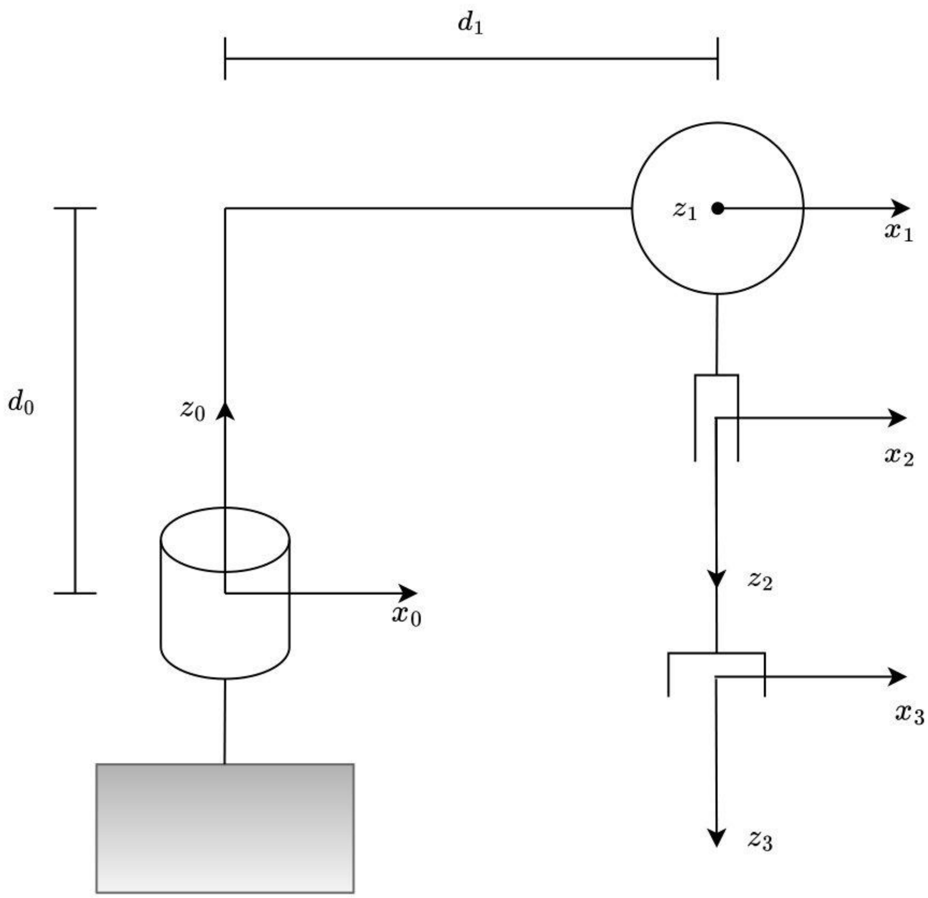
Cinematica diretta completa

Manipolatore sferico di primo tipo




```
syms q1 q2 q3 d0 d1 real
DHsferico1tipo = [d1, pi/2, d0, q1;
                0, pi/2, 0, q2;
                0, 0, q3, 0]
```

DHsferico1tipo =

$$\begin{pmatrix} d_1 & \frac{\pi}{2} & d_0 & q_1 \\ 0 & \frac{\pi}{2} & 0 & q_2 \\ 0 & 0 & q_3 & 0 \end{pmatrix}$$

```
TsfericoList = cinDirDH(DHsferico1tipo);
T01 = TsfericoList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & 0 & \sin(q_1) & d_1 \cos(q_1) \\ \sin(q_1) & 0 & -\cos(q_1) & d_1 \sin(q_1) \\ 0 & 1 & 0 & d_0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TsfericoList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & 0 & \sin(q_2) & 0 \\ \sin(q_2) & 0 & -\cos(q_2) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TsfericoList{3}
```

