

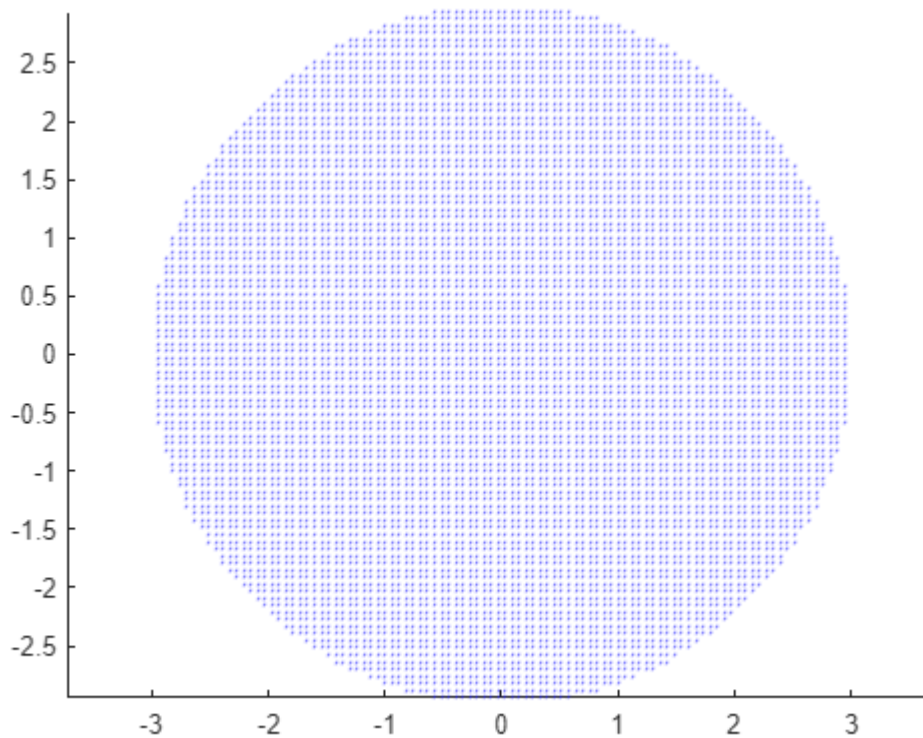
Spazio operativo primario e secondario del robot RRR planare

Spazio operativo primario

Lo spazio operativo primario è $WS_1 = \{\underline{p} \in \mathbb{R}^2 : \|\underline{p}\| \leq 3l\}$.

```
% parametri cerchio
center1 = [0, 0];
radius1 = 1+1+1;

figure
drawWS(center1, radius1, 'b');
axis equal
```

