 + 2n - 5)]}{6(n-3)(n+1)(n^2 + 15n - 4)}.$$

The normality hypothesis of the data is rejected for large values of the test statistic and, according to [48], DH is also approximately chi-squared distributed with two degrees of freedom.

2.2.4. *The Gel–Gastwirth robust Jarque–Bera test*

Gel and Gastwirth [5] recently proposed a robust version of the Jarque–Bera test. Stemming from the fact that sample moments are, among other things, known to be sensitive to outliers, see e.g. [36], Gel and Gastwirth have proposed a modification of JB that uses a robust estimate of the dispersion in the skewness and kurtosis definitions given in Equation (1) instead of the second order central moment m_2. The selected robust dispersion measure is the average absolute deviation from the median and leads to the following statistic of the robust Jarque–Bera test RJB given by

$$\mathrm{RJB} = \frac{n}{6} \left(\frac{m_3}{J_n^3} \right)^2 + \frac{n}{64} \left(\frac{m_4}{J_n^4} - 3 \right)^2$$

with J_n obtained by

$$J_n = \frac{\sqrt{\pi/2}}{n} \sum_{i=1}^{n} |x_i - M| \tag{3}$$

in which M is the sample median. The normality hypothesis of the data is rejected for large values of the test statistic and, according to [5], RJB asymptotically follows the chi-square distribution with two degrees of freedom.

2.2.5. *The Hosking L-moments based test*

Given several disadvantages associated with the use of central moments [36,50–52], Hosking [50] advocated the use of linear combinations of the order statistics instead, termed L-moments, which are less affected by sample variability and, therefore, are more robust to outliers and better for making inferences about an underlying probability distribution. Hosking [50] has shown the rth order sample L-moment can be estimated by

$$l_r = \sum_{k=0}^{r-1} p^*_{r-1,k} \cdot b_k$$

where $p^*_{r,k}$ and b_k are obtained by

$$p^*_{r,k} = (-1)^{r-k}\binom{r}{k}\binom{r+k}{k}; \quad b_k = n^{-1}\sum_{i=1}^{n}\frac{(i-1)(i-2)\cdots(i-k)}{(n-1)(n-2)\cdots(n-k)}x_{(i)}.$$

Based on the second, third and fourth sample L-moments, which have similarities with the corresponding central moments, Hosking [50] also defines new measures of skewness and kurtosis, termed L-skewness τ_3 and L-kurtosis τ_4, and given by

$$\tau_3 = \frac{l_3}{l_2}; \quad \tau_4 = \frac{l_4}{l_2}.$$

The value of τ_3 is bounded between -1 and 1 for all distributions and is close to zero for the normal distribution, while the value of τ_4 is ≤ 1 for all distributions and is close to 0.1226 for the normal distribution. As referred in [36], Hosking has suggested that normality could be tested based on τ_3 and τ_4 according to the following statistic T_{Lmom}

$$T_{\text{Lmom}} = \frac{\tau_3 - \mu_{\tau_3}}{\text{var}(\tau_3)} + \frac{\tau_4 - \mu_{\tau_4}}{\text{var}(\tau_4)} \tag{4}$$

where μ_{τ_3} and μ_{τ_4} are the mean of τ_3 and τ_4, and $\text{var}(\tau_3)$ and $\text{var}(\tau_4)$ are their corresponding variances. The values of μ_{τ_3}, μ_{τ_4}, $\text{var}(\tau_3)$ and $\text{var}(\tau_4)$ can be obtained by simulation. Nonetheless, μ_{τ_3} and μ_{τ_4} are expected to be close to 0 and 0.1226, and Hosking [50] provides an approximation for $\text{var}(\tau_3)$. For the case of $\text{var}(\tau_4)$ there is no approximation currently available. Details concerning the values of these parameters considered in this study are presented in Section 4. The normality hypothesis of the data is rejected for large values of T_{Lmom}, which is also approximately chi-squared distributed with two degrees of freedom according to [36].

2.2.6. *The Hosking test based on trimmed L-moments*

Although L-moments exhibit some robustness towards outliers in the data, as previously referred, they may still be affected by extreme observations [53]. A robust generalization of the sample L-moments has, therefore, been formulated by Elamir and Seheult [53] leading to the development of trimmed L-moments. The proposed formulation for the trimmed L-moments allows for both symmetric and asymmetric trimming of the smallest and largest sample observations. For the case of normality testing suggested herein, only symmetric trimming is considered.

Considering an integer symmetric trimming level t, Elamir and Seheult [53] have shown the rth order sample trimmed L-moment $l_r^{(t)}$ can be estimated by

$$l_r^{(t)} = \frac{1}{r}\sum_{i=t+1}^{n-t}\left\{\frac{\sum_{k=0}^{r-1}\left[(-1)^k\binom{r-1}{k}\binom{i-1}{r+t-1-k}\binom{n-i}{t+k}\right]}{\binom{n}{r+2t}}\right\}x_{(i)}.$$

Based on the second, third and fourth sample trimmed L-moments, Elamir and Seheult [53] also define new measures of skewness and kurtosis, termed TL-skewness $\tau_3^{(t)}$ and TL-kurtosis

$\tau_4^{(t)}$, given by

$$\tau_3^{(t)} = \frac{l_3^{(t)}}{l_2^{(t)}}; \quad \tau_4^{(t)} = \frac{l_4^{(t)}}{l_2^{(t)}}.$$

Based on these new measures, the following test, similar to that given by Equation (4), is considered in the present study:

$$T_{\text{TLmom}}^{(t)} = \frac{\tau_3^{(t)} - \mu_{\tau_3}^{(t)}}{\text{var}(\tau_3^{(t)})} + \frac{\tau_4^{(t)} - \mu_{\tau_4}^{(t)}}{\text{var}(\tau_4^{(t)})}$$

where, for a selected trimming level t, $\mu_{\tau_3}^{(t)}$ and $\mu_{\tau_4}^{(t)}$ are the mean of $\tau_3^{(t)}$ and $\tau_4^{(t)}$, and $\text{var}(\tau_3^{(t)})$ and $\text{var}(\tau_4^{(t)})$ are their corresponding variances. As for the previous test, the values of $\mu_{\tau_3}^{(t)}$, $\mu_{\tau_4}^{(t)}$, $\text{var}(\tau_3^{(t)})$ and $\text{var}(\tau_4^{(t)})$ can be obtained by simulation. Details concerning the values of these parameters considered in this study are presented in Section 4.

Three versions of this test are considered in the proposed study, which correspond to symmetric trimming levels t of 1, 2 and 3. For each test, the normality hypothesis of the data is rejected for large values of the statistic $T_{\text{TLmom}}^{(t)}$.

2.2.7. *The Bontemps-Meddahi tests*

Bontemps and Meddahi [54] recently proposed a family of normality tests based on moment conditions known as Stein equations and their relation with Hermite polynomials. The test statistics are developed using the generalized method of moments approach [55] associated with Hermite polynomials, which leads to test statistics that are robust against parameter uncertainty. The general expression of the test family is thus given by

$$\text{BM}_{3-p} = \sum_{k=3}^{p} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^{n} H_k(z_i) \right)^2 \tag{5}$$

where $z_i = (x_i - \bar{x})/s$ and $H_k(\cdot)$ represents the kth order normalized Hermite polynomial having the general expression given by the following recursive formulation

$$\forall i > 1, \quad H_i(u) = \frac{1}{\sqrt{i}}[u \cdot H_{i-1}(u) - \sqrt{i-1} \cdot H_{i-2}(u)], \quad H_0(u) = 1, \quad H_1(u) = u. \tag{6}$$

It can be seen from Equation (5) that a number of different tests can be obtained by assigning different values to p, which represents the maximum order of the considered normalized Hermite polynomials. Based on the results presented in [54], two different tests are considered in the study presented herein. Following the terminology provided in Equation (5), these tests are termed BM_{3-4} and BM_{3-6}. In both cases, the normality hypothesis of the data is rejected for large values of the test statistic and, according to [54], the general BM_{3-p} family of tests asymptotically follows the chi-square distribution with $p-2$ degrees of freedom.

2.2.8. *The Brys–Hubert–Struyf MC–LR test*

Brys, Hubert and Struyf [56] have proposed a goodness-of-fit test based on robust measures of skewness and tail weight. The considered robust measure of skewness is the medcouple

MC [57,58] defined as

$$\mathrm{MC} = \operatorname*{med}_{x_{(i)} \le m_F \le x_{(j)}} h(x_{(i)}, x_{(j)})$$

where med stands for the median, m_F is the sample median and the kernel function h is given by

$$h(x_{(i)}, x_{(j)}) = \frac{(x_{(j)} - m_F) - (m_F - x_{(i)})}{x_{(i)} - x_{(j)}}$$

and for which a fast computation algorithm is provided in [58]. For the case where $x_{(i)} = x_{(j)} = m_F$, h is then set by

$$h(x_{(i)}, x_{(j)}) = \begin{cases} 1 & i > j \\ 0 & i = j \\ -1 & i < j. \end{cases}$$

The left medcouple (LMC) and the right medcouple (RMC) are the considered robust measures of left and right tail weight [59], respectively, and are defined by

$$\mathrm{LMC} = -\mathrm{MC}(x < m_F); \quad \mathrm{RMC} = \mathrm{MC}(x > m_F).$$

The test statistic $T_{\mathrm{MC-LR}}$ is then defined by

$$T_{\mathrm{MC-LR}} = n(w - \omega)^t \cdot V^{-1} \cdot (w - \omega)$$

in which w is set as $[\mathrm{MC},\mathrm{LMC},\mathrm{RMC}]^t$, and ω and V are obtained based on the influence function of the estimators in w [58,59]. For the case of a normal distribution, ω and V are defined as [56]

$$\omega = [0, 0.199, 0.199]^t; \quad V = \begin{bmatrix} 1.25 & 0.323 & -0.323 \\ 0.323 & 2.62 & -0.0123 \\ -0.323 & -0.0123 & 2.62 \end{bmatrix}.$$

The normality hypothesis of the data is rejected for large values of $T_{\mathrm{MC-LR}}$ and, according to [56], it is suggested that $T_{\mathrm{MC-LR}}$ approximately follows the chi-square distribution with three degrees of freedom.

2.2.9. *The Bonett–Seier test*

Bonett and Seier [60] have suggested a modified measure of kurtosis for testing normality, which is based on a modification of Geary's proposal [61]. The test statistic of the new kurtosis measure T_w is thus given by:

$$T_w = \frac{\sqrt{n+2} \cdot (\hat{\omega} - 3)}{3.54}$$

in which $\hat{\omega}$ is set by

$$\hat{\omega} = 13.29 \left[\ln \sqrt{m_2} - \ln \left(n^{-1} \sum_{i=1}^{n} |x_i - \bar{x}| \right) \right].$$

The normality hypothesis of the data is rejected for both small and large values of T_w using a two-sided test and, according to [60], it is suggested that T_w approximately follows a standard normal distribution.

2.2.10. *The Brys–Hubert–Struyf–Bonett–Seier joint test*

Considering that the Brys–Hubert–Struyf MC–LR test is, mainly, a skewness associated test and that the Bonett–Seier proposal is a kurtosis based test, a joint test, termed $T_{\mathrm{MC-LR}}$–T_w, considering both these measures is proposed herein for testing normality. This joint test attempts to make use of the two referred focused tests in order to increase the power to detect different kinds of departure from normality. This joint test is proposed herein based on the assumption that the individual tests can be considered independent. This assumption is based on a simulation of the two statistics considering 200,000 samples of size 100 drawn from a standard normal distribution that yielded a correlation coefficient of approximately -0.06. In order to control the overall Type I error at the nominal level α, the normality hypothesis of the data is rejected for the joint test when rejection is obtained for either one of the two individual tests for a significance level of $\alpha/2$.

2.2.11. *The Cabaña-Cabaña tests*

Cabaña and Cabaña [62] have recently proposed four families of normality tests based on transformed empirical processes. Two test families are of the Kolmogorov–Smirnov type while the other two are of the Cramér–von Mises type. One family of each type of test focuses on changes on skewness and the other one is sensitive to changes in kurtosis. Considering the results provided in [62], the power of the Kolmogorov–Smirnov type tests is seen to be very similar to that of the Cramér–von Mises type tests. Therefore, only the Kolmogorov–Smirnov type tests were selected in the proposed study, as their implementation complexity is comparatively lower than that of the Cramér–von Mises type tests.

The test statistics proposed in [62] are based on the definition of approximate transformed estimated empirical processes (ATEEP) sensitive to changes in skewness or kurtosis. The proposed ATEEP sensitive to changes in skewness is defined as:

$$w_{S,\ell}(x) = \Phi(x) \cdot \bar{H}_3 - \phi(x) \cdot \sum_{j=1}^{\ell} \frac{1}{\sqrt{j}} H_{j-1}(x) \cdot \bar{H}_{j+3}$$

where ℓ is a dimensionality parameter, $\phi(x)$ is the probability density function of the standard normal distribution, $H_j(\cdot)$ represents the jth order normalized Hermite polynomial given by Equation (6) and $\bar{H}_j$ is the jth order normalized mean of the Hermite polynomial defined as

$$\bar{H}_j = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} H_j(x_i).$$

The proposed ATEEP sensitive to changes in kurtosis is defined as:

$$w_{K,\ell}(x) = -\phi(x) \cdot \bar{H}_3 + [\Phi(x) - x \cdot \phi(x)] \cdot \bar{H}_4 - \phi(x) \cdot \sum_{j=2}^{\ell} \left(\sqrt{\frac{j}{j-1}} H_{j-2}(x) \cdot H_j(x) \right) \cdot \bar{H}_{j+3}.$$

According to [62], the dimensionality parameter ℓ ensures that the test is consistent against alternative distributions differing from the normal distribution having the same mean and variance in at least one moment of order not greater than $\ell + 3$. The Kolmogorov–Smirnov type test statistics sensitive to changes in skewness and in kurtosis, $T_{S,\ell}$ and $T_{K,\ell}$, respectively, are defined as

$$T_{S,\ell} = \max |w_{S,\ell}(x)|; \quad T_{K,\ell} = \max |w_{K,\ell}(x)|.$$

For both cases, the normality hypothesis of the data is rejected for large values of the test statistic. Based on results presented in [62], parameter ℓ was considered to be five.

2.3. *Regression and correlation tests*

2.3.1. *The Shapiro–Wilk test*

The Shapiro and Wilk W statistic [63] is a well-established and powerful test of normality. The statistic W represents the ratio of two estimates of the variance of a normal distribution and is obtained by

$$W = \frac{\left(\sum_{i=1}^{n} a_i \cdot x_{(i)}\right)^2}{n \cdot m_2}$$

where the vector of weights a is obtained by $(a_1, \ldots, a_n) = m \cdot V^{-1} \cdot (m \cdot V^{-1} \cdot V^{-1} \cdot m^t)^{-0.5}$, in which m and V are the mean vector and covariance matrix of the order statistics of the standard normal distribution. The computation of the vector of weights a considered herein is defined according to the improved algorithm presented by Royston [64], which considers the methodology described in [65,66]. Given the definition of W, it is intuitive to observe the normality hypothesis of the data is rejected for small values of W. In order to simplify the application of this test, transformations g have been defined in [66] for different sample sizes such that $g(W)$ approximately follows a standard normal distribution.

2.3.2. *The Shapiro–Francia test*

Since explicit values of m and V are not readily available and the computation of V^{-1} is time consuming for large samples, Shapiro and Francia [67] have proposed a modification of the Shapiro–Wilk test, hereon termed W_{SF}, based on the fact that, for large samples, the sample observations can be treated as being independent. In this context, Shapiro and Francia suggested to replace V^{-1} by an identity matrix, which leads to a vector of weights a defined as $m \cdot (m \cdot m^t)^{-0.5}$. The computation of the W_{SF} test considered herein is defined according to the procedure proposed in [68]. Similarly to the W test, the normality hypothesis of the data is rejected for small values of W_{SF}. As for the previous test, transformations g have also been defined in [66] for different sample sizes such that $g(W_{\mathrm{SF}})$ approximately follows a standard normal distribution.

2.3.3. *The Rahman-Govindarajulu modification of the Shapiro–Wilk test*

Rahman and Govindarajulu [69] have proposed a modification to the Shapiro–Wilk test, hereon termed W_{RG}, which is simpler to compute and relies on a new definition of the weights a using the approximations to m and V suggested in [70,71]. According to these proposals, each element a_i of the new vector of weights becomes

$$a_i = -(n+1)(n+2)\phi(m_i)[m_{i-1}\phi(m_{i-1}) - 2m_i\phi(m_i) + m_{i+1}\phi(m_{i+1})]$$

where it is assumed that $m_0\phi(m_0) = m_{n+1}\phi(m_{n+1}) = 0$. With this modification, the new test statistic W_{RG} assigns larger weights to the extreme order statistics than the original W test, which has been seen to result in higher power against short tailed alternative distributions [69,72]. As for the original W test, the normality hypothesis of the data is rejected for small values of W_{RG}.

2.3.4. *The D'Agostino D test*

D'Agostino [73] proposed the D test statistic as an extension of the Shapiro–Wilk test. The D'Agostino proposal eliminates the need to define the vector of weights a of the Shapiro–Wilk test and is obtained by

$$D = \frac{\sum_{i=1}^{n}(i - (n+1)/2) \cdot x_{(i)}}{n^2 \cdot \sqrt{m_2}}.$$

The normality hypothesis of the data is rejected for both small and large values of D using a two-sided test.

2.3.5. *The Filliben correlation test*

Filliben [74] described the probability plot correlation coefficient r as a test for normality. The correlation coefficient is defined between the sample order statistics and the estimated median values of the theoretical order statistics.

Considering that $m_{(1)}, m_{(2)}, \ldots, m_{(n)}$ represent the estimated median values of the order statistics from a uniform distribution $U(0;1)$, each $m_{(i)}$ is obtained by

$$m_{(i)} = \begin{cases} 1 - 0.5^{(1/n)} & i = 1 \\ \dfrac{(i - 0.3175)}{(n + 0.365)} & 1 < i < n \\ 0.5^{(1/n)} & i = n \end{cases}$$

upon which the estimated median values of the theoretical order statistics can be obtained using the transformation $M_{(i)} = \Phi^{-1}(m_{(i)})$. The correlation coefficient r is then defined as

$$r = \frac{\sum_{i=1}^{n} x_{(i)} \cdot M_{(i)}}{\sqrt{\sum_{i=1}^{n} M_{(i)}^2} \cdot \sqrt{(n-1) \cdot s^2}}$$

leading to the rejection of the normality hypothesis of the data for small values of r.

2.3.6. *The Chen–Shapiro test*

Chen and Shapiro [75] introduced an alternative test statistic CS based on normalized spacings and defined as

$$\mathrm{CS} = \frac{1}{(n-1) \cdot s} \sum_{i=1}^{n-1} \frac{x_{(i+1)} - x_{(i)}}{M_{i+1} - M_i}$$

in which M_i is the ith quantile of a standard normal distribution obtained by $\Phi^{-1}[(i - 0.375)/(n + 0.25)]$. Since a close relation between CS and the Shapiro–Wilk test has been shown to exist [72], their performance is expected to be similar also. According to [75], the normality hypothesis of the data is rejected for small values of CS.

2.3.7. *The Zhang Q tests*

Zhang [76] introduced the Q test statistic based on the ratio of two unbiased estimators of standard deviation, q_1 and q_2, and given by $Q = \ln(q_1/q_2)$. The estimators q_1 and q_2 are obtained by

$q_1 = \sum_{i=1}^{n} a_i x_{(i)}$ and $q_2 = \sum_{i=1}^{n} b_i x_{(i)}$ where the ith order linear coefficients a_i and b_i result from

$$a_i = [(u_i - u_1)(n-1)]^{-1}, \quad for\ i \neq 1;\ \ a_1 = \sum_{i=2}^{n} a_i$$

$$b_i = \begin{cases} -b_{n-i+1} = [(u_i - u_{i+4})(n-4)]^{-1} & i = 1, \ldots, 4 \\ (n-4)^{-1} \cdot [(u_i - u_{i+4})^{-1} - (u_{i-4} - u_i)^{-1}] & i = 5, \ldots, n-4 \end{cases}$$

where the ith expected value of the order statistics of a standard normal distribution, u_i, is defined by $\Phi^{-1}[(i - 0.375)/(n + 0.25)]$. According to [76], Q is less powerful against negatively skewed distributions. Therefore, Zhang [76] has also proposed the alternative statistic $Q*$ by switching the ith order statistics $x_{(i)}$ in q_1 and q_2 by $x^*_{(i)} = -x_{(n-i+1)}$. Based on the definition of both Q and Q^*, the normality hypothesis of the data is rejected for both small and large values of the statistic using a two-sided test.

In addition to these two tests, Zhang [76] has also proposed a joint test $Q{-}Q^*$, stemming from the fact that Q and Q^* are approximately independent. Therefore, for the case of the joint test $Q{-}Q^*$, the normality hypothesis of the data is rejected at the significance level α when rejection is obtained for either one of the two individual tests for a significance level of $\alpha/2$.

According to [76], both Q and Q^* approximately follow a normal distribution. However, Hwang and Wei [77] have proven otherwise and state that the performance of these tests is better when based on their empirical distribution. Since the joint test has shown to be more powerful than the individual tests [76,77], the joint test $Q{-}Q^*$ is the primary choice for the proposed study. Nonetheless, the Q test is also included for comparison purposes.

2.3.8. *The del Barrio-Cuesta-Albertos-Matrán-Rodríguez-Rodríguez quantile correlation test*

A novel approach for normality testing, based on the L_2-Wasserstein distance, has been proposed by del Barrio, Cuesta-Albertos, Matrán and Rodríguez-Rodríguez [78,79]. The BCMR test statistic is defined by

$$\text{BCMR} = \frac{m_2 - \left[\sum_{i=1}^{n} x_{(i)} \cdot \int_{(i-1)/n}^{i/n} \Phi^{-1}(t)dt\right]^2}{m_2}$$

where, according to [78], the numerator represents the squared L_2-Wasserstein distance. The normality hypothesis of the data is rejected for large values of the test statistic.

2.3.9. *The β_3^2 Coin test*

Coin [80] has recently proposed a normality test based on a polynomial regression focused on detecting symmetric non-normal alternative distributions. According to [80], the analysis of standard normal Q–Q plots of different symmetric non-normal distributions suggests that fitting a model of the type

$$z_{(i)} = \beta_1 \cdot \alpha_i + \beta_3 \cdot \alpha_i^3,$$

where β_1 and β_3 are fitting parameters and α_i represent the expected values of standard normal order statistics, leads to values β_3 different from zero when in presence of symmetric non-normal distributions. Therefore, Coin [80] suggests the use of β_3^2 as a statistic for testing normality, thus rejecting the normality hypothesis of the data for large values of β_3^2. As suggested in [80], the values of α_i are obtained using the approximations provided in [81].

2.4. *Other tests*

2.4.1. *The Epps–Pulley test*

Epps and Pulley [82,83] have proposed a test statistic T_{EP} based on the following weighted integral

$$T_{\mathrm{EP}} = \int_{-\infty}^{\infty} |\varphi_n(t) - \hat{\varphi}_0(t)|^2 \mathrm{d}G(t)$$

where $\varphi_n(t)$ is the empirical characteristic function given by $n^{-1}\sum_{j=1}^{n}\exp(itx_j)$, $\hat{\varphi}_0(t)$ is the sample estimate of the characteristic function of the normal distribution given by $\exp(it\bar{x} - 0.5m_2t^2)$ and $G(t)$ is an adequate function chosen according to several considerations [82]. By setting $\mathrm{d}G(t) = g(t)\mathrm{d}t$ and selecting $g(t) = \sqrt{m_2/2\pi}\cdot\exp(-0.5m_2t^2)$ the following statistic can be obtained [82] by

$$T_{\mathrm{EP}} = 1 + \frac{n}{\sqrt{3}} + \frac{2}{n}\sum_{k=2}^{n}\sum_{j=1}^{k-1} \mathrm{e}^{(-(x_j-x_k)^2)/(2m_2)} - \sqrt{2}\sum_{j=1}^{n} \mathrm{e}^{(-(x_j-\bar{x})^2)/(4m_2)}$$

for which the normality hypothesis of the data is rejected when large values of T_{EP} are obtained. To simplify the use of this test by eliminating the need for tables of percentage points of T_{EP}, an approximation to the limit distribution of T_{EP} has been presented in [84].

2.4.2. *The Martinez–Iglewicz test*

Martinez and Iglewicz [85] have proposed a normality test based on the ratio of two estimators of variance, where one of the estimators is the robust biweigth scale estimator S_b^2

$$S_b^2 = \frac{n\cdot\sum_{|\tilde{z}_i|<1}(x_i - M)^2(1-\tilde{z}_i^2)^4}{\left[\sum_{|\tilde{z}_i|<1}(1-\tilde{z}_i^2)(1-5\tilde{z}_i^2)\right]^2}$$

where M is the sample median, $\tilde{z}_i = (x_i - M)/(9A)$, with A being the median of $|x_i - M|$, and when $|\tilde{z}_i| > 1$, $\tilde{z}_i$ is set to 0. The Martinez–Iglewicz test statistic I_n is then given by

$$I_n = \frac{\sum_{i=1}^{n}(x_i - M)^2}{(n-1)\cdot S_b^2}$$

for which the normality hypothesis of the data is rejected for large values of I_n.