T23 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T03 = TsfericoList{4}
```

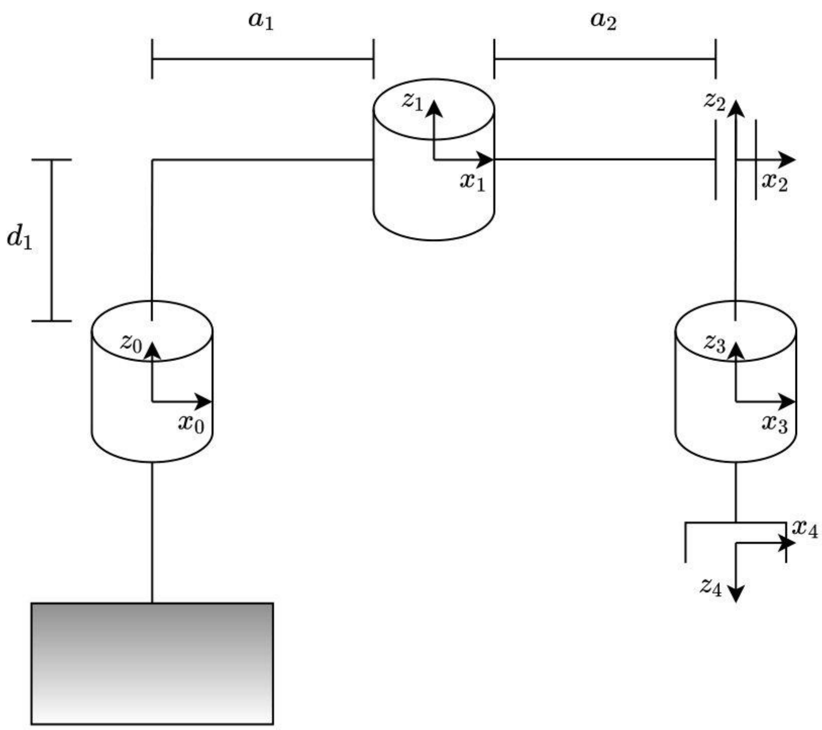
T03 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & \sin(q_1) & \cos(q_1)\sin(q_2) & \cos(q_1)\,\sigma_1 \\ \cos(q_2)\sin(q_1) & -\cos(q_1) & \sin(q_1)\sin(q_2) & \sin(q_1)\,\sigma_1 \\ \sin(q_2) & 0 & -\cos(q_2) & d_0 - q_3\cos(q_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = d_1 + q_3 \sin(q_2)$$

Manipolatore SCARA




```
syms q1 q2 q3 q4 d1 d4 a1 a2 real
DHscara = [a1, 0, d1, q1;
           a2, 0, 0, q2;
           0, 0, q3, 0;
           0, pi, d4, q4]
```

DHscara =

$$\begin{pmatrix} a_1 & 0 & d_1 & q_1 \\ a_2 & 0 & 0 & q_2 \\ 0 & 0 & q_3 & 0 \\ 0 & \pi & d_4 & q_4 \end{pmatrix}$$

```
TscaraList = cinDirDH(DHscara);
T01 = TscaraList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & a_1 \cos(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & a_1 \sin(q_1) \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TscaraList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & a_2\cos(q_2) \\ \sin(q_2) & \cos(q_2) & 0 & a_2\sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TscaraList{3}
```

T23 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T34 = TscaraList{4}
```

T34 =

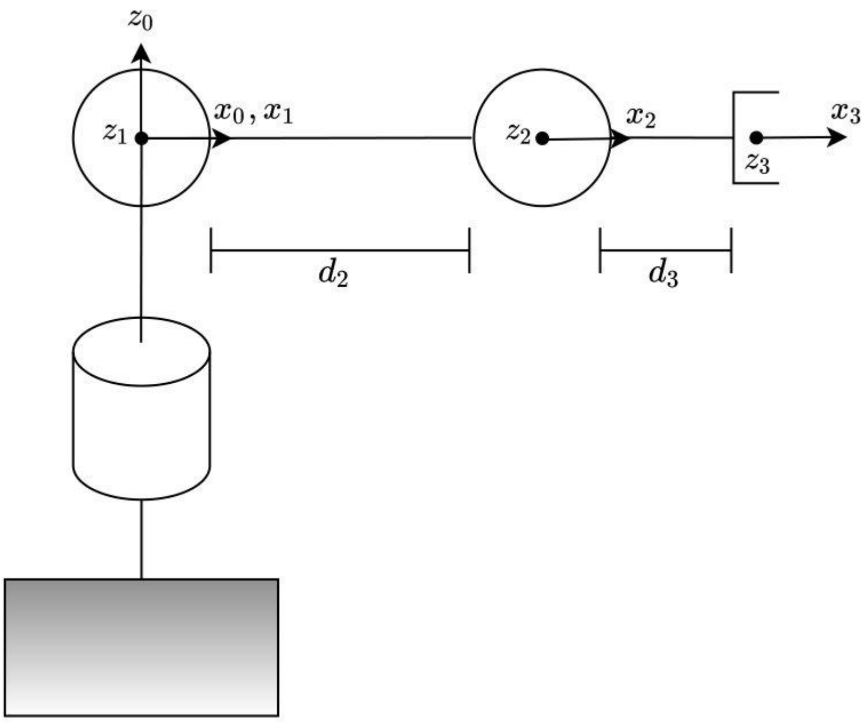
$$\begin{pmatrix} \cos(q_4) & \sin(q_4) & 0 & 0 \\ \sin(q_4) & -\cos(q_4) & 0 & 0 \\ 0 & 0 & -1 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T04 = TscaraList{5}
```

T04 =

$$\begin{pmatrix} \cos(q_1+q_2+q_4) & \sin(q_1+q_2+q_4) & 0 & a_2\cos(q_1+q_2)+a_1\cos(q_1) \\ \sin(q_1+q_2+q_4) & -\cos(q_1+q_2+q_4) & 0 & a_2\sin(q_1+q_2)+a_1\sin(q_1) \\ 0 & 0 & -1 & d_1+d_4+q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Manipolatore antropomorfo