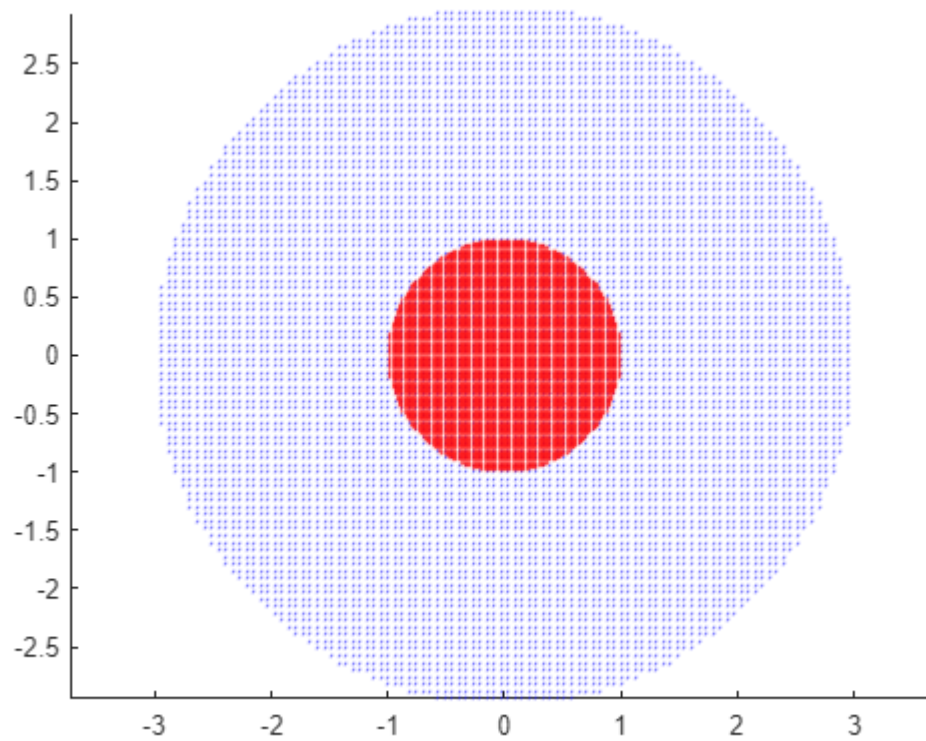


Spazio operativo secondario

Lo spazio operativo secondario è $WS_2 = \{\underline{p} \in \mathbb{R}^2 : \|\underline{p}\| \leq l\}$.

```
center2 = [0, 0];
radius2 = 1;

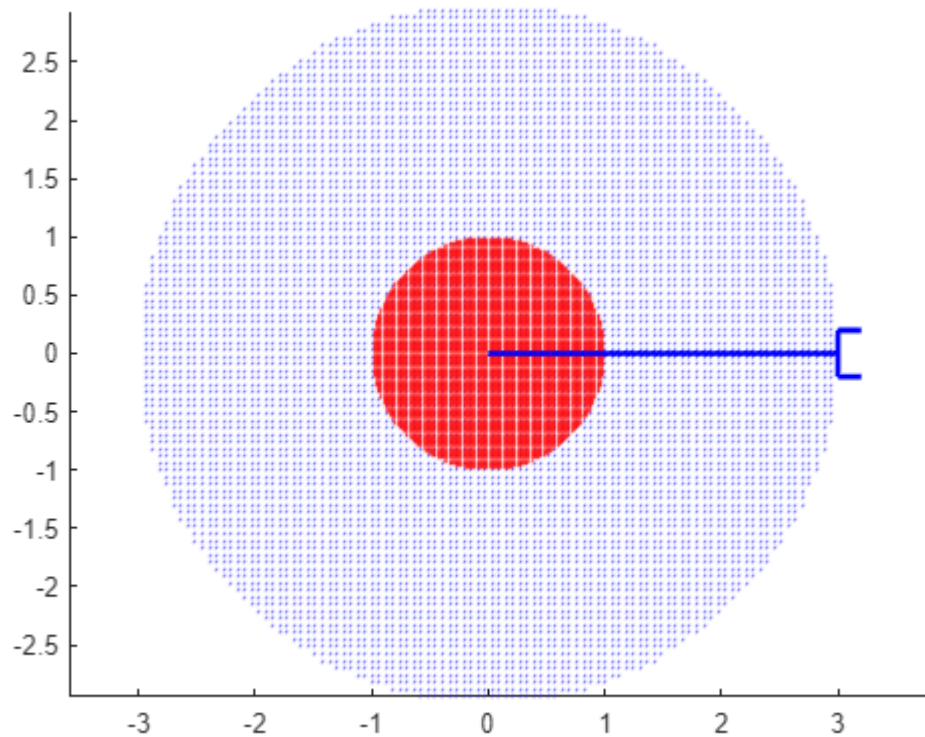
figure
drawWS(center1, radius1, 'b')
hold on
drawWS(center2, radius2, 'r')
```



Alcune configurazioni

Configurazione limite per WS_1

```
figure
drawWS(center1, radius1, 'b')
hold on
drawWS(center2, radius2, 'r')
drawPlanarRobot([1,1,1],[0,0,0])
```



Esempio in WS_2

Si vuole posizionare l'end-effector in $[p_x, p_y] = [0.1, -0.5] \in WS_2$ con orientamento $\phi = \frac{\pi}{2}$. Per fare ciò è necessario ricavare le variabili di giunto $[q_1, q_2, q_3]$.

Dalla cinematica inversa sappiamo che

$$q_2 = \text{atan2}(s_2, c_2) \text{ dove } \begin{cases} c_2 = \frac{\hat{x}^2 + \hat{y}^2 - l_1^2 - l_2^2}{2l_1l_2} = \frac{\hat{x}^2 + \hat{y}^2 - 2}{2} = \frac{(p_x^2 + p_y^2 + 1 - 2(p_x c_\phi + p_y s_\phi)) - 2}{2} \\ s_2 = \sqrt{1 - c_2^2} \end{cases}$$

dato che $\hat{x}^2 + \hat{y}^2 = p_x^2 + p_y^2 + 1 - 2(p_x c_\phi + p_y s_\phi)$ essendo $\hat{x} = p_x - l_3 c_\phi$ e $\hat{y} = p_y - l_3 s_\phi$.

```
% q2
px = 0.1; % x desiderata
py = -0.5; % y desiderata
phi = pi/2; % orientamento desiderato
c2 = (px^2+py^2+1-2*(px*cos(phi)+py*sin(phi))-2)/2;
s2 = sqrt(1-c2^2);
q2 = atan2(s2, c2)
```

```
q2 = 1.4404
```