2.4.3. *The Gel–Miao–Gastwirth test*

Gel, Miao and Gastwirth [86] have recently proposed a directed normality test, which focuses on detecting heavier tails and outliers of symmetric distributions. The test is based on the ratio of the standard deviation and on the robust measure of dispersion J_n defined in Equation (3). The normality test statistic R_{sJ} is therefore given by $R_{sJ} = s/J_n$ which should tend to one under a normal distribution. According to [86], the normality hypothesis of the data is rejected for large values of R_{sJ} and the statistic $\sqrt{n}(R_{sJ} - 1)$ is seen to asymptotically follow the normal distribution $N(0; \pi/2 - 1.5)$. However, it has been empirically found that rejecting the normality hypothesis using a two-sided test extends the range of application of this test, namely to light-tailed distributions, without a significant reduction of its power towards heavy-tailed distributions. Given its enhanced behaviour, the two-sided test is the primary choice for the proposed study. Nonetheless, a detailed power comparison of the two-sided test with the one-sided test, hereon termed $R_{sJ,1}$, is also presented.

3. Comparison of empirical and asymptotical critical values

For many normality tests, the sampling distributions of their corresponding statistics are intractable, for both finite and large sample situations. Nonetheless, in cases where such limit distribution can be approximated, it is of interest to determine how close the simulated percentile values of such distributions are to the corresponding asymptotic values and how fast is the convergence to such values. The tests presented in the previous section for which the limit distribution has been examined in previous studies are referred herein. Reference to previous works on the adequacy of such limit distributions is also made when available. For the remaining cases, comparison results from this study clarifying the suitability of their asymptotical critical values are presented herein. For a given test, the referred results correspond to the comparison of the asymptotical critical values to the empirical ones based on 1,000,000 samples drawn from the standard normal distribution, for five sample sizes n ($n = 25$, $n = 50$, $n = 100$, $n = 200$ and $n = 500$) and considering several significance levels α.

The matter of the limit distribution has been previously addressed for statistics BCMR, D, $T_{S,\ell}$, $T_{K,\ell}$, T_{EP}, JB, RJB, W, W_{SF}, R_{sJ}, *DH*, K^2, T_{Lmom}, BM_{3-4}, BM_{3-6}, $T_{\mathrm{MC-LR}}$ and T_w. Furthermore, as stated before, a limit distribution was also proposed for statistics Q and Q^*, but was rejected based on subsequent studies [77].

According to [79], the asymptotic distribution of BCMR can be obtained numerically by computing its characteristic function and performing a numerical inversion. The convergence of the numerically simulated critical values to those obtained by the asymptotic distribution is very slow, and the use of the asymptotic critical values generally yields conservative results (i.e. the normality hypothesis is rejected more times when considering the asymptotical values than when using the simulated ones) for sample sizes as low as 10. Hence, the use of asymptotic critical values is not recommended for this test. With respect to the D statistic, a standardized version can be defined in order to transform D into a standard normal variable [73,87]. Nonetheless, studies have shown that, even for a sample size of 1000, the percentiles of this new standard statistic do not converge to those of the standard normal distribution and exhibit asymmetric behaviour [34]. Hence, there is evidence of low convergence to the asymptotic distribution, and the use of empirical critical values is recommended. For the case of statistics $T_{S,\ell}$ and $T_{K,\ell}$, the theoretical basis of their limit distribution is addressed in [62] but a closed analytical expression is not available. Nonetheless, through some refined numerical analysis, the asymptotic critical percentiles were determined and compared with those obtained through simulation. Despite the relative proximity of the asymptotic and simulated values, the latter are seen to be more conservative and their use is recommended for a more accurate performance of these tests [62]. With respect to the limit distribution of T_{EP}, a numerical definition of the first four moments of the referred distribution as well as approximations to the limit distribution, obtained by fitting members of the Johnson and of the Pearson system of distributions, can be found in [84]. By comparing the upper percentiles given by the approximated limit distributions with those simulated numerically, it can be observed that for moderate sample sizes ($n \geq 50$) the simulated upper percentiles are very close to the asymptotic values and convergence to these values is seen to be rather fast. Furthermore, a transformation to normality of T_{EP} that enables the use of standard normal percentiles to apply the test is also proposed. Nonetheless, it should be noted that, in either case, for smaller sample sizes, the use of numerically simulated critical values is recommended. According to results presented for JB in [9], for $\alpha \geq 0.05$ the normality hypothesis is rejected more times when considering the simulated values than when using the asymptotical ones, especially for small sample sizes, while for $\alpha < 0.05$ the pattern is not as definite. Figure 1a presents the comparison of the JB asymptotical critical values to the empirical ones, which leads to conclude that, in the overall, the chi-squared distribution approximation of the limit distribution does not work well, even for large sample sizes, and that the speed of convergence is slow. Thus, for a meaningful application of JB, empirical

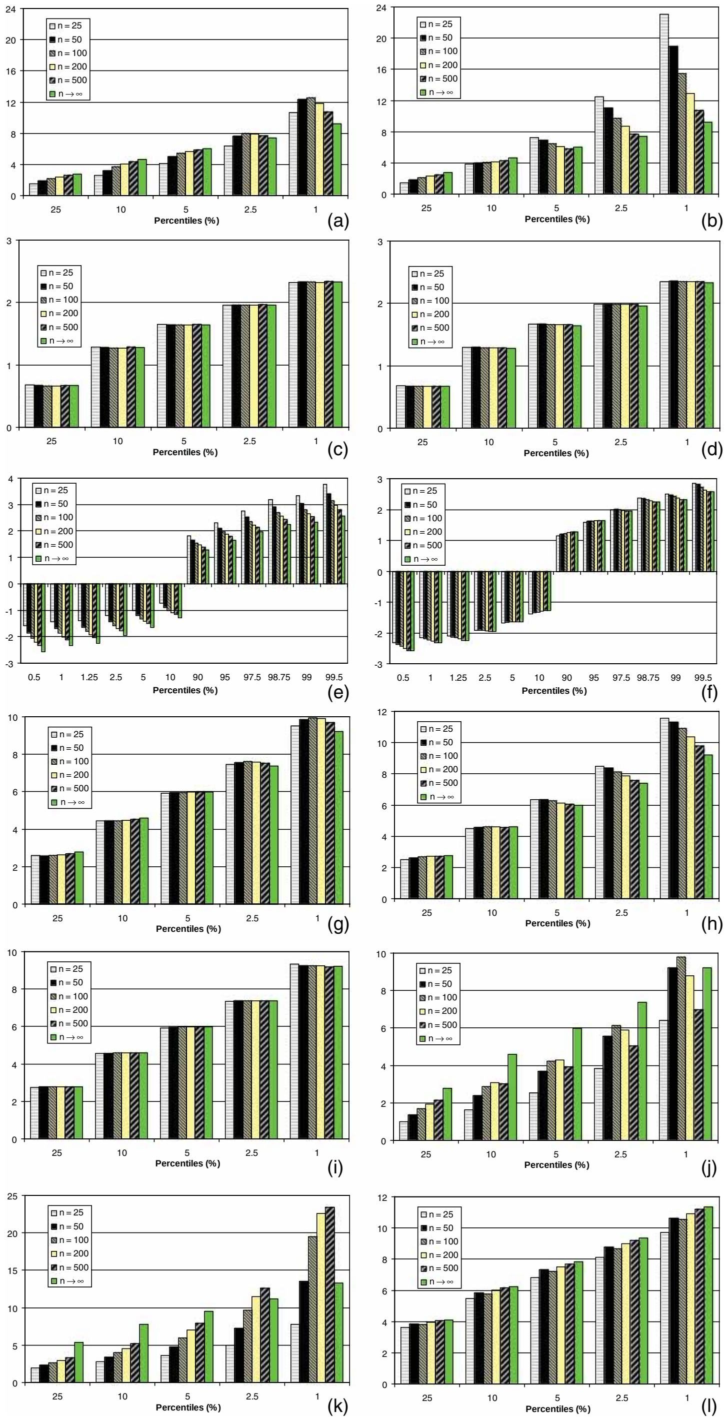

Figure 1. Comparison of empirical and asymptotical critical values for JB (a), RJB (b), W (c), W_{SF} (d), R_{sJ} (e), T_w (f), DH (g), K^2 (h), T_{Lmom} (i), BM_{3-4} (j), BM_{3-6} (k) and $T_{\mathrm{MC-LR}}$ (l).

critical values have to be used [9]. Similar results were also reported for RJB in [5], where it is concluded that the asymptotic chi-squared approximation of critical values is not accurate enough in small to moderate sized samples. Figure 1b presents the comparison of the RJB asymptotical critical values to the empirical ones where it can be seen that the asymptotical approximation is inadequate, even for large sample sizes, and that the speed of convergence is also slow. Hence, the use of empirical critical values is also recommended for RBJ. The problem of finding the limit distribution of W and W_{SF} has been addressed over the years by several researchers using different approaches [66,88–91]. Special emphasis is given to [89] where their asymptotic null distributions are derived and are seen to be identical, though convergence to their critical values is seen to be very slow. As previously referred, transformations g have been defined in [66] for different sample sizes such that $g(W)$ and $g(W_{\mathrm{SF}})$ have an approximately standard normal distribution. According to this approximation, the normality hypothesis is rejected if the transformed variable is larger than the upper percentile of the standard normal distribution. As can be seen from the results presented in Figures 1c and 1d for both tests, the transformed statistics have empirical critical values very close to the standard normal percentiles. Hence, these transformations to normality and the use of critical values obtained from the standard normal distribution are recommended for practical use of these tests. With respect to R_{sJ}, although the statistic $\sqrt{n}(R_{sJ} - 1)$ asymptotically follows the normal distribution as previously referred, observation of Figure 1e shows that the asymptotical critical values differ from the empirical ones, the latter being more conservative. In the overall, the empirical values can be seen to exhibit an asymmetric distribution and a slow convergence towards the asymptotical values. Hence, the use of empirical critical values is recommended for R_{sJ}. Similar conclusions can be drawn from Figure 1f for the case of T_w. Nonetheless, empirical and asymptotical critical values of T_w are closer and the speed of convergence is higher than that of R_{sJ}. For the case of DH, results presented in Figure 1g show that, for $\alpha \geq 0.05$, the asymptotical and empirical critical values are very close and that convergence is fast. On the other hand, for $\alpha < 0.05$ the empirical critical values are more conservative. Similar conclusions can be drawn from the results of test K^2 presented in Figure 1h, although convergence to the asymptotical values is slower for this test, especially for $\alpha < 0.05$. Thus, for DH and K^2, the use of asymptotical critical values is recommended for $\alpha \geq 0.05$, and empirical ones should be used for $\alpha < 0.05$. For the case of T_{Lmom}, results presented in Figure 1i show that the asymptotical and the empirical critical values are very close and that convergence is very fast for all significance levels. Hence, the use of asymptotical critical values is recommended for this test. For the case of tests BM_{3-4} and BM_{3-6}, results presented in Figures 1j and k show an overall low agreement between asymptotical and empirical critical values. In general, the speed of convergence is slow and convergence does not appear to increase with sample size, especially for BM_{3-4}. Hence, for an adequate application of these tests, empirical critical values have to be used. Finally, with respect to $T_{\mathrm{MC-LR}}$, results presented in Figure 1l show that, although the convergence speed is slow, the empirical critical values are close to the asymptotical ones. Nevertheless, the use of empirical critical values is recommended for an adequate use of this test.

4. Statistical distributions considered in the simulation study

As previously stated, the simulation study considers a number of statistical distributions over which the performance of the presented normality tests is to be assessed. The selected alternative distributions were chosen in order to be a representative set exhibiting different values of important properties such as skewness and kurtosis, as found in available power studies. These alternative distributions are categorized into three sets. The first set includes several types of symmetric distributions, the second set includes several types of asymmetric distributions and the third set comprises a number of modified normal distributions with various shapes. A brief description of these distributions is presented in the following.

4.1. *Symmetric distributions*

The considered symmetric distributions are:

- Three cases of the Beta(a, b) distribution, where a and b are the shape parameters, defined as Beta(0.5;0.5), Beta(1;1) and Beta(2;2);
- Three cases of the Cauchy(t, s) distribution, where t and s are the location and scale parameters, respectively, defined as Cauchy(0;0.5), Cauchy(0;1) and Cauchy(0;2);
- One case of the Laplace(t, s) distribution, where t and s are the location and scale parameters, respectively, defined as Laplace(0;1);
- One case of the Logistic(t, s) distribution, where t and s are the location and scale parameters, respectively, defined as Logistic(2;2);
- Four cases of the t-Student(ν) distribution, where ν is the number of degrees of freedom, defined as t(1), t(2), t(4) and t(10);
- Five cases of the Tukey(λ) distribution, where λ is the shape parameter, defined as Tukey(0.14), Tukey(0.5), Tukey(2), Tukey(5) and Tukey(10); and
- One case of the normal distribution, corresponding to the standard normal distribution defined as $N(0;1)$. This distribution is included in order to confirm the nominal significance levels.

4.2. *Asymmetric distributions*

The considered asymmetric distributions are:

- Four cases of the Beta(a,b) distribution, defined as Beta(2;1), Beta(2;5), Beta(4;0.5) and Beta(5;1);
- Four cases of the chi-squared(ν) distribution, where ν is the number of degrees of freedom, defined as $\chi^2(1)$, $\chi^2(2)$, $\chi^2(4)$ and $\chi^2(10)$;
- Six cases of the Gamma(a, b) distribution, where a and b are the shape and scale parameters, respectively, defined as Gamma(2;2), Gamma(3;2), Gamma(5;1), Gamma(9;1), Gamma(15;1) and Gamma(100;1);
- One case of the Gumbel(t, s) distribution, where t and s are the location and scale parameters, respectively, defined as Gumbel(1;2);
- One case of the lognormal(t, s) distribution, where t and s are the location and scale parameters, respectively, defined as LN(0;1); and
- Four cases of the Weibull(a, b) distribution, where a and b are the scale and shape parameters, respectively, defined as Weibull(0.5;1), Weibull(1;2), Weibull(2;3.4) and Weibull(3;4).

4.3. *Modified normal distributions*

The considered modified normal distributions are:

- Six cases of the standard normal distribution truncated at a and b Trunc$(a; b)$, where a and b are the lower and upper truncations points, respectively, defined as Trunc(−1;1), Trunc(−2;2), Trunc(−3;3), Trunc(−2;1), Trunc(−3;1) and Trunc(−3;2);
- Nine cases of a location-contaminated standard normal distribution, hereon termed LoConN$(p; a)$, consisting of randomly selected observations with probability $1 - p$ drawn from a standard normal distribution and with probability p drawn from a normal distribution with mean a and standard deviation 1. The selected cases are defined as LoConN(0.3;1), LoConN(0.4;1), LoConN(0.5;1), LoConN(0.3;3), LoConN(0.4;3), LoConN(0.5;3), LoConN(0.3;5), LoConN(0.4;5) and LoConN(0.5;5);

- Nine cases of a scale-contaminated standard normal distribution, hereon termed ScConN(p; b), consisting of randomly selected observations with probability $1 - p$ drawn from a standard normal distribution and with probability p drawn from a normal distribution with mean 0 and standard deviation b. The selected cases are defined as ScConN(0.05;0.25), ScConN(0.10;0.25), ScConN(0.20;0.25), ScConN(0.05;2), ScConN(0.10;2), ScConN(0.20;2), ScConN(0.05;4), ScConN(0.10;4) and ScConN(0.20;4);
- Twelve cases of a mixture of normal distributions, hereon termed MixN(p; a; b), consisting of randomly selected observations with probability $1 - p$ drawn from a standard normal distribution and with probability p drawn from a normal distribution with mean a and standard deviation b. The selected cases are defined as MixN(0.3;1;0.25), MixN(0.4;1;0.25), MixN(0.5;1;0.25), MixN(0.3;3;0.25), MixN(0.4;3;0.25), MixN(0.5;3;0.25), MixN(0.3;1;4), MixN(0.4;1;4) , MixN(0.5;1;4), MixN(0.3;3;4), MixN(0.4;3;4) and MixN(0.5;3;4); and
- Five cases of standard normal distributions with outliers, hereon termed Nout1 to Nout5, consisting of observations drawn from a standard normal distribution where some of the values are randomly replaced by extreme observations. The extreme observations are separated into upper and lower extreme observations, x^*_{up} and x^*_{low}, respectively. An observation x^*_{up} is defined as $x_{q3} + k\cdot$ IQR, where IQR represents the inter-quartile range of the standard normal distribution, x_{q3} is the 75% quartile of the standard normal distribution and k is a selected constant. An observation x^*_{low} is defined as $x_{q1} - k\cdot$IQR, where x_{q1} is the 25% quartile of the standard normal distribution. The distribution Nout1 has one extreme observation x^*_{up} with $k = 2$, the distribution Nout2 has one extreme observation x^*_{up} with $k = 3$, the distribution Nout3 has two extreme observations x^*_{up} with $k = 2$ and $k = 3$, the distribution Nout4 has one extreme observation x^*_{up} and one extreme observation x^*_{low} both with $k = 2$ and the distribution Nout5 has two extreme observations x^*_{up} and two extreme observations x^*_{low}, with $k = 2$ and $k = 3$. This set of distributions was specifically considered in order to identify which normality tests are less sensitive to extreme observations that may be present in an underlying normal data sample.

5. Simulation study and power results

An extensive simulation study is presented in the following to estimate the power of the selected normality tests. The effects on the power of the tests due to the sample size, the selected significance level and the type of alternative distribution are considered in the simulation study. The study is

Table 1. Empirical values of μ_{τ_3}, μ_{τ_4}, var(τ_3), var(τ_4), $\mu^{(t)}_{\tau_3}$, $\mu^{(t)}_{\tau_4}$, var($\tau^{(t)}_3$) and var($\tau^{(t)}_4$).

Sample size n		μ_{τ_3}	μ_{τ_4}	var(τ_3)	var(τ_4)
25		−1.3015E−5*	1.2383E−1	8.8038E−3	4.9295E−3
50		−2.5783E−5*	1.2321E−1	4.0493E−3	2.0802E−3
100		7.9729E−6*	1.2291E−1	1.9434E−3	9.5785E−4
Sample size n	Trimming level t	$\mu^{(t)}_{\tau_3}$	$\mu^{(t)}_{\tau_4}$	var($\tau^{(t)}_3$)	var($\tau^{(t)}_4$)
25	1	2.8032E−5*	6.7077E−2	8.1391E−3	4.2752E−3
	2	3.0692E−5*	4.4174E−2	8.6570E−3	4.2066E−3
	3	2.0512E−5*	3.3180E−2	9.5765E−3	4.4609E−3
50	1	−3.7182E−5*	6.4456E−2	3.4657E−3	1.5699E−3
	2	−1.4220E−5*	4.0389E−2	3.3818E−3	1.3301E−3
	3	8.5138E−6*	2.8224E−2	3.3813E−3	1.1823E−3
100	1	−1.7081E−5*	6.3424E−2	1.6064E−3	6.8100E−4
	2	−1.4710E−5*	3.9030E−2	1.5120E−3	5.4207E−4
	3	−3.5160E−6*	2.6645E−2	1.4547E−3	4.5107E−4

*These values are considered to be zero in the remaining of the power study.

Table 2. Empirical power results for symmetrical distributions ($\alpha = 0.05$ and $n = 25$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\mathrm{TLmom}}$	$T^{(2)}_{\mathrm{Lmom}}$	$T^{(3)}_{\mathrm{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\mathrm{MC-LR}}$
Beta(0.5;0.5)	0	1.50	41.1	**75.9**	**90.5**	**85.0**	**75.8**	*69.4*	0.5	*62.1*	0.2	**83.7**	38.9	18.1	10.8	0.0	0.0	45.8
Beta(1;1)	0	1.80	12.1	22.9	32.3	22.2	22.8	23.9	0.1	13.2	0.1	28.1	9.4	5.3	4.4	0.0	0.0	12.9
Tukey(2)	0	1.80	12.0	23.1	32.5	22.4	23.0	24.0	0.2	13.4	0.1	28.4	9.4	5.2	4.4	0.0	0.0	13.0
Tukey(0.5)	0	2.08	6.3	8.1	8.2	5.2	8.0	6.4	0.2	3.2	0.2	7.8	4.8	4.0	4.0	0.1	0.1	7.4
Beta(2;2)	0	2.14	5.7	6.7	6.6	4.2	6.7	5.0	0.3	2.6	0.3	6.2	4.4	3.9	4.0	0.1	0.1	6.8
Tukey(5)	0	2.90	13.6	13.3	4.0	4.4	13.4	2.4	2.9	4.9	7.0	9.6	25.1	23.7	19.9	1.9	2.0	5.3
Tukey(0.14)	0	2.97	5.0	4.9	4.7	4.8	4.9	4.7	4.7	4.7	4.8	4.9	4.8	4.7	4.7	4.6	4.6	4.9
$N(0;1)$	0	3.00	5.0	5.1	5.0	5.0	5.1	5.0	5.0	5.0	5.0	5.0	4.8	4.7	4.7	5.0	5.0	4.9
$t(10)$	0	4.00	7.6	9.4	11.4	11.2	9.5	13.0	13.9	13.4	14.2	11.2	6.1	5.5	5.2	13.8	13.9	4.8
Logistic(0;2)	0	4.20	9.1	11.8	13.6	13.6	11.9	15.8	17.0	16.7	17.9	14.1	7.2	6.2	5.7	16.6	16.8	4.8
Tukey(10)	0	5.38	**95.4**	**96.2**	*71.9*	**80.6**	**96.2**	*51.6*	*58.8*	*68.4*	**89.3**	**92.7**	**98.3**	**96.9**	**93.7**	41.2	44.7	*55.7*
Laplace(0;1)	0	6.00	25.8	32.3	28.7	29.9	32.8	32.5	35.3	36.6	41.7	38.1	22.5	16.4	13.0	32.1	32.9	5.4
$t(4)$	0	∞	19.7	26.1	28.6	28.8	26.4	32.0	34.0	33.9	36.0	30.4	11.0	8.0	6.9	32.6	33.2	4.9
$t(2)$	0	∞	*52.5*	*61.3*	*59.7*	*61.0*	*61.6*	*62.7*	*65.1*	*66.1*	*69.3*	*65.9*	29.1	16.6	12.4	*61.5*	*62.5*	6.6
$t(1)$	0	∞	**90.6**	**93.5**	**90.7**	**91.9**	**93.7**	**90.7**	**92.1**	**93.0**	**95.0**	**94.7**	*74.1*	*51.3*	38.8	**88.9**	**89.7**	17.0
Cauchy(0;0.5)	–	–	**90.7**	**93.6**	**90.7**	**91.9**	**93.7**	**90.7**	**92.1**	**93.1**	**95.0**	**94.8**	*74.0*	*51.1*	38.6	**88.9**	**89.8**	17.0
Cauchy(0;1)	–	–	**90.8**	**93.5**	**90.6**	**91.8**	**93.6**	**90.6**	**92.0**	**92.9**	**94.9**	**94.6**	*74.2*	*51.3*	38.6	**88.9**	**89.7**	17.1
Cauchy(0;2)	–	–	**90.7**	**93.4**	**90.6**	**91.8**	**93.5**	**90.6**	**92.0**	**92.9**	**94.8**	**94.6**	*73.9*	*51.1*	38.5	**88.9**	**89.7**	17.1