```
syms q1 q2 q3 d2 d3 real
DHant = [0, pi/2, 0, q1;
            d2, 0, 0, q2;
            d3, 0, 0, q3]
```

DHant =

$$\begin{pmatrix} 0 & \frac{\pi}{2} & 0 & q_1 \\ d_2 & 0 & 0 & q_2 \\ d_3 & 0 & 0 & q_3 \end{pmatrix}$$

```
TantList = cinDirDH(DHant);
T01 = TantList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & 0 & \sin(q_1) & 0 \\ \sin(q_1) & 0 & -\cos(q_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TantList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & d_2\cos(q_2) \\ \sin(q_2) & \cos(q_2) & 0 & d_2\sin(q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TantList{3}
```

T23 =

$$\begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & d_3\cos(q_3) \\ \sin(q_3) & \cos(q_3) & 0 & d_3\sin(q_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T03 = TantList{4}
```

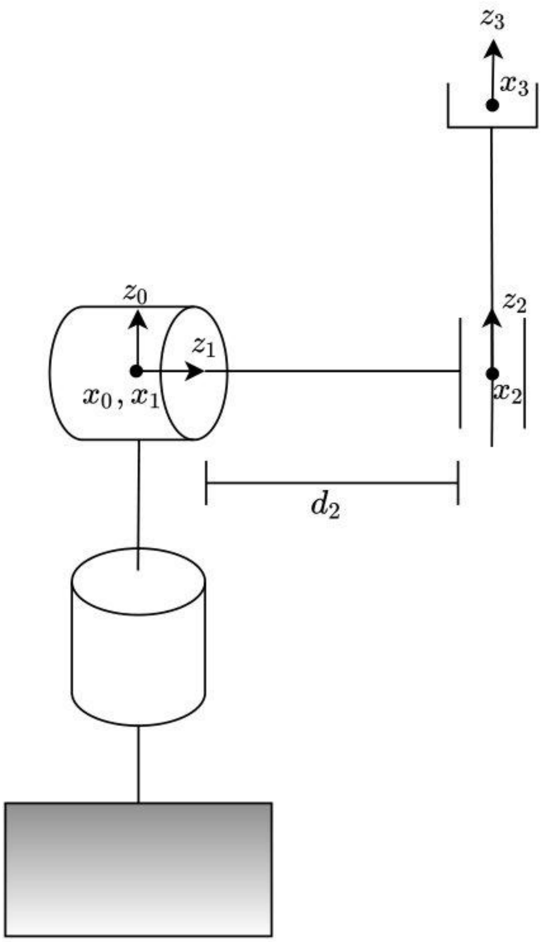
T03 =

$$\begin{pmatrix} \cos(q_2+q_3)\cos(q_1) & -\sin(q_2+q_3)\cos(q_1) & \sin(q_1) & \cos(q_1)\,\sigma_1 \\ \cos(q_2+q_3)\sin(q_1) & -\sin(q_2+q_3)\sin(q_1) & -\cos(q_1) & \sin(q_1)\,\sigma_1 \\ \sin(q_2+q_3) & \cos(q_2+q_3) & 0 & d_3\sin(q_2+q_3)+d_2\sin(q_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = d_3\cos(q_2+q_3)+d_2\cos(q_2)$$

Manipolatore di Stanford (solo struttura portante = manipolatore sferico di II tipo)




```
syms q1 q2 q3 d2 real
DHsferico = [0, -pi/2, 0, q1;
             0, pi/2, d2, q2;
             0, 0, q3, 0]
```