Dalla cinematica inversa sappiamo che

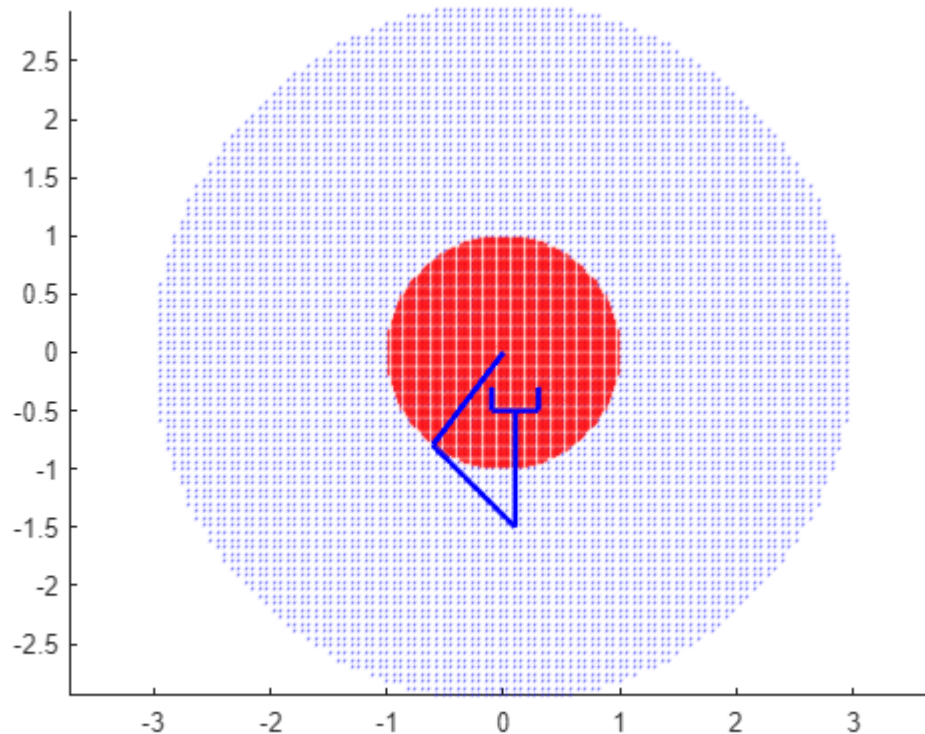
$$q_1 = \text{atan2}(s_1, c_1) \text{ dove } \begin{cases} c_1 = \frac{\hat{x}(l_1 + l_2 c_2) - \hat{y} l_2 s_2}{\hat{x}^2 + \hat{y}^2} = \frac{(p_x - l_3 c_\phi)(l_1 + l_2 c_2) + (p_y - l_3 s_\phi) l_2 s_2}{p_x^2 + p_y^2 + 1 - 2(p_x c_\phi + p_y s_\phi)} = \\ s_1 = \frac{\hat{y}(l_1 + l_2 c_2) - \hat{x} l_2 s_2}{\hat{x}^2 + \hat{y}^2} = \frac{(p_y - l_3 s_\phi)(l_1 + l_2 c_2) - (p_x - l_3 c_\phi) l_2 s_2}{p_x^2 + p_y^2 + 1 - 2(p_x c_\phi + p_y s_\phi)} \end{cases}$$

dato che $\hat{x}^2 + \hat{y}^2 = p_x^2 + p_y^2 + 1 - 2(p_x c_\phi + p_y s_\phi)$ essendo $\hat{x} = p_x - l_3 c_\phi$ e $\hat{y} = p_y - l_3 s_\phi$.

```
% q1
c1 = ((px-cos(phi))*(1+c2)+(py-sin(phi))*s2)/(px^2+py^2+1-2*(px*cos(phi)+py*sin(phi)));
s1 = ((py-sin(phi))*(1+c2)-(px-cos(phi))*s2)/(px^2+py^2+1-2*(px*cos(phi)+py*sin(phi)));
q1 = atan2(s1, c1);
% q3
q3 = phi-q1-q2
```

```
q3 = 2.3548
```

```
figure
drawWS(center1, radius1, 'b')
hold on
drawWS(center2, radius2, 'r')
drawPlanarRobot([1,1,1],[q1,q2,q3])
```



Si vuole ora posizionare l'end-effector nella stessa posizione $[p_x, p_y] = [0.1, -0.5] \in WS_2$ ma con orientamento

$$\phi = \frac{5\pi}{6}.$$

```

% q2
px = 0.1; % x desiderata
py = -0.5; % y desiderata
phi = 5*pi/6; % orientamento desiderato
c2 = (px^2+py^2+1-2*(px*cos(phi)+py*sin(phi))-2)/2;
s2 = sqrt(1-c2^2);
q2 = atan2(s2, c2);
% q1
c1 = ((px-cos(phi))*(1+c2)+(py-sin(phi))*s2)/(px^2+py^2+1-2*(px*cos(phi)+py*sin(phi)));
s1 = ((py-sin(phi))*(1+c2)-(px-cos(phi))*s2)/(px^2+py^2+1-2*(px*cos(phi)+py*sin(phi)));
q1 = atan2(s1, c1);
% q3
q3 = phi-q1-q2;

figure
drawWS(center1, radius1, 'b')
hold on
drawWS(center2, radius2, 'r')
drawPlanarRobot([1,1,1],[q1,q2,q3])

```