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(0.5;0.5)	0	1.50	*67.7*	*67.6*	3.3	5.7	**86.5**	*65.5*	**96.1**	3.9	*60.9*	**88.7**	*61.5*	**86.0**	**82.4**	**90.5**	*63.2*	0.8	*60.1*
Beta(1;1)	0	1.80	30.8	25.6	1.1	3.0	28.2	11.2	*51.3*	14.3	9.3	32.0	20.5	24.0	23.2	44.0	18.0	0.2	29.1
Tukey(2)	0	1.80	30.9	25.7	1.1	3.0	28.4	11.4	*51.4*	14.3	9.5	32.3	20.8	24.2	23.4	44.0	18.3	0.2	29.2
Tukey(0.5)	0	2.08	13.5	11.0	0.8	2.4	7.9	2.8	17.9	11.0	2.3	9.3	6.5	6.4	6.2	13.4	6.5	0.3	13.3
Beta(2;2)	0	2.14	11.3	9.4	0.8	2.5	6.4	2.4	14.5	9.8	2.0	7.6	5.5	5.2	5.0	10.5	5.5	0.4	11.2
Tukey(5)	0	2.90	11.2	8.8	3.3	12.5	7.3	8.0	5.6	6.3	8.1	6.6	2.4	2.0	7.4	7.8	7.5	10.0	11.1
Tukey(0.14)	0	2.97	4.8	4.7	4.8	4.9	4.8	4.8	4.8	4.8	4.8	4.8	4.7	4.6	4.8	4.8	4.9	4.8	4.8
$N(0;1)$	0	3.00	5.0	4.8	5.0	5.0	5.0	5.0	5.0	5.1	5.0	5.0	5.0	4.9	5.0	5.0	5.0	5.0	5.0
$t(10)$	0	4.00	9.4	8.0	12.5	10.1	10.8	12.8	6.8	10.6	13.0	10.4	9.2	11.1	11.5	11.3	9.9	13.0	9.5
Logistic(0;2)	0	4.20	11.9	9.6	14.9	12.1	13.2	15.9	7.7	13.2	16.2	12.6	10.9	13.2	14.1	14.3	12.1	16.3	11.9
Tukey(10)	0	5.38	**83.8**	**90.4**	48.8	*50.9*	**89.0**	**91.7**	*72.5*	**89.7**	**91.9**	**87.2**	37.9	46.7	**89.9**	**82.2**	**80.4**	14.7	**95.5**
Laplace(0;1)	0	6.00	34.4	28.2	30.0	25.3	30.9	36.9	17.1	33.8	37.5	29.2	19.7	27.1	32.8	36.3	30.7	33.2	35.7
$t(4)$	0	∞	26.8	22.8	29.9	24.4	28.4	32.8	17.9	29.5	33.2	27.3	19.8	28.0	29.8	30.3	26.8	27.5	27.4
$t(2)$	0	∞	*61.8*	*57.0*	*58.9*	*51.3*	*61.4*	*66.2*	47.4	*64.0*	*66.6*	*60.1*	39.2	*57.9*	*63.0*	*63.9*	*61.2*	31.9	*63.4*
$t(1)$	0	∞	**92.8**	**91.3**	**88.0**	**82.0**	**92.6**	**94.2**	**86.0**	**93.8**	**94.3**	**92.0**	*64.9*	**88.4**	**93.1**	**92.9**	**92.7**	11.2	**94.3**
Cauchy(0;0.5)	–	–	**92.9**	**91.5**	**88.2**	**82.0**	**92.6**	**94.2**	**86.0**	**93.8**	**94.3**	**92.0**	*64.9*	**88.5**	**93.1**	**92.9**	**92.7**	11.1	**94.3**
Cauchy(0;1)	–	–	**92.8**	**91.5**	**88.0**	**82.1**	**92.5**	**94.1**	**85.9**	**93.7**	**94.2**	**91.9**	*64.7*	**88.4**	**93.0**	**92.8**	**92.6**	11.2	**94.3**
Cauchy(0;2)	–	–	**92.8**	**91.4**	**87.9**	**82.0**	**92.4**	**94.0**	**85.9**	**93.7**	**94.2**	**91.8**	*64.8*	**88.5**	**93.0**	**92.8**	**92.5**	11.1	**94.3**

Table 3. Empirical power results for symmetrical distributions ($\alpha = 0.05$ and $n = 50$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\text{TLmom}}$	$T^{(2)}_{\text{Lmom}}$	$T^{(3)}_{\text{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Beta(0.5;0.5)	0	1.50	**80.0**	**99.1**	**100.0**	**100.0**	**99.2**	**99.5**	38.8	**97.6**	0.0	**99.6**	**82.2**	*51.3*	30.9	0.0	0.0	*69.8*
Beta(1;1)	0	1.80	26.0	*57.6*	**82.6**	**79.4**	*57.9*	**77.8**	0.8	45.7	0.0	*70.5*	25.9	11.7	7.3	0.0	0.0	19.0
Tukey(2)	0	1.80	26.0	*57.7*	**82.7**	**79.5**	*58.1*	**77.8**	0.8	46.0	0.0	*70.7*	26.0	11.8	7.5	0.0	0.0	19.0
Tukey(0.5)	0	2.08	9.8	17.2	24.3	20.2	17.4	28.8	0.1	8.7	0.0	23.1	8.9	5.5	4.6	0.0	0.0	8.6
Beta(2;2)	0	2.14	8.2	13.3	17.4	14.3	13.4	21.4	0.1	6.0	0.0	17.3	7.4	5.0	4.3	0.0	0.0	7.7
Tukey(5)	0	2.90	23.6	24.9	4.1	3.5	25.2	0.8	1.1	1.7	6.3	13.2	44.4	45.2	40.1	0.2	0.2	7.7
Tukey(0.14)	0	2.97	5.0	4.9	4.4	4.6	4.9	4.3	4.3	4.4	4.4	4.8	4.8	4.8	4.8	4.1	4.1	5.0
$N(0;1)$	0	3.00	5.0	5.0	5.0	5.0	5.0	5.1	5.0	5.1	5.0	5.0	4.8	4.9	4.9	5.1	5.1	5.0
$t(10)$	0	4.00	8.8	12.0	16.2	14.6	12.1	17.9	20.5	19.9	21.2	14.5	7.1	6.1	5.7	19.9	20.0	4.7
Logistic(0;2)	0	4.20	11.3	15.9	19.7	18.0	16.0	22.2	25.8	25.2	27.4	19.6	9.1	7.3	6.5	24.1	24.4	4.8
Tukey(10)	0	5.38	**100.0**	**100.0**	**94.9**	**97.4**	**100.0**	*66.0*	**78.9**	**84.7**	**99.3**	**99.8**	**100.0**	**100.0**	**99.9**	42.6	46.2	**83.6**
Laplace(0;1)	0	6.00	43.2	*54.6*	45.6	45.4	*55.2*	48.8	*55.5*	*56.8*	*65.6*	*61.7*	41.4	29.6	23.0	47.5	48.3	7.2
$t(4)$	0	∞	30.6	42.1	46.0	44.1	42.5	49.3	*54.0*	*54.0*	*56.9*	47.4	16.7	10.6	8.6	*50.4*	*50.9*	5.4
$t(2)$	0	∞	**77.8**	**85.9**	**84.0**	**84.1**	**86.1**	**85.2**	**88.1**	**88.7**	**91.1**	**88.8**	*53.2*	29.4	20.2	**84.0**	**84.6**	9.4
$t(1)$	0	∞	**99.4**	**99.7**	**99.3**	**99.5**	**99.7**	**99.2**	**99.5**	**99.6**	**99.8**	**99.8**	**96.8**	**83.4**	*66.9*	**98.7**	**98.9**	34.2
Cauchy(0;0.5)	–	–	**99.4**	**99.7**	**99.4**	**99.5**	**99.7**	**99.3**	**99.6**	**99.6**	**99.8**	**99.8**	**96.8**	**83.4**	*66.9*	**98.8**	**98.9**	34.2
Cauchy(0;1)	–	–	**99.4**	**99.7**	**99.3**	**99.5**	**99.7**	**99.2**	**99.5**	**99.6**	**99.8**	**99.8**	**96.8**	**83.4**	*66.9*	**98.8**	**98.9**	34.2
Cauchy(0;2)	–	–	**99.4**	**99.7**	**99.3**	**99.4**	**99.7**	**99.2**	**99.5**	**99.6**	**99.8**	**99.8**	**96.8**	**83.4**	*67.0*	**98.8**	**98.9**	34.1

Distribution	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(0.5;0.5)	0	1.50	**96.4**	**95.7**	14.4	5.9	**99.9**	**99.2**	**100.0**	11.8	**98.8**	**100.0**	**94.1**	**99.9**	**99.9**	**100.0**	**97.7**	0.1	**93.2**
Beta(1;1)	0	1.80	*64.7*	*56.0*	2.3	2.3	*74.9*	46.2	**93.8**	*56.3*	40.6	**80.6**	*55.5*	*69.2*	*68.7*	**90.7**	*54.3*	0.0	*62.2*
Tukey(2)	0	1.80	*64.8*	*55.9*	2.3	2.4	*75.0*	46.4	**93.8**	*56.1*	40.8	**80.8**	*55.4*	*69.4*	*68.8*	**90.7**	*54.5*	0.0	*62.3*
Tukey(0.5)	0	2.08	29.2	22.6	0.8	1.6	20.8	7.3	45.5	36.4	5.8	26.0	14.3	14.9	16.7	40.8	16.5	0.0	28.7
Beta(2;2)	0	2.14	23.7	18.2	0.7	1.5	15.2	5.0	35.9	30.0	3.9	19.3	11.0	11.0	12.0	31.1	12.6	0.0	23.0
Tukey(5)	0	2.90	19.8	15.0	1.9	16.9	13.0	10.7	13.7	7.2	10.5	11.8	1.1	0.8	11.9	5.4	9.5	9.7	20.0
Tukey(0.14)	0	2.97	4.7	4.7	4.4	4.5	4.6	4.5	4.9	4.5	4.5	4.6	4.5	4.2	4.6	4.6	4.9	4.5	4.7
$N(0;1)$	0	3.00	5.1	4.9	5.0	5.0	5.0	5.0	5.0	5.1	5.0	5.0	5.1	5.0	5.0	5.0	5.0	5.1	5.1
$t(10)$	0	4.00	13.7	11.2	16.3	16.0	15.5	18.8	7.4	16.1	19.2	14.2	12.0	15.3	16.6	17.1	12.5	20.8	13.8
Logistic(0;2)	0	4.20	18.7	15.2	19.6	19.1	19.5	24.0	8.5	21.3	24.5	17.7	14.0	18.1	21.0	22.9	16.4	27.2	18.8
Tukey(10)	0	5.38	**98.6**	**99.7**	*64.7*	*60.5*	**99.7**	**99.7**	**97.9**	**99.5**	**99.8**	**99.5**	33.6	36.3	**99.7**	**97.0**	**98.2**	10.6	**100.0**
Laplace(0;1)	0	6.00	*62.9*	*55.0*	41.5	40.1	*52.1*	*59.2*	26.7	*59.9*	*60.0*	48.4	26.9	37.6	*54.4*	*60.9*	*51.5*	*54.9*	*63.9*
$t(4)$	0	∞	47.1	41.5	43.5	42.6	46.8	*52.7*	27.4	*51.1*	*53.3*	44.3	29.2	42.5	48.8	*52.1*	42.9	44.7	47.6
$t(2)$	0	∞	**88.6**	**85.8**	**79.5**	**77.8**	**86.4**	**89.2**	*72.1*	**89.4**	**89.4**	**84.8**	*55.3*	**78.7**	**87.3**	**89.3**	**85.9**	33.3	**88.8**
$t(1)$	0	∞	**99.8**	**99.7**	**98.4**	**97.5**	**99.7**	**99.8**	**98.6**	**99.8**	**99.8**	**99.6**	**78.8**	**98.0**	**99.7**	**99.7**	**99.7**	2.9	**99.8**
Cauchy(0;0.5)	–	–	**99.8**	**99.7**	**98.4**	**97.5**	**99.7**	**99.8**	**98.7**	**99.8**	**99.8**	**99.6**	**78.6**	**98.1**	**99.7**	**99.7**	**99.7**	2.8	**99.8**
Cauchy(0;1)	–	–	**99.8**	**99.7**	**98.4**	**97.4**	**99.6**	**99.8**	**98.6**	**99.8**	**99.8**	**99.6**	**78.9**	**98.1**	**99.7**	**99.7**	**99.7**	2.8	**99.8**
Cauchy(0;2)	–	–	**99.8**	**99.7**	**98.3**	**97.4**	**99.6**	**99.7**	**98.6**	**99.8**	**99.8**	**99.6**	**78.7**	**98.1**	**99.7**	**99.7**	**99.7**	2.9	**99.8**

Table 4. Empirical power results for symmetrical distributions ($\alpha = 0.05$ and $n = 100$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\text{TLmom}}$	$T^{(2)}_{\text{Lmom}}$	$T^{(3)}_{\text{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Beta(0.5;0.5)	0	1.50	**99.4**	**100.0**	**100.0**	**100.0**	**100.0**	**96.8**	**100.0**	**100.0**	*62.8*	**100.0**	**99.3**	**88.4**	*67.1*	0.0	0.0	**93.4**
Beta(1;1)	0	1.80	*58.7*	**95.0**	**99.9**	**99.9**	**95.2**	**99.7**	*74.4*	**95.2**	0.8	**97.7**	*59.2*	28.6	16.0	0.0	0.0	32.3
Tukey(2)	0	1.80	*58.8*	**95.0**	**99.9**	**100.0**	**95.2**	**99.7**	*74.4*	**95.3**	0.8	**97.7**	*59.4*	28.6	16.2	0.0	0.0	32.2
Tukey(0.5)	0	2.08	19.5	42.7	*70.9*	*70.5*	43.3	**75.7**	8.9	38.5	0.0	*56.8*	19.7	9.8	6.6	0.0	0.0	11.3
Beta(2;2)	0	2.14	15.0	31.9	*55.6*	*55.0*	32.3	*63.0*	4.6	26.4	0.0	44.4	15.1	7.9	5.8	0.0	0.0	9.6
Tukey(5)	0	2.90	44.8	*51.9*	*16.4*	11.3	*52.7*	0.4	0.4	0.5	6.9	22.2	*72.7*	**76.1**	*71.5*	0.0	0.0	16.0
Tukey(0.14)	0	2.97	4.9	4.9	3.9	4.2	4.9	3.9	3.8	3.9	4.0	4.7	4.9	4.9	4.9	3.4	3.4	5.0
$N(0;1)$	0	3.00	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	4.8	4.8	4.8	5.0	5.1	4.9
$t(10)$	0	4.00	10.8	16.2	23.6	19.4	16.3	26.0	30.8	29.5	31.9	19.8	8.4	6.7	6.1	28.7	28.4	5.0
Logistic(0;2)	0	4.20	15.6	24.0	29.7	25.0	24.2	33.3	39.5	38.5	42.4	29.6	12.3	8.7	7.3	35.0	34.6	5.3
Tukey(10)	0	5.38	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**89.4**	**96.4**	**97.6**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	43.0	41.0	**98.3**
Laplace(0;1)	0	6.00	*70.4*	**82.6**	*69.5*	*68.9*	**83.0**	*72.1*	**80.0**	**80.7**	**88.8**	**87.4**	*70.2*	*53.2*	41.3	*66.4*	*65.5*	13.1
$t(4)$	0	∞	49.0	*65.2*	*68.7*	*65.5*	*65.6*	*72.0*	**77.4**	**77.4**	**80.4**	*70.6*	27.6	15.0	11.0	*71.3*	*70.7*	6.9
$t(2)$	0	∞	**95.9**	**98.5**	**97.7**	**97.7**	**98.5**	**98.0**	**98.8**	**98.9**	**99.3**	**99.0**	**82.1**	*50.7*	32.5	**97.0**	**96.8**	17.1
$t(1)$	0	∞	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.7**	**91.6**	**100.0**	**100.0**	*65.2*
Cauchy(0;0.5)	–	–	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.7**	**91.6**	**100.0**	**100.0**	*65.1*
Cauchy(0;1)	–	–	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.7**	**91.5**	**100.0**	**100.0**	*65.3*
Cauchy(0;2)	–	–	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.6**	**91.4**	**100.0**	**100.0**	*65.0*

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(0.5;0.5)	0	1.50	**100.0**	**100.0**	*71.0*	7.8	**100.0**	**100.0**	**100.0**	37.0	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	0.0	**99.9**
Beta(1;1)	0	1.80	**94.2**	**90.5**	15.8	2.6	**99.6**	**96.5**	**100.0**	**95.6**	**95.0**	**99.8**	**95.9**	**99.6**	**99.3**	**100.0**	**94.3**	0.0	**93.6**
Tukey(2)	0	1.80	**94.3**	**90.5**	15.7	2.5	**99.6**	**96.5**	**100.0**	**95.5**	**95.1**	**99.8**	**95.8**	**99.6**	**99.3**	**100.0**	**94.3**	0.0	**93.6**
Tukey(0.5)	0	2.08	*59.1*	48.5	2.5	1.3	*60.4*	32.3	**88.3**	**79.6**	27.5	*69.7*	40.9	46.2	*53.6*	**86.0**	45.6	0.0	*58.5*
Beta(2;2)	0	2.14	49.0	39.2	1.7	1.2	45.7	21.1	**78.1**	*70.6*	17.4	*55.2*	30.4	33.2	39.2	**75.1**	34.3	0.0	48.6
Tukey(5)	0	2.90	38.9	31.7	1.4	27.2	37.8	25.1	*53.1*	9.8	23.6	39.1	2.6	1.8	32.8	3.1	15.2	8.4	39.4
Tukey(0.14)	0	2.97	4.7	4.7	4.3	4.0	4.3	4.1	4.8	4.4	4.1	4.3	4.1	3.8	4.2	4.4	4.8	4.2	4.8
$N(0;1)$	0	3.00	5.0	4.9	5.0	5.0	5.0	5.0	4.9	5.0	5.0	5.0	5.1	4.9	5.0	5.0	5.0	5.0	5.0
$t(10)$	0	4.00	21.1	17.1	20.6	23.2	23.1	28.0	8.4	25.7	28.6	20.4	15.7	20.8	24.7	27.1	17.1	32.6	21.3
Logistic(0;2)	0	4.20	31.7	25.6	25.3	27.8	30.7	36.9	10.7	36.4	37.6	27.1	18.4	24.7	32.7	38.1	25.1	44.1	31.7
Tukey(10)	0	5.38	**100.0**	**100.0**	**91.0**	*70.5*	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	24.3	21.6	**100.0**	**99.9**	**100.0**	4.4	**100.0**
Laplace(0;1)	0	6.00	**90.2**	**86.3**	*57.3*	*56.9*	**79.7**	**84.1**	48.8	**87.2**	**84.7**	**75.8**	36.5	*50.7*	**81.1**	**86.1**	**80.1**	*74.7*	**90.6**
$t(4)$	0	∞	*73.5*	*68.2*	*59.8*	*63.1*	*71.2*	**76.3**	44.8	**77.6**	**76.9**	*67.8*	41.8	*60.1*	*72.9*	**78.1**	*66.7*	*61.3*	*73.6*
$t(2)$	0	∞	**99.2**	**98.8**	**94.7**	**94.6**	**98.6**	**99.0**	**93.8**	**99.3**	**99.1**	**98.2**	*73.1*	**93.6**	**98.7**	**99.2**	**98.5**	24.9	**99.2**
$t(1)$	0	∞	**100.0**	**100.0**	**100.0**	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**89.3**	**99.9**	**100.0**	**100.0**	**100.0**	0.3	**100.0**
Cauchy(0;0.5)	–	–	**100.0**	**100.0**	**100.0**	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**89.2**	**99.9**	**100.0**	**100.0**	**100.0**	0.2	**100.0**
Cauchy(0;1)	–	–	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**89.3**	**99.9**	**100.0**	**100.0**	**100.0**	0.2	**100.0**
Cauchy(0;2)	–	–	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**89.3**	**99.9**	**100.0**	**100.0**	**100.0**	0.3	**100.0**

Table 5. Empirical power results for asymmetrical distributions ($\alpha = 0.05$ and $n = 25$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\text{TLmom}}$	$T^{(2)}_{\text{Lmom}}$	$T^{(3)}_{\text{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Beta(4;0.5)	−1.79	6.35	**89.5**	**97.9**	**99.2**	**99.6**	**97.7**	**76.3**	**83.4**	**95.7**	**75.1**	**98.2**	*64.2*	*62.6*	*62.2*	49.8	*51.0*	*73.8*
Beta(5;1)	−1.18	4.20	47.0	*69.3*	**78.1**	**83.3**	*68.4*	43.3	48.3	*64.9*	39.2	*70.5*	24.5	23.0	22.5	26.0	26.5	28.7
Beta(2;1)	−0.57	2.40	22.1	34.3	40.5	43.6	33.8	11.9	8.5	25.6	5.7	34.6	11.9	10.3	9.9	2.6	2.6	16.9
Weibull(3;4).	−0.09	2.75	4.8	4.6	4.0	4.0	4.6	3.2	2.8	3.2	2.8	4.1	4.5	4.6	4.6	2.5	2.5	5.1
Weibull(2;3.4)	0.05	2.71	4.5	4.3	3.6	3.5	4.3	3.0	2.4	2.8	2.4	3.6	4.4	4.6	4.6	2.3	2.3	5.2
Gamma(100;1)	0.20	3.06	5.5	5.8	6.2	6.2	5.8	6.2	6.2	5.8	6.0	5.9	4.9	4.9	4.9	6.0	5.9	5.2
Gamma(15;1)	0.52	3.40	9.4	11.4	13.5	13.7	11.3	12.4	12.9	11.2	11.7	11.6	6.4	6.3	6.1	10.6	10.5	6.1
Beta(2;5)	0.60	2.88	13.7	18.6	21.3	23.3	18.3	11.9	11.8	14.7	9.3	17.4	7.8	7.5	7.4	7.1	7.1	9.0
Weibull(1;2)	0.63	3.25	12.0	15.7	19.3	20.6	15.5	14.3	14.5	14.1	12.1	15.4	6.7	6.6	6.5	10.7	10.6	7.6
Gamma(9;1)	0.67	3.67	12.3	15.9	18.9	19.6	15.7	16.6	17.3	15.5	15.6	16.1	7.3	7.0	6.8	13.7	13.7	6.9
$\chi^2(10)$	0.89	4.20	18.5	25.5	30.4	31.9	25.1	24.7	26.1	25.0	22.9	25.6	9.6	9.1	8.9	19.2	19.2	8.6
Gamma(5;1)	0.89	4.20	18.5	25.5	30.3	31.9	25.2	24.8	26.3	25.0	23.1	25.7	9.8	9.2	8.9	19.2	19.2	8.6
Gumbel(1;2)	1.14	5.40	24.5	33.6	38.7	40.2	33.2	33.6	35.4	33.8	32.2	34.3	12.6	11.5	11.0	26.5	26.7	9.5
$\chi^2(4)$	1.14	6.00	39.8	*57.1*	*64.8*	*68.8*	*56.4*	46.2	49.8	*56.2*	43.4	*57.7*	19.4	17.9	17.3	33.3	33.7	17.4
Gamma(3;2)	1.15	5.00	27.9	40.2	46.8	49.9	39.6	35.3	37.7	39.6	32.9	40.4	13.6	12.6	12.2	26.1	26.2	11.9
Gamma(2;2)	1.41	6.00	39.6	*57.1*	*64.7*	*68.8*	*56.3*	46.3	49.9	*56.1*	43.6	*57.6*	19.4	17.8	17.2	33.4	33.8	17.5
$\chi^2(2)$	2.00	9.00	*69.4*	**87.6**	**92.3**	**94.7**	**87.0**	*68.3*	*73.6*	**85.3**	*66.0*	**88.3**	39.5	36.7	35.9	48.8	49.7	40.2
Weibull(0.5;1)	2.00	9.00	*69.4*	**87.4**	**92.2**	**94.6**	**86.8**	*68.4*	*73.6*	**85.3**	*66.0*	**88.2**	39.4	36.7	36.0	48.9	49.8	40.3
$\chi^2(1)$	2.83	15.00	**95.2**	**99.3**	**99.7**	**99.9**	**99.2**	**87.8**	**92.2**	**98.3**	**87.5**	**99.4**	*74.6*	*73.0*	*72.8*	*67.2*	*68.5*	**79.8**
LN(0;1)	6.18	113.90	**88.0**	**96.0**	**97.5**	**98.2**	**95.8**	**87.4**	**90.5**	**95.4**	**86.9**	**96.3**	*62.2*	*57.5*	*56.5*	*72.3*	*73.3*	*51.0*