DHsferico =

$$\begin{pmatrix} 0 & -\frac{\pi}{2} & 0 & q_1 \\ 0 & \frac{\pi}{2} & d_2 & q_2 \\ 0 & 0 & q_3 & 0 \end{pmatrix}$$

```
TsfericoList = cinDirDH(DHsferico);
T01 = TsfericoList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & 0 & -\sin(q_1) & 0 \\ \sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TsfericoList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & 0 & \sin(q_2) & 0 \\ \sin(q_2) & 0 & -\cos(q_2) & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TsfericoList{3}
```

T23 =

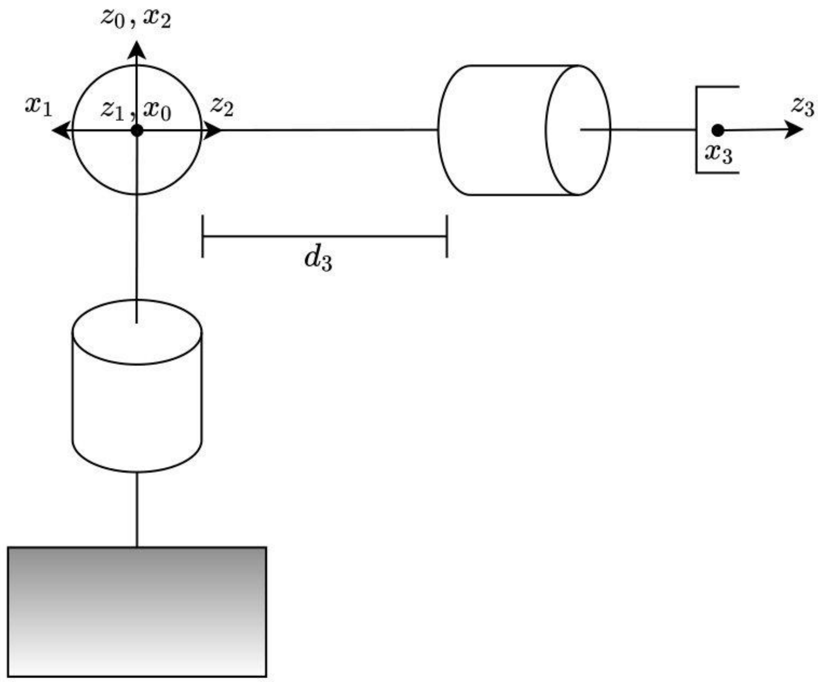
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T03 = TsfericoList{4}
```

T03 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & \cos(q_1)\sin(q_2) & q_3\cos(q_1)\sin(q_2) - d_2\sin(q_1) \\ \cos(q_2)\sin(q_1) & \cos(q_1) & \sin(q_1)\sin(q_2) & d_2\cos(q_1) + q_3\sin(q_1)\sin(q_2) \\ -\sin(q_2) & 0 & \cos(q_2) & q_3\cos(q_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Polso sferico




```
syms q1 q2 q3 d3 real
DHpolso = [0, -pi/2, 0, q1;
           0, pi/2, 0, q2;
           0, 0, d3, q3]
```

DHpolso =

$$\begin{pmatrix} 0 & -\frac{\pi}{2} & 0 & q_1 \\ 0 & \frac{\pi}{2} & 0 & q_2 \\ 0 & 0 & d_3 & q_3 \end{pmatrix}$$