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(4;0.5)	−1.79	6.35	21.3	*71.5*	**89.7**	39.4	**99.1**	**98.2**	**99.4**	**76.8**	**98.0**	**99.2**	*50.3*	**99.8**	**99.0**	14.5	**96.4**	39.7	*50.6*
Beta(5;1)	−1.18	4.20	10.8	24.6	*60.1*	18.6	**78.3**	*71.2*	**80.1**	31.5	*69.8*	**79.2**	23.7	**82.7**	**76.7**	8.0	*69.5*	34.8	18.2
Beta(2;1)	−0.57	2.40	13.7	16.7	15.4	1.4	40.8	27.8	*52.0*	6.2	25.9	42.9	2.5	49.7	37.4	15.5	34.0	6.1	11.5
Weibull(3;4).	−0.09	2.75	4.7	4.7	3.2	3.0	4.2	3.4	5.5	4.4	3.3	4.4	3.5	3.8	4.0	4.0	4.5	3.0	4.7
Weibull(2;3.4)	0.05	2.71	4.8	4.9	2.8	3.3	3.8	3.0	5.4	4.6	2.9	4.1	3.5	3.6	3.6	4.0	4.1	2.6	4.8
Gamma(100;1)	0.20	3.06	5.2	5.1	6.4	5.7	6.1	6.1	5.8	5.6	6.1	6.1	5.1	5.8	6.1	5.2	6.0	5.7	5.3
Gamma(15;1)	0.52	3.40	6.4	6.5	14.7	9.1	13.3	13.1	11.4	8.7	12.9	13.2	8.5	10.8	13.4	5.9	12.9	10.1	7.2
Beta(2;5)	0.60	2.88	7.1	8.3	18.2	11.8	21.9	17.3	24.5	7.6	16.6	22.5	25.5	18.9	20.7	5.6	20.9	8.2	7.4
Weibull(1;2)	0.63	3.25	6.8	7.4	18.7	10.4	19.2	16.9	19.0	9.3	16.5	19.5	17.6	15.8	18.7	5.4	18.0	10.2	7.7
Gamma(9;1)	0.67	3.67	7.3	7.5	20.5	11.4	18.8	18.1	16.2	11.1	17.9	18.7	11.8	14.8	18.7	6.3	18.2	13.2	8.6
$\chi^2(10)$	0.89	4.20	9.1	9.9	31.8	15.9	30.4	28.7	27.0	16.1	28.3	30.3	20.2	23.2	30.1	7.3	29.0	19.1	11.9
Gamma(5;1)	0.89	4.20	9.1	9.9	31.8	15.9	30.4	28.8	27.0	16.1	28.4	30.3	20.3	23.2	30.1	7.4	29.1	19.2	11.9
Gumbel(1;2)	1.14	5.40	12.9	13.4	41.3	20.7	38.9	37.8	33.8	23.4	37.5	38.7	21.8	29.4	38.8	11.2	37.7	25.7	17.5
$\chi^2(4)$	1.14	6.00	15.0	19.8	*61.0*	28.3	*65.1*	*60.6*	*63.7*	34.0	*59.8*	*65.5*	*60.1*	*58.2*	*64.2*	11.1	*60.7*	34.8	23.0
Gamma(3;2)	1.15	5.00	11.6	13.9	46.5	21.8	47.1	43.9	44.2	24.0	43.2	47.3	36.6	37.6	46.5	9.0	44.5	27.0	16.7
Gamma(2;2)	1.41	6.00	14.9	19.8	*60.8*	28.3	*65.0*	*60.6*	*63.5*	34.1	*59.7*	*65.5*	*60.1*	*58.1*	*64.1*	11.1	*60.6*	34.9	23.2
$\chi^2(2)$	2.00	9.00	23.6	42.7	**84.9**	42.6	**92.4**	**89.1**	**92.9**	*60.7*	**88.4**	**92.8**	**95.9**	**94.3**	**91.8**	17.2	**87.3**	44.5	41.8
Weibull(0.5;1)	2.00	9.00	23.9	42.8	**85.1**	42.7	**92.3**	**89.0**	**92.8**	*60.7*	**88.3**	**92.6**	**95.8**	**94.3**	**91.6**	17.6	**87.2**	44.3	41.7
$\chi^2(1)$	2.83	15.00	39.4	**82.2**	**97.4**	*58.0*	**99.7**	**99.4**	**99.8**	**89.0**	**99.3**	**99.8**	**100.0**	**100.0**	**99.7**	30.9	**98.6**	27.6	*70.9*
LN(0;1)	6.18	113.90	*50.7*	*67.3*	**95.4**	*61.7*	**97.6**	**96.6**	**97.5**	**84.6**	**96.3**	**97.7**	**97.6**	**97.2**	**97.4**	43.6	**96.1**	30.6	*72.5*

Table 6. Empirical power results for asymmetrical distributions ($\alpha = 0.05$ and $n = 50$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_{C}	Z_{A}	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\mathrm{TLmom}}$	$T^{(2)}_{\mathrm{Lmom}}$	$T^{(3)}_{\mathrm{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\mathrm{MC-LR}}$
Beta(4;0.5)	−1.79	6.35	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	**98.9**	**99.7**	**100.0**	**97.3**	**100.0**	**96.4**	**96.6**	**96.6**	*67.8*	*68.8*	**95.6**
Beta(5;1)	−1.18	4.20	**81.4**	**96.7**	**99.2**	**99.8**	**96.2**	**77.3**	**85.0**	**96.7**	*70.4*	**97.7**	*58.0*	*56.9*	*56.4*	35.5	36.1	*53.0*
Beta(2;1)	−0.57	2.40	45.7	*72.2*	**85.2**	**91.8**	*71.4*	29.0	20.0	*67.9*	8.5	**77.8**	30.6	25.4	23.5	0.7	0.7	29.5
Weibull(3;4).	−0.09	2.75	5.1	5.1	3.7	4.1	5.0	3.1	1.8	2.7	1.8	4.4	4.8	4.8	4.9	1.4	1.4	5.4
Weibull(2;3.4)	0.05	2.71	4.7	4.6	3.6	3.9	4.5	3.3	1.6	2.3	1.6	4.0	4.4	4.5	4.5	1.5	1.5	5.2
Gamma(100;1)	0.20	3.06	6.3	6.9	7.7	7.8	6.9	7.5	7.6	7.1	7.3	7.2	5.4	5.4	5.4	6.6	6.6	5.2
Gamma(15;1)	0.52	3.40	14.2	18.7	23.2	24.4	18.2	20.1	21.1	20.4	18.9	19.9	8.7	8.6	8.5	14.4	14.4	7.2
Beta(2;5)	0.60	2.88	25.7	39.6	49.1	*58.6*	38.6	21.9	22.3	39.8	15.7	43.0	15.0	14.2	13.8	7.1	7.0	13.4
Weibull(1;2)	0.63	3.25	20.8	31.0	41.0	47.6	30.0	25.7	26.5	33.8	21.3	33.8	11.5	11.2	11.1	14.2	14.1	10.4
Gamma(9;1)	0.67	3.67	20.4	28.6	35.5	37.9	27.9	29.2	31.0	31.8	27.4	30.8	11.7	11.5	11.3	19.7	19.7	8.9
$\chi^2(10)$	0.89	4.20	33.6	48.6	*58.1*	*62.5*	47.5	44.9	48.1	*53.8*	42.2	*51.8*	18.4	17.9	17.8	28.7	28.8	13.0
Gamma(5;1)	0.89	4.20	33.2	48.4	*57.8*	*62.3*	47.3	44.7	47.9	*53.5*	41.9	*51.6*	18.4	17.9	17.7	28.5	28.6	13.0
Gumbel(1;2)	1.14	5.40	43.8	*60.3*	*67.6*	*70.4*	*59.3*	*58.4*	*61.7*	*65.0*	*56.8*	*63.4*	24.8	23.4	22.8	40.4	40.6	14.9
$\chi^2(4)$	1.14	6.00	*69.9*	**89.1**	**94.8**	**97.1**	**88.2**	**77.3**	**82.4**	**91.6**	*73.9*	**91.3**	44.1	43.0	42.8	49.6	50.0	32.4
Gamma(3;2)	1.15	5.00	*51.4*	*72.2*	**81.4**	**86.1**	*70.8*	*62.6*	*67.1*	**77.0**	*59.1*	**75.5**	29.4	28.5	28.3	39.2	39.4	20.5
Gamma(2;2)	1.41	6.00	*69.9*	**89.3**	**94.8**	**97.1**	**88.4**	**77.4**	**82.4**	**91.8**	*73.9*	**91.4**	44.3	43.2	42.9	49.5	49.9	32.6
$\chi^2(2)$	2.00	9.00	**96.1**	**99.7**	**99.9**	**100.0**	**99.6**	**95.4**	**97.6**	**99.7**	**93.3**	**99.8**	**79.0**	**78.6**	**78.7**	*69.4*	*70.1*	*70.2*
Weibull(0.5;1)	2.00	9.00	**96.2**	**99.7**	**100.0**	**100.0**	**99.6**	**95.4**	**97.6**	**99.7**	**93.3**	**99.8**	**79.1**	**78.7**	**78.8**	*69.5*	*70.2*	*70.2*
$\chi^2(1)$	2.83	15.00	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.8**	**100.0**	**100.0**	**99.4**	**100.0**	**98.6**	**98.8**	**98.8**	**86.8**	**87.5**	**97.5**
LN(0;1)	6.18	113.90	**99.5**	**100.0**	**100.0**	**100.0**	**100.0**	**99.5**	**99.8**	**100.0**	**99.3**	**100.0**	**94.7**	**94.3**	**94.4**	**91.3**	**91.8**	**82.6**

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\text{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(4;0.5)	−1.79	6.35	30.5	**95.0**	**99.8**	*70.2*	**100.0**	**100.0**	**100.0**	**96.7**	**100.0**	**100.0**	*68.8*	**100.0**	**100.0**	16.1	**100.0**	49.4	*73.8*
Beta(5;1)	−1.18	4.20	11.5	45.6	**92.0**	38.2	**99.2**	**98.0**	**99.4**	*53.2*	**97.7**	**99.3**	33.0	**99.8**	**99.0**	8.8	**96.1**	*58.4*	24.3
Beta(2;1)	−0.57	2.40	25.7	32.3	34.1	1.0	**84.2**	*69.7*	**92.3**	6.7	*66.6*	**86.8**	0.8	**94.5**	**81.2**	40.4	*71.9*	5.0	18.3
Weibull(3;4).	−0.09	2.75	4.9	5.0	2.8	2.1	4.2	2.9	6.8	4.8	2.7	4.7	2.7	3.7	3.8	4.0	5.1	1.8	4.9
Weibull(2;3.4)	0.05	2.71	5.3	5.2	2.4	1.9	3.9	2.5	6.8	5.4	2.4	4.4	3.7	3.8	3.5	4.3	4.4	1.5	5.3
Gamma(100;1)	0.20	3.06	5.4	5.3	8.3	5.2	7.7	7.6	6.8	5.9	7.5	7.6	5.7	6.7	7.7	5.3	7.4	6.6	5.5
Gamma(15;1)	0.52	3.40	7.1	7.6	26.7	9.8	23.6	22.4	19.6	11.2	22.1	23.5	14.2	16.8	23.4	6.3	21.6	14.7	8.2
Beta(2;5)	0.60	2.88	9.1	12.3	40.4	10.0	*50.2*	39.6	*56.0*	8.2	37.7	*52.4*	*62.2*	49.3	47.7	9.3	44.6	9.9	8.4
Weibull(1;2)	0.63	3.25	7.7	9.5	39.1	10.3	41.5	35.0	42.3	11.8	33.8	42.7	43.4	35.6	40.0	6.6	35.7	14.7	8.4
Gamma(9;1)	0.67	3.67	8.6	9.6	39.7	13.4	36.2	34.0	31.2	15.7	33.4	36.2	22.7	25.1	35.9	7.1	33.2	20.9	10.6
$\chi^2(10)$	0.89	4.20	11.5	14.4	*60.7*	19.8	*59.2*	*55.1*	*54.7*	25.4	*54.3*	*59.5*	43.5	44.1	*58.4*	8.8	*54.2*	32.3	16.0
Gamma(5;1)	0.89	4.20	11.4	14.4	*60.4*	19.6	*58.9*	*55.0*	*54.4*	25.1	*54.1*	*59.3*	43.3	43.8	*58.2*	8.7	*54.0*	32.0	15.8
Gumbel(1;2)	1.14	5.40	19.3	21.8	*71.8*	29.1	*69.0*	*66.7*	*62.7*	39.4	*66.1*	*68.9*	42.3	*50.4*	*68.7*	15.8	*65.4*	43.8	27.1
$\chi^2(4)$	1.14	6.00	21.4	34.8	**91.9**	37.4	**94.9**	**92.3**	**94.8**	*56.8*	**91.7**	**95.3**	**95.8**	**93.9**	**94.4**	14.3	**90.7**	*57.1*	35.4
Gamma(3;2)	1.15	5.00	15.8	22.5	**80.0**	28.3	**82.1**	**77.8**	**80.4**	39.9	**76.8**	**82.7**	**75.4**	*72.4*	**81.2**	11.3	**76.3**	45.8	24.3
Gamma(2;2)	1.41	6.00	21.4	35.0	**91.9**	37.5	**95.0**	**92.4**	**94.9**	*56.7*	**91.8**	**95.4**	**95.8**	**93.9**	**94.5**	14.4	**90.8**	*57.1*	35.2
$\chi^2(2)$	2.00	9.00	37.5	*73.2*	**99.5**	*56.7*	**99.9**	**99.8**	**100.0**	**87.5**	**99.8**	**100.0**	**100.0**	**100.0**	**99.9**	24.6	**99.5**	*60.9*	*64.5*
Weibull(0.5;1)	2.00	9.00	37.4	*73.4*	**99.5**	*56.7*	**99.9**	**99.8**	**100.0**	**87.6**	**99.8**	**100.0**	**100.0**	**100.0**	**99.9**	24.5	**99.6**	*60.8*	*64.5*
$\chi^2(1)$	2.83	15.00	*63.0*	**98.4**	**100.0**	**76.7**	**100.0**	**100.0**	**100.0**	**99.4**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	47.6	**100.0**	21.6	**92.1**
LN(0;1)	6.18	113.90	**77.2**	**93.4**	**100.0**	**84.3**	**100.0**	**100.0**	**100.0**	**98.4**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	*67.1*	**100.0**	23.3	**93.3**

Table 7. Empirical power results for asymmetrical distributions ($\alpha = 0.05$ and $n = 100$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T_{\text{TLmom}}^{(1)}$	$T_{\text{Lmom}}^{(2)}$	$T_{\text{Lmom}}^{(3)}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Beta(4;0.5)	−1.79	6.35	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**85.5**	**84.7**	**100.0**
Beta(5;1)	−1.18	4.20	**99.4**	**100.0**	**100.0**	**100.0**	**100.0**	**99.8**	**99.9**	**100.0**	**98.7**	**100.0**	**92.3**	**91.9**	**91.6**	48.3	47.3	**86.0**
Beta(2;1)	−0.57	2.40	**81.8**	**98.3**	**99.9**	**100.0**	**98.0**	**90.0**	**84.8**	**98.5**	34.7	**99.0**	*65.6*	*56.1*	*51.8*	0.0	0.0	*56.2*
Weibull(3;4).	−0.09	2.75	5.9	6.3	4.4	5.7	6.3	4.0	1.5	3.2	1.4	5.8	5.4	5.4	5.4	0.6	0.6	5.6
Weibull(2;3.4)	0.05	2.71	4.9	5.2	4.5	5.5	5.2	4.8	1.4	2.6	1.2	5.1	4.7	4.5	4.5	1.1	1.2	5.3
Gamma(100;1)	0.20	3.06	7.7	8.9	10.3	10.5	8.7	10.0	10.2	9.8	9.8	9.4	6.1	6.1	6.1	7.6	7.5	5.6
Gamma(15;1)	0.52	3.40	23.9	33.9	42.5	45.3	32.7	36.6	38.6	41.0	35.0	38.0	14.4	14.1	14.1	20.0	19.8	10.2
Beta(2;5)	0.60	2.88	*50.0*	**75.9**	**90.5**	**96.2**	*74.2*	*56.6*	*59.4*	**83.4**	38.7	**82.3**	32.4	30.2	29.4	6.4	6.3	24.7
Weibull(1;2)	0.63	3.25	39.2	*61.1*	**79.6**	**88.2**	*59.0*	*53.2*	*56.3*	*72.5*	44.3	*68.5*	22.6	21.9	21.7	18.9	18.6	17.1
Gamma(9;1)	0.67	3.67	36.9	*53.1*	*64.1*	*68.3*	*51.5*	*54.2*	*57.2*	*62.9*	*51.4*	*58.6*	21.9	21.5	21.4	28.1	27.8	14.4
$\chi^2(10)$	0.89	4.20	*59.6*	**80.6**	**89.6**	**92.8**	**79.0**	**78.0**	**81.6**	**88.4**	*74.4*	**85.2**	37.6	36.9	36.9	41.6	41.0	24.4
Gamma(5;1)	0.89	4.20	*59.7*	**80.7**	**89.7**	**92.8**	**79.2**	**78.3**	**81.8**	**88.6**	*74.7*	**85.3**	37.8	37.3	37.1	41.5	41.0	24.5
Gumbel(1;2)	1.14	5.40	*73.2*	**89.0**	**93.4**	**94.7**	**88.0**	**88.6**	**90.7**	**93.6**	**87.2**	**91.8**	49.8	48.4	48.0	*58.6*	*57.9*	29.6
$\chi^2(4)$	1.14	6.00	**95.3**	**99.8**	**100.0**	**100.0**	**99.7**	**99.1**	**99.5**	**99.9**	**97.9**	**99.9**	**80.1**	**79.9**	**79.9**	*69.3*	*68.5*	*63.0*
Gamma(3;2)	1.15	5.00	**82.3**	**96.5**	**99.1**	**99.7**	**95.8**	**94.1**	**95.9**	**98.6**	**91.2**	**97.8**	*59.7*	*59.3*	*59.2*	*56.3*	*55.4*	41.8
Gamma(2;2)	1.41	6.00	**95.3**	**99.8**	**100.0**	**100.0**	**99.7**	**99.1**	**99.5**	**99.9**	**98.0**	**99.9**	**80.0**	**79.9**	**79.9**	*69.3*	*68.5*	*62.8*
$\chi^2(2)$	2.00	9.00	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.9**	**99.0**	**99.0**	**88.1**	**87.5**	**95.6**
Weibull(0.5;1)	2.00	9.00	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**98.9**	**98.9**	**98.9**	**88.1**	**87.5**	**95.7**
$\chi^2(1)$	2.83	15.00	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**97.7**	**97.5**	**100.0**
LN(0;1)	6.18	113.90	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.1**	**99.0**	**98.9**

	$\sqrt{\beta_1}$	β_2	T_w	$T_{MC-LR} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Beta(4;0.5)	−1.79	6.35	45.6	**99.9**	**100.0**	**93.3**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**85.4**	**100.0**	**100.0**	18.7	**100.0**	*56.9*	**93.3**
Beta(5;1)	−1.18	4.20	12.2	**80.8**	**99.9**	*65.0*	**100.0**	**100.0**	**100.0**	**80.8**	**100.0**	**100.0**	44.5	**100.0**	**100.0**	10.7	**100.0**	**84.2**	34.3
Beta(2;1)	−0.57	2.40	48.4	*62.5*	*72.5*	0.5	**99.9**	**99.2**	**100.0**	7.0	**98.9**	**99.9**	0.3	**100.0**	**99.8**	**82.2**	**97.8**	3.8	31.6
Weibull(3;4).	−0.09	2.75	5.8	5.7	2.8	1.3	5.3	3.1	9.5	6.0	2.8	6.3	2.3	4.3	4.6	5.1	6.6	1.0	5.8
Weibull(2;3.4)	0.05	2.71	6.9	6.0	2.5	1.1	4.6	2.5	9.5	7.7	2.3	5.7	5.3	4.6	3.9	6.2	5.1	0.8	6.8
Gamma(100;1)	0.20	3.06	5.4	5.5	12.3	4.7	10.6	10.2	8.8	6.3	10.1	10.5	7.1	8.1	10.5	5.3	9.9	7.4	5.6
Gamma(15;1)	0.52	3.40	7.8	10.0	49.9	12.1	44.3	41.4	37.2	14.8	40.7	44.4	25.1	26.8	43.8	6.7	39.3	21.6	9.5
Beta(2;5)	0.60	2.88	13.0	22.0	**79.1**	8.1	**89.9**	**81.8**	**92.8**	8.3	**79.8**	**91.7**	**97.0**	**92.8**	**88.3**	19.3	**80.1**	11.8	10.0
Weibull(1;2)	0.63	3.25	8.5	14.7	*73.6*	11.4	**79.1**	*71.1*	**80.6**	14.3	*69.3*	**81.1**	**85.7**	**77.2**	**77.5**	9.2	*67.0*	20.8	8.9
Gamma(9;1)	0.67	3.67	9.9	14.2	*70.4*	18.0	*65.9*	*62.0*	*59.7*	23.0	*61.1*	*66.4*	42.5	43.3	*65.2*	7.8	*59.6*	32.4	13.3
$\chi^2(10)$	0.89	4.20	14.4	24.4	**90.6**	28.8	**90.3**	**87.3**	**88.0**	40.0	**86.6**	**90.8**	**76.6**	*74.4*	**89.7**	10.2	**84.8**	*50.7*	21.8
Gamma(5;1)	0.89	4.20	14.4	24.4	**90.6**	28.8	**90.4**	**87.4**	**88.1**	40.1	**86.8**	**90.9**	**76.4**	*74.2*	**89.8**	10.1	**85.0**	*51.0*	22.1
Gumbel(1;2)	1.14	5.40	29.6	38.8	**95.5**	45.6	**94.3**	**93.1**	**91.5**	*62.3*	**92.8**	**94.4**	*68.9*	**75.1**	**94.1**	22.9	**91.8**	*66.0*	42.2
$\chi^2(4)$	1.14	6.00	31.7	*64.6*	**99.9**	*56.4*	**100.0**	**99.9**	**100.0**	**82.7**	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	18.8	**99.8**	**79.2**	*54.2*
Gamma(3;2)	1.15	5.00	21.8	42.2	**98.5**	42.5	**99.1**	**98.4**	**99.0**	*63.1*	**98.2**	**99.3**	**98.7**	**97.7**	**99.0**	13.7	**97.4**	*69.0*	36.4
Gamma(2;2)	1.41	6.00	31.5	*64.4*	**99.9**	*56.1*	**100.0**	**99.9**	**100.0**	**82.7**	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	18.6	**99.8**	**79.3**	*54.0*
$\chi^2(2)$	2.00	9.00	*57.7*	**96.8**	**100.0**	**80.2**	**100.0**	**100.0**	**100.0**	**99.1**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	35.6	**100.0**	*71.8*	**87.2**
Weibull(0.5;1)	2.00	9.00	*57.5*	**96.7**	**100.0**	**80.3**	**100.0**	**100.0**	**100.0**	**99.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	35.6	**100.0**	*71.8*	**87.2**
$\chi^2(1)$	2.83	15.00	**86.6**	**100.0**	**100.0**	**95.5**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	*69.3*	**100.0**	14.8	**99.5**
LN(0;1)	6.18	113.90	**95.6**	**99.9**	**100.0**	**98.2**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**88.9**	**100.0**	14.6	**99.6**

Table 8. Empirical power results for normal modified distributions ($\alpha = 0.05$ and $n = 25$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\mathrm{TLmom}}$	$T^{(2)}_{\mathrm{Lmom}}$	$T^{(3)}_{\mathrm{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\mathrm{MC-LR}}$
Trunc(−3;1)	−0.55	2.78	10.7	14.8	18.4	20.4	14.6	10.1	9.3	11.3	7.1	14.1	6.1	5.9	6.0	6.1	6.0	8.3
Trunc(−2;1)	−0.32	2.27	8.3	11.0	12.0	11.3	10.8	5.3	1.9	6.0	1.5	9.8	5.3	5.1	5.1	0.8	0.8	7.9
Trunc(−3;2)	−0.18	2.65	4.7	4.6	4.0	3.8	4.6	3.1	2.4	2.6	2.3	3.8	4.4	4.4	4.5	2.1	2.1	5.3
Trunc(−1;1)	0	1.94	7.5	12.2	16.1	9.9	12.1	12.1	0.1	5.7	0.1	13.9	5.6	4.0	3.9	0.0	0.0	8.7
Trunc(−2;2)	0	2.36	4.3	4.2	3.2	2.5	4.1	2.0	0.5	1.5	0.6	3.0	4.1	4.3	4.4	0.3	0.3	5.3
Trunc(−3;3)	0	2.84	4.8	4.5	3.7	3.8	4.5	3.2	3.0	3.4	3.2	4.1	4.6	4.7	4.6	2.8	2.8	5.0
LoConN(0.5;5)	0	1.51	**78.7**	**92.0**	**84.1**	*71.6*	**92.0**	*73.8*	0.8	*72.0*	0.8	**84.2**	**84.4**	*70.1*	*53.7*	0.0	0.0	*52.3*
LoConN(0.5;3)	0	2.04	15.3	19.4	15.5	10.7	19.4	13.4	0.5	8.1	0.4	17.1	14.4	9.8	7.4	0.2	0.2	13.0
LoConN(0.5;1)	0	2.92	4.9	4.8	4.6	4.6	4.8	4.4	4.3	4.4	4.3	4.6	4.6	4.6	4.7	4.3	4.3	5.0
LoConN(0.4;1)	0.04	2.93	4.9	4.8	4.6	4.7	4.8	4.5	4.4	4.4	4.4	4.6	4.6	4.7	4.6	4.4	4.4	5.1
LoConN(0.3;1)	0.06	2.96	5.0	4.9	4.7	4.8	4.9	4.7	4.6	4.6	4.6	4.8	4.7	4.7	4.8	4.5	4.5	5.0
LoConN(0.4;3)	0.23	2.14	18.0	22.4	17.2	13.6	22.2	11.1	1.6	10.6	1.4	17.6	15.7	12.2	10.8	0.6	0.5	13.8
LoConN(0.4;5)	0.32	1.65	**81.3**	**93.1**	**85.3**	**75.2**	**93.1**	*60.2*	4.1	**75.4**	3.9	**82.4**	**84.8**	*72.7*	*59.9*	0.2	0.2	*57.0*
LoConN(0.3;3)	0.46	2.47	23.8	29.0	21.9	20.6	28.8	9.4	6.7	16.6	6.5	20.6	19.2	18.3	17.6	2.4	2.4	13.9
LoConN(0.3;5)	0.67	2.13	**86.6**	**95.2**	**88.7**	**83.5**	**95.2**	38.6	21.1	**82.7**	20.9	**82.0**	**84.8**	**78.7**	*72.1*	2.2	2.4	*56.8*
ScConN(0.05;0.25)	0	3.14	5.6	5.8	5.4	5.5	5.8	5.8	6.2	6.3	6.8	6.1	6.0	6.0	5.8	6.1	6.2	4.7
ScConN(0.10;0.25)	0	3.29	7.1	7.3	6.4	6.5	7.4	7.2	7.9	8.0	9.4	8.2	8.4	8.1	7.5	7.5	7.7	4.4
ScConN(0.20;0.25)	0	3.64	12.7	13.6	9.5	10.0	13.9	11.2	12.6	13.3	16.9	15.5	16.4	14.3	12.1	11.4	11.7	4.6
ScConN(0.05;2)	0	3.97	7.0	8.7	10.9	10.7	8.8	12.0	12.5	11.9	12.2	9.7	5.2	5.0	4.9	12.6	12.7	5.0
ScConN(0.10;2)	0	4.43	8.6	11.4	15.0	14.6	11.5	16.8	17.7	16.8	17.6	13.3	5.8	5.3	5.1	17.8	18.0	5.0
ScConN(0.20;2)	0	4.68	10.6	14.8	18.6	18.5	15.0	21.4	22.9	22.2	23.5	18.1	7.0	5.8	5.4	22.4	22.8	4.9
ScConN(0.20;4)	0	9.75	*52.3*	*65.1*	*64.1*	*66.5*	*65.5*	*66.6*	*70.1*	*71.5*	*74.5*	*69.6*	26.3	13.2	9.9	*64.0*	*65.7*	6.9
ScConN(0.10;4)	0	12.75	37.1	47.5	*52.7*	*52.8*	47.8	*55.3*	*57.0*	*56.4*	*57.6*	*50.9*	12.7	7.4	6.5	*55.3*	*56.0*	5.7
ScConN(0.05;4)	0	13.55	23.6	30.5	35.5	35.1	30.6	37.4	38.2	37.2	38.0	32.4	7.4	5.6	5.4	38.2	38.4	5.3
MixN(0.5;1;0.25)	−1.02	3.87	*72.6*	**79.1**	*66.9*	*69.4*	**79.0**	39.8	45.9	*62.5*	45.1	*71.2*	*63.9*	*60.8*	*58.3*	20.9	21.5	38.8
MixN(0.4;1;0.25)	−0.78	3.34	*53.3*	*60.4*	48.8	*50.9*	*60.2*	24.6	27.5	43.1	25.3	*51.4*	45.0	42.7	40.8	12.8	13.0	31.3
MixN(0.3;1;0.25)	−0.57	3.03	32.2	37.5	30.4	31.8	37.3	14.7	14.8	24.7	12.4	31.0	25.3	23.4	22.0	7.9	8.0	21.0
MixN(0.5;3;0.25)	−0.46	1.78	**94.9**	**96.8**	**96.4**	**94.2**	**96.6**	*56.2*	8.6	**78.3**	6.4	**93.9**	**88.7**	**80.7**	*72.6*	0.7	0.7	*69.8*
MixN(0.4;3;0.25)	−0.16	1.67	**79.4**	**86.9**	**85.6**	**77.9**	**86.3**	*55.9*	1.2	*52.9*	0.8	**76.3**	*69.6*	*54.4*	43.5	0.1	0.1	*57.1*
MixN(0.3;3;0.25)	0.12	1.81	*52.6*	*68.3*	*65.0*	*51.5*	*67.2*	35.7	0.8	31.0	0.9	46.4	45.7	34.6	29.3	0.1	0.1	48.7
MixN(0.5;1;4)	0.44	5.21	47.6	*55.5*	38.3	42.7	*56.1*	38.1	42.5	46.7	*55.3*	*57.9*	46.7	33.6	25.8	32.9	34.3	10.2
MixN(0.4;1;4)	0.56	6.17	*56.6*	*67.1*	*52.0*	*57.1*	*67.7*	*51.8*	*56.6*	*60.8*	*69.0*	*70.5*	48.4	31.7	23.6	44.7	46.7	10.6
MixN(0.3;1;4)	0.7	7.58	*60.2*	*71.8*	*62.8*	*67.1*	*72.3*	*63.4*	*67.9*	*71.0*	**76.9**	**75.9**	42.1	24.8	18.1	*56.6*	*58.8*	9.6
MixN(0.5;3;4)	0.96	4.37	*65.1*	*72.2*	*56.7*	*60.0*	*72.3*	41.5	46.8	*55.1*	*50.8*	*65.6*	*59.1*	*54.2*	*50.6*	25.9	26.7	27.7
MixN(0.4;3;4)	1.21	5.29	**75.9**	**83.3**	*71.1*	*74.5*	**83.4**	*59.3*	*64.9*	*70.1*	*69.9*	**80.0**	*66.4*	*57.7*	*52.1*	38.7	40.2	27.7
MixN(0.3;3;4)	1.52	6.76	**79.1**	**86.9**	**80.5**	**83.1**	**87.0**	*74.5*	**78.9**	**80.7**	**83.2**	**87.0**	*63.2*	49.7	42.6	*54.7*	*56.6*	23.3
Nout1	0	3.00	10.0	13.9	26.1	24.6	14.2	34.4	37.3	31.5	34.9	20.5	6.2	5.4	5.2	41.0	40.9	5.2
Nout2	0	3.00	29.6	*57.1*	**91.1**	**86.2**	*57.5*	**95.8**	**96.6**	**92.9**	**94.2**	*64.7*	6.4	5.5	5.3	**98.2**	**98.1**	5.4
Nout3	0	3.00	43.2	*64.5*	*74.3*	**75.9**	*64.6*	**85.9**	**87.4**	*72.0*	**85.9**	**75.7**	17.2	8.7	7.5	**79.1**	**79.0**	6.8
Nout4	0	3.00	8.6	15.3	7.3	15.2	16.5	9.9	19.7	50.0	30.8	26.1	7.4	5.6	5.1	22.7	28.5	4.6
Nout5	0	3.00	32.3	*50.8*	10.9	31.3	*52.3*	10.2	24.2	*74.6*	*63.6*	*73.5*	29.7	8.2	5.9	12.4	18.2	4.7

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Trunc(−3;1)	−0.55	2.78	7.2	7.9	12.8	4.0	18.4	13.8	22.1	6.8	13.1	19.2	5.6	19.0	17.1	6.4	16.1	6.0	7.4
Trunc(−2;1)	−0.32	2.27	9.7	9.0	3.6	1.0	12.2	6.3	21.2	7.1	5.7	13.5	1.8	13.6	10.5	10.0	11.0	1.5	9.3
Trunc(−3;2)	−0.18	2.65	4.7	4.8	3.0	3.7	4.2	3.1	6.1	4.8	3.0	4.5	3.6	3.7	3.8	3.7	4.5	2.4	4.9
Trunc(−1;1)	0	1.94	19.3	15.5	0.7	2.5	14.1	4.8	30.8	13.6	3.9	16.6	11.3	11.8	11.1	25.0	9.6	0.2	18.7
Trunc(−2;2)	0	2.36	5.7	5.3	1.1	3.0	3.5	1.7	7.2	6.0	1.5	4.0	3.3	3.1	2.9	4.5	3.7	0.8	5.8
Trunc(−3;3)	0	2.84	4.5	4.4	3.5	4.1	4.0	3.5	4.8	4.2	3.5	4.1	3.9	3.6	3.8	4.1	4.3	3.5	4.4
LoConN(0.5;5)	0	1.51	**88.4**	**89.7**	5.1	8.3	**87.4**	*71.3*	**94.0**	1.5	*67.4*	**89.7**	27.4	35.9	**84.7**	*73.4*	**84.6**	2.1	*64.0*
LoConN(0.5;3)	0	2.04	30.0	25.3	1.6	2.8	16.4	7.4	28.5	8.6	6.4	18.9	8.8	9.6	13.9	19.6	14.6	0.6	26.5
LoConN(0.5;1)	0	2.92	4.8	4.8	4.4	4.6	4.6	4.4	5.0	4.9	4.4	4.7	4.7	4.5	4.5	4.6	4.7	4.3	4.8
LoConN(0.4;1)	0.04	2.93	4.9	4.8	4.5	4.8	4.7	4.5	5.0	4.9	4.5	4.7	4.7	4.5	4.7	4.8	4.8	4.4	4.9
LoConN(0.3;1)	0.06	2.96	4.9	4.8	4.9	5.2	4.8	4.7	5.0	4.9	4.7	4.8	4.8	4.7	4.7	4.8	4.9	4.6	4.9
LoConN(0.4;3)	0.23	2.14	23.8	21.3	5.1	10.7	19.1	10.4	29.4	7.1	9.3	21.2	14.9	10.9	16.7	15.8	19.2	1.9	19.8
LoConN(0.4;5)	0.32	1.65	*73.8*	**80.2**	19.3	33.2	**89.1**	**75.8**	**94.3**	5.0	*72.5*	**90.9**	45.5	39.6	**86.7**	*59.5*	**87.8**	7.2	46.7
LoConN(0.3;3)	0.46	2.47	11.8	13.7	15.1	24.2	25.7	18.6	31.2	6.3	17.6	26.8	18.8	13.3	23.8	8.5	29.6	7.3	9.9
LoConN(0.3;5)	0.67	2.13	39.6	*58.4*	*51.8*	*68.1*	**92.5**	**85.0**	**94.9**	21.5	**83.1**	**93.4**	*55.2*	45.7	**91.3**	29.7	**93.7**	25.8	27.2
ScConN(0.05;0.25)	0	3.14	5.6	5.0	6.0	6.2	5.6	6.1	4.6	5.6	6.2	5.4	5.3	5.3	5.7	6.2	5.7	6.8	5.5
ScConN(0.10;0.25)	0	3.29	7.4	6.1	7.3	7.8	6.7	7.9	4.5	6.8	8.1	6.3	5.8	6.0	7.0	8.0	6.9	9.2	7.3
ScConN(0.20;0.25)	0	3.64	14.5	10.9	10.9	12.1	11.1	14.0	5.8	11.8	14.3	10.2	7.7	8.5	12.0	14.6	11.5	16.5	14.5
ScConN(0.05;2)	0	3.97	8.7	7.7	11.4	9.6	10.2	11.4	7.3	10.0	11.5	10.0	8.9	10.9	10.6	9.8	9.1	10.9	8.7
ScConN(0.10;2)	0	4.43	11.5	9.9	15.7	12.5	14.0	16.1	8.9	13.7	16.3	13.5	11.6	14.8	14.7	13.5	12.0	15.4	11.8
ScConN(0.20;2)	0	4.68	15.1	12.3	19.9	15.0	17.6	21.1	10.2	17.8	21.5	17.0	14.4	18.3	18.8	18.2	15.7	20.8	15.5
ScConN(0.20;4)	0	9.75	*64.3*	*58.8*	*60.9*	49.2	*66.5*	*72.0*	47.4	*69.5*	*72.5*	*65.0*	42.7	*60.6*	*68.4*	*66.4*	*65.5*	43.7	*66.4*
ScConN(0.10;4)	0	12.75	47.4	43.5	*51.5*	43.4	*51.9*	*55.6*	39.5	*52.8*	*55.9*	*51.1*	35.0	*51.7*	*53.2*	49.8	48.9	33.3	48.5
ScConN(0.05;4)	0	13.55	30.5	28.0	35.4	30.9	34.3	36.6	26.9	34.3	36.8	33.9	23.4	35.4	35.2	32.4	31.4	22.9	31.0
MixN(0.5;1;0.25)	−1.02	3.87	16.2	35.1	*58.3*	13.4	*74.2*	*72.2*	*68.7*	40.3	*71.5*	*74.2*	20.7	42.9	*74.1*	14.2	**76.3**	37.7	32.3
MixN(0.4;1;0.25)	−0.78	3.34	13.1	26.8	38.1	7.4	*55.4*	*51.3*	*52.5*	20.9	*50.3*	*55.9*	12.1	31.9	*54.8*	9.6	*56.6*	26.1	18.3
MixN(0.3;1;0.25)	−0.57	3.03	11.7	18.8	21.2	4.3	34.5	29.9	34.7	10.0	28.9	35.3	7.3	21.5	33.5	7.6	34.5	14.4	12.0
MixN(0.5;3;0.25)	−0.46	1.78	**81.8**	**91.6**	13.3	1.0	**96.8**	**91.1**	**98.6**	14.5	**89.5**	**97.5**	4.0	**87.2**	**96.0**	*65.9*	**84.5**	7.0	*55.9*
MixN(0.4;3;0.25)	−0.16	1.67	*70.7*	**78.4**	3.0	5.5	**86.6**	*72.2*	**93.0**	2.4	*68.7*	**88.6**	8.2	*63.9*	**84.0**	*70.4*	*63.6*	2.1	*56.5*
MixN(0.3;3;0.25)	0.12	1.81	39.3	*53.3*	4.2	20.2	*67.0*	47.3	**78.7**	1.4	43.6	*69.9*	13.7	33.7	*62.8*	*50.1*	45.6	1.7	30.6
MixN(0.5;1;4)	0.44	5.21	46.8	41.0	37.3	39.1	47.2	*54.1*	27.9	48.8	*54.7*	44.6	19.5	30.0	49.4	46.8	49.1	43.4	*53.2*
MixN(0.4;1;4)	0.56	6.17	*58.6*	*52.5*	48.8	44.3	*60.8*	*67.7*	39.1	*63.4*	*68.3*	*58.3*	25.8	42.3	*63.1*	*58.8*	*62.7*	47.6	*65.0*
MixN(0.3;1;4)	0.7	7.58	*65.7*	*60.1*	*59.0*	47.6	*69.1*	**75.1**	47.8	*71.9*	**75.7**	*67.0*	31.6	*54.4*	*71.1*	*66.3*	*70.1*	47.4	*71.2*
MixN(0.5;3;4)	0.96	4.37	22.5	32.0	*56.6*	46.7	*65.5*	*66.5*	*54.9*	44.6	*66.3*	*64.6*	27.7	35.0	*66.2*	23.2	*69.0*	40.7	39.8
MixN(0.4;3;4)	1.21	5.29	36.1	43.6	*70.7*	*51.6*	**78.7**	**80.7**	*67.1*	*64.3*	**80.7**	**77.6**	32.2	47.3	**79.5**	35.7	**81.4**	45.3	*59.1*
MixN(0.3;3;4)	1.52	6.76	*53.6*	*56.2*	**79.4**	*53.5*	**85.2**	**87.4**	*74.2*	**78.5**	**87.5**	**84.2**	34.6	*60.4*	**86.1**	*50.7*	**86.8**	43.9	*73.9*
Nout1	0	3.00	14.7	10.8	29.4	5.9	21.1	28.6	8.0	19.5	29.4	20.0	6.1	26.1	23.7	21.1	15.2	30.2	14.8
Nout2	0	3.00	*59.3*	47.8	**89.5**	*59.5*	**83.8**	**91.2**	*50.8*	**82.6**	**91.6**	**82.7**	7.7	**93.1**	**86.8**	*66.4*	*62.5*	*74.5*	*61.3*
Nout3	0	3.00	46.0	35.1	**84.1**	7.5	*73.7*	**80.0**	49.7	*67.3*	**80.5**	*71.2*	12.5	*58.5*	**76.0**	44.1	*72.4*	*69.3*	53.9
Nout4	0	3.00	31.4	22.4	3.8	6.2	10.8	24.2	0.8	29.8	26.1	9.3	20.6	16.0	14.5	*57.9*	13.8	42.2	28.7
Nout5	0	3.00	*74.6*	*62.1*	5.7	35.5	29.7	*50.6*	3.2	*64.7*	*53.1*	24.4	23.0	15.6	35.8	**91.2**	43.4	*72.1*	*70.9*

Table 9. Empirical power results for normal modified distributions ($\alpha = 0.05$ and $n = 50$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\text{TLmom}}$	$T^{(2)}_{\text{Lmom}}$	$T^{(3)}_{\text{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Trunc(−3;1)	−0.55	2.78	19.3	31.5	45.8	*58.1*	30.5	18.4	16.6	31.2	11.1	35.7	10.5	9.6	9.4	5.6	5.5	11.2
Trunc(−2;1)	−0.32	2.27	14.5	24.8	35.6	41.2	24.4	16.2	2.0	19.1	1.1	28.7	9.3	7.5	7.1	0.1	0.1	10.1
Trunc(−3;2)	−0.18	2.65	5.2	5.4	4.7	5.4	5.4	4.0	1.9	2.8	1.8	5.0	4.5	4.6	4.7	1.1	1.1	5.4
Trunc(−1;1)	0	1.94	14.0	31.0	*53.6*	48.5	31.3	*51.7*	0.1	20.3	0.0	42.0	12.7	6.5	4.9	0.0	0.0	10.9
Trunc(−2;2)	0	2.36	4.7	5.5	5.3	4.5	5.5	5.9	0.1	1.8	0.1	5.2	4.4	4.3	4.3	0.0	0.0	5.4
Trunc(−3;3)	0	2.84	4.7	4.4	2.9	3.4	4.4	2.4	1.8	2.4	2.0	3.9	4.7	4.7	4.7	1.2	1.3	5.0
LoConN(0.5;5)	0	1.51	**99.3**	**100.0**	**99.6**	**98.9**	**100.0**	**99.4**	43.6	**98.1**	0.2	**99.9**	**99.9**	**99.5**	**98.1**	0.0	0.0	*67.7*
LoConN(0.5;3)	0	2.04	31.7	44.4	34.4	29.8	44.8	43.4	0.3	20.3	0.1	47.4	39.0	27.8	20.4	0.1	0.1	17.9
LoConN(0.5;1)	0	2.92	4.9	4.8	4.4	4.6	4.8	4.3	3.9	4.1	3.9	4.6	4.6	4.7	4.7	3.9	3.9	5.1
LoConN(0.4;1)	0.04	2.93	5.0	4.9	4.6	4.8	4.9	4.5	4.1	4.3	4.0	4.7	4.7	4.7	4.7	4.1	4.0	5.1
LoConN(0.3;1)	0.06	2.96	5.1	5.1	4.9	5.0	5.1	4.8	4.6	4.6	4.5	5.0	4.8	4.8	4.8	4.5	4.5	5.1
LoConN(0.4;3)	0.23	2.14	37.0	49.2	37.0	33.8	49.4	33.9	1.9	26.4	0.9	46.2	40.6	31.6	26.2	0.2	0.2	22.7
LoConN(0.4;5)	0.32	1.65	**99.4**	**100.0**	**99.6**	**99.2**	**100.0**	**96.5**	*51.0*	**98.6**	3.3	**99.8**	**99.9**	**99.4**	**98.1**	0.0	0.0	**83.9**
LoConN(0.3;3)	0.46	2.47	47.7	*59.3*	43.9	43.8	*59.1*	19.1	11.4	40.1	8.6	48.3	45.2	42.6	41.3	1.0	1.0	25.4
LoConN(0.3;5)	0.67	2.13	**99.7**	**100.0**	**99.8**	**99.6**	**100.0**	**87.2**	*74.4*	**99.4**	31.8	**99.7**	**99.9**	**99.6**	**99.1**	0.3	0.3	**91.7**
ScConN(0.05;0.25)	0	3.14	6.1	6.2	5.6	5.3	6.2	5.9	6.7	6.5	7.7	6.6	6.8	6.8	6.6	6.4	6.4	4.6
ScConN(0.10;0.25)	0	3.29	8.9	9.1	6.7	6.3	9.2	7.4	9.0	8.8	11.7	10.1	11.2	10.8	10.2	8.0	8.1	4.6
ScConN(0.20;0.25)	0	3.64	20.0	21.6	11.5	11.0	22.0	13.0	16.2	16.5	24.4	23.9	27.6	24.8	21.2	13.0	13.3	5.9
ScConN(0.05;2)	0	3.97	7.6	10.3	16.3	14.6	10.3	17.3	18.7	17.9	18.3	11.7	5.4	5.1	5.0	19.3	19.3	5.0
ScConN(0.10;2)	0	4.43	10.1	15.0	23.5	21.2	15.1	25.2	27.9	26.9	27.6	17.6	6.4	5.6	5.4	27.9	28.1	5.0
ScConN(0.20;2)	0	4.68	13.7	21.4	29.1	26.7	21.6	32.1	36.4	35.8	37.4	25.6	8.8	6.6	6.0	34.6	35.0	5.1
ScConN(0.20;4)	0	9.75	**78.0**	**89.0**	**89.2**	**89.7**	**89.2**	**89.6**	**92.9**	**93.4**	**94.5**	**90.8**	46.5	21.3	14.3	**86.6**	**87.7**	8.9
ScConN(0.10;4)	0	12.75	*56.7*	*70.2*	**78.0**	**76.6**	*70.4*	**79.4**	**81.6**	**81.2**	**81.5**	*71.9*	18.9	9.4	7.8	**80.4**	**80.7**	5.9
ScConN(0.05;4)	0	13.55	35.3	47.1	*57.5*	*55.3*	47.2	*58.7*	*60.3*	*59.5*	*59.8*	48.3	9.1	6.2	5.7	*60.7*	*60.8*	5.2
MixN(0.5;1;0.25)	−1.02	3.87	**96.4**	**98.2**	**89.5**	**91.6**	**98.2**	*69.5*	**76.8**	**84.7**	*72.2*	**94.5**	**95.7**	**95.5**	**95.2**	24.6	25.2	*74.3*
MixN(0.4;1;0.25)	−0.78	3.34	**85.3**	**90.3**	*72.7*	**75.7**	**90.3**	44.6	*51.5*	*68.3*	42.6	**81.5**	**83.3**	**83.2**	**82.5**	14.1	14.3	*60.7*
MixN(0.3;1;0.25)	−0.57	3.03	*60.5*	*68.0*	48.4	*51.4*	*67.9*	24.9	26.5	44.6	19.0	*57.6*	*56.4*	*55.8*	*54.2*	8.5	8.5	37.3
MixN(0.5;3;0.25)	−0.46	1.78	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**94.6**	*62.8*	**97.7**	6.4	**99.9**	**99.8**	**98.9**	**96.9**	0.2	0.2	*73.4*
MixN(0.4;3;0.25)	−0.16	1.67	**99.2**	**99.8**	**99.7**	**99.6**	**99.8**	**92.3**	23.8	**85.1**	0.2	**97.4**	**96.7**	**88.7**	**77.7**	0.0	0.0	*69.6*
MixN(0.3;3;0.25)	0.12	1.81	**90.9**	**97.2**	**96.2**	**93.7**	**96.8**	**75.5**	6.7	*61.5*	0.3	**79.9**	**82.7**	*69.0*	*58.7*	0.0	0.0	**78.5**
MixN(0.5;1;4)	0.44	5.21	**77.8**	**85.9**	*59.6*	*64.6*	**86.3**	*55.7*	*64.3*	*65.8*	**81.5**	**86.1**	**81.5**	*65.9*	*52.6*	43.3	44.8	21.9
MixN(0.4;1;4)	0.56	6.17	**85.8**	**93.3**	**77.6**	**82.0**	**93.5**	*73.6*	**81.1**	**82.4**	**92.4**	**94.3**	**82.0**	*60.6*	45.8	*60.0*	*61.9*	22.4
MixN(0.3;1;4)	0.7	7.58	**87.3**	**94.6**	**88.4**	**90.7**	**94.8**	**86.5**	**91.6**	**92.3**	**96.4**	**96.0**	*72.9*	46.4	33.3	**76.4**	**78.2**	17.8
MixN(0.5;3;4)	0.96	4.37	**92.7**	**96.0**	**80.8**	**84.2**	**96.0**	*67.4*	*73.3*	**75.3**	**77.6**	**90.7**	**93.1**	**91.5**	**90.0**	31.9	32.8	*56.6*
MixN(0.4;3;4)	1.21	5.29	**97.1**	**98.9**	**92.6**	**94.5**	**98.9**	**85.8**	**89.3**	**88.6**	**93.3**	**97.6**	**95.8**	**92.2**	**89.2**	*50.0*	*51.4*	*57.4*
MixN(0.3;3;4)	1.52	6.76	**97.5**	**99.2**	**97.3**	**98.1**	**99.2**	**95.4**	**96.8**	**96.0**	**98.6**	**99.2**	**92.4**	**83.5**	**77.6**	*71.0*	*72.6*	49.3
Nout1	0	3.00	7.6	9.7	21.1	17.1	9.8	25.4	31.1	26.4	28.7	14.1	5.7	5.2	5.1	36.7	36.7	5.0
Nout2	0	3.00	19.2	40.1	**97.7**	**89.6**	40.5	**98.3**	**99.1**	**98.0**	**97.3**	46.5	5.7	5.3	5.1	**99.9**	**99.9**	5.1
Nout3	0	3.00	34.6	*62.8*	**93.1**	**88.1**	*62.3*	**97.3**	**98.1**	**94.5**	**96.9**	*72.8*	10.3	7.0	6.5	**98.4**	**98.4**	5.7
Nout4	0	3.00	7.5	13.6	10.4	15.3	14.5	12.7	29.7	*50.1*	33.8	20.3	6.7	5.4	5.1	34.0	35.0	4.8
Nout5	0	3.00	42.2	**78.6**	*67.0*	**80.9**	**80.2**	*74.8*	**94.5**	**99.0**	**96.0**	**86.7**	17.0	7.0	5.7	**92.7**	**93.6**	5.1