```
TpolsoList = cinDirDH(DHpolso);
T01 = TpolsoList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & 0 & -\sin(q_1) & 0 \\ \sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TpolsoList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & 0 & \sin(q_2) & 0 \\ \sin(q_2) & 0 & -\cos(q_2) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TpolsoList{3}
```

T23 =

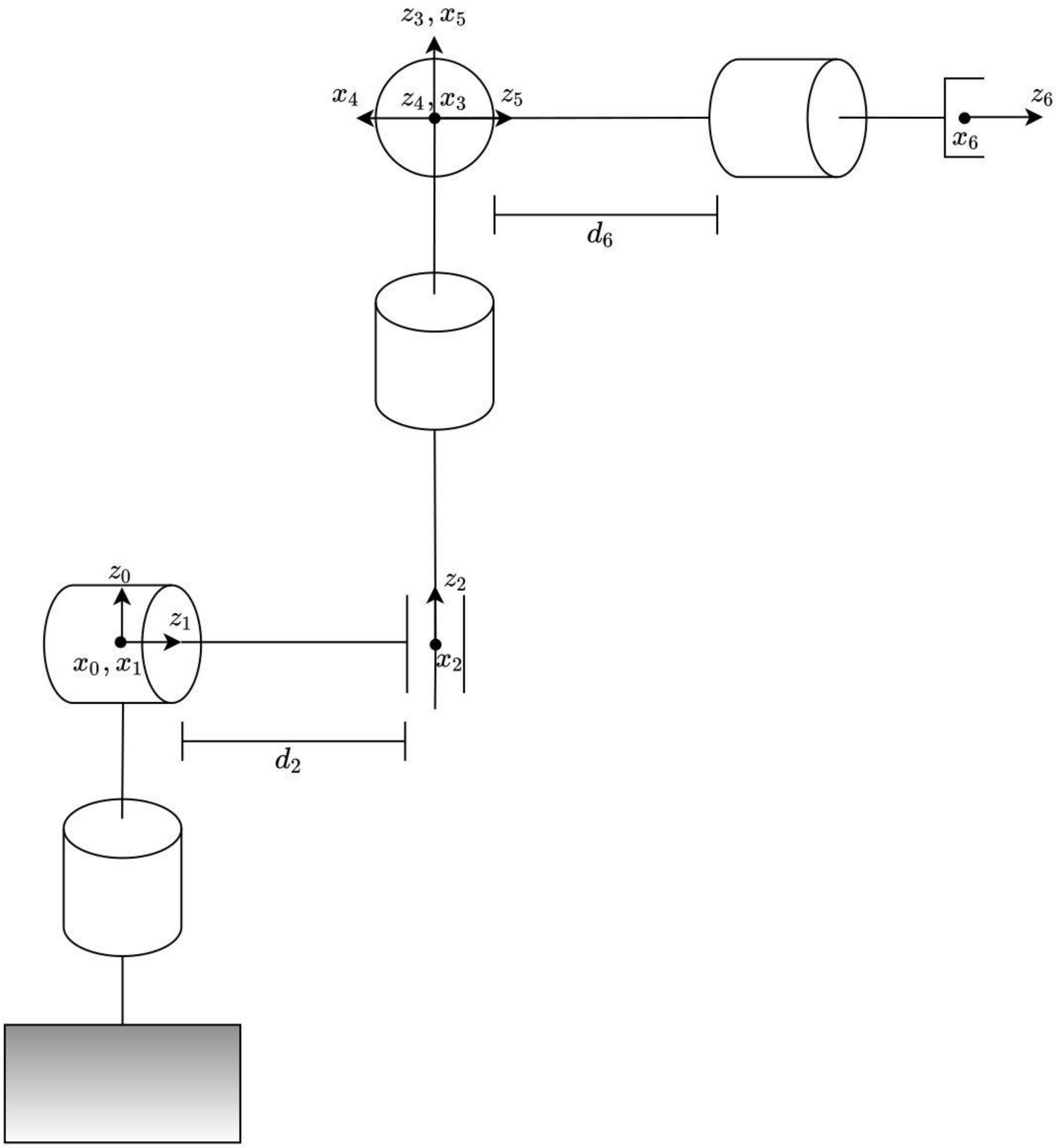
$$\begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & 0 \\ \sin(q_3) & \cos(q_3) & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T03 = TpolsoList{4}
```

T03 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2)\cos(q_3) - \sin(q_1)\sin(q_3) & -\cos(q_3)\sin(q_1) - \cos(q_1)\cos(q_2)\sin(q_3) & \cos(q_1)\sin(q_2) & d_3\,\mathrm{co} \\ \cos(q_1)\sin(q_3) + \cos(q_2)\cos(q_3)\sin(q_1) & \cos(q_1)\cos(q_3) - \cos(q_2)\sin(q_1)\sin(q_3) & \sin(q_1)\sin(q_2) & d_3\,\mathrm{si} \\ -\cos(q_3)\sin(q_2) & \sin(q_2)\sin(q_3) & \cos(q_2) & d \\ 0 & 0 & 0 & \end{pmatrix}$$

Manipolatore di Stanford




```
syms q1 q2 q3 q4 q5 q6 d2 d6 real
DHstanford = [0, -pi/2, 0, q1;
             0, pi/2, d2, q2;
             0, 0, q3, 0;
             0, -pi/2, 0, q4;
             0, pi/2, 0, q5;
             0, 0, d6, q6]
```

```
DHstanford =
```

$$\begin{pmatrix} 0 & -\frac{\pi}{2} & 0 & q_1 \\ 0 & \frac{\pi}{2} & d_2 & q_2 \\ 0 & 0 & q_3 & 0 \\ 0 & -\frac{\pi}{2} & 0 & q_4 \\ 0 & \frac{\pi}{2} & 0 & q_5 \\ 0 & 0 & d_6 & q_6 \end{pmatrix}$$

```
TstanfordList = cinDirDH(DHstanford);
T01 = TstanfordList{1}
```