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Trunc(−3;1)	−0.55	2.78	9.8	11.5	27.5	7.1	45.1	33.1	*53.9*	7.8	31.1	48.0	4.9	*57.8*	42.1	12.2	34.7	6.3	9.0
Trunc(−2;1)	−0.32	2.27	18.6	16.0	6.1	0.3	33.6	17.3	*55.3*	14.8	15.1	38.3	1.2	46.1	29.0	28.7	26.5	0.5	16.9
Trunc(−3;2)	−0.18	2.65	5.6	5.3	3.2	3.3	5.2	3.2	9.3	6.3	3.0	5.9	2.5	4.6	4.6	4.9	5.6	1.3	5.7
Trunc(−1;1)	0	1.94	42.9	34.3	1.1	1.8	44.7	19.2	**75.7**	*50.7*	15.7	*52.1*	33.7	39.0	37.8	*69.8*	29.3	0.0	41.4
Trunc(−2;2)	0	2.36	9.1	7.4	0.7	2.0	5.1	1.7	14.5	12.6	1.4	6.7	4.8	4.4	4.0	10.0	5.1	0.1	9.1
Trunc(−3;3)	0	2.84	4.2	4.4	2.8	2.9	3.5	2.7	5.2	3.8	2.6	3.8	3.2	2.9	3.2	3.5	4.3	2.3	4.3
LoConN(0.5;5)	0	1.51	**99.5**	**99.8**	16.6	7.4	**99.9**	**99.3**	**100.0**	0.8	**99.0**	**99.9**	46.3	*61.2*	**99.8**	**98.4**	**99.9**	0.3	**90.1**
LoConN(0.5;3)	0	2.04	*62.4*	*54.3*	2.0	1.7	37.7	20.7	*54.8*	26.6	17.9	43.2	14.5	16.6	33.7	46.8	38.3	0.1	*57.2*
LoConN(0.5;1)	0	2.92	4.9	4.8	4.2	4.2	4.6	4.1	5.4	4.8	4.1	4.7	4.6	4.5	4.4	4.5	4.7	3.8	4.9
LoConN(0.4;1)	0.04	2.93	5.0	5.0	4.4	4.4	4.8	4.4	5.3	4.9	4.3	4.9	4.9	4.6	4.6	4.6	4.9	4.0	4.9
LoConN(0.3;1)	0.06	2.96	5.0	4.8	4.8	4.7	5.0	4.8	5.3	5.0	4.7	5.1	5.0	4.8	4.9	4.8	5.1	4.5	4.9
LoConN(0.4;3)	0.23	2.14	47.5	43.7	11.1	12.1	42.1	26.3	*56.5*	17.0	23.6	46.8	24.5	18.7	38.5	36.8	45.8	0.7	38.0
LoConN(0.4;5)	0.32	1.65	**92.4**	**98.2**	*59.2*	46.1	**99.9**	**99.5**	**100.0**	3.9	**99.3**	**99.9**	*68.3*	*64.3*	**99.8**	**90.2**	**99.9**	3.8	*61.7*
LoConN(0.3;3)	0.46	2.47	17.8	24.7	35.3	33.6	*52.9*	41.2	*60.0*	6.9	38.9	*55.5*	30.2	21.9	*50.2*	15.7	*61.3*	6.6	11.9
LoConN(0.3;5)	0.67	2.13	*52.2*	**88.7**	**93.2**	**89.5**	**100.0**	**99.8**	**100.0**	30.3	**99.8**	**100.0**	**76.6**	*68.5*	**99.9**	*51.6*	**100.0**	31.3	27.8
ScConN(0.05;0.25)	0	3.14	6.4	5.5	6.0	6.6	5.8	6.5	4.3	5.8	6.6	5.4	5.2	5.2	6.0	6.5	5.9	7.9	6.3
ScConN(0.10;0.25)	0	3.29	10.5	8.2	7.5	8.6	7.7	9.3	4.3	8.2	9.5	6.9	5.5	5.7	8.1	9.6	7.8	12.0	10.4
ScConN(0.20;0.25)	0	3.64	26.4	20.1	12.0	14.7	16.0	19.7	6.6	18.8	20.2	13.8	7.7	8.3	17.0	20.7	16.3	25.2	26.6
ScConN(0.05;2)	0	3.97	11.7	10.1	15.8	15.9	14.8	17.1	8.3	14.6	17.3	14.1	12.2	16.1	15.6	14.2	10.7	16.9	11.6
ScConN(0.10;2)	0	4.43	17.1	14.4	22.3	22.2	21.7	25.5	10.6	21.9	25.9	20.3	16.6	22.6	23.0	21.7	15.8	25.8	17.4
ScConN(0.20;2)	0	4.68	24.9	20.3	27.3	26.3	28.0	33.8	12.3	30.3	34.4	25.9	20.0	26.7	30.0	31.3	22.5	36.2	24.9
ScConN(0.20;4)	0	9.75	**90.6**	**88.1**	**80.3**	**75.7**	**91.1**	**93.5**	*74.6*	**93.3**	**93.7**	**89.7**	*59.3*	**79.5**	**91.9**	**92.3**	**89.8**	*51.9*	**91.0**
ScConN(0.10;4)	0	12.75	*72.9*	*69.6*	*73.0*	*73.2*	**77.1**	**80.2**	*61.3*	**78.4**	**80.4**	**75.9**	*53.7*	**75.5**	**78.2**	**76.8**	*71.5*	44.5	*73.5*
ScConN(0.05;4)	0	13.55	49.9	46.5	*54.4*	*55.4*	*55.6*	*58.6*	43.1	*55.8*	*58.8*	*54.6*	38.1	*57.0*	*56.7*	*54.0*	48.2	34.3	*50.1*
MixN(0.5;1;0.25)	−1.02	3.87	18.0	*68.1*	**86.9**	23.5	**96.0**	**95.8**	**91.3**	*65.3*	**95.7**	**95.8**	24.4	45.8	**96.1**	16.1	**96.9**	*59.3*	47.5
MixN(0.4;1;0.25)	−0.78	3.34	15.7	*53.9*	*66.2*	12.6	**84.1**	**82.6**	**77.0**	33.6	**81.9**	**84.1**	13.6	33.9	**84.2**	9.9	**86.2**	42.5	21.9
MixN(0.3;1;0.25)	−0.57	3.03	17.0	34.5	39.3	7.3	*59.7*	*55.7*	*54.3*	12.6	*54.5*	*60.4*	7.9	23.8	*59.4*	8.6	*62.0*	20.2	13.7
MixN(0.5;3;0.25)	−0.46	1.78	**98.7**	**99.8**	23.8	0.3	**100.0**	**100.0**	**100.0**	18.2	**100.0**	**100.0**	3.4	**99.3**	**100.0**	**93.1**	**99.6**	7.3	*74.9*
MixN(0.4;3;0.25)	−0.16	1.67	**94.2**	**98.5**	5.7	5.0	**99.8**	**99.2**	**99.9**	1.3	**98.9**	**99.9**	10.2	**93.0**	**99.8**	**97.2**	**95.8**	0.7	**86.2**
MixN(0.3;3;0.25)	0.12	1.81	*62.9*	**82.0**	14.7	32.2	**97.4**	**92.3**	**98.8**	1.1	**90.7**	**98.0**	19.0	*64.9*	**96.6**	**83.9**	**85.4**	0.3	46.8
MixN(0.5;1;4)	0.44	5.21	**78.0**	*74.5*	49.2	49.3	**77.1**	**81.7**	*50.3*	**80.0**	**82.2**	*73.5*	19.9	33.1	*78.5*	*72.3*	**79.4**	*64.8*	**85.0**
MixN(0.4;1;4)	0.56	6.17	**88.9**	**86.6**	*63.7*	*56.8*	**89.4**	**92.2**	*67.4*	**91.7**	**92.5**	**87.0**	28.3	49.3	**90.3**	**85.7**	**90.9**	*61.9*	**93.3**
MixN(0.3;1;4)	0.7	7.58	**93.3**	**91.4**	**76.0**	*65.5*	**93.8**	**95.8**	**77.7**	**95.7**	**95.9**	**92.4**	38.4	*67.1*	**94.5**	**92.1**	**94.5**	*55.7*	**95.3**
MixN(0.5;3;4)	0.96	4.37	31.0	*59.5*	**79.4**	49.5	**91.6**	**92.3**	**81.3**	*72.4*	**92.3**	**90.8**	24.9	35.7	**92.0**	32.6	**93.7**	*62.0*	*63.6*
MixN(0.4;3;4)	1.21	5.29	*56.0*	**75.4**	**89.1**	*54.9*	**97.6**	**98.1**	**91.7**	**91.1**	**98.1**	**97.2**	30.5	*51.4*	**97.8**	*53.0*	**98.4**	*57.7*	**86.5**
MixN(0.3;3;4)	1.52	6.76	**81.1**	**88.4**	**93.8**	*64.3*	**99.0**	**99.3**	**95.5**	**97.8**	**99.3**	**98.7**	37.0	*69.5*	**99.1**	*73.8*	**99.2**	46.7	**96.2**
Nout1	0	3.00	11.2	8.8	21.3	5.5	16.0	23.1	4.5	14.8	23.9	14.1	5.7	20.3	18.3	18.1	10.4	27.0	11.3
Nout2	0	3.00	*54.6*	43.1	**89.6**	**91.0**	**91.1**	**96.6**	42.9	**86.9**	**96.9**	**89.3**	6.8	**98.9**	**93.6**	*69.1*	42.7	**87.4**	*55.2*
Nout3	0	3.00	*59.2*	47.9	**92.2**	*68.7*	**88.8**	**94.0**	*53.5*	**85.6**	**94.4**	**86.0**	10.1	**91.3**	**90.9**	*68.5*	*69.8*	**88.2**	*62.3*
Nout4	0	3.00	28.0	20.3	3.5	4.8	12.9	28.4	0.5	35.1	30.5	9.9	20.7	13.3	17.3	*57.8*	14.2	*51.5*	27.0
Nout5	0	3.00	**91.9**	**85.7**	31.1	21.6	**78.2**	**92.5**	13.3	**97.1**	**93.5**	*70.5*	**77.8**	*65.8*	**84.0**	**99.5**	**83.4**	**95.7**	**91.0**

Table 10. Empirical power results for normal modified distributions ($\alpha = 0.05$ and $n = 100$).

Distribution	$\sqrt{\beta_1}$	β_2	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\mathrm{TLmom}}$	$T^{(2)}_{\mathrm{Lmom}}$	$T^{(3)}_{\mathrm{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\mathrm{MC-LR}}$
Trunc(−3;1)	−0.55	2.78	38.2	*67.3*	**91.0**	**97.6**	*65.1*	47.4	47.6	*74.9*	28.1	*74.7*	21.2	18.5	17.7	3.9	3.8	18.9
Trunc(−2;1)	−0.32	2.27	30.3	*58.5*	**86.7**	**93.3**	*57.6*	*55.1*	19.7	*62.0*	2.4	*67.2*	19.7	14.1	12.6	0.0	0.0	15.6
Trunc(−3;2)	−0.18	2.65	6.3	7.6	9.2	13.0	7.5	7.1	2.2	5.4	1.8	8.3	5.0	5.0	5.0	0.2	0.2	5.6
Trunc(−1;1)	0	1.94	31.1	*72.0*	**97.3**	**97.7**	*72.5*	**94.9**	32.4	*71.9*	0.0	**82.6**	29.8	12.8	7.7	0.0	0.0	15.8
Trunc(−2;2)	0	2.36	6.0	9.1	16.8	16.9	9.2	19.7	0.3	5.0	0.0	11.4	5.2	4.4	4.3	0.0	0.0	5.4
Trunc(−3;3)	0	2.84	4.7	4.5	2.6	3.5	4.5	2.3	1.1	1.9	1.3	3.9	4.6	4.7	4.7	0.2	0.2	4.9
LoConN(0.5;5)	0	1.51	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**97.3**	**100.0**	**100.0**	**84.5**	**100.0**	**100.0**	**100.0**	**100.0**	0.0	0.0	**82.3**
LoConN(0.5;3)	0	2.04	*64.2*	**81.9**	*69.4*	*66.5*	**82.4**	**82.5**	22.7	*56.6*	0.1	**86.6**	**78.0**	*64.0*	*51.5*	0.0	0.0	28.3
LoConN(0.5;1)	0	2.92	4.8	4.8	4.4	4.9	4.9	4.4	3.6	3.9	3.5	4.7	4.7	4.6	4.7	3.6	3.6	5.0
LoConN(0.4;1)	0.04	2.93	5.0	4.9	4.5	4.9	4.9	4.5	3.8	4.1	3.7	4.7	4.8	4.8	4.8	3.8	3.8	5.1
LoConN(0.3;1)	0.06	2.96	5.3	5.3	5.0	5.4	5.2	4.9	4.6	4.8	4.6	5.2	5.0	5.0	5.0	4.3	4.3	5.1
LoConN(0.4;3)	0.23	2.14	*70.7*	**85.5**	*72.1*	*70.3*	**85.8**	*74.2*	27.7	*65.0*	1.4	**85.4**	**79.2**	*67.8*	*58.7*	0.0	0.0	44.8
LoConN(0.4;5)	0.32	1.65	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.4**	**100.0**	**100.0**	*61.1*	**100.0**	**100.0**	**100.0**	**100.0**	0.0	0.0	**97.1**
LoConN(0.3;3)	0.46	2.47	**81.5**	**91.5**	**78.6**	**78.5**	**91.4**	*58.3*	47.9	**79.0**	20.7	**86.0**	**82.3**	**79.2**	**77.7**	0.4	0.4	*52.8*
LoConN(0.3;5)	0.67	2.13	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**85.4**	**100.0**	**100.0**	**100.0**	**100.0**	0.0	0.0	**99.9**
ScConN(0.05;0.25)	0	3.14	6.8	7.0	5.7	5.1	7.0	6.1	7.4	7.0	9.1	7.5	8.1	8.2	8.0	6.7	6.7	4.6
ScConN(0.10;0.25)	0	3.29	12.4	12.8	7.3	6.2	13.0	8.4	11.0	10.2	15.6	14.0	16.9	16.4	15.3	8.8	8.7	5.3
ScConN(0.20;0.25)	0	3.64	35.4	39.2	15.2	13.9	39.7	17.0	22.9	22.4	38.1	41.0	49.2	45.4	39.5	15.1	14.8	10.2
ScConN(0.05;2)	0	3.97	8.3	12.5	24.9	20.6	12.6	25.6	28.4	27.3	27.6	14.3	5.6	5.2	5.1	29.5	29.4	5.0
ScConN(0.10;2)	0	4.43	12.2	20.4	36.1	30.4	20.5	37.9	42.7	41.6	42.2	23.6	7.2	5.9	5.6	42.4	42.0	5.1
ScConN(0.20;2)	0	4.68	19.6	33.0	44.8	39.3	33.2	48.6	*55.7*	*55.0*	*56.8*	38.3	11.5	7.6	6.6	*50.9*	*50.2*	5.4
ScConN(0.20;4)	0	9.75	**96.1**	**99.1**	**99.2**	**99.2**	**99.1**	**99.3**	**99.6**	**99.7**	**99.7**	**99.2**	*73.2*	34.1	19.6	**98.0**	**97.8**	12.9
ScConN(0.10;4)	0	12.75	**79.5**	**90.2**	**95.0**	**94.0**	**90.3**	**95.4**	**96.4**	**96.3**	**96.3**	**90.1**	29.5	11.6	8.7	**95.9**	**95.7**	6.4
ScConN(0.05;4)	0	13.55	*51.7*	*67.3*	**80.5**	**77.7**	*67.4*	**81.0**	**82.6**	**82.1**	**82.0**	*66.3*	11.1	6.6	6.0	**83.1**	**83.0**	5.3
MixN(0.5;1;0.25)	−1.02	3.87	**100.0**	**100.0**	**99.5**	**99.7**	**100.0**	**95.7**	**96.9**	**97.0**	**96.3**	**99.9**	**100.0**	**100.0**	**100.0**	30.2	29.6	**98.0**
MixN(0.4;1;0.25)	−0.78	3.34	**99.3**	**99.7**	**94.1**	**95.4**	**99.7**	**80.8**	**84.2**	**88.5**	**76.3**	**98.0**	**99.3**	**99.4**	**99.3**	16.3	16.1	**91.8**
MixN(0.3;1;0.25)	−0.57	3.03	**90.5**	**94.3**	*73.4*	**76.5**	**94.3**	*53.0*	*56.0*	*68.8*	39.9	**86.6**	**89.8**	**90.4**	**89.6**	9.3	9.2	*65.3*
MixN(0.5;3;0.25)	−0.46	1.78	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.6**	**99.8**	**100.0**	**94.0**	**100.0**	**100.0**	**100.0**	**100.0**	0.1	0.1	**82.7**
MixN(0.4;3;0.25)	−0.16	1.67	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.3**	**96.3**	**99.3**	45.4	**100.0**	**100.0**	**99.7**	**97.9**	0.0	0.0	**83.9**
MixN(0.3;3;0.25)	0.12	1.81	**99.9**	**100.0**	**100.0**	**100.0**	**100.0**	**97.5**	**77.1**	**92.7**	3.7	**98.3**	**99.1**	**96.2**	**90.8**	0.0	0.0	**95.3**
MixN(0.5;1;4)	0.44	5.21	**97.7**	**99.3**	**87.5**	**90.6**	**99.3**	**81.1**	**88.5**	**88.0**	**97.7**	**99.1**	**98.7**	**93.1**	**83.0**	*57.3*	*56.2*	*53.2*
MixN(0.4;1;4)	0.56	6.17	**99.2**	**99.9**	**97.2**	**98.2**	**99.9**	**94.5**	**97.5**	**97.4**	**99.7**	**99.9**	**98.5**	**88.7**	*73.7*	**77.3**	**76.2**	*51.7*
MixN(0.3;1;4)	0.7	7.58	**99.2**	**99.9**	**99.4**	**99.6**	**99.9**	**98.9**	**99.7**	**99.7**	**100.0**	**99.9**	**95.1**	*73.9*	*55.1*	**92.1**	**91.5**	36.9
MixN(0.5;3;4)	0.96	4.37	**99.9**	**100.0**	**97.8**	**98.5**	**100.0**	**92.8**	**94.4**	**93.0**	**96.8**	**99.6**	**99.9**	**99.9**	**99.8**	40.9	40.1	**90.6**
MixN(0.4;3;4)	1.21	5.29	**100.0**	**100.0**	**99.8**	**99.9**	**100.0**	**99.0**	**99.3**	**98.7**	**99.8**	**100.0**	**100.0**	**99.8**	**99.6**	*64.5*	*63.3*	**91.8**
MixN(0.3;3;4)	1.52	6.76	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**99.9**	**100.0**	**99.9**	**100.0**	**100.0**	**99.7**	**98.5**	**96.9**	**86.9**	**86.0**	**85.0**
Nout1	0	3.00	6.3	7.4	12.8	10.8	7.4	15.0	20.2	17.0	19.5	9.8	5.4	5.1	5.0	23.0	22.7	5.1
Nout2	0	3.00	12.0	22.2	**99.4**	**87.0**	22.4	**98.6**	**99.5**	**99.0**	**97.5**	28.6	5.3	5.0	5.0	**100.0**	**100.0**	5.1
Nout3	0	3.00	22.1	44.3	**98.5**	**90.8**	43.3	**99.0**	**99.6**	**98.8**	**98.6**	*53.8*	7.2	6.0	5.7	**99.9**	**100.0**	5.3
Nout4	0	3.00	6.6	10.2	8.6	9.8	10.6	12.3	28.0	37.8	28.3	14.0	5.9	5.1	4.9	32.6	27.1	4.9
Nout5	0	3.00	35.2	**80.5**	**96.9**	**96.6**	**82.2**	**98.7**	**99.9**	**100.0**	**99.6**	**79.1**	10.2	6.1	5.5	**100.0**	**99.9**	5.0

	$\sqrt{\beta_1}$	β_2	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Trunc(−3;1)	−0.55	2.78	15.0	19.3	*59.2*	12.2	**88.9**	**78.4**	**93.3**	7.8	**75.9**	**91.3**	3.2	**98.0**	**86.8**	26.1	*69.4*	6.5	12.2
Trunc(−2;1)	−0.32	2.27	37.7	32.1	15.7	0.1	**81.4**	*59.4*	**95.3**	31.1	*54.7*	**86.4**	1.7	**94.9**	**76.7**	*71.8*	*61.2*	0.1	33.2
Trunc(−3;2)	−0.18	2.65	7.7	6.7	4.9	3.4	9.3	4.7	18.9	9.8	4.2	11.4	1.6	11.1	7.9	9.0	8.5	0.6	7.6
Trunc(−1;1)	0	1.94	**77.7**	*68.7*	5.5	1.7	**92.9**	*73.1*	**99.6**	**92.4**	*67.7*	**96.0**	**83.0**	**92.3**	**89.5**	**98.8**	*72.2*	0.0	**77.2**
Trunc(−2;2)	0	2.36	17.0	12.4	0.8	1.6	12.5	3.7	38.4	30.4	2.8	17.6	11.3	10.5	9.5	30.0	9.5	0.0	17.1
Trunc(−3;3)	0	2.84	4.4	4.4	2.4	1.9	3.3	2.1	5.8	3.9	2.0	3.8	2.8	2.7	2.9	3.2	4.4	1.4	4.3
LoConN(0.5;5)	0	1.51	**100.0**	**100.0**	*74.2*	8.8	**100.0**	**100.0**	**100.0**	1.5	**100.0**	**100.0**	*71.1*	**87.1**	**100.0**	**100.0**	**100.0**	0.0	**99.3**
LoConN(0.5;3)	0	2.04	**92.8**	**88.9**	6.9	1.3	**75.2**	*58.6*	**85.2**	*70.1*	*54.5*	**80.3**	24.3	30.3	*72.3*	**82.8**	**78.7**	0.0	**90.6**
LoConN(0.5;1)	0	2.92	5.0	4.9	4.1	4.0	4.5	4.0	5.6	5.0	3.9	4.8	4.6	4.5	4.3	4.6	4.8	3.3	5.1
LoConN(0.4;1)	0.04	2.93	5.0	4.9	4.5	4.0	4.6	4.1	5.7	4.9	4.0	4.9	4.9	4.6	4.5	4.6	4.9	3.5	5.0
LoConN(0.3;1)	0.06	2.96	4.9	4.9	5.3	4.2	5.2	4.8	5.7	4.9	4.8	5.3	5.3	4.8	5.1	4.7	5.4	4.2	5.0
LoConN(0.4;3)	0.23	2.14	**78.6**	**77.4**	37.1	18.3	**79.5**	*65.9*	**86.9**	38.6	*62.5*	**83.5**	38.4	32.2	**77.1**	*70.5*	**84.5**	0.1	*64.5*
LoConN(0.4;5)	0.32	1.65	**99.4**	**100.0**	**99.0**	*71.7*	**100.0**	**100.0**	**100.0**	3.4	**100.0**	**100.0**	**87.0**	**86.6**	**100.0**	**99.6**	**100.0**	1.2	**78.4**
LoConN(0.3;3)	0.46	2.47	29.1	49.3	**75.8**	*51.5*	**87.5**	**80.2**	**89.7**	7.3	**78.4**	**89.2**	44.5	35.0	**86.2**	32.6	**92.7**	5.4	14.2
LoConN(0.3;5)	0.67	2.13	*69.7*	**99.7**	**100.0**	**99.3**	**100.0**	**100.0**	**100.0**	47.1	**100.0**	**100.0**	**90.7**	**86.4**	**100.0**	**80.6**	**100.0**	34.5	27.9
ScConN(0.05;0.25)	0	3.14	7.9	6.5	6.2	6.8	6.3	7.1	4.3	6.6	7.3	5.7	5.1	5.1	6.5	7.1	6.3	9.3	7.9
ScConN(0.10;0.25)	0	3.29	16.9	12.9	7.8	9.3	9.8	11.6	4.6	11.3	11.9	8.3	5.5	5.6	10.2	11.9	9.9	16.1	16.8
ScConN(0.20;0.25)	0	3.64	49.3	40.6	13.9	16.9	26.7	30.7	10.5	33.2	31.3	23.0	7.7	8.5	27.6	31.1	27.4	39.1	49.6
ScConN(0.05;2)	0	3.97	16.0	13.5	21.0	24.6	22.2	26.0	9.6	21.8	26.4	20.6	17.2	24.1	23.6	21.7	13.2	26.2	16.0
ScConN(0.10;2)	0	4.43	26.4	22.3	29.7	34.5	33.4	39.3	13.1	35.1	39.9	30.5	23.7	33.4	35.5	35.5	22.0	41.2	26.5
ScConN(0.20;2)	0	4.68	41.3	35.0	35.7	40.2	44.8	*52.2*	17.0	*50.7*	*53.1*	40.6	27.2	37.2	47.3	*52.1*	35.4	*57.9*	41.5
ScConN(0.20;4)	0	9.75	**99.4**	**99.1**	**95.8**	**93.9**	**99.5**	**99.6**	**96.2**	**99.7**	**99.7**	**99.3**	*74.8*	**91.7**	**99.5**	**99.6**	**99.2**	*57.7*	**99.4**
ScConN(0.10;4)	0	12.75	**92.5**	**91.0**	**90.4**	**93.2**	**94.7**	**95.8**	**85.0**	**95.4**	**95.9**	**94.0**	*73.3*	**92.5**	**95.1**	**95.1**	**91.1**	*51.1*	**92.8**
ScConN(0.05;4)	0	13.55	*72.2*	*69.0*	*74.8*	**79.3**	**78.8**	**81.2**	*63.9*	**79.0**	**81.5**	**77.6**	*57.1*	**79.5**	**79.7**	**77.9**	*68.5*	43.9	*72.1*
MixN(0.5;1;0.25)	−1.02	3.87	18.8	**97.0**	**99.2**	37.0	**100.0**	**100.0**	**99.7**	**90.3**	**100.0**	**100.0**	29.3	44.1	**100.0**	19.2	**100.0**	**82.1**	*68.6*
MixN(0.4;1;0.25)	−0.78	3.34	19.3	**89.1**	**92.6**	19.3	**98.9**	**98.7**	**96.4**	*55.5*	**98.7**	**98.8**	15.6	32.1	**98.9**	9.9	**99.1**	*65.3*	26.9
MixN(0.3;1;0.25)	−0.57	3.03	26.3	*63.1*	**68.7**	10.5	**88.3**	**86.9**	**80.3**	18.3	**86.3**	**88.3**	8.7	22.3	**88.4**	9.3	**90.0**	28.7	15.7
MixN(0.5;3;0.25)	−0.46	1.78	**100.0**	**100.0**	*53.4*	0.1	**100.0**	**100.0**	**100.0**	**26.7**	**100.0**	**100.0**	4.1	**100.0**	**100.0**	**99.8**	**100.0**	**6.8**	**87.9**
MixN(0.4;3;0.25)	−0.16	1.67	**99.8**	**100.0**	29.0	7.2	**100.0**	**100.0**	**100.0**	1.0	**100.0**	**100.0**	15.0	**99.8**	**100.0**	**100.0**	**100.0**	0.1	**98.8**
MixN(0.3;3;0.25)	0.12	1.81	**87.1**	**94.9**	*56.0*	*60.5*	**100.0**	**100.0**	**100.0**	2.9	**100.0**	**100.0**	27.2	**94.5**	**100.0**	**98.9**	**99.7**	0.0	*69.0*
MixN(0.5;1;4)	0.44	5.21	**97.1**	**97.1**	*66.8*	*56.5*	**97.6**	**98.2**	**86.7**	**98.0**	**98.3**	**96.7**	22.1	38.5	**97.8**	**93.0**	**98.1**	**80.7**	**99.0**
MixN(0.4;1;4)	0.56	6.17	**99.5**	**99.5**	**83.7**	*68.5*	**99.7**	**99.8**	**96.1**	**99.8**	**99.8**	**99.5**	32.6	*58.4*	**99.7**	**98.5**	**99.8**	*72.1*	**99.9**
MixN(0.3;1;4)	0.7	7.58	**99.9**	**99.8**	**93.7**	**83.2**	**99.9**	**99.9**	**98.3**	**100.0**	**99.9**	**99.8**	45.5	**78.7**	**99.9**	**99.7**	**99.9**	*62.9*	**99.9**
MixN(0.5;3;4)	0.96	4.37	44.7	**92.7**	**94.9**	49.1	**99.8**	**99.8**	**98.5**	**94.8**	**99.8**	**99.8**	19.4	35.6	**99.8**	48.8	**99.9**	**81.6**	**87.5**
MixN(0.4;3;4)	1.21	5.29	**78.7**	**98.0**	**98.5**	*61.3*	**100.0**	**100.0**	**99.9**	**99.7**	**100.0**	**100.0**	24.3	*55.1*	**100.0**	*74.9*	**100.0**	*67.7*	**98.8**
MixN(0.3;3;4)	1.52	6.76	**97.1**	**99.7**	**99.6**	**81.5**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	31.6	**77.7**	**100.0**	**92.6**	**100.0**	*50.5*	**100.0**
Nout1	0	3.00	8.5	7.1	13.7	2.8	11.1	15.6	3.0	10.5	16.2	9.2	5.5	9.3	12.5	13.7	7.9	20.0	8.6
Nout2	0	3.00	40.6	30.7	**81.7**	**97.2**	**93.4**	**98.2**	25.6	**81.0**	**98.5**	**90.8**	6.0	**99.9**	**95.9**	*63.2*	23.4	**91.8**	40.5
Nout3	0	3.00	*53.5*	42.6	**90.7**	**93.0**	**93.9**	**97.9**	43.3	**88.2**	**98.2**	**90.9**	7.9	**99.1**	**95.8**	*74.4*	49.7	**94.9**	*54.9*
Nout4	0	3.00	20.2	14.7	3.9	4.6	10.9	23.6	0.3	29.3	25.6	7.3	11.8	5.5	14.4	46.5	11.0	47.4	19.9
Nout5	0	3.00	**93.6**	**89.0**	41.1	**84.1**	**96.0**	**99.4**	19.1	**99.7**	**99.6**	**92.2**	**97.0**	**95.9**	**97.9**	**99.9**	**87.6**	**99.2**	**93.3**

carried out for three sample sizes ($n = 25$, $n = 50$ and $n = 100$) and considering significance levels α of 0.10, 0.05, 0.025 and 0.01.

Although critical values or limiting distributions of the tests statistics are available for some of the tests considered herein, critical values for each sample size under consideration were, nonetheless, derived empirically for each test for the considered nominal significance levels, before carrying out the power study. These critical values were based on 1,000,000 samples drawn from the standard normal distribution. In addition to the referred critical values, the values of μ_{τ_3}, μ_{τ_4}, $\text{var}(\tau_3)$ and $\text{var}(\tau_4)$, for the Hosking L-moments based test, and the values of $\mu_{\tau_3}^{(t)}$, $\mu_{\tau_4}^{(t)}$, $\text{var}(\tau_3^{(t)})$ and $\text{var}(\tau_4^{(t)})$, for the Hosking trimmed L-moments based test, were also determined for each sample size by simulation from 1,000,000 samples drawn from the standard normal distribution. For the latter test, the parameters were obtained for each of the previously referred trimming levels t of 1, 2 and 3. The values resulting from this empirical evaluation are presented in Table 1. As can be seen, the values of μ_{τ_3} and of $\mu_{\tau_3}^{(t)}$ for the different trimming levels are very close to zero, and are considered to be zero in the subsequent power study.

Since complete lists of the simulated power values of several normality tests for the different sample sizes and significance levels represent a prohibitive amount of data, only a sample of these results, considered to be representative of the general trend of results, is presented herein. Hence, Tables 2–10 present the power results for the symmetric, asymmetric and modified normal distribution sets, considering samples sizes of 25, 50 and 100 and a significance level of 0.05. Within the symmetric and asymmetric sets, distributions are ordered according to their skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) values, whereas for the modified normal distributions set this ordering is performed for each group of distributions. To complement these results, Tables 11 and 12 and Figure 2 present the average power results of the different tests over each distribution set and for

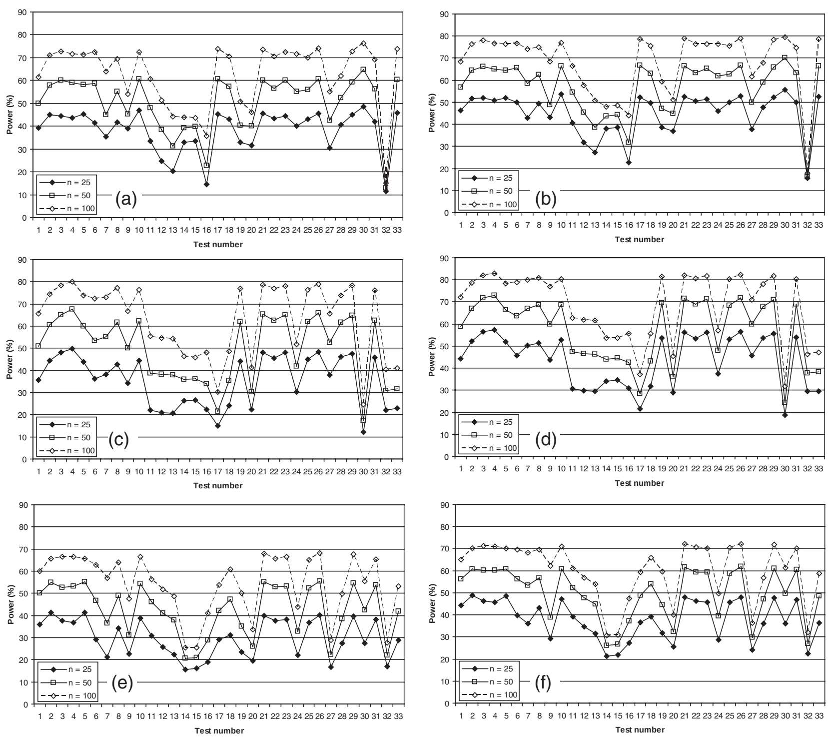

Figure 2. Average empirical power results, for all sample sizes, for the symmetric distributions with $\alpha = 0.05$ (a) and $\alpha = 0.10$ (b); for the asymmetric distributions with $\alpha = 0.05$ (c) and $\alpha = 0.10$ (d); for the modified normal distributions with $\alpha = 0.05$ (e) and $\alpha = 0.10$ (f).

Table 11. Average empirical power results by distribution type for all sample sizes ($\alpha = 0.05$).

Distribution	Sample size	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\text{TLmom}}$	$T^{(2)}_{\text{Lmom}}$	$T^{(3)}_{\text{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\text{MC-LR}}$
Symmetric	25	39.3	45.1	44.4	43.6	45.2	41.5	35.4	41.8	38.9	47.0	33.4	24.6	20.2	32.9	33.5	14.5
	50	49.9	*57.9*	*60.0*	*59.0*	*58.1*	*58.6*	45.1	*55.2*	45.4	*60.6*	47.9	38.4	31.3	39.3	39.7	22.8
	100	*61.3*	*71.1*	*72.7*	*71.6*	*71.2*	*72.3*	*64.1*	*69.5*	*54.0*	*72.3*	*60.6*	*51.4*	44.3	43.8	43.5	35.6
Asymmetric	25	36.1	45.3	49.2	*50.9*	44.9	37.3	39.4	43.8	35.4	45.6	22.0	20.8	20.4	27.3	27.6	22.1
	50	*52.3*	*62.2*	*66.8*	*69.2*	*61.6*	*55.8*	*57.5*	*63.7*	*52.6*	*63.9*	39.4	38.7	38.4	37.9	38.2	33.9
	100	*67.9*	**76.8**	**80.6**	**82.1**	**76.2**	*74.9*	**75.6**	**79.6**	*70.1*	**78.7**	*56.8*	*55.9*	*55.6*	49.2	48.7	48.5
Modified normal	25	35.9	41.3	37.7	36.7	41.3	29.0	21.3	34.2	22.6	38.8	30.7	25.6	22.4	15.6	16.0	19.0
	50	50.0	*55.0*	*52.7*	*53.1*	*55.1*	46.8	36.4	49.0	31.0	*54.4*	46.0	41.1	38.0	20.5	20.9	28.9
	100	*59.9*	*65.8*	*66.5*	*66.6*	*65.7*	*62.9*	*56.9*	*63.9*	47.7	*66.6*	*56.4*	*51.8*	48.7	25.6	25.4	40.9

	Sample size	T_w	$T_{\text{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Symmetric	25	45.3	43.2	33.1	31.5	45.5	43.5	44.4	40.0	43.1	45.6	30.5	40.6	45.0	48.5	42.1	11.6	45.8
	50	*60.7*	*57.4*	40.3	40.1	*60.1*	*56.6*	*60.1*	*55.2*	*55.9*	*60.6*	42.5	*52.4*	*59.4*	*64.8*	*56.2*	12.8	*60.4*
	100	*73.9*	*70.6*	*50.6*	46.0	*73.6*	*70.6*	*72.4*	*71.7*	*70.0*	*74.0*	*55.1*	*62.1*	*72.8*	**76.3**	*69.2*	15.0	*73.8*
Asymmetric	25	14.9	23.9	44.3	22.5	48.2	45.5	48.1	30.4	45.0	48.5	38.0	46.1	47.6	12.0	45.8	22.1	22.9
	50	21.6	35.4	*62.0*	30.4	*65.5*	*62.5*	*65.2*	41.8	*61.9*	*65.9*	*52.7*	*61.7*	*64.9*	17.3	*62.5*	30.9	31.8
	100	30.2	48.7	**76.9**	41.4	**78.7**	**76.9**	**78.2**	*51.8*	**76.5**	**79.1**	*65.8*	*73.9*	**78.3**	24.7	**76.2**	40.4	41.2
Modified normal	25	29.3	31.2	23.5	19.5	40.0	37.5	38.1	22.0	36.9	40.2	16.8	27.4	39.6	27.6	38.3	16.9	28.8
	50	42.3	47.2	35.0	26.1	*55.2*	*52.9*	*53.1*	32.9	*52.5*	*55.5*	22.2	38.4	*54.8*	42.4	*53.8*	22.2	42.0
	100	*53.7*	*61.0*	*50.2*	33.8	*67.9*	*65.7*	*66.4*	43.8	*65.1*	*68.2*	28.9	49.9	*67.6*	*55.4*	*65.4*	27.8	*53.2*

Table 12. Average empirical power results by distribution type for all sample sizes ($\alpha = 0.10$).

Distribution	Sample size	K–S	AD*	Z_C	Z_A	P_s	K^2	JB	DH	RJB	T_{Lmom}	$T^{(1)}_{\mathrm{TLmom}}$	$T^{(2)}_{\mathrm{Lmom}}$	$T^{(3)}_{\mathrm{Lmom}}$	BM_{3-4}	BM_{3-6}	$T_{\mathrm{MC-LR}}$
Symmetric	25	46.3	*51.8*	*52.0*	*50.8*	*51.9*	49.8	42.8	49.3	43.1	*53.6*	40.5	31.9	27.2	38.0	38.7	22.6
	50	*56.9*	*64.3*	*66.3*	*65.0*	*64.5*	*65.6*	*58.6*	*62.5*	48.9	*66.4*	*54.5*	45.4	38.5	43.6	44.4	31.8
	100	*68.3*	**76.3**	**78.0**	**76.8**	**76.5**	**76.8**	*74.0*	*75.0*	*68.3*	**76.8**	*66.5*	*57.5*	*50.9*	48.0	48.5	44.0
Asymmetric	25	44.4	*52.2*	*56.6*	*57.3*	*51.9*	45.7	*50.2*	*51.3*	43.8	*52.9*	30.8	29.9	29.6	34.2	34.5	31.0
	50	*58.9*	*67.0*	*71.8*	*73.0*	*66.5*	*63.7*	*66.9*	*68.7*	*59.9*	*68.8*	47.3	46.6	46.4	44.0	44.5	42.4
	100	*72.1*	**78.8**	**82.2**	**82.8**	**78.3**	**79.0**	**80.1**	**81.0**	**77.1**	**80.4**	*62.7*	*61.9*	*61.6*	*53.5*	*53.6*	*55.5*
Modified normal	25	44.3	48.7	46.3	45.6	48.7	39.6	35.9	43.0	29.3	47.0	39.2	34.6	31.6	21.4	21.9	27.3
	50	*56.3*	*60.8*	*60.1*	*60.1*	*60.8*	*56.1*	*53.3*	*56.7*	38.8	*60.7*	*52.4*	47.7	44.8	26.1	26.7	37.1
	100	*65.2*	*70.2*	*71.3*	*71.0*	*70.1*	*69.5*	*68.2*	*69.6*	*62.2*	*70.9*	*61.2*	*56.9*	*54.1*	30.7	31.0	47.5

	Sample size	T_w	$T_{\mathrm{MC-LR}} - T_w$	$T_{S,\ell}$	$T_{K,\ell}$	W	W_{SF}	W_{RG}	D	r	CS	Q	Q–Q^*	BCMR	β_3^2	T_{EP}	I_n	$R_{S,J}$
Symmetric	25	*52.2*	49.6	38.6	36.8	*52.6*	*50.4*	*51.3*	46.1	49.9	*52.7*	37.7	47.6	*52.2*	*55.6*	49.9	15.7	*52.4*
	50	*66.8*	*63.1*	47.1	44.9	*66.5*	*63.3*	*65.4*	*62.0*	*62.6*	*66.7*	49.9	*59.1*	*65.8*	*70.1*	*63.3*	16.2	*66.5*
	100	**78.5**	**75.4**	*59.2*	*51.1*	**78.9**	**76.3**	**76.3**	**76.4**	**75.7**	**79.0**	*61.7*	*67.9*	**78.3**	**79.6**	*74.7*	17.9	**78.5**
Asymmetric	25	21.6	31.8	*53.7*	29.1	*56.2*	*53.5*	*56.1*	37.5	*53.0*	*56.4*	45.8	*53.5*	*55.6*	18.7	*54.0*	29.5	29.6
	50	28.5	43.1	*69.6*	36.2	*71.5*	*69.0*	*71.2*	47.9	*68.5*	*71.9*	*59.8*	*67.8*	*71.0*	24.4	*69.1*	37.9	38.3
	100	37.1	*55.8*	**81.4**	45.5	**82.0**	**80.6**	**81.9**	*57.1*	**80.3**	**82.4**	*70.9*	**78.2**	**81.7**	31.8	**80.3**	46.3	47.3
Modified normal	25	36.7	39.2	31.9	25.6	48.0	46.3	45.6	28.8	45.8	48.1	24.1	36.0	47.8	36.0	47.0	22.5	36.3
	50	48.9	*54.0*	44.6	32.3	*61.6*	*59.4*	*59.4*	39.4	*58.9*	*61.8*	29.9	47.0	*61.2*	49.7	*60.3*	27.1	48.5
	100	*59.3*	*65.8*	*59.6*	40.0	*72.1*	*70.8*	*70.2*	49.7	*70.3*	*72.2*	36.5	*56.9*	*71.9*	*61.2*	*70.1*	32.1	*58.7*

Table 13. Numbering of the tests.

Test	Test number
K–S	1
AD*	2
Z_{C}	3
Z_{A}	4
P_s	5
K^2	6
JB	7
DH	8
RJB	9
T_{Lmom}	10
$T_{\mathrm{TLmom}}^{(1)}$	11
$T_{\mathrm{TLmom}}^{(2)}$	12
$T_{\mathrm{TLmom}}^{(3)}$	13
BM_{3-4}	14
BM_{3-6}	15
$\mathrm{T}_{\mathrm{MC-LR}}$	16
T_w	17
$T_{\mathrm{MC-LR}} - T_w$	18
$T_{S,l}$	19
$T_{K,l}$	20
W	21
W_{SF}	22
W_{RG}	23
D	24
r	25
CS	26
Q	27
Q–Q^*	28
$BCMR$	29
β_3^2	30
T_{EP}	31
I_n	32
$R_{s,J}$	33

significance levels of 0.05 and 0.10. For Figure 2, the numbering of the tests is defined according to Table 13. In the definition of the average powers, the $N(0;1)$ distribution case is not considered for the case of symmetric distributions and distributions Nout1 to Nout5 are also not considered for the modified normal distributions case. The format of the performance results of Tables 2 to 12 is defined such that power values above 75% are in bold and values between 50% and 75% are in italic.

6. Discussion of the results

A summary of the power results is presented in the following comprising several different levels of comparison. A preliminary general assessment of the results is presented based on the values of $\sqrt{\beta_1}$ and β_2. Then, comparisons are performed by type of normality test, by type of simulated distribution, by sample size and also considering the totality of results. An additional comparison is also performed with respect to the outlier sensitivity of the tests, by specifically addressing the power results obtained for the Nout1 to Nout5 distributions.

Regarding the influence of $\sqrt{\beta_1}$ and β_2, it is observed that when their corresponding values are near those of the normal distribution, none of the tests produces significant power results.

With respect to symmetric distributions, most of the tests are seen to yield better performance when β_2 is either significantly lower or higher than three. In terms of asymmetric distributions, the influence of $\sqrt{\beta_1}$ appears to be slightly larger than that of β_2 over the power of the tests. In general terms, the power of the tests appears to increase with skewness increase. With respect to the modified normal distributions, the influence of $\sqrt{\beta_1}$ and β_2 is not easily identified due to the additional influence of the considered level of contamination.

In terms of the selected normality tests based on the EDF, with the exception of K–S, the remaining tests generally exhibit similar power over the range of selected distributions. In general, the powers of AD* and P_s are very similar, while those of Z_C and Z_A are closer to each other. For the case of the symmetric distributions, and disregarding K–S, there is no clear advantage of one test over the others as their relative performance varies according to sample size and significance level. On the other hand, for the asymmetric distributions, Z_C and Z_A are best, with Z_A presenting a slight edge over Z_C. For the modified normal distributions AD* and P_s are better when sample size decreases, while for the larger sample size again all tests except K–S yield similar results.

In terms of the selected normality tests based on measures of the moments, T_w, T_{Lmom} and K^2 generally exhibit better performance for the symmetric distributions. Nonetheless, T_{Lmom} is best when sample size decreases and K^2 loses more power for the smaller sample size, for which tests such as $T_{\mathrm{MC-LR}}$–T_w and DH present similar or slightly better power. For the case of asymmetric distributions, DH, T_{Lmom} and $T_{S,\ell}$ are seen to have better power, with DH being somewhat better when the sample size is larger, while T_{Lmom} is slightly better for smaller n. For the modified normal distributions, T_{Lmom}, DH and K^2 appear to be the best choices. Nonetheless, for smaller n, $T_{\mathrm{MC-LR}}$–T_w presents, in some cases, better performance than K^2.

In terms of regression and correlation tests, β_3^2, CS and W exhibit better performance for the case of symmetric distributions, with β_3^2 showing an increasing relative power towards the other two tests as n decreases. For the asymmetric distributions, CS and W are generally better, with W_{RG} and BCMR closely following with similar power, while for the modified normal distributions, CS, W and BCMR present the best performance.

When considering all the normality tests for the selected alternative symmetric distributions, β_3^2, CS, T_w and R_{sJ} are generally better, with β_3^2 having a slight edge over the others. For smaller sample sizes β_3^2 is still the best choice and T_{Lmom} also shows good power, with CS, W, T_w and R_{sJ} close behind. A similar analysis for the case of asymmetric distributions shows that Z_A, Z_C, CS and W appear to be the best choices, with relative performance depending on the selected sample size and significance level. For the modified normal distributions, CS, BCMR and W are generally better although, as n decreases, AD*, P_s and also T_{Lmom} become more significant.

To allow for a clearer view of the individual power results of the best tests identified for each distribution set, Figure 3 presents their corresponding power results for the selected distributions of each set, a significance level of 0.05 and a sample size of 50. Similar relative trends were observed for the other significance levels and sample sizes.

When considering all the normality tests against all the non-normal alternative distributions, but excluding the Nout1 to Nout5 distributions, it can be seen that, for the smaller sample size, CS, W and BCMR are generally the best choices, though Z_A, Z_C, T_{Lmom}, W_{RG} , AD* and P_s follow closely. For the sample size of 50, CS, W and Z_A are better, with BCMR, Z_C, W_{RG} and T_{Lmom} also following closely. For the larger sample size, CS, W and BCMR are again the best choices with close performances of Z_A, Z_C and W_{RG}. Considering the whole range of sample sizes, CS, W and BCMR emerge as the best choices, although tests such as Z_A, Z_C, W_{RG}, T_{Lmom}, AD*, P_s, W_{SF} and T_{EP} also show an overall comparable average power.