T01 =

$$\begin{pmatrix} \cos(q_1) & 0 & -\sin(q_1) & 0 \\ \sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = TstanfordList{2}
```

T12 =

$$\begin{pmatrix} \cos(q_2) & 0 & \sin(q_2) & 0 \\ \sin(q_2) & 0 & -\cos(q_2) & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = TstanfordList{3}
```

T23 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T34 = TstanfordList{4}
```

T34 =

$$\begin{pmatrix} \cos(q_4) & 0 & -\sin(q_4) & 0 \\ \sin(q_4) & 0 & \cos(q_4) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T45 = TstanfordList{5}
```

T45 =

$$\begin{pmatrix} \cos(q_5) & 0 & \sin(q_5) & 0 \\ \sin(q_5) & 0 & -\cos(q_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T56 = TstanfordList{6}
```

`T56 =`

$$\begin{pmatrix} \cos(q_6) & -\sin(q_6) & 0 & 0 \\ \sin(q_6) & \cos(q_6) & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T06 = TstanfordList{7}
```

`T06 =`

$$\begin{pmatrix} -\cos(q_6)\,\sigma_8 - \sin(q_6)\,\sigma_4 & \sin(q_6)\,\sigma_8 - \cos(q_6)\,\sigma_4 & \sigma_5 - \sin(q_5)\,\sigma_{10} & q_3 \cos(q_1) \sin(q_2) \\ \cos(q_6)\,\sigma_6 + \sin(q_6)\,\sigma_3 & \cos(q_6)\,\sigma_3 - \sin(q_6)\,\sigma_6 & \sigma_7 & d_2 \cos(q_1 \\ \sin(q_2)\sin(q_4)\sin(q_6) - \cos(q_6)\,\sigma_2 & \sin(q_6)\,\sigma_2 + \cos(q_6)\sin(q_2)\sin(q_4) & \sigma_1 & \\ 0 & 0 & 0 & \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_2)\cos(q_5) - \cos(q_4)\sin(q_2)\sin(q_5)$$

$$\sigma_2 = \cos(q_2)\sin(q_5) + \cos(q_4)\cos(q_5)\sin(q_2)$$

$$\sigma_3 = \cos(q_1)\cos(q_4) - \cos(q_2)\sin(q_1)\sin(q_4)$$

$$\sigma_4 = \cos(q_4)\sin(q_1) + \cos(q_1)\cos(q_2)\sin(q_4)$$

$$\sigma_5 = \cos(q_1)\cos(q_5)\sin(q_2)$$

$$\sigma_6 = \cos(q_5)\,\sigma_9 - \sin(q_1)\sin(q_2)\sin(q_5)$$

$$\sigma_7 = \sin(q_5)\,\sigma_9 + \cos(q_5)\sin(q_1)\sin(q_2)$$

$$\sigma_8 = \cos(q_5)\,\sigma_{10} + \cos(q_1)\sin(q_2)\sin(q_5)$$

$$\sigma_9 = \cos(q_1)\sin(q_4) + \cos(q_2)\cos(q_4)\sin(q_1)$$

$$\sigma_{10} = \sin(q_1)\sin(q_4) - \cos(q_1)\cos(q_2)\cos(q_4)$$

Lo stesso risultato poteva essere ottenuto concatenando la cinematica del manipolatore sferico con la cinematica del polso sferico ${}^{0}\mathbf{T}_6 = \underbrace{{}^{0}\mathbf{T}_3}_{\text{man. sferico}} \underbrace{{}^{0}\mathbf{T}_3}_{\text{polso sferico}}$.

```
DHsferico = [0, -pi/2, 0, q1;
             0, pi/2, d2, q2;
             0, 0, q3, 0];
DHpolso = [0, -pi/2, 0, q4;
           0, pi/2, 0, q5;
           0, 0, d6, q6];

TsfericoList = cinDirDH(DHsferico);
TpolsoList = cinDirDH(DHpolso);

T03_manSferico = TsfericoList{4};
T03_polso = TpolsoList{4};
T06_2 = simplify(T03_manSferico*T03_polso);
isequal(expand(T06), expand(T06_2))
```

```
ans = logical
   1
```