When analysing the power results for the Nout1 to Nout5 distributions, the main objective is not to find tests that reject the normality hypothesis as many times as possible. Instead, the search is for tests whose power is close to the nominal significance level, therefore implying a low sensitivity to outliers. Observation of the power results of all the normality tests against

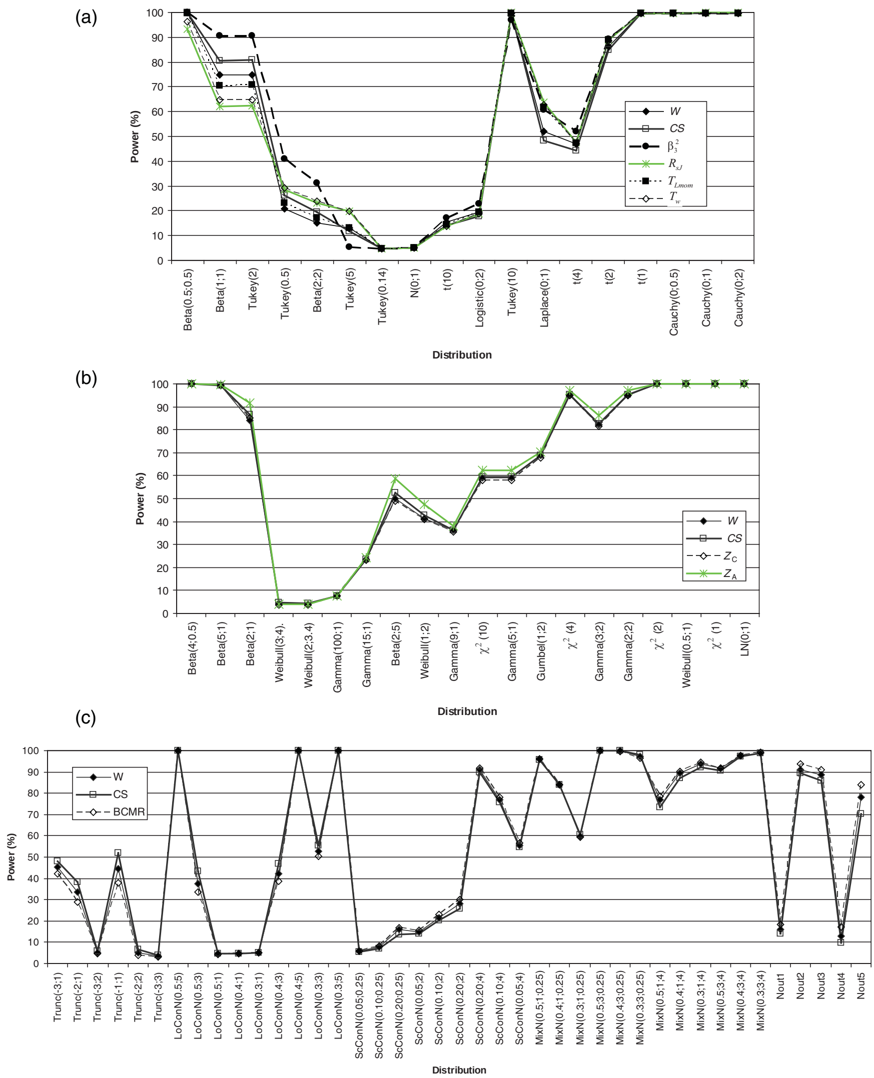

Figure 3. Power of the best tests for with $\alpha = 0.05$, a sample size of 50, and for the symmetric distributions (a), for the asymmetric distributions (b) and for the modified normal distributions (c).

these distributions leads to conclude that $T_{\text{MC-LR}}$, $T^{(3)}_{\text{TLmom}}$ and $T^{(2)}_{\text{TLmom}}$ are the best choices over the complete set of selected normal distributions with outliers. Nonetheless, it is noted that both $T^{(1)}_{\text{TLmom}}$ and Q also have a low sensitivity to few outliers, i.e. a single outlier or two outliers defined as one lower and one upper extreme observations.

With respect to the power of the proposed joint test $T_{\text{MC-LR}}$–T_w, observation of the power of this test for the different distributions shows an advantage over the performance of the individual tests for the asymmetric and modified normal distributions, excluding the Nout1 to Nout5 distributions. For the symmetric distributions, the individual test T_w is generally better than $T_{\text{MC-LR}}$–T_w.

As previously referred, a comparison of the two-sided R_{sJ} test with the one-sided version $R_{sJ,1}$ was carried out for each distribution set in order to verify the advantages of the former. Figure 4 presents the corresponding power results for a significance level of 0.05 and a sample size of 100. Similar relative trends were observed for the other significance levels and sample sizes. When comparing R_{sJ} with $R_{sJ,1}$, the former can be seen to extend the range of application of

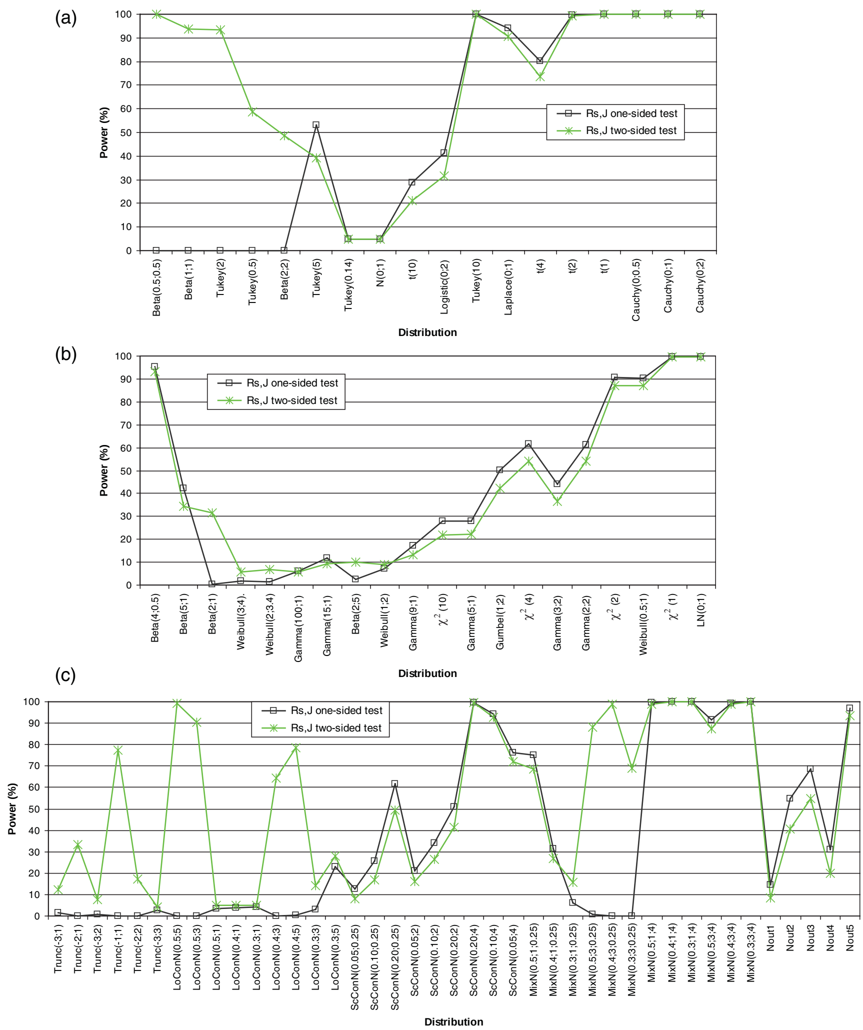

Figure 4. Comparison of power of tests R_{sJ} and $R_{sJ,1}$ for with $\alpha = 0.05$, a sample size of 100, and for the symmetric distributions (a), for the asymmetric distributions (b) and for the modified normal distributions (c).

this test, namely to light-tailed distributions, without a significant reduction of its power towards heavy-tailed distributions. Therefore, when the nature of the non-normality is unknown R_{sJ} is considered to be more adequate than $R_{sJ,1}$.

Finally, a last remark on the performance obtained for the tests K–S, JB, D and W and corresponding comparison with results available in [37]. Considering the common sample size and significance level, results presented herein do not corroborate the findings in [37] but are in much larger agreement with results of other previous studies referenced herein.

7. Concluding remarks

A comprehensive power comparison of existing tests for normality has been performed in the presented study. Given the importance of this subject and the widespread development of normality tests, comprehensive descriptions and power comparisons of such tests are of considerable interest.

Since recent comparison studies do not include several interesting and more recently developed tests, a further comparison of normality tests, such as the one presented herein, is considered to be of foremost interest.

The study addresses the performance of 33 normality tests, for various sample sizes n, considering several significance levels α and for a number of symmetric, asymmetric and modified normal distributions.

General recommendations stemming from the analysis of the power of the selected tests indicate the best choices for normality testing are β_3^2, CS, T_w and R_{sJ} for symmetric distributions, Z_A, Z_C, CS and W for asymmetric distributions and CS, BCMR and W for modified normal distributions, excluding normal distributions with outliers. For this latter case, the tests $T_{\mathrm{MC-LR}}$, $T_{\mathrm{TLmom}}^{(3)}$ and $T_{\mathrm{TLmom}}^{(2)}$ are recommended since they exhibit less sensitivity to outliers. When the nature of the non-normality is unknown, the tests CS, W and BCMR appear to be the best choices.

Acknowledgements

Financial support of the Portuguese Foundation for Science and Technology, through the PhD grant of the first author (SFRH/BD/32820/2007) is gratefully acknowledged.

References

[1] M. Costa, G. Cavaliere, and S. Iezzi, *The role of the normal distribution in financial markets*, in *New Developments in Classification and Data Analysis*, M. Vichi, P. Monari, S. Mignani, and A. Montanari, eds., Springer, Berlin, 2005, pp. 343–350.

[2] I. Min, *A nonparametric test of the conditional normality of housing demand*, Appl. Econ. Lett. 14(2) (2007), pp. 105–109.

[3] R. Liesenfeld and R.C. Jung, *Stochastic volatility models: conditional normality versus heavy-tailed distributions*, J. Appl. Econom. 15(2) (2000), pp. 137–160.

[4] A.H. Herbst, *Lunacy in the stock market – What is the evidence?* J. Bioeconomics 9(1) (2007), pp. 1–18.

[5] Y.R Gel and J.L. Gastwirth, *A robust modification of the Jarque–Bera test of normality*, Econom. Lett. 99(1) (2008), pp. 30–32.

[6] A.R. Nematollahi and L. Tafakori, *On comparison of the tail index of heavy-tail distributions using Pitman's measure of closeness*, Appl. Math. Sci. 1(19) (2007), pp. 909–914.

[7] D. Giles, *Spurious regressions with time-series data: Further asymptotic results*, Comm. Stat. Theory Methods 36(5) (2007), pp. 967–979.

[8] J.-M. Dufour, A. Farhat, L. Gardiol, and L. Khalaf, *Simulation-based finite sample normality tests in linear regressions*, Econom. J. 1(1) (1998), pp. 154–173.

[9] T. Thadewald and H. Büning, *Jarque–Bera test and its competitors for testing normality – a power comparison*, J. Appl. Stat. 34(1) (2007), pp. 87–105.

[10] J. Wilde, *A simple representation of the Bera–Jarque–Lee test for probit models*, Econom. Lett. 101(2) (2008), pp. 119–121.

[11] A.Ö. Önder and A. Zaman, *Robust tests for normality of errors in regression models*, Econom. Lett. 86(1) (2005), pp. 63–68.

[12] M.A. Quddus, *Time series count data models: An empirical application to traffic accidents*, Accid. Anal. & Prev. 40(5) (2008), pp. 1732–1741.

[13] V. Schoder, A. Himmelmann, and K.P. Wilhelm, *Preliminary testing for normality: Some statistical aspects of a common concept*, Clin. Exp. Dermatol. 31(6) (2006), pp. 757–761.

[14] B. Sürücü and E. Koç, *Assessing the validity of a statistical distribution: Some illustrative examples from dermatological research*, Clin. Exp. Dermatol. 33(3) (2007), pp. 239–242.

[15] S. Mathur and S. Dolo, *A new efficient statistical test for detecting variability in the gene expression data*, Stat. Methods Med. Res. 17(4) (2008), pp. 405–419.

[16] Y. Tsong, J. Zhang, and M. Levenson, *Choice of δ noninferiority margin and dependency of the noninferiority trials*, J. Biopharm. Stat. 17(2) (2007), pp. 279–288.

[17] K. Vännman, and M. Albing, *Process capability indices for one-sided specification intervals and skewed distributions*, Qual. Reliab. Eng. Int. 23(6) (2007), pp. 755–765.

[18] H.A. Muttlak and W.S. Al-Sabah, *Statistical quality control based on ranked set sampling*, J. Appl. Stat. 30(9) (2003), pp. 1055–1078.

[19] A. Madan, J.J. Borckardt, and M.R. Nash, *A parametric control chart adjustment for handling serial dependence in health care quality data*, Qual. Manag. Health Care 17(2) (2008), pp. 154–161.

[20] J.S. Oakland, *Statistical Process Control*, 6th ed., Butterworth-Heinemann, UK, 2008.

[21] E. Limpert, W.A. Stahel, and M. Abbt, *Log-normal distributions across the sciences: keys and clues*, BioScience 51(5) (2001), pp. 341–352.
[22] J. Halley and P. Inchausti, *Lognormality in ecological time series*, Oikos 99(3) (2002), pp. 518–530.
[23] M. Bengtsson, A. Ståhlberg, P. Rorsman, and M. Kubista, *Gene expression profiling in single cells from the pancreatic islets of Langerhans reveals lognormal distribution of mRNA levels*, Genome Res. 15(10) (2005), pp. 1388–1392.
[24] A.K. Singh, A. Singh, and M. Engelhardt, *The lognormal distribution in environmental applications*, EPA/600/S-97/006, Environmental Protection Agency, Washington, DC, USA, 1997.
[25] S.S. Shapiro, M.B. Wilk, and H.J. Chen, *A comparative study of various tests for normality*, J. Am. Statist. Assoc. 63(324) (1968), pp. 1343–1372.
[26] M.A. Stephens, *EDF statistics for goodness of fit and some comparisons*, J. Am. Statist. Assoc. 69(347) (1974), pp. 730–737.
[27] E.S. Pearson, R.B. D'Agostino, and K.O. Bowman, *Tests for departure from normality: Comparison of powers*, Biometrika 64(2) (1977), pp. 231–246.
[28] R. D'Agostino and M. Stephens, *Goodness-of-fit Techniques*, Marcel Dekker, Inc., New York, 1986.
[29] L. Baringhaus, R. Danschke, and N. Henze, *Recent and classical tests for normality – a comparative study*, Comm. Statist. Simulation Comput. 18(1) (1989), pp. 363–379.
[30] F.F. Gan and K.J. Koehler, *Goodness-of-fit tests based on P–P probability plots*, Technometrics 32(3) (1990), pp. 289–303.
[31] P. Royston, *Estimating departure from normality*, Stat. Med. 10(8) (1991), pp. 1283–1293.
[32] L. Landry and Y. Lepage, *Empirical behavior of some tests for normality*, Comm. Statist. Simul. Comput. 21(4) (1992), pp. 971–999.
[33] E. Seier, *Comparison of tests for univariate normality*, Interstat 2002. Available at http://interstat.statjournals.net/YEAR/2002/abstracts/0201001.php.
[34] H. Thode, *Testing for Normality*, Marcel Dekker, Inc., New York, 2002.
[35] P.J. Farrell and K. Rogers-Stewart, *Comprehensive study of tests for normality and symmetry: extending the Spiegelhalter test*, J. Stat. Comput. Simul. 76(9) (2006), pp. 803–816.
[36] A.R. Henderson, *Testing experimental data for univariate normality*, Clin. Chim. Acta 366(1/2) (2006), pp. 112–129.
[37] B. Yazici and S. Yolacan, *A comparison of various tests of normality*, J. Stat. Comput. Simul. 77(2) (2007), pp. 175–183.
[38] H. Lilliefors, *On the Kolmogorov-Smirnov test for normality with mean and variance unknown*, J. Am. Statist. Assoc. 62(318) (1967), pp. 399–402.
[39] T.W. Anderson and D.A. Darling, *Asymptotic theory of certain "goodness-of-fit" criteria based on stochastic processes*, Ann. Math. Stat. 23(2) (1952), pp. 193–212.
[40] T.W. Anderson and D.A. Darling, *A test of goodness of fit*, J. Amer. Statist. Assoc. 49(268) (1954), pp. 765–769.
[41] J. Zhang and Y. Wu, *Likelihood-ratio tests for normality*, Comput. Stat. Data Anal. 49(3) (2005), pp. 709–721.
[42] A.G. Glen, L.M. Leemis, and D.R. Barr, *Order statistics in goodness-of-fit testing*, IEEE Trans. Reliab. 50(2) (2001), pp. 209–213.
[43] R.B. D'Agostino and E.S. Pearson, *Tests for departure from normality: Empirical results for the distributions of b2 and* $\sqrt{b1}$, Biometrika 60(3) (1973), pp. 613–622.
[44] R.B. D'Agostino, A. Belanger, and R.B. D'Agostino Jr., *A suggestion for using powerful and informative tests of normality*, Amer. Statist. 44(4) (1990), pp. 316–321.
[45] K. Bowman and L. Shenton, *Omnibus test contours for departures from normality based on* $\sqrt{b1}$ *and b2*, Biometrika 62(2) (1975), pp. 243–250.
[46] C. Jarque and A. Bera, *Efficient tests for normality, homoscedasticity and serial independence of regression residuals*, Econom. Lett. 6(3) (1980), pp. 255–259.
[47] C.M. Urzúa, *On the correct use of omnibus tests for normality*, Econom. Lett. 53(3) (1996), pp. 247–251.
[48] J.A. Doornik and H. Hansen, *An omnibus test or univariate and multivariate normality*, Working Paper, Nuffield College, Oxford, 1994.
[49] K. Bowman and L. Shenton, *A bivariate model for the distribution of b1 and b2*, J. Amer. Statist. Assoc. 72(357) (1977), pp. 206–211
[50] J.R.M. Hosking, *L-moments: Analysis and estimation of distributions using linear combinations of order statistics*, J. R. Statist. Soc. Ser. B 52(1) (1990), pp. 105–124.
[51] J.R.M. Hosking, *Moments or L-moments? An example comparing two measures of distributional shape*, Amer. Statist. 46(3) (1992), pp. 186–189.
[52] T.J. Ulrych, D.R. Velis, A.D. Woodbury, and M.D. Sacchi, *L-moments and C-moments*, Stoch. Environ. Res. Risk Assess. 14(1) (2000), pp. 50–68.
[53] E.A.H. Elamir and A.H. Seheult, *Trimmed L-moments*, Comput. Stat. Data Anal. 43(3) (2003), pp. 299–314.
[54] C. Bontemps and N. Meddahi, *Testing normality: a GMM approach*, J. Econom. 124(1) (2005), pp. 149–186.
[55] L.P. Hansen, *Large sample properties of generalized method of moments estimators*, Econometrica 50(4) (1982), pp. 1029–1054.
[56] G. Brys, M. Hubert, and A. Struyf, *Goodness-of-fit tests based on a robust measure of skewness*, Comput. Stat. 23(3) (2007), pp. 429–442.
[57] G. Brys, M. Hubert, and A. Struyf, *A comparison of some new measures of skewness*, in *Developments in Robust Statistics, ICORS 2001*, R. Dutter, P. Filzmoser, U. Gather, and P.J. Rousseeuw, eds., Springer, Berlin, 2003, pp. 98–113.
[58] G. Brys, M. Hubert, and A. Struyf, *A robust measure of skewness*, J. Comput. Graph. Stat. 13(4) (2004), pp. 996–1017.

[59] G. Brys, M. Hubert, and A. Struyf, *Robust measures of tail weight*, Comput. Stat. Data Anal. 50(3) (2006), pp. 733–759.
[60] D.G. Bonett and E. Seier, *A test of normality with high uniform power*, Comput. Stat. Data Anal. 40(3) (2002), pp. 435–445.
[61] R.C. Geary, *Moments of the ratio of the mean deviation to the standard deviation for normal samples*, Biometrika 28(3/4) (1936), pp. 295–307.
[62] A. Cabaña and E.M. Cabaña, *Tests of normality based on transformed empirical processes*, Methodol. Comput. Appl. Probab. 5(3) (2003), 309–335.
[63] S.S. Shapiro and M.B. Wilk, *An analysis of variance test for normality (complete samples)*, Biometrika 52(3/4) (1965), pp. 591–611.
[64] P. Royston, *Remark AS R94: A remark on Algorithm AS 181: The W test for normality*, Appl. Stat. 44(4) (1995), pp. 547–551.
[65] P. Royston, *Approximating the Shapiro–Wilk W-test for non-normality*, Stat. Comput. 2(3) (1992), pp. 117–119.
[66] P. Royston, *A toolkit for testing for non-normality in complete and censored samples*, Statistician 42(1) (1993), pp. 37–43.
[67] S.S. Shapiro and R.S. Francia, *An approximate analysis of variance test for normality*, J. Amer. Statist. Assoc. 67(337) (1972), pp. 215–216.
[68] P. Royston, *A pocket-calculator algorithm for the Shapiro–Francia test for non-normality: An application to Medicine*, Stat. Med. 12(2) (1993), pp. 181–184.
[69] M.M. Rahman and Z. Govindarajulu, *A modification of the test of Shapiro and Wilk for normality*, J. Appl. Stat. 24(2) (1997), pp. 219–235.
[70] F. Mosteller, *On some useful 'inefficient' statistics*, Ann. Math. Stat. 17(4) (1946), pp. 377–408.
[71] G. Blom, *Statistical Estimates and Transformed Beta Variables*, Wiley, NewYork, 1958.
[72] Z.D. Bai and L. Chen, *Weighted W test for normality and asymptotics a revisit of Chen–Shapiro test for normality*, J. Statist. Plann. Inference 113(2) (2003), pp. 485–503.
[73] R.B. D'Agostino, *An omnibus test of normality for moderate and large size samples*, Biometrika 58(2) (1971), pp. 341–348.
[74] J.J. Filliben, *The probability plot correlation coefficient test for normality*, Technometrics 17(1) (1975), pp. 111–117.
[75] L. Chen and S.S. Shapiro, *An alternative test for normality based on normalized spacings*, J. Stat. Comput. Simul. 53(3) (1995), pp. 269–287.
[76] P. Zhang, *Omnibus test of normality using the Q statistic*, J. Appl. Stat. 26(4) (1999), pp. 519–528.
[77] Y.T. Hwang and P.F. Wei, *A Remark on the Zhang omnibus test for normality*, J. Appl. Stat. 34(2) (2007), pp. 177–184.
[78] E. del Barrio, J.A. Cuesta-Albertos, C. Matrán, and J.M. Rodríguez-Rodríguez, *Tests of goodness of fit based on the L_2-Wasserstein distance*, Ann. Stat. 27(4) (1999), pp. 1230–1239.
[79] É. Krauczi, *A study of the quantile correlation test for normality*, Test, 2007. DOI 10.1007/s11749-007-0074-6.
[80] D. Coin, *A goodness-of-fit test for normality based on polynomial regression*, Comput. Stat. Data Anal. 52(4) (2008), pp. 2185–2198.
[81] J.P. Royston, *Algorithm AS 177: Expected normal Order Statistics (Exact and Approximate)*, Appl. Stat. 31(2) (1982), pp. 161–165.
[82] T.W. Epps and L.B. Pulley, *A test for normality based on the empirical characteristic function*, Biometrika 70(3) (1983), pp. 723–726.
[83] BS 2846-7, *Guide to Statistical Interpretation of Data. Part 7: Tests for Departure from Normality*, British Standards Institution, London, UK, 1997.
[84] N. Henze, *An approximation to the limit distribution of the Epps–Pulley test statistic for normality*, Metrika 37(1) (1990), pp. 7–18.
[85] J. Martinez and B. Iglewicz, *A test for departure from normality based on a biweight estimator of scale*, Biometrika 68(1) (1981), pp. 331–333.
[86] Y.R. Gel, W. Miao, and J.L. Gastwirth, *Robust directed tests of normality against heavy-tailed alternatives*, Comput. Stat. Data Anal. 51(5) (2007), pp. 2734–2746.
[87] R.B. D'Agostino, *Small sample probability points for the D test of normality*, Biometrika 59(1) (1972), pp. 219–221.
[88] J.R. Leslie, M.A. Stephens, and S. Fotopoulos, *Asymptotic distribution of the Shapiro–Wilk W for testing for normality*, Ann. Stat. 14(4) (1986), pp. 1497–1506.
[89] S. Verrill and R.A. Johnson, *Tables and large-sample distribution theory for censored-data correlation statistics for testing normality*, J. Amer. Stat. Assoc. 83(404) (1988), pp. 1192–1197.
[90] P.K. Sen, *Shapiro–Wilk-type goodness-of-fit tests for normality: Asymptotics revisited*, in *Goodness-of-fit Tests and Model Validity*, C. Huber-Carol, N. Balakrishnan, M. Nikulin, and M. Mesbah, eds., Birkhäuser, Boston, 2002, pp. 73–88.
[91] E. del Barrio, E. Giné, and F. Utzet, *Asymptotics for L2 functionals of the empirical quantile process, with applications to tests of fit based on weighted Wasserstein distances*, Bernoulli 11(1) (2005), pp. 131–189.